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Ron Smith and M. Hashem Pesaran

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Monetary Policy Transmission and the Phillips Curve in a Global Context

M. Hashem Pesaran
Faculty of Economics and CIMF, Cambridge University

Ron P. Smith
Birkbeck College, London

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Abstract

The standard derivation of a Phillips curve from a DSGE model requires that all variables are measured as deviations from their steady states. But in practice this is not done. The steady state for output is estimated by some statistical procedure, such as the HP filter, and the steady state for other variables, including inflation, is treated as a constant. This is inconsistent with the theory and raises econometric problems since inflation, for instance, is a very persistent series. We argue that the natural definition of the steady state is the long-horizon forecast and estimate these permanent components from a cointegrating VAR that takes account of global interactions. This estimate of the steady state will reflect any long-run theoretical relationships embodied in the cointegrating vectors. We then estimate Phillips Curves and other standard monetary transmission equations using deviations from the steady states on US data. This is both consistent with the theory and uses the relevant economic information about steady states.

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This is a very preliminary and incomplete draft, please do not quote.

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1 Introduction

The standard macroeconomic model, which is widely used to study the monetary policy transmission mechanism, has three 'structural' equations derived from a DSGE model of optimising agents. These equations are for the output gap (an IS curve), inflation (a Phillips curve) and short interest rates (a Taylor rule). This model has a number of well known shortcomings. It either ignores the rest of the world or treats it as strictly exogenous to obtain the full rational expectations solution of the model. Apart from the short interest rate there is no treatment of the financial system, but the financial system is likely to be an important transmission mechanism both for monetary policy and international shocks. In particular it is the long rather than the short interest rate that is likely to be important in the IS curve, so some term structure relationship is needed, not least for judging the effects of policy. The model provides no explanation of the permanent components, such as the natural rate of output, which are provided by statistical trends such as Hodrick Prescott type filters.

To match the observed dynamics of output and inflation the dynamic stochastic general equilibrium, DSGE, model is also often augmented with ad hoc lags or frictions, since the rational expectations solution by construction eliminates intrinsic dynamics. This makes the interpretation of impulse response functions and the analysis of learning and credibility difficult. These problems interact. For instance, while it might be possible to agree an ordering to different variables within the same country to identify impulse response functions, agreeing an ordering of the same variables in different countries is likely to be impossible.

In this paper we will focus on the role of the Phillips curve. In the almost 50 years since the publication of Phillips (1958) the Phillips Curve has been a central relationship in macroeconomics\(^1\). Although central, the nature of the relationship has been problematic and Section 2 discusses the evolution of the interpretation of the Phillips Curve. A common contemporary interpretation is that the estimated Phillips curve is a structural relationship derived from a DSGE, model. The estimated coefficients are then known functions of some deep parameters. In Section 3, we argue that the empirical implementation of this interpretation is flawed because it requires that all variables are measured as deviations from equilibrium or steady state, both for theoretical and econometric reasons. This is not usually done, partly because of the difficulty of defining the equilibrium or steady state. In Section 4, we suggest a way of measuring the steady state, as a long horizon forecast, and show how this can be done in the context of a particular model, the Global VAR, GVAR, of Dees, di Mauro, Pesaran and Smith, DdPS, (2007). Our argument is, however, more general than the application to this particular model. If economic models are informative about long-run steady states then such information about the steady state, whether it comes from a DSGE or VAR based model, should be used in estimation. Section 5, illustrates this approach, using deviations from steady states, in the estimation of the Phillips Curve and other equations of the

\(^1\)It is at least 50 years since there are a number of plausible precursors for a similar relationship, including Tinbergen.
standard monetary policy transmission model, using US data. Section 6 has some concluding comments.

2 The Evolution of the Phillips Curve

We briefly review the literature to illustrate the very wide range of issues that are raised by the interpretation of the Phillips Curve. In Phillips original article, the curve was a labour market relationship between wage inflation and unemployment; there was considerable emphasis on the non-linearity of the relationship; there was a distinction between the long-run pattern, that Phillips believed was measured by the curve and the short run cyclical loops around the curve\(^2\); and there was some discussion of the impact of cost shocks coming from import prices. Expected inflation was treated as a given constant, which seemed a reasonable assumption for the pre World War I gold standard data that he used. The curve was also treated as a demand relationship: excess demand in the labour market pushed up wages. With the assumption that supply was relatively inelastic given by labour force, it was assumed that most of the variations in unemployment would reflect demand and that these would be uncorrelated with the exogenous cost shocks, e.g. from import prices. This allowed estimation by OLS, though Phillips himself used graphical methods. Subsequently the addition of a couple of side relationships (Okun’s Law linking output and unemployment and treating price inflation as equal to wage inflation minus productivity growth) allowed the equation to be turned into a product market Phillips Curve, linking price inflation to the output gap. This has probably been the dominant form of the Phillips Curve. Again it was initially seen as an excess demand relationship within a Keynesian framework, with supply, potential output, being relatively inelastic. The relationship between labour market Phillips curves and product market Phillips curves has been a matter of continued interest. Sargan (1964) is an early study which had a substantial influence on applied econometrics and drew conclusions similar to New Keynesian Phillips curve about the importance of the share of wages in output or real marginal cost.

