Comparative Advantages of Public Loan and Public Equity Schemes in Venture Capital Markets

by
Andrea Schertler

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Abstract:
Using an agency framework, this paper examines advantages of offering venture capitalists a choice between public equity and loan schemes. Both schemes can be used to promote venture capital investments in high-technology enterprises since they encourage inexperienced venture capitalists to accumulate experience. However, under both schemes, experienced venture capitalists save on management support. This paper shows that offering venture capitalists a choice between these two schemes can lead to a positive self-selection. Inexperienced venture capitalists choose public equity under which they have higher incentives to enter the market. Experienced venture capitalists choose public loans under which they have lower incentives to save on their management support.

Keywords: public equity schemes, public loan schemes, venture capital

JEL classification codes: D82, G24, G28, L14

Andrea Schertler
Kiel Institute for World Economics
24100 Kiel, Germany
Telephone: ++49 431 8814 496
Fax: ++49 431 8814 502
E-mail: a.schertler@ifw.uni-kiel.de
1 Introduction

The German government promotes venture capital investments in young high-technology enterprises with the program *Beteiligungskapital für kleine Technologieunternehmen* (private equity for small high-technology enterprises, BTU). This program contains a loan and an equity scheme. Under the loan scheme, a publicly supported bank covers a share of the venture capitalists’ realized losses by refinancing and by partly guaranteeing the venture capitalists’ participations. Under the equity scheme, a publicly supported co-investor invests in high-technology enterprises if a lead venture capitalist is involved in the respective enterprise. This co-investor demands in exchange for its investment a profit participation. The purpose of this paper is to identify possible advantages of offering venture capitalists a choice between public equity and public loan schemes.

Recent literature has identified important effects of public subsidies promoting venture capital investments in high-technology enterprises. The main arguments in favour of public subsidies are positive externalities in research and development activities, and capital shortages for young high-technology enterprises as a result of asymmetric information problems (Lerner 2002). Theoretical arguments against public subsidies are the crowding out of private investments (European Commission 2000), and induced changes in the venture capitalists’ investment incentives with respect to the selection of investment projects and with respect to the supply of non-financial resources (Keuschnigg and Nielson 2001, Murray and Marriot 1998). However, the literature has not addressed the impact of public subsidies on venture capital markets if venture capitalists are just accumulating experience to successfully select, monitor, and support high-technology enterprises, i.e., if these markets are just emerging.

In emerging venture capital markets, venture capitalists have not yet built reputation, and they have not yet accumulated experience. Venture capitalists need experience to add value to young high-technology enterprises. They accumulate experience by being involved in the management of young high-technology enterprises. Moreover, venture capitalists need reputation, i.e., a track record of successfully financing young high-technology enterprises, in order to raise capital from outside
investors for their investment activities. A priori, outside investors have little information about the profitability of venture capital investments.

Because of venture capitalists’ lack in experience and reputation in emerging venture capital markets, successive waves of failed venture-capital-backed enterprises can undermine the development of a liquid venture capital market (Schertler 2002a). In the case of successive waves of failed venture-capital-backed enterprises, venture capitalists are incapable of raising new funds for their further investment activities, because outside investors supplying capital to venture capital funds believe that high-technology enterprises in the economy have only moderate growth prospects and therefore supply only small amounts of capital. In this situation, a subsidy designed as push-up investment might help to establish venture capital finance.

Schertler (2000, 2002b) has addressed the question whether public loan and public equity schemes can promote experience accumulation in emerging venture capital markets if successive waves of unsuccessful venture-capital-backed enterprises hamper experience accumulation. In these papers, it has been shown that both schemes can encourage inexperienced venture capitalists to enter the market, and that experienced venture capitalists have incentives to save on their management support. Saving on management support can lower the speed of experience accumulation. By contrast, this paper addresses the question whether offering venture capitalists a choice between public loan and equity schemes is more efficient in emerging venture capital markets than offering only one scheme.

In this paper, I show that an equity scheme seems more favourable than a loan scheme to encourage inexperienced venture capitalists to enter the market for financing high-technology enterprises. Inexperienced venture capitalists do not care about sharing the revenue in the high performance state of the project. The reason for this is that the enterprise’s revenue is moderate in this state because of the venture capitalists’ low experience. However, a loan scheme seems favourable since experienced venture capitalists have lower incentives to reduce their management support than under an equity scheme. The reason for this is that experienced venture
capitalists reduce their management support more under the equity scheme because of the co-investor’s participation in revenue.

The paper is organized as follows. Section 2 offers a description of the German BTU-program. Section 3 summarizes the effects of loan and equity schemes on venture capitalists’ investment behaviour. In Section 4, I examine venture capitalist’s choice on the basis of his expected pay-off. In Section 5, I examine which scheme causes the larger reduction in venture capitalist’s management support. In Section 6, the choice on the basis of the total surplus of the enterprises is discussed. Section 7 summarizes the main results.

2  The German BTU-Program and Its Predecessors

In 1995, the German government introduced the BTU-program to improve access to capital for young high-technology enterprises. Supporting young high-technology enterprises via venture capitalists was also hoped to add resources such as contact to customers and suppliers to the enterprises. Under the loan scheme of the BTU-program, venture capitalists and private equity investors can refinance their equity participations where a part of the investment risk is taken on by the government. Under the co-investment scheme (i.e., an equity scheme), they can use a co-investment offered by a publicly supported co-investor.

The BTU-program is, however, not the first attempt to improve the capital access for young high-technology enterprises. A comprehensive description of current government schemes in Germany offers Schmeisser and Galler (2001) and Güllmann (2000).
firms in their portfolios in addition to offering capital in order to receive public support, these two characteristics of venture capital finance were separated under the program *Förderung technologieorientierter Unternehmensgründungen* (support of technology-oriented start-up enterprises, FTU) which existed between 1983 and 1988.

The strict separation of offering capital and supporting the start-ups’ inexperienced management teams seems to have induced adverse incentives (see, for example, Kulicke 1993, 1997). The FTU-program attracted financial intermediaries that did not necessarily have the technological experience to select the most promising high-technology enterprises. Because the government partly covered losses, these financial intermediaries also had relatively few incentives to carefully select and monitor the enterprises. The experts who supported the management teams had few incentives (at least in monetary terms) to do their jobs well. The recognition of the incentive problems caused by the separation of supporting the management teams and offering capital resulted in a reorientation of public policies under the BJTU- and also under the BTU-program.

Under the loan scheme of the BTU-program, the Kreditanstalt für Wiederaufbau (KfW), Germany’s state development bank, refines seventy per cent² of venture capitalists’ and private equity investors’ participations in small and often young high-technology enterprises.³ The loans involved have a maturity of ten years, and can amount up to a maximum of 1.4 million euros. In 2001, the nominal interest rates on the loans were about nine per cent including a risk premium of two per cent.

