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**Abstract**

This paper examines the interactions between employment and training policies. Their effectiveness in stimulating income and employment may be interdependent for various important reasons. For example, the more employment policies stimulate the employment rate, the greater the length of time over which workers use the human capital generated by training policies. Moreover, the greater the government expenditures on employment and training policies, the higher the taxes required to finance these expenditures and these higher taxes reduce aggregate income. On account of such effects, employment and training policies may be complementary or substitutable with respect to aggregate income. To analyze these interactions, we construct a simple, dynamic model of hiring decisions, derived from microfoundations. The model is calibrated with German data. The simulation shows that there are significant interactions between both policies. In the absence of government-budget-constraint effects, there are complementarities, but the government budget constraint introduces substitutabilities. The analysis provides a methodology for examining policy interactions which may be useful well beyond the bounds of employment and training policies.

*Keywords:* interactions; hiring subsidies; training subsidies; employment; income; complementarities;

*JEL classification: J21, J23, J24, J64, J68*

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# 1 Introduction[1]

When analyzing the impact of labor market policies, it is important to take into account possible interactions between different policies. In this context, one important aspect are complementarities between two policies, i.e. the effect of each policy on e.g. unemployment is greater when it is implemented in conjunction with the other policy than in isolation. However, both subsidies might be substitutes because (*i*) they both increase the trained labor force and (*ii*) they both might cause an increase of the tax rate which reduces income in the presence of the government budget constraint. Ignoring the possibility of interactions would distort the evaluation of the performance of policies. This paper offers a methodology which captures potential interactions of labor market policies.

As an example, we consider two important policies: training subsidies and hiring subsidies[2]. Most OECD countries have implemented such policies to encourage both employment and training. To varying degrees, both policies serve a similar purpose, namely, to improve the employment and income perspectives, particularly for low-skilled workers. However, the policies focus on different transitions in the labor market. Whereas training policies are meant to ease the transition from school to training, hiring subsidies are meant to facilitate the transition from training to work. This paper deals with two questions: (*i*) How does the interaction of hiring subsidies and training subsidies look like? (*ii*) Given the existence of complementarities, on what institutional and policy features of the economy does the size of complementarities depend?

One possible channel whereby both policies are complementary is the following: hiring subsidies facilitate the transition from training to work and thereby stimulate the employment rate, as they increase the probability that an apprentice continues working after having finished training successfully. Hence, hiring subsidies improve the effectiveness of training policies. As hiring subsidies increase the expected profits being generated by a former apprentice, the training incentives of the firms raise. Thus, hiring subsidies increase the number of people being hired as apprentices. This broadens the target group for training subsidies.

However, both subsidies might be substitutes because they both increase the trained labor force and reduce the non-trained labor force. They only differ with respect to the transition they are targeted at. Whereas training subsidies aim at increasing the number of people being in training (first transition), hiring subsidies aim at increasing the hiring of successful apprentices as trained employees (second transition). Moreover, the greater the government expenditures for hiring subsidies and training subsidies, the higher the taxes required to finance these expenditures could be and higher taxes reduce the expected life time income. On account of these effects, employment and training policies may be complementary or substitutable with respect to the expected life time income.

Our analysis tackles these issues by presenting a macro model of the labor market and training system that is rich enough to capture the various groups of workers being targeted by the alternative

[1]We would like to thank Alessio J.G. Brown, Wolfgang Lechthaler and Christian Merkl for valuable comments. Moreover we would like to thank the participants of the 2007 Annual Meeting of the Verein für Socialpolitik, of the 3rd Annual IZA Conference on the Evaluation of Labor Market Programs, in particular Jose Galdo, and the participants of the IAB workshop on "Evaluation of Passive and Active Labour Market Policies for the Long-Term Unemployed and Social Benefit Recipients" for the discussion. Besides, we are very grateful to Markus Hummel (IAB) and Jessica Erbe (BIBB) for providing us data and valuable information.

[2]Sometimes, "hiring subsidies" are also called "employment subsidies" or "employment vouchers" and are implemented through a wide variety of policy instruments, such as tax breaks or grants. As they all - given that they are awarded only for a limited period of time - have analogous effects on labor market activities and government budgetary outlays, this paper groups them together under the heading of "hiring subsidies".

policy options, while at the same time being simple enough to generate straightforward, intuitively transparent, policy guidelines. The model allows to identify and qualify each effect being associated with the subsidies. The analysis is based on a model in which the transition probabilities between the different states of the labor market and training system are governed by a Markov Process. The transition probabilities which are affected by the different subsidies are analyzed in detail. Moreover, the model takes some important labor market imperfections – such as wage bargaining, hiring and separation costs as well as imperfections related to the tax and transfer system – as given. Thus, we do not intend to derive policies as first-best responses to labor market failures. In fact, we suppose that the institutions being responsible for labor market failures can be modified only successively and not in the short term. Given this, we analyze the impact of the two subsidies in the presence of these institutions. We calibrate the model for the German labor market and training system.

The main message of the paper is that there are significant interactions between both policies. In the absence of government-budget-constraint effects, there are complementarities with respect to aggregate income, however, the government budget constraint introduces substitutabilities. The net effect is small. The analysis provides a methodology for examining policy interactions which may be useful well beyond the bounds of hiring and training subsidies.

This paper is organized as follows. Section 2 provides the underlying ideas and the relation to the literature. Section 3 describes the theoretical model of the labor market and the training system. In section 4, we present an purely analytical evaluation of a simplified model. In section 5 we calibrate the model. In section 6, we simulate a variety of policies, and discuss the numerical results. We start with the simplified version of the model and enlarge it gradually. Finally, section 7 concludes.

## 2 Underlying ideas and relation to the literature

The analysis of complementarities between labor market institutions and policies is prevalent in the literature (see e.g. Belot and van Ours, 2001). Theoretical analyses of complementarities can be found in Coe and Snower (1997), Orszag and Snower (1999a), Burda and Weder (2002) as well as in L'Haridon (2002). However, they focus on complementarities between institutions or policies other than in this paper. In particular, they do not deal with the question by how far the size of complementarities is affected by different features of the economy.

In our paper, we analyze the interactions between employment and training policies. As an example, we take hiring subsidies and training subsidies because they play a prominent role within the active labor market policy in OECD countries.

Both, hiring subsidies and training subsidies, have been analyzed in detail in the literature, in particular the hiring subsidies. In this context, our study is related to a variety of previous studies analyzing the impact and optimal design of employment subsidies, both, theoretically and empirically. The initial work was done by Pigou (1933) and Kaldor (1936).[3]

Often, the search and matching framework of Mortensen and Pissarides (1994) has been used to analyze connections between the labor market and the education sector (see e.g. Charlot et al., 2005) or the effect of labor market policies (see e.g. Cardullo and van der Linden, 2006, Danthine,

[3] For a survey of the empirical literature, see for example Katz (1998). For US evidence, see Woodbury and Spiegelman (1987) and O'Leary et al. (2005). For British evidence, see Bell et al. (1999). For an analysis of training subsidies, see e.g. Görg and Strobl (2006) as well as Filges et al. (2007).

2005, Boone and van Ours, 2004 as well as Mortensen and Pissarides, 2003). However, the matching technology[4] is assumed to be stable through time. This assumption is acceptable given that the matching technology can be considered independent of input and output of the matching process. However, some empirical studies estimating search and matching functions (see Blanchard and Diamond, 1989, for the United States, and Fahr and Sunde, 2001, 2004, for Germany) have not confirmed the stability through time but have found a negative time trend.[5] Moreover, given that the matching process itself may not be invariant to a policy change, it is not admissible to use the matching function to analyze labor market policies. To prevent running afoul of the Lucas Critique, we do not base our analysis on a policy-invariant matching function. Instead, similar to Brown et al. (2007), we analyze explicitly how policies affect people's incentives given an intertemporal maximization of economic agents. In our analysis, the focus is on the firm side. This has two reasons: (*i*) labor demand, especially with respect to the low-skilled labor force, is the short side of the market in economies with high unemployment and (*ii*) the subsidies which are analyzed are paid to the firms. The household side gets involved through the wage bargaining.

Many studies in this area are static and only account for the short-run effect of an employment policy.[6] There are, however, good theoretical and empirical reasons to believe that longer-run effects are important, often more important than the short-run effects.[7] In this context, our study differs from the literature, as we explicitly capture the dynamic effects of subsidies by specifying the transition rates between employment, unemployment and training as a function of the hiring incentives of the firm. This approach allows us to capture the adjustment processes and thereby to analyze the long-run effects effects.

The existing dynamic frameworks for evaluating subsidies are not well suited to analyze the effects of the policies we are interested in. Mortensen and Pissarides (2003) explore the effects of taxes and subsidies on job creation, job destruction, employment and wages in a search and matching equilibrium model. However, in their model, like in the models of Albrecht and Vroman (2002) as well as Cardullo and van der Linden (2006), migration between skill groups, which is an essential component in our model, does not take place. In this context, we contribute to the existing literature by explicitly allowing for migration from the low-skilled to the medium-skilled labor force.

