KIEL INSTITUTE FOR THE WORLD ECONOMY

KIEL WORKING PAPER

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No. 2181 April 2021

Gabriel J. Felbermayr and Alexander Tarasov

Kiel Institute for the World Economy
ISSN 1862–1155

Leibniz Association

KIEL INSTITUTE FOR THE WORLD ECONOMY

ABSTRACT

TRADE AND THE SPATIAL DISTRIBUTION OF TRANSPORT INFRASTRUCTURE

Gabriel J. Felbermayr and Alexander Tarasov[1]

The distribution of transport infrastructure across space is the outcome of deliberate government planning that reflects a desire to unlock the welfare gains from regional economic integration. Yet, despite being one of the oldest government activities, the economic forces shaping the endogenous emergence of infrastructure have not been rigorously studied. This paper provides a stylized analytical framework of open economies in which planners decide non-cooperatively on transport infrastructure investments across continuous space. Allowing for *intra*- and *inter*national trade, the resulting equilibrium investment schedule features underinvestment that turns out particularly severe in border regions and that is amplified by the presence of discrete border costs. In European data, the mechanism explains about 16% of the border effect identified in a conventionally specified gravity regression.

Keywords: international trade, infrastructure investment, economic geography, border effect

JEL classification: F11, R42, R13

Gabriel J. Felbermayr
Kiel Institute for the World Economy
Kiellinie 66
D-24105 Kiel, Germany
felbermayr@ifw-kiel.de
www.ifw-kiel.de

Alexander Tarasov
National Research University
Shabolovka 26
Moscow, Russia
atarasov@hse.ru

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[1]*We thank Kristian Behrens, Holger Breinlich, Lorenzo Caliendo, Kerem Cosar, Dalia Marin, Philippe Martin, Pierre Picard, Diego Puga, Esteban Rossi-Hansberg, seminar participants at the universities of Zurich, St. Gallen, Konstanz, Tuebingen, Giessen, TU Darmstadt, Linz, Rotterdam, Luxembourg, Ifo Munich, Paris School of Economics, Nottingham, Hohenheim, Milano, and the European University Institute, at the Conference of the Society of Economic Dynamics (SED) in Warsaw, the Congress of the European Economic Association (Geneva), the European Trade Study Group (Munich), the 17th Conference of the SFB/TR (Munich), the Congress of the German Economic Association (Hamburg), and the workshop "Imperfect Competition and Spatial Economics: Theoretical and Empirical Aspects" (Saint-Petersburg) for valuable comments and discussions. We are especially grateful to Philip Ushchev for discussions on variation calculus and to Kharis Sokolov for excellent research assistance. This work is supported by the Russian Science Foundation under grant Project 18-18-00253.*

1 Introduction

The provision of transport infrastructure is one of the oldest and most basic government activities. The roads built by the Inca in South America or by the Romans in Europe bear testimony to this fact. Indeed, any known civilization has devoted resources to the construction of roads; presumably with the objective to unlock the welfare gains from regional integration.[1] On average, OECD countries spend about 1% of GDP on inland transportation infrastructure and maintenance. This amounts to about 3% of countries' public budgets. Emerging or transition economies spend up to 10% of their budgets on transport infrastructure.[2]

A recent World Bank study by Roberts et al. (2018) surveys empirical research and finds that transport infrastructure is an important determinant of economic growth.[3] While the growth literature often does not make the channels explicit through which infrastructure enhances prosperity, a large, mostly empirical literature demonstrates the important role of infrastructure for trade costs, trade flows, and welfare. It makes massive efforts to address the suspected endogeneity of infrastructure but does not model the processes that determine these costs. So, in their authoritative handbook chapter, Redding and Turner (2014) ask for "*further research ... examining the political economy of transport infrastructure investments*". The present paper is a first step towards endogenizing the spatial distribution of transport infrastructure.[4] It rationalizes the finding of Roberts et al. (2018) that placement and design of transport infrastructures systematically influence the growth effects.

We start off with a number of intriguing stylized facts. Using data on bilateral connections between 220 cities in 22 EU countries, we show that travel time is longer between two cities in two different countries as compared to cities within the same country, holding city characteristics and great-circle distances constant. Moreover, we run gravity models at the country-level and find an important role for transport infrastructure in shaping the border effect as defined by Anderson and van Wincoop (2003). When replacing great-circle distance, the usual proxy for transportation costs, by actual road distance or, even better, travel time, the estimated trade-inhibiting effect of a national border falls by about a half. Based on these findings we hypothesize that national governments may have systematic incentives to underinvest in border regions and we develop a general equilibrium model of inter- and intranational trade to rationalize such an

[1] A fascinating account of the history of road construction and operation is provided by Lay (1992).

[2] ITF-OECD (2012), p. 4. These investment costs pale in comparison to total *social* road transport costs (including time costs and externalities), which have been estimated to amount to 20-25% of GDP (Persson and Song, 2010).

[3] Also see Farhadi (2015) for a survey and additional results.

[4] We formalize the non-cooperative behavior of welfare-maximizing national governments in continuous space. However, we argue that our main results can be potentially generalized to the median voter model.

outcome.

In this paper we focus on land-borne transportation, by far the most important mode for intracontinental trade in the EU or in North America. Much transport infrastructure spending is decided in a decentralized manner, in particular in the EU, where only about 1% of total spending is at the Union level and central planning is limited to a small number of projects. For this reason, we assume that welfare-maximizing national governments allocate infrastructure spending over space in a non-cooperative fashion. However, while political space is fragmented, consumers demand goods from all locations in our continuous, linear, two-country economy. National governments do not internalize the benefits from reductions in domestic transportation costs that accrue to foreign consumers. Clearly, in such a setup, there is inefficiently low global investment in infrastructure. More interestingly, however, underinvestment has a spatial structure: it is particularly severe in border regions, where the positive externality of domestic investment on foreign welfare is largest.

As a consequence, trade across national borders entails higher transportation costs than trade within countries, holding bilateral distances and market sizes constant. This effect materializes even in the absence of discrete border costs caused by tariffs or non-tariff measures. However, the effect is magnified by the existence of such costs. The endogenous emergence of broad border zones may contribute towards explaining the empirical fact that international borders tend to restrict international trade much more severely than what observable border costs together with plausible trade elasticities would suggest (Anderson and van Wincoop, 2003).

We employ data on intra-EU trade flows and transportation costs to discipline a calibration of our stylized model. We use the theory to simulate a data set of bilateral intranational and international trade flows, and apply a conventional gravity model. The obtained border effect significantly overestimates the true border costs; about quarter of the bias is due to omitting infrastructure; the rest is a statistical artefact that arises from the high correlation between distance and border. Hence, it appears that endogenous infrastructure investment can explain part of the border effect.

To advance our theoretical understanding of the mechanisms shaping transport infrastructure provision we start by proposing a useful mapping between the spatial distribution of infrastructure spending within an interval into transportation costs between the two endpoints of this interval. This mapping is consistent with the concave relationship between geographical distance and transport costs documented in the data. The link between infrastructure investment over space and transportation costs is shaped by the elasticity of substitution between investment at different locations. We embed this structure into a simple model of intra- and international

trade, where each location produces a unique differentiated product which is subject to transportation costs. Our formalization of the infrastructure-trade cost nexus should turn out useful in other applications as well.

We find that the optimal infrastructure investment at some point in space is not only determined by local conditions at that point, but also – and predominantly – by the situation in other locations that produce and demand goods which transit through that point. The enlargement of a country – e.g., the reunification of Germany – leads to a reallocation of investment away from formerly central regions towards the former border. A higher degree of substitutability between investments at different addresses has an ambiguous effect on investment while a higher elasticity of substitution between goods produced at different location reduces investment.

We extend our analysis by adding a non-contiguous country which supplies and demands goods to and from our two-country continent. We find that an increase in the economic mass of this overseas trading partner induces a reallocation of spending towards coastal regions and away from the landlocked country, strengthening the border effect.

Our paper is related to at least four important strands of literature. First, it connects with papers that study the importance of geographical frictions and transportation costs for trade and welfare. Typically, the literature has treated those costs as exogenous. Limao and Venables (2001) find that up to 60 percent of the cross-country variation in transport costs is due to transport infrastructure and that high cost of land-borne transportation is a more relevant trade barrier than the costs of maritime transportation. Venables (2005) argues that infrastructure explains a larger share of spatial income inequality than sheer geography. Many existing papers assume that countries (or regions) do not have a geographical extension. Recent work provides more spatial detail, but continues to treat infrastructure as exogenous. Cosar and Demir (2014) show that the upgrading of motorways in Turkey significantly increases exports of transport-intensive goods of landlocked cities. Allen and Arkolakis (2014) incorporate realistic topographical features into a spatial model of trade. They find that the introduction of the US interstate highway system has reduced the costs of a coast-to-coast shipment by about a third. Duranton et al. (2014) use data on US interstate highways to show that highways within cities cause them to specialize in sectors that have high weight to value ratios. Allen and Arkolakis (2019) and explore the welfare effects of transport infrastructure investments in a quantitative general equilibrium framework. However, in their paper they do not address the role of non-cooperation across regional planners. Using a multi-region general equilibrium model of trade, Donaldson (2014) and Donaldson and Hornbeck (2015) analyze the welfare gains from railroads

in India and the United States, respectively.[5] They find that improved market access through reduced transport costs creates trade and generates welfare gains, but that it also leads to trade diversion. For Britain, Gibbons et al. (2019) find positive effects of road infrastructure on the regional level of employment and the number of establishments. Behar and Venables (2011) and Redding and Turner (2014) provide excellent surveys. While they cite empirical work on the determinants of transportation costs, they do not provide theoretical references on their endogenous emergence.

Second, our paper is related to a small literature that endogenizes transportation costs, usually by introducing a proper transportation sector. Using an economic geography model, Behrens and Picard (2011) show that the prices for transporting differentiated goods increase in the degree of spatial specialization of the economy and that this channel dampens core-periphery patterns. While their model has a competitive transport sector, Hummels et al. (2009) provide evidence that monopolistic market structure in the transport sector restricts trade. Brancaccio et al. (2019) construct a spatial model of world trade and explore the quantitative role of the transportation sector.[6] These papers do not analyze the endogenous emergence of road infrastructure. Finally, Fajgelbaum and Schaal (2017) provides a framework to solve for the endogenous road infrastructure network and implements it to European countries. However, they do not consider the cross-border issues addressed in our paper.

Third, our paper is related to literature that jointly considers international and intranational aspects of trade. Courant and Deardorff (1992) emphasize the importance of trade *within* countries for trade patterns across countries. Rossi-Hansberg (2005) studies the effects of small border costs on the regional distribution of workers within a country and assesses the implications of the equilibrium population distribution on intra- versus international trade flows. However, his focus is not on infrastructure investment. Bruelhart et al. (2019) empirically show that international trade leads to economic development of border regions relative to interior regions, which is in line with our theory.

Fourth, our paper relates to work on the border puzzle. Transport costs are usually related to geographical distance while the border effect is attributed to some lumpy cost that materializes when crossing a border. Anderson and van Wincoop (2003) estimate that the US-Canadian border reduces international trade relative to intranational trade by a factor of 4.7.[7] Explana-

[5] See also Alder (2019), who explores the quantitative distributional effects of transport infrastructure in India in a general equilibrium trade framework

[6] See also Asturias (2019) for the quantitative role of a oligopolistically competitive transportation sector in a Armington model and Wong (2019) for the effect of round trips between destinations.

[7] Prior to Anderson and van Wincoop (2003), McCallum (1995) compares trade flows within Canada to flows between Canadian provinces and U.S. states, controlling for distance and regional GDPs. Everything else equal,

tions for fixed border costs abound. Among other things, they are related to informational costs (Casella and Rauch, 2003), networks (Combes et al., 2005), or exchange rates (Rose and van Wincoop, 2001). Border effects exist also *within* countries, where the above explanations do not help.[8] Our setup shows that border effects can arise even in the absence of explicit costs at the border.

The structure of the paper is as follows. Section 2 presents some stylized facts on the transportation sector that motivate our modeling choices. Section 3 explains our formulation of the mapping between the spatial distribution of infrastructure investment and transport costs. Section 4 sets up the general equilibrium environment which motivates intra- and international trade and analyzes the optimal infrastructure investment schedule in a closed economy. Section 5 moves to a setting of two symmetric open economies and derives our core results on the endogenous emergence of border regions. Section 6 concludes.

2 Stylized facts

In this section, we use data from the European Union to motivate our analysis, back up our modeling choices, and generate moments to be matched in our quantitative analysis.

2.1 The spatial variation of transportation costs

Direct data on transportation costs is scarce. Combes and Lafourcade (2005) provide generalized transportation cost data for a sample of 8742 French city pairs in 1993. Average road transportation costs are 5.16 French Francs per kilometer.[9] The data also reveals a strong degree of variation in bilateral transportation costs: transportation costs at the 5% percentile are 4.54 while at the 95% percentile they are 5.86 per kilometer. The standard deviation is 0.43. Looking at the average per kilometer cost of transiting one of the 94 French departments,[10] one discovers an even higher standard deviation of 1.12. Only part of this variation is explained by

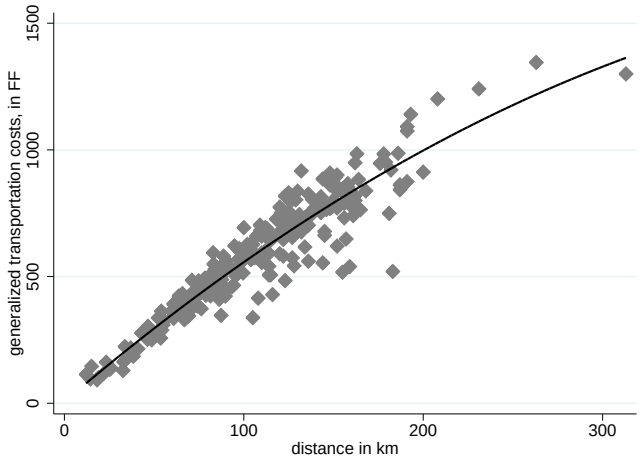
crossing the border reduces trade by a factor as high as 22. For Europe, Nitsch (2000) finds that on average intranational trade is about 10 times higher than international one. Nitsch arrives at his results after controlling for cultural proximity (language), along other conventional gravity covariates. Wei (1996) constructs measures for imports of countries to themselves and compares this with imports from a statistically identical foreign country. He finds that the former magnitude is 2.5 times larger than the latter. Helliwell (1998) offers a comprehensive overview of the pre Anderson and van Wincoop state of the econometric literature.

[8] Okubo (2004) shows a border effect for trade between Japanese regions, Wolf (2000) for the US, and Combes et al. (2005) for their sample of French departments.

[9] Assuming an inflation rate of 3% a year and applying the FF/Euro conversion rate of 6.55957, this would amount to about 1.25 Euro in 2008 current prices.

[10] To obtain a measure of transit costs, we average total variable transport costs per kilometer between neighboring departements, using the neighbors' areas as weights. We disregard the island of Corsica.

Figure 1: Transportation costs and distance: a concave relationship



Notes. Generalized transportation costs between capitals of French departements in French Francs (1993) as a function of geographical distance in km.

variables such as economic activity, density, or topography.[11] We conclude that there is variation to explain, and we conjecture that the spatial distribution of infrastructure investment can contribute to understanding this variation.

However, whatever the quality of roads or rails, geographical distance is likely to play a crucial role. While economic geography models (Fujita et al., 1999) usually model transportation costs as exponential functions of distance, the data reject convexity.[12] Figure 1 provides an illustration based on data by Combes and Lafourcade (2005). Regressing the log of transportation costs between capitals of French *départements* on the log of geographical distance reveals an elasticity of 0.90 with a standard error of 0.02. The hypothesis of the elasticity being equal to unity is rejected at the 1% level.[13] We will target this number in our quantitative analysis.

2.2 The organization of transport infrastructure (in Europe)

Intracontinental trade flows are mostly land-borne. Data from France and the USA for the year of 2010 show that between 72 and 75% of the value of total regional trade (within Europe or

[11] In Appendix E (Table 12), we relate average transit costs to GDP, area and population of the respective department, the ruggedness of territory, and to geographical remoteness. We find that average transportation costs are higher in geologically difficult, less densely populated and poorer departments. Everything else equal, they are also higher in less central regions. Geography explains about 16% of transport cost variation; adding GDP, area and population drives the explanatory power of the model up to 58%. Hence, such a 'naive' model leaves about 40% of variance unexplained.

[12] See McCann (2005) for criticism on the modeling of transportation costs in the literature.

[13] Using robust regression methods to punish outliers leads to an elasticity of 0.92, still different from 1.00 at the 1% level. The same holds true if one restricts the sample to distances below 200, 150 or 100 km.

North America) is transported on roads or railways.[14] Measured by tonnage, the shares are lower as much bulk transportation is water-borne. While the importance of air-borne traffic is increasing, it still is almost irrelevant in intracontinental trade. Hence, in our analysis, we focus on land-borne trade.

Transport infrastructure is publicly provided. Almost all spending on roads and railways is financed by governments, even if private public partnership agreements are becoming increasingly popular.[15] And it absorbs substantial amounts of resources. Infrastructure and maintenance spending on roads amounts to about 1% of GDP across OECD countries (average across 2001-2009).[16] Countries with difficult territory (such as Japan, Switzerland or Italy) have higher than average spending.

In Europe, however, member states decide almost exclusively on their own about transport infrastructure investment projects. There is some coordination at the Union level through the so called Trans-European Transport Network, but the budget has been limited to about 1 billion Euro per year over the 2007-2013 spending period. This is less than 1% of overall spending on infrastructure in the EU (120 billion Euro).[17] Therefore, in our analysis, which focuses on Europe, we assume that countries decide non-cooperatively about infrastructure.[18]

There is no scarcity of anecdotal evidence on underinvestment in border regions. For example, Switzerland has invested heavily to build the new Gotthard-Tunnel, the world's longest railroad tunnel crossing the Alps. However, Germany has not upgraded the railroad links that feed the new tunnel. At the opening of the new tunnel, chancellor Merkel admitted that the tunnel is "the heart" but the "aorta is still missing".[19] Similar observations can be made regarding other important transportation projects in the EU, such as the construction of the high-speed rail link from Paris to Strasbourg (LGV-Est) in France, which is not (yet) properly connected to the German network, or the integration of the high-speed network between Spain and Portugal.[20]

[14] See Table 11 in Appendix E for details and sources.

[15] Applying conventional discount and depreciation rates to the flow investment data, the present value of the stock of public transport infrastructure amounts to between 10% and 20% of GDP in most countries.

[16] OECD Economic Outlook 91 database. There is some disagreement about measurement; the Congressional Budget Office puts total public spending for transportation and water infrastructure between 1.2% and 1.5% of GDP in the 1956-2007 period.

[17] http://ec.europa.eu/transport/infrastructure/tentec/tentec-portal/site/en/abouttent.htm. In the period 2014-2020, centralized spending rises about threefold. However, this still keeps decentralized spending at about 97% of total spending.

[18] In the US, the role of central government in financing and deciding on infrastructure and equipment is more important; however, states play non-trivial roles in financing and planning too, contributing about one quarter to the overall budget; see CBO (2016, Exhibit 6).

[19] Frankfurter Allgemeine Zeitung, June 1st, 2016.

[20] Spain has developed one of the most extensive high-speed railroad networks in Europe; yet, despite a decision of the European Commission from 1992 to connect Spain's capital Madrid with neighboring Portugal's capital Lisbon, the link has not been built yet (https://www.railwaygazette.com/news/infrastructure/single-view/view/esi-

Another telling example is provided by the different density of river crossings for rail or road traffic across the Upper Rhine when it forms the border between France and Germany (179 kilometers, 10 bridges) and when it runs within Germany (180 kilometers, 16 bridges).[21] Interestingly, at the German-French border there are many bridges for pedestrians and bicycles, but these crossings are not for transportation of goods. So, there is not a lack of cooperation per se, but a failure to cooperate on facilitating long-distance traffic. A final example that illustrates the importance of institutions is found at the US-Canadian border. The US interstate highway system was federally funded and the Peace Bridge between Buffalo, NY and Niagara Falls, CA, is cooperatively managed but agents on only one side of the border built other crossings.

2.3 Distance versus travel time in intra- and international bilateral links

In this subsection, we show that locations within jurisdictions are better connected by transport infrastructure than locations lying in two different jurisdictions.

We use data provided by Pisu and Braconnier (2013) on bilateral minimum road distances and minimum travel times between 220 cities in 22 countries in continental Europe.[22] The data come from Bing Maps Route Service and relate to the year 2012. The authors provide population weighted proxies at the country-pair level which we use in our analysis. We merge this information with CES weighted geographical distance measures (based on great-circle distances between major cities), common language and contiguity measures from Mayer and Zignago (2011).[23]

We start by showing direct evidence on relative underinvestment in European border regions. The left-hand diagram in Figure 2 shows that the ratio of road distance to the straight-line distance (the so called divergence) is on average higher between cities in different countries than between cities within the same country. *Intra*national road trips are 9% longer than what the great-circle (bird-flight) distance would indicate; *inter*national road trips are 30% longer. The difference is statistically significant at the 1% level. As the mid panel in Figure 2 shows, the average travel speed on domestic roads is 88 km/h while it is 100 km/h on international road trips; the difference again being significant at the 1% level. However, this disadvantage for

funds-to-improve-madrid-lisboa-connection.html).

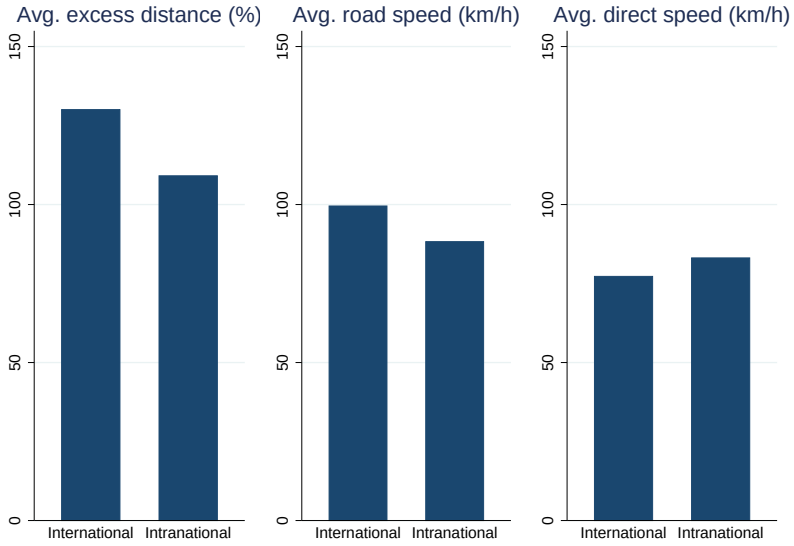
[21] See https://de.wikipedia.org/wiki/Liste_der_Rheinbrücken; the count does not include passages on dams or passages for pedestrians or bicycles only.

[22] The countries are: Austria, Belgium, Bulgaria, Czech Republic, Denmark, Estonia, France, Germany, Greece, Italy, Hungary, Latvia, Lithuania, Luxembourg, Netherlands, Poland, Portugal, Romania, Slovenia, Slovak Republic, Spain, Switzerland.

[23] Head and Mayer (2002) show that measuring distance between countries by a geometric (CES) average of distances between major cities dramatically reduces the border effects in European data. These authors, however, do not account for real transportation costs.

domestic trips does not outweigh the advantage of shorter connections. The hypothetical speed of traffic for overcoming the bird-flight distance is 83 km/h in domestic links versus 77 km/h in international links. This difference is statistically significant at the 5% level.[24]

Figure 2: Distance versus travel time in intra- and international bilateral links



Notes. Average excess distance is the %-ratio between road distance and bird-flight distance, both in km. Avg. road speed denotes the average travel speed on the fastest road connection, in km/h, while avg. direct speed refers to the average theoretical speed on the most direct (= bird-flight distance), in km/h.

Table 1 conditions the difference between domestic and international trips on distance and also controls for destination and origin fixed effects, thus accounting for all possible city characteristics which may affect a place's connectedness to other places. Column (1) reports a simple linear regression that correlates the log of travel time (in minutes) and the log of geographical (great-circle) distance. The obtained elasticity is 0.96. Moreover, column (2) shows that travel time is significantly larger between different countries than within countries, the difference being equal to about 28%. Including the dummy variable for domestic connections brings down the elasticity of travel time with respect to distance to 0.89, again hinting at some concavity in the relationship between distance and trade costs.[25] Columns (3) and (4) repeat this exercise, but use the log of road distance (in km/h) as the dependent variable. It turns out that road distances that cross a political border are longer by some 22%, controlling for bird-flight distance. Finally, columns (5) and (6) show that domestic travel time is longer by 8% given road distances.

Finally, we turn to an investigation of the border effect and show that accounting for the

[24] Note that the hypothetical travel speed on the bird-flight distance is given by the ratio between the travel speed on the road and the excess length of the road trip over the bird-flight distance.

[25] The hypothesis of a unitary elasticity is rejected at the 1% level.