Phelps (1967) and Friedman (1968) emphasised the importance of inflation expectations and a natural rate of unemployment, giving a vertical long run Phillips curve: no long-run trade-off. Unemployment or the output gap only influenced the difference between actual and expected inflation. The issue of testing whether the coefficient of expected inflation was unity raised some difficult conceptual issues. This is most obvious when expected inflation is constant, then the coefficient is not identified without some further assumptions. One can only test that the coefficient of expected inflation is unity in conjunction with a particular auxiliary assumption about how expectations are formed. It has been a matter of dispute as to whether expectations should be regarded as adaptive, perhaps with some form of learning, or rational. There is also an issue as to

\(^2\)Although the Phillips Curve is now usually treated as a short-run relation, there is still controversy about whether it exists as a long-run relation, e.g. Schreiber and Wolters (2007).
whether the relevant expectations should refer to current or future expectations and the timing of the information set on which it was based. In a wage Phillips curve, the inflation rate during the period of the wage bargain is relevant, in an inflation Phillips Curve, it is less obvious. Together with the rational expectations hypothesis one can estimate the coefficient of expected inflation and test the cross-equation restrictions implied by rational expectations.

Lucas (1972) changed the interpretation of the relationship to a supply curve, the amount of output produced depended on the difference between actual and expected prices (inflation). In econometric terms the issue is whether inflation or output is regarded as the independent variable and the extent of the correlation of either or both with the error. There was a considerable literature explaining output by the money supply surprises that drove unexpected inflation. This literature terminated abruptly when it was appreciated that since output was a very persistent series, probably I(1), it could not be explained by surprises which by construction were white noise. Pesaran and Smith (1995) discuss this issue in detail. King and Watson (1994) discuss the effect of stochastic trends on estimation. There were also debates on how to measure the natural rate a point we return to, and whether other variables, overseas influences on costs or financial or monetary influences, should be included in the Phillips curve.

Recently, the Phillips Curve has tended to be regarded a structural equation (in that its coefficients are functions of deeper parameters) that emerges from a DSGE model. Such models are designed to capture the decision rules of some representative agent. Of course, because of aggregation over heterogeneous agents the equation of the representative agent will not look like that of any particular agent and will depend on distributional features, e.g. variances. Aggregation over labour markets with different degrees of tightness is also likely to induce non-linearity. In the DSGE context the Phillips curve, making inflation as a function of the output gap, was one equation of a new consensus macro model, along with an IS curve and a Taylor Rule. As noted above, an important issue in this debate was whether the independent variable should be a measure of the output gap or marginal costs as measured by the labour share of wages. There are also issues in estimation, in particular the econometric properties of estimates of the forward looking component of expected inflation.

3 DSGE based Phillips Curves

We will focus on the hybrid New Keynesian Phillips curve, e.g. Gali et al. (2005 p1108). This is derived from the solution of a DSGE model, which is obtained by log linearising it around its steady states. Thus all variables are expressed as deviations from their steady states. However, in empirical work this is never done. The steady states are assumed either to be constants or, for trended variables like output, the steady state is measured by a statistical procedure such as the HP filter. The standard procedure thus does not use any economic information about the steady state and this is likely to produce misspecification of the estimated equations. Below, we argue that for any particular model it
is feasible to estimate steady states and thus measure the variables in terms of deviations from steady states. For output, statistical filters, like the HP filter, are unlikely to provide a good estimate of the steady state, since it is not based on any economic theory and does not make use of the economic relations that might under the steady state values at a given point in time. Most statistical filters, like the HP filter, are two sided, using information about future values of the variables in calculation of the steady state values, rather than using the information available to agents at the time. This not only raises problems for forecasting with models using HP filtered data, it does not represent people’s judgement about equilibrium output at the time. This would be most noticeable if output dropped sharply because of an unpredicted disaster such as a hurricane, tsunami or a terrorist attack. The HP filter would show equilibrium output dropping smoothly before the unpredicted crisis hit. The HP filter may also not remove the unit root effectively and suffers other problems discussed by Harvey and Jaeger (1993). For inflation, similar problems arise when its steady state is treated as a constant, which implicitly treats it as I(0) though it is very persistent. This is a problem for GMM estimation, which is commonly used, since this requires that the variables are stationary, e.g. Hall (2005). Persistent variables also cause problems for inference, e.g. Li (2007). Since the steady state for a variable has to have the same low frequency characteristics as the variable, there is no doubt that deviations from the steady states would be stationary. The same argument applies to other variables included in the model.

If it is believed that economics is informative about long-run steady states, then this information should be used in estimation rather than being ignored. For instance, were steady state money supply growth to change, steady state inflation would change. If we are to take theory seriously we have to think seriously about the steady states. The relevant steady state for economic agents is the long-horizon forecast that they would make for the variables: their estimate of what the system is adjusting to in the absence of further shocks. Given a particular model, this is a well defined concept. We will illustrate it with one particular model, but our main point is that a theoretically coherent method of estimating the Phillips curve, or other structural relations from a DSGE, also requires a coherent treatment of the steady states.

