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ABSTRACT

ENDOGENOUS GROWTH, SKILL OBSOLESCENCE AND OPTIMAL MONETARY POLICY[1,2]

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We analyze Ramsey optimal monetary policy in a New-Keynesian model with search and matching frictions featuring (i) training costs due to skill loss from long-term unemployment and (ii) endogenous growth through learning-by-doing externalities. In a simplified two-period version of the model, the competitive equilibrium is shown to be inefficient due to two externalities: i) firms do not internalize the effects that hiring has on labor productivity through learning-by-doing; ii) firms do not fully internalize the effects that hiring has on future training costs. These externalities lead to inefficient fluctuations, thereby justifying deviations from price stability in response to productivity shocks. In a calibrated version of the full model we show significant deviations from price stability and significant differences between optimal monetary policy and monetary policy that follows a Taylor rule.

Keywords: skill loss, human capital growth, unemployment, Ramsey optimal monetary policy, labor market frictions, policy trade-off.

JEL: E24; E52

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Endogenous Growth, Skill Obsolescence and Optimal Monetary Policy[1,2]

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Abstract

We analyze Ramsey optimal monetary policy in a New-Keynesian model with search and matching frictions featuring (i) training costs due to skill loss from long-term unemployment and (ii) endogenous growth through learning-by-doing externalities. In a simplified two-period version of the model, the competitive equilibrium is shown to be inefficient due to two externalities: i) firms do not internalize the effects that hiring has on labor productivity through learning-by-doing; ii) firms do not fully internalize the effects that hiring has on future training costs. These externalities lead to inefficient fluctuations, thereby justifying deviations from price stability in response to productivity shocks. In a calibrated version of the full model we show significant deviations from price stability and significant differences between optimal monetary policy and monetary policy that follows a Taylor rule.

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1 Introduction

The past two recessions have been challenging for macroeconomic stabilization policy. Both recessions featured an unusually strong increase in the share of long-term unemployed workers. As Figure 1 illustrates the share of long-term unemployed in the US peaked at about 45% during the Great Recession and during the COVID-19 pandemic, while in previous recessions the share of long-term unemployed never exceeded 25%. Furthermore, while the long-run effects of the COVID-19 pandemic are still unclear, most commentators agree that the Great Recession had permanent negative effects for GDP. This has led to a renewed interest in the hysteresis effects of deep recessions (see Cerra, Fatas, and Saxena (2020) for a recent survey). The implications of these two phenomena for macroeconomic stabilization policy are still underexplored.

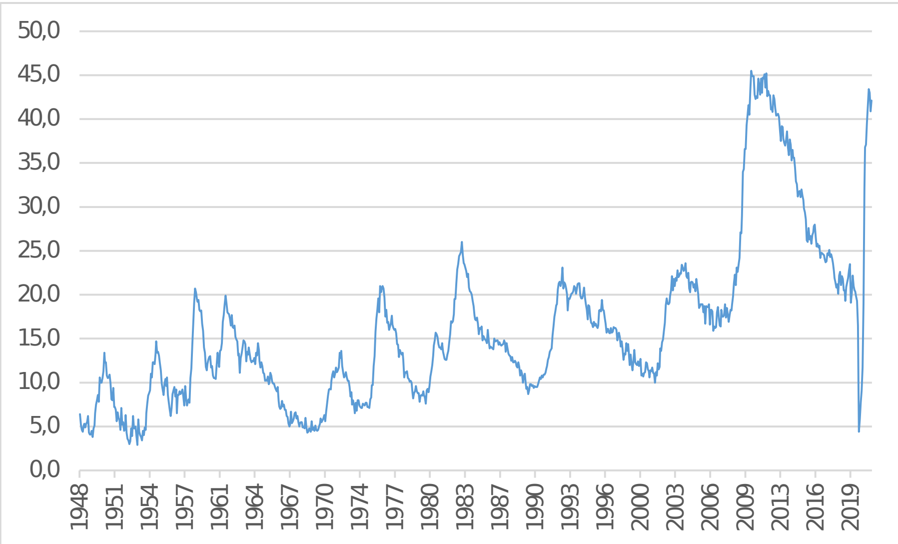

Figure 1: The share of long-term unemployment in total unemployment in the US. Source: FRED

In this paper we analyze the role of endogenous growth and skill-loss from long-term unemployment for optimal monetary policy. We do this in a tractable way, by introducing training costs associated with skill upgrading and learning-by-doing externalities into a New Keynesian model with search and matching frictions. These two features of the model are complementary to each other. Our model is based on Lechthaler and Tesfaselassie (forthcoming), who show that the model can account for key features of the Great

Recession: (i) the "productivity puzzle"–the permanent gap between productivity and output relative to pre-crisis trends, and (ii) the "missing disinflation puzzle"–the relative stability of inflation despite the pronounced fall in output.

Our analysis of optimal monetary policy proceeds in two stages. First, we consider a simple two-period model, and show the inefficiency of the competitive equilibrium analytically by comparing it to the outcome under the planner's problem. We identify two externalities: i) firms do not internalize the effects that hiring has on labor productivity through learning-by-doing; ii) firms do not fully internalize the effects that hiring has on future training costs. The importance of the latter externality depends on the degree to which training costs are reflected in negotiated wages. Interestingly, even if wage bargaining if efficient, the externality still matters because future employers are not represented in the bargaining process.

Second, in a calibrated version of the full, infinite-horizon, model we illustrate our results quantitatively and conduct sensitivity analysis. Using impulse response functions we show that the Ramsey optimal policy deviates from price stability so as to reduce inefficient fluctuations in response to a temporary productivity shock. We also analyze the sensitivity of optimal policy to the degree of sunkness in training costs and the strength of the learning-by-doing externality from aggregate employment to human capital accumulation and productivity growth. We show optimal inflation volatility is lower the lower the degree of sunkness in training costs and the weaker the positive externality from aggregate employment to productivity growth.

Related literature. A number of studies examine Ramsey optimal monetary policy in the presence of frictions in the labor market. The main finding in these studies is that optimal monetary policy deviates from price stability in response to inefficient employment fluctuations implied by labor market distortions. Faia (2009) shows the deviation from price stability when relaxing the Hosios condition for efficiency of the competitive equilibrium, which is that workers' bargaining power should equal the elasticity of the matching function with respect to unemployment. Thomas (2008) derives a quadratic approximation of the welfare function around a non-distorted steady state in a search and matching model with real-wage staggering and convex costs of posting vacancies, which generate monetary trade-offs. Ravenna and Walsh (2011) also derive a quadratic approximation of household welfare under flexible real wages and rationalize monetary trade-offs by assuming stochastic fluctuations in worker-firm bargaining shares. Faia, Lechthaler, and Merkl (2014) study optimal monetary policy in a labor selection model and show that optimal inflaton volatility rises with firing costs. Lechthaler and Snower (2013) study optimal monetary policy in a model with quadratic employment adjustment costs, where these costs depend on aggregate employment, thereby implying externalities in hiring decisions. Lechthaler and Tesfaselassie (2019) extend the search and matching model to include exogenous productivity growth and show that higher productivity growth exacerbates the effects of labor market distortions, thus calling for larger deviations from price stability. None of these papers consider endogenous growth and/or skill loss through long term unemployment as we do. Closer to our paper, Annicchiarico and Rossi (2013) consider optimal monetary policy in the presence of learning-by-doing but within the standard New-Keynesian model with competitive labor markets. Our analysis allows us to consider not only the role of labor market frictions and skill loss but also the complementarity between endogenous growth and skill loss.

The positive and normative implications of sunk costs have been studied within the labor search literature but, to our knowledge, not within the optimal monetary policy and business cycle literature. For instance, Acemoglu and Shimer (1999) study the efficiency of the search and matching model under the assumption that a firm makes ex ante investments before matching with a worker. They show that there is inefficiency provided investment costs are sunk, and the inefficiency can only be prevented by removing all the bargaining power from the worker. Cheron (2005) shows that when fixed match-specific costs are not sunk, the Hosios condition guarantees efficiency of the decentralized economy. Miyamoto (2011) finds similar results in the case where match-specific costs are endogenously determined. Pissarides (2009) introduces sunk matching costs as an amplification mechanism within the search and matching model, with the purpose of matching key labor market facts (in particular the volatility of unemployment).

There exists a small body of theoretical work that examines the relation between business cycle persistence and long-run output in the presence of endogenous growth. Chang, Gomes, and Schorfheide (2002) use learning-by-doing as a propagation mechanism in a real business cycle model. In their model, an increase in the number of hours worked contributes to future improvements in labor skills. Stadler (1990) compares the properties of real and monetary business cycle models in the presence of endogenous growth arising from learning-by-doing. A temporary shock is shown to induce a permanent upward shift in the aggregate production function, thus having long-run effects. Engler and Tervala (2018) use a two-country New-Keynesian model to show that the fiscal output multiplier is significantly larger in the presence of learning-by-doing. Jordà, Singh, and Taylor (2020) demonstrate that monetary policy shocks can have long-lasting effects on productivity and output. Unlike our paper all these models abstract from labor market frictions and/or optimal monetary policy.

