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Endogenizing Prospect Theory's Reference Point

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1 Introduction

Prospect theory is currently one of the most influential models of decision making under uncertainty and has been applied in various fields like finance, consumer choice, and political decision making. Apart from probability weighting, the central innovation of prospect theory is reference-dependence. Reference-dependence means that people do not evaluate final outcomes but instead they base decisions on gains and losses relative to a reference point. Empirically well documented facts supporting reference-dependence comprise diminishing sensitivity (people are more sensitive to changes near their reference points than to changes remote from it) and loss aversion (a negative deviation from the reference point has a higher impact than a positive deviation of equal size).

Unlike original prospect theory (Kahneman and Tversky, 1979), modern variants like cumulative prospect theory and the rank- and sign-dependent utility model (Luce, 1991; Luce and Fishburn, 1991; Tversky and Kahneman 1992; Wakker and Tversky 1993; Chateauneuf and Wakker 1999; Luce, 2000; Zank, 2001; Wakker and Zank 2002; Schmidt and Zank 2009) have been derived from behavioural foundations in terms of preference conditions. Behavioural foundations are desirable because they reveal the underlying assumptions of a model and set the ground for its empirical testing.

It can be argued that the existing axiomatizations of prospect theory (PT) should be extended. One reason, already noted in the literature, is the fact that the reference point is assumed to be given exogenously. While this may have some advantages, models with an endogenous reference point (e.g., Köszegi and Rabin 2006, 2007) have additional flexibility for analyzing behaviour. In our view there exists a second important and possibly more fundamental issue: current axiomatizations of PT assume the existence of a preference relation defined on gains and losses relative to an exogenously fixed reference point and then impose behavioural conditions on this (reference-dependent) preference relation. This means that reference-dependence is not derived form preference conditions but is assumed beforehand and could be regarded as an ad hoc assumption. As a consequence, PT can neither be tested nor applied to concrete choice problems without making prior assumptions about the location of the reference point. This is sometimes interpreted as a major shortcoming of prospect theory (e.g., Fudenberg (2006), Footnote 2 on page 696; see also Pesendorfer (2006) for a discussion).

The goal of the present note is to derive a new foundation for the model. We call the new approach endogenous prospect theory (EPT) because reference-dependence is not assumed beforehand but derived from a behavioural foundation. Our preferences conditions imply the existence of a reference point and determine the location of the reference point endogenously. This requires a criterion for identifying the location of the reference point since reference-dependence becomes meaningless if behavior would not change at the reference point. As mentioned above, according to previous models of prospect theory two criteria can be used to identify the reference point, diminishing sensitivity and loss aversion. In EPT we focus on diminishing sensitivity. Evidence supporting diminishing sensitivity has frequently been reported (Kahneman and Tversky, 1979; Tversky and Kahneman, 1981, 1992; Hershey and Schoemaker, 1985; Budescu and Weiss, 1987; Camerer, 1989; Currim and Sarin, 1989; Heath, Huddart, and Lang, 1999; Luce, 2000; Abdellaoui, 2000; Abdellaoui, Vossmann, and Weber, 2005; Abdellaoui, Bleichrodt, and Paraschiv, 2007; studies in the field of neuroeconomics include Dickhaut et al., 2003; de Martino et al., 2006). The alternative approach, using loss aversion to identify the reference point, is left for future research. The implementation of this approach might not be straightforward. For example, loss aversion implies the existence of a kink at the reference point which needs to be distiguished from possible genuine kinks of the utility function. Further, there exist different definition of loss aversion in the literature (for an overview see Abdellaoui, Bleichrdot, and Paraschiv, 2007), and behavioral foundations of these definitions of loss aversion are often missing or have model-dependent implications (Schmidt and Zank, 2005, 2008). Nonetheless, loss aversion is an important feature of PT and is incorporated in our model.