Table 1: Bird-flight distance, road distance, and travel time across EU countries

	(1)	(2)	(3)	(4)	(5)	(6)
Dep.var.:	log travel time		log road distance		log travel time	
Border (0,1)		0.282***		0.217***		0.0799***
		(0.0418)		(0.0325)		(0.0232)
ln bird-flight distance	0.959***	0.893***	1.041***	0.990***		
	(0.0137)	(0.0128)	(0.0113)	(0.00945)		
ln road distance					0.924***	0.905***
					(0.0104)	(0.00987)
Constant	-0.0795	0.344***	-0.0253	0.300***	-0.0721	0.0526
	(0.0890)	(0.0837)	(0.0727)	(0.0613)	(0.0682)	(0.0657)
R-squared	0.983	0.986	0.992	0.994	0.994	0.994

Notes: OLS regressions. All regressions contain complete sets of origin and destination fixed effects. Robust standard errors (clustered at the pair-level) in parentheses, *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. $N = 441$. We use the data provided by Pisu and Braconnier (2013) on bilateral minimum road distances and minimum travel times between 220 cities in 22 countries in continental Europe. The data come from Bing Maps Route Service and relate to the year 2012. We also use population weighted proxies at the country-pair level. We merge this information with CES weighted geographical distance measures (based on great-circle distances between major cities) from Mayer and Zignago (2011).

quality of transport infrastructure in a conventional gravity equation lowers the estimated border effect. We use bilateral trade data for the year of 2010 provided in the World Input Output Database (WIOD) project (Timmer et al., 2015). Crucially, that data base provides accurate measures for domestic trade within countries.[26]

We estimate a standard gravity equation (Head and Mayer, 2014)

$$X_{ij} = \exp\left[\mu_1 \ln T_{ij} + \mu_2 B_{ij} + ex_i + im_j + \epsilon_{ij}\right].$$

Bilateral trade costs are T_{ij} and B_{ij}, where the latter is a dummy that takes value one for an *inter*national and value zero for an *intra*national transaction. $T_{ij} = (1 + D_{ij})^{\delta_0}\, e^{\delta_1 CL_{ij} + \delta_2 CONT_{ij}}$ models trade costs as a function of distance-related transportation costs D_{ij}, common language CL_{ij} and contiguity $CONT_{ij}$. D_{ij} can either be the weighted great-circle distance between i and j, or the shortest (similarly weighted) road distance, or the minimum (similarly weighted) travel time. Following current practice, we estimate Poisson models, as this setup is best suited do deal with zero trade flows and heteroskedasticity arising from additive error terms.[27]

[26] Unfortunately, in Europe, we have no bilateral trade data at the sub-national level. We have no information on Switzerland, so that our analysis rests on $21^2 = 441$ country pairs. We focus on the 16 non-service sectors contained in WIOD.

[27] See Head and Mayer (2014). As a default, we use a Poisson pseudo maximum likelihood estimator (PPML), but for the instrumental variable estimation we estimate the Poisson model using a a generalized methods of

Table 2: The border effect and the role of infrastructure: Poisson gravity models

	(1)	(2)	(3)
Border (0,1)	-0.804***	-0.547***	-0.422**
	(0.170)	(0.180)	(0.176)
ln great-circle distance	-1.112***		
	(0.128)		
ln road distance		-1.171***	
		(0.123)	
ln travel time			-1.361***
			(0.124)
Common language (0,1)	0.761***	0.840***	0.841***
	(0.132)	(0.134)	(0.125)
Contiguity (0,1)	0.326**	0.215	0.134
	(0.157)	(0.154)	(0.141)
Constant	14.92***	15.58***	16.07***
	(0.881)	(0.872)	(0.802)
Observations	441	441	441
R-squared	0.989	0.990	0.991

Notes: Pseudo Maximum Likelihood (PPML) estimations of Poisson models. All models contain complete sets of separate exporter and importer fixed effects. Robust standard errors in parentheses, *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Column (1) of Table 2 reports a frequently observed pattern in gravity models: the trade elasticity of bird-flight distance is close to -1, and a common language or a adjacency (contiguity) have strongly positive effects on trade flows. The estimated border effect is -0.804, signalling that international trade flows are by about 55% $(1 - \exp(-0.804) = 0.55)$ smaller than intranational ones, holding other trade determinants (such as market sizes, multilateral distances – all captured by fixed effects – geographical distance, contiguity, and common language) constant.[28]

Using the log of road distance or travel time instead of great-circle distance as a proxy of transportation costs (columns (2) and (3)), the estimated border effect increases from -0.804 to -0.547 and to -0.422, respectively. Hence, international trade is only by 42% and 34% smaller, respectively, than intranational trade, holding other determinants of trade constant.[29] Hence,

moments (PGMM) approach. Note that results do not qualitatively depend on using the Poisson approach. Log-linear OLS models yield qualitatively similar results; see Table 13 in Appendix *E*.

[28] The measured border effect seems small compared to earlier findings in the literature, e.g., by Nitsch (2000). This has several reasons. First, we use the more consistent distance measure proposed by Head and Mayer (2002), which has been shown to shrink the border effect in a sample of European data from the early 1990s (see footnote 23). Second, the use of PPML yields a substantially smaller border effect than OLS, which dominated the earlier literature; compare Table 13 in Appendix *E*. Third, thanks to the WIOD project, we have more accurate information on within-country trade. And fourth, our data refers to a very recent period (2010) and lower border effects probably simply show the effects of ongoing economic integration in the EU.

[29] $1 - \exp(-0.547) = 0.42$, and $1 - \exp(-0.422) = 0.34$, respectively.

it appears that, in Europe, using proxies of transportation costs that account for the quality of infrastructure brings down the estimated border effect by between 24% and 38%.

In Appendix E, we show that these results continue to hold in a sample of non-contingent country pairs. We conjecture that in such a sample endogeneity concerns are less important than in Table 2, because countries in non-contingent pairs have no control over the transportation costs between them. Our results remain intact when we turn from aggregate to sectoral data. Estimated coefficients change only minimally. If we drop all trade relationships of any country that account for more than 5% of total sector-level trade and which are, thus presumably important enough to affect investment decisions, we find a substantially higher border effect, but replacing great-circle distance by road distance or travel time shrinks the effect; see Table 15 in Appendix E.[30] Finally, we also check whether our results are driven by the recent eastward enlargement of the EU. Excluding the 10 new eastern European EU member countries present in our data, we find an algebraically smaller border effect. Still, the measured border effect changes upon replacing great-circle distance by road distance or travel time in the way suggested by our model.

3 Modeling transportation costs

Models of international trade and economic geography typically employ Samuelson's (1952) iceberg trade costs assumption: in order to receive one unit of the good at some location x, $T(x,z) \geq 1$ units of that good have to leave the factory at location z.[31] A share $1 - T(x,z)^{-1}$ of the good 'melts' in transport; the share $T(x,z)^{-1}$ arrives at x when one unit of the good is shipped at z. This formulation represents a dramatic simplification (McCann, 2005). But it has proved convenient, because it makes the introduction of a specific transportation sector redundant.

In this paper, we stick to the iceberg formulation, but we let transportation costs between two addresses x and z depend on *cumulative* infrastructure investment. Public infrastructure investment refers to the process of investing some resource at specific locations s with the aim of reducing transportation costs.[32] We assume that the set of geographic locations, S, is given

[30] Table 16 in Appendix E runs sector-level regressions and finds statistically significant border effects in 12 out of 16 sectors. In all of these sectors, the border effect is (algebraically) smaller when road distance or travel time is used instead of great-circle distance.

[31] A popular specification in continuous space is $T(x,z) = e^{a|x-z|}$, with $a > 0$; see Krugman (1991) or Fujita et al. (1999).

[32] Our model is static; hence, we use the terms infrastructure investment and stock of infrastructure interchangeably.

by an interval, $[0, \bar{s}]$, where $\bar{s}$ characterizes the geographical size of the economy. In contrast to a circular economy, the linear specification appears preferable since it features a natural periphery.[33]

We model the effectively available stock of infrastructure over some interval $[x, z] \in S$ using a constant elasticity of substitution aggregator function

$$I(x,z) = \left[\int_x^z i(s)^{1-\delta} ds\right]^{\frac{1}{1-\delta}}, \delta > 1, x \leq z, \tag{1}$$

where $i(s)$ is the level/stock of infrastructure at location $s \in [x, z]$ and $\delta > 1$ is a constant technology parameter (which will turn out to be the inverse of the elasticity of substitution between investments at different locations). $I(x, z)$ decreases in distance for a given $\{i(s)\}$. This formulation has the natural implication that spreading a constant investment budget B over increasing distance $z - x$ lowers the available stock of $I(x, z)$. Note also that if $i(s)$ is equal to zero on some subset (with a positive measure) of $[x, z]$, then the available stock of infrastructure over the whole interval $[x, z]$ is zero.

The costs of transporting a product from z to x is linked to infrastructure by

$$T(x,z) = \left(1 + \frac{1}{\delta - 1} I(x,z)^{1-\delta}\right)^{\gamma}, \delta > 1, \gamma > 0, x \leq z. \tag{2}$$

Transportation costs are symmetric in the sense that delivering a product from x to z costs the same as delivering a product from z to x. The parameter γ governs the effect of the total stock of infrastructure on transportation costs.

The choice of functional forms (1) and (2) proves convenient, as will become transparent below. Moreover, the formulation has properties long discussed (but rarely formally modelled in a general equilibrium context) by transport economists (Winston, 1985, or Gramlich, 1994).

Lemma 1 *Generalized iceberg trade costs $T(x, z)$ have the following properties:*

(i) $T(x, z) \geq 1$ *with* $T(x, x) = 1$ *and* $T(x, y)T(y, z) \geq T(x, z)$.

(ii) $T(x, z) = (1 + \theta|z - x|)^{\gamma}$, *if* $i(s)$ *is a constant* $\bar{\imath}$ *over the interval* $[x, z]$, *and* $\theta = \bar{\imath}^{1-\delta}/(\delta - 1)$.

(iii) $T(x, z) = \infty$, *if* $i(s) = 0$ *on some subset (with a positive measure) of* $[x, z]$.

(iv) $T(x, z)$ *is increasing in distance: i.e. for fixed* x *a more distant location* z *results in higher transportation costs. Moreover, if* $i(s)$ *is a differentiable function on* S, $T(x, z)$ *is concave*

[33] Unlike in a symmetric circular economy, in a linear economy, symmetry in terms of endowments or available technologies does not entail symmetry in infrastructure investments.

in distance if the distance-induced increment to the trade cost gradient is outweighed by an improvement in infrastructure. That is, if $(\gamma-1)\left[i(z)^{1-\delta}/(\delta-1)\right]^2/(T(x,z))^{1/\gamma} < i'(z)\,i(z)^{-\delta}$.

(v) *The (interregional) elasticity of substitution between infrastructure investment at different locations is* $0 < 1/\delta < 1$, so *that investments at different locations are gross complements.*

(vi) *Investment smoothing property: if investment costs do not vary across locations, then the cost-efficient way to achieve some exogenous target level of transportation costs involves a flat spatial investment profile on* $[x,z]$:

$$i(s) = \left\{(z-x)/\left[(\delta-1)\left(\bar{T}^{1/\gamma}-1\right)\right]\right\}^{1/(\delta-1)},$$

where $\bar{T} > 1$ *is the target level of iceberg transportation costs.*

Proof The first three properties directly follow from the definition of the trade costs in (2). The last three properties are proved in Appendix A.

Property (i) establishes that, for any x, y, and z, the triangle inequality holds: $T(x,y)T(y,z) \geq T(x,z)$ (the strict inequality holds, if x, y, and z represent different locations). So, it is cheaper to transport products directly to a destination address, rather than through some intermediate address.

Property (ii) shows that equation (2) collapses to the specification almost exclusively used in empirical gravity models (see Head and Mayer, 2014). Our specification also allows for concavity of transportation costs with respect to distance documented in the data. Iceberg transportation costs are concave in distance, as long as the component of $\partial T(x,z)/\partial z$ driven by variation in infrastructure investment outweighs the pure distance component of $\partial T(x,z)/\partial z$ (see property (iv) in the above lemma). Hence, in general, whether $T(x,z)$ is concave in distance depends on the spatial allocation of infrastructure investment and the values of γ and δ.

Properties (v) and (vi) exploit an isomorphism between (2) and the usual representation of utility in an optimal growth model. The parameter δ measures the ease with which infrastructure investment at some address can substitute for investment at another place. The restriction $\delta > 1$ ensures that investments at different places are gross complements: investment at some address makes investment at some other place more worthwhile, which seems realistic and is consistent with the data.[34]

[34] We do not allow for incremental transport costs incurred at address s to depend on the volume of traffic transiting through s. Actually, in equilibrium, the contrary will hold: more traffic at s will encourage the planner

4 Infrastructure investment and *intra*regional trade

In a first step, we incorporate our modeling of transportation costs and infrastructure into a simple model of *intra*national trade. Later we introduce *inter*national trade.

4.1 Geographical space and goods space

Each location $s \in S$ is home of consumers and producers. In particular, we assume that, at each location s, there are $m(s)$ consumers/workers who inelastically supply one unit of labor each. The total endowment of labor in the economy is then equal to $\int_0^{\bar{s}} m(s)ds$, which we define as L. In addition, each location produces a tradable spatially differentiated industrial good/variety. Consumers perceive industrial goods produced at specific locations as imperfect substitutes.

To build transport infrastructure in the economy, the government hires a fraction t of workers from each location s. To simplify the analysis, we assume that these workers can be sent to any locations in the economy (but not abroad) to build infrastructure projects there. The absence of a transportation network does not matter here. The complementary fraction $(1-t)$ of non-construction workers at location s is hired by private firms at this location to produce an industrial good (meaning that non-construction workers are not mobile across locations).

We assume that locations may differ with respect to topological circumstances. Specifically, infrastructure at address s is produced according to a linear production function $i(s) = l^I(s)/q(s)$, where $l^I(s)$ denotes the labor input used for infrastructure investment, and $1/q(s) > 0$ measures the rate at which that labor is transformed into infrastructure. We restrict $q(s)$ to be continuously differentiable. Feasibility of an investment policy $\{i(s)\}_{s\in S}$ requires $\int_0^{\bar{s}} q(s)\, i(s)\, ds \leq t\int_0^{\bar{s}} m(s)ds = tL$.

4.2 Consumption

In the below analysis, we denote addresses of consumers by x and addresses of producers by z. The utility function of a representative consumer at location x is a Dixit-Stiglitz aggregate over industrial goods:

$$U(x) = \left(\int_0^{\bar{s}} c(x,z)^{\frac{\sigma-1}{\sigma}}\, dz\right)^{\frac{\sigma}{\sigma-1}}, \tag{3}$$

with $\sigma > 1$. In the above, $c(x,z)$ denotes the quantity of an industrial good produced at address z and consumed at x.

to invest more in infrastructure, thereby driving down the gradient of T. This lowers the incremental trade costs at s for all units of goods that transit through s.

We assume that construction workers and workers employed in the industrial sector get the same wage, $w(x)$, which is location specific, and that construction workers consume at their original location. To pay to construction workers, the government imposes a proportional income tax t on each consumer, which appears to be equal to the share of construction workers. Thus, the net income of consumers, including construction workers, at location x is $(1-t)w(x)m(x)$. The budget constraint of consumers at x is then $\int_0^{\bar{s}} c(x,z)\, p(x,z)\, dz \leq (1-t)w(x)m(x)$, where $p(x,z)$ is the price of a good imported from location z and consumed at x.

The utility maximization problem implies that the demand function at x for a variety produced at z is

$$c(x,z) = \frac{(1-t)w(x)m(x)P(x)^{\sigma-1}}{p(x,z)^{\sigma}}, \tag{4}$$

where

$$P(x) = \left[\int_0^{\bar{s}} p(x,z)^{1-\sigma}\, dz\right]^{\frac{1}{1-\sigma}} \tag{5}$$

is the CES price index for industrial goods at location x. The indirect utility at location x can be then written as

$$V(x) = \frac{(1-t)w(x)m(x)}{P(x)}. \tag{6}$$

4.3 Production and Equilibrium

At each location $z \in S$, the industrial good is produced under conditions of perfect competition. The only input of production is labor. The production function is linear: $y(z) = a(z)l(z)$, where $l(z)$ is the labor input and $a(z) > 0$ is an exogenous productivity at location z.

We assume that, within addresses, workers are perfectly mobile across industrial firms. This in turn implies that the f.o.b. price, $p(z)$, is equal to $w(z)/a(z)$, where $w(z)$ is the wage rate at address z. Industrial goods bear transportation costs. We assume that there are no trade costs other than transportation costs.[35] Hence, the c.i.f. prices faced by consumers differ from the f.o.b. (ex-factory) prices. In particular, a consumer at x faces the price $p(x,z) = p(z)\,T(x,z)$ for a variety of the industrial good imported from location z.

In the equilibrium, the total revenues of firms located at z are equal to the total labor income of non-construction workers at this location. As the share of workers employed in the industrial

[35] In Section 5, where we consider international trade, we introduce a discrete trade friction at the border.

sector is $(1-t)$, this implies the following system of equations for wages:

$$\begin{aligned}(1-t)w(z)m(z) &= \int_0^{\bar{s}} p\left(x,z\right)c\left(x,z\right)dx \iff \\ w(z)^{\sigma}m(z)a(z)^{1-\sigma} &= \int_0^{\bar{s}} w(x)m(x)P\left(x\right)^{\sigma-1}T\left(x,z\right)^{1-\sigma}dx, \end{aligned} \tag{7}$$

where

$$P\left(x\right)^{1-\sigma} = \int_0^{\bar{s}} \left(\frac{w(z)}{a(z)}T\left(x,z\right)\right)^{1-\sigma} dz. \tag{8}$$

Thus, for any distribution of transport infrastructure, one can solve for equilibrium wages and price indexes.

4.4 The choice of infrastructure investment

In this section, we characterize optimal policies $\left\{i^{aut}(s), t^{aut}\right\}_{s\in S}$ in a closed economy. The social planner chooses the infrastructure investment and the tax rate to maximize total welfare – the sum of individual indirect utilities – in the economy.[36] Note that, in our framework, wages are endogenous and determined by (7) (with $w(0)$ normalized to unity). This implies that, choosing the infrastructure investment, the social planner needs to take into account the effects of infrastructure on the equilibrium wages. Hence, the maximization problem can be written as follows:

$$\max_{\{i(x),t\}_{x\in S}} \left\{(1-t)\int_0^{\bar{s}} \frac{w(x)m(x)}{P(x)}dx\right\} \tag{9}$$

[36] An alternative way of modelling the endogenous infrastructure distribution can be a majority rule voting system. For any distribution of the transport infrastructure, each household derives a certain utility level which is represented by its indirect utility function. As a result, each household has its own preferences over the continuum of infrastructure distributions that are determined by the geographical location of this household: a household prefers more infrastructure investments around its location. For instance, for a household located at the periphery ($x = 0$), the optimal infrastructure profile on $[0, \bar{s}]$ is a decreasing curve with maximum at $x = 0$. If it is possible to show that the household preferences defined above are single peaked, the political equilibrium infrastructure profile is the profile chosen by the median voter, which is the household located at $x = \bar{s}/2$. So, in the absence of asymmetries across locations, the median household will choose a hump shaped profile with maximum at $\bar{s}/2$. In other words, the median voter solution (if it exists) is qualitatively equivalent to the social planner solution. Thus, all the qualitative results derived in the paper can be extended to the framework with a majority rule voting system.

subject to

$$\int_0^{\bar{s}} q(x)\, i(x)\, dx \leq tL, \tag{10}$$

$$w(x)^{\sigma} m(x) a(x)^{1-\sigma} = \int_0^{\bar{s}} w(z) m(z) P(z)^{\sigma-1} T(x,z)^{1-\sigma}\, dz, \tag{11}$$

$$P(x)^{1-\sigma} = \int_0^{\bar{s}} \left(\frac{w(z)}{a(z)} T(x,z)\right)^{1-\sigma} dz. \tag{12}$$

In the below analysis, we assume that

$$\gamma(\delta - 1) < 1. \tag{13}$$

This condition implies that investing in infrastructure at a subset of S with non-zero measure that has no infrastructure yields unbounded positive returns (see details in the proof of Proposition 1).[37] This in turn means that the optimal infrastructure investment, $i^{aut}(s)$, is greater than zero almost everywhere (there are no corner solutions). Note that the inequality in (13) is a sufficient condition. That is, $i^{aut}(s)$ can be greater than zero almost everywhere, even if $\gamma(\delta - 1) \geq 1$.

Next, we characterize the solution of the planner's problem. Note that, without loss of generality, we find the solution among continuous non-negative functions on $[0, \bar{s}]$. That is, social welfare is maximized with respect to $i^{aut}(x)$, where $i^{aut}(x)$ is continuous.

Proposition 1 *If $\gamma(\delta - 1) < 1$, the optimal allocation of infrastructure spending across space and the optimal tax rate chosen by a social planner under autarky are implicitly determined by the following system of equations:*

$$i^{aut}(s)^{\delta} = \frac{\gamma(\sigma-1)L}{q(s)} \left(\int_0^{\bar{s}} \frac{m(x)w(x)}{P(x)} dx\right)^{-1} \left[\phi^L(s) + \phi^R(s)\right], \tag{14}$$

$$t^{aut} = \frac{\int_0^{\bar{s}} q(s)\, i^{aut}(s)\, ds}{L}, \tag{15}$$

where $\phi^L(s)$ and $\phi^R(s)$ denote aggregate marginal utilities from investing to the left or the right of s, respectively, that are determined by the whole distribution of infrastructure investments

[37] The idea is based on the fact that the function $\left(1 + \frac{1}{\delta-1} x^{1-\delta}\right)^{-\gamma}$ has an infinite derivative at $x = 0$, if $\gamma(\delta - 1) < 1$. This allows to show that the marginal change in $T(x,z)^{-1}$ is equal to infinity, if one increases infrastructure investment from zero to some $\varepsilon > 0$ on the subset of $[x, z]$ with non-zero measure. This in turn implies infinite returns in terms of the social welfare.

(see the exact formulas in Appendix C).

Proof In *Appendix C.*

The investment at location s is higher, the larger the sum $\phi^L(s) + \phi^R(s)$ or the lower the cost of infrastructure at the location, $q(s)$. It is straightforward to see that $i^{aut}(\bar{s}) = i^{aut}(0)$ - the infrastructure investments are zero at the borders of the region. In general, the analysis of the properties of $i^a(s)$ is complicated, as the optimal infrastructure profile depends on $w(x)$, $\lambda^{aut}(x)$, and $\mu^{aut}(x)$ that cannot be found explicitly (see the proof of the proposition). To better understand the properties of $i^{aut}(s)$, in the next subsection, we consider a setup where wages are exogenous (fixed at certain arbitrary values), formulate some properties of the optimal infrastructure in this case, and discuss the effects of wage endogeneity on the optimal choice of transport infrastructure.

4.4.1 The Role of Endogenous Wages

Let us assume for the moment that the social planner chooses the optimal policies $\left\{i^{aut,p}(s), t^{aut,p}\right\}_{s\in S}$ without taking into account the effects of infrastructure investment on wages (the "partial" equilibrium). That is, the social planner treats wages as given. In this case, the social planner's maximization problem can be written as follows:

$$\max_{\{i(x),t\}_{x\in S}} \left\{(1-t)\int_0^{\bar{s}} \frac{w(x)m(x)}{P(x)}dx\right\} \tag{16}$$

subject to

$$\int_0^{\bar{s}} q(x)\, i(x)\, dx \leq tL, \tag{17}$$

$$P(x)^{1-\sigma} = \int_0^{\bar{s}} \left(\frac{w(z)}{a(z)}T(x,z)\right)^{1-\sigma} dz. \tag{18}$$

The following proposition holds.

Proposition 2 *If $\gamma(\delta-1) < 1$, the optimal allocation of infrastructure spending across space and the optimal tax rate chosen by a social planner under autarky when the wage schedule*

is taken as given are implicitly determined by the following system of equations:

$$i^{aut,p}(s)^{\delta} = \frac{\gamma(1-t^{aut,p})L}{q(s)}\left(\int_0^{\bar{s}}\frac{m(x)w(x)}{P(x)}dx\right)^{-1}\left[\phi^{L,p}(s)+\phi^{R,p}(s)\right], \quad (19)$$

$$t^{a,p} = \frac{\int_0^{\bar{s}} q(s)\, i^{aut,p}(s)\, ds}{L}, \quad (20)$$

where $\phi^{L,p}(s)$ and $\phi^{R,p}(s)$ denote aggregate marginal utilities from investing to the left or the right of s, respectively.

Proof In Appendix A.

When the effects of infrastructure investment on wages are assumed away, it becomes less complicated to explore the properties of the optimal $i(x)$. In particular, it is straightforward to see that $i^{aut,p}(\bar{s}) = i^{aut,p}(0)$. Moreover, if $q(s)$ is continuously differentiable, $i^{aut,p}(x)$ is increasing in the neighborhood of zero and decreasing in the neighborhood of $\bar{s}$: specifically, $\left(i^{aut,p}(s)\right)'_{s=0} = \infty$ and $\left(i^{aut,p}(s)\right)'_{s=\bar{s}} = -\infty$. It is also possible to show that, in a "flat" economy, when $a(s) = a$, $w(s) = w$, $m(s) = m$, $q(s) = q$ for all s, the optimal infrastructure profile, $i^{aut,p}(s)$, is symmetric around $s = \bar{s}/2$ and has a local extremum at this point. The next proposition summarizes these findings.

