Kiel Institute of World Economics
Duesternbrooker Weg 120
24105 Kiel (Germany)

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The Impact of Public Subsidies on Venture Capital Investments in Start-Up Enterprises

by
Andrea Schertler

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The Impact of Public Subsidies on Venture Capital Investments in Start-Up Enterprises

Abstract:

The relationship between a venture capitalist and an entrepreneur is modeled to investigate the impact of public subsidies on venture capital investments in start-up enterprises. In this model, the venture capitalist only finances start-up enterprises if he has sufficient expertise to make high-risk investments in new technology profitable in terms of their expected value. It is shown that a venture capitalist who already has sufficient expertise reduces his management support in the start-up enterprise under a public subsidy. Moreover, venture capitalists who do not have sufficient expertise may finance start-up enterprises if future losses of the start-up investment are partly covered by the government.

Keywords: venture capital, double-sided moral hazard, public subsidies, start-up investments

JEL classification: D82, G24, G28, L14

Andrea Schertler
Kiel Institute of World Economics
24100 Kiel, Germany
Telephone: ++49 431 8814 496
Fax: ++49 431 8814 502
E-mail: a.schertler@ifw.uni-kiel.de
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1 Motivation

Several European governments have introduced public subsidies in order to encourage venture capital investments in young high-technology enterprises. Governments frequently use loan and guarantee schemes to encourage investments in start-ups. Under a loan scheme, governments refinance investments at a favorable interest rate and they often cover a share of the credit risk. Under a guarantee scheme, governments guarantee to take on a share of the investors’ realized losses.

The rationale for subsidizing enterprises via venture capitalists is that high-technology start-ups are believed to be a prerequisite for productivity and employment growth and that venture capitalists are believed to increase the expected profit of these enterprises. Indeed, venture capitalists take on more functions in their portfolio firms and add more valuable resources than traditional intermediaries.

Venture capitalists do not only provide financial means and take on the control of the managerial decisions in a way similar to passive equity holders. But they also exert consultancy services; for example, they use their industrial contacts to develop customers relations and recruit key employees. These services are important for the overall success of young high-technology enterprises, since the founders have the technological expertise for realizing their innovative idea, but they often lack the managerial skills and industrial contacts to run their businesses effectively.

The impact of public subsidies on venture capital investments in general, and on the management support extended by venture capitalists in particular, has neither theoretically nor empirically been examined in depth. However, the work by Schertler and Stolpe (2000) points out the channels through which public subsidies affect the behavior of venture capitalists. Under a public subsidy, venture capitalists might have lower incentives to carefully monitor the enterprises’ progress and they might have less incentives to carefully select their portfolio firms. For this reason, public subsidies may reduce the intensity of venture capitalists’ control and selection and so let the efficiency of venture capital finance deteriorate. But in an emerging venture capital market, public subsidies might provide a positive stimulus if inexperienced venture capitalists still lack the necessary reputation and knowledge which they can build through a series of subsidized financing episodes. How large this positive stimulus of public subsidies will be may be difficult to determine ex ante because governments usually cannot distinguish between venture capitalists seeking to build up a long-term reputation and those who do not.

This paper develops a theoretical model to investigate the impact of public subsidies when venture capitalists use their acquired expertise to make high-risk investments profitable. The impact is primarily on venture capitalists’ effective management support for the start-ups they finance. Public subsidies increase venture capitalists’ expected pay-offs from a given investment and hence attract venture capitalists with relatively little experience to invest their money in start-up enterprises. But more experienced venture capitalists also welcome the government’s offer to cover part of their potential future losses which is a popular form of subsidies for venture capital. However, experienced venture capitalists reduce their management support in response to these subsidies. The effects of public subsidies are derived in a theoretical framework which takes two
important characteristics of venture capital finance in accordance with empirical findings into account. Only over longer periods of time can venture capitalists accumulate the specific technological knowledge which helps them to manage investments in enterprises’ early stages of growth. At every point in time, venture capitalists differ with respect to their individual knowledge and expertise and so does the value of management support they proffer to enterprises.

Since both contracting parties’ efforts affect the value of the enterprise, there exists a double-sided moral hazard problem between the venture capitalist and the entrepreneur. When a double-sided moral hazard problem exists, both contracting parties have to give the opposite party sufficient incentives for doing something after the contract has been signed. In this model, the entrepreneur needs incentives for spending her technological expertise in the development of the prototype, and the venture capitalist needs incentives for pledging his contacts to customers and suppliers as well as his specific technological expertise to the enterprise he has chosen to finance.

The recent literature on venture capital has discussed the double-sided moral hazard problem between the venture capitalist and the entrepreneur in some detail. Schmidt (1999) uses an incomplete contract approach in which the efforts of the contracting parties increase the pay-off of the enterprise. He shows that a convertible security contract is efficient while a combination of debt and equity is not. In a related model by Repullo and Suarez (1998), the efforts of the contracting parties increase the probability to reach the good state of the project financed in stages. They show that the investor of the first capital infusion has to be protected against a dilution of his shares. In the model by Marx (1998), the venture capitalist’s effort has only an effect on the pay-off in the bad state of the project. She shows in her model that a contract combining debt with equity is more efficient than a pure equity or a pure debt contract. All these studies assume a homogenous group of venture capitalists, while in the model presented below, venture capitalists differ with respect to their expertise and experience accumulated in the past.

The paper is organized as follows. Section 2 discusses the financing problems of young innovative enterprises, and venture capitalists’ role in financing these enterprises. Section 3 introduces a contract theoretical model in which the entrepreneur seeks financial means and the venture capitalist offers management support in addition to financial means. The impact of public subsidies on the management support exerted by the venture capitalist, and the venture capitalist’s expertise in making start-up investments profitable are examined in section 4. Section 5 concludes and addresses some further research questions which have to be solved to evaluate the impact of public subsidies more generally.