The analysis is based on an ex ante policy evaluation. In this context, the paper contrasts with the ex post policy evaluation which is omnipresent in the literature. However, according to Wolpin (2007), there is "little methodological or applied research explicitly concerned with ex ante policy evaluation using nonexperimental methods, ...".[8] The following analysis wants to contribute to fill the gap by using a macro approach with a special focus on complementarities.

This detailed grid allows us to analyze and contrast the effects of training and hiring subsidies

[4] Like a production function, the matching technology describes the relation between input – the number of unemployed ($U$) and the number of vacancies ($V$) – and output given by the number of matches ($M$): $M = f(U, V)$. Often, a Cobb Douglas function is used: $M = U^{\alpha} V^{\beta}$. However, if $\alpha + \beta$ do not sum up to 1, the results are input dependent.

[5] Moreover, many empirical studies reject the hypothesis of constant returns to scale, see e.g. Warren (1996) for the United States as well as Fahr and Sunde (2001) for Germany.

[6] See e.g. Layard et al. (1991) and Snower (1994).

[7] Orszag and Snower (2000) show that the dynamic, long-run effects of employment subsidies differ from what may be expected in the short run, once the corresponding lagged adjustment processes have worked themselves out.

[8] One example is Brown et al. (2007). Another example in this context is the analysis by Todd and Wolpin (2006). They assess the impact of a school subsidy program in Mexico by using a theoretical framework for an ex ante evaluation. However, their model is restricted to the micro level.

in a common framework, explicitly taking the budgetary effects into account[9].

We now proceed to present the underlying model.

# 3 The model

The analysis is based on a Markov model of the labor market and the training system. The dynamics are governed by a Markov matrix that summarizes the transition probabilities among the different states. The transition probabilities are the result of an optimization principle of the firms and the individuals.

The model is meant to provide an framework for analyzing the interaction of hiring and training subsidies with respect to income and employment. In this context, it contains a variety of common labor market imperfections like insider wage bargaining as well as hiring and firing costs. Our model is meant to be rich enough to capture the relevant heterogeneity of the labor market but it also aims to be simple enough to generate straightforward, intuitively transparent, policy guidelines. Thus, the model involves some judicious compromises between analytical simplicity and the depiction of heterogeneous labor market behaviors. Specifically, the labor force is differentiated according to its different skill levels, which are defined by the level of educational attainment. We assume, that each skill level corresponds to a certain productivity level. Total population is divided into eight groups (see table (1)): people being in school ($S$), people joining vocational training ($T$) and those being either employed ($N_i$) or unemployed ($U_i$). The employed and unemployed, respectively, are divided into three subgroups according to the skill level $i = l, m, h$. Here, as well as for other variables below, the subscript $l$ stands for "low-skilled"; the subscript $m$ for "medium-skilled", the subscript $h$ for "high-skilled".

| state | variable |
|---|---|
| low-skilled employment | $N_l$ |
| low-skilled unemployment | $U_l$ |
| medium-skilled employment | $N_m$ |
| medium-skilled unemployment | $U_m$ |
| high-skilled employment | $N_h$ |
| high-skilled unemployment | $U_h$ |
| vocational training | $T$ |
| school | $S$ |

Table 1: The Labor Market States.

Vocational training takes $p$ periods, so that there are $p$ cohorts. In each period, a fraction $\rho + \theta$ leaves the vocational training where $\rho$ is the mortality rate and $\theta$ is the breaking off rate of training. So, given the inflow into vocational training, $T_1$, the outflow $p$ period later is given by $T_1(1-\rho-\theta)^p$. The stock of people being in vocational training is given by $T = T_1 \sum_{c=1}^{p} (1-\rho-\theta)^{c-1}$.[10]

For simplicity, there is no capital. Moreover, we assume constant returns to labor. Let $a_i$ be the productivity of an employee with a skill level $i = l, m, h$. When making their employment

[9] Orszag and Snower (2003a and 2003b) stress that the literature has disregarded the total impact of employment subsidies on the government budget constraint. On the one hand, aggregate payroll taxes finance subsidies but on the other hand, payroll taxes can be reduced if the subsidies reduce unemployment and thereby unemployment benefit payments. In this paper we follow their line of reasoning.

[10] In the initial steady state, $T$ is exogenously given. In the presence of subsidies, $T$ is determined endogenously.

decisions, firms face a random operation cost $\epsilon_t$ which is *iid* across workers and time. The cost may be interpreted as an operating cost or a productivity shock. With respect to all employees, its mean of future values is normalized to zero and its cumulative distribution $\Gamma(\epsilon_t)$ is time-invariant.

In the model, training takes place within the dual system of vocational training, the dominant form in Germany. This is a combination of vocational training provided by a private employer (training on-the-job) and theoretical education in vocational schools. With respect to the latter, the associated costs (e.g. for school buildings, salaries of the teachers) are distributed among the population. We assume that these costs have a highly fix character, so that a change in the number of apprentices does not influence the level. For the sake of simplicity, they are ignored in the remainder. The costs which are important in the model are the costs for the employer which are caused by the provision of vocational training (e.g. wages of additional employees being in charge of the instruction of apprentices within the firm). With respect to the distribution of these costs, it is necessary to distinguish two types of training: general training and specific training.[11] With respect to the German system of vocational training, one can argue, that training has a mostly general character. Due to detailed plans determining the content of training and central examinations, vocational training within the dual system is highly standardized on a national level.[12] According to the original theory by Becker (1964), in the presence of competitive markets, the employee receives all the returns from general training and thus also has to pay for training. However, German employers also pay a substantial amount of money for the training of apprentices and thus – from the theoretical point of view – also pay for general training.[13] It can be shown, that firms – in contrast to the original theory – have an incentive to invest in general vocational training, given that there are imperfections in the labor market.[14] In addition, the hold-up problem (see Williamson, 1985, and Malcomson, 1997) does not occur if the investor (i.e. the firm paying for general education) receives the full marginal return on investment and therefore will not under-invest. However, a hold-up problem may arise if the costs of general education for the firms arise.[15] Without a shift of financial burden away from the firm sector, the fraction of firms providing vocational training would decrease. The firms would under-invest.

We now continue by presenting the dynamic structure.

## 3.1 The dynamic structure

The transitions among the labor market states are summarized in Figure (1). In all states besides $S$, people face a probability $\rho$ of dying. With respect to the school leavers, a fraction $\eta_{S,T}$ is hired as apprentice.[16] The residual part $(1 - \eta_{S,T})S$ tries to get a job as a low-skilled employee; only a fraction $\eta_l$ is hired. A low-skilled employee faces a probability $\varphi_l$ of being fired and a probability $\eta_{N,T}$ of being hired as apprentice, a low-skilled unemployed faces a probability $\eta_{U,T}$ of being hired as

[11] See Becker (1964). For a recent survey of the literature on private sector training see Leuven (2005).

[12] See for further details Harhoff and Kane (1993) and Lindner (1998).

[13] According to Beicht and Walden (2002), in 2000 an employer being engaged in vocational training faced on average additional, non-wage costs of 8,166 EUR per apprentice.

[14] See e.g. Acemoglu and Pischke (1999) as well as Harhoff and Kane (1993). Appendix A1 contains an analysis showing that firms have an incentive to invest in general vocational training.

[15] The costs of education for the firm could rise e.g. because the apprentice spends more time for training and less for productive workplace activities. The additional financial burden (or lower profit) could induce firms to reduce vocational training. See for an analysis of the changing environment and its consequences for vocational training e.g. Büchel (2002) and Woessmann (2004, 2006).

[16] In Appendix A2, we present in model, in which $\eta_{S,T}$ is supply-driven. See also Oskamp and Snower (2008) for a model with a supply-driven transition rate.

apprentice and a probability $(1-\eta_{U,T})\eta_l$ of being hired as low-skilled employee. With a probability $\theta$ per period, an apprentice breaks off training, however. We assume that an apprentice cannot be fired. An apprentice who has finished training successfully and survives, is hired as a medium-skilled employee with a probability $\eta_{T,N_m}$. A medium-skilled employee faces a probability $\varphi_m$ of being fired. An unemployed medium-skilled is hired with a probability $\eta_m$.

