A Tale of Two Policies: Prudential Regulation and Monetary Policy with Fragile Banks

by Ignazio Angeloni and Ester Faia

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Keywords: capital requirements, leverage, bank runs, combination policy, market liquidity.
A Tale of Two Policies: Prudential Regulation and Monetary Policy with Fragile Banks*

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Abstract

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1 Introduction

The financial crisis is producing, among other consequences, a change in perception on the respective roles of financial regulation and monetary policy. The pre-crisis common wisdom sounded roughly like this. Capital requirements and other prudential instruments were supposed to ensure, at least with high probability, the solvency of individual banks, with the implicit tenet that stable banks would automatically translate into a stable financial system. At the other corner, monetary policy should largely disregard financial matters and concentrate on pursuing price stability (a low and stable consumer price inflation) over some appropriate time horizon. The recent experience is changing this accepted wisdom in two ways. On the one hand, the traditional formal requirements for individual bank solvency (asset quality and adequate capital) are no longer seen as sufficient for systemic stability; regulators are increasingly called to adopt a macro-prudential approach (Borio [8], Morris and Shin [27]). On the other, monetary policy is asked to contribute to control systemic risks in the financial sector. This crisis has demonstrated that such risks can have disruptive implications for output and price stability down the road, and there is increasing evidence that monetary policy influences the degree of riskiness of the financial sector (the "risk-taking channel" of Borio and Zhu [9], to which Maddaloni and Peydró Alcalde [26] and Altunbas et. al [1] have recently provided supporting evidence). These ideas suggest the possibility of useful interactions between the conduct of monetary policy and that of systemic prudential regulation.

With this in mind, in this paper we move some steps towards studying, in an integrated framework, how bank regulation and monetary policy interact in fragile banking systems. Our first step is to propose a model that is simple enough and yet incorporates some key elements of financial fragility experienced in the recent crisis. In our model banks provide liquidity to both depositors and entrepreneurs. As in Diamond and Rajan ([14], [15]) banks have special skills in redeploying the projects’ assets in case of early liquidation. The firms’ cash flow is uncertain and this introduces uncertainty in the bank balance sheets. Banks, financed with deposits and capital, are exposed to runs, with a probability that increases with their deposit ratio or leverage (in our simple construct the two are directly related). Our arguments apply equally to traditional banks and to other leveraged entities, issuing uninsured short-maturity debt. The relationship between the bank and its "outside" financiers (depositors and capitalists) is disciplined by two incentives:
depositors can run the bank, forcing early liquidation of the loan and depriving bank capital of its return; and the bank can withhold its special skills, forcing a costly liquidation of the loan. The desired capital ratio is determined by trading-off balance sheet risk with the ability to obtain higher returns for outside investors in "good states" (no run), which increase with the share of deposits in the bank’s liability side.

Introducing these elements in the standard model provides a characterization of financial sector that is, we think, more apt to interpret the recent experience than traditional "financial accelerator" formulations\(^1\) where the transmission from the financial to the real sector takes place via the value of collateral rather than explicitly through banks. Endogenizing the banks’ capital structure also provides a natural way to bring in capital requirements and study their links with monetary policy. Our model allows, inter alia, to study how capital regulation, and potentially also liquidity ratios and other prudential instruments, influence economic performance, collective welfare and the optimal monetary policy.

Other papers have examined optimal monetary policy design and bank regulation, with specific reference to the pro-cyclicality of capital requirements (Blum and Hellwig [7], and Cecchetti and Li [11]). Two main elements differentiate our work. First, the previous studies take capital requirements as given and study the optimal monetary policy response, while we consider their interaction and possible combinations. Second, in earlier studies the loan market and bank capital structure were specified exogenously or ad hoc, while we incorporate optimizing bank behavior explicitly. Gertler and Karadi [19] have recently proposed a model with micro-founded banks related in spirit to ours. But their approach to modelling the bank is different, and, more importantly, their aim is to look at the effects of unconventional monetary policies, while we explore the interplay between (conventional) monetary policy and bank regulation. Their focus is more on crisis management, ours on crisis prevention.

Our main conclusions are as follows. From the theoretical model we find that in the optimal bank deposit ratio (complement to one of the bank capital ratio) is positively related to: 1) the bank expected return on assets (ROA); 2) the uncertainty of the projects outcomes; 3) the banks’ special skills in liquidating projects, and negatively related to 4) the return on bank deposits. These

\(^1\) See Bernanke, Gertler and Gilchrist [3] for a pioneering work and later formulations from Christiano, Motto and Rostagno [13] among others.
properties echo the main building blocks of the Diamond-Rajan banking model. The intuition, roughly speaking, is that increases in 1), 2) and 3) raise the return to outside bank investors of a unitary increase in deposits, the first by increasing the expected return in good states (no run), the second by reducing its cost in bad states (run), the third by increasing the expected return relative to the cost between the two states. A higher bank deposit interest rate reduces deposits from the supply side, because it increases, ceteris paribus, the probability of run. From the empirical analysis of the calibrated model, a number of results emerge. A monetary expansion or a positive productivity boom increase bank leverage and risk. The transmission from productivity changes to bank risk is stronger when the riskiness of the projects financed by the bank is low. Pro-cyclical capital requirements are destabilising; they amplify the response of output and inflation to other shocks and may generate unstable dynamics. Monetary policy cannot neutralise this effect fully. Anti-cyclical ratios have the opposite effect (stabilising). The optimal policy combination includes mildly anti-cyclical capital ratios and a monetary policy rule that reacts to inflation and "leans-against-the-wind". Two alternative forms of “leaning” are examined: a positive response of the policy-determined interest rate to asset prices or to bank leverage. The second tends to be marginally better in presence of regulated capital ratios, assuming capital regulation is not pro-cyclical.

The rest of the paper is as follows. Section 2 describes the model. Section 3 characterizes the transmission mechanism with and without banks. Section 4 examines the sensitivity to investment risk and the performance of leaning-against-the-wind monetary policy. Section 5 discusses the role of Basel capital ratios and how they affect the transmission mechanism in our model. Sections 6 and 7 deal with optimal policy and welfare. Section 8 concludes.

2 The Baseline Model

The starting point is a conventional DSGE model with nominal rigidities. To this, we add optimizing banks and subsequently a prudential regulatory authority setting capital ratios on banks.

The economy is populated by workers/depositors/bank capitalists, entrepreneurs and bankers (meaning, bank managers). Workers are risk averse, while entrepreneurs and bankers are risk neutral. The central bank sets the nominal interest rate.
Entrepreneurs launch projects that require an initial investment; this is financed by the bank, that raises money from depositors and bank capitalists. Bank capitalists claims the residual value after depositors are paid out. As in Diamond and Rajan ([14], [15]), the bank capital structure is determined by the bank managers, who act on behalf of outside investors (depositors and capitalists combined) by maximizing their overall return. If the return on bank assets is low and the bank is not able to pay depositors in full there is a run on the bank, in which case the bank capital holders get zero while depositors get the market value of the liquidated loan.

2.1 Households

There is a continuum of identical households who consume, save and work. Households include real sector "workers" and bankers. Following Gertler and Karadi [19] we assume that in every period a fraction $\gamma$ of household members are bankers and a fraction $(1 - \gamma)$ workers. Bankers have a finite tenure in their job; with probability $\theta$ they remain bankers every period, otherwise they become workers. A corresponding fraction of workers become bankers every period, so that the share of bankers $\gamma$ remains constant over time. This finite survival scheme is needed to avoid that bankers accumulate enough wealth to ease up the liquidity constraint; we will return on this point later. Workers earn wages and return them to the household; similarly bankers earn a fee from their services, that is returned to the household. Consumption and investment decisions are made by the household, pooling all available resources.

Households maximize the following discounted sum of utilities:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

where $C_t$ denotes aggregate consumption and $N_t$ denotes labour hours. The households receive at the beginning of time $t$ a real labour income $\frac{W_t}{p_t} N_t$. As households can work both in the industrial and in the banking sector, we consider labour income as inclusive of the fees workers receive as managers of the banks. Those fees are determined in the next section.