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1 This results form the assumption that the entrepreneur always offers the contract to a venture capitalist who acts on a competitive venture capital market, i.e. the venture capitalists’ expected pay-offs are always equal zero.

2 For an overview of the recent literature on venture capital contracts see Schertler (2000).
2 Innovative enterprises and the role of venture capitalists

Innovative young enterprises have only a very limited access to public equity and credit markets. Because they predominantly invest in intangible assets, by spending a large share of their capital on research and development activities, they cannot offer the collateral often required to get a bank credit. Empirical studies find evidence that young high-technology enterprises indeed have difficulties to get capital in credit markets. The higher the shares of intangible firm assets, the lower the overall debt shares of enterprises are (Friend and Lang 1988, Rajan and Zingales 1995). Moreover, young enterprises use relatively expensive trade credits more often than established enterprises. This indicates that young enterprises have still not build up a long-term, and therefore credible, relationship to creditors (Petersen and Rajan 1994, Harhoff and Körting 1998).

But young innovative enterprises also have limited access to traditional equity markets. Certainly, they cannot sell their shares on public equity markets since they have not build up a track record. But, they may also not attract passive outside equity holders on private markets, because they appear as too risky investments. The risk to lose an investment spent in high-technology enterprises in the early stage of development, that is before production is started, is over 60 per cent (Ruhnka and Young 1987). Internal factors, such as developing a prototype that does not work, predominantly affect this risk. Therefore, the risk to lose an investment decreases with enterprises’ development progress. External factors, such as unanticipated competition, constantly affect the risk over the development stages of an enterprise; the impact only increases in the exit stage, in which the shares of the venture capitalists are sold to other share holders (Ruhnka and Young 1991).

But the risk to lose an investment infused in young high-technology enterprises is not exogenously given. The risk ascribable to internal factors and external factors depends on the behavior, expertise and experience of the management team and the entrepreneur. Opportunistic behavior of the entrepreneur can be mitigated by contract design such as the design used in venture capital finance. But an entrepreneur who has an innovative idea often does not have sufficient expertise and experience, necessary to manage her enterprise, to organize the production process in an efficient manner, and to develop an advertising strategy. Therefore, passive outside equity holders can only reduce the risk to lose an investment ascribable to incentive problems by using a contract similar to venture capital contracts, but they cannot reduce the probability ascribable to entrepreneur’s insufficient management know-how.

Venture capital contracts contain a multitude of control mechanisms to mitigate various incentive problems arising if information about several key characteristics of the enterprise are asymmetrically distributed between venture capitalists and entrepreneurs. Entrepreneur’s compensation with a basic salary and a profit participation, gives the entrepreneur incentives to exert high effort (Baker and Gompers 1999). Venture capital investments are almost exclusively financed with convertible securities (Kaplan and Strömberg 2000), so that the entrepreneur substantially participates in increasing profits but does not benefit from increasing risks (Gompers 1997). Therefore, the entrepreneur has incentives to avoid high risks. Furthermore, venture capitalists infuse their capital in stages and not at once (Sahlman 1990). After each capital infusion, they have the opportunity to abandon the project, if contractually specified financial or non-financial criteria are not met. This sets strong incentives to entrepreneurs to exert high effort and to
avoid high risks as well (Gompers 1995). But nevertheless venture capitalists need substantial technological knowledge to control the progress of enterprises’ development.

The advantage of venture capital finance compared to other financial sources mainly stems from venture capitalists’ technological knowledge. Venture capitalists do not only use their knowledge and expertise to control the development progress of their portfolio firms. Indeed, with their knowledge and expertise, venture capitalists can select the most fruitful ideas out of a large number of entrepreneurs seeking financial means. Out of 100 enterprises demanding venture capital, only between two and three receive funding in Germany (Mann 1999). Moreover, they do not only control the business policies, but they also affect these policies by direct interventions. Since they take on several functions in their portfolio firms, they only invest in a small number of enterprises at the same time. By building a network with other venture capitalists, they are capable to recruit key employees, and to build up contacts to customers and suppliers (Barry 1994, Gorman and Sahlman 1989).

Venture capitalists’ management support can create a surplus in venture capital-backed firms as several empirical studies indicate. Even after the initial public offering, venture capital-backed enterprises outperform non-venture capital-backed ones (Brav and Gompers 1997). Furthermore, the total costs of going public are lower for venture capital-backed enterprises: the degree of underpricing and the compensation for the underwriter are lower (Megginson and Weiss 1991). Venture capitalists also affect the patenting behavior of their portfolio firms: venture capital-backed enterprise significantly patent more than other comparable enterprises (Kortum and Lerner 1998). Therefore, passive equity holders cannot be seen as a substitute for venture capitalists. Even if passive equity holders infuse the capital and a consultancy firm offers the necessary management support, this may only a very imperfect substitute for venture capitalists, because of incentive problems among the three contracting parties.

Evidence confirms the importance of venture capitalists’ technological knowledge in financing young high-technology enterprises. Barry et al. (1990) for the US and Bergström et al. (1995) for Sweden find evidence that venture capitalists’ quality in controlling and supporting the management team negatively affects the degree of underpricing. Venture capitalists’ quality is approximated by different variables, such as the length of time which the lead venture capitalist has served on the board of directors. Furthermore, as the study by Gompers (1996) indicates, venture capitalists take care of signaling their knowledge and expertise to the market: young venture capitalists bring their portfolio firms public earlier than older venture capitalists do. The benefit of bringing their portfolio firms public earlier, i.e. signaling their true quality or knowledge to outside investors, seems to exceed the costs of doing this, i.e. the greater underpricing of the enterprises’ shares.