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² In 1995, investments were refinanced up to 85 (75) per cent in the new (old) Laender. After the 1 July 1999, this ratio was reduced to 80 (70) per cent in the new (old) Laender. Since 1 January 2000, the ratio of refinancing has been identical in the new and old Laender.

³ An enterprise is defined here as being small if it has fewer than 50 employees, total annual revenues of less than seven million euros or a balance sheet of less than five million euros. This definition of small enterprises is in accordance with EU guidelines (Europäische Kommission 2001). Moreover, enterprises must not be older than five years.
The KfW has the option to demand a lower risk premium in exchange for profit participation.

The co-investment scheme is managed by the Technologie-Beteiligungs-Gesellschaft (technology participation company, TBG), an affiliate of the Deutsche Ausgleichsbank. The TBG enters silent partnerships in high-technology enterprises if a lead investor, which can be a venture capitalist or a bank subsidiary, invests at least the same amount in form of equity and if this lead investor takes on the support and monitoring of the enterprise. The TBG invests a maximum amount of up to 1.5 million euros that the high-technology enterprise must repay within ten years. In exchange, the TBG receives once one per cent of the volume that is invested in the high-technology enterprise, an annual interest payment of seven per cent, and profit participation, which must be individually specified in the cooperation contract (Europäische Kommission 2001).

Under the co-investment scheme, the TBG has the possibility to take on a part of the lead investor’s investment risk. The TBG guarantees 50 per cent of its own equity to the lead investor (thus, not more than 50 per cent of the lead investor’s investment amount), so that the TBG bears up to 75 per cent of the total investment risk. In exchange for this guarantee, the lead investor has to pay a fee that amounts to 15 per cent of the guaranteed investment volume.

By the end of 2002, the total mobilized private equity of the BTU-program is expected to be 2.7 billion euros. The German government needs 195.2 million euros to mobilize this amount of private equity (Europäische Kommission 2001). It is important to note that the failure of a supported high-technology enterprise causes not only a loss for the government under the co-investment scheme but also under the loan scheme. The reason for this is that the KfW guarantees each single loan offered to venture capitalists. To put it differently, if the venture-capital-backed high-technology enterprise fails whose venture capitalist has partly re-financed his participation under the loan scheme, the venture capitalist has not to repay the loan to the KfW independent of whether his portfolio return is positive.
The expected costs of the co-investment scheme for the German government amount to 6.8 per cent of the total mobilized private equity if the TBG makes with its portfolio positive returns. The TBG has to carry 13 percentage points of its failed participations that are expected to be about 30 per cent.\footnote{Failure rates, expected and realized ones, vary substantially (Kirchhoff and Greene 1995). Failure rates of high-technology enterprises vary among industries and years because they depend on the development of product as well as stock markets. The TBG bases its calculation on a failure rate of 30 per cent because this number is often cited in the literature (see, for example, Schröder (1992)). But this number predominantly bases on studies for the United States. Moreover, 30 per cent does not seem within reach after the bursting of stock market bubbles.} Moreover, of the 17 per cent failed participations left, the TBG has to carry additionally 20 per cent up to five per cent of its participation volume. Thus, the expected costs of the co-investment scheme amount to 13.6 per cent of the participations of the TBG. Since the lead investor invests at least the same amount as the TBG the costs for the German government are 6.8 per cent of the total mobilized private equity (Europäische Kommission 2001) if the TBG has positive profits. The costs of the guarantee of the lead investor’s investment should be covered by the fee, which the lead investor has to pay.

The expected costs of the loan scheme for the German government amount to 7.62 per cent of the total mobilized private equity if the KfW does not realize losses. The two per cent risk premium, which investors have to pay for a loan, should cover up to 16.4 per cent of the failed refinanced loans. The KfW must also carry 20 per cent of the further failed loans up to five per cent of its total refinanced loan volume. Thus, the expected costs of the loan scheme amount to 10.88 per cent of the refinanced loans. Since private equity investors can only refinance participations up to 70 per cent, the expected costs are 7.62 per cent of the total mobilized private equity (Europäische Kommission 2001).
3 The Set-Up: Public Equity and Public Loan Schemes

Public equity and loan schemes change the incentives of experienced venture capitalists to support the management teams of high-technology enterprises. Under a loan scheme, the government partly covers venture capitalist’s realized losses, while, under an equity scheme, a publicly supported co-investor invests capital in venture-capital-backed enterprises in exchange for profit participation.

Under the public equity scheme, the venture capitalist has only to carry a part of the investment risk, since the co-investor carries the risk of his investment himself. In exchange for his investment, the co-investor demands participation on the enterprise’s profit when the performance is high, while the revenue of the enterprise is solely distributed among the venture capitalist and the entrepreneur when the performance is low. The venture capitalist has fewer incentives to support the management team, since the profit of the enterprise has to be shared with an additional contracting party (the co-investor) when the performance is high, while the venture capitalist has to carry his costs for doing management support alone.

Under public equity, the reaction curve of the venture capitalist, which is defined as the venture capitalist’s effort as a function of the entrepreneur’s effort, shifts inwards since the profit of the enterprise has to be shared with an additional party (Figure 1). The venture capitalist’s reaction curve has a positive slope and the marginal product decreases in the entrepreneur’s effort. Under public equity, the entrepreneur also exerts for each level of venture capitalist’s effort less effort in the development of the enterprise since the enterprise’s profit must be shared with the co-investor. Thus the entrepreneur’s reaction curve shifts inwards as well. The equilibrium under public equity is characterized by lower effort amounts \((E_{ES}^*, V_{ES}^*)\) of the contracting parties in comparison to the equilibrium without public subsidy \((E^*, V^*)\).

Under the loan scheme, the venture capitalist has to re-pay the loan, if the performance of the enterprise is high. If he realizes a loss with the participation, he has only to re-pay a part of the initial loan. Thus, his investment risk is lower with than without public loan. Since the
investment risk is lower, he has lower incentives to carefully monitor and support the enterprise. The venture capitalist’s reaction curve shifts inwards lowering the effort amounts of both contracting parties \((E^*_LS, V^*_LS)\) in the new equilibrium, although the entrepreneur’s reaction curve does not shift inwards (Figure 1). By contrast, under the equity scheme both reaction curves shifts inwards. Under both schemes, the reduction of management support results in a lower total surplus of the enterprise which contains the venture capitalist’s, the entrepreneur’s and the government’s expected pay-offs.

