Oligopolistic Competition and Optimal Monetary Policy

by Ester Faia

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Oligopolistic Competition and Optimal Monetary Policy

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Abstract

The literature has shown that product market frictions and firms' dynamic play a crucial role in reconciling standard DSGE with several stylized facts. This paper studies optimal monetary policy in a DSGE model with sticky prices and oligopolistic competition. In this model firms' monopolistic rents induce both intra-temporal and intertemporal time-varying wedges which induce inefficient fluctuations of employment and consumption. The monetary authority faces a trade-off between stabilizing inflation and reducing inefficient fluctuations, which is resolved by using consumer price inflation as a state contingent sale subsidy. An analysis of the welfare gains of alternative rules shows that targeting mark-ups and asset prices might improve upon a strict inflation targeting.

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1 Introduction

It is empirically well established that firms’ mark-ups behave countercyclically and that firms’ profits behave pro-cyclically\(^1\). Several recent studies show that accounting for firm dynamic and endogenous mark-ups movements\(^2\) helps in improving the performance of DSGE models in several directions and is consistent with empirical evidence.

A model featuring frictions in the product market raises questions on the appropriate design of optimal monetary policy. Time-varying wedges in the form of endogenous mark-up movements and oligopolistic competition induce significant welfare costs that might call for deviating from price stability policies. In this paper a DSGE model with oligopolistic firms and firms’ adjustment costs on pricing à la Rotemberg 1982 is used to answer this question. Firms in the model engage in oligopolistic competition: this leads to endogenous variations in mark-up even in the flexible price case\(^3\). Specifically monopolistic mark-ups depend on firms market share: an increase in the number of firms, increases competition and demand elasticity, therefore reduces mark-ups. Time-varying monopolistic rents reduce output below the efficient level and induce inefficient fluctuations in employment and consumption. Those elements induce a trade-off for the monetary authority between reducing the cost of adjusting prices and smoothing inefficient fluctuations in output.

The design of optimal policy follows the Ramsey approach (Atkinson and Stiglitz 1976, Lucas and Stokey 1983, Chari, Christiano and Kehoe 1991) in which the optimal path of all variables is obtained by maximizing agents’ welfare subject to the relations describing the competitive economy and via an explicit consideration of all wedges that characterize both the long run and the cyclical dynamics. Recent studies apply this approach to the analyses of optimal policy in the context of models with nominal and real rigidities\(^4\).

In the present model, oligopolistic competition introduces three types of time-varying wedges.

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\(^3\)Notice that for this effect to hold the model does not need to assume firm heterogeneity, despite this the models equilibrium conditions are iso-morphic to the ones in Bilbiie, Ghironi and Melitz 2007a.

First, firms’ monopolistic mark-ups induce a time-varying wedge between the marginal rate of substitution between consumption and labour and the marginal productivity of labour. Because of this output and employment are inefficiently low and the monetary authority is tempted to increase the number of varieties and to reduce mark-ups, by increasing demand. In this respect consumer inflation can be used to smooth and reduce firms’ monopolistic rents and to boost demand. Second, there is an intra-temporal wedge on the evolution of firms’ values, as the number of new entrants is dampened by the increase in monopolistic rents of the existing firms. This wedge distorts the allocation of consumption between two different dates. Once again the policy maker would like to use consumer price inflation to boost consumption demand. Finally, nominal rigidities and increasing return to scale interact to induce a third type of time-varying distortion. Due to increasing return to scale, when the number of varieties increases the price of each variety increases relative to the price of the consumption basket. When firms face adjustment costs on producer prices, increases in producer price inflation act as sales taxes. This creates a bias for the policy maker toward PPI stabilization rather than CPI stabilization. Overall analytical and quantitative (the economy is simulated under productivity and government expenditure shocks) results show that the Ramsey planner deviates form full consumer price stabilization for all types of shocks considered.

To provide a full assessment of optimal monetary policy design the analysis compares welfare costs of alternative monetary policy rules. Welfare in this context is computed using second order approximations methods which, in models with large distortions, allow to account for the effects of volatilities on mean welfare. Furthermore we consider conditional welfare metrics, which allow to account for the transitional dynamic from one policy regime to the others. Results show rules placing high weight on PPI inflation rather than CPI inflation are welfare improving. Second, rules targeting mark-ups perform better than rules responding solely to inflation. This is so as this allows to reduce the distortion caused by the oligopolistic wedge. Third, asset price targeting, in terms of \textit{lean against the wind policy}, is welfare enhancing. Firms’ rents affect the evolution of asset prices and through this they distort the intertemporal allocation of consumption between two different dates. For this reason it is welfare enhancing to target asset prices.

\footnote{See Schmitt-Grohe and Uribe 2004a, Faia and Monacelli 2007, and Faia 2008b.}
This paper is related to a recent literature which introduces endogenous entry and strategic interactions into DSGE models\(^6\). Most of the studies have focused on analyzing the dynamic properties of those models and their ability to replicate stylized facts. Some studies analyze the role of stabilization policies in those type of models: Bergin and Corsetti 2009, Bilbiie, Ghironi and Melitz 2008. Recently Lewis 2009 has analyzed the design of optimal policy in presence of firms’ heterogeneity and one period sticky wages.

This paper is structured as follows. Section 2 presents the model. Section 3 presents the Ramsey plan. Section 4 discusses the welfare costs of alternative monetary policy rules. Section 5 concludes.