Proposition 3 *1) $i^{aut,p}(\bar{s}) = i^{aut,p}(0)$. 2) If $q(s)$ is continuously differentiable, then $\left(i^{aut,p}(s)\right)'_{s=0} = \infty$, $\left(i^{aut,p}(s)\right)'_{s=\bar{s}} = -\infty$. 3) If $(\gamma(\delta-1))^{-1} > \sigma - 1 > 1$, then, in a flat economy, the optimal infrastructure investment function, $i^{aut,p}(s)$, is symmetric around $s = \bar{s}/2$ with $\left(i^{aut,p}(s)\right)'_{s=\bar{s}/2} = 0$.*

Proof 1) and 2) directly follow from the expression for $i^{aut,p}(s)$ in Proposition 2. The statement in 3) is based on the fact that if $(\gamma(\delta-1))^{-1} > \sigma - 1 > 1$, the objective function in the maximization problem (16)-(18) is strictly concave in $i(s)$ (see the proof in Appendix A), implying unique and, therefore, symmetric $i^{aut,p}(s)$ (since symmetric $i^{aut,p}(s)$ is the solution of the first order conditions).

Note that $(\gamma(\delta-1))^{-1} > \sigma - 1 > 1$ implies that $\gamma(\delta-1) < 1$. The former (stronger) condition implies global concavity of the objective function, while the latter (weaker) condition just guarantees the absence of corner solutions. Again, we emphasize that if $(\gamma(\delta-1))^{-1} > \sigma - 1 > 1$ does not hold, the objective function in (16)-(18) can be still concave with respect to $i(s)$. That is, the condition is sufficient.

According to the proposition, if the addresses are symmetric in everything, except their locations on $[0, \bar{s}]$, the optimal infrastructure is symmetric around the middle point of the $[0, \bar{s}]$-interval. Our simulations show that besides the symmetry property the infrastructure profile has a hump shape with maximum at $\bar{s}/2$ (unfortunately, it is very problematic to strictly prove this property). In this case, transportation costs in central parts of the country are lower than in peripheral parts and, thereby, consumers located closer to the middle point have higher indirect utility. The intuition is that, at the optimum, the *sum* of marginal benefits from investment needs to be equalized to the constant cost of investment at each location and this is achieved by higher investment in central locations.

To explore the role of wage endogeneity in determining the optimal infrastructure schedule, we consider a flat economy with an arbitrary symmetric transportation cost function $T(x, y)$ and compare two scenarios. In the first scenario, wages are exogenous and the same across all locations. In the second scenario, wages are determined by the good market clearing condition. Our simulations show that both scenarios lead to quantitatively almost identical values of the total welfare (the average real income). That is, if we define for arbitrary $T(x, y)$

$$W_1 = \int_0^{\bar{s}} \frac{w(x)m(x)}{P(x)} dx,$$

where $w(x) = w$, $m(x) = m$, $a(x) = a$, and

$$W_2 = \int_0^{\bar{s}} \frac{w(x)m(x)}{P(x)} dx,$$

where $m(x) = m$, $a(x) = a$, and wages are determined by

$$w(x)^{\sigma} m(x) a(x)^{1-\sigma} = \int_0^{\bar{s}} w(z)m(z)P(z)^{\sigma-1} T(x, z)^{1-\sigma} dz,$$

then the values of W_1 and W_2 will be quantitatively very close to each other (computing W_2 we normalize $w(0)$ to unity), which is, perhaps, due to relatively little variation in $w(x)$ in a flat economy.[38] This quantitative finding implies that the optimal distribution of transport infrastructure is almost identical under both scenarios. In other words, endogeneity of wages has a very minor impact on the optimal infrastructure profile in a closed flat economy. As a result, the solution of (9)-(12) is negligibly different from the solution of (16)-(18) and, therefore, has

[38] We run a number of numerical simulations with random symmetric transportation costs (alternatively, we assumed a random distribution of transport infrastructure and constructed corresponding transportation costs). In all cases we considered, the difference between W_1 and W_2 was minor, up to 0.19%.

the same properties. Numerical simulations in Section 5.3 support this conclusion. Note also that this quantitative outcome takes place only in a closed economy. If there are two countries and the local governments decide non-cooperatively on the infrastructure investments, the above scenarios do differ (see Section 5.4).

The difference between two scenarios in the closed economy lies in the distribution of real wages across locations. Since the optimal infrastructure profile has a hump shape, the demand for labor in central locations is higher. When wages are endogenous, this results in relatively higher nominal wages in central locations than at the periphery and, thereby, higher real wages there compared to the case when wages are fixed. As a result, wage endogeneity leads to more unequal distribution of real wages, while keeping the average real wage the same.

4.5 Comparative statics

In this section, we discuss how changes in the parameters in the model can affect the optimal infrastructure profile. We first consider uniform changes in $m(x)$, $q(x)$, and $a(x)$. Assume that $m(x) = \kappa m_0(x)$, where $m_0(x)$ is some "initial" distribution of the population size. In this case, it is straightforward to see that the only impact of κ on $i^{aut}(s)$ is through the government tax revenues tL: a rise in κ increases L. As a result, since investments at different places are gross complements, a rise in κ increases $i^{aut}(s)$ for all $s \neq 0, \bar{s}$. In the same way, a uniform rise in $q(x)$ has a negative impact on $i^{aut}(s)$ for all $s \neq 0, \bar{s}$. Finally, it can be clearly seen from the optimization problem that uniform changes in $a(x)$ have no impact on $i^{aut}(s)$. The next lemma summarizes these findings.

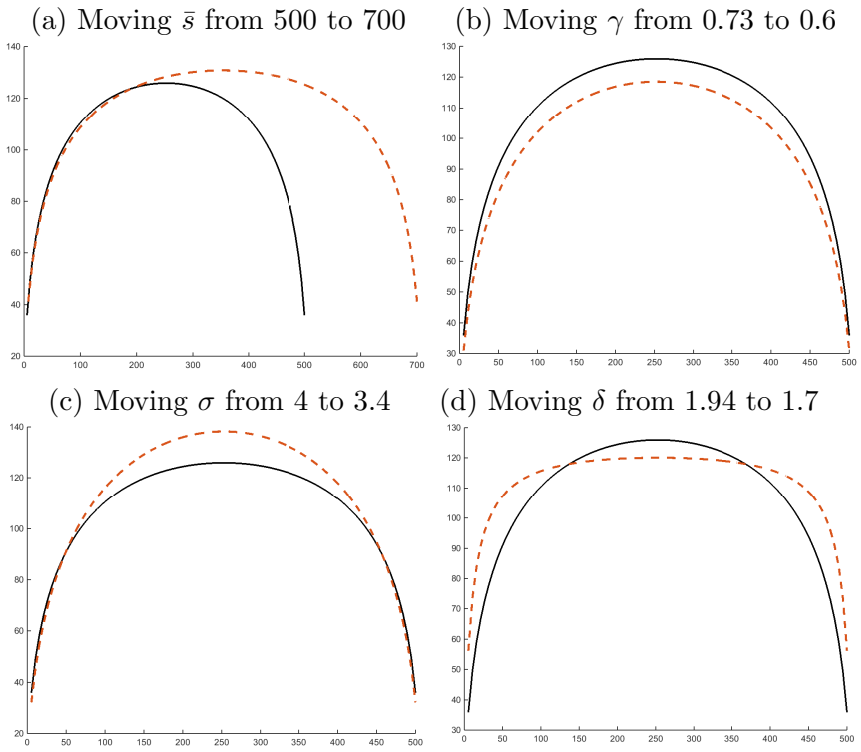
Lemma 2 *1) A uniform rise in $m(x)$ ($q(x)$) increases (decreases) $i^{aut}(s)$ for all $s \neq 0, \bar{s}$. 2) Assuming that $a(x) = a$ for all locations, changes in a do not affect $i^{aut}(s)$.*

Unfortunately, due to complexity of the functional equations that determine $i^{aut}(s)$ (or $i^{aut,p}(x)$), the formal analysis of the comparative statics with respect to $\bar{s}$, γ, δ, and σ does not seem tractable: so, we turn to numerical experiments. Clearly, a rise in $\bar{s}$ increases infrastructure investment in the neighborhood of $\bar{s}$, as there are more locations there. However, the effect on the whole distribution of infrastructure is not obvious. Figure 3(a) provides a numerical illustration of the effect of a rise in $\bar{s}$ on the infrastructure profile.[39] As can be seen, enlargement of a country slightly diverts investment away from locations sufficiently far from the added

[39] In the figure we consider a discrete variation of the model that has similar properties as the continuous model (see more on the choice of the parameterization in Section 5.3).

territory. This is potentially consistent with the German post-reunification experience.[40]

Figure 3: Closed economy equilibrium investment loci: comparative statics



Notes. Solid black curve: default investment distribution with $\sigma = 4$ (the elasticity of substitution between varieties), $\delta = 1.94$ (the inverse of the elasticity of substitution between investments at different locations), $\gamma = 0.73$ (governs the effect of the total stock of infrastructure on transportation costs), $m(x)/q(x) = m/q = 500$, and $\bar{s} = 500$. Dashed red curves: distributions resulting from alternative parameterizations. See Table 3 for calibration details.

A lower value of γ makes the dependence of transportation costs on infrastructure investments weaker. Therefore, the returns from investing in infrastructure at all locations are smaller, leading to lower $i^{aut}(x)$ at all locations (Figure 3(b)). Lower σ makes the varieties of the differentiated product less substitutable, increasing the benefits of intra- and international trade. As a result, incentives to invest in infrastructure go up and, therefore, $i^{aut}(x)$ rises (mostly in central locations, see Figure 3(c)). In contrast, changing δ has an ambiguous impact on infrastructure investment. With the chosen parameterization, a decrease in δ from 1.94 to 1.7 decreases infrastructure investments at central locations and increases at peripheral ones. The change in δ also lowers the spatial variance of investment, as lower δ makes investments at different locations

[40] See, for instance, the information on the development of motorways in Germany since the reunification in 1990 at https://www.bmvi.de/SharedDocs/DE/Artikel/StB/entwicklung-der-autobahnen-in-deutschland-seit-der-wiedervereinigung.html.

more easily substitutable (Figure 3(d)).

5 Infrastructure investment and *inter*national trade

Now, we assume that the world economy consists of two independent countries, each with its own government that decides on infrastructure investment in a non-cooperative way. However, consumers demand goods from all over the world. Thus, we have a situation with 'globalized markets, but regional politics'. Otherwise, we maintain all earlier assumptions. To start with, we assume that countries are symmetric. In particular, the world geography is described by the $[0, 2\bar{s}]$-interval, where locations from $[0, \bar{s}]$ and $(\bar{s}, 2\bar{s}]$ represent the home and the foreign country, respectively. In Appendix D, we explore how the emergence of *inter*continental trade affects the *intra*continental distribution of infrastructure investment, trade, welfare, and the size of the border effect.

To isolate a pure border effect on the equilibrium infrastructure profile we consider a flat economy as a benchmark: i.e., $q(x) = q$, $m(x) = m$, and $a(x) = 1$ for all $x \in [0, 2\bar{s}]$ (we normalize $a(x)$ to unity, as uniform changes in $a(x)$ do not affect the optimal distribution of infrastructure).[41] Finally, we suppose that trade between locations in different countries is subject to an exogenous discrete (i.e., independent from distance) border friction denoted by $\tau \geq 1$. That is, the cost of delivering one unit of a product produced at foreign location z to domestic location x is $\tau T(x, z)$, where $T(x, z)$ is modeled as before. Here, τ can represent both unmeasured trade frictions (contractual issues, currency exchange, languages, etc.) and differences in tastes (home market bias).

5.1 World planner problem

We start with a description of the first-best situation, in which economic and political space coincide. Such a world planner problem is characterized as follows:

$$\max_{\{i(x),t\}_{x\in S}} \left\{ (1-t) \int_0^{2\bar{s}} \frac{w(x)}{P(x)} dx \right\}$$

[41] The framework can be easily extended to the case when $q(x)$, $m(x)$, $a(x)$ are symmetric around $x = \bar{s}$.

subject to

$$\begin{aligned} q\int_0^{2\bar{s}} i\left(x\right)dx &\leq t\int_0^{2\bar{s}} m(x)dx = 2tL, \\ w(x)^{\sigma} &= \int_0^{2\bar{s}} w(z)P\left(z\right)^{\sigma-1}\tau^{(1-\sigma)ind(x,z)}T\left(x,z\right)^{1-\sigma}dz, \qquad (21) \\ P\left(x\right)^{1-\sigma} &= \int_0^{2\bar{s}} \tau^{(1-\sigma)ind(x,z)}\left(w(z)T\left(x,z\right)\right)^{1-\sigma}dz, \qquad (22) \end{aligned}$$

where $ind(x,z)$ is an indicator function, which is equal to 1 if locations x and z belong to different countries and 0 otherwise, and $L = \bar{s}m$ is the population size of each country.

The world planner problem looks very similar to the social planner problem in the case of a closed economy. The only difference is that geographical space is now given by the $[0, 2\bar{s}]$-interval and trade between locations in different countries incurs extra costs. Hence, we can formulate the following proposition.

Proposition 4 *If $\gamma\left(\delta-1\right)<1$, the optimal allocation of infrastructure spending across the world geography and the optimal tax rate chosen by the world planner are implicitly determined by the following system of equations:*

$$\begin{aligned} i^{w}\left(s\right)^{\delta} &= \frac{2\gamma(\sigma-1)L}{q}\left(\int_0^{2\bar{s}}\frac{w(x)}{P(x)}dx\right)^{-1}\left[\phi^{L,w}(s,\tau)+\phi^{R,w}(s,\tau)\right], \\ t^{w} &= \frac{q\int_0^{2\bar{s}} i^{w}\left(s\right)ds}{2L}, \end{aligned}$$

where $\phi^{L,w}\left(s,\tau\right)$ and $\phi^{R,w}\left(s,\tau\right)$ denote aggregate marginal utilities from investing to the left or the right of s, respectively.

Proof The proof is exactly the same as that for Proposition 1.

When τ is equal to unity, the properties of the world planner solution are the same as those for the autarky planner case. In particular, the infrastructure profile has a hump shape with maximum at $\bar{s}$. However, when $\tau > 1$, the infrastructure profile chosen by the world planner has a double-hump shape (see the details in Appendix A). At Home it is decreasing around the border $\bar{s}$, at Foreign it is increasing. Due to the presence of the border friction, the infrastructure profile in the countries is skewed towards internal locations compared to the case with no border friction.

5.2 Global economics, regional politics

We assume that two independent governments play a Nash game and set their infrastructure investment schedules non-cooperatively.

The game is defined as $\Gamma = (I, U_i, \Theta_i)$, where $I = \{H, F\}$ represents the set of countries, Θ_i is country i's strategy set ($i \in I$), and U_i is country i's payoff functional defined on $\Theta_H \times \Theta_F$. In the context of the present framework, Home's strategy set, Θ_H, is given by $\left\{i^H(x), t^H\right\}_{x\in[0,\bar{s}]}$, where $i^H(x) \geq 0$, $\int_0^{\bar{s}} i^H(x)\,dx \leq Lt^H/q$, and $t^H \in [0,1]$. Similarly, Θ_F is the set of $\left\{i^F(x), t^F\right\}_{x\in[\bar{s},2\bar{s}]}$, where $i^F(x) \geq 0$, $\int_{\bar{s}}^{2\bar{s}} i^F(x)\,dx \leq Lt^F/q$, and $t^F \in [0,1]$. Here, $i^H(x)$ and $i^F(x)$ are continuous on $[0,\bar{s}]$ and $[\bar{s},2\bar{s}]$, respectively. Finally, the countries' payoffs are represented by the corresponding countries' total welfare functions.[42]

Since the countries are symmetric, in the following analysis we focus on the home country only. Given the infrastructure profile and the tax rate in the foreign country, the optimization problem of the social planner at Home can be written as follows:

$$\max_{\{i(x),t\}_{x\in[0,\bar{s}]}} \left\{(1-t)\int_0^{\bar{s}} \frac{w(x)}{P(x)}dx\right\}, \tag{23}$$

subject to

$$q\int_0^{\bar{s}} i(x)\,dx \leq tL, \tag{24}$$

for $x \in [0,\bar{s}]$,

$$w(x)^\sigma = \int_0^{\bar{s}} w(z)\left(\frac{P(z)}{T(x,z)}\right)^{\sigma-1}dz + \frac{(1-t^F)\tau^{1-\sigma}}{1-t}\int_{\bar{s}}^{2\bar{s}} w(z)\left(\frac{P(z)}{T(x,z)}\right)^{\sigma-1}dz, \tag{25}$$

$$P(x)^{1-\sigma} = \int_0^{\bar{s}} (w(z)T(x,z))^{1-\sigma}\,dz + \tau^{1-\sigma}\int_{\bar{s}}^{2\bar{s}} (w(z)T(x,z))^{1-\sigma}\,dz, \tag{26}$$

for $x \in [\bar{s},2\bar{s}]$,

$$w(x)^\sigma = \int_{\bar{s}}^{2\bar{s}} w(z)\left(\frac{P(z)}{T(x,z)}\right)^{\sigma-1}dz + \frac{(1-t)\tau^{1-\sigma}}{1-t^F}\int_0^{\bar{s}} w(z)\left(\frac{P(z)}{T(x,z)}\right)^{\sigma-1}dz, \tag{27}$$

$$P(x)^{1-\sigma} = \tau^{1-\sigma}\int_0^{\bar{s}} (w(z)T(x,z))^{1-\sigma}\,dz + \int_{\bar{s}}^{2\bar{s}} (w(z)T(x,z))^{1-\sigma}\,dz. \tag{28}$$

In the above, the transportation costs between two locations from different countries depend

[42] As the space of strategies is infinitely dimensional, the proof of existence of the Nash equilibrium in the game is quite complex. However, in all our simulations, the system of functional equations that defines the symmetric Nash outcome in pure strategies has a solution. Moreover, a discrete analogue of the game (with a discrete number of locations) has Nash equilibrium in pure strategies.

on both domestic and foreign infrastructure investment. As a result, foreign infrastructure investment affects the choice of infrastructure investment in the home country, and vice versa. Moreover, when choosing an infrastructure profile for her country, the social planner takes into account the effects of domestic infrastructure investment on home and foreign wages and, through wages, on home and foreign price indexes that are, in turn, affect the terms of trade of the countries. As a result, the choice of the infrastructure profile in an open economy is determined by the size of trade with Foreign, the fact that investments in different countries are chosen non-cooperatively, and the terms of trade effects.

Next, we formulate the solution of the game described above and discuss some of its properties. The following proposition describes the best response of the home social planner given a certain infrastructure profile in the foreign country.

Proposition 5 *If $\gamma(\delta-1)<1$, then, given an infrastructure profile in the foreign country, $i^F(x)$, and a tax rate, t^F, the home social planner chooses an infrastructure investment schedule and a tax rate, $\left\{i^H(x), t^H\right\}_{x\in[0,\bar{s}]}$, such that the following functional equations are satisfied:*

$$\begin{aligned} i^H(x)^\delta &= \frac{\gamma(\sigma-1)}{\lambda q}\left(\tilde{\phi}^{L,N}(x,\tau)+\tilde{\phi}^{R,N}(x,\tau)\right), \\ t^H &= \frac{q\int_0^{\bar{s}} i^H(x)\,dx}{L}, \end{aligned}$$

where $\tilde{\phi}^{L,N}(x,\tau)$ and $\tilde{\phi}^{R,N}(x,\tau)$ denote aggregate marginal utilities from investing to the left or the right of s, respectively (see Appendix C for details)..

Proof In Appendix C.

Compared to the closed economy case, the optimal infrastructure profile in an open economy is affected by the additional gains from investing in infrastructure caused by changes in the exports and imports of countries. These gains are represented by the second terms in $\tilde{\phi}^{L,N}(x,\tau)$ and $\tilde{\phi}^{R,N}(x,\tau)$. Moreover, these gains are partly determined by the terms of trade effects that are captured by the Lagrange multipliers $\lambda(x)$ and $\mu(x)$. Finally, the choice of the optimal tax t^H is not only based on the trade off between better infrastructure and a higher income tax, but also on the effect of the tax on the country's terms of trade. As a result, the expression for λ, the Lagrange multiplier for the infrastructure spending constraint, is more complicated compared to the closed economy case (or the open economy case with the social planner). It is straightforward to see that if $\tau=\infty$, the solution in Proposition 5 is exactly the same as the solution for the closed economy case, where $\lambda=\left(\int_0^{\bar{s}} w(x)/P(x)dx\right)/L$.

In the symmetric Nash equilibrium, countries' equilibrium infrastructure profiles are symmetric around $x = \bar{s}$, such that $i^F(2\bar{s} - x) = i^H(x)$ for $x \in [0, \bar{s}]$ and $t^F = t^H$. Substituting these conditions into the expression for $i^H(x)$ in Proposition 5 gives the conditions that define the symmetric Nash equilibrium of our game. It is straightforward to see that $i^H(\bar{s}) > 0$.[43] That is, even though $\bar{s}$ is a peripheral location for the social planner, to facilitate trade, she chooses some positive infrastructure investment at the border between the countries. As in the closed economy, $i^H(0) = 0$.

It appears to be quite complicated to analytically explore the properties of the infrastructure distribution in the case of the Nash outcome. However, it is intuitively clear that, in the presence of a foreign country, the distribution of infrastructure investment has more mass in regions closer to the border (compared to the closed economy case). In other words, a decrease in τ ($\tau = \infty$ corresponds to the closed economy case) skews the infrastructure investment towards the border, as the gains from trade increase. Nevertheless, our simulations show that the infrastructure profile still is hump-shaped (even when τ is equal to unity). Hence, in the Nash equilibrium, investment close to the border is lower than in central locations.

Simulations also show that, compared to the first-best solution, the non-cooperative solution features underinvestment at the border. This outcome is based on a network externality: local governments do not internalize the benefits from reductions in domestic transportation costs that accrue to foreign consumers, and these unaccounted benefits are largest the closer a location is to the border. There is also the role of the terms of trade effects. Instead of setting non-cooperative tariffs, the social planners can use infrastructure investment (and the income tax) strategically to manipulate the cost of delivering goods over space. Higher transport costs close to the border increase the price of imports, reducing their demand. This can improve the terms of trade. We quantitatively explore the role of the terms of trade in determining the distribution of infrastructure in Section 5.4.

Another possible explanation for low investment close to national borders may coincide with geographical frictions such as mountain ranges or rivers. In this case, investment is low because of high investment costs, and not due to the network externality. We explore the quantitative role of this possibility in Appendix *D*.

Note that the existence of explicit border costs τ skews the equilibrium infrastructure profile away from the border as gains from trade with the foreign country are lower and the marginal returns from investing around the border are smaller. This offers a natural explanation for the large border effect observed in the data. Indeed, a rise in τ not only increases the trade

[43] Given that there is a continuous space, there is no location at the border, meaning that $i^H(\bar{s}) = \lim_{s \longrightarrow \bar{s}} i^H(s)$.

costs with the foreign country, but also decreases the stock of infrastructure around the border. As a result, the effect of the discrete border cost is larger than the one implied by τ alone. Moreover, even when there is no explicit border cost ($\tau = 1$), the infrastructure profile in an non-cooperative equilibrium has a hump shape, which in turn can generate a statistical border effect in the world with no border costs. This suggests that failing to account for infrastructure biases the border effect measured in the traditional econometric specifications of the gravity model.

5.3 Calibration and numerical illustrations

To provide numerical illustrations and some ideas about the relevance of our mechanism, we calibrate our model. We think it is useful to discipline our numerical exercise with the help of available data, but we are fully aware of the fact that our framework is too stylized to provide a full empirical analysis of the spatial distribution of infrastructure investment in Europe. We work with a discrete formulation of our model (see Appendix B for details) and set parameters such that the model replicates some key moments estimated on European data.

For the calibration, we normalize some parameters. Recall that assuming symmetry within countries ($m(x) = m$, $q(x) = q$) implies that only m/q matters for infrastructure investments. So, we set this ratio to 500. For the cumulative stock of transport infrastructure to matter, the geographical size of a country must be large enough: we set $\bar{s}$ at 500. We assume that, in our discrete formulation, the number of locations within each country is 94, which is the number of the departments in France. Finally, we set the elasticity of substitution σ to 4. In Table 5 we report robustness checks pertaining to these choices.

Particularly crucial parameters such as δ, γ and τ are set such that the model replicates key empirical moments obtained from EU data. To this end, we apply a gravity equation of the type

$$\ln X_{ij} = \mu_1 \ln D_{ij} + \mu_2 B_{ij} + ex_i + im_j + \epsilon_{ij}, \tag{29}$$

to data generated by our model and to actual data. X_{ij} represents trade volumes from location i to j, D_{ij} denotes the distance between the locations, B_{ij} is a border dummy that is equal to one if i and j belong to different countries, ex_i and im_j are exporter and importer fixed effects, and ϵ_{ij} is the error term. We use European trade data to estimate the empirical counterpart of (29); see Section 2.3. This yields estimates $\hat{\mu}_1 = -1.1$ and $\hat{\mu}_2 = -0.8$. We parameterize the model such that the trade data generated from it produces exactly the same values for μ_1 and μ_2 as

Table 3: Calibration strategy

(a) Normalizations		
Parameter	Value	Explanation
m/q	500	arbitrary choice
$\bar{s}$	500	arbitrary choice
n	94	the number of French departments
(b) Parameters calibrated to match EU data		
Parameter	Value	Moment
γ	0.73	Distance elasticity of transport costs in French data (Combes and Lafourcade, 2005): 0.9
δ	1.94	Distance elasticity μ_1 estimated in EU trade data: -1.1
τ	1.09	Border effect μ_2 estimated in EU trade data: -0.8
σ	4	Taken from the literature

those obtained from real data.[44] The parameter γ is chosen such that the generated elasticity of *internal* transportation costs with respect to distance matches the value of 0.9 estimated in French data.[45]

Table 3 summarizes the calibration strategy and outcomes. The border friction implies an ad valorem tax equivalent of 9%, which seems to be a reasonable magnitude for Europe. The interregional elasticity of substitution is $1/\delta = 0.52$, implying a fairly strong degree of complementarity of investments between different locations. The parameter governing the curvature of the transportation cost – distance relationship is $\gamma = 0.73$. We use this parameterization as our benchmark configuration.