The rest of the paper proceeds as follows. In section 2 we present the simple, two-period model and analytically show the inefficiency of the decentralized economy in the presence of the two externalities of the model. In section 3 we present the full, infinite-horizon, model. In section 4 we derive the objective of the Ramsey planner in the infinite-horizon model. Section 5 discusses the calibration of the model and presents the main results. Section 6 shows the relationship between the two externalities and optimal inflation volatility. Section 7 concludes.

2 A two-period model

2.1 General setup

In this section we develop a simple model with search and matching frictions, endogenous growth through learning-by-doing, and skill-loss through long-term unemployment and demonstrate that this model features two externalities: i) private firms do not internalize the effects of their decision on human capital growth; ii) private firms do not (fully) internalize the effects of their decisions of future training costs. These two externalities imply that even the economy without rigid prices and without monopolistic distortion is inefficient, giving the Ramsey planner a motive to deviate from price stability.[1]

To be able to show these two externalities analytically and to develop the intuition behind these results, we use a model with the minimal structure that allows us to develop these insights. Most importantly, we restrict the model to two periods which has the advantage of simplifying the wage bargaining process substantially. After having developed these insights we will use a full-fledged model for numerical analysis in section 3.

Workers are assumed to be risk neutral and live for two periods. The mass of workers is normalized to unity. The labor market is characterized by search and matching frictions. In the first period all workers start unemployed but have sufficient human capital and thus do not need training. All jobs last for only one period so that the number of searchers in both periods is unity. Those workers that find a job in the first period maintain their human capital and in aggregate generate endogenous growth in productivity through learning-by-doing. However, after production takes place all employed workers lose their job and start looking for a new job in period 2 (i.e., the separation rate is unity). Those workers that do not find a job in the first period lose their human capital and need training in the second period before production takes place (if they find a job).

The number of matches, M_t, $t \in \{1, 2\}$, is determined by a constant returns-to-scale matching function, with the number of searching workers S_t, and the number of posted vacancies V_t as its arguments

$$M_t = \mu S_t^{\alpha} V_t^{1-\alpha} = \mu V_t^{1-\alpha}, \tag{1}$$

where $\mu > 0$ is a scale parameter describing the efficiency of the labor market and $\alpha > 0$ is the elasticity of the matching function. The second equality follows from our assumption that all workers have to search for a job in both periods, i.e., $S_t = 1$. Dividing equation

[1] If the economy with flexible prices was efficient and the monopolistic distortion can be offset by a subsidy, the Ramsey planner would find it optimal to mimic the economy with flexible prices by holding the price level constant.

(1) by V_t and defining labor market tightness as $\theta_t \equiv V_t/S_t$ we can write the vacancy filling rate as

$$q(\theta_t) \equiv \frac{M_t}{V_t} = \mu\theta_t^{-\alpha}. \tag{2}$$

Similarly, the job-finding rate is given by $\theta_t q(\theta_t)$.

Learning-by-doing as a driver of endogenous growth is introduced in a standard way: higher aggregate employment N_t generates a positive externality on the accumulation of aggregate human capital H_{t+1} (due to enhanced opportunities of learning-by-doing). To capture this phenomenon aggregate human capital in period 2 is given by[2]

$$H_2 = BN_1H_1 \equiv h(N_1, H_1), \tag{3}$$

where $B > 0$ is a scale parameter, and H_1 is exogenously given.

If in period 2 a firm is matched with a worker that was unemployed in period 1, the firm needs to upgrade the matched worker's skill at a cost of χ. The expected training cost in period 2 per hired worker TC_2 is thus an increasing function in the share of period 1 unemployed $u_1 = 1 - N_1$ in total job searchers in period 2,

$$TC_2 = \frac{u_1}{S_2}\chi = u_1\chi. \tag{4}$$

Each worker is member of a risk-neutral representative household with a continuum of members. Unemployed worker receive unemployment benefits equal to u_b. As is common in the literature, we assume that income is pooled within the household so that consumption is equalized across employed and unemployed members. Each period the household consumes all income. It discounts future consumption by the subjective discount factor, β.

[2] As is common in the endogenous growth literature the change in human capital is linear in the level of human capital. It is the absence of diminishing returns in human capital accumulation that allows the model to generate sustained growth.

2.2 Decentralized economy

This section illustrates the solution of the decentralized, competitive economy in which wages are determined via Nash-bargaining. The model is solved backwards, starting with period 2.

Second period. Since in period 2 we do not need to take account of future values the firm values are simply given by the following static equations

$$J_2^S = H_2 - w_2^S \quad and \quad J_2^L = H_2 - w_2^L,$$

where J^S and J^L denote, respectively, gross firm value from hiring short-term and long-term unemployed workers, while w^S and w^L are the corresponding wages. The price of output is normalized to unity and the output/productivity of workers is solely determined by their human capital, which is H_2. The human capital of a worker who was unemployed in period 1 is also H_2 because the training upgrades the worker's skills before production takes place. The value of hiring a long-term unemployed, net of the training cost χ, is then $J_2^L - \chi$.

The corresponding worker values are

$$W_2^S = w_2^S, \quad W_2^L = w_2^L \quad and \quad U_2 = u_b,$$

where the value of unemployment U_2 is equal across workers, i.e., independent of employment in period 1.

Wages are set according to Nash bargaining so that the optimal surplus sharing rule for matches with the short-term unemployed (respectively, long-term unemployed) is given by $W_2^S - U_2 = (1-\nu)/\nu J_2^S$, (respectively, $W_2^L - U_2 = (1-\nu)/\nu(J_2^L - \xi\chi)$), where ν is the bargaining power of the firm. The parameter ξ governs the extent to which training costs are sunk at the time of wage bargaining with a long-term unemployed worker. When $\xi = 0$ training costs are fully sunk and not at all reflected in the negotiated wage. By contrast, when $\xi = 1$ training costs are not sunk at all and fully reflected in the negotiated wage (and borne by the worker according his bargaining power). Combining the surplus sharing rules, the job values and worker values gives the wage rules

$$w_2^S = \nu u_b + (1-\nu) H_2 \quad and \quad w_2^L = \nu u_b + (1-\nu)(H_2 - \xi\chi).$$

The bargained wages are a weighed average of unemployment benefits and period-2 productivity net of training costs that are part of the wage bargaining with the long-term unemployed. Here it is the main advantage of the two-period structure that future values do not enter the wage bargaining, implying simpler wage equations.

Free entry of firms implies that the value of a vacancy is zero. This implies the following vacancy creation condition for period 2

$$\frac{\kappa}{q(\theta_2)} = (1-u_1) J_2^S + u_1(J_2^L - \chi). \tag{5}$$

Substituting out the value functions, equation (5) can be rewritten as

$$\frac{\kappa}{q(\theta_2)} + u_1\chi(1-(1-\nu)\xi) = \nu(H_2 - u_b). \tag{6}$$

That is, the expected cost of hiring a worker, including the firm's share of the expected training cost equals the expected firm-profit generated by the worker.

First period. In period 1 all workers have equal human capital, so there is only one wage level, one equation defining firm-value J_1, and two equations defining worker values, W_1 and U_1,

$$\begin{aligned} J_1 &= H_1 - w_1, \\ U_1 &= u_b + \beta\left(\theta_2 q(\theta_2) w_2^L + (1-\theta_2 q(\theta_2))\, u_b\right), \\ W_1 &= w_1 + \beta\left(\theta_2 q(\theta_2) w_2^S + (1-\theta_2 q(\theta_2))\, u_b\right). \end{aligned}$$

The value of a firm is just given by its contemporaneous profits. There is no continuation value because all jobs are destroyed at the end of period 1. For workers the continuation values depend on the employment status, because unemployed workers need retraining in period 2 if they are to become productive, implying a lower wage if training costs are shared.

As before, the surplus sharing rule is $W_1 - U_1 = [(1-\nu)/\nu]J_1$ so that the bargained wage is given by

$$w_1 = \nu \left[u_b - \beta\theta_2 q(\theta_2)(1-\nu)\xi\chi \right] + (1-\nu)H_1.$$

Note that, unless $\xi = 0$ (training costs are sunk) the threat point of a worker is lower than the unemployment benefit u_b. Thus by accepting a lower wage the worker compensates the firm for the benefit that current employment eliminates the need to pay training costs in period 2.