Households save and invest in bank deposits and bank capital. Deposits, $D_t$, pay a gross nominal return $R_t$ one period later. Alternatively, and without qualitative change in the analysis, we could assume the existence of another asset, say government bonds; in this case deposits can be
assumed to be held for their transaction or liquidity services, that justify a wedge between the bond rate and the deposit rate. Finally, households are the owners of both the monopolistic competitive sector and the banking sector. Because of this they are entitled to receive from the monopolistic sector nominal profits for an amount, $\Theta_t$, and from any banker who ceases activity nominal profits of an amount, $\Pi_t$. The budget constraint reads as follows:

$$P_tC_t + T_t + D_{t+1} \leq W_tN_t + \Theta_t + \Pi_t + R_tD_t$$

Note that the return from, and the investment in, bank capital do not appear in equation 2. The reason is that we have assumed, as explained later, that all returns on bank capital are reinvested every period.

Households choose the set of processes $\{C_t, N_t\}_{t=0}^\infty$ and assets $\{D_{t+1}, B_{t+1}\}_{t=0}^\infty$, taking as given the set of processes $\{P_t, W_t, R_t\}_{t=0}^\infty$ and the initial wealth $D_0, B_0$ so as to maximize 1 subject to 2. The following optimality conditions hold:

$$\frac{W_t}{P_t} = \frac{U_{n,t}}{U_{c,t}}$$

$$U_{c,t} = \beta E_t\{R_tU_{c,t+1}\}$$

Equation 3 gives the optimal choice for labour supply. Note that, since labor income includes the banker’s fee, the supply of labor determined in 3 depends also on this fee. Equation 4 gives the Euler condition with respect to deposits and government bonds. Optimality requires that the first order conditions and No-Ponzi game conditions are simultaneously satisfied.

### 2.2 Banks

There is in the economy a large number ($L_t$) of investment projects, each run by an entrepreneur. The project lasts two periods and requires an initial investment. Each project’s size is normalized to unity (think of one machine) and its price is $Q_t$. The entrepreneur has no internal funds, but receives finance from a bank. We assume a competitive banking system: bank profits are driven to

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2 In principle this fee is endogenous and depends (as will be seen later in the paper) on the bank capital structure. In practice, given the small ratio of bankers relative to workers, this component is negligible and we neglect its endogeneity for simplicity.
zero except for a fee, specified below. Likewise, banks have no internal funds but receive finance from two classes of agents: holders of demand deposits and capitalists. Total bank loans (equal to the number of projects multiplied by their unit price) are equal to the sum of deposits \(D_t\) and bank capital, \((BK_t)\). The aggregate bank balance sheet is:

\[ Q_tL_t = D_t + BK_t \]  

(5)

The capital structure is determined by the banker, whose function is to optimize ex-ante the bank capital structure (share of demand deposits and of capital) on behalf of depositors and bank capitalists. The banker’s task is to find the capital structure that maximizes the combined expected return of depositors and capitalists, in exchange for a fee. Individual depositors are served sequentially and fully as they come to the bank for withdrawal; capitalists instead are rewarded pro-quota after all depositors are served. This payoff mechanism exposes the bank to runs, that occur when the return from the project is insufficient to reimburse all depositors. As soon as they realize that the payoff is insufficient they run the bank and force the liquidation of the project. The timing is as follows. At time \(t\), the banker decides the optimal capital structure, expressed by the ratio of deposits to the total cost of the project, \(d_t\), collects the funds, lends, and then the project is undertaken. At time \(t+1\), the project’s outcome is known and payments to depositors, capitalists and the banker are made, as discussed below. A new round of projects starts.

Generalizing Diamond and Rajan [14], [15], we assume that the return of each project for the bank is equal to an expected value, \(R_{A,t}\), plus a random shock with a uniform distribution with dispersion \(h\). Therefore, the project outcome is \(R_{A,t} + x_{j,t}\), where \(x_{j,t}\) spans across the interval \([-h; h]\) with probability \(\frac{1}{2h}\). We assume \(h\) to be constant across projects and time, but will run sensitivity analyses on its value.

Each project is financed by one bank. Our bank is a \textit{relationship lender}: by lending it acquires a specialized non-sellable knowledge of the characteristics of the project. This knowledge determines an advantage in extracting value from it before the project is concluded, relative to other agents. Let the ratio of the value for the outsider (liquidation value) to the value for the bank be \(0 < \lambda < 1\). Again we assume \(\lambda\) to be constant, but we will examine the sensitivity of the results to changes in its value.
Consider the payoffs to each of our players, in the situation where the realisation of \( x_{j,t} \) is negative, as depicted in graph 1, point C (time subscripts are omitted in the graph for brevity). There are three cases.

Case A: Run for sure. The outcome of the project is too low to pay depositors. This happens if gross deposits (including interest) are located to the right-hand-side of C in the graph, where \( R_{A,t} + x_{j,t} < R_t d_t \). Payoffs in case of run are distributed as follows. Capitalists receive the leftover after depositors are served, so they get zero in this case. Depositors alone (without bank) would get only a fraction \( \lambda(R_{A,t} + x_{j,t}) \) of the project’s outcome; the remainder \((1 - \lambda)(R_{A,t} + x_{j,t})\) is shared between depositors and the bank depending on bargaining power. Following Diamond and Rajan [14] and [15] we assume this extra return is split in half (other assumptions are possible without qualitative change in the results). Therefore, depositors end up with

\[
\frac{(1 + \lambda)(R_{A,t} + x_{j,t})}{2}
\]

and the bank with

\[
\frac{(1 - \lambda)(R_{A,t} + x_{j,t})}{2}
\]

Note that we have assumed that bank runs do not destroy value per se; liquidation by depositors alone is equivalent to liquidation by the bank. The model can easily be extended to include a specific extra cost from bank runs.

Case B: Run only without the bank. The project outcome is high enough to allow depositors to be served if the project’s value is extracted by the bank, but not otherwise. This happens if gross deposits (including interest) are located in the segment BC in the graph, i.e the range where \( \lambda(R_{A,t} + x_{j,t}) < R_t d_t \leq (R_{A,t} + x_{j,t}) \). In this case, the capitalists alone cannot avoid the run, but with the bank they can. So depositors are paid in full, \( R_t d_t \), and the remainder is split in half between the banker and the capitalists, each getting \( \frac{R_{A,t} + x_{j,t} - R_t d_t}{2} \). Total payment to outsiders is \( \frac{R_{A,t} + x_{j,t} + R_t d_t}{2} \).

Case C: No run for sure. The project’s outcome is high enough to allow all depositors to be served, with or without the bank’s participation. This happens in the zone AB, where \( R_t d_t \leq \lambda(R_{A,t} + x_{j,t}) \). Depositors get \( R_t d_t \). However, unlike in the previous case, now the capitalists have a
higher bargaining power because could decide to liquidate the project alone and pay the depositors in full, getting \( \lambda(R_{A,t} + x_{j,t}) - R_t d_t \); this is thus a lower threshold for them. The banker can extract \((R_{A,t} + x_{j,t}) - R_t d_t\), and again we assume that the capitalist and the bank split this extra return in half. Therefore, the bank gets:

\[
\frac{\{(R_{A,t} + x_{j,t}) - R_t d_t\} - [\lambda(R_{A,t} + x_{j,t}) - R_t d_t]}{2} = \frac{(R_{A,t} - \lambda)(1 + x_{j,t})}{2}
\]

This is less than what the capitalist gets. Total payment to outsiders is:

\[
\frac{(1 + \lambda)(R_{A,t} + x_{j,t})}{2}
\]

We can now write the expected value of total payments to outsiders as follows:

\[
\frac{1}{2h} \int_{-h}^{R_t d_t - R_{A,t}} \left( \frac{1 + \lambda}{2} \right)(R_{A,t} + x_{j,t}) dx_{j,t} + \frac{1}{2h} \int_{R_t d_t - R_{A,t}}^{h} \left( \frac{1 + \lambda}{2} \right)(R_{A,t} + x_{j,t}) + R_t d_t dx_{j,t} + \\
+ \frac{1}{2h} \int_{h}^{R_t d_t - R_{A,t}} \left( \frac{1 + \lambda}{2} \right)(R_{A,t} + x_{j,t}) dx_{j,t}
\]

The three terms express the payoffs to outsiders in the three cases described above, in order. The banker’s problem is to maximise expected total payments to outsiders by choosing the suitable value of \( d_t \).