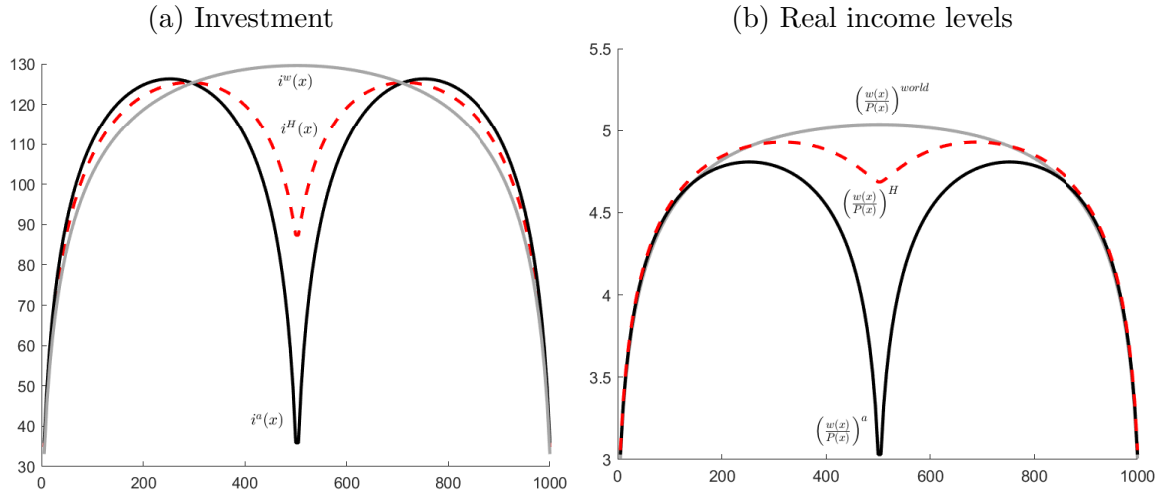
We are now ready to run numerical exercises. We start with the counterfactual situation where τ is equal to unity. Figure 4 compares optimal schedules for infrastructure investment and aggregate price levels, denoting the outcome under autarky by $i^a(x)$, the world-planner solution by $i^w(x)$, and non-cooperative Nash outcome by $(i^H(x), i^F(x))$. In line with our analytical results, the world-planner schedule has a unique maximum at the location of the border, while the non-cooperative schedule has a double-hump shape with underinvestment around the border (compared to the world-planner outcome). This shape can generate a border effect even without having any discrete border costs.

Panel *b* in Figure 4 depicts the distribution of the price index across locations in the autarky,

[44] Note that our approach in calibrating the model implies that we attribute all of iceberg costs to transportation costs. That is, there are standard for the structural gravity equation literature identification issues. For instance, we cannot rule out that the variation in trade across pairs of addresses is due to variation in taste parameters or trade costs, rather than due to variation in transportation costs.

[45] We do not match spending on infrastructure investment t, because due to the one-period nature of our model $i(x)$ measures capital stocks while the empirical value of t discussed in Section 2 refers to the flow of (gross) investment.

Figure 4: Infrastructure investment and real income levels across space in the absence of explicit (discrete) border frictions (i.e., $\tau = 1$)



Notes. $i^a(x), i^w(x)$, and $i^H(x)$ refer to the autarky, central-planer, and non-cooperative optimal infrastructure investment distributions. Similarly for real income levels. See Table 3 for the parameterization and Table 4 for numerical results.

world-planner, and Nash outcomes. In our framework, the price index at location x depends on the 'remoteness' of the location (which determines the level of 'global' trade) and the level of infrastructure in the neighborhood (which determines the level of 'local' trade). Thus, in the world-planner outcome the price index is the lowest at the border, since the border is the least remote location in the world economy and the infrastructure investment is the highest there. In contrast, in the Nash equilibrium the price index has the lowest value at some internal location, rather than at the border (meaning that the local effect is stronger than the global one). This is caused by the skewness of the infrastructure profile towards internal locations in the Nash outcome. If we decrease the geographical size of countries $\bar{s}$ (making 'local' trade less important), the lowest value of the price index will be closer to the border.

Given our parameterization, Table 4 shows that underinvestment in border regions is quantitatively quite substantial, while global underinvestment is not. Compared to the first-best situation (row (2)), the non-cooperative outcome (row (3)) displays an investment gap of 33% at the border (compare 86.7 to 129.2). Overall spending on infrastructure investment (measured by the equilibrium tax rate), falls only 2.7% short of the first-best.[46] Compared to autarky, total

[46] At first glance, the implied tax rate of about 22% might seem high compared to the flow of public spending in relation to GDP as reported in Section 2, Fact 5. Note, however, that the fact refers to the flow of investment while in our static model flows and stocks are indistinguishable. Translating the flows into stocks, one would obtain much higher values that are not unreasonably high relative to the outcomes reported in column (3) of

investment is about 6.5% higher in the first-best situation and about 3.7% in the Nash equilibrium. Real net per capita income at location x being given by $W(x)= (1-t)w(x)/P(x)$, we use average real per capita income, $E[W]$, as a measure of welfare (in Table 4 we normalize autarky welfare to unity).[47] The gains from trade are about 4.3% when the infrastructure investment schedule is first-best and about 4.1% when it is Nash.[48]

The cost of non-cooperation therefore amounts to about 0.2% of autarky welfare, or, equivalently to 4.7% of the gains from trade achievable when investment is first-best. In fact, compared to the first best, the absence of cooperation redistributes the infrastructure profile without substantially changing the average investment. In the Nash outcome, there are more investment in central locations and less around the border. For instance, in the first best outcome the welfare of a consumer living at the border is by 6.5% higher than that in the case of the Nash equilibrium. Note that, in both cases, the level of investment at the periphery is rather the same, which is one of the reasons behind the minor difference in the average welfare.

Inequality (as measured by the variance of the log of real net income multiplied by 100) is lower in the first-best situation compared to autarky and is even lower in Nash, where the income differential between the border region and the periphery is much smaller. Interestingly, the level of inequality seems to be non-monotonic in the level of openness: when $\tau = 1.09$, inequality is lower than in the autarky and free trade cases.

Note that the degree of inequality generated by the model in the Nash outcome (0.72%) is actually quite comparable to the empirical counterpart for France (0.9%) (see the data from INSEE – the French national statistical office – for 2011).[49] That is, despite the stylized nature of our model, we can actually capture the degree of regional inequality in our quantitative exercise, which can be also considered as an external validity check for the numerical exercise we perform. The fact that the inequality generated by the model is slightly smaller than that in the data is fine, since there are many reasons for spatial inequality that matter in reality but that are absent in our framework.

In Figure 5 we set $\tau = 1.09$. Taking into account the size of the border friction, the quantitative implications in this case are not much different from those discussed in the previous paragraph. The world planner spends 1.4% more on infrastructure than country-level govern-

Table 3.

[47] Recall that $m(x)$ is constant across all locations and exogenous.

[48] Note that one of the reasons of the small gains from trade is their spatial dimension. In particular, the gains in the remote from the border locations are very small. This in turn affects the average (across all the locations) gains from trade.

[49] Recall that the number of locations within each country is chosen to match the number of the departments in France, while γ is chosen to fit the elasticity of *internal* transportation costs with respect to distance in French data.

Table 4: Autarky versus first-best and Nash outcomes under trade

	(1) border inv. $i(\bar{s})$	(2) max. inv. $\arg\max i(x)$	(3) tax rate t	(4) real income $E[W]$	(5) inequality $V[\ln W], \%$
Autarky: $\tau = \infty$					
(1) $t^a, i^a(x)$	34.6	250	21.4%	1.0000	1.10
No discrete border frictions: $\tau = 1$ (counterfactual)					
(2) $t^w, i^w(x)$	129.2	500	22.8%	1.0425	0.96
(3) $t^H, i^H(x)$	86.7	290	22.2%	1.0407	0.76
Discrete border frictions: $\tau = 1.09$ (data)					
(4) $t^w, i^w(x)$	113.2	340	22.5%	1.0321	0.81
(5) $t^H, i^H(x)$	77.6	280	22.0%	1.0309	0.72

Notes: The parameterization is as in Figure 4. Two symmetric countries, $S = [0, 1000]$. Autarky welfare is normalized to unity.

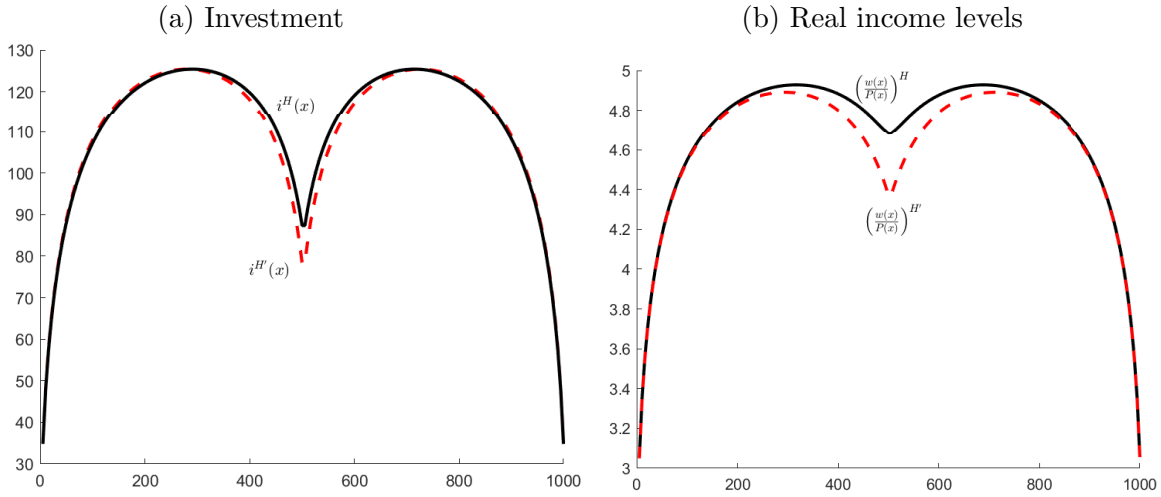
ments would, while the shortfall of investment at the border is around 31%. The border cost decreases average real per capita income by around 0.9%. The presence of the border cost τ also changes the distribution of the price index (see panel b of Figure 5). If there is a border friction, the prices of the foreign varieties rise compared to the prices of the local varieties and, infrastructure investment moves away from the border. As a result, the price index achieves its minimum in more remote (from the border) locations. In the extreme case with prohibitive border costs, we have the autarky case where the price index has a U-shape with the minimum at the central location.

To assess the role of infrastructure in explaining the border effect, we perform the following experiment. We set the infrastructure investment $i(x)$ at all locations to its average value under the benchmark parameterization $\bar{\imath}$ (which is 110.2). In this case, the distribution of infrastructure is flat and does not affect the size of the border effect. All other parameters and variables remain fixed. Then, we simulate trade volumes within and between countries and estimate the border effect generated by the model. We find that the estimate of μ_2 drops from 0.80 to 0.67. This means that the variation in infrastructure investments explains around 16% of the border effect. The rest is explained by the border friction τ and the correlation between the border dummy and distance generated by the model.[50] The discrete border friction $\tau = 1.09$ explains about 32% of the border effect, and 52% is due to the correlation between the border dummy and distance.[51] These findings suggest that the border effect identified in a gravity model that

[50] That correlation amounts to 71%. It arises from the fact, that pairs of locations characterized by short distances tend to be within-country and, thus, have a border dummy of zero while the opposite is true for long distances.

[51] This is a statistical artefact that arises from the mismeasurement of the distance function. In our framework,

Figure 5: How higher discrete border costs change the distribution of infrastructure investment and price levels across space



Notes. $i^H(x)$ refers to the non-cooperative (Nash) infrastructure investment distributions with τ=1, and $i^{H'}(x)$ shows the same loci with a higher border cost: τ=1.09. See Table 3 for the parameterization and Table 4 for numerical results.

ignores variation in transport infrastructure over space and the potential correlation between distance and a border dummy in the data is biased upward. In our case, the size of the bias is about 200%: the real border effect is $(\sigma-1)\ln\tau \approx 0.26$, while the estimated border effect is 0.8.

We also perform robustness checks regarding the chosen values of m/q, $\bar{s}$, n, and σ. In particular, we change the value of one of these parameters, re-calibrate the model, and assess the role of infrastructure in explaining the border effect as well as the size of welfare losses. Table 5 summarizes the results. As can be seen, a rise in m/q (see row (1) in the table) barely changes the calibrated values of γ and τ and the welfare losses. The value of δ decreases from 1.94 to 1.85. The role of infrastructure in explaining the border effect becomes slightly less pronounced: it falls from 16% to 15%. An increase in the geographical size of countries $\bar{s}$ (row (2)) raises the value of δ, while the role of infrastructure decreases from 16% to 15%. The values of γ and τ and the welfare losses do not substantially change. A decrease in the number of internal regions n from 94 to 70 (row (3)) has a minor impact on the values of the calibrated parameters, welfare losses, and the role of infrastructure. Finally, a rise in σ from 4 to 5 (row (4)) has a relatively

the link between distance and transportation costs is more complicated than usually assumed because of the presence of an endogenously arising infrastructure schedule that shapes this link. As a result, the model can generate a statistical border effect even when $i(x) = \bar{\imath}$ and $\tau = 1$ (since the correlation between distances and the border dummy is high).

Table 5: Robustness checks

		(1) γ	(2) δ	(3) τ	(4) $\triangle W$	(5) $Infra$
Benchmark:	$m/q = 500, \bar{s} = 500,$ $n = 94,\ \sigma = 4$	0.73	1.94	1.09	0.1%	16%
(1)	$m/q = 1000$	0.73	1.85	1.09	0.1%	15%
(2)	$\bar{s} = 700$	0.73	1.99	1.09	0.1%	15%
(3)	$n = 70$	0.73	1.95	1.1	0.1%	16%
(4)	$\sigma = 5$	0.66	2.04	1.04	0.1%	15%

Notes: We fix the moments measured in European and French data, modify our choice of free parameters, and obtain new calibrated values for γ, δ, τ, (columns (1), (2), (3)). Columns (4) and (5) show the welfare costs resulting from an inefficient infrastructure investment, and the relative importance of infrastructure in explaining the border effect, respectively.

Table 6: Robustness checks with respect to the values of the moments in the data

		(1) γ	(2) δ	(3) τ	(4) $\triangle W$	(5) $Inf.$	(6) $V[\ln W]$
Benchmark:	$\hat{\mu}_1 = -1.1,\ \hat{\mu}_2 = -0.8$	0.73	1.94	1.09	0.11%	16%	0.72
OLS	$\hat{\mu}_1 = -1.7,\ \hat{\mu}_2 = -1.4$	0.81	1.77	1.35	0.01%	11%	0.46
HM (2014)	$\hat{\mu}_1 = -1.1,\ \hat{\mu}_2 = -1.6$	0.74	1.94	1.38	0.03%	12.5%	0.79
"old" EU	$\hat{\mu}_1 = -1.08,\ \hat{\mu}_2 = -0.69$	0.73	1.95	1.06	0.14%	16%	0.74

Notes: We change the measured gravity moments and obtain new calibrated values for γ, δ, τ, (columns (1), (2), (3)). Columns (4), (5), and (6) show the welfare costs resulting from an inefficient infrastructure investment, the relative importance of infrastructure in explaining the border effect, and the level of inequality, respectively.

substantial impact on the calibrated values of the parameters (especially, on γ and τ), while the impact on the role of infrastructure and welfare losses is small. The role of infrastructure decreases from 16% to 15%, while the welfare losses do not change.

In the above calibration procedure, we use the estimates of the border effect and the distance elasticity derived from applying the PPML estimator to bilateral trade data for Europe. Below, we consider several alternative values of these estimates. First, we consider the estimates derived from the OLS estimator. Second, we take the estimates from the meta-analysis in Head and Mayer (2014). Finally, we consider the estimates of the border effect and the distance elasticity derived from applying the PPML estimator to the "old" EU members only. Table 6 reports the results.

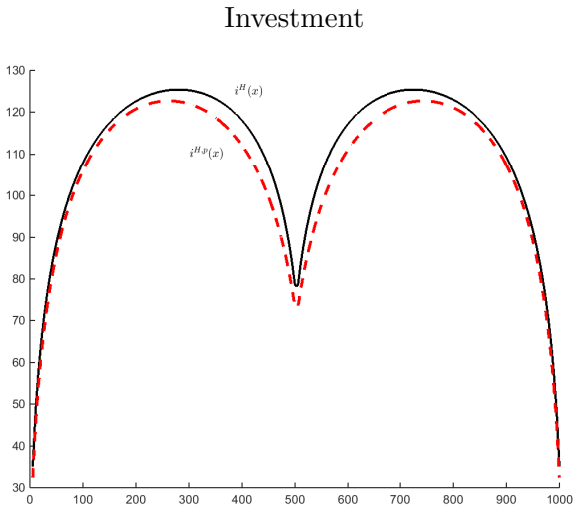
As can be seen from the table, taking different gravity moments does not substantially changes the calibrated values of the parameters. The exception is the value of τ that is higher

in the cases when the border effect in the data is equal to -1.4 and -1.6, respectively. In these cases, the role of infrastructure in explaining the border effect is lower: 11% and 12.5%. Overall, according to our model, the role of an exogenous border friction becomes relatively more important, when the border effect is larger. The welfare losses from the misallocation of infrastructure remain minor. An interesting thing to notice is the dependence of the inequality on the value of δ. The idea is that when the distance elasticity is higher, the calibrated value of δ is lower: the infrastructure investments are more substitutable. Not-surprisingly, this leads to a lower level of inequality in the economy.

5.4 The role of the wage endogeneity

In this section, we explore the role of wage endogeneity in determining the infrastructure profile. Recall that when wages are endogenous, the local social planner takes into the effects of infrastructure investment on the wage schedule in both countries, which in turn affects the countries' terms of trade. When wages are exogenous (set to unity), there are no terms of trade effects in the model, as we consider a perfectly competitive environment. Next, we perform two experiments. First, given the benchmark configuration, we assess how the infrastructure profile and spending on infrastructure change, if we set all the wages to unity. Second, we re-calibrate the model assuming exogenous wages and compare the quantitative implications of both specifications (with and without wage endogeneity).

Figure 6: The distribution of infrastructure with and without wage endogeneity



Notes. $i^H(x)$ and $i^{H,p}(x)$ refer to the non-cooperative (Nash) infrastructure investment distributions with and without wage endogeneity. See Table 3 for the parameterization.

Table 7: First-best and Nash outcomes with and without wage endogeneity under trade

		(1) border inv. $i(\bar{s})$	(2) max. inv. $\arg\max i(x)$	(3) tax rate t	(4) real income $E[W]$	(5) inequality $V[\ln W]$, %
Discrete border frictions: $\tau = 1.09$ (data)						
(1)	$t^w, i^w(x)$	113.2	340	22.5%	1.0321	0.81
(2)	$t^H, i^H(x)$	77.6	280	22.0%	1.0309	0.72
(3)	$t^{H,p}, i^{H,p}(x)$	72.5	260	21.3%	1.0306	0.44

Notes: The parameterization is as in Figure 4. Two symmetric countries, $S = [0, 1000]$. Autarky welfare is normalized to unity.

As can be seen from Figure 6, under the benchmark configuration, the Nash specification with endogenous wages results in a slightly higher level of investments in all locations. In particular, the level of investment at the border is 7% higher when wages are endogenous (see row (3) in Table 7). Moreover, when wages are exogenous, the infrastructure profile is skewed a bit more towards central locations: the maximum investment is at $s = 260$ (compared to 280, when wage are endogenous). Spending on infrastructure, the welfare losses from non-cooperation, and the role of infrastructure in explaining the border effect do not substantially change. An interesting observation to notice is that the level of inequality is much lower in the case with exogenous wages: 0.44 versus 0.72. In other words, general equilibrium changes in nominal wages result in a more unequal distribution of the real income across locations.

Next, we calibrate the model assuming that wages are exogenous and set to unity at all locations. As one can see from Table 8 (see row (1)), there are relatively minor changes in the calibrate values of the parameters: τ decreases from 1.09 to 1.08, δ and γ increase to 1.95 and 0.74, respectively. The role of infrastructure in explaining the border effect rises from 16% to 18%. The losses from non-cooperation rise as well (from 0.1% to 0.2%).

The findings above show that accounting for wage endogeneity does not substantially change the quantitative implications of our framework. This in turn suggests that the terms of trade effects do not play a decisive role in determining the distribution of transport infrastructure across space.

6 Conclusions

This paper develops a simple theoretical model where consumers demand goods from the entire world, but the world is fragmented into jurisdictions which set infrastructure investment schedules in a non-cooperative way. Governments care only for the welfare of their own constituency

Table 8: Calibrated values of the parameters with and without wage endogeneity

		(1) γ	(2) δ	(3) τ	(4) $\triangle W$	(5) $Infra$
Benchmark:	$m/q = 500, \bar{s} = 500,$ $n = 94,\ \sigma = 4$	0.73	1.94	1.09	0.1%	16%
(1)	$m/q = 500, \bar{s} = 500,$ $n = 94,\ \sigma = 4,\ w(x) = 1$	0.74	1.95	1.08	0.2%	18%

Notes: We fix the moments measured in European and French data, assume exogenous and identical wages, and obtain new calibrated values for γ, δ, τ, (columns (1), (2), (3)). Columns (4) and (5) show the welfare costs resulting from an inefficient infrastructure investment, and the relative importance of infrastructure in explaining the border effect, respectively.

and ignore the effects that their decisions have on foreign consumers; this basic externality leads to global underinvestment. The lower the geographical distance of a location is to the political border, the bigger is the externality since a larger share of the benefits caused by investment accrues to foreign consumers. Hence, the externality has a spatial structure: infrastructure underinvestment is stronger in politically peripheral regions of jurisdictions rather than in central ones.

The local lack of infrastructure investment makes imports from other countries more expensive than imports from other regions from the same country, even if geographical distance or incomes of trading partners are the same. Our infrastructure story may therefore contribute towards unpacking trade costs and explaining the border effect first highlighted by McCallum (1995) and since then discussed in a voluminous empirical literature. We use intra- and international trade flows data from Europe to calibrate the model. We find that about 16% of the empirically observable border effect is due to low infrastructure investment around national borders. In our simulations, the border effect is magnified by the emergence of intercontinental trade.

Information from the Bing Maps Route Service allows us to show that international road distances are about 22% longer than intranational ones and that travel is 28% longer, holding bird-flight distance constant. The econometrically identified border effect falls from -0.80 in a conventional setup to -0.55 when road distance is used rather than bird-flight distance. Hence, using transportation cost proxies that account for infrastructure brings down the border effect, as implied by our theoretical argument.

Future work should endogenize the distribution of economic activity by allowing intranational migration of workers or/and firms. Moreover, to carry out a full-fledged quantitative analysis,

it would be needed to conduct the analysis on a plane rather than a line with realistic topology as in Allen and Arkolakis (2014); this is a non-trivial generalization, since it requires not only to characterize optimal investment schedules for given routes, but also to solve for the layout of optimal transportation networks.

References

Alder, S. (2019). Chinese Roads in India: The Effect of Transport Infrastructure on Economic Development. mimeo.

Allen, T. and C. Arkolakis (2014). Trade and the Topography of the Spatial Economy. Quarterly Journal of Economics, 129(3): 1085-1140.

Allen, T. and C. Arkolakis (2019). The Welfare Effects of Transportation Infrastructure Improvements. NBER Working Paper 25487.

Anderson, J.E. and E. van Wincoop, E. (2003). Gravity with Gravitas: A Solution to the Border Puzzle. American Economic Review 93(1): 170-92.

Asturias, J. (2019). Endogenous transportation costs. European Economic Review, forthcoming.

Behar, A. and A. Venables (2011). Transport Costs and International Trade. In: de Palma, A., R. Lindsey, E. Quinet, and R. Vickerman, eds., A Handbook Of Transport Economics, Edgar Elgar.

Behrens, K., P. Picard (2011). Transportation, freight rates, and economic geography. Journal of International Economics 85(2): 280-291.

Brancaccio, G., Kalouptsidi, M., and Papageorgiou, T. (2019). Geography, Transportation, and Endogenous Trade Costs. Econometrica, forthcoming.

Bruelhart, M., Cadot, O., and Himbert, A. (2019). Let There Be Light: Trade and the Development of Border Regions. Working Papers P251, FERDI.

Casella, A., and J. Rauch, Overcoming Informational Barriers to International Resource Allocation: Prices and Ties, Economic Journal 113(1): 21-42.

Cazala, A., J. Deterne, G. Crespy, P. Garnier, G. De Monchy, and P. Rimattei (2006), Rapport sur la comparaison au niveau européen des coûts de construction, d'entretien et d'exploitation des routes, Conseil général des ponts et chaussées, available on http://www.ladocumentationfrancaise.fr/var/storage/rapports-publics/074000038.pdf.

Chen, N. (2004). Intra-national versus international trade in the European Union: why do national borders matter? Journal of International Economics 63(1): 93-118.

Combes, P-P., M. Lafourcade and T. Mayer, 2005. The Trade Creating Effects of Business and Social Networks: Evidence from France, Journal of International Economics 66:1-29.