3 The model

The impact of public subsidies on venture capital investments in start-up enterprises is examined in a model of the relationship between a single entrepreneur and a single venture capitalist. An entrepreneur ( EN ) has an innovative product idea, but she lacks the necessary financial means to finance the start-up investment $ I > 0 $ herself, and the
expertise and experience necessary to manage the enterprise in an efficient manner. Therefore, she prefers funding by a venture capitalist (VC) who offers a profit increasing management support. Before the start-up investment is made, the venture capitalist and the entrepreneur sign a contract, i.e. they specify the equity allocation \((1-\alpha, \alpha)\), which determines the profit allocation between them.

*Figure 1: Structure of the model*

![Diagram](image)

After the capital has been infused, both contracting parties exert their efforts, without observing the effort amount of the respective complementary party. Neither the specific effort of the entrepreneur nor the venture capitalist’s effort can be contracted upon. While none of the contract parties can affect the probability of product innovation’s technical success, both can affect the product innovation’s expected profit. The revenue of the enterprise can take two states.\(^3\) In the state with low performance \(l\), reached with probability \(p_l\ (0 < p_l < 1)\), the venture capitalist’s pay-off does not cover the investment costs independent of the effort amounts of the contracting parties. While in the state with high performance \(h\), reached with probability \(p_h\ (0 < p_h < 1)\), the enterprise’s revenue exceeds the effort and investment costs. The sum of the probabilities is equal one \((p_l + p_h = 1)\). Figure 1 depicts the time structure of the model in the traditional manner.

The expected revenue \(\hat{R}\) of the enterprise is given by \(\hat{R} = (p_l + A p_h) E^\beta \tilde{V}^{\alpha}\), with \(1 > \beta + \omega\), \(0 < \beta, \omega < 1\), and \(A = (1 - p_l)/p_h\) as a shift parameter ensuring that the revenue of the enterprise is larger than the effort costs in the good state of the project. We assume \(I > p_l\) to ensure a positive shift parameter. \(E\) denotes entrepreneur’s effort, and \(\tilde{V}\) is the venture capitalist’s management support. The management support offered by the venture capitalist is a function of his time spent in the enterprise \(V\) and his expertise \(H\), \(\tilde{V} = V^\rho H^{1-\rho}\), with \(0 < \rho < 1\). The venture capitalist has accumulated his expertise by

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\(^3\) Considering a third state of the project in which the start-up investment is totally lost does not change the main results of the model. Since including a third state increases the complexity of the model, it is not taken into account.
involving in high-technology start-up enterprises financed by him in the past. Therefore, his expertise is exogenously given in the short-term; the venture capitalist maximizes his expected pay-off by choosing his time for doing management support. Inserting the venture capitalist’s management support function and the shift parameter $A$ into the expected revenue function of the enterprise, and defining $\lambda := (1 - \rho) \delta$ and $\delta := \rho \omega$, it follows for the expected revenue $\hat{R} = I E^\beta V^\delta H^\lambda$, with $\lambda > \delta$. That is, increasing the venture capitalist’s expertise a marginal unit, raises the expected revenue more than increasing his time which he spends in the enterprise.

Although we only explicitly consider a single venture capitalist in the model, there are in fact many venture capitalists in the market who differ with respect to their accumulated expertise $H$. Venture capitalists who financed a multitude of high-technology start-ups in the past have more expertise than venture capitalists who only financed some start-up enterprises. If the entrepreneur meets a venture capitalist without sufficient expertise, she may look for another venture capitalist who has sufficient expertise to make the investment profitable. Therefore, two important effects of public subsidies can be investigated. How do public subsidies affect the management support exerted by venture capitalists who have sufficient expertise to make high-risk start-up investments profitable? And how do public subsidies affect the investment decision of venture capitalists who do not have yet build up sufficient expertise to make high-risk start-ups profitable?

3.1 Expected pay-offs of the contracting parties

The venture capitalist receives the share $\alpha$ of the enterprise’s expected revenue. He has to carry the costs of the start-up investment $I$, since the entrepreneur has no funds on her own. Furthermore, he takes the costs for doing management support into account (the value of his outside option). It is assumed that he cannot offer his time without his expertise. The higher his time $V$, or the higher his expertise accumulated in the past $H$, the higher is the value of his outside option given by $HV$. For simplicity, the interest rate is set equal zero. His expected pay-off function is given by:

\[ \hat{U}_{\text{VC}} = \alpha I E^\beta V^\delta H^\lambda - HV - I. \]

The entrepreneur receives the share $(1 - \alpha)$ of the enterprise’s expected revenue. Since she has only to carry her effort costs, her expected pay-off function is given by:

\[ \hat{U}_{\text{EN}} = (1 - \alpha) I E^\beta V^\delta H^\lambda - E. \]

This simultaneous move game between the venture capitalist and the entrepreneur has two Nash equilibria. In one of these, the efforts by the contracting parties are zero, and this is not of economic interest. Since the expected pay-offs and the realized pay-offs in all states of the project are always zero, nobody would provide venture capital. However, there exists another Nash equilibrium in which both contracting parties offer positive effort amounts, if the venture capitalist has sufficient expertise to make the high-risk investment profitable. By differentiating the expected pay-off functions of the entrepreneur [2] and of the venture capitalist [1] with respect to their effort amounts $E$ and $V$ and
setting them to zero, we obtain the optimal effort amounts as a function of the equity allocation \((1-\alpha, \alpha)\):

\[ E^* = \left[(1-\alpha)^{1-\delta} \alpha^\delta \beta^{1-\delta} h I H^{\lambda-\delta}\right]^{1/(1-\beta-\delta)} \]

and

\[ V^* = \left[(1-\alpha)^{\beta} \alpha^{1-\beta} \beta^{\delta} h I H^{\beta+\lambda-1}\right]^{1/(1-\beta-\delta)}. \]