In order to keep the model simple, we assume that the number of breakups and deaths is equal to the number of school leavers, so that the relevant population $(N_l+U_l+N_m+U_m+T+B)$ is constant. Moreover, we treat the high-skilled labor force $(N_h+U_h)$ as a quasi-fix factor, i.e. we assume, that these states are not affected by the introduction of subsidies. However, given the government budget constraint, it is necessary to take these states into account, because $N_h$ also carries a part of the fiscal burden and $U_h$ is responsible for a part of the fiscal burden. Ignoring it would bias the amount of the fiscal burden which has to be carried by the low-skilled and medium-skilled employees. In

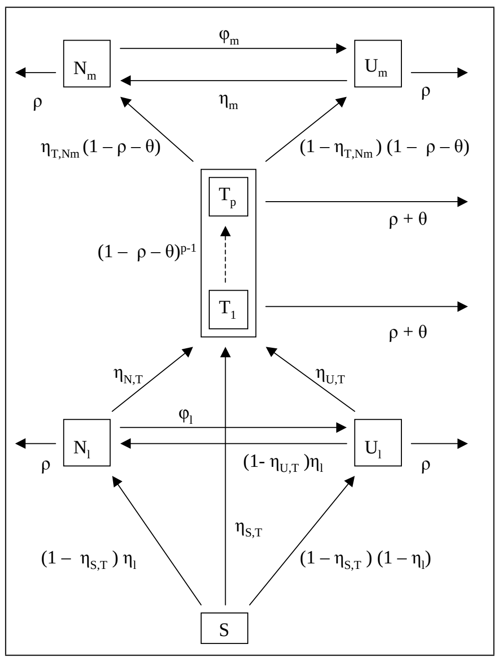


Figure 1: The dynamic structure of the model.

order to keep the model simple, we treat the high-skilled labor force $(N_h+U_h)$ as a quasi-fix factor, i.e. we assume, that these states are not affected by the introduction of subsidies. However, given the government budget constraint, it is necessary to take these states into account, because $N_h$ also carries a part of the fiscal burden and $U_h$ is responsible for a part of the fiscal burden. Ignoring it would bias the amount of the fiscal burden which has to be carried by the low-skilled and medium-skilled employees. Moreover, we assume that the number of breakups and deaths is equal to the number of people being in state $S$, so that the relevant population $(N_l+U_l+N_m+U_m+T+S)$ is constant. Given this, the model containing the labor market and the training system can be described as follows:

$$Z_{t+1} = MT_{t+1}\ Z_t \tag{1}$$

where $Z_t$ is a vector of the different states:

$$Z_t = (N_{m,t},\ U_{m,t},\ N_{l,t},\ U_{l,t},\ T_{1,t},\ S_t) \tag{2}$$

and $MT$ is a Markov matrix of transition probabilities: (3)

$$MT = \begin{pmatrix} (1-\phi_m-\rho) & \eta_m & 0 & 0 & \eta_{T,N_m}(1-\rho-\theta)^p & 0 \\ \phi_m & (1-\eta_m-\rho) & 0 & 0 & (1-\eta_{T,N_m})(1-\rho-\theta)^p & 0 \\ 0 & 0 & (1-\phi_l-\eta_{N,T}-\rho) & (1-\eta_{U,T})\eta_l & 0 & (1-\eta_{S,T})\eta_l \\ 0 & 0 & \phi_l & (1-(1-\eta_{U,T})\eta_l-\eta_{U,T}-\rho) & 0 & (1-\eta_{S,T})(1-\eta_l) \\ 0 & 0 & \eta_{N,T} & \eta_{U,T} & 0 & \eta_{S,T} \\ \rho & \rho & \rho & \rho & 1-(1-\rho-\theta)^p & 0 \end{pmatrix}$$

With respect to the transition rates $\eta_{N,T}$ and $\eta_{U,T}$, we assume that a transition is connected with a change of the firm. This is based on the assumption that there are two types of firms: (1.) firm which employ only low-skilled and (2.) firms which employ only medium-skilled and can be engaged in vocational training.[17]

## 3.2 The characteristics of the subsidies

In the following, we analyze two kinds of subsidies:

- Hiring subsidies ($\sigma^{\eta_{T,N_m}}$) are paid to firms which hire successful apprentices as medium-skilled employees. The subsidy is paid during the first period of the employment spell.
- Training subsidies ($\sigma^{\eta_{S,T}}$) are paid to firms which hire school leavers as apprentices. The training subsidies are paid per apprentice and per period over the whole phase of vocational training.

Hiring subsidies aim at improving the employment situation of the successful apprentices by increasing the hiring incentive of the firm. In contrast, training subsidies aim at improving the human capital in a first step and then in a second step, the long-term employment perspective, given that the employment rate of the medium-skilled labor force is higher than the employment rate of the low-skilled labor force.

Agents in our model pursue the following sequence of decisions. First, the government sets the income tax rate to ensure that its tax receipts are equal to its expenditures. Second, wages are determined through bargaining. Third, the random operating costs are revealed and finally, employment decisions are made.

[17] This can be justified by the assumption that the firms are engaged in different sectors with different requirements with respect to human capital.

### 3.3 Government Budget Constraint

When analyzing the government budget constraint, four policy instruments are taken into account: $(i)$ a payroll tax with a tax rate $t_i$, $(ii)$ an unemployment benefit $b_i$, $(iii)$ a hiring subsidy, $\sigma^{\eta_{T,N_m}}$ and $(iv)$ a training subsidy, $\sigma^{\eta_{S,T}}$. Our model has three levels of payroll tax rates, in order to match a progressive tax system $(t_h > t_m > t_l)$. The ratios are assumed to be exogenous, whereas the levels are set so that the tax receipts are equal to the government's expenditures. We assume that people being engaged in vocational training do not pay taxes. Given the presence of the subsidies above, the government budget constraint is expressed as follows:

$$\begin{aligned}\textstyle\sum_{i=l,m,h} t_i w_i N_i = \textstyle\sum_{i=l,m,h} \beta_i w_i (1-t_i) U_i \\ + \sigma^{\eta_{S,T}}\ \eta_{S,T}\ S \textstyle\sum_{c=1}^{p} (1-\rho-\theta)^{c-1} \\ + \sigma^{\eta_{T,N_m}}\ \eta_{T,N_m}\ (1-\rho-\theta)^p\ T_1\end{aligned} \tag{4}$$

where the left-hand side stands for the tax receipts, to be paid by the employees of different skill groups. The term on the right-hand side represents the sum of the unemployment benefits, with the net replacement rate $\beta_i$, the training subsidies $\sigma^{\eta_{S,T}}$ paid to firms for hiring school leavers as apprentices and finally the hiring subsidies, $\sigma^{\eta_{T,N_m}}$, paid to firms which hire successful apprentices as medium-skilled employees.[18]

### 3.4 Wage Determination

For simplicity, we assume that the wage of the apprentice, $w_v$, is given exogenously. In the remainder, we therefore focus on the wage $w_i$ for each skill level $i$, which is the outcome of a Nash bargain between the median employee of that skill level and the firm. The median insider faces no risk of dismissal at the negotiated wage. The wage is renegotiated in each period. We start by calculating the relevant surplus for both, the employee and the firm.

#### 3.4.1 Surplus of the employee

A person with a skill level $i$ has the following utility function:

$$u_{i,t}(C) = \frac{1}{1-\gamma}[C_{i,t}]^{1-\gamma} \tag{5}$$

where $\gamma$ is the coefficient of constant relative risk aversion (CRRA). Utility depends positively on consumption $C_{i,t}$.[19] For simplicity, we assume that there is no disutility of labor. Under bargaining agreement, the employee receives the net wage $w_i(1-t_l)$ in each period. The expected present value of the employee´s life time utility, $V_{i,t}^N$, is:

$$V_{i,t}^N = \frac{1}{1-\gamma}[w_{i,t}(1-t_{i,t})]^{1-\gamma} + \delta[(1-\varphi_i-\rho)V_{i,t+1}^N + \varphi_i V_{i+1}^U] \tag{6}$$

where $\delta$ is the discount factor and $V_{i+1}^U$ is the expected future life time utility for an unemployed. (Here, as well as for other variables below, the superscript $N$ stands for "employed"; the superscript

[18] Recall, that $T_1$ is the inflow into training and $(1-\rho-\theta)^p T_1$ is the outflow from training.

[19] In this model, for simplicity, workers consume all their income, i.e. either the net wage in the case of employment or the unemployment benefit in the case of unemployment.