**Figure 1:** The impact of subsidies on effort of the contracting parties

![Diagram](image)

Public loans and public equity cause a lower total surplus of the enterprises only if venture capitalists’ experience is in a certain range, i.e., above the minimum level \(H\) and below the critical level \(H_S\) (Figure 2). The reason for this is that only venture capitalists whose experience is within this range have incentives to use the subsidy. Venture capitalists whose experience is not lower than the minimum level \(H\) are capable of financing high-technology start-ups without public subsidy since their expected pay-offs are non-negative. Venture capitalists whose experience exceeds the critical level \(H_S\) have no incentives to use the government’s subsidy since their expected pay-offs are higher without than with subsidy. In this case,
government’s monetary transfer negatively affects the venture capitalist’s expected pay-offs by reducing the effort of both contracting parties more than the government’s monetary transfer increases the venture capitalist’s expected pay-off.

But public loan and equity schemes cause still another effect in venture capital markets. Venture capitalists who have not yet accumulated sufficient experience to finance high-technology enterprises in a profitable way have incentives to infuse capital in high-technology enterprise under loan and equity schemes. In Figure 2, venture capitalists whose experience is below the minimum level $\bar{H}$ but above the minimum level under a subsidy $\bar{H}_S$ have incentives to enter the market for high-technology investments. Due to the subsidies, these inexperienced venture capitalists can finance high-technology enterprises and by being involved in the management of these enterprises they can accumulate additional experience so that after some periods of time, they may be capable of financing high-technology enterprises without subsidies.

*Figure 2: The impact of subsidies on venture capitalist’s expected pay-off*

Since Schertler (2000, 2002b) analyses the effects of public loan schemes and public equity schemes on venture capitalist’s management support, on venture capitalist’s experience, and on the total surplus of the enterprise in
detail, only a short summary of the analytical framework is given in the following.

An entrepreneur (EN) has an innovative product idea, but she lacks the necessary financial means to finance the start-up investment \( I > 0 \) herself, and the expertise to manage the enterprise in an efficient manner. Therefore, she prefers funding by a venture capitalist (VC) who offers a revenue-increasing management support. Before the start-up investment is made, the venture capitalist and the entrepreneur sign a contract, i.e. they specify the revenue allocation \((1 - \alpha, \alpha)\), where \( \alpha \) denotes the revenue share of the venture capitalist.

After the capital has been infused, both contracting parties exert their non-contradictable effort, without observing the effort amount of the respective complementary party. Both contracting parties can affect the product innovation’s expected revenue. The revenue of the enterprise can take two states. In the state with low performance \( l \), reached with probability \( p_l \) \((0 < p_l < 1)\), the venture capitalist’s pay-off does not cover the investment costs independent from effort amounts of the contracting parties. While in the state with high performance \( h \), reached with probability \( p_h \) \((0 < p_h < 1)\), the enterprise’s revenue can exceed the effort and investment costs. The sum of the probabilities is equal one \((p_l + p_h = 1)\).

The expected revenue \( \hat{R} \) of the enterprise is given by

\[
\hat{R} = (p_l + Ap_h)E^\beta \tilde{V}^{\omega}, \quad \text{with } 1 > \beta + \omega, \quad 0 < \beta, \omega < 1, \quad \text{and } A = (I - p_l)/p_h \text{ as a shift parameter ensuring that the revenue of the enterprise is larger in the state with high performance than in the state with low performance.} \quad E \text{ denotes entrepreneur’s effort, and } \tilde{V} \text{ is the venture capitalist’s management support that is a function of his time spent in the enterprise } V \text{ and his experience } H, \quad \tilde{V} = V^\rho H^{1-\rho}, \quad \text{with } 0 < \rho < 1. \text{ The venture capitalist has accumulated his experience by being involved in the management of high-technology start-up enterprises in the past. Therefore, his experience is exogenously given in the short-term; the venture capitalist maximizes his expected pay-off by choosing his time for doing management support.}

The venture capitalist’s expected pay-off function without public subsidy, which results from his expected revenue minus the costs of the start-up
investment $I$ and the costs for doing management support given by $HV$, is given by:

$$[1] \quad \hat{U}^{VC} = \alpha I E^{\beta} V^{\delta} H^{\lambda} - HV - I, \text{ with } \lambda := (1 - \rho) \omega \text{ and } \delta := \rho \omega. \quad (1)$$

Venture capitalists differ with respect to their experience $H$. Those who financed a multitude of high-technology start-ups in the past have more experience than venture capitalists who only financed some start-up enterprises.

The entrepreneur receives the share $(1 - \alpha)$ of the enterprise’s expected revenue. Since she has only to carry her effort costs, her expected pay-off function is given by:

$$[2] \quad \hat{U}^{EN} = (1 - \alpha) I E^{\beta} V^{\delta} H^{\lambda} - E. \quad (2)$$

In the stable Nash equilibrium of this simultaneous move game, both contracting parties have positive effort amounts, if the venture capitalist has sufficient experience to make the high-risk investment profitable. Optimisation of expected pay-offs yields the following optimal effort amounts:

$$[3] \quad E^* = \left[ (1 - \alpha)^{1 - \delta} \alpha^{1 - \delta} \beta^{1 - \delta} \delta^{1 - \delta} IH^{\lambda - \delta} \right]^{\frac{1}{1 - \beta - \delta}} \quad \text{and}$$

$$[4] \quad V^* = \left[ (1 - \alpha)^{\beta} \alpha^{1 - \beta} \beta^{1 - \beta} \delta^{1 - \beta} IH^{\beta + \lambda - 1} \right]^{\frac{1}{1 - \beta - \delta}}. \quad (3, 4)$$

Inserting the optimal effort amounts by the entrepreneur and by the venture capitalist without subsidy in the expected pay-off functions gives the expected pay-offs of the contracting parties as a function of the equity allocation $(1 - \alpha, \alpha)$ and of the venture capitalist’s experience $H$:

$$[5] \quad \hat{U}^{VC} = \left[ (1 - \alpha)^{\beta} \alpha^{1 - \beta} \beta^{\beta \delta} \delta^{\delta} IH^{\lambda - \delta} \right]^{\frac{1}{1 - \beta - \delta}} (1 - \delta) - I \quad \text{and}$$

$$[6] \quad \hat{U}^{EN} = \left[ (1 - \alpha)^{1 - \delta} \alpha^{\delta} \beta^{\beta \delta} \delta^{\delta} IH^{\lambda - \delta} \right]^{\frac{1}{1 - \beta - \delta}} (1 - \beta). \quad (5, 6)$$

The equity allocation preferred by the venture capitalist, which must be used to determine minimum experience level, is given by $\alpha = 1 - \beta$. 


Setting equation [5] equal zero and solving for venture capitalist’s experience gives the minimum level of experience. All venture capitalists who have less experience than the minimum level $H$ have no incentive to finance high-technology enterprises, since their expected pay-offs are always negative. All venture capitalists who have at least this experience are capable of financing start-up investments, since their expected pay-offs are non-negative.