The models we shall use are the country specific models of the Global Var of DdPS. Pesaran and Smith (2006) discuss how this can be interpreted as the solution of a rational expectations DSGE model, without the short run restrictions imposed by the DSGE model. The long run restrictions can be imposed on the cointegrating vectors. The short run restrictions are on deviations from the steady state. It is these deviations from steady state that are used below in estimating a short-run Phillips curve and other equations of the standard monetary transmission process. One could also ask whether there was a long-run Phillips curve, e.g. cointegration between steady state inflation and steady state unemployment, but we do not pursue that issue here. The proposed approach to the estimation of a short run Phillips curve has the advantage that it combines the long-run information in the cointegrating VAR with the more ‘structural’ estimation of the short run relations.
4 Steady States and Model

The approach we use to derive the long-horizon forecasts follows Garratt, Robertson and Wright, GRW, (2006). This is a form of multivariate Beveridge-Nelson (1982) decomposition, which has the advantage that it can be expressed directly in terms of the observables. If there is no cointegration, the GRW decomposition will be identical to the multivariate Beveridge-Nelson decomposition. If there is cointegration, it differs from the standard Beveridge-Nelson decomposition. Suppose that we have a vector $z_t$, this can be decomposed into permanent ($z_t^P$) and transitory (or cyclical, $z_t^C$) components, $z_t = z_t^P + z_t^C$, where the permanent component has a deterministic ($z_t^{DP}$) and a stochastic ($z_t^{SP}$) component, $z_t^P = z_t^{DP} + z_t^{SP}$. The GRW estimate of the permanent component is the long horizon forecast for $z_t$ less the deterministic component.

$$z_t^{DP} = g_0 + g_1 t$$

$$z_t^P = \lim_{h \to \infty} E_t(z_{t+h} - z_t^{DP})$$

$$z_t^P = \lim_{h \to \infty} E_t(z_{t+h} - g_1 h).$$

Our estimate of the steady state will then be $z_t^P$. The cyclical component, representing deviation from the steady state is then given by $\tilde{z}_t = z_t - z_t^P$. If there is a stochastic trend, any permanent shocks will be embodied in the steady state as they are realized. If there are cointegrating relations, these can reflect the theoretical long-run relations embodied in the cointegrating vectors. For any given model this long-horizon forecast can be calculated. The details of the calculations are described in Garratt et al. (2006, Ch. 10).

Note that unlike the HP filter, the GRW estimate of the permanent component does not use any information about future values of the variables.

Suppose there are a set of countries $i = 0, 1, 2, ..., N$, with country 0, say the US, as the numeraire country. The objective is to model a particular country, say $i$. As an example a second-order country-specific VARX*(2,2) model with deterministic trends can be written as

$$x_{it} = B_{i0}d_t + B_{i1}x_{i,t-1} + B_{i2}x_{i,t-2} + B_{i0}^*x_{it}^* + B_{i1}^*x_{i,t-1}^* + B_{i2}^*x_{i,t-2}^* + u_{it}, \quad (1)$$

where $x_{it}$ is a $k_i \times 1$ (usually five or six) vector of domestic variables, $x_{it}^*$ a $k_i^* \times 1$ vector of foreign variables specific to country $i$, and $d_t$ an $s \times 1$ vector of deterministic trends as well as observed common variables such as oil prices.

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3This is related to the argument in Lee and Nelson (2007) but they use the long-horizon forecasts as a measure of expectations rather than steady state and estimate them using a multivariate state space model.

4The calculation of the permanent and transitory components and the estimation in this paper were carried out in Microfit 5 (see Pesaran and Pesaran (2007)).

5It does use future information in the estimation of the parameters of the model, but this is likely to be a second order effect which could be dealt with by recursive estimation of the parameters.
typically \((1, t, p_i^r)\)', but could contain seasonal or break dummy variables. The unknown coefficients are the \(k_i \times s\) matrix \(B_{id}\), the \(k_i \times k_i\) matrices \(B_{11}\) and \(B_{12}\) of the lagged coefficients of domestic variables, \(B_{00}^*, B_{11}^*\) and \(B_{12}^*\) matrix of coefficients of foreign variables specific to country \(i\), and \(u_{it}\) is a \(k_i \times 1\) vector of idiosyncratic country-specific shocks, with \(E(u_{it}u_{jt}) = \Sigma_{ij} = \Sigma_{ji}^t\) and \(E(u_{it}u_{jt'}) = 0\), for all \(i, j, t \neq t'\).

The cointegrating VARX* can be written as a VECM

\[
\Delta x_{it} = B_{id}d_t - \Pi_iz_{i,t-1} + B_{00}^*\Delta x_{it}^* + \Gamma_i\Delta z_{i,t-1} + u_{it},
\]

where \(z_{it} = (x_{it}', x_{it}^*)'\) and

\[
\Pi_i = (I - B_{11} - B_{12}, -B_{00}^* - B_{11}^* - B_{12}^*),
\]

\[
\Gamma_i = (-B_{12}, -B_{12}^*).\]

To ensure that the same deterministics prevail under different rank restrictions on \(\Pi_i\), the coefficients of the determinsitic components, \(B_{id}\), need to be restricted so that they lie in the cointegrating space, namely we must have

\[
B_{id} = \Pi_i\gamma_i,
\]

where \(\gamma_i\) is an unrestricted \((k_i + k_i^*) \times s\) matrix of constant coefficients. Under these restrictions and assuming that \(\text{rank}(\Pi_i) = r_i < k_i + k_i^*\), we have \(\Pi_i = \alpha_i\beta_i^t\), where \(\beta_i\) is the \((k_i + k_i^*) \times r_i\) matrix of the cointegrating coefficients and

\[
\Delta x_{it} = -\alpha_i\beta_i^t(z_{i,t-1} - \gamma_i d_{t-1}) + B_{00}^*\Delta x_{it}^* + \Gamma_i\Delta z_{i,t-1} + \Pi_i\gamma_i\Delta d_t + u_{it}. \tag{2}
\]

The \(r_i\) error correction terms of the model can now be written as

\[
\xi_{it} = \beta_i'z_{it} - \beta_i'\gamma_i d_t = \beta_{ix}^t x_{it} + \beta_{ix}^s x_{it}^* + \gamma d_t.
\]

The \(\xi_{it}\) are mean zero \(r_i \times 1\) vector of disequilibrium deviations from the long run relationships.