Combes, P-P. and M. Lafourcade, 2005. Transport Costs. Measures, Determinants, and Regional Policy Implications for France. Journal of Economic Geography 5:319-349.

Congressional Budget Office, 2015. Public Spending on Transportation and Water Infrastructure: 1956-2014. US Congress, Washington, DC (https://www.cbo.gov/publication/49910).

Cosar, K. and B. Demir (2015). Domestic Road Infrastructure and International Trade: Evidence from Turkey, *Journal of Development Economics*, forthcoming.

Costinot, A. and A. Rodriguez-Clare (2014), Trade Theory with Numbers: Quantifying the Consequences of Globalization, Handbook of International Economics, Vol. 4, Chapter 4, Gita Gopinath, Elhanan Helpman, and Kenneth Rogoff (eds.), Elsevier: Holland.

Courant, P.N., and A.V. Deardorff (1992). International Trade with Lumpy Countries. Journal of Political Economy 100(1): 198-210.

Donaldson, D. (2014). Railroads and Raj: Estimating the Impact of Transportation Infrastructure. American Review of Economics, forthcoming.

Donaldson, D., and R. Hornbeck, Railroads and American Economic Growth: a "Market Access" Approach, Quarterly Journal of Economics, forthcoming.

Duranton, G., P. Morrow, and M. A. Turner (2014). Roads and Trade: Evidence from the US. The Review of Economic Studies 81 (2): 681-724.

Fajgelbaum, P.D., and E. Schaal (2017), Optimal Transport Networks In Spatial Equilibrium, NBER Working Paper 23200.

Farhadi, M. (2015). Transport infrastructure and long-run economic growth in OECD countries. Transportation Research Part A: Policy and Practice 74: 73-90.

Fujita, M., P.R. Krugman, and A.J. Venables (1999). The Spatial Economy. Cambridge, MA: MIT Press.

Gibbons, S., Lyytikaeinen, T., Overman, Henry G., and Sanchis-Guarner, R. (2019). New Road Infrastructure: The Effects on Firms. Journal of Urban Economics 110: 35–50.

Gramlich, E.M. (1994). Infrastructure Investment: A Review Essay. Journal of Economic Literature 32(3): 1176-1196.

Head, K., and T. Mayer (2014), Gravity Equations: Workhorse, Toolkit, and Cookbook, Handbook of International Economics, Vol. 4, Chapter 3, Gita Gopinath, Elhanan Helpman, and Kenneth Rogoff (eds.), Elsevier: Holland.

Head, K., and T. Mayer (2002), Illusory Border Effects: Distance mismeasurement inflates estimates of home bias in trade, CEPII Working Paper 2002-1.

Helliwell, J.F. (1998). How Much Do National Borders Matter? The Brookings Institution.

Hummels, D., V. Lugovskyy and A. Skiba (2009). The trade reducing effects of market power in international shipping, Journal of Development Economics 89: 84–97.

ITF-OECD (International Transport Forum) (2012), Statistics Brief-Infrastructure Investment, Paris.

Krugman, P. (1991). Increasing Returns and Economic Geography. Journal of Political Economy 99: 383-499.

Lay, M.G. (1992), Ways of the World: A History of the World's Roads and of the Vehicles That Used Them, New Brunswick: Rutgers University Press.

Limao, N., and A.J. Venables (2001). Infrastructure, Geographical Disadvantage, Transport Costs, and Trade. World Bank Economic Review 15(3): 451-479.

Mayer, T., and S. Zignago (2011), Notes on CEPII's distance measures: the GeoDist Database, CEPII Working Paper 2011-25.

McCallum, J. (1995). National Borders Matter: Canada-U.S. Regional Trade Patterns. American Economic Review 85(3): 615-23.

McCann, Ph. (2005). Transport costs and new economic geography. Journal of Economic Geography 5: 305–318.

Nitsch, V. (2000). National Borders and International Trade. Evidence from the European Union. Canadian Journal of Economics 33(4): 1091-1105.

Okubo, T. (2004). Border Effect in the Japanese Market – A Gravity Model Analysis. Journal of the Japanese and International Economies 18: 1-11.

Persson, J. and D. Song (2010), The Land Transport Sector: Policy and Performance, OECD Economics Department Working Papers, No. 817, OECD Publishing.

Pisu, M. and H. Braconier (2013), Road Connectivity and the Border Effect: Evidence from Europe, GEP Research Paper Series 2013/06.

Redding, S. and M. Turner (2015), Transportation Costs and the Spatial Organization of Economic Activity, in G. Duranton, J. Henderson and W. Strange (eds.), Handbook of Urban and Regional Economics, Chapter 20, 1339-1398.

Roberts, M., M. Melecky, T. Bougna and Y. Xu (2018), Transport Corridors and Their Wier Economic Benefits: A Critical Review of the Literature. World Bank Policy Research Working Paper 8302.

Rose, A.K., and E. van Wincoop (2001). National Money as a Barrier to International Trade: The Real Case for Currency Union. American Economic Review 91(2): 386-390.

Rossi-Hansberg, E. (2005). A Spatial Theory of Trade. American Economic Review 95(5): 1464-1491.

Samuelson, P. (1952). The Transfer Problem and Transport Costs: the Terms of Trade when Impediments are Absent. Economic Journal 62: 278–304.

Timmer, M., E. Dietzenbacher, B. Los, R. Stehrer, G. de Vries (2015), "An Illustrated User Guide to the World Input-Output Database: the Case of Global Automotive Production", Review of International Economics, doi: 10.1111/roie.12178.

Wei, S. (1996). Intra-National versus International Trade: How Stubborn are Nations in Global Integration? NBER Working Paper 553.

Winston, C. (1985). Conceptual Developments in the Economics of Transportation: An Interpretative Survey. Journal of Economic Literature 23: 57-94.

Wolf, H. (2000). Intranational Home Bias in Trade. Review of Economics and Statistics 82(4): 112-128.

Wong, W. F. (2019). The Round Trip Effect: Endogenous transport costs and international trade. mimeo.

Venables, A. (2005). Spatial disparities in developing countries: cities, regions, and international trade. Journal of Economic Geography 5(1): 3 - 21.

Appendix A: Proofs

In this Appendix, we provide some proofs for the lemmas and propositions in the paper. The proofs of Propositions 1 and 5 (that are longer and more technical) can be found in Appendix C.

Proof of Lemma 1

Property (i)

For three different locations x, y, and z such that $x < y < z$,

$$\begin{aligned}
T(x,y)T(y,z) &= \left(1+\frac{1}{\delta-1}\int_x^y i\left(s\right)^{1-\delta}ds\right)^{\gamma}\left(1+\frac{1}{\delta-1}\int_y^z i\left(s\right)^{1-\delta}ds\right)^{\gamma} \\
&= \left(1+\frac{1}{\delta-1}\int_x^y i\left(s\right)^{1-\delta}ds+\frac{1}{\delta-1}\int_y^z i\left(s\right)^{1-\delta}ds+\frac{1}{\left(\delta-1\right)^2}\int_x^y i\left(s\right)^{1-\delta}ds\int_y^z i\left(s\right)^{1-\delta}ds\right)^{\gamma} \\
&> \left(1+\frac{1}{\delta-1}\int_x^y i\left(s\right)^{1-\delta}ds+\frac{1}{\delta-1}\int_y^z i\left(s\right)^{1-\delta}ds\right)^{\gamma} \\
&= \left(1+\frac{1}{\delta-1}\int_x^z i\left(s\right)^{1-\delta}ds\right)^{\gamma} = T(x,z).
\end{aligned}$$

Property (iv)

The behavior of $T\left(x,z\right)$ with respect to geographical distance can be checked by looking at the derivative of $T\left(x,z\right)$ with respect to z. Specifically, we have that

$$\begin{aligned}
T\left(x,z\right) &= \left(1+\frac{1}{\delta-1}\int_x^z i\left(s\right)^{1-\delta}ds\right)^{\gamma}, \\
T_z\left(x,z\right) &= \gamma\left(1+\frac{1}{\delta-1}\int_x^z i\left(s\right)^{1-\delta}ds\right)^{\gamma-1}\frac{1}{\delta-1}\left[i\left(z\right)^{1-\delta}\right]>0, \\
T_{zz}\left(x,z\right) &= \gamma\left(1+\frac{1}{\delta-1}\int_x^z i\left(s\right)^{1-\delta}ds\right)^{\gamma-1}\left(\left(\gamma-1\right)\frac{\left(\frac{1}{\delta-1}\left[i\left(z\right)^{1-\delta}\right]\right)^2}{1+\frac{1}{\delta-1}\int_x^z i\left(s\right)^{1-\delta}ds}-i\left(z\right)^{-\delta}i'\left(z\right)\right)\gtrless 0 \\
&\iff \frac{\left(\gamma-1\right)}{\left(T\left(x,z\right)\right)^{1/\gamma}}\left(\frac{i\left(z\right)^{1-\delta}}{\delta-1}\right)^2 \gtrless i'\left(z\right)i\left(z\right)^{-\delta}.
\end{aligned}$$

The left-hand side shows the effect of a marginal increase in distance on $T_z\left(x,z\right)$ under the assumption that $i\left(z+dz\right)=i\left(z\right)$. It reflects variation in trade costs due to an increase in distance, holding infrastructure constant. The right-hand side makes the opposite assumption and reports the change in $T_z\left(x,z\right)$ due to the difference in infrastructure investment between z and $z+dz$, holding the sheer costs of distance constant. $T\left(x,z\right)$ is convex (as in Krugman), if the left-hand side dominates.

Property (v)

We compute the elasticity of substitution between investment at two different locations $s', s'' \in [x, z]$ as follows

$$-\frac{d\ln\left[i\left(s'\right)/i\left(s''\right)\right]}{d\ln\left|\frac{\partial T(x,z)}{\partial i(s')}/\frac{\partial T(x,z)}{\partial i(s'')}\right|},$$

where for any $s \in [x, z]$

$$\frac{\partial T(x,z)}{\partial i\left(s\right)} \overset{d.}{\equiv} -\gamma\left(1+\frac{1}{\delta-1}\int_x^z i\left(s\right)^{1-\delta}ds\right)^{\gamma-1} i\left(s\right)^{-\delta}.$$

As a result, the elasticity of substitution is given by

$$-\frac{d\ln\left[i\left(s'\right)/i\left(s''\right)\right]}{-d\delta\ln\left|i\left(s'\right)/i\left(s''\right)\right|} = \frac{1}{\delta} < 1.$$

Property (vi)

The Lagrangian for the cost minimizing problem can be written as follows:

$$\Lambda\left(\left\{i\left(s\right)\right\},\lambda\right) = q\int_x^z i\left(s\right)ds + \lambda\left[1+\frac{1}{\delta-1}\int_x^z i\left(s\right)^{1-\delta}ds - \bar{T}^{1/\gamma}\right],$$

where q is the cost of infrastructure investment at location s. It is straightforward to see that the first order conditions imply that, for any two locations $k, l \in [x, z]$,

$$i\left(k\right) = i\left(l\right).$$

Taking into account the budget constraint, we derive that

$$i\left(k\right) = i\left(l\right) = \left\{\left(z-x\right)/\left[\left(\delta-1\right)\left(\bar{T}^{1/\gamma}-1\right)\right]\right\}^{1/(\delta-1)}.$$

Proof of Proposition 2

As in the proof of Proposition 1, we consider the Lagrange function for the optimization problem formulated in Proposition 2:

$$\begin{aligned}\mathcal{L} &= \int_0^{\bar{s}}\frac{(1-t)m(x)w(x)}{P(x)}dx - \lambda\left(\int_0^{\bar{s}} q\left(x\right)i\left(x\right)dx - tL\right)\\ &\quad + \int_0^{\bar{s}}\mu(x)\left(\int_0^{\bar{s}}\left(\frac{w(y)}{a(y)}T(x,y)\right)^{1-\sigma}dy - P(x)^{1-\sigma}\right)dx,\end{aligned}$$

where λ and $\mu(x)$ are corresponding Lagrange multipliers. Using the results derived in the proof of Proposition 1 (see Appendix C), the FOC's for $i(x)$, t, and $P(x)$ can be written as follows:

$$\begin{aligned}
\frac{\lambda q(s) i(s)^{\delta}}{\gamma(\sigma-1)} &= \int_{s}^{\bar{s}} \mu(x) \int_{0}^{s} \left(\frac{w(y)}{a(y)}\right)^{1-\sigma} \left[1+\frac{1}{\delta-1}\int_{y}^{x} i(s)^{1-\delta} ds\right]^{\gamma(1-\sigma)-1} dydx \\
&\quad + \int_{0}^{s} \mu(x) \int_{s}^{\bar{s}} \left(\frac{w(y)}{a(y)}\right)^{1-\sigma} \left[1+\frac{1}{\delta-1}\int_{x}^{y} i(s)^{1-\delta} ds\right]^{\gamma(1-\sigma)-1} dydx, \\
\lambda &= \left(\int_{0}^{\bar{s}} \frac{m(x)w(x)}{P(x)} dx\right) L^{-1}, \\
\frac{(1-t)m(x)w(x)}{(P(x))^{2}} &= (\sigma-1)\frac{\mu(x)}{P(x)^{\sigma}} \iff \mu(x) = \frac{(1-t)m(x)w(x)}{(\sigma-1)P(x)^{2-\sigma}}.
\end{aligned}$$

Thus, we have that, *and are defined as*

$$\begin{aligned}
\phi^{L,p}(s) &= \int_{0}^{s}\int_{s}^{\bar{s}} \frac{m(x)w(x)}{P(x)^{2-\sigma}} \left(\frac{w(y)}{a(y)}\right)^{1-\sigma} T(x,y)^{1-\sigma-1/\gamma} dydx, \\
\phi^{R,p}(s) &= \int_{s}^{\bar{s}}\int_{0}^{s} \frac{m(x)w(x)}{P(x)^{2-\sigma}} \left(\frac{w(y)}{a(y)}\right)^{1-\sigma} T(x,y)^{1-\sigma-1/\gamma} dydx.
\end{aligned}$$

Proof of Proposition 3

Note that the maximization problem given by $(16)-(18)$ can be written in the following way:

$$\max_{\{i(x),t\}_{x\in S}} \left\{(1-t)\int_{0}^{\bar{s}} w(x)m(x)\left(\int_{0}^{\bar{s}} \left(\frac{w(z)}{a(z)}T(x,z)\right)^{1-\sigma} dz\right)^{1/(\sigma-1)} dx\right\}$$

subject to

$$\int_{0}^{\bar{s}} q(x)\, i(x)\, dx \leq tL.$$

Hence, it is sufficient to show that $\left(\int_{0}^{\bar{s}} \left(\frac{w(z)}{a(z)}T(x,z)\right)^{1-\sigma} dz\right)^{1/(\sigma-1)}$ is strictly concave in $i(x)$. In doing so, we consider the following function:

$$K(\xi) = \left(K_1(\xi) + K_2(\xi)\right)^{1/(\sigma-1)},$$

where

$$\begin{aligned}
K_1(\xi) &= \int_{0}^{x} \left(\frac{w(z)}{a(z)}\right)^{1-\sigma} \left[1+\frac{1}{\delta-1}\int_{z}^{x} \left(i(s)+\xi h(s)\right)^{1-\delta} ds\right]^{\gamma(1-\sigma)} dz, \\
K_2(\xi) &= \int_{x}^{\bar{s}} \left(\frac{w(z)}{a(z)}\right)^{1-\sigma} \left[1+\frac{1}{\delta-1}\int_{x}^{z} \left(i(s)+\xi h(s)\right)^{1-\delta} ds\right]^{\gamma(1-\sigma)} dz,
\end{aligned}$$

and $h(s)$ is an arbitrary continuous on $[0, \bar{s}]$ function. To show strict concavity, it is sufficient to show that $K''(0) < 0$ for any $i(s)$ and $h(s)$. We have

$$\begin{aligned} K'(\xi) &= \frac{1}{\sigma-1}\left(K_1(\xi)+K_2(\xi)\right)^{1/(\sigma-1)-1}\left(K_1'(\xi)+K_2'(\xi)\right), \\ K''(\xi) &= \frac{1}{\sigma-1}\left(\frac{1}{\sigma-1}-1\right)\left(K_1(\xi)+K_2(\xi)\right)^{1/(\sigma-1)-2}\left(K_1'(\xi)+K_2'(\xi)\right)^2 \\ &\quad +\frac{1}{\sigma-1}\left(K_1(\xi)+K_2(\xi)\right)^{1/(\sigma-1)-1}\left(K_1''(\xi)+K_2''(\xi)\right). \end{aligned}$$

Since we assume that $\sigma > 2$, the first term in $K''(\xi)$ evaluated at $\xi = 0$ is strictly negative. Thus, $K''(0)$ is strictly negative if $K_1''(0) + K_2''(0)$ is strictly negative. To show that $K_i''(0) < 0$, it is sufficient to show that for any $i(s)$, $h(s)$, x and z such that $z \le x$,

$$\left(\left[1+\frac{1}{\delta-1}\int_z^x \left(i(s)+\xi h(s)\right)^{1-\delta} ds\right]^{\gamma(1-\sigma)}\right)''_{\xi=0} < 0.$$

It is straightforward to show that

$$\begin{aligned} &\left(\left[1+\frac{1}{\delta-1}\int_z^x \left(i(s)+\xi h(s)\right)^{1-\delta} ds\right]^{\gamma(1-\sigma)}\right)''_{\xi=0} \\ = \;& \frac{\gamma(\sigma-1)}{\left[1+\frac{1}{\delta-1}\int_z^x i(s)^{1-\delta}ds\right]^{\gamma(\sigma-1)+1}}\left[\left(\gamma(\sigma-1)+1\right)\frac{\left(\int_z^x i(s)^{-\delta}h(s)ds\right)^2}{1+\frac{1}{\delta-1}\int_z^x i(s)^{1-\delta}ds} - \delta\int_z^x i(s)^{-\delta-1}h^2(s)ds\right]. \end{aligned}$$

The final step is to show that

$$\left(\gamma(\sigma-1)+1\right)\frac{\left(\int_z^x i(s)^{-\delta}h(s)ds\right)^2}{1+\frac{1}{\delta-1}\int_z^x i(s)^{1-\delta}ds} - \delta\int_z^x i(s)^{-\delta-1}h^2(s)ds < 0.$$

To show the inequality, it is sufficient to show that

$$\begin{aligned} \left(\gamma(\sigma-1)+1\right)\frac{\left(\int_z^x i(s)^{-\delta}h(s)ds\right)^2}{\frac{1}{\delta-1}\int_z^x i(s)^{1-\delta}ds} &< \delta\int_z^x i(s)^{-\delta-1}h^2(s)ds \iff \\ \frac{\left(\delta-1\right)\left(\gamma(\sigma-1)+1\right)}{\delta}\left(\int_z^x i(s)^{-\delta}h(s)ds\right)^2 &< \left(\int_z^x i(s)^{-\delta-1}h^2(s)ds\right)\left(\int_z^x i(s)^{1-\delta}ds\right). \end{aligned}$$

The condition of the lemma implies that $(1-1/\delta)\left(\gamma(\sigma-1)+1\right) < 1$. Moreover, the Cauchy–Bunyakovsky–Schwarz inequality implies that

$$\left(\int_z^x i(s)^{-\delta}h(s)ds\right)^2 \le \left(\int_z^x i(s)^{-\delta-1}h^2(s)ds\right)\left(\int_z^x i(s)^{1-\delta}ds\right).$$

That is, $K_1''(0) + K_2''(0)$ is strictly negative, implying strict concavity of the considered objective function with respect to $i(s)$.

Note that the derivative of $i^{aut,p}(x)$ with respect to x can be written as follows:

$$\begin{aligned} q(x)\delta i^{aut,p}(x)^{\delta-1}\left(i^{aut,p}(x)\right)' + q'(x)i^{aut,p}(x)^{\delta} &= bL\gamma\left(1-\alpha\right)\left(1-t^{aut}\right)\left(\left(\phi^{L}(x)\right)' + \left(\phi^{R}(x)\right)'\right) \iff \\ \left(i^{aut,p}(x)\right)' &= \frac{bL\gamma\left(1-\alpha\right)\left(1-t^{aut}\right)\left(\left(\phi^{L}(x)\right)' + \left(\phi^{R}(x)\right)'\right) - q'(x)i^{aut,p}\left(x\right)^{\delta}}{q(x)\delta i^{aut,p}\left(x\right)^{\delta-1}}. \end{aligned}$$

We have that

$$\begin{aligned} \left(\phi^{L}(x)\right)' &= \frac{\tilde{m}\left(x\right)\left(\int_{x}^{\bar{s}}\left(1+\frac{1}{\delta-1}\int_{x}^{t} i\left(r\right)^{1-\delta}dr\right)^{\gamma(1-\sigma)-1}dt\right) - \int_{0}^{x}\tilde{m}\left(s\right)\left(1+\frac{1}{\delta-1}\int_{s}^{x} i\left(r\right)^{1-\delta}dr\right)^{\gamma(1-\sigma)-1}ds}{\int_{0}^{\bar{s}}\tilde{m}\left(s\right)v(s)ds}, \\ \left(\phi^{R}(x)\right)' &= \frac{-\tilde{m}\left(x\right)\left(\int_{0}^{x}\left(1+\frac{1}{\delta-1}\int_{t}^{x} i\left(r\right)^{1-\delta}dr\right)^{\gamma(1-\sigma)-1}dt\right) + \int_{x}^{\bar{s}}\tilde{m}\left(s\right)\left(1+\frac{1}{\delta-1}\int_{x}^{s} i\left(r\right)^{1-\delta}dr\right)^{\gamma(1-\sigma)-1}ds}{\int_{0}^{\bar{s}}\tilde{m}\left(s\right)v(s)ds}. \end{aligned}$$

It is straightforward to see that

$$\begin{aligned} \left(\phi^{L}(0)\right)' + \left(\phi^{R}(0)\right)' &> 0, \\ \left(\phi^{L}(\bar{s})\right)' + \left(\phi^{R}(\bar{s})\right)' &< 0. \end{aligned}$$

Since $q'(x)$ is continuous on $[0,\bar{s}]$ and $i^{aut,p}\left(0\right) = i^{aut,p}\left(\bar{s}\right) = 0$, $q'(0)i^{aut,p}\left(0\right)^{\delta} = q'(\bar{s})i^{aut,p}\left(\bar{s}\right)^{\delta} = 0$. This in turn immediately implies that

$$\begin{aligned} \left(i^{aut,p}\left(0\right)\right)' &= \infty \\ \left(i^{aut,p}\left(\bar{s}\right)\right)' &= -\infty. \end{aligned}$$

Finally, when $i^{aut,p}(x)$ is symmetric, it is straightforward to see that $\left(i^{aut,p}\left(\bar{s}/2\right)\right)' = 0$, implying a local extremum at this point.

The World Planner Solution

As discussed in Section 4.4.1, the optimal infrastructure profile chosen by the world planner almost coincides with that chosen by the world planner in the case when wages are exogenous and identical across all locations. Therefore, exploring the effects of τ on $i^{w}\left(s\right)$, we consider the case with exogenous wages, which is easier to analyze. When wages are fixed (without the loss of generality, we assume that $w(s) = 1$ for all $s \in [0, 2\bar{s}]$), the optimal infrastructure profile solves

$$
\begin{aligned}
i\left(s\right)^{\delta} &= \frac{2\gamma L(1-t)}{q}\left(\int_0^{2\bar{s}}\frac{dx}{P(x)}\right)^{-1}\left[\phi^L(s,\tau)+\phi^R(s,\tau)\right], \text{ where}\\
t &= \frac{q\int_0^{2\bar{s}} i\left(s\right)ds}{2L}, \text{ and}
\end{aligned}
$$

$$
\begin{aligned}
\phi^L(s,\tau) &= \int_0^s\int_s^{2\bar{s}} P(x)^{\sigma-2}\tau^{(1-\sigma)ind(x,y)}T(x,y)^{1-\sigma-1/\gamma}dydx,\\
\phi^R(s,\tau) &= \int_s^{2\bar{s}}\int_0^{s} P(x)^{\sigma-2}\tau^{(1-\sigma)ind(x,y)}T(x,y)^{1-\sigma-1/\gamma}dydx.
\end{aligned}
$$

It is straightforward to see that

$$
\begin{aligned}
\left(\phi^L(s,\tau)\right)_s' &= \int_s^{2\bar{s}} P(s)^{\sigma-2}\tau^{(1-\sigma)ind(s,y)}T(s,y)^{1-\sigma-1/\gamma}dy - \int_0^{s} P(x)^{\sigma-2}\tau^{(1-\sigma)ind(x,s)}T(x,s)^{1-\sigma-1/\gamma}dx,\\
\left(\phi^R(s,\tau)\right)_s' &= \int_s^{2\bar{s}} P(x)^{\sigma-2}\tau^{(1-\sigma)ind(x,s)}T(x,s)^{1-\sigma-1/\gamma}dx - \int_0^{s} P(s)^{\sigma-2}\tau^{(1-\sigma)ind(s,y)}T(s,y)^{1-\sigma-1/\gamma}dy.
\end{aligned}
$$

For sufficiently small $\varepsilon > 0$,

$$
\begin{aligned}
\left(\phi^L(\bar{s}-\varepsilon,\tau)\right)_s' + \left(\phi^R(\bar{s}-\varepsilon,\tau)\right)_s' &\approx \tau^{1-\sigma}\int_{\bar{s}}^{2\bar{s}} P(\bar{s})^{\sigma-2}T(\bar{s},y)^{1-\sigma-1/\gamma}dy - \int_0^{\bar{s}} P(\bar{s})^{\sigma-2}T(\bar{s},y)^{1-\sigma-1/\gamma}dy\\
&\quad +\tau^{1-\sigma}\int_{\bar{s}}^{2\bar{s}} P(x)^{\sigma-2}T(x,\bar{s})^{1-\sigma-1/\gamma}dx - \int_0^{\bar{s}} P(x)^{\sigma-2}T(x,\bar{s})^{1-\sigma-1/\gamma}dx.
\end{aligned}
$$

Since $i\left(s\right)$ and, therefore, $P(s)$ are symmetric around $\bar{s}$,

$$
\left(\phi^L(\bar{s}-\varepsilon,\tau)\right)_s' + \left(\phi^R(\bar{s}-\varepsilon,\tau)\right)_s' < 0, \text{ if } \tau > 1.
$$

Similarly,

$$
\begin{aligned}
\left(\phi^L(\bar{s}+\varepsilon,\tau)\right)_s' + \left(\phi^R(\bar{s}+\varepsilon,\tau)\right)_s' &\approx \int_{\bar{s}}^{2\bar{s}} P(\bar{s})^{\sigma-2}T(\bar{s},y)^{1-\sigma-1/\gamma}dy - \tau^{1-\sigma}\int_0^{\bar{s}} P(\bar{s})^{\sigma-2}T(\bar{s},y)^{1-\sigma-1/\gamma}dy\\
&\quad +\int_{\bar{s}}^{2\bar{s}} P(x)^{\sigma-2}T(x,\bar{s})^{1-\sigma-1/\gamma}dx - \tau^{1-\sigma}\int_0^{\bar{s}} P(x)^{\sigma-2}T(x,\bar{s})^{1-\sigma-1/\gamma}dx > 0.
\end{aligned}
$$

The above findings mean that the infrastructure profile is decreasing around $\bar{s}$ at Home and increasing around $\bar{s}$ in Foreign and, therefore, has a double-hump shape.