The total surplus of the enterprise without subsidy is the sum of the entrepreneur’s and the venture capitalist’s expected pay-offs. Using the venture capitalist’s preferred revenue allocation yields:

$$\hat{U}_{\text{Total}} = \hat{U}_{\text{VC}} + \hat{U}_{\text{EN}} = CH^{1-\delta} (1-\delta + \beta) I^{1-\beta-\delta} - I,$$

with $C := (1 - \beta)^{1-\beta} \beta^{2\beta-\delta} \delta^{-\beta-\delta}$. where $0 < C < 1$.

What happens now under a public loan scheme? Suppose that under the loan scheme the government covers the share $\theta$ of the venture capitalist’s total losses given by $I - \alpha E^\beta V^\delta H^\lambda$ in the low performance state of the project. Then, the expected pay-off of the venture capitalist [1] has to be extended by $p_i \theta I - p_i \theta \alpha E^\beta V^\delta H^\lambda$, while the entrepreneur’s expected pay-off [2] does not change. Optimising the expected pay-offs under the loan scheme yields the optimal effort levels:

$$E_{LS}^* = \left(1 - \alpha \right)^{1-\delta} \alpha^{\delta} \beta^{1-\delta} \delta^{-\beta-\delta} I^{1-\delta} (1 - p_i \theta)^\delta H^{\lambda-\delta} \right)^{\frac{1}{1-\beta-\delta}} \text{ and}$$

$$V_{LS}^* = \left(1 - \alpha \right)^{\beta} \beta^{1-\beta} \beta^{1-\beta} I^{\beta} (1 - p_i \theta)^{1-\beta} H^{\beta+\lambda-1} \right)^{\frac{1}{1-\beta-\delta}}.$$

Inserting the optimal effort amounts of the venture capitalist and the entrepreneur under a public loan scheme in the expected pay-off functions of the contracting parties gives the expected pay-off under the loan scheme as a function of the equity allocation:

$$\hat{U}_{LS}^{\text{vc}} = \left(1 - \alpha \right)^{\beta} \beta^{1-\beta} \beta^{1-\beta} I^{\beta} (1 - p_i \theta)^{1-\beta} H^{\lambda-\delta} \right)^{\frac{1}{1-\beta-\delta} (1 - \delta)} + \frac{(p_i \theta - 1) I}{p_i \theta}.$$
and

\[ [11] \hat{U}_{LS}^{EN} = \left[ (1-\alpha)\delta^\beta \delta^\delta I^{1-\delta} (I-p_1 \theta)^\delta H^{\lambda-\delta} \right] \frac{1}{1-\beta^\delta} \left( 1-\beta \right). \]

The experience level that leads to zero profits of venture capitalist’s expected pay-off given in equation [16] with the preferred revenue allocation of the venture capitalist is the minimum level of experience under the loan scheme. The critical level of experience under the loan scheme is determined using the difference between venture capitalist’s expected pay-off with public loan and without public loan.

In order to determine the change in the enterprise’s total surplus analytically, government’s costs have to be specified. Under the loan scheme, the government expects to carry the share \( \theta \) of venture capitalist’s looses only realized in the low performance state of the project reached with probability \( p_l \):

\[ [12] \hat{U}_{LS}^{G} = -p_1 \theta \left[ I - (1-\alpha)\delta^\beta \delta^\delta I^{1-\delta} (I-p_1 \theta)^\delta H^{\lambda-\delta} \right] \frac{1}{1-\beta^\delta}. \]

Under the loan scheme, the total surplus of the enterprise is given by the sum of the venture capitalist’s expected pay-off, the entrepreneur’s expected pay-offs, and the government’s expected cost. Using the equity allocation preferred by the venture capitalist, the total surplus is given by:

\[ [13] \hat{U}_{LS}^{Total} = \hat{U}_{LS}^{VC} + \hat{U}_{LS}^{EN} + \hat{U}_{LS}^{G} = C \left[ H^\lambda \delta^\beta \delta^\delta I^{1-\delta} (I-p_1 \theta)^\delta \right] \frac{1}{1-\beta^\delta} \left[ (1-\delta + \beta)I - \delta (I-p_1 \theta) \right] - I. \]

Under the public equity scheme, the co-investor takes on a share \( \mu \) of the start-up investment \( I \) in exchange for a profit share \( 1-\kappa \). Thus the expected pay-off of the venture capitalist [1] has to be extended by \( -(I-p_1)(1-\kappa)E^\beta V^\delta H^\lambda + \mu I \) while the entrepreneur’s expected pay-off [2] has to be extended by \( -(I-p_1)(1-\kappa)E^\beta V^\delta H^\lambda \). Optimisation of expected pay-offs gives the optimal effort levels under public equity:

\[ [14] E_{ES}^* = \left[ (1-\alpha)\delta^\beta \delta^\delta \left( p_1 +(1-p_1)\kappa \right) H^{\lambda-\delta} \right] \frac{1}{1-\beta^\delta} \quad \text{and} \]

...
Inserting the optimal effort amounts under the public equity scheme in the expected pay-off functions of the venture capitalist and the entrepreneur gives the expected pay-offs under the equity scheme as a function of the equity allocation:

\[
\hat{U}_{ES}^{\text{VC}} = \left(1 - \alpha\right)^{1 - \beta} \alpha^{1 - \beta} \beta^{\delta} \delta^{1 - \beta} \left(\frac{p_l + (1 - p_l) \kappa}{H^{\beta + \lambda - 1}}\right) \left[\frac{1}{1 - \beta - \delta}\right].
\]

and

\[
\hat{U}_{ES}^{\text{EN}} = \left(1 - \alpha\right)^{1 - \delta} \alpha^{\delta} \beta^{\delta} \delta^{1 - \beta} \left(\frac{p_l + (1 - p_l) \kappa}{H^{\lambda - \delta}}\right) \left[\frac{1}{1 - \beta - \delta}\right] (1 - \beta),
\]

where \(\kappa\) denotes the remaining profit share allocated between the venture capitalist and the entrepreneur, with \(0 \leq \kappa \leq 1\).

The minimum level of experience under the equity scheme is determined by equation [16], with the revenue allocation preferred by the venture capitalists.

Under the equity scheme, the public co-investor expects a monetary loss in the state with low performance, while in the state of high performance he participates in profits instead of receiving a re-payment of his capital infused. His expected pay-off is given by:

\[
\hat{U}_{ES}^{G} = -\mu I + (1 - \kappa)(I - p_l) \left(1 - \alpha\right)^{1 - \beta} \alpha^{1 - \beta} \beta^{\delta} \delta^{1 - \beta} \left(\frac{p_l + (1 - p_l) \kappa}{H^{\beta + \lambda - 1}}\right) \left[\frac{1}{1 - \beta - \delta}\right].
\]

The total surplus of the enterprise is again given by the sum the venture capitalist’s and the entrepreneur’s expected pay-offs under the equity scheme, and the public co-investor’s expected costs. With venture capitalist’s preferred equity allocation, it follows:

\[
\hat{U}_{ES}^{\text{Total}} = \hat{U}_{ES}^{\text{VC}} + \hat{U}_{ES}^{\text{EN}} + \hat{U}_{ES}^{G}
\]
In order to analyse the venture capitalist’s choice regarding public loan and equity schemes, it is useful to define first a relationship between the remaining profit share, which is distributed among the venture capitalist and the entrepreneur under the equity scheme, and the co-investor’s share of the start-up investment since this reduces the number of parameters and, thus, makes the comparison of the two schemes more tractable.