It can be shown (see Appendix) that the value of \( d_t \) that maximises equation 8 is comprised in the interval \( \lambda \frac{R_{A,t} + h}{R_t} < d_t < \frac{R_{A,t} + h}{R_t} \). In this zone, the third integral in the equation vanishes and the expression reduces to

\[
\frac{1}{2h} \int_{-h}^{R_t d_t - R_{A,t}} \left( \frac{1 + \lambda}{2} \right)(R_{A,t} + x_{j,t}) dx_{j,t} + \frac{1}{2h} \int_{R_t d_t - R_{A,t}}^{R_{A,t}} \left( \frac{1 + \lambda}{2} \right)(R_{A,t} + x_{j,t}) + R_t d_t dx_{j,t}
\]

Consider equation 9 in detail. A marginal increase in the deposit ratio has three effects. First, it increases the range of \( x_{j,t} \) where a run occurs, by raising the upper limit of the first integral; this effect increases the overall return to outsiders by \( \frac{1}{2h} \left( \frac{1 + \lambda}{2} R_t d_t \right) R_t \). Second, it decreases the range
of \(x_{j,t}\) where a run does not occur, by raising the lower limit of the second integral; the effect of this on the return to outsiders is negative and equal to \(-\frac{1}{2\pi}R_t^2d_t\). Third, it increases the return to outsiders for each value of \(x_{j,t}\) where a run does not occur; this effect is \(\frac{1}{2\pi} \left( \int_{R_t d_t - R_{A,t}}^{h} \frac{1}{2} dx_{j,t} \right) R_t = \frac{1}{2\pi} \left( \frac{h - R_t d_t + R_{A,t}}{2} \right) R_t\). Equating to zero the sum of the three effects and solving for \(d_t\) yields the following equilibrium condition

\[
d_t = \frac{1}{R_t} \frac{R_{A,t} + h}{2 - \lambda}.
\]

Since the second derivative is negative, this is the optimal value of \(d_t\).

The optimal deposit ratio depends positively on \(h, \lambda\) and \(R_{A,t}\), and negatively on \(R_t\). An increase of \(R_t\) reduces deposits because it increases the probability of run. Moreover, an increase in \(R_{A,t}\) raises the marginal return in the no-run case (the third effect just mentioned), while it does not affect the other two effects, hence it raises \(d_t\). An increase in \(\lambda\) reduces the cost in the run case (first effect), while not affecting the others, so it raises \(d_t\). The effect of \(h\) is more tricky. At first sight it would seem that an increase in the dispersion of the project outcomes, moving the extreme values of the distribution both upwards and downwards, should be symmetric and have no effect. But this is not the case. When \(h\) increases, the probability of each given project outcome \(\frac{1}{2\pi}\) falls. Hence the expected loss stemming from the change in the relative probabilities (sum of the first two effects) falls, but the marginal gain in the no-run case (third term) does not, because the upper limit increases. The marginal effect is \(\frac{R_t}{2}\), because depositors get the full return, but half is lost by the capitalist to the banker. Hence, the increase of \(h\) has on \(d_t\) a positive effect, as \(R_{A,t}\).

### 2.2.1 A measure of bank fragility

A natural measure of bank riskiness is the probability of a run occurring. This can be written as:

\[
z_t = \frac{1}{2h} \int_{-h}^{R_t - R_{A,t}} dx_{j,t} = \frac{1}{2} \left( 1 - \frac{R_{A,t} - R_t d_t}{h} \right) = \frac{1}{2} - \frac{R_{A,t}(1 - \lambda) - h}{2h(2 - \lambda)}
\]

Note that \(z_t\) ranges between 0 and 1 for plausible values of \(\lambda\) and \(h\), between 0.3 and 1 (the chart below, where \(x\) stands for \(h\) and \(y\) stands for \(\lambda\), is constructed for \(R_{A,t} = 1.03\)). As \(\lambda\) and
\( h \) fall below 0.3 the probability or run tends to fall below zero; at this point \( \frac{R_{A,t} + h}{2 - \lambda} \) falls below \( R_{A,t} + h \), and the marginal equilibrium condition 10 ceases to hold. Deposits can never fall below the level where a run becomes impossible. Some degree of bank risk is always optimal in this model.

\[
\text{2.2.2 Aggregation}
\]

In the aggregate, the amount invested in every period is \( Q_t L_t \). The total amount of deposits in the economy is

\[
D_t = \frac{Q_t L_t R_{A,t} + h}{R_t \frac{R_{A,t} + h}{2 - \lambda}}
\]

and the bank’s optimal capital is:

\[
BK_t = (1 - \frac{1}{R_t \frac{R_{A,t} + h}{2 - \lambda}})Q_t L_t
\]

Firms are financed by the intermediary for an amount:

\[
Q_t L_t = Q_t K_t
\]
The above expressions suggest that following a contractionary monetary policy, raising \( R_t \), the optimal amount of bank capital increases on impact. The effect of cyclical up or downswings on the capital structure is more complex, as it depends on several counterbalancing factors, including the dynamics, as the empirical results will show.

2.2.3 Accumulation of bank capital

Equation 13 is a demand for bank capital for any given level of investment, \( Q_t \), and interest rate structure \( (R_t, R_{\lambda t}) \). As to the supply, we assume that bank capital accumulates in the form of undistributed dividends. After remunerating depositors and paying the competitive fee to the banker, a return accrues to the bank capitalist as retained earning (including any reinvested dividends). Bank capital accumulates from retained earnings as follows:

\[
BK_{t+1} = \theta[\text{BK}_t + \text{RITCAP}_{t+1}Q_{t+1}K_{t+1}]
\]

where \( \text{RITCAP}_{t+1} \) is the unitary return to the capitalist. The parameter \( \theta \) is a decay rate, inclusive of both the bank survival rate (Gertler and Karadi [19]) and bank capital depreciation. \( \text{RITCAP}_{t+1} \) can be derived from equation 9 as follows:

\[
\text{RITCAP}_{t+1} = \frac{1}{2h} \int_{R_{t+1}d_{t+1} - R_{\lambda t+1}}^{h} \frac{(R_{\lambda t+1} + x_{j,t+1}) - R_{t+1}d_{t+1}}{2} dx_{j,t+1} = \frac{(R_{\lambda t+1} + h - R_{t+1}d_{t+1})^2}{8h}
\]

Note that this expression considers only the no-run state because if a run occurs the capitalist receives no return. The accumulation of bank capital obtained substituting 16 into 15:

\[
BK_{t+1} = \theta[\text{BK}_t + \frac{(R_{\lambda t+1} + h - R_{t+1}d_{t+1})^2}{8h}Q_{t+1}K_{t+1}]
\]

2.3 Producers

Each firm \( i \) has monopolistic power in the production of its own variety and therefore has leverage in setting the price. In changing prices it faces a quadratic cost equal to \( \vartheta \left( \frac{P_{i(t)}}{P_{i(t-1)}} - \pi \right)^2 \), where \( \pi \) is the steady state inflation rate and where the parameter \( \vartheta \) measures the degree of nominal price rigidity. The higher \( \vartheta \) the more sluggish is the adjustment of nominal prices. In the particular case
of \( \vartheta = 0 \), prices are flexible. Each firm assembles labour (supplied by the workers) and (finished) entrepreneurial capital to operate a constant return to scale production function for the variety \( i \) of the intermediate good:

\[
Y_t(i) = A_t F(N_t(i), K_t(i))
\]  

(18)

Each monopolistic firm chooses a sequence \( \{K_t(i), L_t(i), P_t(i)\} \), taking nominal wage rates \( W_t \) and the rental rate of capital \( Z_t \), as given, in order to maximize expected discounted nominal profits:

\[
E_0 \{ \sum_{t=0}^{\infty} \Lambda_{0,t} [P_t Y_t(i) - (W_t N_t(i) + Z_t K_t(i))] - \frac{\vartheta}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - \pi \right)^2 \}
\]

(19)

subject to the constraint \( A_t F_t(\bullet) \leq Y_t(i) \), where \( \Lambda_{0,t} \) is the households’ stochastic discount factor.