In the GVAR these national systems are stacked to form a global VAR, here we will work just with the national systems by supplementing (2) by a marginal model for the \(x_{it}^*\) of the form

\[
\Delta x_{it}^* = \mu^* + \Gamma_i^*\Delta z_{i,t-1} + u_{it}^*. \tag{3}
\]

In the DdPS version \(x_{it}\) are a \(k_i \times 1\) subset of the logarithm of real output, \(y_{it}\), inflation, \(\pi_{it} = p_{it} - p_{it-1}\), where \(p_{it}\) is the logarithm of a general price index; the exchange rate variable, which is defined as \(e_{it} = p_{it}\), where \(e_{it}\) is the logarithm of the nominal exchange rate against the dollar; a short interest rate, \(r_{it}^S = 0.25\log(1+R_{it}^S/100)\), where \(R_{it}^S\) is a short interest rate measured in percent per annum; a long interest rate, \(r_{it}^L = 0.25\log(1+R_{it}^L/100)\); and the logarithm of real equity prices, \(q_{it}\), and the logarithm of real oil prices \(p_{it}^s\). The variables included in the different country models are not always the same, e.g. there are
no equity price or long-term interest rate data for some and the US model being the base country is treated differently as discussed in the next section.

The \( x^*_t \) are calculated as country specific trade weighted averages of the corresponding variables of the other countries, \( x^*_t = \sum_{j=0}^{N} w_{ij} x_{jt} \), where \( w_{ij} \) represents the share of country \( j \) in the trade (exports plus imports) of country \( i \) (with \( w_{ii} = 0 \)). The weak exogeneity of the \( x^*_t \) can be tested and imposed when accepted. The VARX* models can be estimated separately for each country, taking into account the possibility of cointegration between \( x^*_t \) and \( x^*_t \).

In this exercise, which is a preliminary investigation of this approach we calculate the steady states or permanent components from (2) using just identifying restrictions for the cointegrating vectors. The estimates of the steady states are invariant to the form of the just identifying restrictions chosen. We supplement the VARX* with the simple marginal model (3). Natural extensions are to impose over-identifying restrictions on (2) as in Dees, Holly, Pesaran and Smith (2007) or to derive the steady states from the Global VAR as a whole, which includes 26 countries including the Euro area.

5 A Short Run Phillips Curve for the US

We drop country subscripts to focus on a single country, the US. The US model includes \( y_t, \pi_t, r^{S}_t, r^{L}_t, q_t, \) and \( p^o_t \) as endogenous variables and \( y^*_t, \pi^*_t \) (reflecting the effective real exchange rate) and \( r^{S}_t \) as exogenous variables. When tested these are found to be weakly exogenous, but the foreign financial variables \( r^{S}_t, r^{L}_t, q_t \), perhaps not surprisingly, are not weakly exogenous and therefore are not included in the US model. The price of oil is regarded as endogenous to the US model. Using a VARX*(2,2) as in (2) the first two maximal eigenvalue statistics were 91.28 and 51.09. The 5% asymptotic critical values are 54.24 and 47.99 suggesting two cointegrating vectors. However the bootstrapped critical values are 64.18 and 55.78. Similarly the trace statistics were 221.46 and 130.18, with asymptotic critical values 158.01 and 122.96 and bootstrapped critical values of 187.27 and 145.14. Thus, it is unclear whether there is one or two cointegrating vectors. We will follow DdPS and assume two. The marginal model is of the same form as (3) except that the intercepts in the equations explaining the first differences of domestic and foreign inflation were set to zero.

The US VARX* allows for restricted trends in the cointegrating vectors, but their coefficients were not significantly different from zero, so the cotrending restrictions, that there are no trends in the cointegrating relations, are imposed. In calculating the permanent components, the growth rates, \( g_1 \) of all variables were set to zero except for domestic and foreign output, real equity prices, and the real effective exchange rate variable. None of the growth rates set to zero

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6 The data used in this exercise are available on the Journal of Applied Econometrics data archive (http://qed.queensu.ca/jae/).

7 The co-trending test statistic (distributed asymptotically as a Chi squared with two degrees of freedom) is 5.46 compared to its asymptotic 5% critical value of 5.99, and the bootstrapped critical value of 11.89.
were significantly different from zero. The transitory or cyclical component is only defined up to a linear transformation so we standardised it to have mean zero over the sample period. Therefore, we need not include an intercept in regressions involving the cyclical variables.

The correlation between the permanent and transitory components is not constrained to be either zero or one. The permanent component of inflation has roughly constant mean (because we have imposed that there is no deterministic trend) but the stochastic trend is noticeable. The permanent component of the inflation series varies over the range 0.005-0.014, whereas inflation itself varies over the wider range of 0 to 0.038. The correlation between inflation and its permanent component is 0.45, whilst the correlation between inflation and its transitory component is as much as 0.96. In contrast the correlation of the permanent and transitory components of inflation at 0.19 is rather low. Figure 1 gives the transitory component of inflation, $\pi_t$. Notice these are a quite different decomposition of US inflation, from that given by Lee and Nelson (2007). How one models the long-run has implications for how one estimates the steady state.