For any equity allocation \((1-\alpha, \alpha)\) fulfilling \(0 < \alpha < 1\), the effort amounts by the contracting parties are positive if the venture capitalist infuses the start-up investment. Entrepreneur’s effort increases with venture capitalist’s expertise, since \(\lambda > \delta\), while the time which the venture capitalist spends in the enterprise decreases with his expertise, since \(\beta + \omega - \rho_0 - 1 < 0\). Therefore, venture capitalists with long experience in financing start-up investments need less time to control and support the management team compared to relatively inexperienced venture capitalists.\(^4\)

Inserting the optimal effort amounts by the entrepreneur \([3]\) and by the venture capitalist \([4]\) in the expected pay-off functions \([1]\) and \([2]\) gives the expected pay-offs of the contracting parties as a function of the equity allocation \((1-\alpha, \alpha)\) and of the venture capitalist’s expertise \(H\):

\[ \hat{U}^{EN} = \left[(1-\alpha)^{1-\delta} \alpha^\delta \beta^{1-\delta} h I H^{\lambda-\delta}\right]^{1/(1-\beta-\delta)} (1-\beta) \]

and

\[ \hat{U}^{VC} = \left[(1-\alpha)^{\beta} \alpha^{1-\beta} \beta^{\delta} h I H^{\beta+\lambda-1}\right]^{1/(1-\beta-\delta)} (1-\delta) - I. \]

For any equity allocation \((1-\alpha, \alpha)\) fulfilling \(0 < \alpha < 1\), the entrepreneur has a positive expected pay-off, while the venture capitalist’s expected pay-off depends on the amount of the start-up investment \(I\) and on his expertise \(H\). As noted above, in the state of the project with low performance \(l\), the venture capitalist realizes a negative pay-off, while in the state of the project with high performance \(h\), he may receive a positive pay-off depending on his expertise and on the equity allocation. Certainly, if his expertise is

\(^4\) This relationship between venture capitalist’s expertise and the time spent for doing management support implies that venture capitalists can finance more and more enterprises with increasing expertise, since they need less and less time for doing management support. Therefore, the observed development in the US had not to come along with reduced management support. Since the late 1970s, the amount of money managed by venture capital funds has increased dramatically, while the number of professional staff in these funds has not increased proportionally (Fried and Hisrich 1991). As argued by Flynn (1991), the higher the number of active investments, the lower the venture capitalist’s management support in a single investment. But if venture capitalists’ expertise is taken into account, the value of management support has not necessarily decreased.
insufficient, he receives a negative pay-off even in the good state of the project, and therefore, he does not finance the start-up investment of the enterprise.

3.2 Pay-offs in different project states and the optimal allocation of equity

In the state with low performance \( I \), the pay-off of the venture capitalist is given by:

\[
U^{VC} = \left[ (1 - \alpha)^{\bar{h}} \alpha^{1 - \bar{h}} \right] \left[ \frac{1}{1 - \delta} \right] \left[ \frac{1 - p_i}{p_h} \right] - I.
\]

In order to ensure that the venture capitalist’s pay-off in the bad state of the project is smaller than zero, the start-up investment has to be chosen sufficiently large: \( I > 1/\delta \).

Comparing this restriction on the amount of the start-up investment and the first restriction which ensures that the shift parameter \( A \) is positive \((I > p_i)\), we obtain the result that the first restriction is not longer necessary, because \( I > 1/\delta \) requires that \( I > 1 \), since \( 0 < \delta < 1 \), while fulfilling the first restriction \( I > p_i \) only needs an amount of the investment never larger than one, since \( 0 < p_i < 1 \).

In the good state of the project, venture capitalist’s pay-off is given by:

\[
U^{VC} = \left[ (1 - \alpha)^{\bar{h}} \alpha^{1 - \bar{h}} \right] \left[ \frac{1}{1 - \delta} \right] \left[ \frac{1 - p_i}{p_h} \right] - I.
\]

Necessary but not sufficient for a positive pay-off of the venture capitalist in the good state of the project is the restriction that \((I - p_i)/(1 - \delta + p_i \delta) > 0\). Rearranging the terms results in \( I > p_i/(1 - \delta + p_i \delta) \). Again, this restriction is less restrictively than \( I > 1/\delta \), i.e. this restriction requires a lower amount of the start-up investment, since \( 1 - \delta > 0 \).

In order to investigate the impact of public subsidies on expertise of venture capitalists which is at least necessary to make a high-risk start-up investment profitable, we do not require an explicit assumption on which of the contracting parties offer the contract. However, we have to determine which equity allocations are possible and desirable for the entrepreneur and for the venture capitalist. Let us first assume that the entrepreneur offers the contract specifying the equity allocation \((1 - \alpha, \alpha)\). She maximizes her expected pay-off \[5\] under the condition that the venture capitalist’s expected pay-off \[6\] is non-negative. Since the venture capitalist does not pay the entrepreneur a basic salary, there are several parameter constellations ensuring that the venture capitalist’s expected pay-off is non-negative as long as \( 0 < \alpha < 1 \). Under these parameter constellations, the entrepreneur chooses \( \alpha = \delta \). \[5\] If the venture capitalist’s expected pay-off is negative for this equity allocation, the entrepreneur increases \( \alpha \) until the venture capitalist’s expected pay-off is non-negative.