Optimal vacancy posting for period one then is

$$\frac{\kappa}{q(\theta_1)} = J_1 = \nu \left[H_1 - u_b + \beta\theta_2 q(\theta_2)(1-\nu)\xi\chi \right]. \tag{7}$$

2.3 Planner economy

The social planner chooses the number of vacancies in both periods, V_1 and V_2 so as to maximize the discounted sum of consumption in period 1 and period 2, $C_1^* + \beta C_2^*$ (where the superscript * indicates the planner economy). In period 1 consumption is equal to output less vacancy creation costs, $C_1^* = \mu V_1^{*1-\alpha} H_1 - \kappa V_1^*$. In period 2 consumption is equal to output less vacancy creation costs and training costs, $C_2^* = \mu V_2^{*1-\alpha} H_2^* - \kappa V_2^* - \left(1 - \mu V_1^{*1-\alpha}\right)\mu V_2^{*1-\alpha}\chi$. The first-order conditions with respect to V_1^* and V_2^* are, respectively,

$$\frac{\kappa}{(1-\alpha)\,\mu V_1^{*-\alpha}} = H_1 + \beta\mu V_2^{*1-\alpha}\chi + \beta B H_1 \mu V_2^{*1-\alpha}, \tag{8}$$

$$\frac{\kappa}{(1-\alpha)\,\mu V_2^{*-\alpha}} + u_1^*\chi = H_2^*. \tag{9}$$

In both equations (8) and (9) the left-hand side is the expected cost of hiring a worker. In period 1 this is simply the expected vacancy posting cost, whereas in period 2 it also includes the expected training cost. Note that the planner takes account of the congestion externality by pre-multiplying the worker-finding rate with $(1-\alpha)$. On the right hand side we see the benefits of hiring, which in period 2 is simply given by output. In period one the benefit includes two additional terms: the increase in period 2 productivity (the learning-by-doing effect, last term on the right-hand side) and the reduction in training costs in period 2 due to a smaller share of job searchers who lost human capital (the second term on the right-hand side).

2.4 Comparison

Comparing equations (6) and (9) it can be seen that second-period vacancy posting in the decentralized economy is optimal if the Hosios condition is satisfied ($\nu = 1-\alpha$), unemployment benefits are zero ($u_b = 0$), there is full sharing of training costs ($\xi = 1$), and unemployment in period 1 is optimal. The first two conditions are the well-known conditions for optimal vacancy posting in the standard search and matching model. The third condition extends this to the presence of training costs. It assures that training costs are shared between both parties in accordance with their shares of profits. If training costs were partly sunk (i.e., $\xi < 1$), firms would have to bear a disproportionate share of the training cost, leading to the creation of too few vacancies.

Using these conditions in the vacancy posting condition for period 1 (equation 7), it becomes

$$\frac{\kappa}{q(\theta_1)} = (1-\alpha)\, H_1 + \alpha\beta\, (1-\alpha)\, \mu V_2^{1-\alpha}\chi, \tag{10}$$

where we substituted the job-finding rate $\theta_2 q(\theta_2)$ by $\mu V_2^{1-\alpha}$. There are two differences compared to the planner's solution (8). First, unlike the planner, firms in the competitive economy do not internalize the effect of vacancy creation in period 1 on productivity in period 2. Second, private firms internalize only partly the effect of hiring in period 1 on training costs in period 2. In other words, firms are not fully compensated for the positive externality that job creation maintains the human capital of workers reducing the need for training in the future.

To see the training cost effect more clearly, assume $B = 0$, in which case the planner's optimality condition (8) simplifies to

$$\frac{\kappa}{\mu V_1^{-\alpha}} = (1-\alpha) H_1 + \beta (1-\alpha) \mu V_2^{1-\alpha} \chi, \tag{11}$$

This equation is very similar to the competitive equilibrium—equation (10) but note that the second term on the right-hand side of equation (10) is multiplied by α while this is not the case in equation (11). A private firm partially internalizes the reduction in the training cost, because it benefits the worker (as the worker doesn't need to pay training costs in the future) and it can participate in this gain through a reduced wage payment. However, it does not fully internalize the effect because part of the benefit of reduced training accrues to the future employer of the worker, which is not represented in period 1 bargaining.

Thus, both features, learning-by-doing and skill loss through long term unemployment introduce an inefficiency into the decentralized economy. Private firms post too few vacancies, because they do not fully internalize the beneficial effects of posting vacancies. While we have demonstrated this using a simplified model, the same mechanisms are at play in the full model. The Ramsey planner thus has a motive to use monetary policy in response to business cycle shocks to reduce inefficient fluctuations.

3 The full dynamic model

We now present the baseline model—an infinite-horizon New-Keynesian model featuring search and matching frictions, endogenous growth from learning-by-doing, and skill loss from long-term unemployment and associated training costs. Following the pioneering work of Walsh (2003) the model economy has two sectors: a retail sector and a wholesale sector. Firms in the wholesale sector combine raw labor and human capital to produce output and sell their output to the retail sector in a perfectly competitive market. The labor market is subject to search frictions. As in the previous section endogenous growth is generated by learning-by-doing and long-term unemployed workers need retraining.

Each retail firm transforms the wholesale good into a differentiated final good and sells it to households in a monopolistically competitive market. Retails firms set prices under Calvo-type nominal price staggering. Each household consists of a continuum of employed and unemployed (and searching) workers who pool their income.[3] Household utility depends on consumption only.

3.1 Labor market and human capital dynamics

We start by describing the aggregate relationships in the labor market within the wholesale sector and the endogeneity of aggregate human capital dynamics. The size of the labor force is normalized to one. At the beginning of each period a fraction δ of previously employed workers are separated from their jobs. These unemployed workers immediately engage in job search. As a result aggregate employment evolves according to the dynamic equation

$$N_t = (1-\delta)N_{t-1} + M_t, \tag{12}$$

where M_t is the number of newly formed matches in period t, which become productive immediately. Moreover, the number of searching workers in period t is given by

$$S_t = 1 - (1-\delta)N_{t-1}, \tag{13}$$

and the unemployment rate after hiring takes place is $u_t = 1 - N_t$.

The number of newly created matches, M_t, and the job-filling rate, $q(\theta_t)$, are given, respectively, by equation (1) and equation (2), but for ease of reading we repeat them here,

$$M_t = \mu S_t^{\alpha} V_t^{1-\alpha}, \tag{14}$$

$$q(\theta_t) \equiv \frac{M_t}{V_t} = \mu\theta_t^{-\alpha}. \tag{15}$$

[3] As is well-known locating labor market frictions and nominal price rigidities in different sectors as well as income pooling by workers make the model tractable.

The accumulation of aggregate human capital is generalized to

$$H_{t+1} = (1 - \delta_H)H_t + BN_tH_t, \tag{16}$$

where δ_H is the depreciation rate of human capital. One can rewrite equation (16) in terms of the gross growth rate of human capital

$$\Gamma_{H,t+1} \equiv \frac{H_{t+1}}{H_t} = 1 - \delta_H + BN_t, \tag{17}$$

which shows that a fall in aggregate employment today leads to a fall in future productivity growth.

3.2 Households

There is a representative household with a continuum of members over the unit interval. The period utility function is given by

$$U_t = \log C_t. \tag{18}$$

Household consumption C_t is a Dixit-Stiglitz composite of a continuum of differentiated goods $C_t = \left(\int_0^1 C_{k,t}^{1/\mu_p} dk\right)^{\mu_p}$ where each good is indexed by k, $\mu_p = \frac{\epsilon}{\epsilon - 1}$ and ϵ is the elasticity of substitution between goods. Optimal consumption allocation across goods gives the demand equation $C_{k,t} = \left(\frac{P_{k,t}}{P_t}\right)^{-\epsilon} C_t$ where $P_t = \left(\int_0^1 P_{k,t}^{1-\epsilon} dk\right)^{\frac{1}{1-\epsilon}}$ is the price index.

In any given period a fraction N_t of household members are employed by firms and earn a nominal wage W_t. The rest earn nominal unemployment benefits of $P_t u_b H_t$, $u_b > 0$, and search for work.[4] As with the two-period model income is pooled within the household so that consumption is equalized across employed and unemployed workers. The household maximizes the lifetime utility $E_t \sum_{i=0}^{\infty} \beta^i U_{t+i}$, where β is the subjective discount factor and ζ_t is a discount factor shock. The household's budget constraint is

$$P_tC_t + A_t = W_tN_t + P_tu_bH_t(1 - N_t) + R_{t-1}A_{t-1} + D_t, \tag{19}$$

[4]The presence of H_t ensures that along a balanced growth path real unemployment benefits grow at the same rate as the real wage.

where R_t is the nominal interest rate on bond holdings A_t, and D_t is aggregate nominal profit from ownership of retails firms.

It is straightforward to derive the familiar consumption Euler equation

$$1 = E_t \left(Q_{t,t+1} \frac{R_t}{\Pi_{t+1}} \right), \tag{20}$$

where $\Pi_t \equiv P_t / P_{t-1}$ is gross inflation rate and $Q_{t,t+1} \equiv \beta \left(C_{t+1}/C_t \right)^{-1}$ is the household's stochastic discount factor.

3.3 Firms

Next we describe the structure of the intermediate goods sector followed by the final goods sector.

3.3.1 Intermediate goods sector

Intermediate-goods firms can employ only one worker and produce with aggregate human capital H_t. The firms face standard search and matching frictions as well as frictions related to skill obsolescence and associated training costs incurred for skill upgrading.[5] There is an unlimited number of potential entrants that need to post a vacancy at real cost $H_t \kappa$ to have the chance to find a worker and enter the market. In addition, potential entrants anticipate to pay training costs if the matched worker needs skill upgrade.