The next section introduces our framework of decision making under uncertainty and some basic concepts. Section 3 contains our behavioral conditions and the main result: By imposing our central axiom –termed consistent diminshing sensitivity– reference-dependence arises endogenously in our model and the reference point is located at the position at which sensitivity towards changes in outcomes is maximal. Concluding remarks are presented in Section 4.

2 Notation and Basic Concepts

We analyze decision problems under uncertainty and consider a finite set S of *states* of nature.[2] That is, $S = \{s_1, \ldots, s_n\}$ for a natural number $n \geq 3$, and $\mathcal{A} = 2^S$ is the algebra of subsets of S. Elements of $\mathcal{A}$ are called *events*. An *act* f assigns to each state a real valued *outcome*. The set of acts $\mathcal{F}$ can be identified with the Cartesian product space $I\!R^n$, and hence, we write $f = (f_1, \ldots, f_n)$, where f_i is short for $f(s_i)$. An act f is *rank-ordered* if its outcomes are ordered from best to worst: $f_1 \geq \cdots \geq f_n$. For each act f there exists

[2]Our results can be extended to infinite state spaces by using tools presented in Wakker (1993). Identical results for the case of decision under risk, that is, when (objective) probabilities are given, can be derived by applying the procedure of Köbberling and Wakker (2003, Section 5.3).

a permutation ρ of $\{1, \ldots, n\}$ such that $f_{\rho(1)} \geq \cdots \geq f_{\rho(n)}$, i.e. such that the outcomes are *rank-ordered with respect to* ρ. For each permutation ρ of $\{1, \ldots, n\}$ the set $I\!R^n_\rho$ consists of those acts which are rank-ordered with respect to ρ. Acts that can be rank-ordered with respect to the same permutation are called *comonotonic.*

We use the notation $f_E g$ for an act that agrees with the act f on event E and with the act g on the complement E^c. Also, we use $h_i f$ instead of $h_{\{s_i\}} f$ for any state $s_i \in S$. Sometimes we identify constant acts with the corresponding outcome. We may thus write $f_E x$ for an act agreeing with f on E and giving outcome x for states $s \in E^c$.

We consider a preference relation $\succcurlyeq$ on the set of acts. As usually, $f \succcurlyeq g$ means that act f is weakly preferred to act g. The symbols $\succ$ and $\sim$ denote strict preference and indifference, respectively. The preference relation $\succcurlyeq$ is a *weak order* if it is *complete* ($f \succcurlyeq g$ or $g \succcurlyeq f$ for any acts f, g) and transitive. A functional $V : \mathcal{F} \rightarrow I\!R$ *represents* the preference relation $\succcurlyeq$ if for all $f, g \in \mathcal{F}$ we have $f \succcurlyeq g \Leftrightarrow V(f) \geqslant V(g)$.

An example of a representing functional is Choquet expected utility (CEU) introduced by Schmeidler (1989) and Gilboa (1987). It extends the classical subjective expected utility of Savage (1954) by introducing a non-additive measure for events: a *capacity* v satisfies $v(S) = 1, v(\emptyset) = 0$, and $v(A) \geq v(B)$ if $A \supseteq B$ and $A, B \in \mathcal{A}$. A capacity v is *strictly monotonic* if $v(A) > v(B)$ for $A \supsetneq B$ and $A, B \in \mathcal{A}$.

Choquet expected utility holds if the preference relation can be represented by the functional

$$CEU(f) = \sum_{i=1}^{n} U(f_i)\pi_i \quad \text{with} \quad \pi_i = v(\{s_{\rho(1)}, \ldots, s_{\rho(i)}\}) - v(\{s_{\rho(1)}, \ldots, s_{\rho(i-1)}\}). \qquad (1)$$

The strictly increasing and continuous *utility*, U, is cardinal (i.e., it can be replaced by a positive linear transformation of U) and the capacity, v, is unique. In terms of behavioral conditions, CEU can be derived by restricting Savage (1954)'s sure-thing principle to acts which are pairwise comonotonic, and further by requiring a consistent ordering of utility differences accross states (see Köbberling and Wakker 2003).