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\[5\] Taking the derivation of entrepreneur’s expected pay-off with respect to \( \alpha \) results in:

\[
\delta (1 - \alpha)^{\bar{h}} \alpha^{\bar{h}} = (1 - \delta)^{(1 - \alpha)^{\bar{h}} - \delta^{\bar{h}}} \cdot \alpha^{\bar{h}}.
\]

Solving this equation for \( \alpha \) gives:

\[
\alpha = \delta.
\]
pay-off is equal zero. Note, that for any \( \alpha \) fulfilling \( 0 < \alpha < 1 \), the entrepreneur has a positive expected pay-off.

More important for our analysis is the equity allocation preferred by the venture capitalist, since this equity allocation must be used to determine venture capitalist’s expertise at least necessary to make a start-up investments profitable. Venture capitalist’s expected pay-off reaches its maximum at \( \alpha = 1 - \beta \) which ensures that entrepreneur’s expected pay-off is positive. So far, we have examined the equity allocation without considering under which conditions the venture capitalist finances the start-up investment. Whether the venture capitalist finance start-up investments depends on his expertise. Even if the equity allocation preferred by the venture capitalist is contractually specified, he can have a negative expected pay-off so that he does not infuse capital. Only if venture capitalist’s expertise is sufficiently high, he finances the enterprise’s start-up investment.

### 3.3 The role of accumulated expertise in making a profit from high-tech investments

With a sufficient level of expertise, the venture capitalist can make an investment profitable. The sufficient level can be determined using the venture capitalist’s expected pay-off given in [6] which still depends on the equity allocation \((1 - \alpha, \alpha)\). Since the entrepreneur starts her enterprise for all \( \alpha \) fulfilling \( 0 < \alpha < 1 \), we can and have to use the equity allocation preferred by the venture capitalist \((\alpha = 1 - \beta)\) to determine the sufficient level of venture capitalist’s expertise which leads to a profitable investment. Inserting this equity allocation in equation [6] results in:

\[
\tilde{U}_{VC} = \left[ (1 - \beta)^{1 - \beta} \delta \beta^{2\delta} \delta^{1 - \delta} H^{\delta - 1} \right]^{1 - \beta - \delta} (1 - \delta) - 1.
\]

Solving for the venture capitalist’s expertise \( H \) gives:

\[
[8] \quad \tilde{H} = \left[ \frac{1}{(1 - \delta)} \left( (1 - \beta)^{1 - \beta} \delta \beta^{2\delta} \delta^{1 - \delta} \right) \right]^{1 - \beta - \delta} \delta^{1 - \delta}.
\]

All venture capitalists who have less expertise than this sufficient level \( \tilde{H} \) have no incentive to finance start-up enterprises, since their expected pay-offs are always negative. All venture capitalists who have at least this expertise are capable to finance start-up investments, since their expected pay-offs are non-negative. Venture capitalists whose expertise is equal to this sufficient level only finance start-up investments if the equity allocation preferred by them \((\alpha = 1 - \beta)\) is contractually specified, while venture capitalists whose expertise exceeds this sufficient level, may also accept equity stakes which are lower than their preferred stakes \((\alpha < 1 - \beta)\).

### 3.4 A numerical example

Figure 2 depicts the expected pay-offs of the entrepreneur and of two venture capitalists with expertise \( H^* \) and \( H \) where \( H^* > \tilde{H} > H \) for a parameter sample which fulfills
the assumptions made. The venture capitalist with expertise $H^*$ finances the start-up investment even if the equity allocation preferred by the entrepreneur $α = δ$ is contractually specified. While the venture capitalist with expertise $H'$ does not finance the start-up investment, since his expected pay-off is negative for any equity allocation $(1 - α, α)$ fulfilling $0 < α < 1$, i.e. even the equity allocation that maximizes his expected pay-off, $α = 1 - β$, does not result in a positive expected pay-off. Since the expected pay-off of the venture capitalist with expertise $H'$ is for any equity allocation negative, the entrepreneur can only start her business if the venture capitalist with expertise $H^*$ finances the start-up investment. Venture capitalists predominantly accumulate expertise through their involvement in high technology start-ups. Therefore, in a relative young venture capital market, in which venture capitalists have not yet build up their specific knowledge necessary to finance high-technology high-risk start-ups, there might be a demand for venture capital from the founders of high-technology start-ups but not a supply.

Figure 2: Expected pay-offs depending on the equity allocation and venture capitalists’ expertise

![Figure 2: Expected pay-offs depending on the equity allocation and venture capitalists’ expertise](image)

Parameters used: $δ = 0.3$, $β = 0.4$, $λ = 0.6$, $p_t = 0.8$, $I = 5$, $H^* = 7$, $H = 2$.

4 The incidence of public subsidies

Several European governments have introduced various subsidy schemes to increase the supply of venture capital for high-technology start-ups. These schemes have in common, that venture capitalists’ losses of investments in start-up enterprises are partly covered by the government, either by giving a guarantee to take on part of the losses which a venture capitalist may realize or by refinancing venture capitalists’ investments with credits at favorable interest rates which also involves the sharing of risk by the government. Given the model described in section 3, we can examine how public subsidies affect the effort
amounts by the contracting parties, and how the sufficient level of venture capitalist’s expertise changes.

In our model, the venture capitalist who finances a start-up investment only realizes a loss in the bad state of the project. A public subsidy changes the venture capitalist’s expected pay-off [1], since the government covers a share $\theta$ of the venture capitalist’s total losses given by $I - \alpha E^{\beta} V^{\delta} H^\lambda$ in the bad state of the project. Therefore, the expected pay-off of the venture capitalist [1] has to be extended by $p_i \theta I - p_i \theta \alpha E^{\beta} V^{\delta} H^\lambda$.