From a comparison of the minimum level without public equity and the minimum level with public equity it follows that \( \kappa > (1 - \mu)^{1-\beta-\delta} \) must be fulfilled in order to encourage inexperienced venture capitalists to enter the market for high-risk investments. The smaller the term on the right-hand side for a given remaining revenue share, the higher the subsidized amount under the public equity scheme is. In the following, I assume that \( \kappa = \left(1 - \mu\right)^{1-\beta-\delta} / 2 \) which ensures that the scheme encourages inexperienced venture capitalists to enter the market for high-risk investments.

The venture capitalist bases his decision regarding public loan and equity schemes on the difference in his expected pay-offs. The difference between the venture capitalist’s expected pay-offs under the equity scheme [16] and under the loan scheme [10] is given by:

\[
\hat{U}_{DIF}^{VC} = \hat{U}_{ES}^{VC} - \hat{U}_{LS}^{VC} = \\
C(1-\delta)H^{1-\beta-\delta} \left[ (p_t + (I - p_t)\kappa)\frac{1}{1-\beta-\delta} - (I^\beta (I - p_t\theta))^{1-\beta} \frac{1}{1-\beta-\delta} \right] \\
- \left( \kappa^{1-\beta-\delta} + p_t\theta - 1 \right) I
\]

Thus, which scheme a venture capitalist prefers does not only depend on the policy parameters of the public loan and equity scheme \((\kappa, \theta)\) but also
on his level of experience accumulated in the past and on the level of the innovation risk which is measured by the probability to reach the low performance state of the project.

In the following, I will analyse first how the venture capitalist’s choice changes if his experience increases. Thereafter, I will analyse how his choice changes if the innovation risk of the enterprise increases.

The impact of an increase in the venture capitalist’s experience on the difference in expected pay-offs results from the partial derivative of equation [20] with respect to the experience level:

\[
\frac{\partial \hat{U}_{DIF}^{VC}}{\partial H} = C (1-\delta) \frac{\lambda - \delta}{1-\beta - \delta} \frac{\lambda - \delta}{H^{1-\beta - \delta}} \left( p_l + (I - p_l) \kappa \right)^{1-\beta - \delta} - \\
C (1-\delta) \frac{\lambda - \delta}{1-\beta - \delta} \frac{\lambda - \delta}{H^{1-\beta - \delta}} \left( I^\beta (I - p_l \theta) \right)^{1-\beta} .
\]

The effect of an increase in the level of the experience depends on the sign of the term in the square bracket. Setting the square bracket term smaller than zero and rearranging gives

\[
0 < \frac{p_l (1-\kappa) + I \kappa - I^\beta (I - p_l \theta)^{1-\beta}}{I^\beta (I - p_l \theta)^{1-\beta}} \frac{1}{1-\beta - \delta},
\]

which is fulfilled if the start-up investment is sufficiently large (as indicated by the partial derivative of the square bracket term with respect to the start-up investment) and if the remaining profit share is smaller than one.\(^5\)

To put it differently, the change in the venture capitalist’s expected pay-off in the low performance state of the project (‘his additional loss’) is lower than the change in his expected pay-off in the high performance state of the project (‘his additional revenue’) if his experience increases by a marginal unit. Therefore, the difference in expected pay-offs decreases with increasing experience because the venture capitalist does not have to share the enterprise’s revenue with a third party under the loan scheme.

\(^5\) The third term in the inequality is larger than the second term if the remaining profit share is smaller than one and if \(I\) is sufficiently large:

\[
I \kappa < I^\beta (I - p_l \theta)^{1-\beta} .
\]

Rearranging gives:

\[
0 < I \left( 1 - \kappa^{1-\beta} \right) - p_l \theta .
\]
However, the partial derivative does not show whether there exists a parameter constellation for which the difference in expected pay-off is negative and a parameter constellation for which the difference is positive. Alternatively put, the partial derivative does not indicate whether the venture capitalist really changes his choice of public subsidy if his level of experience changes.

A parameter constellation under which the venture capitalist chooses the public equity scheme results from an experience level equal to the minimum level under the public loan scheme $H_{LS}$, and a probability to reach the low performance state of the project that is equal to zero. Using equation [20], inserting the minimum experience level under the loan scheme and setting the probability to reach the low performance state of the project equal to zero results in:

\[
\hat{U}_{VC}^{DIF}\Big|\left(H = H_{LS}, P_l = 0\right) = I \left(\frac{1}{\beta - \delta} I^2 - \kappa \frac{1}{\beta - \delta} I\right).
\]

Rearranging the right-hand side shows that this difference in expected pay-offs is always larger than zero, since $\kappa \frac{1}{\beta - \delta} - \kappa \frac{2}{\beta - \delta} > 0$. Therefore, venture capitalists whose experience is not yet high enough to make high-risk investments profitable prefer the public equity scheme if the probability to reach the project’s low performance state is equal to zero independent from the other parameters of the model, i.e., the start-up investment and the effort elasticity of both contracting parties.

Note, that if the probability to reach the project’s low performance state is equal to zero, venture capitalists always prefer the public equity scheme independent from their experience. Inserting the critical level under the public loan scheme $H_{LS}$, and setting the probability to reach the project’s low performance state equal to zero in equation [20] shows that a venture capitalist with a high level of experience also prefers the public equity scheme.

Thus, an increase in the venture capitalist’s experience does not necessarily initiate a change in the venture capitalist’s choice of a public scheme.
Let me now analyse the impact of an increase in the innovation risk, i.e., in the probability to reach the low performance state of the project, on the venture capitalist’s choice.

The partial derivative of the difference in expected pay-offs with respect to the probability to reach the low performance state of the project \( p_l \) is given by:

\[
\frac{\partial \hat{U}_{VC}^{\text{DIFF}}}{\partial p_l} = CH^{\frac{\lambda - \delta}{1 - \beta - \delta}} \frac{1 - \delta}{1 - \beta - \delta} \left( p_l + (I - p_l) \kappa \right)^{\frac{\beta + \delta}{1 - \beta - \delta}} (1 - \kappa) + \]

\[
CH^{\frac{\lambda - \delta}{1 - \beta - \delta}} \frac{1 - \delta}{1 - \beta - \delta} (1 - \beta) (I - p_l \theta)^{\frac{\delta}{1 - \beta - \delta}} I^{\frac{\beta}{1 - \beta - \delta}} - \theta I
\]

Thus, the effect of an increase in the probability to reach the low performance state of the project depends on the experience of the venture capitalist. The higher the experience is, the larger the first term in equation [22] becomes which can lead to a positive impact of an increase in the probability to reach the low performance state on the difference in expected pay-offs.