![Figure 1: Transitory (cyclical) component of inflation.](image)

For output all the correlations tend to be high because of common trends but the correlation between the GRW transitory component and the deviation from an HP trend is quite low, 0.38, so they are picking up different features and measuring different concepts. Figure 2 gives the transitory component of output measured both as the deviation from the GRW permanent component, $y_t$, and the deviation from a HP filter. The major difference between them may arise from the fact that the HP filter uses future information, unavailable to economic
agents, in determining the trend level. At some points the differences are very marked, in the early 1980s, the HP filter indicated that output was above trend, the GRW estimate that it was well below trend. There are also points where they move in opposite directions. The reason for the differences deserves further investigation, but for the moment we will use the GRW estimate since it best corresponds to the economic concept of steady state.

![Figure 2 GRW and HP estimates of the transitory component of output.](image)

Of course, all the variables will have steady states, not just output and inflation. The transitory component of long term interest rate is shown in Figure 3. The correlation between the cyclical component of output and the cyclical component of long interest rates is -0.74. In particular, it is noticeable that long term interest rates were well above their steady state values in the early 1980s, when output was below its steady state value. We present estimates below using deviations from steady states, which we would argue are the appropriate measures, both for reasons of economic and econometric theory. Of course, because we are using quite different measures of the variables, they will not be comparable with standard estimates.

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8 This also applies to less formal methods of determining the cycle, like the NBER reference cycle, which uses subsequent events to determine whether a turning point has occurred.
Before presenting results using our approach, we will first estimate a standard new Keynesian hybrid Phillips Curve using HP filtered output. This makes inflation a linear function of forward and lagged inflation and current and lagged output gap and takes the form:

\[
\pi_t = a_0 + a_1 \pi_{t+1} + a_2 \pi_{t-1} + c_1 \hat{y}_{t}^{HP} + c_2 \hat{y}_{t-1}^{HP} + u_t,
\]

where \( \pi_{t+1} = E(\pi_{t+1} \mid I_{t-1}) \), and \( \hat{y}_{t}^{HP} \) is the deviation of real output, \( y_t \), from an HP filtered trend. Throughout we treat all variables dated \( t \) and later as endogenous. As instruments for this equation we use a constant and \( \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, \pi_{t-4}, \hat{y}_{t-1}^{HP}, \hat{y}_{t-2}^{HP}, r_{t-1}^{s}, r_{t-1}^{L}, \Delta p_{t-1}, y_{t-1}^{*}, \pi_{t-1}^{*}, \pi_{t-2}^{*}. \) The estimates (with standard errors in parentheses) for 1980Q2 to 2003Q3\(^9\) are

\[
\begin{align*}
\pi_t &= 0.0003 + 0.47 \pi_{t+1}^{*} + 0.50 \pi_{t-1}^{*} \\
&\quad + 0.22 \hat{y}_{t}^{HP} - 0.17 \hat{y}_{t-1}^{HP} + res_{\pi_t} \\
&\quad (0.13) \quad (0.09) \quad (0.16) \quad (0.15)
\end{align*}
\]

\( \bar{R}_{\pi}^2 = 0.56, G\bar{R}_{\pi}^2 = 0.58, SER \times 100 = 0.406. \) \( \bar{R}_{\pi}^2 \) is the standard IV \( \bar{R}_{\pi}^2 \) using the actual values of the right hand side endogenous variables, \( G\bar{R}_{\pi}^2 \) is the generalised

\(^9\)The sample periods are the longest possible with the DdPS data.
\( \hat{R}^2 \) of Pesaran and Smith (1994) which uses the predicted values of the right hand side endogenous variables; \( SER \) is the standard error of the regression. The residual is indicated by \( res_{\pi t} \). The current and lagged values of output gap variables are not individually significant, although they have the correct signs. Replacing \( \hat{y}^H_{t+1} \) and \( \hat{y}^H_{t-1} \) in ( ) by \( \Delta \hat{y}^H_t \) does not alter the main conclusion that the output gaps measured using HP filter are not statistically significant in the New Keynesian Phillips Curve. Similar results, small and insignificant output slope coefficients, are quite common in the literature\(^{10}\). A Wald test of the hypothesis that the the forward and backward inflation coefficients sum to unity is not rejected with a p value of 0.772. Below are given various misspecification test statistics, with p values in brackets.

\[
\begin{array}{cccccc}
\text{Sargan} & \chi^2(9) & \text{SC} & \chi^2(4) & \text{FF} & \chi^2(1) & \text{N} & \chi^2(2) & \text{H} & \chi^2(1) \\
20.12 & 49.23 & 1.38 & 11.18 & 13.54 & . \\
[0.017] & [0.000] & [0.238] & [0.004] & [0.000] & \\
\end{array}
\]

The Sargan statistic (which is the same as Hansen’s J statistic) is a test of the over-identifying restrictions, used to check the validity of the instruments; SC is a LM test for fourth order residual serial correlation; FF is a RESET test for functional form; N is a test for normality of the errors; H is a RESET test for heteroskedasticity. The overidentifying restrictions are rejected, and there is evidence of strong negative residual serial correlation, non-normality and heteroskedasticity.