4.1 Lowering venture capitalists’ effort in case of sufficient expertise

The expected pay-off of the venture capitalist is now given by:

$$U_{S}^{VC} = \alpha (I - p_i \theta) E^{\beta} V^{\delta} H^\lambda - HV - I + p_i \theta I,$$

while the entrepreneur’s expected pay-off does not change. Since the efforts of both contracting parties are interdependent, a public subsidy does not only affect venture capitalist’s effort amount but also the effort amount of the entrepreneur. Under a public subsidy, effort amounts of the contracting parties are given by:

$$E_{S}^* = \left( (1 - \alpha)^{1 - \delta} \beta^{1 - \delta} \delta^\delta (I - p_i \theta)^\delta H^{\lambda - \delta} \right) \frac{1}{1 - \beta - \delta}$$

and

$$V_{S}^* = \left( (1 - \alpha)^{\beta} \delta^{1 - \beta} \delta (I - p_i \theta)^{1 - \beta} H^{\beta + \lambda - 1} \right) \frac{1}{1 - \beta - \delta}.$$

Both contracting parties reduce their effort amounts under a public subsidy independent of the venture capitalist’s expertise. Comparing effort amounts by the entrepreneur with and without public subsidy, i.e. comparing equations [3] and [10], we receive the following inequality $(I - p_i \theta) < I$. Since this inequality is always fulfilled, the entrepreneur offers less effort if the government covers a share of venture capitalist’s expected losses. Comparing venture capitalist’s efforts with and without public subsidy, i.e. comparing equation [4] and [11], leads to the result that the venture capitalist reduces his effort under a public subsidy as well.

Under a public subsidy, the expected pay-offs of the contracting parties are given by:

$$U_{S}^{EN} = \left( (1 - \alpha)^{1 - \delta} \beta^{\delta} \delta (I - p_i \theta)^{\delta} H^{\lambda - \delta} \right) \frac{1}{1 - \beta - \delta} (1 - \beta)$$

and

$$U_{S}^{VC} = \left( (1 - \alpha)^{\beta} \alpha^{1 - \beta} \delta^{\beta} (I - p_i \theta)^{1 - \beta} H^{\beta + \lambda - 1} \right) \frac{1}{1 - \beta - \delta} (1 - \delta) + (p_i \theta - 1)I.$$

Comparing venture capitalist’s expected pay-off with [13] and without public subsidy [6] shows that there exists a critical level of expertise $\overline{H}$, with $\overline{H} > H$, above which the venture capitalist does not prefer government’s covering of realized losses, since his expected pay-off is lower with than without a public subsidy. In this case, a public
subsidy negatively affects the venture capitalist’s expected pay-off by reducing the effort of both contracting parties more than the government’s monetary transfer increases the venture capitalist’s expected pay-off.

4.2 Substituting for expertise in case of an inexperienced venture capitalist

A public subsidy reduces the sufficient level of venture capitalist’s expertise at least necessary to make high-risk start-up investments profitable. Since the equity allocations preferred by the contracting parties are not affected by a public subsidy, the examination of the equity allocations given in section 3.2 further holds. Using the expected pay-off of the venture capitalist under a public subsidy \( \alpha = 1 - \beta \) and solving for the expertise \( H \) results in:

\[
H = \left[ \frac{(1 - \beta)I}{(1 - \delta)} \left( 1 - \beta \right)^{1 - \beta} \beta^{2 \beta - \delta} I^\beta (1 - p, \theta)^{1 - \beta} \right]^{\frac{1}{\beta + \delta - 1}}.
\]

Comparing the sufficient level of expertise with \( H \) and without public subsidy \( \overline{H} \) under the assumption that the sufficient level with public subsidy is lower than without public subsidy results in the following restriction on the amount of the start-up investment:

\[
I > p, \theta \left( 1 - (1 - p, \theta)^{1 - \beta - \delta} \right).
\]

Therefore, the impact of a public subsidy depends on whether this inequality is fulfilled and on the venture capitalist’s expertise. If the venture capitalist’s expertise exceeds the sufficient level \( \overline{H} \), he also finances high-risk start-up investments without a public subsidy. If the venture capitalist’s expertise is equal or exceeds the critical level \( \overline{H} \), he finances start-up investments and he does not make use of a public subsidy, since this reduces his expected pay-off.

There are still two more expertise levels of particular interest. If the inequality given in \( H \) holds, the venture capitalist’s expertise at least necessary to make a start-up investment profitable is lower with than without a public subsidy. Therefore, a venture capitalist, whose expertise \( \overline{H}_s \) is lower than the sufficient level \( \overline{H} \) (see figure 3), does not finance start-up investments without a public subsidy, but if the government covers a share of his realized losses he makes use of this offer and infuses capital in start-up enterprises.

If the inequality given in \( H \) does not hold, the venture capitalist’s expertise at least necessary to make a start-up investment profitable is higher with than without a public subsidy. Therefore, in this case, a public subsidy sets no incentives for relatively inexperienced venture capitalists to infuse capital in start-ups. But if there exist a venture capitalist with expertise \( \overline{H}_s' \) fulfilling \( \overline{H} < \overline{H}_s' < \overline{H} \), this one makes use of government’s offer to cover a share of realized losses so that a public subsidy only results in an
efficiency loss, because this venture capitalist also finances start-ups without public subsidy and because he reduces his effort under a public subsidy.