Appendix B: Discrete variant of the model

To match the moments, we consider a discrete version of our framework. We assume that there are $2n$ locations uniformly distributed on $[0, 2\bar{s}]$, with n locations in each country. We define $\triangle_n = \bar{s}/n$ as the distance between any two adjacent locations. In discrete space, the costs of transportation costs between i and j $(i \leq j)$ are

$$T(i,j) = \left(1 + \frac{\triangle_n}{\delta - 1} \sum_{k=i}^{j} i\,(k)^{1-\delta}\right)^{\gamma}. \tag{30}$$

Such a specification implies that the transportation costs from i and j are determined by infrastructure investments in all locations between them as well as investments at i and j and, moreover, that there are some endogenous costs of transporting products within each location. Specifically, these costs are determined by the level of infrastructure at a location:

$$T(j,j) = \left(1 + \frac{\triangle_n}{\delta - 1} i\,(j)^{1-\delta}\right)^{\gamma} > 1.$$

For sufficiently large n, the above discrete version of the model approximates the continuous specification and $T(j,j) \approx 1$ for any j. In our exercise, we set n at 94, meaning that there are 94 regions within each country which trade with each other and foreign locations. Note that given $n = 94$ and $\bar{s} = 500$, the discrete version of the model does not perfectly approximate the continuous one and, therefore, should be considered as a discrete variation of the benchmark continuous model which is used to fit the data.

Appendix C: More Technical Proofs

In this Appendix, we provide more technical proofs of Propositions 1 and 5.

Proof of Proposition 1

First, we show that if $\gamma(\delta-1)<1$, then investing in infrastructure at a subset of S with non-zero measure that has no infrastructure yields unbounded positive returns. Assume that the optimal infrastructure profile $i^{aut}(s)$ is equal to zero on some subset of S with non-zero measure. As $i^{aut}(s)$ is continuous, without loss of generality, we can assume that the subset is given by $(x-\Delta, x+\Delta)$, where x is a certain location. Let us consider the indirect utility at x, which is given by $V(x)=(1-t)m(x)w(x)/P(x)$, and let us compute the change in the indirect utility if $i(s)=\varepsilon>0$ on $(x-\Delta, x+\Delta)$ instead of zero. In doing so, we assume that wages do not change. Note that taking into account changes in wages makes the returns at x even greater. Thus, we consider how $P(x)^{-1}$ changes, if $i(s)$ increases from zero to ε on $(x-\Delta, x+\Delta)$. Specifically, we consider the following function of ε:

$$F(\varepsilon)=\left(\begin{array}{c}\int_0^{x-\Delta}\left(\frac{w(z)}{a(z)}\right)^{1-\sigma}\left(1+\frac{1}{\delta-1}\int_z^{x-\Delta} i\left(s\right)^{1-\delta}ds+\frac{\Delta\varepsilon^{1-\delta}}{\delta-1}\right)^{\gamma(1-\sigma)}dz\\ +\int_{x-\Delta}^{x}\left(\frac{w(z)}{a(z)}\right)^{1-\sigma}\left(1+\frac{(x-z)\varepsilon^{1-\delta}}{\delta-1}\right)^{\gamma(1-\sigma)}dz\\ +\int_{x+\Delta}^{\bar{s}}\left(\frac{w(z)}{a(z)}\right)^{1-\sigma}\left(1+\frac{1}{\delta-1}\int_{x+\Delta}^{z} i\left(s\right)^{1-\delta}ds+\frac{\Delta\varepsilon^{1-\delta}}{\delta-1}\right)^{\gamma(1-\sigma)}dz\\ +\int_{x}^{x+\Delta}\left(\frac{w(z)}{a(z)}\right)^{1-\sigma}\left(1+\frac{(z-x)\varepsilon^{1-\delta}}{\delta-1}\right)^{\gamma(1-\sigma)}dz\end{array}\right)^{1/(\sigma-1)}.$$

Next, we take the derivative of $F(\varepsilon)$ and evaluate it at zero. We have

$$\frac{dF(\varepsilon)}{d\varepsilon}=\gamma\varepsilon^{-\delta}F(\varepsilon)^{2-\sigma}\left(\begin{array}{c}\Delta\int_0^{x-\Delta}\left(\frac{w(z)}{a(z)}\right)^{1-\sigma}\left(1+\frac{1}{\delta-1}\int_z^{x-\Delta} i\left(s\right)^{1-\delta}ds+\frac{\Delta\varepsilon^{1-\delta}}{\delta-1}\right)^{\gamma(1-\sigma)-1}dz\\ +\int_{x-\Delta}^{x}(x-z)\left(\frac{w(z)}{a(z)}\right)^{1-\sigma}\left(1+\frac{(x-z)\varepsilon^{1-\delta}}{\delta-1}\right)^{\gamma(1-\sigma)-1}dz\\ +\Delta\int_{x+\Delta}^{\bar{s}}\left(\frac{w(z)}{a(z)}\right)^{1-\sigma}\left(1+\frac{1}{\delta-1}\int_{x+\Delta}^{z} i\left(s\right)^{1-\delta}ds+\frac{\Delta\varepsilon^{1-\delta}}{\delta-1}\right)^{\gamma(1-\sigma)-1}dz\\ +\int_{x}^{x+\Delta}(z-x)\left(\frac{w(z)}{a(z)}\right)^{1-\sigma}\left(1+\frac{(z-x)\varepsilon^{1-\delta}}{\delta-1}\right)^{\gamma(1-\sigma)-1}dz\end{array}\right).$$

Consider, for instance, the following function of ε (which is the part of the above expression for $dF(\varepsilon)/d\varepsilon$):

$$\begin{aligned}&\frac{\varepsilon^{-\delta}F(\varepsilon)^{2-\sigma}}{\left(1+\frac{1}{\delta-1}\int_z^{x-\Delta} i\left(s\right)^{1-\delta}ds+\frac{\Delta\varepsilon^{1-\delta}}{\delta-1}\right)^{\gamma(\sigma-1)+1}}\\ =&\frac{\varepsilon^{-\delta}F(\varepsilon)^{2-\sigma}\varepsilon^{(\delta-1)(\gamma(\sigma-1)+1)}}{\left(\varepsilon^{\delta-1}+\frac{\varepsilon^{\delta-1}}{\delta-1}\int_z^{x-\Delta} i\left(s\right)^{1-\delta}ds+\frac{\Delta}{\delta-1}\right)^{\gamma(\sigma-1)+1}}\\ =&\frac{F(\varepsilon)^{2-\sigma}\varepsilon^{(\delta-1)\gamma(\sigma-1)-1}}{\left(\varepsilon^{\delta-1}+\frac{\varepsilon^{\delta-1}}{\delta-1}\int_z^{x-\Delta} i\left(s\right)^{1-\delta}ds+\frac{\Delta}{\delta-1}\right)^{\gamma(\sigma-1)+1}}.\end{aligned}$$

Note that the value of $F(\varepsilon)^{2-\sigma}$ at zero is determined by the value of $\varepsilon^{(\delta-1)\gamma(2-\sigma)}$ at zero. Hence, taking into account that $\delta - 1 > 0$, it is straightforward to show that if $(\delta-1)\gamma - 1 < 0$, the value of the above function at zero is equal to infinity. The same outcome takes place, if we consider the other terms in the derivative. As a result, if $(\delta-1)\gamma - 1 < 0$,

$$\frac{dF(\varepsilon)}{d\varepsilon}|_{\varepsilon=0} = \infty.$$

That is, the marginal change in $P(x)^{-1}$ is equal to infinity. The same outcome holds, if we consider a marginal change in $P(x')^{-1}$, where $x' \in (x-\Delta, x+\Delta)$. This implies that investing in infrastructure at $(x-\Delta, x+\Delta)$ leads to infinite social welfare gains. This in turn means that if $(\delta-1)\gamma < 1$, there are no corner solutions.

Consider now the Lagrange function for the optimization problem determined by (9) − (12):

$$\begin{aligned}
\mathcal{L} \quad = \quad & \int_0^{\bar{s}} \frac{(1-t)m(x)w(x)}{P(x)} dx - \lambda \left(\int_0^{\bar{s}} q\left(x\right) i\left(x\right) dx - tL \right) \\
& + \int_0^{\bar{s}} \lambda(x) \left(\int_0^{\bar{s}} w(y)m(y)P(y)^{\sigma-1} T(x,y)^{1-\sigma} dy - w^{\sigma}(x)m(x)a(x)^{1-\sigma} \right) dx \\
& + \int_0^{\bar{s}} \mu(x) \left(\int_0^{\bar{s}} \left(\frac{w(y)}{a(y)} T(x,y) \right)^{1-\sigma} dy - P(x)^{1-\sigma} \right) dx,
\end{aligned}$$

where λ, $\lambda(x)$, and $\mu(x)$ are corresponding Lagrange multipliers. To derive the FOC for the optimal infrastructure profile, one needs to take the Gateaux derivatives of $\mathcal{L}$ with respect to $i\left(x\right)$, $w(x)$, $P(x)$, and t.

The Derivative wrt Infrastructure and Tax Rate

To derive the first order condition with respect to $i(x)$, we consider the following function of ξ, where $h(\cdot)$ is an arbitrary continuous function on $[0, \bar{s}]$:

$$\begin{aligned}
\mathcal{L}^1(\xi) &= \int_0^{\bar{s}} \frac{(1-t)m(x)w(x)}{P(x)}dx - \lambda\left(\int_0^{\bar{s}} q\left(x\right)\left(i\left(x\right)+\xi h(x)\right)dx - tL\right) \\
&+ \int_0^{\bar{s}} \lambda(x)\left(\int_0^{x} w(y)m(y)P(y)^{\sigma-1}\left[1+\frac{1}{\delta-1}\int_y^x \left(i(s)+\xi h(s)\right)^{1-\delta}ds\right]^{\gamma(1-\sigma)}dy\right)dx \\
&+ \int_0^{\bar{s}} \lambda(x)\left(\int_x^{\bar{s}} w(y)m(y)P(y)^{\sigma-1}\left[1+\frac{1}{\delta-1}\int_x^y \left(i(s)+\xi h(s)\right)^{1-\delta}ds\right]^{\gamma(1-\sigma)}dy\right)dx \\
&+ \int_0^{\bar{s}} \mu(x)\left(\int_0^{x} \left(\frac{w(y)}{a(y)}\right)^{1-\sigma}\left[1+\frac{1}{\delta-1}\int_y^x \left(i(s)+\xi h(s)\right)^{1-\delta}ds\right]^{\gamma(1-\sigma)}dy\right)dx \\
&+ \int_0^{\bar{s}} \mu(x)\left(\int_x^{\bar{s}} \left(\frac{w(y)}{a(y)}\right)^{1-\sigma}\left[1+\frac{1}{\delta-1}\int_x^y \left(i(s)+\xi h(s)\right)^{1-\delta}ds\right]^{\gamma(1-\sigma)}dy\right)dx \\
&- \int_0^{\bar{s}} \lambda(x)w^{\sigma}(x)a(x)^{1-\sigma}m(x)dx - \int_0^{\bar{s}} \mu(x)P(x)^{1-\sigma}dx.
\end{aligned}$$

Setting the derivative of the above function with respect to ξ evaluated at $\xi = 0$ to zero for any continuous $h(\cdot)$ gives the first order condition with respect to $i(x)$. Specifically, we have

$$\begin{aligned}
\frac{\frac{d\mathcal{L}^1(\xi)}{d\xi}|_{\xi=0}}{\gamma(\sigma-1)} &= -\frac{\lambda}{\gamma(\sigma-1)}\int_0^{\bar{s}} q\left(x\right)h(x)dx \\
&+ \int_0^{\bar{s}} \lambda(x)\left(\int_0^{x} \frac{w(y)m(y)}{P(y)^{1-\sigma}}\left[1+\frac{1}{\delta-1}\int_y^x i(s)^{1-\delta}ds\right]^{\gamma(1-\sigma)-1}\int_y^x i(s)^{-\delta}h(s)dsdy\right)dx \\
&+ \int_0^{\bar{s}} \lambda(x)\left(\int_x^{\bar{s}} \frac{w(y)m(y)}{P(y)^{1-\sigma}}\left[1+\frac{1}{\delta-1}\int_x^y i(s)^{1-\delta}ds\right]^{\gamma(1-\sigma)-1}\int_x^y i(s)^{-\delta}h(s)dsdy\right)dx \\
&+ \int_0^{\bar{s}} \mu(x)\left(\int_0^{x} \left(\frac{w(y)}{a(y)}\right)^{1-\sigma}\left[1+\frac{1}{\delta-1}\int_y^x i(s)^{1-\delta}ds\right]^{\gamma(1-\sigma)-1}\int_y^x i(s)^{-\delta}h(s)dsdy\right)dx \\
&+ \int_0^{\bar{s}} \mu(x)\left(\int_x^{\bar{s}} \left(\frac{w(y)}{a(y)}\right)^{1-\sigma}\left[1+\frac{1}{\delta-1}\int_x^y i(s)^{1-\delta}ds\right]^{\gamma(1-\sigma)-1}\int_x^y i(s)^{-\delta}h(s)dsdy\right)dx.
\end{aligned}$$

Next, we change the order and limits of integration in the above expression. In particular, the expression can be rewritten as follows:

$$\begin{aligned}
\frac{\frac{d\mathcal{L}^1(\xi)}{d\xi}|_{\xi=0}}{\gamma(\sigma-1)} &= -\frac{\lambda}{\gamma(\sigma-1)}\int_0^{\bar{s}} q\left(x\right)h(x)dx \\
&+\int_0^{\bar{s}}\left(\int_s^{\bar{s}}\lambda(x)\int_0^{s}\frac{w(y)m(y)}{P(y)^{1-\sigma}}\left[1+\frac{1}{\delta-1}\int_y^x i(s)^{1-\delta}ds\right]^{\gamma(1-\sigma)-1}dydx\right)i(s)^{-\delta}h(s)ds \\
&+\int_0^{\bar{s}}\left(\int_0^{s}\lambda(x)\int_s^{\bar{s}}\frac{w(y)m(y)}{P(y)^{1-\sigma}}\left[1+\frac{1}{\delta-1}\int_x^y i(s)^{1-\delta}ds\right]^{\gamma(1-\sigma)-1}dydx\right)i(s)^{-\delta}h(s)ds \\
&+\int_0^{\bar{s}}\left(\int_s^{\bar{s}}\mu(x)\int_0^{s}\left(\frac{w(y)}{a(y)}\right)^{1-\sigma}\left[1+\frac{1}{\delta-1}\int_y^x i(s)^{1-\delta}ds\right]^{\gamma(1-\sigma)-1}dydx\right)i(s)^{-\delta}h(s)ds \\
&+\int_0^{\bar{s}}\left(\int_0^{s}\mu(x)\int_s^{\bar{s}}\left(\frac{w(y)}{a(y)}\right)^{1-\sigma}\left[1+\frac{1}{\delta-1}\int_x^y i(s)^{1-\delta}ds\right]^{\gamma(1-\sigma)-1}dydx\right)i(s)^{-\delta}h(s)ds.
\end{aligned}$$

When $i(s)$ is optimally chosen, $d\mathcal{L}^1(0)/d\xi$ is supposed to be equal to zero for any continuous $h(\cdot)$. This allows us to write the FOC with respect to $i(s)$ in the following way:

$$\begin{aligned}
\frac{\lambda q(s)i(s)^{\delta}}{\gamma(\sigma-1)} &= \int_s^{\bar{s}}\lambda(x)\int_0^{s}\frac{w(y)m(y)}{P(y)^{1-\sigma}}\left[1+\frac{1}{\delta-1}\int_y^x i(s)^{1-\delta}ds\right]^{\gamma(1-\sigma)-1}dydx \\
&+\int_0^{s}\lambda(x)\int_s^{\bar{s}}\frac{w(y)m(y)}{P(y)^{1-\sigma}}\left[1+\frac{1}{\delta-1}\int_x^y i(s)^{1-\delta}ds\right]^{\gamma(1-\sigma)-1}dydx \\
&+\int_s^{\bar{s}}\mu(x)\int_0^{s}\left(\frac{w(y)}{a(y)}\right)^{1-\sigma}\left[1+\frac{1}{\delta-1}\int_y^x i(s)^{1-\delta}ds\right]^{\gamma(1-\sigma)-1}dydx \\
&+\int_0^{s}\mu(x)\int_s^{\bar{s}}\left(\frac{w(y)}{a(y)}\right)^{1-\sigma}\left[1+\frac{1}{\delta-1}\int_x^y i(s)^{1-\delta}ds\right]^{\gamma(1-\sigma)-1}dydx.
\end{aligned}$$

Equating the derivative of the Lagrange function with respect to t to zero implies

$$\int_0^{\bar{s}}\frac{m(x)w(x)}{P(x)}dx = \lambda L \iff \lambda = \frac{\int_0^{\bar{s}}\frac{m(x)w(x)}{P(x)}dx}{L}.$$

The Derivative wrt Wage and Price Index

In a similar way as for $i(x)$, to derive the first order condition with respect to $w(x)$, we consider the following function of ξ:

$$\begin{aligned}\mathcal{L}^2(\xi) &= \int_0^{\bar{s}} \frac{(1-t)m(x)\left(w(x)+\xi h(x)\right)}{P(x)}dx - \lambda\left(\int_0^{\bar{s}} q\left(x\right)i\left(x\right)dx - tL\right) \\ &+ \int_0^{\bar{s}} \lambda(x)\left(\int_0^{\bar{s}} \frac{\left(w(y)+\xi h(y)\right)m(y)}{P(y)^{1-\sigma}}T(x,y)^{1-\sigma}dy - \left(w(x)+\xi h(x)\right)^{\sigma} m(x)a(x)^{1-\sigma}\right)dx \\ &+ \int_0^{\bar{s}} \mu(x)\left(\int_0^{\bar{s}} \left(\frac{w(y)+\xi h(y)}{a(y)}\right)^{1-\sigma} T(x,y)^{1-\sigma}dy - P(x)^{1-\sigma}\right)dx.\end{aligned}$$

Then,

$$\begin{aligned}\frac{d\mathcal{L}^2(\xi)}{d\xi}|_{\xi=0} &= \int_0^{\bar{s}} \frac{(1-t)m(x)}{P(x)}h(x)dx \\ &+ \int_0^{\bar{s}} \lambda(x)\left(\int_0^{\bar{s}} \frac{m(y)}{P(y)^{1-\sigma}}T(x,y)^{1-\sigma}h(y)dy - \sigma w(x)^{\sigma-1}h(x)m(x)a(x)^{1-\sigma}\right)dx \\ &+ (1-\sigma)\int_0^{\bar{s}} \mu(x)\int_0^{\bar{s}} \frac{w(y)^{-\sigma}}{a(y)^{1-\sigma}}T(x,y)^{1-\sigma}h(y)dydx.\end{aligned}$$

As a result, the FOC with respect to $w(x)$ can be written as follows:

$$\begin{aligned}&\frac{(1-t)m(x)}{P(x)} + \frac{m(x)}{P(x)^{1-\sigma}}\left(\int_0^{\bar{s}} \lambda(y)T(x,y)^{1-\sigma}dy\right) \\ =\ &\sigma\lambda(x)m(x)\left(\frac{w(x)}{a(x)}\right)^{\sigma-1} + (\sigma-1)\frac{w(x)^{-\sigma}}{a(x)^{1-\sigma}}\left(\int_0^{\bar{s}} \mu(y)T(x,y)^{1-\sigma}dy\right).\end{aligned}$$

Finally, consider the derivative with respect to $P(x)$. We have

$$\begin{aligned}\mathcal{L}^3(\xi) &= \int_0^{\bar{s}} \frac{(1-t)m(x)w(x)}{P(x)+\xi h(x)}dx - \lambda\left(\int_0^{\bar{s}} q\left(x\right)i\left(x\right)dx - tL\right) \\ &+ \int_0^{\bar{s}} \lambda(x)\left(\int_0^{\bar{s}} \frac{w(y)m(y)}{\left(P(y)+\xi h(y)\right)^{1-\sigma}}T(x,y)^{1-\sigma}dy - w^{\sigma}(x)m(x)a(x)^{1-\sigma}\right)dx \\ &+ \int_0^{\bar{s}} \mu(x)\left(\int_0^{\bar{s}} \left(\frac{w(y)}{a(y)}\right)^{1-\sigma} T(x,y)^{1-\sigma}dy - \left(P(x)+\xi h(x)\right)^{1-\sigma}\right)dx.\end{aligned}$$

Taking the derivative, we obtain

$$\begin{aligned}\frac{d\mathcal{L}^3(\xi)}{d\xi}|_{\xi=0} &= -\int_0^{\bar{s}} \frac{(1-t)m(x)w(x)}{(P(x))^2} h(x)dx \\ &+(\sigma-1)\int_0^{\bar{s}} \lambda(x) \left(\int_0^{\bar{s}} \frac{w(y)m(y)}{P(y)^{2-\sigma}} T(x,y)^{1-\sigma} h(y)dy \right) dx \\ &+(\sigma-1)\int_0^{\bar{s}} \mu(x) \left(P(x)\right)^{-\sigma} h(x)dx.\end{aligned}$$

This implies the following FOC condition:

$$\frac{(1-t)m(x)w(x)}{(P(x))^2} = (\sigma-1)\frac{\mu(x)}{P(x)^{\sigma}} + (\sigma-1)\frac{w(x)m(x)}{P(x)^{2-\sigma}} \left(\int_0^{\bar{s}} \lambda(y) T(y,x)^{1-\sigma} dy \right).$$

Combining all the outcomes above with the equilibrium conditions for wages and price indexes, we obtain a system of functional equations that determines the optimal infrastructure profile where

$$\begin{aligned}\phi^L(s) &= \int_0^{s}\int_s^{\bar{s}} \left[\lambda(x)\frac{w(y)m(y)}{P(y)^{1-\sigma}} + \mu(x)\left(\frac{w(y)}{a(y)}\right)^{1-\sigma} \right] T(x,y)^{1-\sigma-1/\gamma} dydx, \\ \phi^R(s) &= \int_s^{\bar{s}}\int_0^{s} \left[\lambda(x)\frac{w(y)m(y)}{P(y)^{1-\sigma}} + \mu(x)\left(\frac{w(y)}{a(y)}\right)^{1-\sigma} \right] T(x,y)^{1-\sigma-1/\gamma} dydx.\end{aligned}$$

Proof of Proposition 5

The technique of the proof is similar to that of Proposition 1. First, we write down the Lagrange function for the optimization problem:

$$
\begin{aligned}
\mathcal{L} \;=\; & \int_0^{\bar{s}} \frac{(1-t)w(x)}{P(x)}dx - \lambda\left(q\int_0^{\bar{s}} i\,(x)\,dx - tL\right) \\
& + \int_0^{\bar{s}} \lambda(x)(1-t)\left(\int_0^{\bar{s}} w(z)P\,(z)^{\sigma-1}\,T\,(x,z)^{1-\sigma}\,dz - w^{\sigma}(x)\right)dx \\
& + \int_0^{\bar{s}} \lambda(x)(1-t^{F})\tau^{1-\sigma}\left(\int_{\bar{s}}^{2\bar{s}} w(z)P\,(z)^{\sigma-1}\,T\,(x,z)^{1-\sigma}\,dz\right)dx \\
& + \int_{\bar{s}}^{2\bar{s}} \lambda(x)\left(\int_{\bar{s}}^{2\bar{s}} w(z)P\,(z)^{\sigma-1}\,T\,(x,z)^{1-\sigma}\,dz + \frac{(1-t)\tau^{1-\sigma}}{1-t^{F}}\int_0^{\bar{s}} w(z)P\,(z)^{\sigma-1}\,T\,(x,z)^{1-\sigma}\,dz - w^{\sigma}(x)\right)dx \\
& + \int_0^{\bar{s}} \mu(x)\left(\int_0^{\bar{s}} (w(z)T\,(x,z))^{1-\sigma}\,dz + \tau^{1-\sigma}\int_{\bar{s}}^{2\bar{s}} (w(z)T\,(x,z))^{1-\sigma}\,dz - P(x)^{1-\sigma}\right)dx, \\
& + \int_{\bar{s}}^{2\bar{s}} \mu(x)\left(\tau^{1-\sigma}\int_0^{\bar{s}} (w(z)T\,(x,z))^{1-\sigma}\,dz + \int_{\bar{s}}^{2\bar{s}} (w(z)T\,(x,z))^{1-\sigma}\,dz - P(x)^{1-\sigma}\right)dx.
\end{aligned}
$$

where λ, $\lambda(x)$, and $\mu(x)$ are corresponding Lagrange multipliers.