To introduce skill loss from long-term unemployment in a tractable way, a period is taken to represent six months.[6] Similar to Acharya, Bengui, Dogra, and Wee (2018), the long-term unemployed are those job seekers in period t whose last job was in period $t-2$ or earlier (in the US this corresponds to those unemployed for 27 weeks or longer). By contrast, a searching worker in period t whose last job was in period $t-1$ does not need skill upgrade. These two types of workers may be differentiated as long-term unemployed

[5] A detailed discussion of the standard search and matching model can be found in, for e.g., Pissarides (2000).

[6] The main motivation for this assumption is that it simplifies the wage bargaining process because only two types of unemployed workers exist.

vs. short-term unemployed. Consistent with these definitions, the expected training cost per hired worker TC_t is given by

$$TC_t = z_t \chi H_t, \tag{21}$$

where $z_t \equiv u_{t-1}/S_t$ is the ratio of the number of long-term unemployed job seekers to total job seekers. Thus z_t is the probability that a firm matches with a long-term unemployed, who thus needs to upgrade the matched worker's skill at a cost of χH_t.[7]

It is important to note that the training cost is a predetermined endogenous variable (TC_t is given as of period t but responds to shocks with a one-period lag). An adverse shock in period t-1 that lowers employment N_{t-1} and the job-finding rate $\theta_{t-1}q(\theta_{t-1})$ also increases the share of long-term unemployment in total job seekers in period t and thus the expected training cost, as given in equation (21).

Let J_t^S (J_t^L) denote the value to a firm of matching with a short-term (long-term) unemployed worker. The value of a vacancy is then given by $q(\theta_t)\left[z_t\left(J_t^L - \chi H_t\right) + (1 - z_t)J_t^S\right]$. Free entry of firms drives down the value of a vacancy to zero so that the vacancy creation condition, adjusted for the presence of a training cost and a balanced growth path, is

$$\kappa H_t = q(\theta_t)\left[z_t\left(J_t^L - \chi H_t\right) + (1 - z_t)J_t^S\right]. \tag{22}$$

The cost of posting a vacancy equals the expected net benefit of posting a vacancy, the expected profits in case the search for a worker is successful. If the cost of posting a vacancy were lower than the expected profit of posting a vacancy, new vacancies would be posted, lowering the vacancy filling rate and thereby expected profits until the incentive to post further vacancies vanishes. Likewise, an increase in the training cost has similar effects on the incentive to post vacancies. But crucially, the expected training cost depends on the probability that a new hire comes from the long-term unemployed who need skill upgrading.

[7] The presence of H_t ensures that along the balanced growth path the vacancy posting cost and the training cost grow at the same rate as aggregate labor productivity. Without the above assumption vacancies would overtime converge towards infinity and unemployment towards zero, since the ratio of vacancy creation costs to labor productivity would converge towards zero.

Active firms in this sector face a perfectly competitive output market. Let P_t^I denote the nominal market price and $p_t^I \equiv P_t^I / P_t$ the real market price. Then the value of a job filled with a short-term unemployed, respectively, long-term unemployed worker, is defined as

$$J_t^S = a_t H_t p_t^I - w_t^S + (1-\delta) E_t \{Q_{t,t+1} J_{t+1}\} \tag{23}$$

$$J_t^L = a_t H_t p_t^I - w_t^L + (1-\delta) E_t \{Q_{t,t+1} J_{t+1}\}, \tag{24}$$

where a_t is productivity and $w_t^m = W_t / P_t$ is real wage of worker type $m \in \{S, L\}$. The value of a firm is the sum of the current profits and the expected future value of the match discounted by the appropriate discount factor. Note that because of skill upgrade the continuation value of a match with a long-term unemployed is equal to that of a short-term unemployed.

In response to a positive productivity shock (i.e., higher a_t), firms post more vacancies. As in the standard search and matching model, the resulting increase in labor market tightness increases the average duration of vacancies, and thus raises the expected cost of vacancy creation. However, here since training costs are predetermined the total expected cost of hiring does not increase in proportion to the decrease in the job-filling rate. Thus the presence of sunk training costs has an amplification effect on vacancy creation and market tightness.

Note also that, as aggregate vacancies rise, the share of long-term unemployment in total unemployment falls. This implies that firms expect future training costs, tc_{t+1}, to decline. This effect alone reduces the continuation value of a match and thus lowers the incentive to post a vacancy.

The wage rate is set under the standard assumption of Nash bargaining. The real value of employment and unemployment to a continuing worker and a searcher who is short-term unemployed are the same. The real value of employment is given by

$$W_t^S = w_t^S + E_t \left\{ Q_{t,t+1} \left[(1 - \delta(1 - f_{t+1})) W_{t+1}^S + \delta(1 - f_{t+1}) U_{t+1} \right] \right\}, \tag{25}$$

where $f_{t+1} \equiv \theta_{t+1} q(\theta_{t+1})$ is the job finding rate.

The real value of employment to a worker who was long-term unemployed (LTU), i.e., unemployed as of period $t-1$ or earlier is

$$W_t^L = w_t^L + E_t \left\{ Q_{t,t+1} \left[(1-\delta(1-f_{t+1}))W_{t+1}^S + \delta(1-f_{t+1})U_{t+1} \right] \right\}. \tag{26}$$

The corresponding real value of unemployment to short-term and long-term unemployed is the same because both get the same unemployment benefit and will have the same level of skills next period. Thus,

$$U_t = u_b H_t + E_t \left\{ Q_{t,t+1} \left[f_{t+1} W_{t+1}^L + (1-f_{t+1})U_{t+1} \right] \right\}. \tag{27}$$

Under Nash bargaining the optimal surplus sharing rule for new matches with the long-term unemployed (respectively, continuing workers or short-term unemployed) is given by $S_t^L = \bar{\nu}(J_t^L - \xi\chi H_t)$, (respectively, $S_t^S = \bar{\nu} J_t^S$), where $\bar{\nu} \equiv (1-\nu)/\nu$, and ν is the bargaining power of the firm. We have

$$S_t^S = w_t^S - u_b H_t + E_t \left\{ Q_{t,t+1}[(1-\delta)(1-f_{t+1})S_{t+1}^S + f_{t+1}(S_{t+1}^S - S_{t+1}^L)] \right\} \tag{28}$$

$$S_t^L = w_t^L - u_b H_t + E_t \left\{ Q_{t,t+1}[(1-\delta)(1-f_{t+1})S_{t+1}^S + f_{t+1}(S_{t+1}^S - S_{t+1}^L)] \right\}, \tag{29}$$

so that $S_t^S - S_t^L = w_t^S - w_t^L$. Using this equation, the surplus sharing rules and the relation $J_t^S - J_t^L = -(w_t^S - w_t^L)$, we get $J_t^S - J_t^L = -(1-\nu)\xi\chi H_t$ and $S_t^S - S_t^L = (1-\nu)\xi\chi H_t$ and $w_t^L = w_t^S - (1-\nu)\xi\chi H_t$. In the limiting case $\xi = 0$ (i.e., training costs are fully sunk) both types of workers earn the same wage. By contrast, when $\xi > 0$, the long-term unemployed receive lower wages because they bear part of the training costs. Moreover, the larger ν, that is, the higher the bargaining power of the firm, the larger the gap between the wages of workers who were long-term unemployed and short-term unemployed.

3.3.2 Final goods sector

Each firm k in the final goods sector produces a differentiated final good using a linear technology $Y_{k,t} = Y_{k,t}^I$ and receives a subsidity τ so that the firm's real marginal cost, $mc_{k,t}$, is given by $(1-\tau)p_t^I$. Price setting is subject to Calvo-type price staggering, where ω is the fraction of firms whose prices are fixed in any given period. Let $P_{k,t}$ denote firm $k's$ output price. Each firm k maximizes lifetime profit $E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+i} \left(P_{k,t}/P_{t+i} - (1-\tau)p^I_{t+i}\right) Y_{k,t+i}$ subject to the total demand for good k, $Y_{k,t+i} = (P_{k,t}/P_{t+i})^{-\epsilon} Y_{t+i}$, where $Y_{t+i} = C_{t+i} + G_{t+i} + H_{t+i}\kappa V_{t+i} + \chi \frac{u_{t-1+i}}{S_{t+i}} q(\theta_{t+i}) V_{t+i}$ is total aggregate demand that includes consumption, government spending G_t, the vacancy posting costs and training costs. The resulting optimal price is

$$p^*_t = \mu_p \frac{E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+i} (1-\tau) p^I_{t+i} \frac{Y_{t+i}}{Y_t} \left(\frac{P_{t+i}}{P_t}\right)^{\epsilon}}{E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+i} \frac{Y_{t+i}}{Y_t} \left(\frac{P_{t+i}}{P_t}\right)^{\epsilon-1}}, \tag{30}$$

where $p^*_t \equiv P^*_t/P_t$, $y_t = Y_t/H_t$ and μ_p is the price markup in the absence of price staggering. Endogenous growth feeds back into optimal pricing through two counteracting effects. Lower expected growth implies a lower discount rate (higher stochastic discount factor) but also lower expected future demand growth.