**Figure 3: Expertise of venture capitalists**

Let us first discuss the impact of a public subsidy on venture capital investment under the assumption that the inequality given in [15] holds. Then a public subsidy sets incentives for relatively inexperienced venture capitalists to make use of government’s offer to cover a share of realized losses in order to finance high-risk start-up investments, since $\bar{H}_S < \bar{H}$. Note, that these venture capitalists do not finance start-ups without the government covers a share of their realized losses. But a public subsidy provokes still another effect. Venture capitalists whose expertise is at least equal to the sufficient level $\bar{H}$ also make use of government’s offer to cover a share of their realized losses (see appendix 6.1) and hence reduce their effort amounts in the enterprises (see section 4.1). Therefore, a public subsidy causes an inefficiency, since these venture capitalists also finance start-up investments without public subsidy. But in an emerging venture capital market in which the venture capitalists have still not build up their expertise, a public subsidy sets a positive stimulus.

Let us now discuss the impact of a public subsidy on venture capital investment under the assumption that the inequality given in [15] does not hold. In this case, a public subsidy does not offer incentives for relatively inexperienced venture capitalists to finance start-up investments. There might be an impact of a public subsidy on venture capital investments, if there exist venture capitalists with expertise $\bar{H}^+_s$, such that $\overline{H} < \bar{H} < \bar{H}^+_s$. These venture capitalists might have incentives to make use of a public subsidy in order to increase their expected pay-offs. As shown in appendix 6.2, there do not exist venture capitalists with expertise $\bar{H}^+_s$ fulfilling $\overline{H} < \bar{H} < \bar{H}$. Therefore, if the start-up investment of the enterprises is lower than the critical value given in [15] on the right hand side, a public subsidy does not have an impact on venture capital investments at all.

The last step to complete the model is to combine the restrictions on the amount of the start-up investment given in [15] with the first restriction imposed in section 3.2 ($I > 1/\delta$). Both restrictions require that the investment infused is larger than one. Since none of the two restrictions is dominated by the respective other one, we have to scrutinize under which parameter constellation which restriction has to be fulfilled, i.e. under which parameter the following inequality holds:

$$\frac{p_\theta}{1 - (1 - p_\theta)^{1-\beta-\delta}} < \frac{1}{\delta}.$$
Rearranging the terms results in \((1 - \delta p, \theta) > (1 - p, \theta)^{\frac{1-\delta}{1-\beta}}\). In order to ensure that the venture capitalist’s expected pay-off is negative in the state with low performance \(l\) and that a public subsidy has indeed an impact on venture capital investments, the amount of the start-up investment has to fulfill the following inequality:

\[
I > \begin{cases} 
\frac{1}{\delta} & \text{if } (1 - \delta p, \theta)^{\frac{1-\delta}{1-\beta}} > (1 - p, \theta)^{\frac{1-\delta}{1-\beta}} \\
p, \theta & \text{otherwise} 
\end{cases}
\]

### 4.3 A numerical example

As in section 3.4, a numerical example should clarify the model results. Figure 4 depicts the expected pay-offs of the entrepreneur and of the venture capitalist with expertise \(H^* > \bar{H}\) without and with public subsidy (\(S\)) using the parameter sample of section 3.4. The venture capitalist’s expected pay-off is higher with a public subsidy than without it, while the entrepreneur’s expected pay-off is lower, if the government covers a share of venture capitalist’s realized losses. The sum of the contracting parties’ expected pay-offs without public subsidy exceeds the sum of the contracting parties’ expected pay-offs with public subsidy, if the government’s monetary transfer is not taken into account. Therefore, a public subsidy causes an efficiency loss, if the venture capitalist’s expertise exceeds the sufficient level \(\bar{H}\).

Figure 5 depicts the expected pay-offs of the entrepreneur and of the venture capitalist if the expertise of the venture capitalist is below the sufficient level \(H < \bar{H}\). This venture capitalist does not finance the start-up investment of the enterprise without public subsidy. If the government covers a share of his realized losses, he does finance it. Therefore, if venture capitalists lack the sufficient expertise to finance high-risk start-ups, a public subsidy can help to establish a supply for those kind of financing.
Figure 4: Expected pay-offs with and without a public subsidy if the venture capitalist’s expertise exceeds the sufficient level

Parameters used: $\delta = 0.3$, $\beta = 0.4$, $\lambda = 0.6$, $\theta = 1.0$, $p_f = 0.8$, $I = 5$, $H^* = 7$.

Figure 5: Expected pay-offs with and without a public subsidy if the venture capitalist’s expertise does not exceed the sufficient level

Parameters used: $\delta = 0.3$, $\beta = 0.4$, $\lambda = 0.6$, $\theta = 1.0$, $p_f = 0.8$, $I = 5$, $H^* = 2$. 
5 Conclusions

This paper has analyzed the impact of public subsidies on venture capital investments in start-up enterprises. If venture capitalists lack experience making high-technology start-up investment profitable, public subsidies may increase the supply of venture capital invested in early stages of enterprises’ development. But venture capitalists whose expertise already exceeds a sufficient level also welcome public subsidies because they can then save on the effort to support the management of the firms they finance. Venture capitalists spend less management support if the government covers a share of their realized losses. Thus, public subsidies can only make sense when the supply of venture capital for high-technology start-ups is too low and too few venture capitalists have the appropriate expertise important for financing enterprises with high risks.

But our analysis of the impact of public subsidies on venture capital investments in start-up enterprises is still restrictive in several ways. So far, only a simple equity contract between the venture capitalist and the entrepreneur has been considered. Venture capitalists almost exclusively use convertible securities to finance their investments (Kaplan and Strömberg 2000). Under a convertible security contract the entrepreneur has more incentives to exert high effort and to avoid high risks (Gompers 1997). Therefore, one further direction of research is whether the results obtained above also hold for a convertible security contract.