If the partial derivative of the difference in expected pay-offs with respect to the probability to reach the project’s low performance state is positive, the public equity scheme is the dominant scheme of the two schemes analysed. Alternatively put, if the partial derivative is positive, venture capitalists never choose the public loan scheme independent of the experience, which they have accumulated in the past.

However, the public equity scheme does not dominate the public loan scheme for all parameter combinations. If the probability to reach the low performance state of the project is equal to one, and if the share of the venture capitalist’s losses covered by the government is equal to one, i.e., if \( \theta = 1 \), venture capitalists choose independent from their experience the public loan scheme, if the remaining profit share is smaller than one and if the start-up investment is sufficiently high (see equation [21]). Thus, the
choice of a venture capitalist regarding public loan and equity schemes changes with increasing experience and with increasing innovation risks.

Figure 3 depicts the results of a numerical solution of the venture capitalist’s choice for an increase in the probability to reach the project’s low performance state as well as for an increase in the experience. The black area shows parameter constellations for which the equity scheme leads to a higher expected pay-off than the loan scheme (for all possible $(\beta, \delta)$ combinations with $\beta \in [0.1,0.6]$, $\delta \in [0.1,0.6]$, and $\beta + \delta \leq 0.7$). In the case the venture capitalist is inexperienced, there exists an alternative interpretation: the black area shows parameter constellations for which the equity scheme results in a lower minimum level of experience in comparison to the loan scheme (i.e., $H_{ES} < H_{LS}$). The dark grey area gives parameter constellations for which the venture capitalist’s expected pay-off is higher under the loan scheme than under the equity scheme.

The white area can be interpreted as parameter constellations for which the venture capitalist is indifferent between the two schemes. This indifference is usually depicted as a line and not as an area. In this model, however, the use of an area is necessary because of the combination of the effort elasticity of the two contracting parties. A particular parameter constellation in Figure 3 is grey only if the loan scheme is chosen for all possible $(\beta, \delta)$-combinations; a particular parameter constellation is black only if the equity scheme is chosen for all possible $(\beta, \delta)$-combinations. A particular parameter constellation painted white contains some $(\beta, \delta)$-combinations for which the venture capitalist chooses the loan scheme, some $(\beta, \delta)$-combinations for which he chooses the equity scheme, and some $(\beta, \delta)$-combinations for which he is indifferent between the two schemes. Restricting the effort elasticity in the following way $\beta \in [0.1,0.6]$, $\delta \in [0.1,0.6]$, and $\beta + \delta \leq 0.7$ has the advantage that the white area is comparatively small.

An inexperienced venture capitalist chooses for many $(\theta, \kappa)$-combinations the public equity scheme, and only for some $(\theta, \kappa)$-combinations the public loan scheme. If the probability to reach the low performance state of the project is only moderate, the inexperienced venture capitalist almost exclusively chooses the public equity scheme (if the probability to reach
the low performance state is equal to zero, he chooses always the equity scheme). The reason for this is that the subsidized amount is higher under the equity scheme than under the loan scheme, and that the inexperienced venture capitalist does not care about sharing the revenue in the high performance state of the project because the enterprise’s revenue is moderate in this state that is a result of the venture capitalist’s low experience. With increasing innovation risk, the subsidized amount under the loan scheme increases so much that the inexperienced venture capitalist chooses the loan scheme for some $(\theta, \kappa)$-combinations.

A venture capitalist whose experience is equal to the minimum level without public subsidy chooses as his inexperienced counterpart the public equity scheme if the innovation risk is comparatively low. The difference in expected pay-offs of this venture capitalist reacts more sensitive to a change in the probability to reach the low performance state of the project than the difference in expected pay-offs of the inexperienced venture capitalist. If the probability to reach the low performance state of the project is very high, the venture capitalist with the minimum level of experience chooses the loan scheme for much more $(\theta, \kappa)$-combinations than his inexperienced counterpart.
Figure 3: Venture capitalists’ choice of public loan and equity schemes

Venture capitalist’s experience is equal to the minimum level under the loan scheme $\overline{H}_{LS}$

Venture capitalist’s experience is equal to the minimum level $\overline{H}$

Venture capitalist’s experience is equal to the critical level under the loan scheme $\overline{H}_{LS}$

- Public equity scheme leads to a higher expected pay-off
- Public loan scheme leads to a higher expected pay-off

Note: In the graphs, the covered loss and the remaining profit share are always larger than zero.
A venture capitalist whose experience is equal to the critical level under the loan scheme chooses almost exclusively the public loan scheme and only for some extreme \((\theta, \kappa)\)-combinations the public equity scheme. The reason for this is that the experienced venture capitalist has low incentives to share the enterprise’s revenue with a third party because the revenue is comparatively high due to the high experience of the venture capitalist.

Summing up, if the venture capitalists are inexperienced, they choose the public equity scheme for many \((\theta, \kappa)\)-combinations. However, the higher the experience of the venture capitalists is, the higher the number of \((\theta, \kappa)\)-combinations under which they choose the public loan scheme. Moreover, the number of \((\theta, \kappa)\)-combinations under which the loan scheme is chosen increases with the probability to reach the low performance state of the project for all venture capitalists whose experience is between the minimum level under the public loan scheme and the minimum level without public subsidies.

Thus, one advantage of offering venture capitalists a choice between public loan and equity schemes might be that different types of innovation risks are supported. For an appropriately chosen \((\theta, \kappa)\)-combination, inexperienced venture capitalists do not concentrate in a particular type of investment because the equity scheme encourage them to finance low-risk investments, while the loan scheme encourage them to finance high-risk investments. Moreover, offering two schemes may have the advantage that venture capitalists who are very inexperienced are encouraged to finance investments with low risk so that these venture capitalists can first accumulate a basic experience, while their counterparts who are less inexperienced are encouraged to finance investments with higher risks. And this may speed up the experience accumulation in the venture capital market.

5 Impact on Venture Capitalist’s Management Support

Experienced venture capitalists reduce their time for management support under public subsidies and this may slow down the experience accumulation of these experienced venture capitalists not intended by public subsidies. Therefore, the question to be addressed in the following is whether experienced venture capitalists which can choose between the
public loan and the public equity scheme choose the scheme which causes lower reductions in the management support and which, thus, affects less the speed of experience accumulation.