Equation (30) can be rewritten as

$$p^*_t = \mu_p \frac{F_{n,t}}{F_{d,t}}, \tag{31}$$

where $F_{n,t}$ and $F_{d,t}$ are auxiliary variables given by

$$F_{n,t} = (1-\tau) p^I_t y_t c_t^{-1} + \omega\beta \left(\frac{c_{t+1}}{c_t}\right)^{-1} \Pi^{\epsilon}_{t+1} F_{n,t+1}, \tag{32}$$

and

$$F_{d,t} = y_t c_t^{-1} + \omega\beta \left(\frac{c_{t+1}}{c_t}\right)^{-1} \Pi^{\epsilon-1}_{t+1} F_{d,t+1}. \tag{33}$$

Under Calvo-type price staggering the aggregate price index can be rewritten as

$$1 = (1-\omega) p_t^{*(1-\epsilon)} + \omega \Pi_t^{\epsilon-1}. \tag{34}$$

Aggregating both sides of the market clearing condition for the intermediate good and using the demand equation for the final good k leads to a relationship between aggregate final output y_t and intermediate good output y^I_t,

$$y^I_t = \Delta_t y_t, \tag{35}$$

where $\Delta_t \equiv \int_0^1 (P_{k,t}/P_t)^{-\epsilon} df$ is a measure of price dispersion, which can be rewritten as

$$\Delta_t = (1-\omega)p_t^{*-\epsilon} + \omega\Pi_t^{\epsilon}\Delta_{t-1}. \tag{36}$$

As aggregate output in the intermediate good sector is equal to aggregate employment, Eq. (35) can be rewritten as

$$N_t = \Delta_t y_t. \tag{37}$$

Finally, the aggregate resource constraint in stationary form is given by

$$y_t = c_t + \kappa V_t + tc_t q(\theta_t)V_t. \tag{38}$$

4 Ramsey optimal monetary policy

The Ramsey planner maximizes household utility subject to the competitive equilibrium under nominal price rigidity and labor market frictions, i.e., the Ramsey planner takes the distortions on the labor market as given. As is standard, we assume the government subsidy τ is set such that it eliminates monopolistic distortions in steady state.[8] We point out, however, that the government subsidy is not time-varying, and therefore monopolistic distortions might reappear in response to business cycle shocks.

We first transform the objective of the planner into a stationary form. This is necessary, because our model features positive long-run growth. The objective function as expressed in terms of the level of consumption is

$$\Pi_t = E_t \sum_{i=t} \beta^{i-t} \log C_i. \tag{39}$$

When reformulated in recursive form we get

$$\Pi_t \;=\; \log C_t + \beta E_t \Pi_{t+1}. \tag{40}$$

[8]Thus we have the well-known condition that the optimal level of the subsidy rate τ is set equal to $1/\epsilon$.

In appendix A.2 we show that maximizing 40 is equivalent to maximizing the de-trended objective

$$\Pi'_t = \log c_t + \frac{\beta}{1-\beta} \log \Gamma_{H,t+1} + \beta E_t \Pi'_{t+1} \quad (41)$$

expressed in normalized consumption c. The objective function (41) can be written alternatively in terms of employment by using the human capital accumulation equation to substitute out $\Gamma_{H,t+1}$,

$$\Pi'_t = \log c_t + \frac{\beta}{1-\beta} \log(1 - \delta_H + BN_t) + \beta E_t \Pi'_{t+1} \quad (42)$$

Interestingly, in the presence of endogenous growth ($B > 0$) the Ramsey planner's relevant welfare function depends directly on the level of aggregate employment N_t.

The economy under the Ramsey optimal policy is compared to a benchmark, simple Taylor-type rule considered in the related literature (e.g., Blanchard and Galí (2010))

$$R_t = R \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{u_t}{u} \right)^{\phi_u} \quad (43)$$

where the variables without time-subscript denote steady state values.

5 Calibration and main results

5.1 Calibration

Table 1 shows the calibration of the model to the US economy and at a biannual frequency.

The implied biannual values of Γ_H, δ_H, ω, as well as the targeted steady state job-filling rate are based on the quarterly values used in Lechthaler and Tesfaselassie (forthcoming). The steady state unemployment rate is set at 5% and the job separation rate δ is set such that the implied share of long-term unemployment in total unemployment in the US before 2008 is about 20% (Acharya, Bengui, Dogra, and Wee (2018)). The implied steady state job-finding rate is 0.8. The elasticity of the matching function α is set at 0.5,

Table 1: Parameter configuration

Parameter		Calibrated values
β	subjective discount factor	0.99^2
ω	fraction of non-optimizing firms	0.56
ϵ	elasticity of substitution between final goods	6
Γ_H	steady state growth	1.015
δ_H	human capital depreciation rate	0.0375
δ	job separation rate	0.2105
α	elasticity of the matching function	0.5
ν	firm's share of surplus	0.5
u_b	unemployment benefit parameter	0.44
ξ	sunk cost parameter	0
κ	vacancy posting cost	0.4
χ	training cost	0.6
B	learning-by-doing coefficient	0.055
ϕ_π	inflation coefficient	5
ϕ_u	unemployment coefficient	-0.8

values that are common in the literature (see, e.g., Pissarides (2009)). We impose the Hosios condition for efficiency in the absence of sunk training costs and learning-by-doing externalities, so that the firm's share of surplus ν is equal to the elasticity parameter in the matching function α. The scale parameter in the matching function μ and steady state labor market tightness are set given the steady state job-finding rate and the steady state job-filling rate. The replacement rate is set at 0.5 (the implied value of the unemployment benefit parameter u_b is 0.44). This value is well within the range typically used in the literature. For instance, regarding the replacement rate that includes the value of leisure and home production, which we do not model, Shimer (2005), Hagedorn and Manovskii (2008) and Hall and Milgrom (2008) use, respectively, 0.4, 0.71, and 0.95.

As a baseline the sunk cost parameter ξ is set at zero (Pissarides (2009), Acharya, Bengui, Dogra, and Wee (2018) and Lechthaler and Tesfaselassie (forthcoming)) but we also undertake sensitivity analysis with respect to this parameter. We target a steady state ratio of training costs to vacancy posting costs equal to 0.3, which is at the lower end of values considered in Pissarides (2009).[9] The training cost parameter χ and the cost

[9]We think the chosen value is reasonable, as Pissarides (2009) considers fixed matching costs that may also include "costs of finding out about the qualities of the particular worker, of interviews, and of negotiating with her".

of posting a vacancy κ are set consistent with the resulting steady state solution of the model. The scale parameter in the human capital accumulation equation B is consistent with the steady state annualized growth rate and the steady state employment rate. Regarding the Taylor rule coefficients ϕ_π and ϕ_u, we set them at 5 and −0.8 (the optimal simple rule in Blanchard and Galí (2010) under a US labor market).

5.2 Impulse responses

The quantitative analysis of the Ramsey optimal monetary policy is done using impulse responses to a temporary but persistent shock to productivity a_t. To be specific a_t is assumed to follow an autoregressive process of order 1: $a_t = a_{t-1}^{\rho_a} u_{at}$, $0 < \rho_a < 1$. In line with previous studies the autocorrelation coefficient ρ_a is set equal to 0.9[2] while the standard deviation of the innovation u_{at} is set equal to 0.01.

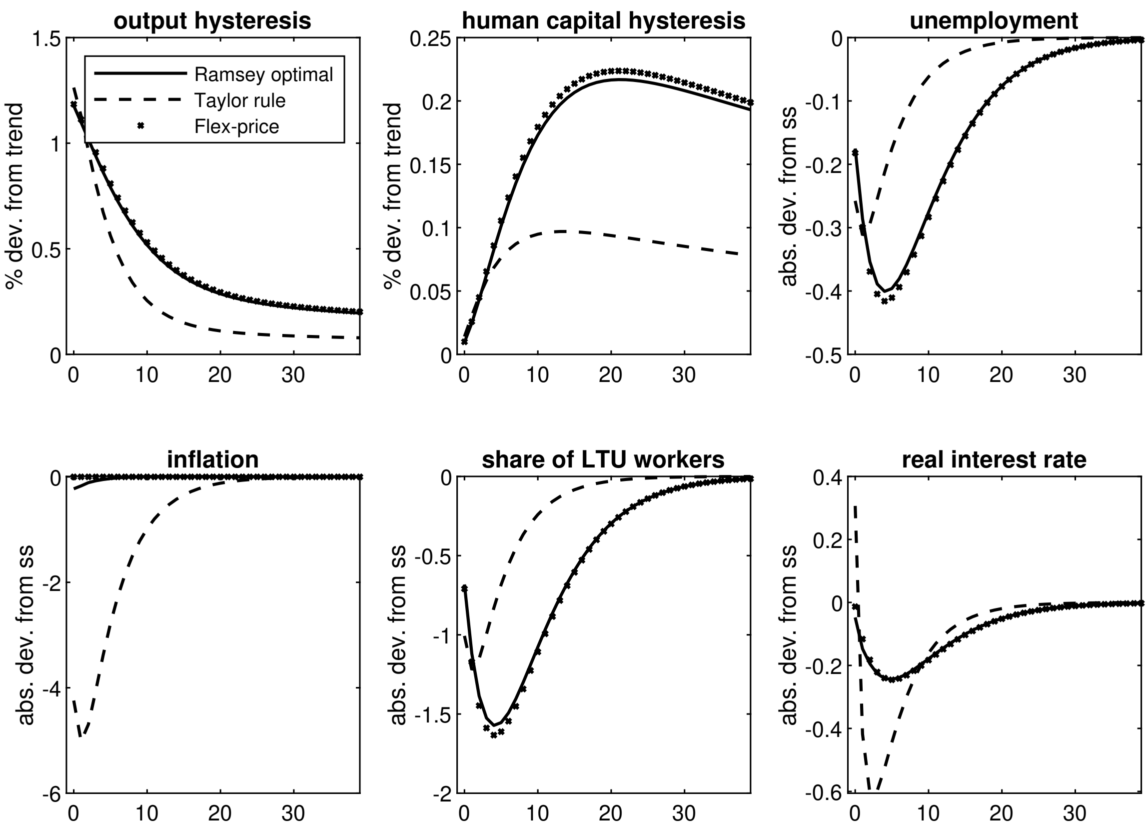

Figure 2: Impulse response to a temporary rise in labor productivity. Comparing Ramsey optimal, Taylor rule and Flexible price.