Furthermore, the examination of the impact of public subsidies lacks a critical consideration of welfare effects. In order to fully evaluate the impact of public subsidies on the overall welfare two further stylized facts of venture capital finance have to be taken into account. First, venture capitalists do not finance a single enterprise at any one time, on the contrary each venture capitalist is responsible for ten enterprises on average (Gorman and Sahlman 1989). Second, venture capitalists accumulate their technological knowledge, expertise and experience important for financing successfully start-up investments by engaging in high-risk enterprises so that expertise is built up over time.

Welfare effects of public subsidies can only adequately be evaluated if we take into account that a venture capitalist finances a multitude of enterprises. The above analysis has shown that a venture capitalist who has sufficient expertise to manage a high-risk start-up investment reduces his time in his portfolio firm. Since his time budget is a binding constraint by maximizing the overall profit of his portfolio consisting of more than one enterprise, and since he reduces his time in a single enterprise under a public subsidy, he can increase the total number of start-up enterprises financed by him. This may or may not improve the overall welfare of an economy.

Moreover, welfare effects of public subsidies can only adequately be evaluated if the accumulation process of venture capitalists’ technological knowledge and expertise is taken into account. The above analysis has shown, that a venture capitalist who does not have sufficient expertise to make a start-up investment profitable can finance this start-up investments if the government covers a share of his realized losses. This on its own may not welfare improving. But venture capitalists’ engagement in start-up enterprises may generate the accumulation of technological knowledge and expertise, so that venture capitalists finance start-up investment also without public subsidy after some time has passed. In order to evaluate the welfare implications of public subsidies, the
intertemporal effects has to be borne in mind. But public subsidies may also interfere the process of technological knowledge accumulation, since venture capitalists may change their technological specialization.

And, guarantee and loan schemes may have different impact on the behavior of venture capitalists. While a loan scheme directly increases the overall available venture capital volume, a guarantee scheme does not. Therefore, the impact of these schemes on the venture capital investment in start-up enterprises depend on whether venture capitalists have sufficient money in their funds invested by outside investors. Finally, there is the question whether a loan scheme tends to replace the funds invested by outside investors. While venture capitalists generally have to share the profits realized by investing the capital of the fund with the outside investors, they do not have to share these profits if the government offers loans at favorable interest rates and partly covers the credit risk.

6 Appendix

6.1 The inefficiency caused by a public subsidy

A venture capitalist whose expertise is as large as the sufficient level $H$ makes use of the public subsidy and by doing so he reduces his time in the enterprise. In order to show that a venture capitalist with expertise $H$ has indeed incentives to make use of the public subsidy offered by the government, we have to show that his expected pay-off under a public subsidy is larger than without. Using the difference between his expected pay-off with [13] and without public subsidy [6], inserting the sufficient level of expertise [8] and the equity allocation preferred by the venture capitalist we get the following inequality ensuring that the expected pay-off under a public subsidy is larger than the expected pay-off without a public subsidy:

$\left[ \frac{(1-\beta)^{1-\beta} \beta^{2p \delta \delta} I^{\beta}}{(1-\beta)^{1-\beta} \beta^{2p \delta \delta} I} \right]^{1 \over 1-\beta-\delta} \left( (I - p, \theta)^{1-\beta} - I^{1-\beta-\delta} \right) + p, \theta > 0.$

Rearranging terms and solving for the start-up investment yields:

$I > \frac{p, \theta}{1-(1-p, \theta)^{1-\beta-\delta}}.$

Therefore, if inequality [15] holds, the venture capitalist whose expertise is equal to the sufficient level of expertise $H$ makes use of a public subsidy. Comparing venture capitalist’s effort amounts with [11] and without public subsidy [4] shows that the venture capitalist with expertise $H$ reduces his effort in the enterprise.

6.2 A public subsidy has no impact under specific conditions

Assume that $I \leq p, \theta \left( 1-(1-p, \theta)^{1-\beta-\delta} \right).$
In order to show that under this condition a public subsidy has no impact on venture capital investments we have to show that there does not exist a venture capitalist with expertise \( \bar{H}_S^+ < \bar{H} \). In order to determine \( \bar{H} \), we set the difference between the venture capitalist’s expected pay-offs with equation \([13]\) and without public subsidies \([6]\) equal zero:

\[
\left(1-\alpha\right)\alpha^{1-\beta}\beta^\delta I^\beta I^{\lambda-\delta}(I-p_\theta)^{1-\beta} \left(1-\left[\frac{I}{(I-p_\theta)}\right]^{1-\beta}\right) + \left(p_\theta I\right) = 0.
\]

Solving for the venture capitalist’s expertise gives:

\[
\bar{H} = \left[\left(p_\theta I\right)\left[I^{\lambda-\delta} - 1\right]^{-1}\right]^{1-\beta}\left[I^{\lambda-\delta} - 1\right]^{-1}
\]

A venture capitalist with expertise \( \bar{H} \) is indifferent between making use of the public subsidy and financing the start-up investment without that the government covers a share of his realized losses. Inserting \( \bar{H} \) and \( \bar{H}_S^+ \) given in equation \([14]\) in \( \bar{H}_S^+ < \bar{H} \) and rearranging the terms yields:

\[
(1-p_\theta)\left[I^{\lambda-\delta} - 1\right]^{1-\beta} < p_\theta
\]

which results in the inequality given in \([15]\) which directly contradicts the assumption with which we have started. Therefore, there does not exist a venture capitalist with expertise \( \bar{H}_S^+ \) fulfilling \( \bar{H} < \bar{H}_S^+ < \bar{H} \). Summing up, a public subsidy does not have an impact on venture capital investments if the inequality \([15]\) does not hold.
7 References