$U$ for "unemployed"). In the case of disagreement, the employee´s fallback position is $b_{i,t}$, assumed to be equal to the unemployment benefit. Assuming that disagreement in the current period does not affect the expected future life time income, the expected present value of life time utility in the case of disagreement is:

$$V_{i,t}^{d,N} = \frac{1}{1-\gamma}[b_{i,t}]^{1-\gamma} + \delta[(1-\varphi_i-\rho)V_{i,t+1}^{N} + \varphi_l V_{i+1}^{U}] \tag{7}$$

Given the expected present values in the cases of agreement and disagreement, we can calculate the bargaining surplus for the employee ($S_{i,t}^{E} = V_{i,t}^{N} - V_{i,t}^{d,N}$):

$$S_{i,t}^{E} = \frac{1}{1-\gamma}[w_{i,t}(1-t_{i,t}+\kappa)]^{1-\gamma} - \frac{1}{1-\gamma}[b_{i,t}]^{1-\gamma} \tag{8}$$

### 3.4.2 Surplus of the firm

Under bargaining agreement, the firm receives the expected profit $(a_{i,t} - w_{i,t})$ in each period $t$. The expected present value of the profit $\pi_{i,t}$ with respect to an employee with skill level $i$ is therefore:

$$\pi_{i,t} = (a_{i,t} - w_{i,t}) + \delta(1-\varphi_i-\rho-\underbrace{\eta_{N,T}}_{\text{only for } i=l})\pi_{i,t+1} + \delta\varphi_l(-\varsigma_{i,t+1}) \tag{9}$$

where $\varsigma_i$ are the firing costs. In the case of a low-skilled we additionally have to take into account $\eta_{N,T}$, illustrating the transition from low-skilled employment to training.[20] In the case of disagreement the employee imposes the maximal cost on the firm (e.g. through strike, work-to-rule, sabotage) short of inducing dismissal. The firm's fallback loss is approximated by the firing costs. Again, we assume that disagreement in the current period does not affect future returns. Thus, the present value of expected future profits in the case of disagreement is:

$$\pi_{i,t}^{d} = -\varsigma_{l,t} + \delta(1-\varphi_i-\rho-\underbrace{\eta_{N,T}}_{\text{only for } i=l})\pi_{i,t+1} + \delta\varphi_l(-\varsigma_{i,t+1}) \tag{10}$$

Finally, we can calculate the bargaining surplus for the firm ($S_{l,t}^{F} = \pi_{i,t} - \pi_{i,t}^{d}$):

$$S_{i,t}^{F} = (a_{i,t} - w_{i,t}) \; + \varsigma_{i,t} \tag{11}$$

### 3.4.3 Bargaining

To determine the wage, we use the Nash bargaining solution. The bargaining power of the employee is denoted by $\mu \,\epsilon\, \langle 0,1 \rangle$, and the bargaining power of the firm by $1-\mu$. The negotiated wage $w_{l,t}$ maximizes the Nash product ($\Lambda_t$):

$$\Lambda_t = \; (S_{i,t}^{E})^{\mu} \; (S_{i,t}^{F})^{1-\mu} \tag{12}$$

[20] The way, how the transition rate $\eta_{N,T}$ is implemented in the calculation of the profit implies, that the transition is connected with a change of the firm. A low-skilled worker who decides to enter training has to leave the current firm. This is based on the assumption that there are two types of firms: (i) firms which only employ low-skilled and (ii) firms which are engaged in training and employ medium-skilled. This can be justified by the assumption that the firms are engaged in different sectors with different requirements with respect to human capital.

We have to solve:

$$\frac{\partial\ \Lambda_t}{\partial\ w_{i,t}} \stackrel{!}{=} 0$$

and get the following relationship:

$$\begin{aligned}
&(1-t_i)(a_i - w_i + \varsigma_i)^{1-\mu}(w_i - t_i w_i)^{-\gamma}(-b_i^{1-\gamma} + (w_i - t_i w_i)^{(1-\gamma)})^{-1+\mu}\ \mu\ (1-\gamma)^{1-\mu} \\
=&(a_i - w_i + \varsigma_l)^{-\mu}\ (-b_i^{1-\gamma} + (w_i - t_i w_i)^{1-\gamma})^{\mu}(1-\mu)\ (1-\gamma)^{-\mu} \qquad (13)
\end{aligned}$$

In the equilibrium, $(i)$ the firing costs are assumed to be $\varsigma_i = c_\varphi\ w_i$, $(ii)$ and $(ii)$ the unemployment benefit level is defined on the base of the economywide average net wage: $b_i = \beta_i\ w_i\ (1-t_i)$. Finally, the negotiated wage is:

$$w_i = \frac{a_i\ \mu\ (1-\gamma)}{(1-\beta_i^{1-\gamma})(1-\mu) + \mu(1-c_\varphi)(1-\gamma)} \quad \text{for } i = l, m, h \qquad (14)$$

## 3.5 Transition Rates

Now, we analyze in detail the two transition rates which are affected by the subsidies. Whereas $\eta_{T,N_m}$ is affected by the hiring subsidy, $\eta_{S,T}$ is in particular affected by the training subsidy.[21] Although both transition rates are can be regarded as hiring rates, in in the remainder, we refer to $\eta_{S,T}$ as *training rate* in order to avoid confusion with the other hiring rate $\eta_{T,N_m}$. All other transition rates, especially the hiring and separation rates of the low-skilled and the medium skilled, respectively, ($\eta_i$ and $\varphi_i$ with $i = l, m$) are not affected by the implementation of subsidies. They are treated as constant factors, so that a microfoundation is not necessary. We now start with the derivation of $\eta_{T,N_m}$.

### 3.5.1 The hiring rate

The expected present value of profit generated by the medium-skilled employee, after the random cost $\epsilon_{\eta_{T,N_m}}$ is observed, is:[22]

$$\begin{aligned}
\pi_{T,N_m,t} = &-\epsilon_{\eta_{T,N_m,t}} + \sigma^{\eta_{T,N_m}} \qquad (15)\\
&+ (a_m - w_m)\sum_{j=0}^{\infty}\delta^t(1-\varphi_m-\rho)^j - \delta\ \varphi_m\ \varsigma_m \sum_{j=0}^{\infty}\delta^j(1-\varphi_m-\rho)^j
\end{aligned}$$

Under the assumption that the hiring decision is made by the firm which also conducted the vocational training, the hiring costs are zero because the successful apprentice to be hired as medium-skilled employee is already in the firm. Given this, the person is hired, if the expected present value of profit is positive: $\pi_{T,N_m,t} > 0$, thus:

$$\epsilon_{\eta_{T,N_m,t}} < \sigma^{\eta_{T,N_m}} + \frac{a_m - w_m - \delta\ \varphi_m\ \varsigma_m}{1-\delta(1-\varphi_m-\rho)} \qquad (16)$$

[21] Recall that training subsidies are not paid for hiring low-skilled employees ($N_l$) or low-skilled unemployed ($U_l$) as apprentices. The corresponding transition rates, which would be affected ($\eta_{N,T}$) and ($\eta_{U,T}$) by the subsidy are very low, in particular in relation to ($\eta_{S,T}$). Thus, the quantitative impact of subsidies would be low. However, the complexity of the model would increase signficantly.

[22] Recall that eq. (15) is equal to eq. (9) for $\sigma^{\eta_{T,N_m}} = 0$, $i = m$ and for $-\epsilon_{\eta_{T,N_m}} = 0$. The hiring decisions are made after the random operating costs are revealed, thus in contrast to eq. (9), they have to be taken into account when deriving the hiring rate.

We assume that $\epsilon_{\eta_{T,N_m},t}$ is uniformly distributed between $\epsilon^-_{\eta_{T,N_m}}$ and $\epsilon^+_{\eta_{T,N_m}}$. The corresponding hiring rate is:

$$\eta_{T,N_m} = \frac{(\sigma^{\eta_{T,N_m}} + \frac{a_m - w_m - \delta\ \varphi_m\ \varsigma_m}{1-\delta(1-\varphi_m-\rho_m)}) - \epsilon^-_{\eta_{T,N_m}}}{\epsilon^+_{\eta_{T,N_m}} - \epsilon^-_{\eta_{T,N_m}}} \tag{17}$$

The critical reader might argue, that not all successful apprentices are taken over by the firm which conduct vocational training. Some of the successful apprentices will get a job in a firm which is not engaged in vocational training. So far, we have assumed that the hiring rate $\eta_{T,N_m}$ was equal to the take over rate. In the remainder, we assume, that there are two hiring rates: (1.) $\eta^1_{T,N_m}$ being associated to the firm which is engaged in vocational training, and (2.) $\eta^2_{T,N_m}$ being associated to the firm which is not engaged in vocational training. Now, only $\eta^1_{T,N_m}$ can be interpreted as take over rate and is given by eq. (17) for $\eta_{T,N_m} = \eta^2_{T,N_m}$. With respect to $\eta^2_{T,N_m}$ and in contrast to eq. (17), hiring costs ($\chi_T$) have to be taken into account, as the apprentice is hired by another firm for which the successful apprentice is an outsider. Therefore, the hiring rate $\eta^2_{T,N_m}$ is calculated as follows:

$$\eta^2_{T,N_m} = \frac{[-\chi_T + \sigma^{\eta_{T,N_m}} + \frac{a_m - w_m - \delta\ \varphi_m\ \varsigma_m}{1-\delta(1-\varphi_m-\rho_m)})] - \epsilon^-_{\eta^2_{T,N_m}}}{\epsilon^+_{\eta^2_{T,N_m}} - \epsilon^-_{\eta^2_{T,N_m}}} \tag{18}$$