2 Model Economy

At each point in time the economy is populated by a continuum of identical households with mass one. The representative households has preferences over consumption and leisure. Households choose consumption, \(c_t\), labour hours, \(h_t\), and investment in risk free bonds, \(b_t\), and firms shares, \(x_t\). They receive labour income, returns on bonds, dividends and capital gains on firms share. Consumption in this economy is given by the following Dixit-Stiglitz aggregator:

\[
\begin{align*}
    c_t &\equiv \sum_{i=0}^{N_t} (c_i^t)^{\frac{-1}{1-\epsilon}} \frac{di}{1-\epsilon} \\
\end{align*}
\]  

(1)

with \(N_{h,t}\) being the number of firms operating into the economy at time \(t\) which is given by:

\[
N_{h,t} = N_t + N_{e,t}
\]

(2)

and evolves according to:

\[
N_{t+1} = (1 - \varrho)(N_t + N_{e,t})
\]

(3)

with \(\varrho\) being an exogenous destruction rate. The optimal allocation of expenditure on each variety yields:

\[
ct = \left( \frac{p_t^i}{p_t^c} \right)^{-\epsilon} c_t
\]

(4)

---

where:

\[ p_t^c = \left( \sum_{i=0}^{N_t} (p_i^t)^{i+1} di \right)^{\frac{1}{i+1}} \]  \hspace{1cm} (5)

is the consumer price index (CPI). There is a continuum of agents who maximize the expected lifetime utility. They choose the set of processes \( \{c_t, h_t, b_t, x_t\} \) to maximize:

\[ E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \log(c_t) - \frac{h_t^{1+\tau}}{1+\tau} \right\} \]  \hspace{1cm} (6)

subject to the following budget constraint (in nominal terms):

\[ p_t^c c_t + p_t^c b_t + p_t^c v_t N_{h,t} x_{t+1} \leq W_t h_t + (1 + r_{t-1}^n) p_t^c b_{t-1} + p_t^c (d_t + v_t) N_t x_t + \Theta_t - \tau_t \]  \hspace{1cm} (7)

and to (3). In the above equations \( v_t \) is firms’ share value, \( d_t \) is firms’ dividends, \( (1 + r_{t-1}^n) \) are nominal returns on risk-free bonds, \( W_t \) are nominal wages, \( \tau_t \) are nominal fiscal transfers. First order conditions to the above problem read as follows:

\[ c_t^{-1} = \beta(1 + r^n_t) E_t \left\{ c_{t+1}^{-1} \frac{p_t^c}{p_{t+1}^c} \right\} \]  \hspace{1cm} (8)

\[ v_t = \beta(1 - \beta) E_t \left\{ \left( \frac{c_{t+1}}{c_t} \right)^{-1} (v_{t+1} + d_{t+1}) \right\} \]  \hspace{1cm} (9)

\[ \psi h_t^c c_t = \frac{W_t}{p_t^c} = w_t \]  \hspace{1cm} (10)

Equation (8) is the classical Euler condition with respect to risk-free bonds. Equation (9) is the optimal investment condition with respect to firms’ share and describes the evolution of firms’ share value. Finally equation (10) is the optimality condition with respect to labour hours. We can solve forward equation (9) to obtain an expression of the asset price as discounted sum of future profits:

\[ v_t = E_t \left\{ \sum_{s=t+1}^{\infty} (\beta(1 - \beta))^{s-t} \left( \frac{c_s}{c_t} \right)^{-1} d_s \right\} \]  \hspace{1cm} (11)

Future dividends, \( d_s \), are given by firms’ profits which in oligopolistic models can be written as a function of number of firms operating in the market and demand elasticity, \( \Pi(N_{h,t}, \varepsilon) \).
Notice that, as in large part of the recent literature, money plays the role of nominal unit of account\textsuperscript{7}. The assumption of a cashless economy implies that zero inflation will be an outcome in the long-run. Departure from price stability occurs in the short run as the monetary authority responds to productivity and government expenditure shocks in order to reduce the impact of oligopolistic externalities.

2.1 Firms’ Optimization under Flexible Prices

Let’s start by analyzing firms’ behavior under flexible prices. This analyses is instructive as it highlights to what extent endogenous variations in mark-up are due to oligopolistic competition and to sticky prices. Firms’ produce variety \( i \) using labour hours according to the following production function:

\[
y^i_t = z_t h^i_t
\]

where \( z_t \) is an aggregate productivity shifter. Firms face the following demand for variety \( i \):

\[
y^i_t = \left( \frac{p^i_t}{\sum_{i=0}^{N_t} (p^i_t)^{\frac{1}{\varepsilon}} d^i_t} \right)^{-\varepsilon} (y^i_{c_t})
\]

with \( y^i_{c_t} = c_t + g_t \) being aggregate demand. Firms engage in oligopolistic competition and maximize the following (nominal) profit function:

\[
d^i_t = \Pi(N_t, \varepsilon) = p^i_t y^i_t - \frac{W_t}{z_t}
\]