Figure 2 shows the impulse responses of output, human capital, unemployment, the share of long-term unemployment (LTU) in total unemployment, the inflation rate, and the real rate of interest to a positive productivity shock under the Ramsey optimal policy (solid line) and, for comparison, under a simple Taylor-type rule (dashed line) and under flexible prices (starred). Reflecting the underlying endogenous growth in the model, the impulse response named 'output hysteresis' shows the gap between actual output and output in the absence of the shock, expressed as a percentage of the latter. The impulse response named 'human capital hysteresis' is defined analogously. The impulse responses of inflation and the nominal interest rate are shown in absolute deviations and annualized.

A temporary increase in labor productivity raises output and human capital above their pre-shock trend, and lowers unemployment, the share of long-term unemployment in total unemployment, and the inflation rate. The lower share of long-term unemployed implies a reduction in expected training costs that amplifies the drop in unemployment (see also Lechthaler and Tesfaselassie (2019)). In turn, the presence of endogenous growth implies that the temporary shock to productivity has permanent effects on the level of human capital and output—the surge in employment enhances learning by doing and pushes the economy to a permanently higher level.

Under the policy that follows a Taylor-rule, the effect of the productivity shock on real variables is much less pronounced than under Ramsey optimal policy. For output and human capital the differences are especially pronounced in the long run, where the more expansionary Ramsey policy implies much larger hysteresis effects. Under Ramsey policy, inflation declines by less and the real interest rate (reflecting the inertial nature of optimal policy–Woodford 2003) is less volatile than is the case under the Taylor rule. We can conclude that in the model under consideration the policy that follows a traditional Taylor rule is sub-optimal, implying too much volatility in inflation but too little volatility in output and unemployment.

The deviation from price stability under the Ramsey optimal policy also implies that the decentralized economy with flexible prices features excess volatility of unemployment and, importantly, an inefficiently strong response of human capital and output along the adjustment to their higher long-run levels.[10] In response, and in the presence of nominal price rigidity, the Ramsey planner uses the aggregate demand channel to reduce demand for the final good and in turn demand for the intermediate good. The resulting reduction in the relative price of the intermediate good p_t^I implies that the marginal revenue product of labor, and thus the match surplus, also decline. At the same time, a reduction in p_t^I implies a reduction in the real marginal cost of final good producers and therefore inflation.

To summarize, the economy with flexible prices exhibits excess volatility in output and unemployment, while the economy with rigid prices and a Taylor-rule exhibits too little volatility. Optimal monetary policy lies between both cases but closer to the model with flexible prices.

5.3 Endogenous growth vs. training costs

As noted above, our baseline model features two separate but closely related deviations from the standard model, endogenous growth based on human capital through learning-by-doing, and training costs related to the skill loss of long-term unemployed workers.[11] In order to see the role of each effect in isolation Figure 3 shows the impulse responses of a model in which one or both of the two features are absent.

The starred plot in Figure 3 shows impulse responses when only the training cost channel is present, the dotted line shows impulse responses when only the endogenous growth channel is present, and the dashed line shows impulse responses when both features are absent. In each case the impulse responses show the difference of a variable under Ramsey policy to that under Taylor rule policy. For instance, the solid line in the lower left panel shows that the Ramsey planner pushes up inflation by four percentage points relative to the economy under a Taylor rule, the difference between both lines in figure 2. We show

[10] If the decentralized economy with flexible prices were efficient, the Ramsey planner would keep inflation constant at zero.

[11] As shown in Lechthaler and Tesfaselassie (forthcoming) both features are necessary to yield impulse responses that are broadly consistent with the recent empirical findings on the fall in productivity growth after deep recessions and the observed relative stability of inflation despite the pronounced fall in GDP during the Great Recession.

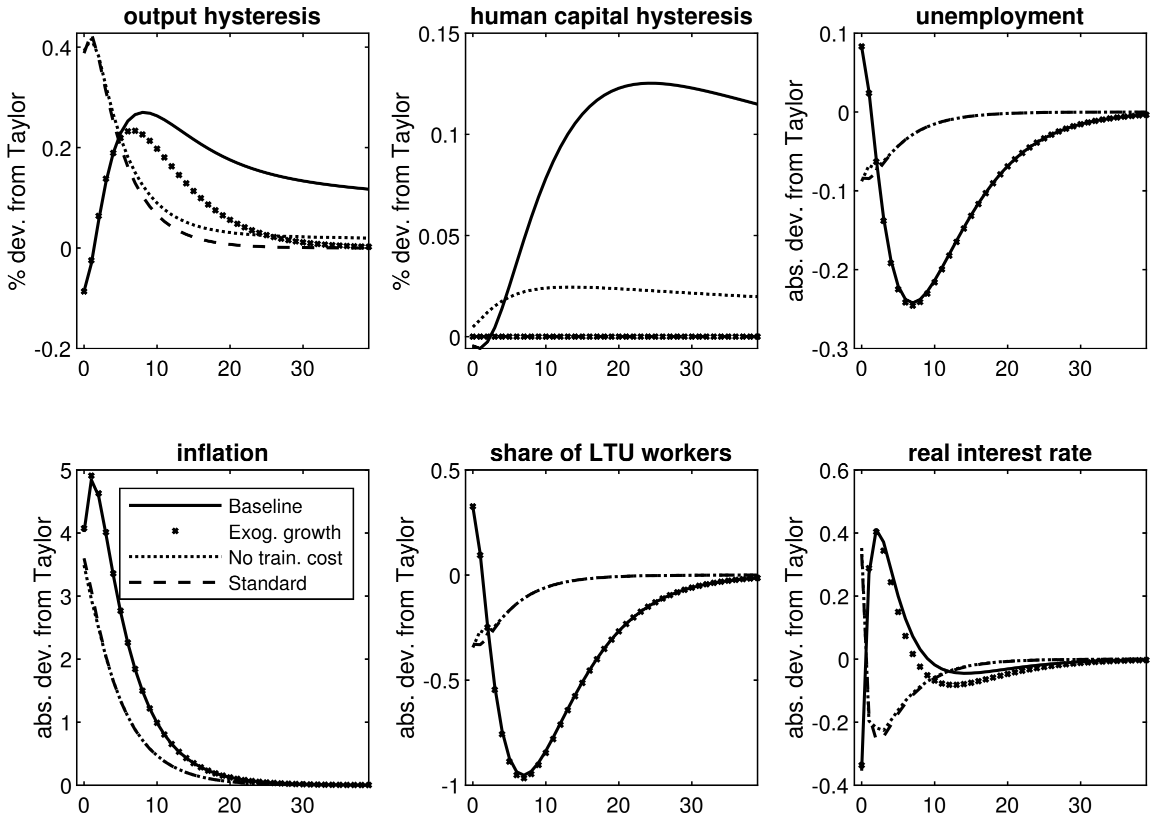

Figure 3: Impulse response to a temporary rise in labor productivity. Comparing baseline with three limiting cases: exogenous growth, no training costs, standard model. Each line shows the difference between the Ramsey optimal and the Taylor-rule in the respective model.

this difference so as to illustrate the effects of Ramsey policy in a compact way. The appendix shows complementary figures that illustrate the full impulse responses (i.e., not the differences).

Three observations can be made from figure 3. First, endogenous growth matters primarily for output and human capital in the medium to long run, because of the hysteresis effects it implies. In the short run, and for the other macroeconomic variables endogenous growth has very little effect (starred and solid lines are almost identical).

Second, when the training cost channel is absent (dotted line), the Ramsey planner raises output much more in the short run, but less in the long run. The hysteresis effects are much smaller than when the training cost is absent (dotted line vs solid line), which suggests the presence of complementarity between the endogenous growth channel and the training cost channel.