### 3.5.2 Training Rate

Now, we consider the hiring of people leaving school ($S$) as apprentices. If the training rate would only depend on the profit in the phase of vocational training, there would be no hiring. As the output $a_v$ generated by the apprentice is supposed to be smaller than the sum of the wage, $w_v$, and the additional, non-wage costs of vocational training, $k_v$, the profit of the firm in the training phase is negative. However, the training decision is not only based on the financial outcome in the training phase, rather the firm regards the costs of vocational training as an investment, which causes a profit, $\pi_{T,N_m}$, (eq. (15)), once the successful apprentice continues working in the firm as a medium-skilled employee with a certain probability $(1-\rho-\theta)^p\ \eta^1_{T,N_m}$. Therefore, also the expected profit in the latter phase has to be taken into account, when deriving the training rate. The expected present value of profit generated by an apprentice is:[23]

$$\begin{aligned}\pi_{S,A,t} = -\epsilon_{\eta_{S,T},t} + (a_v - w_v - k_v + \sigma^{\eta_{S,T}}) \sum_{j=0}^{p-1} \delta^j (1-\rho-\theta)^j \\ + \delta^p\ (1-\rho-\theta)^p\ \eta^1_{T,N_m}\ E(\pi_{T,N_m})\end{aligned} \tag{19}$$

Given the firing costs $\chi_{S,T}$, a school leaver is hired as apprentice if $\pi_{S,T,t} > \chi_{S,T}$. By solving eq. (19) for $\epsilon_{\eta_{S,T},t}$, we get:

$$\begin{aligned}\epsilon_{\eta_{S,T},t} < -\chi_{S,T} + (a_v - w_v - k_v + \sigma^{\eta_{S,T}}) \sum_{i=0}^{p-1} \delta^i (1-\rho-\theta)^i \\ + \delta^p\ (1-\rho-\theta)^p\ \eta^1_{T,N_m}\ E(\pi_{T,N_m})\end{aligned} \tag{20}$$

[23] Note that $E(\pi_{T,N_m})$ is given by equation (15) with $-\ \epsilon_{\eta_{T,N_m}} = 0$.

We assume that $\epsilon_{\eta_{T,S},t}$ is uniformly distributed between $\epsilon^-_{\eta_{S,T}}$ and $\epsilon^+_{\eta_{S,T}}$. The corresponding hiring rate is:

$$\eta_{S,T} = \qquad (21)$$

$$\frac{[-\chi_{S,T} + (a_v - w_v - k_v + \sigma^{k_v}) \sum_{i=0}^{p-1} \delta^i (1-\rho-\theta)^i + \delta^p \ (1-\rho-\theta)^p \ \eta^1_{T,N_m} \ E(\pi_{T,N_m})] - \epsilon^-_{\eta_{S,T}}}{\epsilon^+_{\eta_{S,T}} - \epsilon^-_{\eta_{S,T}}}$$

Whereas training subsidies only have a direct effect – they increase the training rate $\eta_{S,T}$ –, hiring subsidies have a direct and an indirect effect. Primarily, hiring subsidies aim at easing the transition from training to work, i.e. they increase the hiring rates $\eta^1_{T,N_m}$ and $\eta^2_{T,N_m}$ according to eq. (17). Moreover the hiring subsidies given that have affect $\eta^1_{T,N_m}$, also have an indirect effect as they increase the training rate according to eq. (21). The decision to hire an apprentice also depends on the probability that the person continues working in the firm as medium-skilled. The higher the hiring rate, $\eta^1_{T,N_m}$, the higher is this probability and the higher is the incentive to hire a person as apprentice. Thus, hiring subsidies does not only increase the fraction of apprentices who are hired as medium-skilled employees, they also increase the number of apprentices.

### 3.6 Labor Market Equilibrium

The following system of equations constitutes the equilibrium of the model:

- the six labor market dynamic equations given by the transition matrix (3)
- the government budget constraint, equation (4),[24] and finally
- the equations determining the hiring rates, equations (17), (21) and (18).

Now, we continue by presenting an analytical solution of a simplified version of the model.

## 4 A simple analytical evaluation

In the following, we present a simple model, which allows for an analytical solution. The simplified model, is then – after the calibration in the following section – used as a starting point for the numerical analysis. In order to quantify the different effects, we then implement more and more component so that we finally get the full model

### 4.1 The simplified model

In order to get an analytically traceable solution, we make the following simplifications: ($i$) there are no taxes:[25] $t_i = 0$ with $i = l, m, h$, ($ii$) training takes one period: $p = 1$, ($iii$) training is not broken off: $\theta = 0$ and ($iv$) the agents are risk neutral: $\gamma = 0$. With respect to the high-skilled labor force, all variables are treated as constant. With respect to the medium- and low-skilled labor force,

[24] As the equation describing the government budget constraint contains three tax rates, it is necessary to introduce two additional equations in order to close the model. They describe the ratios between the tax rates, $t_h$, $t_m$ and $t_l$.

[25] In the presence of taxes, a purely analytically examination is not possible. Therefore we have done the analysis with a different, 2-period-model, which is presented in Appendix A3.

the labor market states can be expressed as functions of the transition rates, i.e. in particular as functions of the hiring rates (eq. (17) for $\eta_{T,N_m} = \eta^1_{T,N_m}$, (18) and (21)) and thereby as a function of the hiring and training subsidies. The corresponding steady state expressions can be derived on the base of the Markov Matrix.

According to Coe and Snower (1997) "policies are complementary in the sense that the effect of each policy is greater when implemented in conjunction with the other policy than in isolation". In the following we calculate complementarities with respect to aggregate income. Aggregate income, $\Phi$, is calculated as the income aggregated over all states:[26]

$$\Phi = \sum_{i=m,l,h}[N_i\ w_i(1-t_i) + U_i\ \beta_i\ w_i(1-t_i)] + T\ w_v \tag{22}$$

To check the presence of complementarities, we calculate the cross derivative of $\Phi$ with respect to the two subsidies, $\sigma^{\eta_{S,T}}$ and $\sigma^{\eta_{T,N_m}}$:[27]

$$\frac{\partial^2 \Phi}{\partial \sigma^{\eta_{S,T}}\ \sigma^{\eta_{T,N_m}}} = [\ \underbrace{a_m(1-\beta_m)\ \Sigma_\epsilon\ \mu_m\ (1-\rho)\ \rho^2\ (\eta_l + \eta_{N,T}(1-\eta_l) + \rho + \varphi_l)}_{<0}\ ]\ / \tag{23}$$
$$[\ \underbrace{\Pi\epsilon\ (1-\beta_m(1-\mu_m) - c_\phi \mu_m)(1+\rho)}_{<0}$$
$$\underbrace{(\eta_l\ (1-\eta_{U,T})(\eta_{N,T}+\rho) + (\eta_{U,T}+\rho)(\eta_{N,T}+\rho+\varphi_l))}_{>0}\ \underbrace{(\eta_m+\rho+\varphi_m)}_{>0}\ ]$$

with:
$\Sigma_\epsilon = (\epsilon^-_{\eta^1_{T,N_m}} - \epsilon^+_{\eta^1_{T,N_m}} + \epsilon^-_{\eta^2_{T,N_m}} - \epsilon^+_{\eta^2_{T,N_m}}) < 0$ and
$\Pi_\epsilon = (\epsilon^-_{\eta^1_{T,N_m}} - \epsilon^+_{\eta^1_{T,N_m}})(\epsilon^-_{\eta^2_{T,N_m}} - \epsilon^+_{\eta^2_{T,N_m}})(\epsilon^-_{\eta_{S,T}} - \epsilon^+_{\eta_{S,T}}) < 0.$

The cross derivative is unambiguously positive. Hence, the result reveals – at least for a very simple version of the model – the existence of complementarities with respect to aggregate income in the sense that the impact of one subsidy one aggregate income is bigger in the presence of the other subsidy.

The intuition behind this is as follows: hiring subsidies facilitate the transition from vocational training to work. They increase the probability that an apprentice continues working in the firm as a medium-skilled employee after having finished vocational training successfully. Thereby, hiring subsidies improve the effectiveness of training policies, since the higher probability will amplify the positive impact of training subsidies. Moreover, hiring subsidies indirectly increase the number of people being hired as apprentices. This broadens the target group for training subsidies.