Prospect theory generalizes CEU by introducing a reference-point r, which may impact on utility and capacity. In all axiomatic work we are aware of, the existence and location of this reference-point is assumed exogenously. Formally, previous models considered a preference relation $\succcurlyeq_r$ on acts with outcomes being deviations from r, i.e. for the act f the outcome f_i is interpreted as gain (loss) if it is better (worse) than r.

Prospect Theory (PT) holds if the representing functional for $\succcurlyeq_r$ has the form

$$\begin{aligned} PT(f) &= \sum_{i=1}^{n} U(f_i)\pi_i, \\ \text{with } \pi_i &= \begin{cases} v^+(\{s_{\rho(1)},\ldots,s_{\rho(i)}\}) - v^+(\{s_{\rho(1)},\ldots,s_{\rho(i-1)}\}) \text{ if } f_i \geq r \\ v^-(\{s_{\rho(i)},\ldots,s_{\rho(n)}\}) - v^-(\{s_{\rho(i+1)},\ldots,s_{\rho(n)}\}) \text{ if } f_i \leq r\ . \end{cases} \end{aligned}$$

The two (possibly different) capacities v^+ and v^- are uniquely determined and the utility is a ratio scale (i.e., unique up to multiplication by a positive constant) as it is fixed at the reference-point, i.e., $U(r) = 0$.

3 A New Foundation for Prospect Theory

Let us first recall some standard properties for the preference $\succcurlyeq$, before we introduce the main preference condition that allows identifying the reference-point. The preference relation $\succcurlyeq$ on $\mathcal{F}$ satisfies *monotonicity* if $f \succ g$ whenever $f_i \geq g_i$ for all states s_i with a strict inequality for at least one state. By employing this condition we ensure that the capacities, derived later, are stictly monotone because monotonicity excludes null states, that is, states where the preference is independent of the magnitude of outcomes. Formally, a state s_i is *null* if $x_i f \sim y_i f$ for all acts f and all outcomes x, y.

The continuity condition defined here is continuity with respect to the Euclidean topology on $\mathbb{R}^n$: $\succcurlyeq$ satisfies *continuity* if for any act f the sets $\{g \in \mathcal{F} | g \succcurlyeq f\}$ and $\{g \in \mathcal{F} | g \preccurlyeq f\}$ are closed subsets of $\mathbb{R}^n$.

In what follows we use several indifferences of the form $x_i f \sim y_i g$ with the assumption that all acts involved in such indifferences are rank-ordered with respect to the same permutation ρ. We can now introduce the main condition in the paper: *consistent diminishing sensitivity* holds if for each outcome x one of the following holds:

(I) for any $w, z, y > x$

$$\begin{aligned} \text{if } x_j f &\sim y_j g \text{ and } z_j f \sim w_j g, \\ \text{then } w - z &> y - x \\ \text{and further } x_i f' &\sim y_i g' \text{ implies } z_i f' \sim w_i g'; \text{ or} \end{aligned}$$

(II) for any $w, z, y < x$

$$\begin{aligned} \text{if } x_j f &\sim y_j g \text{ and } z_j f \sim w_j g, \\ \text{then } z - w &> x - y \\ \text{and further } x_i f' &\sim y_i g' \text{ implies } z_i f' \sim w_i g'. \end{aligned}$$

In the presence of weak order, monotonicity and continuity, one can always find acts f and g and distinct outcomes w, z, y, x such that the indifferences $x_j f \sim y_j g$ and $z_j f \sim w_j g$ hold. The first indifference says that the difference in preference between f and g outside state j is off-set by receiving x and y, for the respective acts, if state j occurs. The second indifference says that the difference in preference between f and g outside state j is off-set by receiving z and w, for the respective acts, if state j occurs. One observes that the second indifference is obtained from the first by replacing x and y with z and w, respectively. Consistent diminishing sensitivity puts constraints on the relationship between $y - x$ and $w - z$ as explained next.