The difference in the time for management support under the two schemes results from equation [9] and [15]:

\[ V^{*}_{\text{DIF}} = V^{*}_{\text{ES}} - V^{*}_{\text{LS}} = \]

\[
\left[ (1-\alpha)^\beta \alpha^{1-\beta} \beta^\delta \delta^{1-\beta} H^{\beta+\lambda-1} \right]^{\frac{1}{1-\beta-\delta}} \left( p_i + (I - p_i) \kappa \right)^{\frac{1}{1-\beta-\delta}} - \\
\left[ (1-\alpha)^\beta \alpha^{1-\beta} \beta^\delta \delta^{1-\beta} H^{\beta+\lambda-1} \right]^{\frac{1}{1-\beta-\delta}} \left( I - p_i \theta \right)^{\frac{1}{1-\beta-\delta}}.
\]

The sign of the difference in the time for management support does not depend on the venture capitalists’ experience: the venture capitalists’ experience determines only the size of the difference. Whether the sign of the difference in the time for management support is negative or positive depends on the term in the second bracket that is identical to the term in equation [21]. There it has been shown that the sign of the term in the second bracket is negative if the start-up investment is sufficiently large and if the remaining profit share is smaller than one.

Thus, under the public equity scheme, experienced venture capitalists reduce their time for management support more than under the public loan scheme. This result follows from the specification of the revenue function of the enterprise. Without public subsidies, the venture capitalist’s incentives to support the management team are stronger in the high performance state than in the low performance state of the project because of the shift parameter that leads to higher revenue in the high performance state than in the low performance state of the project. Since the costs of the management support are identical in both states of nature, the value of an additional unit management support is higher in the high performance state than in the low performance state of the project. Under the public loan scheme, all available revenue is transferred to the government in the low performance state of the project in which the venture capitalist has low incentives to support the management compared to the high performance state, while under the public equity scheme, a part of the revenue is
transferred to the government in the high performance state of the project lowering the venture capitalist’s incentives for exerting management support significantly. Therefore, experienced venture capitalists reduce their management support less under the loan scheme than under the equity scheme.

In order to minimize the negative impact of public subsidies on the speed of experience accumulation in the case of experienced venture capitalists, the government should only offer the loan scheme because under the loan scheme, these venture capitalists have lower incentives to reduce their management support than under the equity scheme. We have to care about the change in management support induced by public subsidies because a reduction in the time for management support of the experienced venture capitalists can lead to a slow-down in the speed of experience accumulation that contradicts the intention of the public subsidy.

However, the solely offering of a public loan scheme seem not to be necessary since, as Figure 3 suggests, the venture capitalists whose experience is equal to the minimum level without public subsidy choose for many \((\theta, \kappa)\)-combinations the loan scheme, and this number of \((\theta, \kappa)\)-combinations increases if the venture capitalists’ experience increases. Thus, even if the government offers a choice between a public loan and a public equity scheme, experienced venture capitalists predominantly choose the scheme that induces a lower reduction in the management support.

6 Choice on the Basis of the Enterprise’s Surplus

Public loan and equity schemes cause a reduction in the total surplus of the enterprise financed by an experienced venture capitalist and this reduction are also costs of the public subsidies. In the following, I analyse therefore whether experienced venture capitalists choose the public subsidy that causes the lower reduction in the total surplus of the enterprise. Comparing the outcome of the choice on the basis of the difference in total surplus with the one on the basis of venture capitalist’s expected pay-offs might offer some insight in further advantages of offering a choice between these two schemes.
The difference in the total surplus is defined as the total surplus under the equity scheme containing the venture capitalist’s, the entrepreneur’s and the government’s expected pay-offs minus the total surplus under the loan scheme. Inserting the venture capitalist’s preferred revenue allocation, the difference in the enterprise’s total surplus is given by:

\[
\hat{U}^{TOTAL}_{DIF} = \hat{U}^{TOTAL}_{ES} - \hat{U}^{TOTAL}_{LS} = \\
\left( \frac{\lambda - \delta}{CH_{1-\beta - \delta}} \right) \left( p l + (1 - p l) \kappa \right)^{1-\beta - \delta} (1 - \delta + \beta) \left( \frac{p l + (1 - p l) \kappa}{1 - \kappa} \right) - \\
\left( \frac{\lambda - \delta}{CH_{1-\beta - \delta}} \right) \left( I - p l \right)^{1-\beta - \delta} \left( (1 - \delta + \beta) I + \delta p l \right)
\]

with \( H \in \left[ \underline{H}, \overline{H}_{LS} \right] \).

By inspection of equation [24] it follows that the sign of the difference in the total surplus does not depend on the venture capitalist’s level of experience. Alternatively put, the scheme that causes the lower reduction in the total surplus in the case of a venture capitalist whose experience is equal to minimum level without public subsidy also causes the lower reduction in the total surplus in the case of a venture capitalist whose experience is equal to the critical level. The difference in the total surplus does not depend on the venture capitalists’ level of experience since the money transfer by the government is taken into account. By contrast, the choice of the venture capitalist on the basis of the difference in his expected pay-offs depends on his experience.

Let me now turn to the question which scheme causes a lower reduction in the total surplus. In order to show that the public loan scheme causes a lower reduction in the surplus for almost all parameter combinations, I vary the value of the probability to reach the low performance state.

The difference in the total surplus increases monotonically with the probability to reach the low performance state (proof see appendix). Thus, if the difference in the total surplus is non-positive for a probability to
reach the low performance state equal to one, the loan scheme always causes a lower reduction in the total surplus than the equity scheme.  

Inserting $p_l = 1$ into the difference in the total surplus yields:

\[
\hat{U}_{DIF}^{TOTAL}(p_l = 1) = \\
CH^{1-\beta-\delta}(1+(I-1)\kappa)^{\frac{\beta + \delta}{\lambda - \delta}} (1-\delta + \beta)(1+(I-1)\kappa) + \\
CH^{1-\beta-\delta}(1+(I-1)\kappa)^{\frac{\beta + \delta}{\lambda - \delta}} (1-\kappa)(I-1)/(1-\beta) - \\
CH^{1-\beta-\delta} I^{1-\beta-\delta}(1-\theta)^{\delta} [(1-\delta + \beta)I + \delta \theta] 
\]

A sensitivity check of the parameters reveals that the difference is negative for sufficiently large start-up investments and a remaining profit share strictly smaller than one.

Thus, the loan scheme causes a lower reduction in the surplus than the equity scheme does and, therefore, the loan scheme causes lower costs. In order to minimize the costs of public subsidies, governments should only offer a public loan scheme. However, this is not necessary because experienced venture capitalists prefer the loan scheme for almost all $(\theta, \kappa)$-combinations as argued above.

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6 If the probability to reach the low performance state is equal to zero, the difference in the total surplus is non-positive for all parameter combinations (proof see appendix). Therefore, it is more efficient to use the loan scheme for all those enterprises that have low innovation risks. By contrast, inexperienced and experienced venture capitalists favour the equity scheme if the probability to reach the low performance state is equal to zero.