Under strategic pricing the effect of a change in prices on demand can be decomposed as follows:

\[
\frac{\partial y^i_t}{\partial p^i} = -\varepsilon \frac{y^i_t}{p^i_t} + \varepsilon (p^i_t)^{-\varepsilon} y^c (p^c_t)^{-\varepsilon - 1} \frac{\partial p^c_t}{\partial p^i_t}
\]

where:

\[
\frac{\partial p^c_t}{\partial p^i_t} = \frac{1}{1 - \varepsilon} \left[ p^c_t \right]^{\frac{1}{1 - \varepsilon} - 1} (1 - \varepsilon) (p^i_t)^{-\varepsilon} = \left( \frac{p^i_t}{p^c_t} \right)^{-\varepsilon}
\]

Overall the effect on demand of a change in the price of each variety can be written (after substituting (16) into (15)):

\[
\frac{\partial y^i_t}{\partial p^i_t} = \varepsilon \frac{y^i_t}{p^i_t} [\xi^i - 1]
\]

\textsuperscript{7}See Woodford 2003, chapter 3. Thus the present model may be viewed as approximating the limiting case of a money-in-the-utility model in which the weight of real balances in the utility function is arbitrarily close to zero.
where $\xi^i = \frac{p^i_{\text{pc}}}{p^i_{\text{pc}}}$ is firm’s $i$ market share. Importantly demand elasticity in this context depends on firms’ market share. Using equation (17) we can solve firms’ profits optimization and obtain the following first order condition:

$$\frac{\partial \Pi^i_t}{\partial p^i_t} = y^i_t + (p^i_t - \frac{W_t}{z_t}) \frac{\varepsilon y^i_t}{p^i_t} [\xi^i - 1] = 0$$ (18)

By solving (18) we obtain the following optimal price:

$$p^*_t = \frac{(1 - \xi^i_t)^{\varepsilon} W_t}{(1 - \xi^i_t)^{\varepsilon} - 1} z_t$$ (19)

Firms’ mark-up, $\mu(\xi^i) = \frac{(1 - \xi^i)\varepsilon}{(1 - \xi^i)\varepsilon - 1}$, here depends on market share. As $\xi^i$ goes to zero the mark-up tends to $\mu = \frac{\varepsilon}{\varepsilon - 1}$ and the model nests the monopolistic competition case. Let’s define $\tilde{\varepsilon}^i = (1 - \xi^i)^{\varepsilon}$ : market power goes up when the demand elasticity for variety $i$, $\tilde{\varepsilon}^i$, decreases. In a symmetric equilibrium all firms charge the same price $p^i = p^*$ so that $p^e = (N)^{1 - \varepsilon}$. Therefore the expression for the market share becomes:

$$\xi = \frac{p^*}{p^c} = \frac{1}{N}$$ (20)

and the optimal price can be written as follows:

$$p^*_t = \frac{(N_t - 1)^{\varepsilon} W_t}{(N_t - 1)^{\varepsilon} - 1} z_t$$ (21)

Hence the optimal mark-up depends countercyclically on the number of firms operating in the economy at time $t$. As the number of firms increases, the degree of competition increases and the mark-up decreases. Importantly notice that differently from Bilbiie et al. 2007 the mark-up depends on the number of firms even when firms are homogenous. The existence of a mark-up in this context is in fact related to the fact that firms engage in oligopolistic competition rather than to firms heterogeneity.

Let’s now examine the role of endogenous mark-up variations for business cycle fluctuations in a competitive equilibrium. By rewriting condition (21) in terms of CPI index and by imposing labour market equilibrium, we obtain the following:

$$\mu(N_t, \varepsilon) = \frac{(N_t - 1)^{\varepsilon}}{(N_t - 1)^{\varepsilon} - 1} \frac{1}{N_t^{1 - \varepsilon}} = \psi r^e c_t$$ (22)
Equation (22) shows that the mark-up, \( \mu(N_t, \varepsilon) \), represents a wedge on the condition equalizing the marginal rate of substitution between consumption and leisure and labour productivity. This has two consequences. First, in this environment, any shock transmitted to the economy is amplified by endogenous mark-up variations in a way that tend to amplify employment variability. This is the sense in which Rotemberg and Woodford (1995) propose endogenous mark-up variations as a possible solution to the Dunlop-Tarshis puzzle. Second, the presence of this time-varying wedge renders employment fluctuations inefficient.

2.2 Firms’ Optimization under Sticky Prices

Let’s now assume that firms face quadratic costs on price adjustment, \( \varkappa_t \), as in Rotemberg 1982:

\[
\varkappa_t = \frac{\kappa}{2} \left( \frac{p_t}{p_{t-1}} - 1 \right)^2 \frac{p_t}{p_{t-1}} y_t
\]

where \( y_t = c_t + g_t \), with \( g_t \) being exogenous government expenditure, and where \( \psi \) can be thought as the sluggishness in the price adjustment process: as \( \psi \to 0 \) prices become flexible. Such cost induce a sticky price adjustment.