To derive the first order condition with respect to $i(x)$, we consider the following function of ξ, where

$h(\cdot)$ is an arbitrary continuous function on $[0,\bar{s}]$:

$$
\begin{aligned}
\mathcal{L}^1(\xi) \quad = \quad & \int_0^{\bar{s}} \frac{(1-t)w(x)}{P(x)}dx - \lambda\left(q\int_0^{\bar{s}}\left(i\left(x\right)+\xi h(x)\right)dx - tL\right) \\
& +(1-t)\int_0^{\bar{s}}\lambda(x)\left(\begin{array}{c}\int_0^x w(z)P\left(z\right)^{\sigma-1}\left[1+\frac{1}{\delta-1}\int_z^x\left(i(s)+\xi h(s)\right)^{1-\delta}ds\right]^{\gamma(1-\sigma)}dz \\ +\int_x^{\bar{s}} w(z)P\left(z\right)^{\sigma-1}\left[1+\frac{1}{\delta-1}\int_x^z\left(i(s)+\xi h(s)\right)^{1-\delta}ds\right]^{\gamma(1-\sigma)}dz\end{array}\right)dx \\
& +(1-t^F)\tau^{1-\sigma}\int_0^{\bar{s}}\lambda(x)\left(\int_{\bar{s}}^{2\bar{s}}\frac{w(z)}{P\left(z\right)^{1-\sigma}}\left[1+\frac{\int_x^{\bar{s}}\left(i(s)+\xi h(s)\right)^{1-\delta}ds+\int_{\bar{s}}^z i^F(s)^{1-\delta}ds}{\delta-1}\right]^{\gamma(1-\sigma)}dz\right)dx \\
& +\frac{(1-t)\tau^{1-\sigma}}{1-t^F}\int_{\bar{s}}^{2\bar{s}}\lambda(x)\left(\int_0^{\bar{s}}\frac{w(z)}{P\left(z\right)^{1-\sigma}}\left[1+\frac{\int_z^{\bar{s}}\left(i(s)+\xi h(s)\right)^{1-\delta}ds+\int_{\bar{s}}^x i^F(s)^{1-\delta}ds}{\delta-1}\right]^{\gamma(1-\sigma)}dz\right)dx \\
& +\int_0^{\bar{s}}\mu(x)\left(\begin{array}{c}\int_0^x w(z)^{1-\sigma}\left[1+\frac{1}{\delta-1}\int_z^x\left(i(s)+\xi h(s)\right)^{1-\delta}ds\right]^{\gamma(1-\sigma)}dz \\ +\int_x^{\bar{s}} w(z)^{1-\sigma}\left[1+\frac{1}{\delta-1}\int_x^z\left(i(s)+\xi h(s)\right)^{1-\delta}ds\right]^{\gamma(1-\sigma)}dz\end{array}\right)dx \\
& +\tau^{1-\sigma}\int_0^{\bar{s}}\mu(x)\left(\int_{\bar{s}}^{2\bar{s}} w(z)^{1-\sigma}\left[1+\frac{1}{\delta-1}\int_x^{\bar{s}}\left(i(s)+\xi h(s)\right)^{1-\delta}ds+\frac{1}{\delta-1}\int_{\bar{s}}^z i^F(s)^{1-\delta}ds\right]^{\gamma(1-\sigma)}dz\right)dx \\
& +\tau^{1-\sigma}\int_{\bar{s}}^{2\bar{s}}\mu(x)\left(\int_0^{\bar{s}} w(z)^{1-\sigma}\left[1+\frac{1}{\delta-1}\int_z^{\bar{s}}\left(i(s)+\xi h(s)\right)^{1-\delta}ds+\frac{1}{\delta-1}\int_{\bar{s}}^x i^F(s)^{1-\delta}ds\right]^{\gamma(1-\sigma)}dz\right)dx \\
& -(1-t)\int_0^{\bar{s}}\lambda(x)w^{\sigma}(x)dx-\int_{\bar{s}}^{2\bar{s}}\lambda(x)w^{\sigma}(x)dx+\int_{\bar{s}}^{2\bar{s}}\lambda(x)\left(\int_{\bar{s}}^{2\bar{s}} w(z)P\left(z\right)^{\sigma-1}T\left(x,z\right)^{1-\sigma}dz\right)dx \\
& -\int_0^{2\bar{s}}\mu(x)P(x)^{1-\sigma}dx+\int_{\bar{s}}^{2\bar{s}}\mu(x)\left(\int_{\bar{s}}^{2\bar{s}}\left(w(z)T\left(x,z\right)\right)^{1-\sigma}dz\right)dx.
\end{aligned}
$$

Taking the derivative with respect to ξ and evaluating it at $\xi = 0$, we derive

$$\begin{aligned}
\frac{\frac{d\mathcal{L}^1(\xi)}{d\xi}|_{\xi=0}}{\gamma(\sigma-1)} &= -\frac{\lambda q}{\gamma(\sigma-1)}\int_0^{\bar{s}} h(x)dx \\
&+(1-t)\int_0^{\bar{s}}\lambda(x)\left(\begin{array}{c}\int_0^x w(z)P(z)^{\sigma-1}T(x,z)^{1-\sigma-1/\gamma}\int_z^x i(s)^{-\delta}h(s)dsdz \\ +\int_x^{\bar{s}} w(z)P(z)^{\sigma-1}T(x,z)^{1-\sigma-1/\gamma}\int_x^z i(s)^{-\delta}h(s)dsdz\end{array}\right)dx \\
&+(1-t^F)\tau^{1-\sigma}\int_0^{\bar{s}}\lambda(x)\left(\int_{\bar{s}}^{2\bar{s}} w(z)P(z)^{\sigma-1}T(x,z)^{1-\sigma-1/\gamma}\int_x^{\bar{s}} i(s)^{-\delta}h(s)dsdz\right)dx \\
&+\frac{(1-t)\tau^{1-\sigma}}{1-t^F}\int_{\bar{s}}^{2\bar{s}}\lambda(x)\left(\int_0^{\bar{s}} w(z)P(z)^{\sigma-1}T(x,z)^{1-\sigma-1/\gamma}\int_z^{\bar{s}} i(s)^{-\delta}h(s)dsdz\right)dx \\
&+\int_0^{\bar{s}}\mu(x)\left(\begin{array}{c}\int_0^x w(z)^{1-\sigma}T(x,z)^{1-\sigma-1/\gamma}\int_z^x i(s)^{-\delta}h(s)dsdz \\ +\int_x^{\bar{s}} w(z)^{1-\sigma}T(x,z)^{1-\sigma-1/\gamma}\int_x^z i(s)^{-\delta}h(s)dsdz\end{array}\right)dx \\
&+\tau^{1-\sigma}\int_0^{\bar{s}}\mu(x)\left(\int_{\bar{s}}^{2\bar{s}} w(z)^{1-\sigma}T(x,z)^{1-\sigma-1/\gamma}\int_x^{\bar{s}} i(s)^{-\delta}h(s)dsdz\right)dx \\
&+\tau^{1-\sigma}\int_{\bar{s}}^{2\bar{s}}\mu(x)\left(\int_0^{\bar{s}} w(z)^{1-\sigma}T(x,z)^{1-\sigma-1/\gamma}\int_z^{\bar{s}} i(s)^{-\delta}h(s)dsdz\right)dx.
\end{aligned}$$

By changing the order and limits of integration, we have

$$\begin{aligned}
\frac{\frac{d\mathcal{L}^1(\xi)}{d\xi}|_{\xi=0}}{\gamma(\sigma-1)} &= -\frac{\lambda q}{\gamma(\sigma-1)}\int_0^{\bar{s}} h(s)ds \\
&+(1-t)\int_0^{\bar{s}}\left(\int_s^{\bar{s}}\lambda(x)\int_0^{s}\frac{w(z)}{P(z)^{1-\sigma}}T(x,z)^{1-\sigma-1/\gamma}\,dzdx\right)i(s)^{-\delta}h(s)ds \\
&+(1-t)\int_0^{\bar{s}}\left(\int_0^{s}\lambda(x)\int_s^{\bar{s}}\frac{w(z)}{P(z)^{1-\sigma}}T(x,z)^{1-\sigma-1/\gamma}\,dzdx\right)i(s)^{-\delta}h(s)ds \\
&+(1-t^F)\tau^{1-\sigma}\int_0^{\bar{s}}\left(\int_0^{s}\lambda(x)\int_{\bar{s}}^{2\bar{s}}\frac{w(z)}{P(z)^{1-\sigma}}T(x,z)^{1-\sigma-1/\gamma}\,dzdx\right)i(s)^{-\delta}h(s)ds \\
&+\frac{(1-t)\tau^{1-\sigma}}{1-t^F}\int_0^{\bar{s}}\left(\int_{\bar{s}}^{2\bar{s}}\lambda(x)\int_0^{s}\frac{w(z)}{P(z)^{1-\sigma}}T(x,z)^{1-\sigma-1/\gamma}\,dzdx\right)i(s)^{-\delta}h(s)ds \\
&+\int_0^{\bar{s}}\left(\int_s^{\bar{s}}\mu(x)\int_0^{s}w(z)^{1-\sigma}T(x,z)^{1-\sigma-1/\gamma}\,dzdx\right)i(s)^{-\delta}h(s)ds \\
&+\int_0^{\bar{s}}\left(\int_0^{s}\mu(x)\int_s^{\bar{s}}w(z)^{1-\sigma}T(x,z)^{1-\sigma-1/\gamma}\,dzdx\right)i(s)^{-\delta}h(s)ds \\
&+\tau^{1-\sigma}\int_0^{\bar{s}}\left(\int_0^{s}\mu(x)\int_{\bar{s}}^{2\bar{s}}w(z)^{1-\sigma}T(x,z)^{1-\sigma-1/\gamma}\,dzdx\right)i(s)^{-\delta}h(s)ds \\
&+\tau^{1-\sigma}\int_0^{\bar{s}}\left(\int_{\bar{s}}^{2\bar{s}}\mu(x)\int_0^{s}w(z)^{1-\sigma}T(x,z)^{1-\sigma-1/\gamma}\,dzdx\right)i(s)^{-\delta}h(s)ds.
\end{aligned}$$

Hence, the first order condition for $i(x)$ can be written as follows:

$$\begin{aligned}
\frac{\lambda q i(s)^{\delta}}{\gamma(\sigma-1)} &= \int_s^{\bar{s}}\int_0^{s}\left[(1-t)\lambda(x)\frac{w(z)}{P(z)^{1-\sigma}}+\mu(x)w(z)^{1-\sigma}\right]T(x,z)^{1-\sigma-1/\gamma}\,dzdx \\
&+\int_0^{s}\int_s^{\bar{s}}\left[(1-t)\lambda(x)\frac{w(z)}{P(z)^{1-\sigma}}+\mu(x)w(z)^{1-\sigma}\right]T(x,z)^{1-\sigma-1/\gamma}\,dzdx \\
&+\tau^{1-\sigma}\int_0^{s}\int_{\bar{s}}^{2\bar{s}}\left[(1-t^F)\lambda(x)\frac{w(z)}{P(z)^{1-\sigma}}+\mu(x)w(z)^{1-\sigma}\right]T(x,z)^{1-\sigma-1/\gamma}\,dzdx \\
&+\tau^{1-\sigma}\int_{\bar{s}}^{2\bar{s}}\int_0^{s}\left[\frac{1-t}{1-t^F}\lambda(x)\frac{w(z)}{P(z)^{1-\sigma}}+\mu(x)w(z)^{1-\sigma}\right]T(x,z)^{1-\sigma-1/\gamma}\,dzdx.
\end{aligned}$$

The first order condition with respect to t is given by

$$\begin{aligned}\lambda L &= \int_0^{\bar{s}} \frac{w(x)}{P(x)} dx + \int_0^{\bar{s}} \lambda(x) \left(\int_0^{\bar{s}} w(z) P(z)^{\sigma-1} T(x,z)^{1-\sigma} dz - w^{\sigma}(x) \right) dx \\ &\quad + \frac{\tau^{1-\sigma}}{1-t^F} \int_{\bar{s}}^{2\bar{s}} \lambda(x) \int_0^{\bar{s}} w(z) P(z)^{\sigma-1} T(x,z)^{1-\sigma} dz dx.\end{aligned}$$

Next, we derive the first order conditions for $w(x)$ and $P(x)$. Consider the following function:

$$\begin{aligned}\mathcal{L}^2(\xi) &= \int_0^{\bar{s}} \frac{(1-t)(w(x)+\xi h(x))}{P(x)} dx - \lambda \left(q \int_0^{\bar{s}} i(x) dx - tL \right) \\ &\quad + \int_0^{\bar{s}} \lambda(x)(1-t) \left(\int_0^{\bar{s}} (w(z)+\xi h(z)) P(z)^{\sigma-1} T(x,z)^{1-\sigma} dz - (w(x)+\xi h(x))^{\sigma} \right) dx \\ &\quad + \int_0^{\bar{s}} \lambda(x)(1-t^F)\tau^{1-\sigma} \int_{\bar{s}}^{2\bar{s}} (w(z)+\xi h(z)) P(z)^{\sigma-1} T(x,z)^{1-\sigma} dz dx \\ &\quad + \int_{\bar{s}}^{2\bar{s}} \lambda(x) \left(\begin{array}{c} \int_{\bar{s}}^{2\bar{s}} (w(z)+\xi h(z)) P(z)^{\sigma-1} T(x,z)^{1-\sigma} dz \\ + \frac{(1-t)\tau^{1-\sigma}}{1-t^F} \int_0^{\bar{s}} (w(z)+\xi h(z)) P(z)^{\sigma-1} T(x,z)^{1-\sigma} dz - (w(x)+\xi h(x))^{\sigma} \end{array} \right) dx \\ &\quad + \int_0^{\bar{s}} \mu(x) \left(\int_0^{\bar{s}} ((w(z)+\xi h(z)) T(x,z))^{1-\sigma} dz + \tau^{1-\sigma} \int_{\bar{s}}^{2\bar{s}} ((w(z)+\xi h(z)) T(x,z))^{1-\sigma} dz - P(x)^{1-\sigma} \right) dx, \\ &\quad + \int_{\bar{s}}^{2\bar{s}} \mu(x) \left(\tau^{1-\sigma} \int_0^{\bar{s}} ((w(z)+\xi h(z)) T(x,z))^{1-\sigma} dz + \int_{\bar{s}}^{2\bar{s}} ((w(z)+\xi h(z)) T(x,z))^{1-\sigma} dz - P(x)^{1-\sigma} \right) dx.\end{aligned}$$

Taking the derivative with respect to ξ, evaluating it at $\xi = 0$, and changing the order of the integrals,

we derive

$$\begin{aligned}
\frac{d\mathcal{L}^2(\xi)}{d\xi}|_{\xi=0} &= (1-t)\int_0^{\bar{s}} \frac{h(x)}{P(x)}dx - (1-t)\sigma\int_0^{\bar{s}} \lambda(x)w(x)^{\sigma-1}h(x)dx \\
&\quad +(1-t)\int_0^{\bar{s}} \frac{h(x)}{P(x)^{1-\sigma}}\left(\int_0^{\bar{s}} \lambda(z)T(x,z)^{1-\sigma}\,dz + \frac{\tau^{1-\sigma}}{1-t^F}\int_{\bar{s}}^{2\bar{s}} \lambda(z)T(x,z)^{1-\sigma}\,dz\right)dx \\
&\quad +(1-\sigma)\int_0^{\bar{s}} \frac{h(x)}{w(x)^{\sigma}}\left(\int_0^{\bar{s}} \mu(z)T(x,z)^{1-\sigma}\,dz + \tau^{1-\sigma}\int_{\bar{s}}^{2\bar{s}} \mu(z)T(x,z)^{1-\sigma}\,dz\right)dx \\
&\quad -\sigma\int_{\bar{s}}^{2\bar{s}} \lambda(x)w(x)^{\sigma-1}h(x)dx \\
&\quad +\int_{\bar{s}}^{2\bar{s}} h(x)P(x)^{\sigma-1}\left(\int_{\bar{s}}^{2\bar{s}} \lambda(z)T(x,z)^{1-\sigma}\,dz + (1-t^F)\tau^{1-\sigma}\int_0^{\bar{s}} \lambda(z)T(x,z)^{1-\sigma}\,dz\right)dx \\
&\quad +(1-\sigma)\int_{\bar{s}}^{2\bar{s}} \frac{h(x)}{w(x)^{\sigma}}\left(\tau^{1-\sigma}\int_0^{\bar{s}} \mu(z)T(x,z)^{1-\sigma}\,dz + \int_{\bar{s}}^{2\bar{s}} \mu(z)T(x,z)^{1-\sigma}\,dz\right)dx.
\end{aligned}$$

The latter expression is supposed to be equal to zero for any continuous $h(x)$ on $[0, 2\bar{s}]$. As a result, we derive the first order condition for $w(x)$. For $x \in [0, \bar{s}]$,

$$\begin{aligned}
&\frac{1-t}{P(x)} + \frac{1-t}{P(x)^{1-\sigma}}\left(\int_0^{\bar{s}} \lambda(z)T(x,z)^{1-\sigma}\,dz + \frac{\tau^{1-\sigma}}{1-t^F}\int_{\bar{s}}^{2\bar{s}} \lambda(z)T(x,z)^{1-\sigma}\,dz\right) \\
= \;& (1-t)\sigma\lambda(x)w(x)^{\sigma-1} + \frac{\sigma-1}{w(x)^{\sigma}}\left(\int_0^{\bar{s}} \mu(z)T(x,z)^{1-\sigma}\,dz + \tau^{1-\sigma}\int_{\bar{s}}^{2\bar{s}} \mu(z)T(x,z)^{1-\sigma}\,dz\right).
\end{aligned}$$

For $x \in [\bar{s}, 2\bar{s}]$,

$$\begin{aligned}
&P(x)^{\sigma-1}\left(\int_{\bar{s}}^{2\bar{s}} \lambda(z)T(x,z)^{1-\sigma}\,dz + (1-t^F)\tau^{1-\sigma}\int_0^{\bar{s}} \lambda(z)T(x,z)^{1-\sigma}\,dz\right) \\
= \;& \sigma\lambda(x)w(x)^{\sigma-1} + \frac{\sigma-1}{w(x)^{\sigma}}\left(\tau^{1-\sigma}\int_0^{\bar{s}} \mu(z)T(x,z)^{1-\sigma}\,dz + \int_{\bar{s}}^{2\bar{s}} \mu(z)T(x,z)^{1-\sigma}\,dz\right).
\end{aligned}$$

Finally, to derive the first order condition with respect to $P(x)$, we consider

$$\begin{aligned}
\mathcal{L}^3(\xi) &= \int_0^{\bar{s}} \frac{(1-t)w(x)}{P(x)+\xi h(x)}dx - \lambda\left(q\int_0^{\bar{s}} i(x)\,dx - tL\right) \\
&+\int_0^{\bar{s}} \lambda(x)\begin{pmatrix} (1-t)\left(\int_0^{\bar{s}} w(z)\left(P(z)+\xi h(z)\right)^{\sigma-1}T(x,z)^{1-\sigma}\,dz - w^{\sigma}(x)\right) \\ +(1-t^F)\tau^{1-\sigma}\int_{\bar{s}}^{2\bar{s}} w(z)\left(P(z)+\xi h(z)\right)^{\sigma-1}T(x,z)^{1-\sigma}\,dz \end{pmatrix} dx \\
&+\int_{\bar{s}}^{2\bar{s}} \lambda(x)\begin{pmatrix} \int_{\bar{s}}^{2\bar{s}} w(z)\left(P(z)+\xi h(z)\right)^{\sigma-1}T(x,z)^{1-\sigma}\,dz \\ +\frac{(1-t)\tau^{1-\sigma}}{1-t^F}\int_0^{\bar{s}} w(z)\left(P(z)+\xi h(z)\right)^{\sigma-1}T(x,z)^{1-\sigma}\,dz - w^{\sigma}(x) \end{pmatrix} dx \\
&+\int_0^{\bar{s}} \mu(x)\left(\int_0^{\bar{s}} \left(w(z)T(x,z)\right)^{1-\sigma}dz + \tau^{1-\sigma}\int_{\bar{s}}^{2\bar{s}} \left(w(z)T(x,z)\right)^{1-\sigma}dz - \left(P(x)+\xi h(x)\right)^{1-\sigma}\right)dx, \\
&+\int_{\bar{s}}^{2\bar{s}} \mu(x)\left(\tau^{1-\sigma}\int_0^{\bar{s}} \left(w(z)T(x,z)\right)^{1-\sigma}dz + \int_{\bar{s}}^{2\bar{s}} \left(w(z)T(x,z)\right)^{1-\sigma}dz - \left(P(x)+\xi h(x)\right)^{1-\sigma}\right)dx.
\end{aligned}$$

Again, taking the derivative with respect to ξ, evaluating it at $\xi = 0$, and changing the order of the integrals, we derive

$$\begin{aligned}
\frac{d\mathcal{L}^3(\xi)}{d\xi}|_{\xi=0} &= -\int_0^{\bar{s}} \frac{(1-t)w(x)h(x)}{P(x)^2}dx \\
&+(1-t)(\sigma-1)\int_0^{\bar{s}} w(x)P(x)^{\sigma-2}h(x)\left(\int_0^{\bar{s}} \lambda(z)T(x,z)^{1-\sigma}\,dz + \frac{\tau^{1-\sigma}}{1-t^F}\int_{\bar{s}}^{2\bar{s}} \lambda(z)T(x,z)^{1-\sigma}\,dz\right)dx \\
&+(\sigma-1)\int_0^{\bar{s}} \mu(x)P(x)^{-\sigma}h(x)dx, \\
&+(\sigma-1)\int_{\bar{s}}^{2\bar{s}} w(x)P(x)^{\sigma-2}h(x)\left((1-t^F)\tau^{1-\sigma}\int_0^{\bar{s}} \lambda(z)T(x,z)^{1-\sigma}\,dz + \int_{\bar{s}}^{2\bar{s}} \lambda(z)T(x,z)^{1-\sigma}\,dz\right)dx \\
&+(\sigma-1)\int_{\bar{s}}^{2\bar{s}} \mu(x)P(x)^{-\sigma}h(x)dx.
\end{aligned}$$

Thus, we have that for $x \in [0, \bar{s}]$,

$$\begin{aligned}
\frac{(1-t)w(x)}{P(x)^2} &= (1-t)(\sigma-1)w(x)P(x)^{\sigma-2}\left(\int_0^{\bar{s}} \lambda(z)T(x,z)^{1-\sigma}\,dz + \frac{\tau^{1-\sigma}}{1-t^F}\int_{\bar{s}}^{2\bar{s}} \lambda(z)T(x,z)^{1-\sigma}\,dz\right) \\
&+(\sigma-1)\mu(x)P(x)^{-\sigma}.
\end{aligned}$$

For $x \in [\bar{s}, 2\bar{s}]$,

$$\mu(x) = -w(x)P(x)^{2\sigma-2}\left((1-t^F)\tau^{1-\sigma}\int_0^{\bar{s}}\lambda(z)T(x,z)^{1-\sigma}\,dz + \int_{\bar{s}}^{2\bar{s}}\lambda(z)T(x,z)^{1-\sigma}\,dz\right).$$

Combining everything together, we derive the result formulated in the proposition, where

$$\begin{aligned}
\tilde{\phi}^{L,N}(s,\tau) &= \int_0^s\int_s^{\bar{s}}\left[(1-t^H)\lambda(x)\frac{w(z)}{P(z)^{1-\sigma}} + \mu(x)w(z)^{1-\sigma}\right]T(x,z)^{1-\sigma-1/\gamma}\,dzdx \\
&\quad +\tau^{1-\sigma}\int_0^s\int_{\bar{s}}^{2\bar{s}}\left[(1-t^F)\lambda(x)\frac{w(z)}{P(z)^{1-\sigma}} + \mu(x)w(z)^{1-\sigma}\right]T(x,z)^{1-\sigma-1/\gamma}\,dzdx, \\
\tilde{\phi}^{R,N}(s,\tau) &= \int_s^{\bar{s}}\int_0^s\left[(1-t^H)\lambda(x)\frac{w(z)}{P(z)^{1-\sigma}} + \mu(x)w(z)^{1-\sigma}\right]T(x,z)^{1-\sigma-1/\gamma}\,dzdx \\
&\quad +\tau^{1-\sigma}\int_{\bar{s}}^{2\bar{s}}\int_0^s\left[\frac{1-t^H}{1-t^F}\lambda(x)\frac{w(z)}{P(z)^{1-\sigma}} + \mu(x)w(z)^{1-\sigma}\right]T(x,z)^{1-\sigma-1/\gamma}\,dzdx,
\end{aligned}$$

and

$$\begin{aligned}
\lambda L &= \int_0^{\bar{s}}\frac{w(x)}{P(x)}dx - \frac{(1-t^F)\tau^{1-\sigma}}{1-t^H}\int_0^{\bar{s}}\lambda(x)\int_{\bar{s}}^{2\bar{s}}w(z)P(z)^{\sigma-1}T(x,z)^{1-\sigma}\,dzdx \\
&\quad +\frac{\tau^{1-\sigma}}{1-t^F}\int_{\bar{s}}^{2\bar{s}}\lambda(x)\int_0^{\bar{s}}w(z)P(z)^{\sigma-1}T(x,z)^{1-\sigma}\,dzdx.
\end{aligned}$$

Appendix D: Some Extensions

In this Appendix, we explore the quantitative role of intercontinental trade and "natural" borders in our framework.