## 4.2 Robustness Checks

Now, we discuss the impact of different parameter values on the size of complementarity. In this context, we calculate the derivative of the cross derivative, $\frac{\partial^2 \Phi}{\partial \sigma^{\eta_{S,T}}\ \sigma^{\eta_{T,N_m}}}$, with respect to the parameters of interest. In particular, we are interested in the effect of the elasticity of the hiring rate and the training rate, respectively. Moreover, we analyze the impact of two parameters which are crucial for the wage determination: the bargaining power of the medium-skilled, $\mu_m$, and

[26] The income in the labor market state $S$ is 0.
[27] See Appendix A4 for the derivation.

the degree of risk aversion, $\gamma$. We start with the analysis of the elasticity of the hiring rate, $ela(\eta_{T,N_m}) = (\partial\eta_{T,N_m}/\eta_{T,N_m})/(\partial w_m/w_m)$.[28]

### 4.2.1 The hiring elasticity

When analyzing the impact of the hiring elasticity, it has to be taken into account, that the parameter does not directly enter the eq. (17) which determines the hiring rate. Rather the hiring elasticity is implicitly fixed by the choice of the distribution limits $\epsilon^-_{\eta^q_{T,N_m}}$ and $\epsilon^+_{\eta^q_{T,N_m}}$ (for $q = 1, 2$), i.e. a higher $\epsilon^-_{\eta^q_{T,N_m}}$ implies a higher hiring elasticity. As an example, we calculate the effect of $\epsilon^-_{\eta^q_{T,N_m}}$ on the size of the complementarity which is illustrated by $\frac{\partial^2\Phi}{\partial\sigma^{\eta_{S,T}}\ \sigma^{\eta_{T,N_m}}}$.[29] We get the following expression:

$$\partial(\frac{\partial^2\Phi}{\partial\sigma^{\eta_{S,T}}\ \sigma^{\eta_{T,N_m}}})/\partial\epsilon^-_{\eta^q_{T,N_m}} = \frac{\partial^2\Phi}{\partial\sigma^{\eta_{S,T}}\ \sigma^{\eta_{T,N_m}}}\frac{-1}{\Sigma_\epsilon}\frac{\epsilon^-_{\eta^{2/q}_{T,N_m}} - \epsilon^+_{\eta^{2/q}_{T,N_m}}}{\epsilon^-_{\eta^q_{T,N_m}} - \epsilon^+_{\eta^q_{T,N_m}}} > 0 \quad \text{with } q = 1, 2 \tag{24}$$

The impact of a higher $\epsilon^-_{\eta^q_{T,N_m}}$ on the size of complementarity is unambiguously positive. Hence, a higher wage elasticity of the hiring rate amplifies the size of complementarity for a given level of the subsidy.

### 4.2.2 The training elasticity

Also the wage elasticity of the training rate, $ela(\eta_{S,T}) = (\partial\eta_{S,t}/\eta_{S,t})/(\partial w_v/w_v)$,[30] does not directly enter the eq. (21) which determines the training rate. Therefore, similar to the calculation above, we analyze the impact the distribution parameter $\epsilon^-_{\eta_{S,T}}$ on the size of complementarity. We obtain:

$$\partial(\frac{\partial^2\Phi}{\partial\sigma^{\eta_{S,T}}\ \sigma^{\eta_{T,N_m}}})/\partial\epsilon^-_{\eta_{S,T}} = \frac{\partial^2\Phi}{\partial\sigma^{\eta_{S,T}}\ \sigma^{\eta_{T,N_m}}}\frac{-1}{\epsilon^-_{\eta_{S,T}} - \epsilon^+_{\eta_{S,T}}} > 0 \tag{25}$$

Again, the impact of a higher $\epsilon^-_{\eta_{S,T}}$ on the size of complementarity is unambiguously positive. Hence, a higher wage elasticity of the training rate amplifies the size of complementarity for a given level of the subsidy.[31]

### 4.2.3 The bargaining power

Finally, we analyze the impact of the bargaining power of the medium-skilled, $\mu_m$. Again, according to eq. (23), the parameter, has no direct impact on the size of complementarity. Rather it affects the size of complementarity via the wage, $w_m$, directly but also indirectly as the hiring rates $\eta_{N,T}$, $\eta_{U,A}$ and $\eta_m$ also depend on the wage.

$$\partial(\frac{\partial^2\Phi}{\partial\sigma^{\eta_{S,T}}\ \sigma^{\eta_{T,N_m}}})/\partial\mu_m = \frac{\partial^2\Phi}{\partial\sigma^{\eta_{S,T}}\ \sigma^{\eta_{T,N_m}}}\frac{-(-1+\beta_m)\ \mu_m}{(1-\beta_m(1-\mu_m)-c_\phi\mu_m)} > 0 \tag{26}$$

[28] In the remainder, we assume that $\partial w_m$ is given by the subsidy level.

[29] An increase of the upper limit $\epsilon^+_{\eta^q_{A,N_m}}$ has the opposite impact.

[30] In the remainder, we assume that $\partial w_v$ is given by the subsidy level.

[31] An increase of the upper limit $\epsilon^+_{\eta_{S,T}}$ has the opposite impact.

A higher bargaining power of the medium-skilled has a positive effect on the size of the complementarity. In particular, a higher bargaining power increases the wage of the medium-skilled labor force. Hence, the positive effect of the training subsidies on medium-skilled employment is associated with a higher weight when calculating aggregate income. So, the size of the complementarity is increased.

# 5 Calibration

The impact of the subsidies and thereby the credibility of the ex ante policy evaluation heavily depends on the validity of the chosen parameter values. To check the validity, a possible "diagnostic is to test the restrictions the model places on the data" (Wolpin, 2007). Therefore in the remainder, we not only illustrate the calibration, we also make some cross checks in order to test, whether parameter values which can be derived endogenously correspond to the values given by the literature.

With respect to the initial equilibrium (i.e. in the absence of subsidies), we treat some variables as exogenous. However, in the presence of subsidies, these variables are treated as endogenous. One example is the training rate $\eta_{T,N_m}$. With respect to the initial steady state, it is treated as an exogenous variable so that we can derive the limits of the corresponding distribution for the random operating costs. However, in the presence of subsidies, we treat the limits as given and the training rate is a function of the subsidies. The tables (2) and (3) contain a survey of all variables and parameters which are set exogenously in the initial equilibrium.

We calibrate the model for Germany with a year as unit of time. The annual interest rate, $i$, is set at 4.0%,[32] which leads to a discount rate of $\delta = 0.962$. For simplicity, we set the coefficient of relative risk aversion $\gamma = 0$. The death rate is set to 0.023 which corresponds to an average working life time of about 44 years.[33] The number of periods, $p$, a person is engaged in vocational training is set to 3.[34] The break off rate $\theta$, is set to 0.037. The value for the death rate and the break off rate imply, that roughly 17% of the apprentices do not finish vocational training successfully, which is in line with the empirical data.[35] According to Wilke´s (2005) Kaplan-Meier functions for Germany, the hiring rates for the high-skilled, medium-skilled and the low-skilled are set at $\eta_h = 0.55$, $\eta_m = 0.59$ and $\eta_l = 0.49$. Moreover, according to the Bundesministerium für Bildung und Forschung (2004, 2007) about 25% of the people who have successfully finished vocational training become unemployed, thus the corresponding hiring rate, $\eta_{T,N_m}$ is set at 0.75. However, about 53% of all successful apprentices stay in the firm in which they have been trained ($\eta^1_{A,N_m} = 0.4$),[36] this part of the hiring rate can be regarded as take over rate. The residual is hired by another firm ($\eta^2_{A,N_m} = 0.35$).

The labor market states are defined and quantified as follows. The low-skilled labor force ($N_l + U_l$) includes people with an educational attainment corresponding to less than upper-secondary

[32] This is the average real interest rate over the whole period, calculated as the difference between the nominal interest rate of long term government bonds and the inflation rate. Nominal values are transformed to real values by using the consumption deflator.

[33] The value is in line with the empirical data. Working life begins at the age of 17 and according to Brussig and Wojtkowski (2006), retirement takes places between 62 and 63.

[34] This corresponds to the typical length of vocational training within the dual system in Germany.

[35] Given the data for the number of new apprentices and the number of successful apprentices (Statistisches Bundesamt, 2006), the value is confirmed.

[36] Bundesministerium für Bildung und Forschung (2004, 2007).

education.[37] The medium-skilled labor force ($N_m + U_m$) contains all people with vocational upper secondary education. People with post-secondary and tertiary education are considered as being high-skilled ($N_h + U_h$). The corresponding value can be calculated on the basis of data from the OECD (1999 to 2005).[38]

Next, we calibrate the productivities and the wages. Based on data from the German national accounts, the aggregate productivity, $a$, is set to $54,243$ EUR and the aggregate producer wage, $w$, which is calculated as average gross wage per employee plus social security payments, is set equal to $32,520$ EUR.[39] In order to get the wages for different skill groups, OECD indices for the relative earnings of the population with income from employment for different skill groups are used, they yield the following ratios: $w_h/w = 1.27$, $w_m/w = 0.92$ and $w_l/w = 0.72$.[40] According to Beicht and Walden (2002), the wage of a person being engaged in vocational training is set to $w_v = 8269$ EUR and the productivity of an apprentice is set to $a_v = 7,730$ EUR. Moreover, according to Beicht and Walden (2002) the annual (non-wage) costs of an apprentice are $8,166$ EUR. As 30% (Dohmen and Hoi, 2004) are tax-deductible, the relevant costs are set to $k_v = 5,716$.