Third, looking at the dynamics of the real interest rate when the training cost channel is absent (dotted line), the fact that on impact the difference is positive is a result of the Taylor rule generating a more negative real interest rate than does the Ramsey optimal policy (by comparison, in the baseline case shown in Figure 1, on impact the real rate rises under the Taylor rule while it falls under the Ramsey optimal policy).

Finally, overall the standard model (dashed line) is very close to the model with endogenous growth but no training costs (dotted line). Thus, endogenous growth per se does not lead the Ramsey planner to deviate much from his policy pursued in the standard model.

6 Optimal inflation volatility

The two key parameters of the model are the degree of sunkness of training costs (ξ) and the strength of the positive externality from aggregate employment to productivity growth (B).

We first show the relation between ξ and optimal inflation volatility when only skill-loss from long-term unemployment is operative, that is, in the absence of endogenous growth ($B = 0$). Figure 4 shows the optimal inflation volatility as a function of the degree of sunkness of training costs (ξ) around the baseline case $\xi = 0$. The upper bound for ξ is chosen for computational reasons - it is set such that the steady state employment rate remains below unity.

Optimal inflation volatility declines monotonically with ξ, which confirms our analytical results based on the two-period model of section 2. Optimal volatility declines by about 13 percent as ξ increases from zero to 0.15. The intuition is straightforward, the larger is ξ (i.e., the lower the degree of sunkness in training costs), the more efficient is wage bargaining and thus the more efficient is the competitive equilibrium, implying that monetary policy focuses more on offsetting inefficiencies arising from nominal distortions (and in turn implying less deviation from price stability).

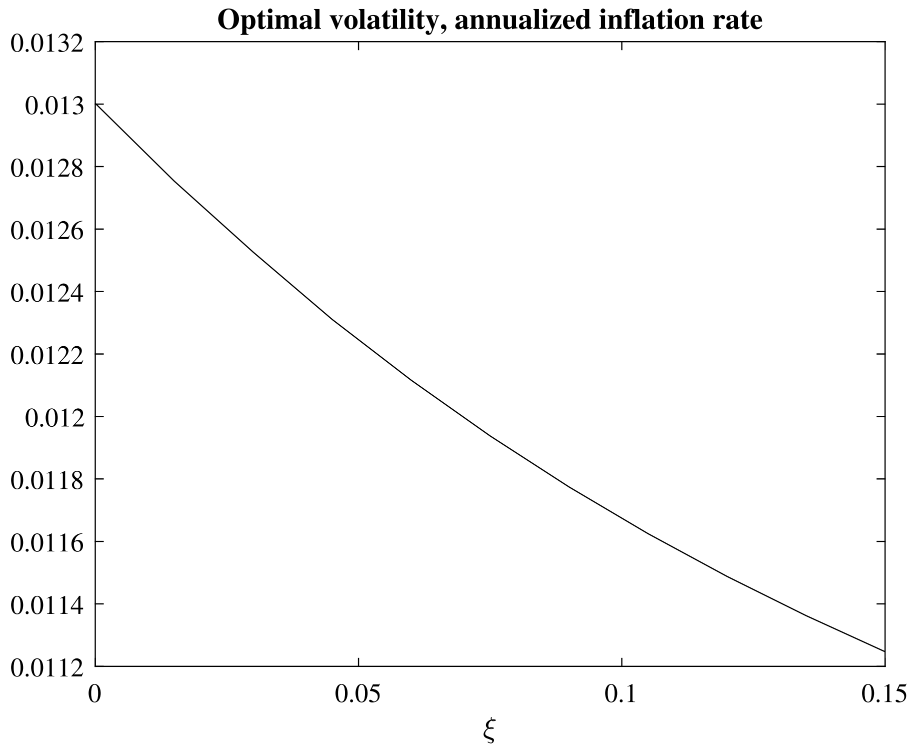

Figure 4: Sunkness of training costs and the optimal volatility of inflation.

Next, we show the relation between B and optimal inflation volatility when only learning-by-doing externality is operative, that is, in the absence of skill loss from long-term unemployment (no training costs are incurred). Figure 5 shows the optimal inflation volatility as a function of the strength of the learning-by-doing externality (controlled by the parameter B in equation (16)) around the baseline calibration $B = 0.055$.

Optimal inflation volatility increases monotonically with B, which also confirms our analytical results based on the two-period model of section 2. The intuition is again straightforward, the larger is B (i.e., the stronger the positive externality from aggregate employment to productivity growth), the less efficient is the competitive equilibrium, implying that monetary policy focuses less on offsetting inefficiencies arising from nominal distortions (and in turn implying greater deviations from price stability).

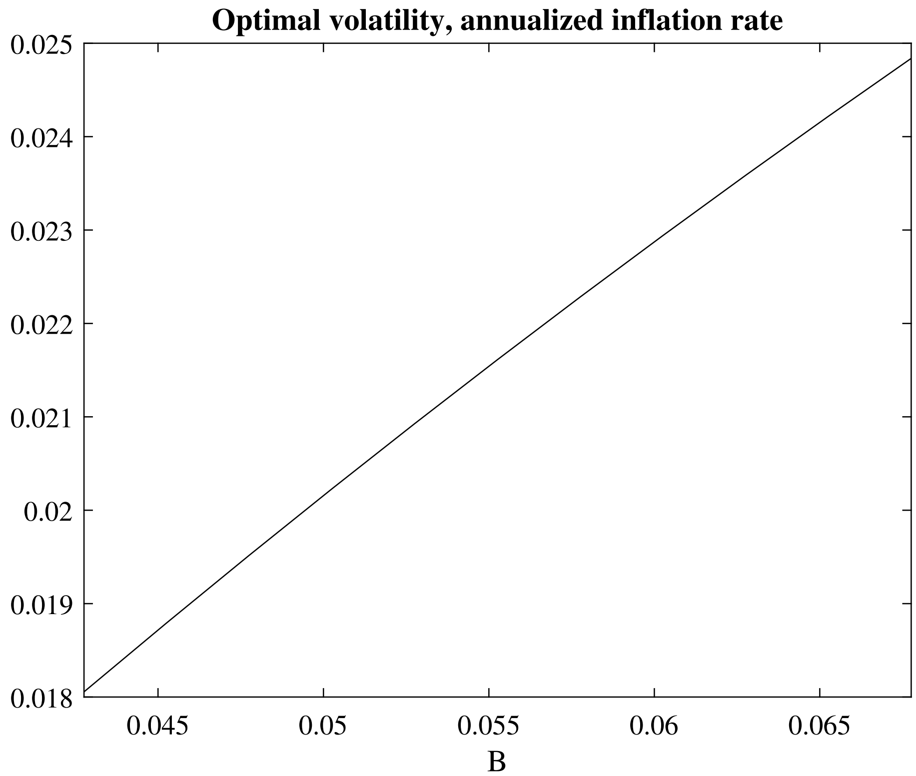

Figure 5: Learning-by-doing externality and the optimal volatility of inflation.

7 Concluding remarks

We analyze Ramsey optimal monetary policy in a New-Keynesian model with search and matching frictions featuring training costs due to skill loss from long-term unemployment and endogenous growth through learning-by-doing externalities. We show that the competitive equilibrium is inefficient due to two externalities, that is, firms fail to internalize the effects that hiring has on labor productivity through learning-by-doing and firms do not fully internalize the effects that hiring has on future training costs. These externalities lead to inefficient fluctuations, thereby justifying deviations from price stability in response to productivity shocks. Optimal inflation volatility is shown to be increasing in the degree of sunkness of training costs and in the strength of the learning-by-doing externality.

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A Appendix

A.1 Detrended model for optimal policy

The planner's problem is to maximize

$$\Pi_t = E_t \sum_{i=t} \beta^{i-t} \log C_i, \tag{A.1}$$

subject to the three constraints

$$
\begin{aligned}
N_t &= (1-\delta)N_{t-1} + \mu \left(1-(1-\delta)N_{t-1}\right)^{\alpha} V_t^{1-\alpha} && \text{(A.2)}\\
C_t &= a_t H_t N_t - H_t \kappa V_t - H_t \chi \left[1-\mu\left(1-(1-\delta)N_{t-2}\right)^{\alpha-1} V_{t-1}^{1-\alpha}\right](1-N_{t-2}) \\
&* \ \mu \left(1-(1-\delta)N_{t-1}\right)^{\alpha-1} V_t^{1-\alpha} && \text{(A.3)}\\
H_{t+1} &= \left(1-\delta_H + BN_t\right) H_t && \text{(A.4)}
\end{aligned}
$$

This is equivalent to maximizing

$$\Pi_t = E_t \sum_{i=t} \beta^{i-t} \left(\log c_i + \log H_i\right), \tag{A.5}$$

subject to the constraints

$$
\begin{aligned}
N_t &= (1-\delta)N_{t-1} + \mu \left(1-(1-\delta)N_{t-1}\right)^{\alpha} V_t^{1-\alpha} && \text{(A.6)}\\
c_t &= a_t N_t - \kappa V_t - \chi \left[1-\mu\left(1-(1-\delta)N_{t-2}\right)^{\alpha-1} V_{t-1}^{1-\alpha}\right](1-N_{t-2}) \\
&* \ \mu \left(1-(1-\delta)N_{t-1}\right)^{\alpha-1} V_t^{1-\alpha} && \text{(A.7)}\\
\Gamma_{H,t+1} &= \left(1-\delta_H + BN_t\right) && \text{(A.8)}
\end{aligned}
$$