It is important to recall that under price stickiness and even in absence of strategic pricing, mark-up vary in response to shocks. Consider a shock that increases demand: as prices adjust slowly firms must adjust mark-ups to make the optimality condition to hold. In the context of the present model mark-ups vary in response to shocks because of both price stickiness and oligopolistic competition. As shown in Jiaimovich and Floteotto 2004 the amplification obtained through countercyclical mark-ups movements can reproduce the volatilities of output and employment found in the data without requiring implausibly large productivity shocks. Let’s define the relative price as:

\[
\omega_t = \left( \frac{p_t}{p_{t-1}} \right)
\]

Firms’ per period profits in this case are given by:

\[
d_t = \Pi(N_t, \varepsilon) = \omega_t y_t - \frac{w_t}{z_t} - \frac{\kappa}{2} \left( \frac{p_t}{p_{t-1}} - 1 \right)^2 \omega_t y_t
\]

where:

\[
y_t = (\omega_t)^{-\varepsilon} (c_t + g_t + \varkappa_t)
\]
Firms in this case choose a sequence of prices, \( \{p_t^i\}_{t=0}^{\infty} \), to maximize the expected future sum of discounted profits:

\[
E_0 \{ \sum_{s=1}^{\infty} (\beta \rho)^{s-1} \left( \frac{c_t^s + 1}{c_t} \right)^{-1} d_t \} 
\]

Firms’ optimization delivers the following optimality conditions:

\[
p_i^t = \frac{\mu_i^t w_t}{\pi_t^i} \quad (27)
\]

\[
\mu_i^t = \frac{\bar{\varepsilon}_i^t}{(\bar{\varepsilon}_i^t - 1)(1 - \frac{\kappa}{2} (\pi_t^i - 1)^2 + \kappa \pi_t^i (\pi_t^i - 1) - \Gamma_t)} \quad (28)
\]

with:

\[
\Gamma_t = \kappa E_t \{ \beta \rho (\frac{c_t + 1}{c_t})^{-1} \pi_t (\pi_t^i - 1) \frac{y_{t+1}^i}{y_t^i} \}
\]

In a symmetric equilibrium \( p_t^i = p_t^1 = \pi_t^1 = \pi_t, \mu_t^1 = \mu_t, y_t^1 = y_t^d, \omega_t = (\frac{p_t}{p_t^1}) = (N_t)^{-1} \) and \( \bar{\varepsilon}_t^i = \frac{1 - N_t}{N_t} \). From equation (28) inflation depends on the number of firms in the market through the demand elasticity. Bilbiie, Ghironi and Melitz 2007b have shown that the dependence of inflation on the number of firms, a state variable, helps in explaining part of observed inflation persistence.

### 2.3 Firms’ profits and asset values

After imposing a symmetric equilibrium and after substituting for the aggregate demand relation, \( y_t^d = \omega_t (c_t + g_t + \varepsilon_t) \), the labour demand relation, \( (\frac{p_t}{p_t^1}) = \omega_t = \mu_t \frac{w_t}{c_t} \), and the resource constraint, \( y_t^d = c_t + \varepsilon_t = N_t \omega_t y_t^d = N_t z_t h_t \), firms profits read as follows:

\[
d_t = \Pi(N_t, \varepsilon_t) = (1 - \frac{1}{\mu_t} - \frac{\kappa}{2} (\pi_t - 1)^2) \frac{y_t^d}{N_t} \quad (29)
\]

Given firms’ profits, firms’ asset value is obtained using equation (11). Recall that the number of firms in the economy evolves according to \( N_{t+1} = (1 - \theta)(N_t + N_{e,t}) \). The number of firms that enter each period the oligopolistic market, \( N_{e,t} \), is determined through an entry condition. To enter the market firms have to pay a fixed entry costs, \( f_{e,t} \), which can be written as \( z_t h_{e,t} \), where \( h_{e,t} \) is the number of hours employed to set up new firms. The entry condition equates the value of a firm to the entry cost and reads as follows:

\[
v_t = f_{e,t} \frac{w_t}{z_t} \quad (30)
\]
2.4 Aggregate Equilibrium Conditions

Aggregate output in this economy is given by:

\[ y_t^c = (1 - \frac{\kappa}{2} (\pi_t - 1)^2)(c_t + g_t) = N_t \omega_t y_t^d \]  \hspace{1cm} (31)

Aggregate accounting in this economy implies:

\[ c_t + g_t + N_{e,t} v_t = w_t n_t + N_t d_t \]  \hspace{1cm} (32)

Due to increasing returns to scale CPI and PPI inflation are different \( t \). It is useful to define the PPI/CPI ratio:

\[ \frac{1 + \pi_t}{1 + \pi_t^c} = \frac{\omega_t}{\omega_t - 1} \]  \hspace{1cm} (33)

Risk free bonds and firms’ share are in zero net supply. Finally the equilibrium in the labour market implies:

\[ N_t h_{d,t} + N_{e,t} h_{e,t} = h_t \]  \hspace{1cm} (34)

and states that total labour supply, \( h_t \), must equalize the number of hours demanded to open new firms plus the number of hours demanded to run production in the existing firms.

3 Optimal Ramsey Policy

The optimal policy plan is determined by a monetary authority that maximizes the discounted sum of utilities of all agents given the constraints of the competitive economy. The next task is to select the relations that represent the relevant constraints in the planner’s optimal policy problem. This amounts to describing the competitive equilibrium in terms of a minimal set of relations involving only real allocations, in the spirit of the primal approach described in Lucas and Stokey 1983. There is a fundamental difference, though, between that classic approach and the one followed here, which stems from the impossibility, in the presence of sticky prices and other frictions, of reducing the planner’s problem to a maximization only subject to a single implementability constraint. Khan, King and Wolman 2003 adopt a similar structure to analyze optimal monetary policy in a closed economy with market power, price stickiness and monetary frictions, while Schmitt-Grohe and Uribe 2002 to analyze a problem of joint determination of optimal monetary and fiscal policy.
The consumers’ optimality conditions for this economy can be summarized as follows:

\[ u_{c,t} \omega_t \frac{e_{t}}{\mu_t} = \beta(1 - \theta)E_t \left( u_{c,t+1} \left( \frac{e_{t+1}}{\mu_{t+1}} + (1 - \frac{1}{\mu_{t+1}} - \frac{\kappa}{2}(\pi_{t+1} - 1)^2) \left( \frac{c_{t+1} + g_{t+1}}{N_{t+1}} \right) \right) \right) \]  

\[ \frac{u_{h,t}}{u_{c,t}} = \omega_t \frac{z_t}{\mu_t} \]  

\[ \mu_t = \frac{\varepsilon_t}{(\varepsilon_t - 1)(1 - \frac{\kappa}{2}(\pi_t - 1)^2 + \kappa \pi_t(\pi_t - 1) - \kappa E_t \left( \beta \rho \left( \frac{\pi_{t+1}}{\pi_t} \right) \right)^{-1} \pi_{t+1}(\pi_t - 1) \frac{zt}{\mu_t}} \]  

\[ p_t = \mu_t \frac{w_t}{z_t} \]  

\[ N_{t+1} = (1 - \theta)(N_t + N_{c,t}) \]  

(1 - \frac{\kappa}{2}(\pi_t - 1)^2)(c_t + g_t) + \omega_t N_{c,t} e_{t} = \omega_t z_t h_t \]

where: \( \omega_t = (N_t)^{-1} \) and \( \frac{1 + \pi_t}{1 + \pi_t} = \frac{\omega_t}{\omega_{t-1}} \). Condition (35) equalizes the intra-temporal allocation of consumption to the marginal rate of transformation and is obtained by merging equation (9), (29), (30), and the labour demand \( \left( \frac{p_t}{w_t} \right) = \omega_t = \mu_t \frac{w_t}{z_t} \). Condition (36) equalizes the marginal rate of substitution between labour and consumption to the marginal rate of transformation and is obtained by merging labour supply, given by equation (10), and labour demand, given by \( \left( \frac{p_t}{w_t} \right) = \omega_t = \mu_t \frac{w_t}{z_t} \). Equations (38) and (37) together give the Phillips curve. Finally equations (39) and (40) are the evolution of the number of firms and the resource constraint. The latter is obtained by merging the accounting condition, (32), the dividends equation, (29), the labour market equilibrium conditions given by (34) and the condition \( \omega_t = \mu_t \frac{w_t}{z_t} \). Notice that the government resource constraints is not included among the equilibrium conditions as fiscal policy is passive due to the absence of distortionary taxation.

**Definition 2.** Let \( \Lambda^n_t = \{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}\}_{t=0}^{\infty} \) represent sequences of Lagrange multipliers on the constraints, (35),(36),(37), (38), (39),(40). Then for given stochastic process \( \{a_t, g_t\}_{t=0}^{\infty} \), plans for the control variables \( \Xi^n_t = \{c_t, h_t, w_t, N_t, N_{c,t}, \mu_t, \pi_t, \omega_t, p_t\}_{t=0}^{\infty} \) and for the co-state variables \( \Lambda^n_t = \{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}\}_{t=0}^{\infty} \) represent a first best constrained allocation if they solve the following maximization problem:
Min\{\lambda_{1,t}\}_{t=0}^{\infty} Max\{\lambda_{3,t}\}_{t=0}^{\infty} E_0 \left( \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \right)

subject to (35), (36), (37), (38), (39), (40).

3.0.1 Non-recursivity and Initial Conditions

As a result of constraints (35) and (37) exhibiting future expectations of control variables, the maximization problem as spelled out in (41) is intrinsically non-recursive. As first emphasized in Kydland and Prescott 1980, and then developed by Marcet and Marimon 1999, a formal way to rewrite the same problem in a recursive stationary form is to enlarge the planner’s state space with additional (pseudo) co-state variables. Such variables, that I denote $\chi_{1,t}$ and $\chi_{3,t}$ for (35) and (37) respectively, bear the crucial meaning of tracking, along the dynamics, the value to the planner of committing to the pre-announced policy plan. Another aspect concerns the specification of the law of motion of these Lagrange multipliers. For in this case both constraints feature a simple one period expectation, the same co-state variables have to obey the laws of motion:

\begin{align*}
\chi_{1,t+1} &= \lambda_{1,t} \\
\chi_{3,t+1} &= \lambda_{3,t}
\end{align*}

Using the new co-state variable so far described the state space of the Ramsey allocation is amplified as follows $\{a_t, \chi_{1,t}, \chi_{3,t}\}_{t=0}^{\infty}$ and a new saddle point problem is derived which is recursive in the new state space. Consistently with a timeless perspective, the values of the three co-state variables are set at time zero equal to their solution in the steady state.