Let’s denote by \( \{mc_t\}_{t=0}^{\infty} \) the sequence of Lagrange multipliers on the above demand constraint, and by \( \tilde{p}_t \equiv \frac{P(i)}{P_t} \) the relative price of variety \( i \). The first order conditions of the above problem read:

\[
\frac{W_t}{P_t(i)} = mc_t A_t F_{n,t}
\]

(20)

\[
\frac{Z_t}{P_t(i)} = mc_t A_t F_{k,t}
\]

(21)

\[
0 = Y_t \tilde{p}_t^{-\varepsilon} ((1 - \varepsilon) + \varepsilon mc_t - \vartheta \left[ \pi_t \frac{\tilde{p}_t}{\tilde{p}_{t-1}} - \pi \right] \frac{\pi_t}{\tilde{p}_{t-1}} +
\]

\[
+ \vartheta \left[ \pi_{t+1} \frac{\tilde{p}_{t+1}}{\tilde{p}_t} - \pi \right] \pi_{t+1} \frac{\tilde{p}_{t+1}}{\tilde{p}_t}
\]

(22)

where \( F_{n,t} \) is the marginal product of labour, \( F_{k,t} \) the marginal product of capital and \( \pi_t = \frac{P_t}{P_{t-1}} \) is the gross aggregate inflation rate (its steady state value, \( \pi \), is equal to 1). Notice that all firms employ an identical capital/labour ratio in equilibrium, so individual prices are all equal in equilibrium. The Lagrange multiplier \( mc_t \) plays the role of the real marginal cost of production. In a symmetric equilibrium \( \tilde{p}_t = 1 \). This allows to rewrite equation 22 in the following form:
\[ U_{c,t}(\pi_t - \pi) = \beta E_t\{U_{c,t+1}(\pi_{t+1} - \pi)\pi_{t+1}\} + \\
+ U_{c,t}A_tF_t(\bullet)\frac{\varepsilon}{\eta}(mc_t - \frac{\varepsilon - 1}{\varepsilon}) \]

The above equation is a non-linear forward looking New-Keynesian Phillips curve, in which deviations of the real marginal cost from its desired steady state value are the driving force of inflation.\(^3\)

### 2.3.1 Capital Producers

A competitive sector of capital producers combine investment (expressed in the same composite as the final good, hence with price \(P_t\)) and existing capital stock to produce new capital goods. This activity entails physical adjustment costs. The corresponding CRS production function is \(\phi(\frac{I_t}{K_t})K_t\), so that capital accumulation obeys:

\[ K_{t+1} = (1 - \delta)K_t + \phi(\frac{I_t}{K_t})K_t \]  

where \(\phi(\bullet)\) is increasing and convex.

Define \(Q_t\) as the re-sell price of the capital good. Capital producers maximize profits \(Q_t\phi(\frac{I_t}{K_t})K_t - P_tI_t\), implying the following first order condition:

\[ Q_t\phi'(\frac{I_t}{K_t}) = P_t \]

The gross (nominal) return from holding one unit of capital between \(t\) and \(t+1\) is composed of the rental rate plus the re-sell price of capital (net of depreciation and physical adjustment costs):

\[ Y^k_t \equiv Z_t + Q_t((1 - \delta) - \phi(\frac{I_t}{K_t})\frac{I_t}{K_t} + \phi(\frac{I_t}{K_t})) \]

The gross (real) return to entrepreneurs from holding a unit of capital between \(t\) and \(t+1\) is equalized in equilibrium to the gross (real) return that entrepreneurs return to banks for their loan services, \(R_{A,t+1}\):

\(^3\)Woodford [33].
\[
\frac{R_{t+1}}{\pi_{t+1}} = \frac{Y_{t+1}^k}{Q_t}
\]  

(27)

### 2.4 Goods Market Clearing

Equilibrium in the final good market requires that the production of the final good equals the sum of private consumption by households and entrepreneurs, investment, public spending, and the resource costs that originate from the adjustment of prices:

\[
Y_t = C_t + I_t + G_t + \frac{\phi}{2} (\pi_t - \pi)^2
\]  

(28)

In the above equation, \(G_t\) is government consumption of the final good which evolves exogenously and is assumed to be financed by lump sum taxes.

### 2.5 Monetary Policy

We assume that monetary policy is conducted by means of an interest rate reaction function of this form:

\[
\ln \left( \frac{1 + R_t}{1 + R} \right) = (1 - \phi_r) \left[ \phi_{\pi} \ln \left( \frac{\pi_t}{\pi} \right) + \phi_q \ln \left( \frac{Y_t}{Y} \right) + \phi_y \ln \left( \frac{Q_t}{Q} \right) + \phi_d \ln \Delta \left( \frac{d_t}{d} \right) \right] + \phi_r \ln \left( \frac{1 + R_{t-1}}{1 + R} \right)
\]  

(29)

All variables are deviations from the target or steady state (symbols without time subscript). Note that the reaction function includes two alternative terms that express leaning-against-the-wind behavior, respectively a reaction to asset prices \((Q_t)\) or to the deposit ratio \((d_t)\). Our approach will consist in finding policy specifications \{\(\phi_{\pi}, \phi_q, \phi_y, \phi_d, \phi_r\)\} that maximize household welfare\(^4\)

We solve the model by computing a second order approximation of the policy functions around the non-stochastic steady state.

2.6 Parameter values

Household preferences and production. The time unit is the quarter. The utility function of households is 
\[ U(C_t, N_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} + \nu \log(1 - N_t), \text{ with } \sigma = 2, \] as in most real business cycle literature. We set \( \nu \) set equal to 3, chosen in such a way to generate a steady-state level of employment \( N \approx 0.3 \). We set the discount factor \( \beta = 0.99 \), so that the annual real interest rate is equal to 4\%.

We assume a Cobb-Douglas production function 
\[ F(\bullet) = K_t^\alpha (N_t)^{1-\alpha}, \text{ with } \alpha = 0.3. \] The quarterly aggregate capital depreciation rate \( \delta \) is 0.025, the elasticity of substitution between varieties 6. The adjustment cost parameter is set so that the volatility of investment is larger than the volatility of output, consistently with empirical evidence: this implies an elasticity of asset prices to investment of 2.