The problem is still not stationary because the object function contains H which is not stationary. Thus our goal is to split the objective into a part that is stationary (Π_t') and a remaining part that only depends on current H_t (which is predetermined state variable as of period t). So let us use the transformation

$$\Pi_t = \Pi'_t + \lambda \log H_t$$

where the transforming factor λ has still to be determined such that it makes Π'_t stationary. Reformulating the above equation

$$\begin{aligned}
\Pi'_t &= \Pi_t - \lambda \log H_t \\
&= \log c_t + \log H_t - \lambda \log H_t + \beta \Pi_{t+1} \\
&= \log c_t + (1-\lambda) \log H_t + \beta\lambda \log H_t - \beta\lambda \log H_t + \beta \Pi_{t+1} \\
&= \log c_t + (1-\lambda+\beta\lambda) \log H_t - \beta\lambda \log \frac{H_{t+1}}{\Gamma_{H,t+1}} + \beta \Pi_{t+1} \\
&= \log c_t + (1-\lambda+\beta\lambda) \log H_t - \beta\lambda \log \frac{1}{\Gamma_{H,t+1}} + \beta \left(\Pi_{t+1} - \lambda H_{t+1}\right) \\
&= \log c_t + (1-\lambda+\beta\lambda) \log H_t + \beta\lambda \log \Gamma_{H,t+1} + \beta \left(\Pi'_{t+1}\right)
\end{aligned}$$

in the last equation we still have included the nonstationary term H_t, which would cause problems since H_{t+1} is contained in Π'_{t+1} and so on. However, if $1 - \lambda + \beta\lambda = 0$, then the H_t drops out (also out of future terms) and thus Π'_t becomes stationary. Thus the condition for stationarity of Π'_t is $\lambda = 1/(1-\beta)$, and then the planner's goal to maximize Π_t is equivalent to maximize

$$\Pi'_t = \log c_t + \frac{\beta}{1-\beta} \log \Gamma_{H,t+1} + \beta \left(\Pi'_{t+1}\right) + \lambda \log H_t$$

and since H_t is predetermined, i.e., out of the control of the planner at time t, this is equivalent to maximizing

$$\Pi'_t = \log c_t + \frac{\beta}{1-\beta} \log \Gamma_{H,t+1} + \beta \left(\Pi'_{t+1}\right)$$

A.2 Limiting cases

This appendix shows complementary figures that illustrate the full impulse responses for generating Figure 3.

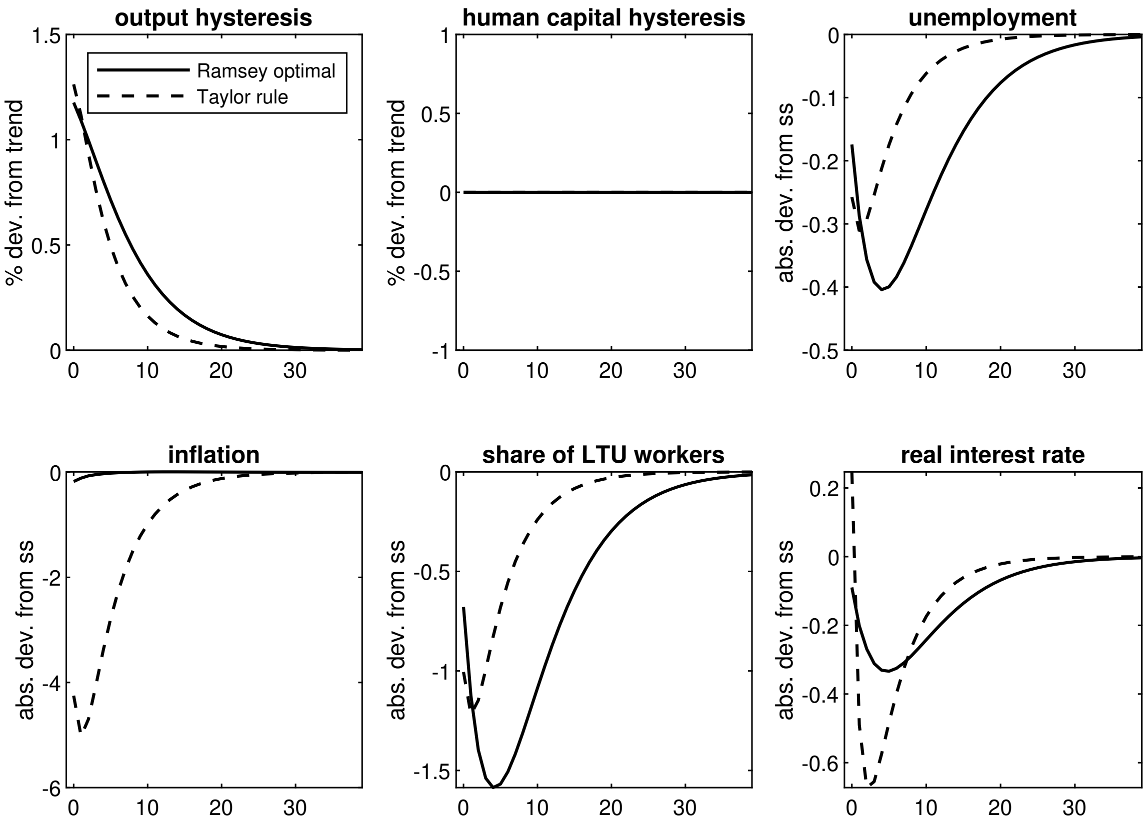

Figure 6: Impulse response to a temporary rise in labor productivity. Shutting down endogenous growth.

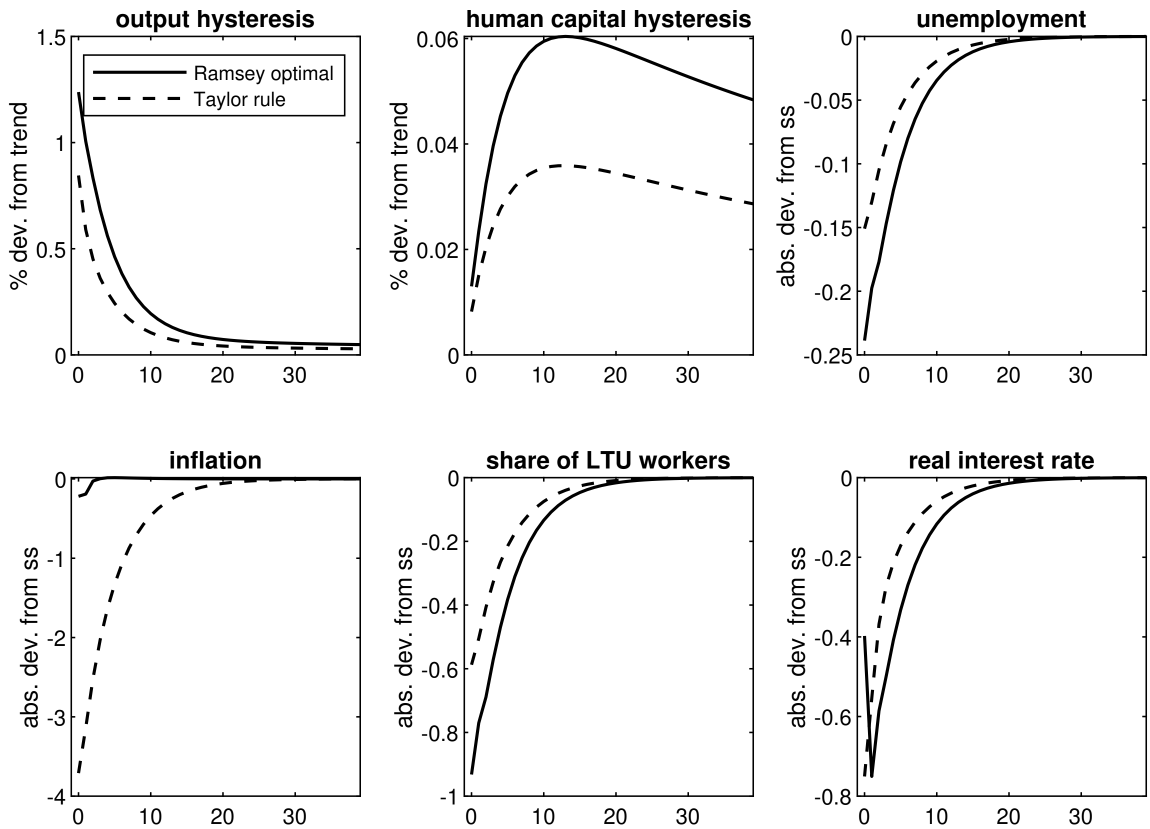

Figure 7: Impulse response to a temporary rise in labor productivity. Shutting down training costs.