The net replacement rates $\beta_i$ are set to 78.25% for the low-skilled unemployed, to 68.25% for the medium-skilled unemployed, and to 64.67% of high-skilled unemployed.[41] According to Chen and Funke (2005), we set the hiring costs to 10 percent of the wage and the firing costs are set to 60 percent of the wage, thus the corresponding parameters are $c_\eta = 0.1$ and $c_\varphi = 0.6$.

In order to calculate the tax rates, we use the income tax scale of the year 2002 described in Boss and Elendner (2003) which illustrates the progressive character of the German tax system to calculate the following ratios: $t_h/t_l = 1.437$ and $t_m/t_l = 1.178$. Then, we can calculate the tax rate. Based on data for transition rates between the education and training system on the one hand and the labor force on the other hand,[42] we calculate (*i*) the training rate: $\eta_{S,T} = 0.70$ and (*ii*) the ratio: $\eta_{U,T}/\eta_{N,T} = 3.6$. Based on these values and the given equations, we can calculate the value of the bargaining power, the productivities for the different skill levels, the tax rates and the missing transition rates for the initial steady state.

Finally, we have to determine the parameters with respect to the uniform distribution of the random operating costs $\epsilon$. For the training rate and the hiring rates, respectively, the lower limits have to be set: $\epsilon^-_{\eta_{S,T}}$ concerning the training rate, $\eta_{S,T}$, and $\epsilon^-_{\eta^q_{T,N_m}}$ concerning the hiring rates, $\eta^q_{T,N_m}$ for $q = 1, 2$. When setting the corresponding values, we take into account the implications for the wage elasticities of the transition rates. For the sake of simplicity we set $\epsilon^-_{\eta^q_{T,N_m}} = 0$ and $\epsilon^-_{\eta_{S,T}} = 0$. The upper limits are not set exogenously, rather they are the implied results of a full calibration. As all the other variables and parameters of the equations determining the hiring rates and the training rate, respectively, as well as the training rate and the hiring rates themselves are given for the initial steady state, one can easily solve the equations determining the hiring rates, eq. (21) for $\epsilon^+_{\eta_{S,T}}$, eq. (17) for $\epsilon^+_{\eta^1_{T,N_m}}$ and eq. (18) for $\epsilon^+_{\eta^2_{T,N_m}}$.

To check, whether the chosen values for the low limits are appropriate, we calculate the wage

[37] This corresponds to the conventional definition which classifies people with an educational attainent corresponding to at most level 2 of the International Standard Classification of Education (ISCED) as low-skilled.

[38] See Appendix A5.

[39] Statistisches Bundesamt (2008), average of annual data for the period 2000 - 2007.

[40] See OECD (1999-2005). These values imply a ratio $w_m/w_l = 1.26$ which is in line with the corresponding data reported by Wienert (2006).

[41] The values are net replacement rates (unweighted average across six family types) of workers with 67, 100 and 150 percent of average productivity. See OECD (2006).

[42] See Reinberg and Hummel (2006).

| variable / parameter | description | value | source |
|---|---|---|---|
| $\delta$ | discount rate | 0.962 | Standard value |
| $\gamma$ | CRRA | 0 | Assumption |
| $p$ | periods of vocational training | 3 | Standard value for Germany |
| $\rho$ | death rate | 0.023 | To match the working life |
| $\theta$ | break off rate of training | 0.037 | To match the empirical data |
| $\eta_h$ | hiring rate (high-skilled) | 0.55 | |
| $\eta_m$ | hiring rate (medium-skilled) | 0.59 | Wilke (2005) |
| $\eta_l$ | hiring rate (low-skilled) | 0.49 | |
| $\eta^1_{T,N_m}$ | take over rate | 0.4 | BMBF (2004, 2007) |
| $\eta^2_{T,N_m}$ | hiring rate after training | 0.35 | To match the data |
| $\eta_{S,T}$ | training rate | 0.70 | Own calculations based on |
| $\eta_{U,T}\ /\eta_{N,T}$ | ratio of transition rates | 3.6 | Reinberg and Hummel (2006) |
| $a$ | aggregate productivity | $54,243$ | Statistisches |
| $w$ | aggregate producer wage | $32,520$ | Bundesamt (2008) |
| $w_h/w$ | relative earning (high-skilled) | 1.27 | |
| $w_m/w$ | relative earning (medium-skilled) | 0.92 | OECD (1999-2005) |
| $w_l/w$ | relative earning (low-skilled) | 0.72 | |

Table 2: Exogenous values.

| variable / parameter | description | value | source |
|---|---|---|---|
| $a_v$ | productivity of an apprentice | $7,730$ | Beicht and Walden (2002) |
| $w_v$ | wage of an apprentice | $8,269$ | |
| $k_v$ | training costs for firm | $5,716$ | Beicht and Walden (2002), Dohmen and Hoi (2004) |
| $\beta_h$ | net repl. rate (high-skilled) | 0.6467 | |
| $\beta_m$ | net repl. rate (medium-skilled) | 0.6825 | OECD (2006) |
| $\beta_l$ | net replacement rate(low-skilled) | 0.7825 | |
| $c_\eta$ | hiring cost in relation to the wage | 0.1 | Chen and Funke (2005) |
| $c_\varphi$ | firing cost in relation to the wage | 0.6 | |
| $t_h\ /\ t_l$ | tax rate ratio (1) | 1.437 | Boss and Elendner (2003) |
| $t_m\ /\ t_l$ | tax rate ratio (2) | 1.178 | |
| $\epsilon^-_{\eta_{S,T}}$ | lower limit of the distribution | 0 | Assumption |
| $\epsilon^-_{\eta^1_{T,N_m}}$ | lower limit of the distribution | 0 | Assumption |
| $\epsilon^-_{\eta^2_{T,N_m}}$ | lower limit of the distribution | 0 | Assumption |

Table 3: Exogenous values.

elasticities and get: $ela(\eta^1_{T,N_m}) = -1.71$, $ela(\eta^2_{T,N_m}) = -1.76$ and $ela(\eta_{S,T}) = -0.81$. Given the empirical estimates, as summarized in Orszag and Snower (1999b) hiring elasticities range from $-0.5$ to $-4.0$. However, these values refer to permanent change of the wages. The elasticities with respect to the short-term subsidies are significantly smaller.[43] Thus, the elasticities are in line with the literature.

All derived values are summarized in table (4).[44]

| variable / parameter | variable | basis of calculation | value |
|---|---|---|---|
| low-skilled unemployment | $U_l$ | | 0.034 |
| low-skilled employment | $N_l$ | | 0.150 |
| medium-skilled unemployment | $U_m$ | see Appendix A5 | 0.067 |
| medium-skilled employment | $N_m$ | | 0.668 |
| people in vocational training | $T$ | | 0.057 |
| people in school | $S$ | | 0.024 |
| bargaining power | $\mu$ | eq.(14) for $i = l, m, h$ | 0.205 |
| productivity (high-skilled) | $a_h$ | and $a =$ | 73,005 |
| productivity (medium-skilled) | $a_m$ | $\frac{a_l N_l + a_m N_m + a_h N_h + a_v T}{N_l + N_m + N_h + T}$ | 48,751 |
| productivity (low-skilled) | $a_l$ | | 29,352 |
| tax rate (high skilled) | $t_h$ | ratios of tax rates | 0.070 |
| tax rate (medium-skilled) | $t_m$ | and eq.(4) | 0.057 |
| tax rate (low-skilled) | $t_l$ | with: $\sigma = 0$ | 0.049 |
| firing rate (low-skilled) | $\varphi_l$ | equations (3) and (4) of | 0.096 |
| firing rate (medium-skilled) | $\varphi_m$ | Matrix $MT$ and the | 0.055 |
| training rate (low-skilled unempl.) | $\eta_{U,T}$ | ratios of the transition rates | 0.041 |
| training rate rate (low-skilled empl.) | $\eta_{N,T}$ | | 0.012 |
| upper limit of the distribution | $\epsilon^+_{\eta_{S,T}}$ | corresponding hiring rate | 41,132 |
| upper limit of the distribution | $\epsilon^+_{\eta^1_{T,N_m}}$ | and corresponding equation | 399,625 |
| upper limit of the distribution | $\epsilon^+_{\eta^2_{T,N_m}}$ | for hiring elasticity | 442,177 |

Table 4: Derived parameter values for the initial steady state.

# 6 Numerical Analysis

In the following calculations, we assume that for each subsidy either 0 or 5,000 EUR can be spent. As the hiring subsidy is paid only for one period the possible amounts are therefore also 0 or 5,000 EUR. In contrast, the training subsidy is paid over the whole period of vocational training, therefore the annual payment is supposed to be 1,771 EUR which implies – for the full model – a total amount of 5,000 EUR.[45] The subsidies are paid per person. We then calculate the

[43] According to Snower (1996), the elasticities with respect to hiring subsidies are 8-10 times smaller than the elasticities with respect to permanent wage subsidies.