Suppose that x is such that the property (I) of consistent diminishing sensitivity holds. Further, assume that increasing x in state j of act f to z is as good as increasing y in state j of act g to a larger outcome w. Then, consistent diminishing sensitivity requires two features. First, a larger increment than $y - x$ is needed to obtain the second indifference and, hence, $w - z > y - x$. Second, this "diminishing sensitivity" is required to be independent of the (pair of) acts f and g and the state j, so that the strict inequality is consistent across states. Such a finding is in agreement with risk aversion in the sense of diminishing marginal utility for increments in outcomes.

Suppose, however, that x is such that the property (II) of consistent diminishing sensitivity holds. Then those indifferences say that decreasing x in state j of act f to z is as bad as decreasing outcome y in state j of act g to a smaller w. The property now requires that a larger decrement than $x - y$ is needed to obtain the second indifference and, hence, $z - w > x - y$. Similarly to the previous case, this "diminishing sensitivity" is required to be independent of the acts f and g and the state j. This latter finding is in agreement with risk seeking in the sense of diminishing marginal utility for decrements in outcomes.

Note, that consistent diminishing sensitivity does not require a distinction of outcomes into gains and losses. It only says that for each outcome x one of the constraints, (I) or (II) above, must hold. It may, therefore, occur that for all outcomes only the first constraint (I) holds. Or, it may be the case that for all outcomes only the second constraint (II) holds. It is worth noting at this stage that, in the presence of the other standard properties, if there exists some x for which constraint (I) is satisfied, then (I) must be satisfied for all $x' > x$; and if there exists some x for which the second constraint (II) is satisfied, then (II) is satisfied for all $x' < x$. It, therefore, follows that if there exists an outcome x^+ for which (I) holds and an outcome x^- for which (II) holds, then there exists a unique outcome r for which both (I) and (II) must hold, and this outcome r acts as a reference point for the preference $\succcurlyeq$.

The following calculus illustrates consistent diminishing sensitivity. We distinguish 3 cases: (A) First, suppose that CEU holds and that utility is strictly concave. Then substitution of CEU for the indifferences $x_j f \sim y_j g$ and $z_j f \sim w_j g$ and subtracting the first resulting equality from the second implies

$$U(y) - U(x) = U(w) - U(z).$$

The additional requirement of strict concavity for utility implies that $w - z > y - x$ must hold. Recall that such preferences can be interpreted as PT preferences with the reference point being at minus infinity (that is, all outcomes are gains). Further, it must hold that $x_i f' \sim y_i g'$ implies $z_i f' \sim w_i g'$ for otherwise the above equality is violated. This implies that for each outcome x constraint (I) of consistent diminishing sensitivity holds.

In the second case (B) we assume that CEU holds with a strictly convex utility. Such preferences can then be interpreted as PT preferences with the reference point being at infinity (that is, all outcomes are seen as being losses). Similarly to case (A) it now follows that for each outcome x constraint (II) of consistent diminishing sensitivity holds.

For the third case (C) suppose that there exists an outcome r such that preferences are represented by PT with U strictly concave (convex) for $f(s) \geq (\leq) r$. Then, substitution of PT for the indifferences $x_j f \sim y_j g$ and $z_j f \sim w_j g$ and subtracting the first resulting equality from the second implies

$$U(y) - U(x) = U(w) - U(z)$$

whenever $w, z, y > x \geq r$ and the strict concavity of U implies $w - z > y - x$. Further, $x_i f' \sim y_i g'$ implies $z_i f' \sim w_i g'$, for otherwise the above equality is violated. We also have

$$U(y) - U(x) = U(w) - U(z)$$

whenever $w, z, y < x \leq r$ and the strict convexity of U implies $z - w > x - y$. Further, $x_i f' \sim y_i g'$ implies $z_i f' \sim w_i g'$, for otherwise the above equality is violated. We conclude that in this case both (I) and (II) of consistent diminishing sensitivity hold at r.