3.1 Planner Solution and The Role of Wedges

Before turning to the quantitative properties of the Ramsey plan, both in the long run and in response to shocks, it is instructive to consider the comparison between the planner solution and the competitive economy. The planner solution is obtained by maximizing agents’ utility under the resource constraint, the equation for the evolution of the number of firms, and by assuming flexible prices. Such plan delivers the pareto optimal allocation. The comparison between our distorted
competitive economy allocation and the efficiency conditions associated with the planner solution will highlight the role of distortions and time-varying wedges in this model. The planner problem is this model can be described as follows:

$$\max_{\{c_t, h_t, N_t, N_{e,t}\}_{t=0}^\infty} E_0 \left\{ \sum_{t=0}^\infty \beta^t u(c_t, h_t) \right\}$$

subject to (39) and:

$$c_t + g_t + \omega_t N_{e,t} f_{e,t} = \omega_t z_t h_t$$

Let's define as $\mu_t$ and $\phi_t$ the Lagrange multipliers associated, respectively, with constraints (44) and (39). After taking first order conditions of the planner problem and after some manipulations, we get the following pareto efficiency conditions:

$$\frac{u_{c,t}}{u_{h,t}} = \omega_t z_t$$

$$u_{c,t} \omega_t f_{e,t} = \beta(1 - \varphi) E_t \left\{ u_{c,t+1}(\omega_{t+1} f_{e,t+1} + \epsilon(N_t) \frac{c_{t+1} + g_{t+1}}{N_{t+1}}) \right\}$$

where $\epsilon(N_t) = (1 - \frac{1}{\mu_{t+1}})$. Condition (45) is a static efficiency condition equalizing the marginal rate of substitution between consumption and labour to the marginal rate of trasformation. Notice that the component $\omega_t = (N_t)^{\frac{1}{\epsilon}}$ does not represent a distortion leading to pareto inefficiency: in presence of increasing returns to scale the marginal rate of transformation efficiently increases with the number of firms. Condition (46) is an intra-temporal efficiency condition equalizing the marginal rate of substitution between consumption at two different dates with the marginal rate of transformation. Let's now compare the pareto efficient solution with the optimality conditions describing the competitive economy under flexible prices. The latter can be summarized as follows:

$$u_{c,t} \omega_t \frac{f_{e,t}}{\mu_t} = \beta(1 - \varphi) E_t \left\{ u_{c,t+1}(\omega_{t+1} \frac{f_{e,t+1}}{\mu_{t+1}} + (1 - \frac{1}{\mu_{t+1}}) \frac{c_{t+1} + g_{t+1}}{N_{t+1}}) \right\}$$

$$-\frac{u_{h,t}}{u_{c,t}} = \omega_t \frac{z_t}{\mu_t}$$

To equalize equations (46) and (45) to equations (47) and (48) one needs to set $\mu_t = 1$ and $E_t(\frac{\mu_{t+1}}{\mu_t}) = 1$. This implies that firms’ monopolistic rents distort the competitive economy compared to the pareto efficient allocation. In particular $\mu_t$ act as a time-varying wedge which distorts the
condition equalizing the marginal rate of substitutions between consumption and labour and the marginal rate of transformation. This renders employment fluctuations pareto inefficient. As firms pay a fixed cost of entry, they are allowed to extract rents. This rent seeking behavior reduces aggregate demand and, consequently, reduces labour demand. The second component, \( E_t \left( \frac{\mu_{t+1}}{\mu_t} \right) \), is a inter-temporal time-varying wedge which distorts the allocation of consumption between two different dates. The monopolistic profits of new entrants in the market reduce at the margin the value of existing and newly created firms. Hence, consumers investing in new firms see their marginal profits reduced to the extent that new entrants acquire monopolistic rents.

Under sticky prices the monetary authority can affect the real allocation and abate the welfare costs of wedges. As the monetary authority is endowed with a single instrument and faces multiple wedges, it will have to trade-off among them. The mechanism through which the monetary policy affects wedges runs as follows. First, notice that the competitive economy allocation under sticky prices, described by equations (35),(36), (37), (38), (39),(40), contains the competitive economy allocation under flexible prices described by equations, (47), (48), (44) and (39). Indeed it is always possible to reach the flexible price allocation by setting \( \pi_t = 1, \pi^c_t = 1 \). However under sticky prices the monetary authority can reach a set of allocation which are pareto superior to the one under flexible prices. The monetary authority can indeed use CPI inflation to increase consumption demand, which in turn increases employment and output. Consider for instance a positive technology shock. Under sticky prices the monetary authority can increase demand, by reducing inflation and mark-ups. This will increase firms’ profits and the number of entrants in the industry. The increase in the number of new firms will increase labour demand and will push employment toward the pareto efficient level. To highlight the transmission mechanism at work in this case, let’s consider the following entry condition:

\[
f_{c,t} \left( N_t \right) \frac{1}{\mu_t} = E_t \left\{ \sum_{s=t+1}^{\infty} (\beta (1 - \varphi))^{s-t} \left( \frac{c_s}{c_t} \right)^{-1} d_s \right\}
\]

For given entry cost, \( f_{c,t} \), an increase in demand can increase total firms discounted profits, \( d_s \), in the market, which in turn, increases the number of firms in the market, \( N_t \), and reduces mark-ups, \( \mu_t \). Recall, from condition (47), that a fall in firms’ mark-ups induces an increase in employment.
3.2 Optimal Policy in the Long Run