In order to parameterize the degree of price stickiness \( \vartheta \), we observe that by log-linearizing equation 23 we can obtain an elasticity of inflation to real marginal cost (normalized by the steady-state level of output)\(^5\) that takes the form \( \frac{\varepsilon - 1}{\vartheta} \). This allows a direct comparison with empirical studies on the New-Keynesian Phillips curve such as Gali and Gertler [18] and Sbordone [29] using Calvo-Yun approach. In those studies, the slope coefficient of the log-linear Phillips curve can be expressed as 
\[ \frac{(1-\hat{\vartheta})(1-\beta \hat{\vartheta})}{\vartheta}, \] where \( \hat{\vartheta} \) is the probability of not resetting the price in any given period in the Calvo-Yun model. For any given values of \( \varepsilon \), which entails a choice of the steady state level of the markup, we can thus build a mapping between the frequency of price adjustment in the Calvo-Yun model \( \frac{1}{1-\vartheta} \) and the degree of price stickiness \( \vartheta \) in the Rotemberg setup. The recent New Keynesian literature has usually considered a frequency of price adjustment of four quarters as realistic. Recently, Bils and Klenow [4] have argued that the observed frequency of price adjustment in the US is higher, in the order of two quarters. As a benchmark, we parameterize 
\[ \frac{1}{1-\vartheta} = 4, \] which implies \( \hat{\vartheta} = 0.75 \). Given \( \varepsilon = 6 \), the resulting stickiness parameter satisfies 
\[ \vartheta = \frac{Y(\varepsilon-1)}{(1-\vartheta)(1-\beta \vartheta)} \approx 30, \] where \( Y \) is steady-state output.

Banks. To calibrate \( h \) we have calculated the average dispersion of corporate returns from the data constructed by Bloom et al. [6] (we are grateful to Nick Bloom for giving us access to his data), which is 0.31, and multiplied this by the square root of 3, the ratio of the maximum

---

\(^5\)To produce a slope coefficient directly comparable to the empirical literature on the New Keynesian Phillips curve this elasticity needs to be normalized by the level of output when the price adjustment cost factor is not explicitly proportional to output, as assumed here.
deviation to the standard deviation of a uniform distribution. The result, 0.5, is our benchmark.

One way to interpret $\lambda$ is to see it as the ratio of two present values of the project, the first at the interest rate applied to firms’ external finance, the second discounted at the bank internal finance rate (the money market rate). A benchmark estimate can be obtained by taking the historical ratio between the money market rate and the lending rate. In the US over the last 20 years, based on 30-year mortgage loans, this ratio has been around 3 percent. This leads to a value of $\lambda$ around 0.6. In the empirical analyses we have chosen 0.5 and then checked the sensitivity to a higher value, 0.8. Finally we parametrize the survival rate of banks at 0.97.

Note that, in principle, $h$ and $\lambda$ could be considered endogenous to the state of the economy. Recent work by Bloom ([5], [6]) has shown that the dispersion of corporate returns is anticyclical: cyclical slowdowns are systematically associated with a higher variance returns (actually, higher uncertainty of corporate returns leads business cycle downturns). The link between $h$ and the cycle is a further element that could be added into our framework. In this paper we have used a fixed $h$ benchmark throughout and done sensitivity analysis around this value.

*Shocks.* Total factor productivity is assumed to evolve as:

$$A_t = A_t^{\rho_\alpha} \exp(\varepsilon_t^{\alpha})$$  \hspace{1cm} (30)

where the steady-state value $A$ is normalized to unity (which in turn implies $\omega_m = 1$) and where $\varepsilon_t^{\alpha}$ is an i.i.d. shock with standard deviation $\sigma_\alpha$. In line with the real business cycle literature, we set $\rho_\alpha = 0.95$ and $\sigma_\alpha = 0.008$. Log-government consumption is assumed to evolve according to the following process:

$$\ln(\frac{G_t}{G}) = \rho_g \ln(\frac{G_t-1}{G}) + \varepsilon_t^{g}$$

where $G$ is the steady-state share of government consumption (set in such a way that $\frac{G}{G} = 0.25$) and $\varepsilon_t^{g}$ is an i.i.d. shock with standard deviation $\sigma_g$. We follow the empirical evidence for the U.S. in Perotti [28] and set $\sigma_g = 0.0074$ and $\rho_g = 0.9$.

We introduce a monetary policy shock as an additive disturbance to the interest rate set through the monetary policy rule. The monetary policy shock is assumed to have zero or
moderate persistence, depending on different cases examined. Following empirical evidence for US and Europe, the standard deviations of the shocks is set to 0.006.

3 Transmission Channels With and Without Banks

To begin with, we look at the responses to three shocks, one at a time, with parameters kept at their benchmark values: a one-time (total factor) productivity rise; a (moderately persistent) monetary restriction (persistence parameter 0.3); a one-time positive shock to the marginal return on capital (MRK), interpreted as a positive asset price shock. The latter shock, in particular, will be useful later when we will analyse monetary rules including a response to asset prices.

As one would expect, the productivity shock (figure 1) reduces inflation and increases output on impact. Investment and Tobin’s Q rise. The policy-driven short term interest rate $R_t$ declines following the fall in inflation, and the bank return on assets ($R_{A,t}$ or ROA) declines more or less in line. The lower interest rates raise deposits and tilt the composition of the bank balance sheet towards higher leverage and risk.

In the monetary shock (figure 2) both inflation and output drop on impact, as in all standard models, with a corresponding fall in investment and Tobin’s Q. ROA rises with the interest rate; in the figure the spread between the two rises, but this is sensitive to parameter values – generally speaking, with a persistent shock or with interest rate smoothing, the spread tends to rise after a monetary restriction. Banks lose deposits and replace them with capital, leading to a less risky balance sheet composition; bank riskiness drops on impact – a "risk taking channel" of monetary policy operating in reverse.

Figure 3 shows that in response to a positive asset market shock output and inflation rise on impact; the rise in investment fuelled by the asset price boom drives up ROA above the short term rate. Bank risk thus declines, but later rises above baseline driven by the high value of the deposit ratio.

The above results together suggest that the co-movements of bank risk on one side, and interest rates and output on the other, are not systematic: they depend on the nature of the shock. Higher policy-driven interest rates lead to lower bank risk, but not if there is a concurrent investment boom, for example generated by asset market exuberance. In this case banks become more risky.
in spite of higher policy rates.

To examine how banks affect the transmission, we compare two models, one with benchmark parameters, the other obtained setting $\lambda = 0.9$. A value of $\lambda$ close to unity means that banks lose most of their advantage as relationship lenders. In figure 4, constructed assuming a TFP shock, the response from the standard model with banks is shown with a solid line, that from the model with a near-unitary $\lambda$ with a dashed line. We can see that a low $\lambda$ amplifies the expansionary effect of this shock on output. The reason is that the decline in ROA tends to be larger on impact than that of the short term rate, so that the spread between the two declines. This result does not, however, generalize to other shocks not shown here – monetary policy, public expenditures, asset prices, etc.; a low $\lambda$ may amplify or dampen the output response depending on parameters. Conversely, the response of investment and capital tends to be always more persistent for high values of $\lambda$.

4 Sensitivity to Key Parameters

4.1 Entrepreneurial risk

Entrepreneurial risk is distinct from bank risk: the first is measured by the parameter $h$, while the second depends endogenously on the bank capital structure. The two are linked, however: a higher $h$ tends to increase the bank leverage and the probability of run on the bank for all values of $\lambda$ below unity, as one can see in equation 11 and the chart attached to it. Moreover, $h$ also affects the response of the bank capital structure and risk to all other shocks.

Figures 5 and 6 report the responses to a TFP and a monetary shock respectively, with values of $h$ equal to 0.5, the baseline, and 0.9, an alternative in which the entrepreneurial risk is higher.

Under the positive productivity shock, the response of output, capital, credit and investment is stronger if the value of $h$ is lower. This highlights a self-reinforcing mechanism that may operate in "exuberant" phases: positive productivity shocks are more expansionary if the perception of investment risk is low.

Under a monetary restriction, on the contrary, the business cycle response is amplified in the high risk case; we have a bigger drop in output, investment and capital, together with a sharper fall in bank leverage. This is due to the fact that the downward effect of a given change in $R_t$ on the deposit ratio is stronger when $h$ is high (see equation 10), hence in presence of a stronger need for
bank capital $R_{it}$ rises more on impact and the transmission to investment and output is higher.