Intercontinental trade

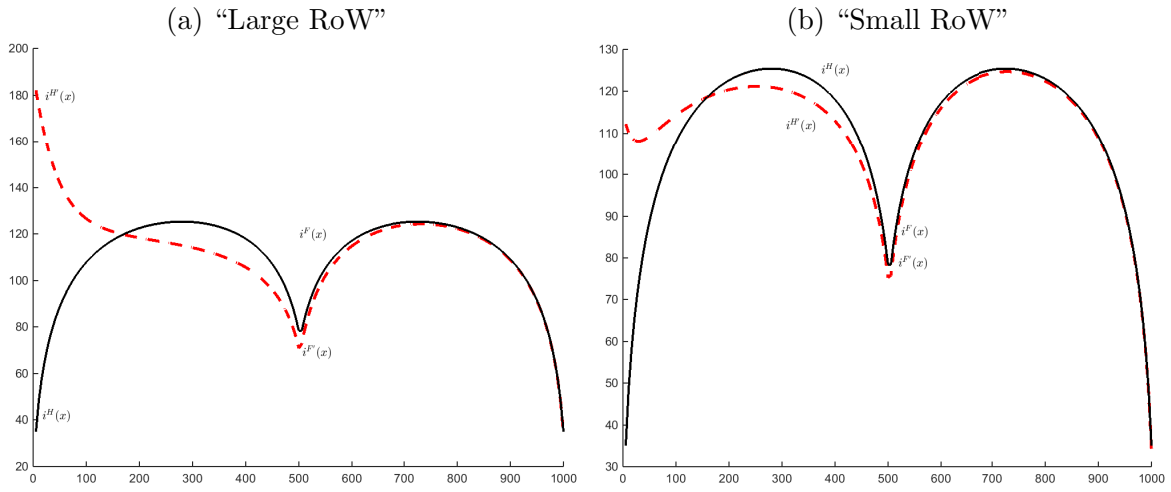
So far, we have studied a world of two countries which are connected by land-borne trade. Now we study how the emergence of *inter*continental trade affects the *intra*continental distribution of infrastructure investment, trade, welfare, and the size of the border effect. We do this by assuming that there is a third country, call it rest of the world (RoW), which supplies and demands goods to and from Home and Foreign via a harbor that is located at address 0 in Home. We also assume that RoW's supply and demand capacity is exogenous to the outcomes in Home and Foreign. In particular, we do not model the spatial extension of RoW, and treat it as a mass point at location 0.

By construction, this constellation creates a coastal country (Home) and a landlocked country (Foreign). It also breaks symmetry between Home and Foreign, as any location in Home now is more central

than any region in Foreign, even with a uniform distribution of infrastructure investment: on average, consumer prices will be lower in Home than in Foreign. However, with endogenous infrastructure investment, there will be additional effects that tend to exacerbate inequality both between Home and Foreign as well as within them.[52]

Panels (a) and (b) in Figure 7 show how the intracontinental infrastructure investment distribution changes due to the emergence of intercontinental trade with RoW. In panel (a), we assume that the number of varieties produced in RoW constitutes a quarter of those produced in Home or Foreign (around 23.5 varieties). In addition, we normalize the market size of RoW (the total income multiplied by the price index to the power of $\sigma-1$) to 23.5. In this case, the share of trade volumes from RoW to Home and Foreign in the total income of those countries constitutes around 10%, reflecting the share of imports (in total income) of EU from the rest of the world in 2010. In Home, there will be a substantial reallocation of spending towards the coastal locations where imports from and exports to RoW crowd out varieties from locations within Home and Foreign. Investment falls with increasing distance from the harbor and is lower than in the benchmark case (intracontinental trade only) in about 65% of all locations in Home. In particular, Home will invest less around the border with Foreign. With the infrastructure schedule more skewed in Home, the distribution of real wages will also become more uneven, so that trade with RoW makes Home a more unequal economy.

Figure 7: Overseas trade and the distribution of intracontinental transport infrastructure investment



Notes. $i^H(x)$ and $i^F(x)$ refer to the Nash infrastructure investment distributions without RoW, while $i^{H\prime}(x)$ and $i^{F\prime}(x)$ refer to those when there is trade with RoW. The parameterization is as follows: $\sigma = 4$, $\delta = 1.94$, $\gamma = 0.73$, $m/q = 500$, $\tau = 1.09$, and $\bar{s} = 500$. In Panel (a) RoW has the "size" of 23.5, in (b) the size is 10.

Simulations show that in Foreign, now a landlocked country, incentives for investment barely change,

[52] Another application of this model variant may be to poor coastal countries in Africa, which trade mostly intercontinentally, and where intracontinental infrastructure and trade are underdeveloped.

Table 9: Overseas trade: Inequality and Welfare

	Home			Foreign		
	Welfare	Inequality	Tax rate	Welfare	Inequality	Tax rate
Size of RoW is 0:	1.00	0.72	22%	1.00	0.72	22%
Size of RoW is 10:	1.03	0.09	22.6%	1.00	0.72	22%
Size of RoW is 23.5:	1.08	1.29	23.5%	1.00	0.72	22%

Notes: Calibration as in Table 3; Rest of the World ('RoW') is modeled as a mass point at location 0. When the size of RoW is zero, welfare is normalized to unity.

because there are two competing factors. On the one hand, all else equal, the presence of RoW should result in more investment. On the other hand, this effect is mitigated by the remoteness of RoW and the reallocation of infrastructure investment in Home away from the border with Foreign.

Note that the emergence of RoW increases the *intra*continental border effect, because it diverts investment away from the border zone. This means that, in a standard gravity equation, the estimated border dummy should grow larger with the increasing importance of intercontinental trade despite the fact that border related trade costs have not changed. Specifically, under the benchmark parameterization the presence of RoW increases the size of the border effect by around 5% (from -0.8 to -0.84).

In panel (b) of Figure 7, we consider a case where RoW is (with a "size" of 10). In this case, the infrastructure profile at Home still has a hump shape. However, there is more investment in coastal areas and less in the landlocked country compared to the benchmark case (of intracontinental trade only).

A rise in trade with RoW increases welfare in the coastal country but has minor effects in the landlocked country (Foreign), where inequality also remains essentially unchanged. In contrast, trade with RoW has a substantial non-monotonic effect on inequality in Home. When the size of RoW is small, inequality in Home decreases (since the distribution of the infrastructure becomes flatter (see panel *b* in Figure 7)). When RoW is big, inequality rises, as the infrastructure profile is highly skewed towards the border with RoW (see panel *a* in Figure 7). Table **??** summarizes the effects of trade with RoW on Home and Foreign under the benchmark parameterization.

The role of "natural" borders

As discussed, infrastructure underinvestment around national borders can be also explained by the fact that borders could endogenously arise in mountain regions or close to rivers. In this case, low investment is caused by high cost of investing in transport infrastructure rather than by the political externality emphasized in our theory. In this section, we perform several numerical experiments that allow us to assess this mechanism quantitatively. In contrast to the calibration exercise in the main body of the paper (where the cost of investing in infrastructure $q(x)$ is flat), we numerically explore the case where $q(x)$ is higher around the border. Specifically, we assume that $q(x)$ equals 2 (and then 2.5) on some

Table 10: The role of "natural" borders

		(1) τ	(2) $Infra$: TE	(3) $Infra$: PME
Benchmark:	$q(x) = 1$ everywhere	1.09	16%	16%
(1)	$q(x) = 2$ on 2%	1.06	25%	14%
(2)	$q(x) = 2$ on 4%	1.04	33%	12.5%
(3)	$q(x) = 2.5$ on 2%	1.05	29%	14%
(4)	$q(x) = 2.5$ on 4%	1.02	40%	12.5%
(5)	$q(x) = 2.5$ on 5%	1.00	46%	11%

Notes: We assume that $q(x)$ is higher on some interval around the border. The rows report the chosen values of $q(x)$ and the length of the interval as a percentage of the total geographical size of a country. Column (1) reports the calibrated values of τ in each case. Columns (2) and (3) report the overall role of infrastructure in explaining the border effect and the role of the political mechanism, respectively. *TE*: total effect; *PME*: effect attributable to the political mechanism.

interval of locations around the border, while at all other locations $q(x)$ remains equal to unity as in the benchmark case. In other words, $q(x)$ is assumed to be a step function. As the length of such an interval we consider 2% (and then 4% and 5%) of the total geographical size of each country.

The choice of the values of $q(x)$ around the border is based on the findings of an official report from the French government (Cazala et al., 2006), which compares the costs of motorway construction within and between 6 European countries. It finds that, for France, the average cost of construction on "easy" terrain is 4.8 mn. Euro while it is 6.6 mn. on "difficult" and 11.4 mn on "very difficult" terrain, so that the ratio between the two costs is 2.4. The report provides evidence for other countries as well; for example, the ratio is 2.0 in Germany, 2.6 in Spain, and 2.4 in Sweden.

We then recalibrate the model matching the same moments in the data and decompose the effect of infrastructure on the border effect into the political and the "natural" border mechanisms. Table 10 summarizes our findings. In particular, column (1) reports the calibrated values of τ for different scenarios (the calibrated values of γ and δ have barely changed). An exogenous border friction τ and higher costs of investing in border regions have similar effects. Thus, a rise in the cost or the length of the interval where the cost is higher leads to a lower calibrated value of τ. For instance, when $q(x)$ is equal 2 on the 2% border interval, the value of τ drops from 1.09 to 1.06. Thus, difficult territory in border regions plays the same role as an exogenous border friction.

As before, to isolate the role of infrastructure in determining the border effect, we assume a flat distribution of infrastructure (keeping all other parameters fixed) and compute the border effect generated by the model. Column (2) reports the results. Not surprisingly the role of infrastructure substantially increases. In the benchmark case, the distribution of infrastructure explains around 16% of the border effect, that share goes up significantly when the difficulty of geography is higher in border regions.

We then explore the pure role of the political mechanism (PME) in explaining the border effect. Given the calibrated values of the parameters derived in the case when $q(x)$ is a step function, we make $q(x)$ a

flat function (setting $q(x)$ to its average value everywhere), find the optimal distribution of infrastructure in this case and then compute the border effect generated by the model. The difference (as a percentage of the actual border effect) between the border effect derived when only $q(x)$ is flat and the one derived when the whole infrastructure profile is flat is considered as the pure effect of the political mechanism. That is, we try to understand by how much the border effect rises compared to the case when the infrastructure profile is flat, if we add only the political mechanism (assuming away the differences in topology across locations). Column (3) in Table 10 reports the findings. As can be seen, the role of the political mechanism is smaller compared to the benchmark case, but nevertheless remains relatively substantial. In the benchmark specification, the political mechanism accounts for 16% of the border effect, in the presence of the "natural" border, it accounts for $11\% - 14\%$. In particular, the magnitude of the network effect falls, when the value of $q(x)$ around the border or the length of the border interval rises. Overall, the calibration results suggest that a more difficult geography around the border which roughly doubles the role of infrastructure in restricting trade leads to a slight reduction in the relative role of the political mechanism by about 3-4 percentage points. Hence, the political mechanism loses significance as natural border barriers grow, but – at least with our calibration – the effect is relatively small.

Finally, we also assess how the presence of a natural border can affect the welfare loss associated with the misallocation of infrastructure investment. As before, we consider the difference between the average real per capita income in the first-best and non-cooperative outcome assuming now that $q(x)$ is equal to 2.5 on the 5% border interval (rather than being flat). Note that, in this case, the calibrated value of τ is close to unity (see row (5) in Table 10), implying that the border effect is solely generated by the distribution of infrastructure and the correlation between the border dummy and distance. We find that the welfare loss is about 0.16%, which is slightly higher than that in the benchmark case, 0.1%.

Table 11: Intercontinental merchandise trade by transport mode: NAFTA and EU

	North America				French intra-EU trade			
	value		weight		value		weight	
	bn. USD	%	mio. tons	%	bn. EUR	%	mio. tons	%
Truck	557	61	187	29	338	69	200	59
Rail	131	14	134	21	15	3	15	4
Air	45	5	1	0	22	4	0	0
Water	81	9	210	32	70	14	77	23
Pipeline	63	7	106	16	n.a.	n.a.	n.a.	n.a.
Other	40	4	9	1	45	9	45	13
Total	918	100	646	100	491	100	336	100

Notes: Data refer to 2010 (except French quantity information: 2006). Sources: US department of transportation, Freight Facts and Figures 2011, Table 2.8., www.ops.fhwa.dot.gov\freight\freight_analysis\nat_freight_stats\docs\11factsfigures\index.htm and Commissariat du development durable, www.statistiques.developpement-durable.gouv.fr\transports\873.html.

Appendix E: Additional Tables and Sensitivity Analysis

In this section, we report some tables and provide the sensitivity analysis we refer to in Section 2

Intercontinental trade by transport mode

What explain variance in transportation costs across space?

Table 12: Explaining variance in transportation costs in France

Dep. var.: Ln variable transport costs per km for transits through a departement						
	(1)	(2)	(3)	(4)	(5)	(6)
ln distance to Paris					0.034**	
					(0.015)	
ln GTC to Paris						0.039**
						(0.015)
ln geography	0.041***	0.029***	0.032***	0.020**	0.004	0.001
	(0.013)	(0.011)	(0.010)	(0.008)	(0.011)	(0.011)
ln GDP		-0.048***	-0.186***	-0.028	-0.019	-0.018
		(0.013)	(0.050)	(0.052)	(0.048)	(0.047)
ln population			0.167***	-0.002	-0.006	-0.007
			(0.056)	(0.059)	(0.056)	(0.055)
ln area (km2)				0.087***	0.068***	0.067***
				(0.013)	(0.015)	(0.014)
Constant	1.590***	2.026***	1.112***	1.110***	1.059**	0.967***
	(0.009)	(0.118)	(0.334)	(0.299)	(0.285)	(0.288)
N	94	94	94	94	94	94
$adj.R^2$	0.158	0.289	0.333	0.583	0.609	0.616
$F-stat.$	10.06	9.443	10.66	41.91	41.25	45.17
$RMSE$	0.097	0.090	0.088	0.070	0.068	0.067

Robust standard errors in parentheses, $^{***}p < 0.01$,$^{**}p < 0.05$,$^{*}p < 0.1$.
GTC: generalized transport cost. "Geography" measures the difference between the points of highest and lowest altitude above sea level in a departement.

Sensitivity Analysis on the role of infrastructure for the border effect.

Treating borders as exogenous to trade shocks ϵ_{ij} (or sectoral trade shocks ϵ_{ijr}), the identification assumption underlying our analysis can be stated as $cov\left(\epsilon_{ijr}, T_{ij} | B_{ij}, ex_{ir}, im_{ir}\right) = 0$. A high realization of ϵ_{ijr} could incite countries i and j to invest into reducing T_{ij}, which could bias the distance effect μ_1 away from zero. How this affects our coefficient of interest, μ_2, is not obvious, though. T_{ij} depends on investment I_i and I_j, but also on investment by countries $k \neq i, j$ located on the shortest travel route between i and j. So, if i and j boost investment due to a positive shock in ϵ_{ijr}, they also lower T_{ii} and T_{jj}, which increases X_{ii} and X_{jj}, and this contributes to moving μ_2 upwards at given I_k.

To contain the danger of biased results, we restrict the sample to non-contingent countries, the transport links between which are importantly shaped by countries other than the trade partners themselves. The assumption is that the covariance of bilateral, sectoral trade shocks ϵ_{ijr} and infrastructure investment $I_k, k \neq i, j$ is zero. Columns (1) to (3) of Table 15 show the results. First, note that excluding the 61 contingent pairs strongly increases the absolute value of the border effect (column (1)).[53] Second, replacing great-circle distance by travel time lowers the absolute value of the border effect quite significantly (column (3)). A similar result obtains when using road distance (column (2)). Interestingly, the elasticity of trade with respect to travel time is almost unchanged by restricting the sample to non-contingent pairs.

Next, we turn from aggregate to sectoral data. Columns (1) to (3) of Table 14 show that our argument remains robust. Estimated coefficients change only minimally. If we drop all trade relationships of any country that account for more than 5% of total sector-level trade and which are, thus presumably important enough to affect investment decisions, we find a substantially higher border effect, but replacing great-circle distance by road distance or travel time shrinks the effect; see columns (7) to (9) of table 15. Table 16 runs sector-level regressions and finds statistically significant border effects in 12 out of 16 sectors. In all of these sectors, the border effect is (algebraically) smaller when road distance or travel time is used instead of great-circle distance.

Finally, we also check whether our results are driven by the recent eastward enlargement of the EU. Excluding the 10 new eastern European EU member countries present in our data, we find an algebraically smaller border effect (see columns (4) to (6) of table 15). Still, the measured border effect changes upon replacing great-circle distance by road distance or travel time in the way suggested by our model.

[53] The common language dummy turns insignificant as we have only very few non-contingent countries with common language in the sample.

Table 13: The border effect and the role of infrastructure: OLS regressions

	(1)	(2)	(3)	(4)	(5)	(6)
Dep.var.:	aggregate trade			sectoral trade		
Border (0,1)	-1.419***	-1.003***	-0.784**	-1.092***	-0.614***	-0.405***
	(0.333)	(0.335)	(0.334)	(0.144)	(0.148)	(0.148)
ln bird-flight distance	-1.707***			-2.069***		
	(0.136)			(0.0521)		
ln road distance		-1.708***			-2.053***	
		(0.147)			(0.0551)	
ln travel time			-1.917***			-2.255***
			(0.136)			(0.0560)
Contiguity (0,1)	0.130	0.117	0.0400	0.122**	0.124**	0.0781
	(0.135)	(0.146)	(0.127)	(0.0563)	(0.0590)	(0.0565)
Common language (0,1)	-0.189	-0.0959	-0.0201	-0.276***	-0.161**	-0.0689
	(0.171)	(0.169)	(0.171)	(0.0760)	(0.0761)	(0.0766)
Constant	18.36***	18.76***	19.06***	16.33***	16.70***	16.74***
	(0.909)	(1.015)	(0.868)	(0.476)	(0.502)	(0.478)
Observations	441	441	441	6,975	6,975	6,975
R-squared	0.949	0.949	0.950	0.894	0.893	0.892

Notes: OLS regressions. All models contain complete sets of separate exporter and importer fixed effects (exporter * sector, importer * sector effects in case of sectoral trade data). Robust standard errors in parentheses, *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table 14: The border effect and the role of infrastructure: Sectoral trade data, PPML models

	(1)	(2)	(3)
Border (0,1)	-0.775***	-0.503***	-0.391***
	(0.0915)	(0.101)	(0.101)
ln great-circle distance	-1.142***		
	(0.0674)		
ln road distance		-1.211***	
		(0.0693)	
ln travel time			-1.391***
			(0.0713)
Common language (0,1)	0.763***	0.843**	0.841***
	(0.0803)	(0.0793)	(0.0792)
Contiguity (0,1)	0.320***	0.200**	0.131
	(0.0845)	(0.0843)	(0.0816)
Constant	12.49***	13.24***	13.63***
	(0.780)	(0.788)	(0.758)
Observations	7,004	7,004	7,004
R-squared	0.959	0.961	0.961

Notes: Pseudo Maximum Likelihood (PPML) estimations of Poisson models. All models contain complete sets of separate exporter × sector, importer × sector effects. Robust standard errors in parentheses, *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table 15: Sensitivity analysis

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Non-contingent pairs only			"Old" EU members only			Small trade links only		
	Aggregate data						Sectoral data		
Border (0,1)	-2.063***	-1.626***	-1.546***	-0.687***	-0.584***	-0.422***	-2.767***	-2.634***	-2.473***
	(0.459)	(0.457)	(0.428)	(0.170)	(0.180)	(0.162)	(0.164)	(0.165)	(0.165)
ln great circle-distance	-1.204***			-1.079***			-0.580***		
	(0.135)			(0.138)			(0.0607)		
ln road distance		-1.264***			-1.021***			-0.564***	
		(0.126)			(0.129)			(0.0582)	
ln travel time			-1.365***			-1.248***			-0.724***
			(0.140)			(0.120)			(0.0607)
Common language (0,1)	-0.295	-0.168	-0.145	0.893***	0.917***	0.892***	0.244	0.295*	0.303*
	(0.415)	(0.404)	(0.355)	(0.147)	(0.152)	(0.136)	(0.166)	(0.163)	(0.161)
Contiguity (0,1)				0.152	0.147	0.0277	0.643***	0.652***	0.567***
				(0.180)	(0.174)	(0.141)	(0.0741)	(0.0745)	(0.0740)
Constant	17.66***	17.90***	17.69***	15.59***	15.34***	16.01***	11.20***	11.08***	11.60***
	(0.837)	(0.784)	(0.744)	(0.824)	(0.775)	(0.653)	(0.438)	(0.424)	(0.405)
Observations	377	377	377	121	121	121	5,324	5,324	5,324
R-squared	0.997	0.998	0.997	0.993	0.993	0.994	0.999	0.999	0.999

Notes: Poisson Pseudo Maximum Likelihood estimations. All models contain complete sets of separate exporter and importer fixed effects (exporter × sector, importer × sector effects in case of sectoral trade data). All models contain a common language dummy and a contiguity dummy (not shown). Robust standard errors in parentheses, *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. $N = 441$, R^2 between 0.90% and 0.99%.

Table 16: The border effect infrastructure - sector by sector: PPML regressions

	Z = ln bird-flight distance															
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
	Agriculture	Mining	Food	Textiles	Leather	Wood	Paper	Fuels	Chemicals	Plastics	Non-metals	Metals	Machinery	Electrical	Transport	Other
Border	-2.346***	-1.260*	-1.416***	0.137	0.462	-1.652***	-1.680***	-0.657	0.0241	-0.676***	-1.701***	-0.742***	-1.110***	-0.528***	-0.509**	-0.966***
	(0.347)	(0.725)	(0.254)	(0.326)	(0.499)	(0.201)	(0.214)	(0.548)	(0.248)	(0.221)	(0.216)	(0.189)	(0.155)	(0.190)	(0.235)	(0.227)
Z	-0.807***	-2.087***	-0.972***	-1.030***	-1.131***	-0.837***	-0.913***	-1.385***	-0.848***	-0.899***	-1.176***	-1.212***	-0.694***	-0.780***	-0.638***	-1.060***
	(0.205)	(0.529)	(0.221)	(0.140)	(0.211)	(0.150)	(0.185)	(0.458)	(0.156)	(0.150)	(0.179)	(0.169)	(0.115)	(0.123)	(0.153)	(0.164)
	Z = ln road distance															
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Border	-2.263***	-1.147	-1.158***	0.509	0.923*	-1.513***	-1.460***	-0.788	0.268	-0.341	-1.501***	-0.540***	-0.874***	-0.224	-0.315	-0.519**
	(0.377)	(0.756)	(0.289)	(0.393)	(0.540)	(0.235)	(0.247)	(0.583)	(0.278)	(0.230)	(0.247)	(0.207)	(0.148)	(0.192)	(0.242)	(0.209)
Z	-0.747***	-1.938***	-1.043***	-1.181***	-1.330***	-0.842***	-0.966***	-1.052**	-0.938***	-1.069***	-1.169***	-1.217***	-0.819***	-0.968***	-0.738***	-1.315***
	(0.213)	(0.502)	(0.224)	(0.167)	(0.234)	(0.160)	(0.189)	(0.410)	(0.166)	(0.143)	(0.182)	(0.165)	(0.104)	(0.114)	(0.148)	(0.129)
	Z = ln travel time															
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Border	-2.084***	-1.207	-1.016***	0.697*	1.022*	-1.407***	-1.316***	-0.585	0.357	-0.167	-1.349***	-0.368*	-0.755***	-0.114	-0.302	-0.335
	(0.408)	(0.787)	(0.273)	(0.407)	(0.563)	(0.237)	(0.242)	(0.607)	(0.283)	(0.232)	(0.227)	(0.194)	(0.155)	(0.198)	(0.253)	(0.206)
Z	-0.948***	-1.930***	-1.244***	-1.418***	-1.419***	-0.986***	-1.170***	-1.319***	-1.092***	-1.290***	-1.392***	-1.456***	-0.971***	-1.113***	-0.784***	-1.557***
	(0.229)	(0.526)	(0.218)	(0.188)	(0.262)	(0.172)	(0.200)	(0.461)	(0.168)	(0.142)	(0.180)	(0.161)	(0.109)	(0.126)	(0.171)	(0.129)

Notes: Poisson Pseudo Maximum Likelihood estimations. All models contain complete sets of separate exporter and importer fixed effects (exporter * sector, importer * sector effects in case of sectoral trade data). All models contain a common language dummy and a contiguity dummy (not shown). Robust standard errors in parentheses, *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. $N = 441$, R^2 between 0.90% and 0.99%.