Optimal monetary policy in the long run amounts at setting the rate of inflation to which the policy maker would like to converge. To develop an analogy with the Ramsey-Cass-Koopmans model, this amounts to computing the *modified golden rule* steady state. To determine the optimal inflation rate in the long run, one needs to solve the first order conditions of the Ramsey plan in the steady-state. In particular the first order condition with respect to inflation is sufficient to determine the long run optimal level of inflation. Taking first order conditions with respect to inflation of the plan described in (41) delivers:

\[
0 = \lambda_{4,t} \left[ \tilde{\varepsilon}_t (\tilde{\varepsilon}_t - 1)(-\kappa (\pi_t - 1) + \kappa (2\pi_t - 1)) \right] \\
- \lambda_{4,t+1} \left[ \tilde{\varepsilon}_t (\tilde{\varepsilon}_t - 1)(-\kappa (2\pi_t - 1))(\frac{c_t}{c_{t-1}})^{-1} \frac{y_t}{y_{t-1}} \right] - \lambda_{6,t}\kappa (\pi_t - 1)
\]  

(49)

After imposing steady state the condition above becomes:

\[
\lambda_{4} \left[ \tilde{\varepsilon}_t (\tilde{\varepsilon}_t - 1)(-\kappa (\pi - 1)) \right] - \lambda_{6}\kappa (\pi - 1) = 0
\]  

(50)

Since \(\lambda_4 > 0, \lambda_6 > 0\) (the constraints must hold with equality), \(\kappa > 0\) (I am not imposing *a priori* that the steady-state coincides with the flexible price allocation), equation (50) implies \(\pi = 1\) or a zero average (net) inflation rate. Recall, by equations (33), that \(\pi = 1\) implies \(\pi^c = 1\). The intuition for this result is simple. Under commitment, the planner cannot resort to ex-post inflation as a device for eliminating market inefficiencies. Hence the planner chooses the inflation rate that allows to minimize the cost of adjusting prices, \(\frac{\kappa}{2} (\pi_t - 1)^2\).

3.3 Optimal Response to Shocks

Let’s now analyze the dynamic properties of the Ramsey plan in a calibrated version of the model. Technically I compute the stationary allocations that characterize the deterministic steady state of the first order conditions to the Ramsey plan. I then compute a second order approximation of the respective policy functions in the neighborhood of the same steady state. This amounts to implicitly assuming that the economy has been evolving and policy been conducted around such a steady already for a long period of time. Calibration is set as follows:
Preferences. The discount factor, $\beta$, is set to 0.99, so that the annual interest rate is equal to 4 percent. Both the elasticity of consumption, $\sigma$, and the Frisch elasticity, $\phi$, are normalized to one, while the elasticity of labour supply, $\tau$, is set to 3. Results are robust to alternative parameters.

Technology. Following Bilbiie, Ghironi and Melitz 2007 the elasticity of product variety, $\varepsilon$, is set to 3.8 and the firms’ destruction rate, $\rho$, to 0.025. Finally the adjustment cost parameter, $\kappa$, is set to 70. This parameter is varied in the simulations to test robustness.

Shocks. The model is simulated under productivity and government expenditure shocks which follow $AR(1)$ processes. Persistence and volatility of the productivity shocks are calibrated as in the RBC literature, $\rho_a = 0.95$ and $\sigma_a = 0.008$. government consumption evolves according to the following exogenous process, \( \ln \left( \frac{g_t}{y_t} \right) = \rho_g \ln \left( \frac{g_{t-1}}{y_{t-1}} \right) + \epsilon_{t}^g \), where the steady-state share of government consumption, $g$, is set so that $\frac{g}{y} = 0.25$ and $\epsilon_{t}^g$ is an i.i.d. shock with standard deviation $\sigma_g$. Empirical evidence for the US in Perotti 2004 suggests $\sigma_g = 0.0074$ and $\rho_g = 0.9$.

Figure 1 shows impulse response functions to a one percent positive productivity shock for a number of selected variables. Due to the increase in productivity output increases. CPI inflation falls and deviates significantly from zero. Optimal monetary policy is pro-cyclical since under sticky prices an decrease in inflation by boosting demand reduces the mark-up. The reduction in mark-up boosts consumption demand, which in turn raises employment demand and the number of firms.

In response to government expenditure shocks, figure 2, optimal monetary policy implies a fall in consumption and in the price level. This is consistent with the findings of Khan, King and Wolman 2003\textsuperscript{8}. In order to generate a fall in consumption the government increases the nominal interest rate and this also implies a fall in the price level. Overall deviations from price stability are rather small for this shocks alone, a result in line with previous literature (see Khan, King and Wolman 2003, Schmitt-Grohe and Uribe 2004, Faia 2009).

Finally, figure 3 shows that the optimal volatility of inflation, under both productivity and government expenditure shocks, decreases when the elasticity of demand increases (the mark-up decreases). This is so since higher elasticity implies that the market is more competitive and firms’ monopolistic rents are lower. The lower the mark-up, the higher the aggregate demand and the

\textsuperscript{8}They argue that the government will want to have less consumption when government purchases are high since this makes the contingent claims value of the public spending high, making it easier to satisfy monopoly producers. This argument is valid when the utility of the representative agent is separable so that the price of the state contingent security only depends on consumption.
closer is the economy to the pareto efficient allocation. This implies that with higher elasticity the monetary authority has lower incentive toward active policies and the use of state contingent inflation subsidies.