By contrast, however, under a monetary shock the decline in bank riskiness is smaller when entrepreneurial risk is higher (a high $h$ dampens the stronger increase in $R_{it}$ in equation 11). This means that the risk taking channel of monetary policy operates more strongly when the ex-ante uncertainty of projects is low (this effect involving two factors, $h$ and the interest rate, should not be confused with the fact that bank risk increases when entrepreneurial risk $h$ rises, as we have already noted). This observation connects with the earlier one concerning "exuberant" states; in these situations, an overly expansionary monetary policy tends to have particularly strong effects on bank leverage, exacerbating the increase of bank risk. Since the empirical evidence shows that entrepreneurial risk tends to be anti-cyclical, we conclude that the strength of the risk taking channel depends on the cyclical position. An expansionary monetary policy when the economy is strong increases bank risk by more than the same policy when the economy is weak.

4.2 Leaning against the wind

Figure 7 compares the benchmark monetary policy rule with two strategies in which the interest rate reacts also to asset prices (more precisely Tobin’s Q, with a coefficient of 0.5) or alternatively to bank leverage (the change in the deposit ratio, with the same coefficient). We regard these as alternative options of using monetary policy (also) to control risks in the financial sector by leaning-against-the-wind in financial markets. Comparing these alternatives can contribute new elements to the old debate on whether monetary policy should react to expected inflation only (see Bernanke and Gertler [2]) or to asset prices as well (Cecchetti, Genberg, Lipsky and Whadhani [12]). Since one argument in that debate was that responding to asset prices would inject volatility in the economy, it is interesting to look at an alternative measure based on bank balance sheets, that should be empirically more stable.

The figure is constructed assuming an asset price shock. As one can see, the two strategies give mixed results. The rule that reacts to Tobin Q is successful in stabilising output and inflation, but on bank risk and the deposit ration the result is less clear. Reacting to leverage instead does not seem to improve the performance relative to a standard Taylor rule, and in some cases (on output and inflation for example) the performance is actually worse. All in all, the results speak in favor of responding to asset prices, not leverage. But this result is obtained under a single shock only.
As we shall see later, using a broader set of calibrated shocks tends to tilt the balance in favor of responding to leverage in some cases.

5  Introducing Bank Capital Requirements

Capital regulation in our model takes the form of an exogenously imposed ratio between banking capital, $BK_t$, and the total amount of bank loans, $Q_tK_t$. The regulatory ratio is either fixed, like in Basel I, or risk-weighted. Since the riskiness of bank assets tends to be (negatively) correlated with the economic cycle, we mimic a Basel II-type regime by introducing a negative response of the capital ratio to output. A negative coefficient means that the capital regime is pro-cyclical (for given loans, regulatory capital decreases in a cyclical upswing); a positive one, that the regime is anticyclical. We use the following iso-elastic formulation:

$$BK_t = b_0^c \left( \frac{Y_t}{Y_{SS}} \right)^{b_1^c} Q_tK_t$$

(31)

We assume that, when imposed, the capital ratio is always binding. The value of $b_1^c$, the elasticity of bank capital relative to deviations of $Y_t$ from its steady state value, $Y_{SS}$, is set to 0 in the "fixed capital requirement" case, -0.1 in the procyclical case and to 0.1 in the anticyclical case. We do not claim that these coefficients are realistic; we use them only to see how economic equilibria change as we move marginally away, in either direction, from a fixed capital ratio.

In addition to replacing equation 13 with 31 we need to modify the accumulation of capital, equation 17, as follows:

$$BK_{t+1} = \theta[BK_t + \frac{\{R_{A,t+1} + h - R_{t+1}[1 - b_0^e (\frac{Y_{t+1}}{Y_{SS}})^{b_1^e}]\}^2}{8h}Q_{t+1}L_{t+1}]$$

(32)

Figure 8 compares the optimal capital (solid line) regime with the procyclical capital ratio (dashed line). A fixed capital ratio is clearly destabilizing. After a slow start due to the gradual accumulation mechanism, bank capital builds up strongly, driven by the higher ROA; the impulse

6In fact, a small degree of pro-cyclicality existed also in Basel I, due to accounting conventions and other factors.

7Kashyap and Stein [21] report very different estimates of the degree of procyclicality of Basel II, depending on methodologies, data, etc. What seems to be very robust is the sign – Basel II is clearly procyclical in the sense that the capital requirements on a given loan pool increase more, when the economy decelerates, relative to what they did under Basel I.
responses oscillate sharply before returning to baseline. We conclude that endogenous changes in the capital ratio act as a dampening factor in this model. The output amplification result under regulatory capital echoes that of Cecchetti and Li [11], but the mechanism here is different (they assume that the supply of capital is proportional to output). It is interesting to see that this capital regime, though having the undesirable flip side of accentuating the business cycle, is actually successful in containing bank risk: riskiness rises following a productivity shock is the free capital regime, while it does not under a fixed capital ratio.

Figure 9 compares three different regulatory regimes (fixed, pro-cyclical and anticyclical). The dampening effect of anti-cyclical capital requirements is evident. The pro-cyclical regime has the opposite effect, accentuating the oscillations of all macro variables sharply. Again, the results for the bank risk are different: it is more stable under a fixed capital ratio, relative to the two alternatives.

6 Welfare Analysis and Optimal Monetary Policy

We analyse optimal policy based on household welfare, which is maximised subject to the competitive equilibrium conditions (or alternatively, the capital regulatory requirement 31 and 32) within the class of monetary policy rules represented by (29). Specifically we search for parametrization of these rules that satisfy the following 3 conditions: a) they are simple and realistic; b) they guarantee uniqueness of the rational expectation equilibrium; c) they maximize the expected life-time utility of the representative agent.

Some observations on the computation of welfare are important since the model features significant frictions operating both in the steady state and over the dynamic. First, we cannot safely rely on first order approximations to compare the welfare associated to monetary policy rules, because in an economy with a distorted steady state stochastic volatility affects both first and second moments. Since in a first order approximation of the model solution the expected value of a variable coincides with its non-stochastic steady state, the effects of volatility on the variables’ mean values is by construction neglected. Policy alternatives can be correctly ranked only by resorting to a higher order approximation of the policy functions.\footnote{See Kim and Kim [22] for an analysis of the inaccuracy of welfare calculations based on log-linear approximations in dynamic open economies.} Additionally one needs
to focus on the *conditional* expected discounted utility of the representative agent. This allows to account for the transitional effects from the deterministic to the different stochastic steady states respectively implied by each alternative policy rule.

Our metric for comparing alternative policies is the fraction of household’s consumption that would be needed to equate conditional welfare $W_0$ under a generic policy to the level of welfare $\bar{W}_0$ implied by the optimal rule. Such fraction, $\Omega$, should satisfy the following equation:

$$W_{0,\Omega} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U((1 + \Omega)C_t) \right\} = \bar{W}_0$$

Under a given specification of utility one can solve for $\Omega$ and obtain:

$$\Omega = \exp \left\{ \left( \frac{\bar{W}_0 - W_0}{1 - \beta} \right) - 1 \right\}$$

We compare the welfare performance of alternative monetary policy rules including three shocks, productivity, government expenditure and monetary policy calibrated as indicated earlier.

We proceed in two steps. First, in the next subsection we compare some policy rules of particular interest. Then, in the next subsection, we compute the optimal policy by optimising the parameter values within a predefined class of rules. We consider both the cases of free and constrained capital. We also distinguish the situation in which monetary policy takes capital regulation as given, from that in which an optimum is sought via a combined policy.

### 6.1 Comparing monetary policy rules

Table 1 summarises our policy rules. The first six are standard variations of the Taylor rule without response to asset markets. We start from the usual Taylor formulation with coefficients of 1.5 and 0.5 on inflation and output and then consider more or less aggressive parameters on inflation, output and the lagged interest rate coefficient (that measures the degree of interest smoothing by the central bank). After this, we examine three groups of rules where monetary policy reacts to financial variables. The groups, denoted by A), B), and C), differ for the size of the inflation and output coefficients: specifically, we consider a "flexible" response to inflation and output (FIR; coefficients of 1.5 and 0.5 respectively), an "aggressive" response to inflation (AIR; coefficients of 2.5 and 0.5), or a "pure" version responding to inflation only (PIR; coefficients of 1.5 and 0
respectively). The leaning-against-the-wind behavior is expressed by a response to the asset price – specifically, Tobin Q – or the (change of) the deposit ratio. In each group we also experiment with two assumptions on interest rate smoothing (coefficient equal to 0 or 0.6).