To allow for cost shocks we added the change in the logarithm of the real oil prices. The estimates for the same sample are

\[
\begin{align*}
\pi_t &= +0.0002 +0.56 \pi_t +0.43 \pi_{t+1} -0.02 \hat{y}^H_{t+1} \\
&+0.017 \hat{y}^H_{t-1} +0.019 \Delta p_t +res_{\pi t} \\
&\quad (0.001) (0.14) (0.11) (0.21) (0.18) (0.011)
\end{align*}
\]

\( R^2 = 0.57, GR^2 = 0.59, SER \times 100 = 0.401 \). The change in the oil prices is just significant at the 10% level, but the effect of the output gap (or its first difference) continues to be statistically insignificant. The restriction that the inflation coefficients sum to unity is again not rejected, p=0.915. The misspecification test statistics are

\[
\begin{array}{cccccc}
\text{Sargan} & \chi^2(7) & \text{SC} & \chi^2(4) & \text{FF} & \chi^2(1) & \text{N} & \chi^2(2) & \text{H} & \chi^2(1) \\
17.84.12 & 15.22 & 0.41 & 1.57 & 11.72 & . \\
[0.013] & [0.004] & [0.520] & [0.455] & [0.001] & \\
\end{array}
\]

The addition of oil price changes seems to have been effective in dealing with the non-normal errors. However, the other two more important specification

\(^{10}\)Lee and Nelson (2007) comment that "Forward-looking specifications, favoured by theory, produce the smallest slope estimates." They give references for a range of insignificant slope estimates in the literature. This is one of the reasons why the use of real marginal cost variables are often favoured over output gap measures (a la HP filters) in the New Keynesian type Phillips curve.
errors (the break down of the Sargan test and the strong evidence of residual serial correlation) continue to be present.

We now apply our approach, using the same specification but measuring all variables as deviations from their steady states. As noted earlier since by construction all deviations have zero means the new regressions are estimated with an intercept term. Using the deviation of the oil price from its steady state gives:

\[
\tilde{\pi}_t = \beta_1 E_t (\tilde{\pi}_{t+1} | I_{t-1}) + \beta_2 \tilde{\pi}_{t-1} + \gamma_1 \tilde{y}_t + \gamma_2 \tilde{y}_{t-1} + \delta \tilde{p}_t + u_t \tag{7}
\]

The instruments used are all measured as deviations from steady state and include a constant, \( \tilde{\pi}_{t-1}, \tilde{\pi}_{t-2}, \tilde{\pi}_{t-3}, \tilde{\pi}_{t-4}, \tilde{y}_{t-1}, \tilde{y}_{t-2}, \tilde{y}_{t-3}, \tilde{y}_{t-4}, \tilde{p}_{t-1}, \tilde{y}_{t-1}, \tilde{y}_{t-2}, \) \( \tilde{\pi}_{t-4}, \) \( \tilde{y}_{t-3}, \) \( \tilde{y}_{t-4}, \) \( \tilde{p}_{t-1}, \) \( \tilde{y}_{t-1}, \) \( \tilde{y}_{t-2}, \) \( \tilde{\pi}_{t-2}, \) \( \tilde{y}_{t-1}, \) \( \tilde{y}_{t-2}, \) and \( \tilde{e}_t. \) The estimates (with standard errors in parentheses) obtained over the period 1980Q4 to 2003Q3 are\(^{11}\)

\[
\begin{align*}
\tilde{\pi}_t = & \quad 0.38 \quad \tilde{\pi}_{t+1} + 0.52 \quad \tilde{\pi}_{t-1} + 0.39 \quad \tilde{y}_t - 0.31 \quad \tilde{y}_{t-1} \\
& \quad (0.13) \quad (0.12) \quad (0.11) \quad (0.10) \\
& + 0.017 \quad \tilde{p}_t + \text{res}_{xt} \\
& \quad (0.009)
\end{align*}
\]

\( \bar{R}_x^2 = 0.53, \quad GR_2 = 0.56, \quad SER \times 100 = 0.354. \) Compared to (5) the output gap has a much more significant effect.\(^{12}\) Although, the total effect of the level of the output gap is positive, as before a lot of the effect on inflation is coming from the change in the output gap. The deviation of the oil price from steady state is just at the edge of significance, \((p=0.059).^{13}\) The coefficients of the forward and backward inflation terms sum to 0.90 and this is not significantly different from unity \((p=0.315). \) The test statistics are

\[
\begin{array}{cccccc}
\text{Sargan} & \chi^2(8) & \text{SC} & \chi^2(4) & \text{FF} & \chi^2(1) & \text{N} & \chi^2(2) & \text{H} & \chi^2(1) \\
10.95 & 4.24 & 0.23 & 8.96 & 0.07 & 0.205 & 0.375 & 0.633 & 0.011 & 0.790
\end{array}
\]

There is some evidence of non-normality, but otherwise the equation seems well specified. Recursive estimation indicates that this equation seems structurally stable, though there is a slight indication that the effect of the oil price may be getting smaller towards the end of the period. Figure 4 gives the recursive estimates of the coefficient on output and their two standard error bands.

\(^{11}\)Two observations are lost to initialise the transitory components.

\(^{12}\)Note that \( \bar{R}^2 \) and \( GR^2 \) of the two Phillips curves in terms of \( \pi_t \) and \( \tilde{\pi}_t \) are not directly comparable, although in the present application they happen to be quite close to one another.