4 Welfare Analysis Under Alternative Rules

Most central banks follow explicitly or implicitly Taylor type rules, hence a comprehensive analysis of the role of monetary policy in a model with product market frictions requires a welfare comparison across different monetary policy rules. For this comparison I consider the following class of rules:

\[
\ln \left( \frac{1 + r^n_t}{1 + r^n} \right) = (1 - \phi_r) \left( \phi_x \ln \left( \frac{\pi_t}{\pi} \right) + \phi_y \ln \left( \frac{y_t}{y} \right) + \phi_x \ln \left( \frac{x_t}{x} \right) \right) + \phi_r \ln \left( \frac{1 + r^n_{t-1}}{1 + r^n} \right)
\]

where \(\phi_x\) represents a response to CPI inflation rate, \(\phi_y\) the response to output and \(\phi_r\) the degree of interest rate smoothing. I therefore consider three alternative targets for the variable \(x_t\): CPI inflation, mark-up and asset prices. For the three variables the parameters in the monetary policy rules are respectively, \(\phi_{cpi}, \phi_{mua}, \phi_{ap}\). The literature on optimal policy has stressed the role of producer price inflation for price stability policies, on the other side central bank usually target consumer price inflation. As in this model the two inflation rates follow different dynamics and are affected by different wedges, it is appropriate to assess whether a welfare based ranking of rules would assign different weights to the two. Since the main wedge in this economy is given by monopolistic rents, mark-up is also a natural candidate as monetary policy target. Finally, the reason for considering asset prices stems from the fact that firms’ rents affect the evolution of asset prices, therefore distort consumption allocation between two different dates. The monetary authority might therefore want to smooth this intra-temporal wedge.

Before turning to the results some observations on the computation of welfare in this context are in order. First, one cannot safely rely on standard first order approximation methods to compare the relative welfare associated to each monetary policy arrangement. Indeed in an economy with a distorted steady state stochastic volatility affects both first and second moments of those variables

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9 All coefficients apart the ones on inflation rates are divided by four to make the rule compatible with a standard Taylor rule at annual frequencies.
Table 1: Welfare comparison of alternative monetary policy rules.

<table>
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<th>$\phi_\pi$</th>
<th>$\phi_y$</th>
<th>$\phi_r$</th>
<th>$\phi_{cpi}$</th>
<th>$\phi_{mu}$</th>
<th>$\phi_{up}$</th>
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<tr>
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<td>0</td>
<td>0.0118</td>
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</table>

that are critical for welfare. Hence policy arrangements can be correctly ranked only by resorting to a higher order approximation of the policy functions\(^{10}\). Additionally one needs to focus on the conditional expected discounted utility of the representative agent. This allows to account for the transitional effects from the deterministic to the different stochastic steady states respectively implied by each alternative policy rule\(^{11}\). Define $\Omega$ as the fraction of household’s consumption that would be needed to equate conditional welfare $W_0$ under a generic interest rate policy to the level of welfare $\tilde{W}_0$ implied by the optimal rule. Hence $\Omega$ should satisfy the following equation:

$$W_{0,\Omega} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U((1 + \Omega)c_t, n_t) \right\} = \tilde{W}_0$$

Under a given specification of utility one can solve for $\Omega$ and obtain:

$$\Omega = \exp \left\{ \left( \tilde{W}_0 - W_0 \right) (1 - \beta) \right\} - 1$$

The model economy is simulated under two sources of aggregate uncertainty, productivity and government consumption shocks. The table below report results\(^{12}\):

First, the best rule is the one that targets mark-ups alongside with PPI inflation and with an interest rate smoothing of 0.9. This rule allows the policy maker top strike an optimal balance

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\(^{10}\)See Kim and Kim (2003) for an analysis of the inaccuracy of welfare calculations based on log-linear approximations in dynamic open economies.

\(^{11}\)See Kim and Levin (2004) for a detailed analysis on this point.

\(^{12}\)Welfare is computed as percentage costs with respect to the rule that maximizes agents’ utility.
between reducing the cost of adjusting prices and smoothing inefficient employment fluctuations induced by monopolistic rents. Second, it appears to be the case that targeting output is always welfare detrimental. This result, consistent with the previous literature (see for instance Schmitt-Grohe and Uribe 2004c, Faia and Monacelli 2007), is explained by the fact that optimality prescribes responding to a measure of the output gap based on the best forecast for potential output, something not available in this context. Third, interest rate smoothing seems always welfare improving. Responding to the interest rate allows to smooth inefficient fluctuations in asset prices, which in this context arise because of monopolistic rents. Finally responding to asset prices is welfare improving compared to a standard Taylor rule that responds to PPI inflation and output or compared to strict inflation targeting.

5 Conclusions

This paper studies optimal monetary policy in a DSGE model with sticky prices and product market frictions. In this model firms’ monopolistic rents induce both intra-temporal and intertemporal time-varying wedges which induce inefficient fluctuations of employment and consumption. The monetary authority faces a trade-off between reducing the cost of adjusting prices and smoothing inefficient fluctuations, which is resolved by using consumer price inflation as a state contingent sale subsidy. An analysis of the welfare gains of alternative rules show that targeting mark-ups, asset prices or other indicators of the real economic activity might improve upon a strict inflation targeting.

Future research should explore the role of alternative oligopolistic settings (Cournot games, collusions, cartels) in helping to reconcile standard DSGE models with stylized facts about inflation and prices and in understanding the monetary policy transmission mechanism.
References


Figure 1: Impulse response of selected variables under Ramsey plan to productivity shocks.
Figure 2: Impulse response of selected variables under Ramsey plan to government expenditure shocks.
Figure 3: Volatility of (annual) consumer price inflation in percentage deviations from the steady state for different values of the elasticity of demand.