<table>
<thead>
<tr>
<th>Table 1. Monetary policy rules</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rule</strong></td>
</tr>
<tr>
<td>------------------------</td>
</tr>
<tr>
<td>Flexible inflation response</td>
</tr>
<tr>
<td>Flexible infl. response with smoothing</td>
</tr>
<tr>
<td>Aggressive infl. response with smoothing</td>
</tr>
<tr>
<td>Pure infl. response with smoothing</td>
</tr>
<tr>
<td>Aggressive output response with smoothing</td>
</tr>
<tr>
<td>Flexible infl. r. and aggressive smoothing</td>
</tr>
</tbody>
</table>

Table 2 shows results for the "unconstrained capital" case, for different combinations of entrepreneurial risk, \( h \), and market liquidity, \( \lambda \). The entries of the table represent the utility loss, expressed in terms of percent of consumption, relative to the optimal rule within each column. By construction, the best rule in each column is denoted by a value of 0. The numbers in the table are comparable only within, and not across, columns.

<table>
<thead>
<tr>
<th>Table 2. Welfare cost ( \Omega ) relative to optimal rule, optimal bank capital</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rule</strong></td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>Flexible inflation response</td>
</tr>
<tr>
<td>Flexible infl. response with smoothing</td>
</tr>
<tr>
<td>Aggressive infl. response with smoothing</td>
</tr>
<tr>
<td>Pure infl. response with smoothing</td>
</tr>
<tr>
<td>Aggressive output response with smoothing</td>
</tr>
<tr>
<td>Flexible infl. r. and aggressive smoothing</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rule</th>
<th>( h, \lambda )</th>
<th>0.5, 0.5</th>
<th>0.5, 0.8</th>
<th>0.8, 0.5</th>
<th>0.8, 0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) Flexible infl. response, asset price</td>
<td>0.126</td>
<td>0.154</td>
<td>0.132</td>
<td>0.153</td>
<td></td>
</tr>
<tr>
<td>A) Flexible infl. response, leverage</td>
<td>0.149</td>
<td>0.169</td>
<td>0.151</td>
<td>0.166</td>
<td></td>
</tr>
<tr>
<td>A) Flexible infl. r., asset price, smoothing</td>
<td>0.100</td>
<td>0.117</td>
<td>0.105</td>
<td>0.117</td>
<td></td>
</tr>
<tr>
<td>A) Flexible infl. r., leverage, smoothing</td>
<td>0.102</td>
<td>0.114</td>
<td>0.104</td>
<td>0.114</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rule</th>
<th>( h, \lambda )</th>
<th>0.5, 0.5</th>
<th>0.5, 0.8</th>
<th>0.8, 0.5</th>
<th>0.8, 0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>B) Aggressive i.r., asset price, smoothing</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>C) Pure infl.r., asset price, smoothing</td>
<td>0.005</td>
<td>0.009</td>
<td>0.006</td>
<td>0.011</td>
<td></td>
</tr>
</tbody>
</table>
Note: $\Omega$ is the (percent) fraction of consumption required to equate welfare in any given policy rule to that of the best policy in each column (see equation 33).

The best rule, AIR with smoothing and a response to the asset price, is invariant with respect to the parameters $h$ and $\lambda$. The ranking across different rules is also broadly consistent for all parameter combinations. The rule with aggressive output response is consistently very bad. Excluding this one, there are two categories: some rules are close to the optimum, others are far behind. The first group includes AIT and PIR with smoothing, also with a response to the asset price. Of the groups B) and C), we have reported for brevity only the result from the best performing rule within each group. FIR performs well only without leaning-against-the-wind. PIR (with or without leaning) and FIR (without leaning) are close behind the best rule. Generally speaking, the strength of the response to inflation of the optimal rule depends on the degree of price stickiness; the higher the latter the lower the optimal response to inflation. The rules incorporating a response to bank leverage perform worse than those responding to the asset price, though the difference is not very large.

In sum, the message is that the best performing rules 1) contain a strong response to inflation (aggressive or pure); 2) include interest rate smoothing; 3) incorporate a response to asset prices. However, we will see in the next section the good properties of smoothing are reduced if the other responses are allowed to increase.

Table 3 considers the case with capital regulation, with the three sun-cases, fixed capital ratio, anti-cyclical and pro-cyclical response to output respectively. In table 3 the numbers are constructed so that the "best rule" serving as benchmark is common to the whole table, so comparison is possible both within and across columns.

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9 The utility shown in the table are very low. It should be kept in mind that welfare comparisons, including relative ones, are sensitive to the parameters of the utility function, risk aversion in particular. The rankings among rules are generally quite robust, however.
Table 3. Welfare cost $\Omega$ relative to optimal rule, Basel capital regimes

<table>
<thead>
<tr>
<th>Rule</th>
<th>$B_1^*=0$</th>
<th>$B_1^*=0.1$</th>
<th>$B_1^*=-0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexible inflation response with smoothing</td>
<td>0.548</td>
<td>0.461</td>
<td>1.252</td>
</tr>
<tr>
<td>Aggressive inflation response with smoothing</td>
<td>0.097</td>
<td>0.090</td>
<td>0.175</td>
</tr>
<tr>
<td>Pure inflation response with smoothing</td>
<td>0.206</td>
<td>0.178</td>
<td>0.394</td>
</tr>
<tr>
<td>A) Flexible inflation response, asset price</td>
<td>0.421</td>
<td>0.411</td>
<td>0.500</td>
</tr>
<tr>
<td>A) Flexible inflation response, leverage</td>
<td>0.488</td>
<td>0</td>
<td>2.780</td>
</tr>
<tr>
<td>A) Flexible infl. resp., asset price, smoothing</td>
<td>0.475</td>
<td>0.416</td>
<td>1.018</td>
</tr>
<tr>
<td>A) Flexible infl. response, leverage, smoothing</td>
<td>0.548</td>
<td>0.083</td>
<td>9.446</td>
</tr>
<tr>
<td>B) Aggressive inflation response, leverage</td>
<td>0.053</td>
<td>0.010</td>
<td>0.390</td>
</tr>
<tr>
<td>B) Aggressive infl. resp., asset price, smoothing</td>
<td>0.095</td>
<td>0.089</td>
<td>0.183</td>
</tr>
<tr>
<td>C) Pure infl. response, asset price, smoothing</td>
<td>0.176</td>
<td>0.160</td>
<td>0.357</td>
</tr>
</tbody>
</table>

Note: $\Omega$ is the (percent) fraction of consumption required to equate welfare in any given policy rule to that of the best policy in the table (see equation 33).

In the table we included, on top, the three best performing rules among the first six from table 2. Then we included all rules in group A) (FIR, with various combinations of smoothing and leaning) plus the best performing of groups B) and C). Finally we have also included AIR with a response to leverage.

The pro-cyclical Basel regime (last column) performs badly for all monetary policy rules. The best monetary policy rule under pro-cyclical Basel is AIR with smoothing, with or without response to the asset price. Likewise, the fixed capital regime (first column) is consistently worse than the anticyclical one (middle column). The table sends a clear message against pro-cyclical capital regulations, that confirms the indications from the impulse responses. Under an anti-cyclical capital regime, the very best rule (and the best in the whole table) is FIR without smoothing and with response to bank leverage. AIR (still without smoothing and with response to bank leverage) is a very close second. As soon as one moves towards more pro-cyclical versions, aggressive anti-inflationary behavior becomes a better choice.