\(^{13}\)If the oil price term is excluded, there is evidence of negative serial correlation and failure on the Sargan test, but the output deviations continue to remain statistically significant.
The results, in particular the weight given to forward and backward inflation, seem to be sensitive to choice of instruments. Instrument selection is an issue that deserves further investigation. Given the structure of the GVAR and the decomposition of the variables into permanent and transitory components, there are a large number of potential instruments. One would not wish to choose the instruments from this large set which best predicted the right hand side endogenous variables, because that would tend to push the estimates towards those from OLS.

In the IS equation, the deviation of output from steady state is made a linear function of its lagged value, the deviation from steady state of the short real rate, \((\bar{r}_t^S - \bar{\pi}_t)\), (the short interest rate and the quarterly inflation rate have the same horizon), the deviation from steady state of the yield spread between long and short interest rates, \((\bar{r}_t^L - \bar{r}_t^S)\), and the deviation from steady state of the oil price. The instruments were a constant and \(\bar{y}_{t-1}, \bar{y}_{t-2}, \bar{p}_{t-1}^L, \bar{p}_{t-1}^S, \bar{\pi}_{t-1}, \cdots (\bar{r}_{t-1}^S - \bar{\pi}_{t-1}), (\bar{r}_{t-2}^S - \bar{\pi}_{t-2}), (\bar{r}_t^L - \bar{r}_{t-1}^S), (\bar{r}_{t-2}^L - \bar{r}_{t-2}^S)\). The estimates over the sample 1980Q2 to 2003Q4 are,

\[
\bar{y}_t = \begin{pmatrix}
0.29 \\
-1.32 \\
-4.24 \\
-0.026
\end{pmatrix} \begin{pmatrix}
\bar{y}_{t-1} \\
(\bar{r}_t^S - \bar{\pi}_t) \\
(\bar{r}_t^L - \bar{r}_t^S) \\
res_t^y
\end{pmatrix} + \begin{pmatrix}
(0.073) \\
(0.223) \\
(0.688) \\
(0.009)
\end{pmatrix}
\]
\( \bar{R}_y^2 = 0.96, \bar{G}R_y^2 = 0.79, SER \times 100 = 0.331 \). The variables are all highly significant and have the right sign. Interest rate effects, both the deviations from steady state of the short-run real rate and the slope of the yield curve are important in explaining deviations of output from steady state, with the yield curve having a much larger effect than the short run real rate. Although the lagged deviation of output from its steady state is significant, the deviation from steady state is much less persistent than output itself or HP filtered output. Again recursive estimates are fairly stable, with some indication that the persistence of output increased as data from the early years of this century are added. The test statistics are

\[
\begin{align*}
\text{Sargan } \chi^2(6) & \quad \text{SC } \chi^2(4) & \quad \text{FF } \chi^2(1) & \quad N \chi^2(2) & \quad H \chi^2(1) \\
6.84 & \quad 2.33 & \quad 4.94 & \quad 6.78 & \quad 0.34 \\
[0.336] & \quad [0.676] & \quad [0.026] & \quad [0.034] & \quad [0.560]
\end{align*}
\]

We also investigated whether the deviation of real equity prices from their steady state, \( \bar{q}_t \), had an effect on \( \bar{y}_t \). This was added to the equation above and \( \bar{q}_{t-1} \) added to the instrument list. The estimates, over the same sample, are

\[
\begin{align*}
\bar{y}_t & = 0.32 - 1.16 (\bar{r}_t^L - \bar{\pi}_t) - 4.12 (\bar{r}_t^L - \bar{\pi}_t) - 0.016 \bar{p}_t^o \\
& + 0.033 \bar{q}_{t-1} + res_{yt} \\
(0.068) & \quad (0.229) & \quad (0.607) & \quad (0.007)
\end{align*}
\]

\( \bar{R}_y^2 = 0.97, \bar{G}R_y^2 = 0.79, SER \times 100 = 0.280 \). The deviation of equity prices from steady state is significant, \( t=2.28 \), indicating that measuring in terms of deviations from steady state can reveal effects that may not be apparent in the original data. The other coefficients do not change very much. The test statistics are

\[
\begin{align*}
\text{Sargan } \chi^2(6) & \quad \text{SC } \chi^2(4) & \quad \text{FF } \chi^2(1) & \quad N \chi^2(2) & \quad H \chi^2(1) \\
9.27 & \quad 5.83 & \quad 2.85 & \quad 0.92 & \quad 0.329 \\
[0.137] & \quad [0.212] & \quad [0.091] & \quad [0.631] & \quad [0.531]
\end{align*}
\]

Adding the deviation of real equity prices from steady state has removed the problems with functional form and normality.