7 Using a start-up investment equal to 100, the difference in the surplus is negative as long as the remaining profit share $\kappa$ is lower than 0.99. Using a lower start-up investment than 100 demands a lower remaining profit share in order to ensure that the loan scheme causes a lower reduction in the total surplus than the equity scheme.
Summary

This paper has analysed possible advantages of offering venture capitalists a choice between public loan and public equity schemes. A loan and an equity scheme are combined in the German BTU-program. Under the loan scheme, the German government covers a share of the venture capitalists’ realized losses by refinancing and partly guaranteeing the venture capitalists’ participations. Under the equity scheme, a publicly supported co-investor invests in high-technology enterprises if the same investment amount is invested by a private venture capitalist.

Offering venture capitalists a choice between a public loan scheme and a public equity scheme may have the advantage that inexperienced venture capitalists have stronger incentives to enter the market for venture capital. This is because several types of enterprises, those with high and those with comparatively low innovation risks, are supported. These inexperienced venture capitalists may start to accumulate additional stage- and technology-specific experience in order to successfully select, monitor and support high-technology enterprises.

If only a public loan scheme is offered, inexperienced venture capitalists have low incentives to finance high-technology enterprises. The reason for this is that this scheme covers only losses realized in the state with low performance. However, inexperienced venture capitalists do not only worry about the high risk but also about the insufficient revenue in the state of the project with high performance that results from their lack in experience. Yet, offering only a public loan scheme would have the advantage, that experienced venture capitalists reduce their management support by less than under the public equity scheme. Moreover, the reduction in the enterprise’s surplus would be lower under the loan scheme than under the equity scheme.

If only a public equity scheme is offered, inexperienced venture capitalists have strong incentives to enter the venture capital market compared to a situation in which only a public loan scheme is offered. However, the public equity scheme reduces the total surplus of the enterprise more than the public loan scheme. This is because experienced venture capitalists
reduce their management support more under the equity scheme than under the loan scheme.

Offering venture capitalists a choice between these two schemes can lead to a positive self-selection. Inexperienced venture capitalists choose the public equity scheme. Under this scheme, inexperienced venture capitalists have higher incentives to enter the market and, thus, to accumulate experience in order to support high-technology enterprises successfully. By contrast, experienced venture capitalists choose the public loan scheme. Under the loan scheme, experienced venture capitalists have lower incentives to save on their management support than under the equity scheme.

8 Appendix

Non-Positive Difference in the Surplus

If the probability to reach the low performance state is equal to zero, the difference in the total surplus is given by:

$$
U_{DIF}^{TOTAL} \left( P_l = 0 \right) = CH^{1-\beta-\delta} I^{1-\beta-\delta} \kappa^{1-\beta-\delta} \left[ (1-\delta + \beta \kappa + (1-\kappa)/(1-\beta) \right] - \frac{1}{CH^{1-\beta-\delta} I^{1-\beta-\delta} (1-\delta + \beta) }
$$

The sign of the difference depends solely on the term in the brackets.

Suppose

$$
f_{DIF}^{TOTAL} = \kappa^{1-\beta-\delta} \left[ (1-\delta + \beta \kappa + (1-\kappa)/(1-\beta) \right] - (1-\delta + \beta).
$$

If $\kappa = 0$ then $f_{DIF}^{TOTAL}$ is smaller than zero. If $\kappa = 1$ then $f_{DIF}^{TOTAL}$ is equal to zero. The difference in the total surplus is non-positive for all parameter combinations fulfilling the restrictions if $f_{DIF}^{TOTAL}$ is monotonic increasing in $\kappa$. The partial derivative with respect to the remaining profit share is given by:
\[
\frac{\partial f_{\text{DIF}}^{\text{TOTAL}}}{\partial \kappa} = \frac{1}{1 - \beta - \delta} (1 - \delta + \beta) \kappa^{\frac{1}{1 - \beta - \delta} - 1} + \frac{\beta + \delta}{(1 - \beta - \delta)(1 - \beta)} \kappa^{\frac{\beta + \delta}{1 - \beta - \delta} - 1} - \frac{1}{(1 - \beta - \delta)(1 - \beta)} \kappa^{\frac{1}{1 - \beta - \delta} - 1}.
\]

Since \(\beta (1 - \beta \kappa) + \delta (1 - \kappa) + \kappa \delta \beta > 0\), \(f_{\text{DIF}}^{\text{TOTAL}}\) is monotonically increasing in \(\kappa\) and thus the difference in the total surplus is non-positive when the probability to reach the low performance state is equal zero.

**Increasing Difference in the Surplus**

The partial derivative of the difference in total surplus with respect to the probability to reach the project’s low performance state is given by:

\[
\frac{\partial \hat{U}_{\text{DIF}}^{\text{TOTAL}}}{\partial p_t} = CH^{1 - \beta - \delta} (p_t + (I - p_t) \kappa) g_{1 - \beta - \delta} \left[ (1 - \delta + \beta) (1 - \kappa) - \frac{(1 - \kappa)}{(1 - \beta)} \right] +
\]

\[
CH^{1 - \beta - \delta} \frac{\beta + \delta}{1 - \beta - \delta} (p_t + (I - p_t) \kappa) g_{1 - \beta - \delta} \left[ (1 - \kappa) (1 - \delta + \beta) (p_t + (I - p_t) \kappa) +
\]

\[
CH^{1 - \beta - \delta} \frac{\beta + \delta}{1 - \beta - \delta} (p_t + (I - p_t) \kappa) g_{1 - \beta - \delta} \left[ (1 - \kappa) (I - p_t) \frac{(1 - \kappa)}{(1 - \beta)} +
\]

\[
CH^{1 - \beta - \delta} I^{1 - \beta - \delta} \frac{\delta}{1 - \beta - \delta} (I - p_t \delta) g_{1 - \beta - \delta} \left[ (1 - \delta + \beta) I + \delta p_t \delta \right] -
\]

\[
CH^{1 - \beta - \delta} I^{1 - \beta - \delta} (I - p_t \delta) g_{1 - \beta - \delta} \delta \theta.
\]

Rearranging the first three terms gives:

\[
(p_t + (I - p_t) \kappa) (\beta + \delta) (1 - \delta + \beta) (1 - \beta) +
\]

\[
(p_t + (I - p_t) \kappa) \left[ (1 - \beta - \delta) (1 - \delta + \beta) (1 - \beta) - (1 - \beta - \delta) \right],
\]

\[
+ (\beta + \delta) (1 - \kappa) (I - p_t) > 0
\]

Rearranging the last two terms gives:

\[
(1 - \delta + \beta) I + \delta p_t \theta - (1 - \beta - \delta) \theta p_t + (1 - \beta - \delta) p_t \theta > 0.
\]
Therefore, the difference in total surplus is monotonically increasing in the probability to reach the low performance state of the project.

References