One can think of the following thought experiment to see how monetary policy may wish to adjust, after a change in the bank capital regime, to compensate for the hypothetical welfare loss. Suppose we start from the best point in the table, FIR with leverage and anti-cyclical capital, and capital regulation is changed towards a fixed capital ratio. If that happens monetary policy needs to become more aggressive on inflation – a result that echoes Cecchetti and Li [11]. But even
doing that the welfare improvement is not sufficient to compensate for the initial loss. The same happens if one moves further towards a pro-cyclical regime.

6.2 Optimal monetary policy

To enhance our assessment of the optimal monetary policy rule we search over a grid of parameters the welfare-maximizing simple rule within the class 29. Computation of conditional welfare is done by considering the three main shocks: productivity, government expenditure and interest rate parametrized as described in the calibration section. The search grid is specified as follows: \( \phi_y \in \{1.5 \text{ to } 3.5\}, \phi_y = 0.5, \phi_q \in \{0 \text{ to } 1\}, \phi_r = \{0 \text{ or } 0.6\} \). The choice of the search grid is motivated by two considerations: to consider empirically plausible values for the operational rules; to avoid indeterminacy regions, which in our model occur for very aggressive values of the response to output and asset prices combined with interest rate smoothing. We use the model with freely determined bank capital.

The values that identify the optimal policy rule are as follows: \( \phi_y = 3.5; \phi_q = 0.8; \phi_r = 0 \).

Figure 10 shows the welfare costs of deviating from the optimal policy for different values of the response to inflation and asset price. Three results emerge. First, optimality requires a rather active policy rule that includes a high response to output and asset prices. The lean-against-the-wind policy seems a rather robust prescription from our model, in contrast with result obtained from financial accelerator-type models (Bernanke and Gertler [2]; Faia and Monacelli [17]). Second, the optimal response to inflation is quite aggressive. Since the model induces amplification of the main macro variables, including inflation, under our combination of shocks, it does not come as a surprise that it calls for aggressive inflation stabilization. Finally, the model prescribes no response to past interest rates, assuming that the coefficients on inflation and the asset price are large enough. There is a trade-off between aggressiveness and smoothing. Our banking model induces a quite persistent dynamics for most variables; in this context, further persistence from the policy is unwarranted.

7 Conclusions

Since the crisis started, the landscape of economic policy has changed; some well established paradigms have collapsed right at the time of their maximum triumph. A casualty concerns some
earlier tenets concerning the interaction between bank regulation and monetary policy. The old consensus, according to which the two policies should be conducted in isolation, each pursuing its own goal using separate sets of instruments, is increasingly challenged. After years of glimpsing at each other from the distance, monetary policy and prudential regulation – though still unmarried – are moving in together. This opens up new research horizons, highly relevant at a time in which central banks on both sides of the Atlantic are acquiring new responsibilities in the area of systemic stability.

We have tried to move a step forward by constructing a new macro-model that integrates banks in a meaningful way and using it to analyze the role of banks in transmitting shocks to the economy, the effect of monetary policy when banks are fragile, and the way monetary policy and bank capital regulation can be conducted as a coherent whole. Our conclusions at this stage are summarized in the introduction, and need not repeating here.

While our model brings into the picture a key source of risk in modern financial system, namely leverage (and implicitly, also the maturity mismatch between bank assets and liabilities), there are also others that we have left out from our highly abstract construct. Of special importance is the interconnection within the banking system. As Morris, Shin, Brunnermeier and others have noted (see e.g. [27], [10]), a system where leveraged financial institutions are exposed against each other and can suddenly liquidate positions under stress is, other things equal, more unstable than one in which banks lend only to entrepreneurs, as in our model. Introducing bank inter-linkages and heterogeneity in macro models is, we believe, the most urgent challenge in this line of research.

While we were finishing this work we came across a very recent paper by Gertler and Kiyotaki [20] that introduces bank heterogeneity and interbank exposure in the Gertler-Karadi model, assuming banks operate in islands subject to idiosyncratic shocks.
We want to show that the value of $\tau$ that maximises equation 8

\[
\frac{1}{2h} \int_{-h}^{h} \frac{(1 + \lambda)(R_{A,t} + x_{j,t})}{2} dx_{j,t} + \frac{\text{R}_d d_t - R_{A,t}}{2} \frac{R_t d_t}{R_t} dx_{j,t} + \frac{\text{R}_d d_t - R_{A,t}}{2} \frac{R_t d_t}{\lambda} dx_{j,t}
\]

is within the interval \((\lambda \frac{R_{A,t} + h}{R_t}; R_{A,t} + h R_t)\). To do this we show first that the optimum is not below \(\frac{R_{A,t} - h}{R_t}\); than that it is not above \(\frac{R_{A,t} + h}{R_t}\); and finally that it cannot be in the interval \((\frac{R_{A,t} - h}{R_t}; \lambda \frac{R_{A,t} + h}{R_t})\).

1. Consider first very low values of $R_d d_t$, below $\lambda(R_{A,t} - h)$. In this case a run is impossible ex-ante, with or without the bank. The return to outsiders is given by \(1 + \lambda \frac{R_{A,t} + x_{j,t}}{2} dx_{j,t}\), which does not depend on $\tau$. Hence the value of equation 8 in this interval is constant, as in the extreme left side of graph 2 (the time subscripts are omitted in the graph for simplicity). As $R_d d_t$ grows above $\lambda(R_{A,t} - h)$, but below $R_{A,t} - h$, the relevant expression becomes

\[
\frac{1}{2h} \int_{h}^{} (R_{A,t} + x_{j,t}) + \frac{R_d d_t}{1} \frac{R_t d_t}{\lambda} dx_{j,t} + \frac{\text{R}_d d_t}{2} \frac{R_t d_t}{R_t} dx_{j,t}
\]

The derivative with respect to $d_t$ is

\[
\frac{R_t}{4h} \left[ \frac{R_t d_t}{\lambda} - (R_{A,t} - h) \right]
\]

which is positive in the interval we consider. Intuitively, in this region, depending on the realisation of $x_{j,t}$, one may fall either in the case where the run is impossible ex-post, or in the case where it is possible without the bank. The return to outside claimants is higher in the second case (because the banker’s fee is smaller), so as $d_t$ increases the overall expected return to outsiders
increases. Hence we conclude that the value of $d_t = \frac{R_{A,t}-h}{R_t}$ dominates all values to the left; see graph 2.

2. Consider now the opposite case, $R_t d_t > (R_{A,t} + h)$. In this case the expression reduces to the first integral:

$$\frac{1}{2h} \int_{-h}^{h} \frac{(1 + \lambda)(R_{A,t} + x_{j,t})}{2} dx_{j,t} = \frac{1}{2} (\lambda + 1) R_{A,t}$$

a constant independent on $d_t$ (graph 2, right-hand side). In this case the run is certain ex-ante, and depositors get always the same, namely the expected liquidation value of the loan $R_{A,t}$ minus the banker’s fee $\frac{1}{2} (1 - \lambda) R_{A,t}$.

3. We are now at the case where $\left(\frac{R_{A,t}-h}{R_t} < d_t < \lambda \frac{R_{A,t}+h}{R_t}\right)$. The derivative of equation 8 with respect to $d_t$ is

$$\frac{2R_t d_t}{8h\lambda} (\lambda - 1)^2 > 0$$

This portion of the curve is upward sloping and convex, see graph 2. We then conclude that the value $d_t = \lambda \frac{R_{A,t}+h}{R_t}$ dominates all points to the left and that the value $d_t = \frac{R_{A,t}+h}{R_t}$ dominates all points to the right, QED.
References


Graph 1: Bank capital structure and the risk of bank run

Graph 2: Return to outside investors
Figure 1: Impulse response to a positive productivity shock
Figure 2: Impulse response to a positive interest rate shock
Figure 3: Impulse response to a positive shock in the marginal return on capital
Figure 4: Alternative values of lambda (positive productivity shock)
Figure 5: Alternative values of h (positive productivity shock)
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