The representing functional that agrees with either (A) or (B) or (C) is called *endogenous prospect theory* (EPT). Note that consistent diminishing sensitivity is a necessary condition for EPT. The following theorem shows that, in the presence of the other standard preference conditions, consistent diminishing sensitivity is also sufficient for EPT. This is the main result of the paper:

THEOREM 1 *Suppose that* $\succcurlyeq$ *is a preference relation on* $I\!R^n, n \geq 3$. *Then the following two statements are equivalent:*

(i) EPT holds with strictly monotone capacities.

(ii) The preference relation $\succcurlyeq$ *is a monotonic, continuous weak order satisfying consistent diminishing sensitivity.*

Utility is a ratio scale and the capacities are unique. If the reference point r *is finite it is uniquely determined.* □

The proof of Theorem 1 is presented in the Appendix.

4 Conclusion

From a theoretical point of view, the mathematical tools used in our theory build on existing tools that were used to derive PT with exogenous reference points. The advances proposed in this paper are conceptually important. For PT to become a valuable tool for economic analyses the model needs a theoretical foundation of how to detect the reference point from preferences. This is a shortcoming in earlier derivations of PT, which has often been criticized, and our note proposes a solution that overcomes this hurdle. We think that this makes PT more sound as a theory and more acceptable. At the same time, this note clarifies on a fundamental aspect on prospect theory: like the classical subjective expected utility and other models of choice under uncertainty and ambiguity, PT belongs to the same family of models which are founded on common assumptions about preferences over uncertain acts.

Appendix: Proof

To prove Theorem 1 we remark that deriving statement (ii) from statement (i) is standard in conjunction with the comments preceding Theorem 1 regarding consistent diminishing sensitivity. Next we assume statement (ii) and derive statement (i). We distinguish three cases:

Case 1: For all outcomes x we have condition (I) of consistent diminishing sensitivity satisfied. In this case the comonotonic tradeoff consistency of Köbberling and Wakker (2003) holds and it follows from their Theorem 8 that CEU holds (with uniqueness results as noted in their Observation 9 (c)). Further, locally, we can always find indifferences $x_j f \sim y_j g$ and $z_j f \sim w_j g$ for acts f, g a state j and outcomes $w, z, y > x$. Substitution of

CEU and subtraction of the first resulting equality from the second implies

$$U(y) - U(x) = U(w) - U(z).$$

Constant diminishing sensitivity demands $w - z > y - x$ in this case. Because this implication must hold for any outcome x (and corresponding $w, z, y > x$), it follows, first locally and then globally, that the utility function must be concave.

<u>Case 2:</u> For all x we have condition (II) of consistent diminishing sensitivity satisfied. Similar to the previous case, the results of Köbberling and Wakker (2003) hold and we obtain CEU. Further, consistent diminishing sensitivity implies, first locally and then globally, that the utility function is convex. Uniqueness results apply as noted in Observation 9 (c) of Köbberling and Wakker (2003).

<u>Case 3:</u> There exist an outcome x^+ for which condition (I) of constant diminishing sensitivity holds and an outcome x^- for which condition (II) of consistent diminishing sensitivity holds. It then follows that there exists a unique outcome r for which both (I) and (II) must hold, which is the reference point for the preference $\succcurlyeq$. In this case consistent diminishing sensitivity implies the sign-comonotonic tradeoff consistency of Köbberling and Wakker (2003), and from their Theorem 12 we obtain that PT holds. By Proposition 8.2 in Wakker and Tversky (1993) the gain-loss consistency requirement can be dropped from statement (ii) in Theorem 12 in Köbberling and Wakker's (2003) when the number of states of nature exceeds 2, which is the case here. Similar to cases 1 and 2 above we derive strict concavity of utility for outcomes above r and strict convexity for utility for outcomes below r, first locally and then globally. Uniqueness results follow from Observation 13 in Köbberling and Wakker (2003).

Together cases 1–3 cover all possibilities and thus statement (i) follows. This completes to proof of the theorem. □

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