Given the importance of the deviation of the long-rate from steady state, through the yield curve term, it is important to model the effect of the short, policy, rate on the deviation of the long-rate from steady state. There is an issue here as to whether it is appropriate to model this in terms of the policy rate itself, or its deviation from steady state. Here, we do it using the policy rate itself, since policy should respond to both transitory and permanent inflation shocks. The estimated equation is just a partial adjustment of the deviation of the long-rate to steady state from the policy rate. The instruments are a constant and \( \bar{\pi}_{t-1}, \bar{\pi}_{t-2}, \bar{y}_{t-1}, \bar{y}_{t-2}, \bar{r}_t^L, \bar{r}_{t-1}^L, \bar{p}_{t-1}^o, \bar{y}_{t-1}^*, \bar{\pi}_{t-1}^*, \bar{\pi}_{t-2}^*, \bar{r}_{t-1}^L \). The estimates for 1980Q2 to 2003Q4 are

\[
\begin{align*}
\bar{r}_t^L & = -0.0014 + 0.087 \bar{r}_t^L + 0.77 \bar{p}_{t-1}^L + res_{Lt} \\
& 0.0004 \quad (0.023) \quad 0.007 \quad (0.059)
\end{align*}
\]
\[ R_{rL}^2 = 0.72, \quad GR_{rL}^2 = 0.74, \quad SER \times 100 = 0.169. \] All the coefficients are significant, including the intercept, which is included since we are not using deviations from steady state for short interest rates. The test statistics are:

<table>
<thead>
<tr>
<th>Sargan ( \chi^2 ) (10)</th>
<th>SC ( \chi^2 ) (4)</th>
<th>FF ( \chi^2 ) (1)</th>
<th>N ( \chi^2 ) (2)</th>
<th>H ( \chi^2 ) (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.31</td>
<td>1.93</td>
<td>0.84</td>
<td>12.96</td>
<td>15.93</td>
</tr>
<tr>
<td>[0.050]</td>
<td>[0.749]</td>
<td>[0.360]</td>
<td>[0.002]</td>
<td>[0.000]</td>
</tr>
</tbody>
</table>

There are issues with the over-identifying restrictions which are on the edge of significance, normality and heteroskedasticity, but this is a very simple term structure equation.

Over the sample used here, the data also show a clear Taylor Rule, for the short rate. The instruments are a constant \( r_{t-1}^S \), \( r_{t-2}^S \), \( \pi_{t-1} \), \( \pi_{t-2} \), \( \Delta y_{t-1} \), \( \Delta y_{t-2} \), \( \Delta y_{t-1}^r \). The estimates for 1980Q1 to 2003Q4 are:

\[
\begin{align*}
    r_t^S &= -0.004 + 0.50 r_{t-1}^S + 0.38 r_{t-2}^S \quad (0.001) \quad (0.201) \quad (0.176) \\
    &\quad +0.36 \Delta y_t + 0.34 \pi_t + \text{res}_{rSt} \quad (0.098) \quad (0.114)
\end{align*}
\]

\[ R_{rs}^2 = 0.91, \quad GR_{rs}^2 = 0.93, \quad SER \times 100 = 0.257. \] The long run response to inflation is much greater than unity and is of a similar magnitude to the response to the growth rate. The test statistics are:

<table>
<thead>
<tr>
<th>Sargan ( \chi^2 ) (3)</th>
<th>SC ( \chi^2 ) (4)</th>
<th>FF ( \chi^2 ) (1)</th>
<th>N ( \chi^2 ) (2)</th>
<th>H ( \chi^2 ) (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.78</td>
<td>0.58</td>
<td>0.06</td>
<td>9.49</td>
<td>12.74</td>
</tr>
<tr>
<td>[0.286]</td>
<td>[0.965]</td>
<td>[0.810]</td>
<td>[0.009]</td>
<td>[0.000]</td>
</tr>
</tbody>
</table>

There are issues with both normality and heteroskedasticity, but these are preliminary estimates and further investigation is required.

These preliminary estimates for the US are suggestive. Changes in the policy rate have a relatively small effect on deviations of the long rate from steady state. But the deviations of the long rate from steady state have a marked effect on deviations of output from steady state and these have effects on the deviation of inflation from steady state. It will be interesting to see the extent to which they get replicated in other industrial countries and the extent to which they are sensitive to the form of the decomposition to determine steady state values.

## 6 Conclusion

Economic models, whether DSGEs or Cointegrating VARs, should be informative about long-run steady states and the derivation of Phillips Curves from DSGEs requires that the variables be measured as deviations from their steady states. Instead of using the information from the economic model about the steady states, it is the practice to approximate the steady state either by constants or statistical procedures like the HP filter. It is natural to regard the
steady state at time $t$, as the long-horizon forecast for the variables in the system in the absence of further shocks given the information at time $t$. For any particular model this can be calculated and we have illustrated the application of this procedure, using deviations from steady states. We have done this for one particular model the GVAR, but the point is more general. The approach also provides a route to combining the benefits of a cointegrating VAR for the long-run with the more easily interpreted forward looking short run models and ensures that the short run models converge to the long-run steady states.

While encouraging, these US estimates are preliminary. There are a range of issues that deserve further investigation. (a) The sensitivity of the results to the calculation of the steady state needs to be investigated. The issues include the difference between the GRW and standard multivariate Beveridge-Nelson decomposition with cointegration; the treatment of deterministic elements; the effect of imposing over-identifying restrictions on the cointegrating vectors; and the use of the whole global model to calculate the permanent components. (b) Following the standard approach to US models, we have emphasised domestic variables and except for their role in calculating steady states and their use as instruments the global variables have played little role. Whether other global variables apart from the price of oil have an impact on the US economy deserves investigation. (c) The results may be sensitive to choice of instruments and the most effective way to choose instruments needs investigation, given that the GVAR structure provides a lot of potential instruments. (d) Forward looking ‘structural’ equations measured as deviations from steady states, of the sort estimated, impose restrictions on the VARX* and these could be tested. (e) The approach needs to be applied to other countries. This is relatively straightforward in the context of the Global VAR.
References