[44] Given all these values, it can be checked easily, that the firms have an incentive to engage in vocational training as it was assumed in section 3 (see Appendix A1 for a detailed calculation).

[45] Recall, that an apprentice continues training with a probability of $(1-\rho-\theta)$. Given the values for $\rho$ and $\theta$ and taking into account that training takes 3 periods, we get: $1,771 = 5,000/(\sum_{c=1}^{3}(1-\rho-\theta)^{c-1})$. The hiring subsidy corresponds to 17% of the wage $w_m$ and the training subsidy corresponds to 24% of the wage $w_v$.

impact on aggregate income for three alternatives: either only one subsidy is implemented or both subsidies are implemented simultaneously. The size of complementarity is calculated as follows: First, we calculate the percentage increase in aggregate income for the two cases, in which only one subsidy is implemented, and then calculate the sum. Second, we compute the percentage increase in aggregate income given that both subsidies are implemented simultaneously. Third, we calculate by how much (in percent) the increase in aggregate income given that both subsidies are implemented simultaneously is higher than the sum of the separate effects. Moreover, we conduct a similar calculation with respect to employment.

Now, we start by analyzing the interactions of the hiring and training subsidy for the simplified model, presented in section 4.1. Then, we gradually increase the complexity of the model. The approach allows us to to analyze the effects of different components of the model. Moreover, we respect to the full model, we also analyze the interactions of the subsidies with respect to welfare. Finally, we do some robustness checks.

## 6.1 Models with gradually increasing complexity

### 6.1.1 Simplified Model

First, we calculate the numerical solution of the simplified model presented above, i.e. we assume, that (*i*) there are no taxes, (*ii*) training is not broken off ($\theta = 0$), (*iii*) the duration of training is one period ($p = 1$) and the individuals are risk neutral ($\gamma = 0$). For this case, the impact of the subsidies on aggregate income and employment is shown in table (5).[46] The analytical solution

| policy<br><br>increase of ... in % | $\sigma^{\eta_{S,T}} = 1,771$<br>$\sigma^{\eta_{T,N_m}} = 0$ | $\sigma^{\eta_{S,T}} = 0$<br>$\sigma^{\eta_{T,N_m}} = 5,000$ | $\sigma^{\eta_{S,T}} = 1,771$<br>$\sigma^{\eta_{T,N_m}} = 5,000$ | size of<br>complementarity<br>in % |
|---|---|---|---|---|
| income | 0.243988 | 0.554453 | 1.0453 | 0.179488 |
| employment | 0.171391 | 0.548224 | 0.724719 | 0.70928 |

Table 5: The size of complementarity in the benchmark case.

is confirmed by the numerical result. With respect to aggregate income, the two subsidies are complementary, however, the complementarity is quite weak. With respect to employment, the size of complementarity is significantly higher.

To explain the result, we first analyze the effect of the three policies on the income of the employed only. For every policy, the increase in income is higher than the increase in employment. The result is not surprising as the employment group which is increasing (the medium-skilled) has the higher weight (i.e. income) when calculating aggregate income. However, the size of complementarity with respect to income is lower compared to the size of complementarity with respect to employment. Recall that the size of complementarity is the relative difference between the effect of a simultaneous implementation and the sum of the separate effects in only caused by the differences in employment. When calculating the complementarity with respect to income, the difference has a relatively low importance.

[46] With respect to the complete model $\sigma^{\eta_{S,T}} = 1,771$ EUR implies that the total amount of training subsidies corresponds to the total amout of hiring subsidies. For $\theta = 0$ and $p = 1$, this is no longer the case. Setting $\sigma^{\eta_{S,T}} = \sigma^{\eta_{T,N_m}} = 5,000$ EUR would increase the size of complementarity to 1.4% with respect to employment and to 0.3% with respect to aggregate income

Second, we analyze the impact of the three policies on the income of the unemployed. With respect to unemployment, all three policies have a negative effect and hence, they also have a negative effect on the aggregate income of the unemployed. Moreover, the size of complementarity (in absolute terms) is now negative.

Finally, both factors, the lower complementarity with respect to the income of the employed and the negative complementarity with respect to the income of the unemployed explain, why the complementarity with respect to aggregate income is lower than the complementarity with respect to employment.

### 6.1.2 Model with realistic calibration of the training phase

Now, we still assume the absence of taxes. However, with respect to vocational training, we adjust the values to realistic ones ($\theta = 0.037$ and $p = 3$). The impact of the subsidies on aggregate income and employment is shown in table (6). As in the previous simulation, there are complementarities

| policy<br><br>increase of ... in % | $\sigma^{\eta_{S,T}} = 1,771$<br>$\sigma^{\eta_{T,N_m}} = 0$ | $\sigma^{\eta_{S,T}} = 0$<br>$\sigma^{\eta_{T,N_m}} = 5,000$ | $\sigma^{\eta_{S,T}} = 1,771$<br>$\sigma^{\eta_{T,N_m}} = 5,000$ | size of<br>complementarity<br>in % |
|---|---|---|---|---|
| income | 0.788739 | 0.500777 | 1.2981 | 0.665327 |
| employment | 0.549082 | 0.388578 | 0.947042 | 1.00055 |

Table 6: The size of complementarity for a realistic calibration of the training phase.

with respect to employment and aggregate income. Moreover, the complementarity with respect to employment is again lower than the complementarity with respect to aggregate income. However, now, the complementarities are significantly higher than in the previous simulation. The result is caused by another remarkable result: Now, as the training takes $p = 3$ periods and training subsidies are paid in every period of training, the amount of subsidies is now higher than in the simplified model (with $p = 1$). The total amount of training subsidies corresponds to the amount of hiring subsidies. Consequently, the effect of training subsidies on employment and aggregate income is significantly higher than in the previous simulation. The higher the impact of the training subsidy is, the higher is also the target group for hiring subsidies. Finally, the size of complementarity is higher than in the previous simulation.

The critical reader might argue that the fraction of apprentices which breaks off vocational training, $\theta$, is not exogenous but has to be treated as a function of the hiring subsidy, $\sigma^{\eta_{T,N_m}}$, i.e. hiring subsidies do not only influence the demand for successful apprentices but via $\theta$ they also influence the supply of apprentices. The reasoning would be as follows: a subsidy rate increases the hiring rate $\eta_{T,N_m}$ and thus the probability of the apprentice to get a medium-skilled job. The higher probability of getting a high, medium-skilled income could be expected to reduce the incentive to break off training. Therefore $\theta$ should be treated as an endogenous variable with $\frac{\partial\theta}{\partial\sigma_{\eta T,N_m}} < 0$. However, studies trying to identify reason for breaking off training do not give any argument to think that financial incentives would reduce $\theta$.[47] One main reason to break off training is a problem in the relationship between apprentice and the instructor in the firm. Many apprentices breaking off training do not intend to finally stop training but they try to get a training position somewhere else. Thus, for the aggregate, the financial incentive cannot be supposed to play a role.

[47] See BMBF (2003) for a survey a study on reasons to breaking off training.

### 6.1.3 Model with taxes

In contrast to the analysis so far, we now implement taxes. For the moment, we assume, that only the expenditures for unemployment benefits have to be financed via taxes. According to eq. (4), the tax rates are set in a way which ensures that the tax receipts of the government are equal to its expenditures for unemployment benefits. However, with respect to the government budget constraint, we ignore the presence of subsidies, they are assumed to be 0 in eq. (4). The impact of the subsidies on aggregate income and employment is shown in table (7). Again, there

| policy<br><br>increase of ... in % | $\sigma^{\eta_{S,T}} = 1,771$<br>$\sigma^{\eta_{T,N_m}} = 0$ | $\sigma^{\eta_{S,T}} = 0$<br>$\sigma^{\eta_{T,N_m}} = 5,000$ | $\sigma^{\eta_{S,T}} = 1,771$<br>$\sigma^{\eta_{T,N_m}} = 5,000$ | size of complementarity in % |
|---|---|---|---|---|
| income | 1.1395 | 0.750095 | 1.90426 | 0.77612 |
| employment | 0.549082 | 0.388578 | 0.947042 | 1.00055 |

Table 7: The size of complementarity in the presence of taxes.

are complementarities and the complementarities with respect to employment are higher than the complementarities with respect to aggregate income. With respect to employment, the results are absolutely equal to the results of the previous simulation. The introduction of taxes does not affect the producer wages according to eq. (14) and thereby the hiring rates, $\eta^1_{T,N_m}$ and $\eta^2_{T,N_m}$, as well as the training rate, $\eta_{S,T}$. Hence, the introduction of taxes has also no effect on employment. With respect to aggregate income, the effect of all three policies is higher than in the previous simulation. The result is not surprising, given that now, unemployment benefits have to be financed via taxes. The implementation of subsidies increases aggregate employment and reduces aggregate unemployment, thereby the number of people getting wages increases and the number of people requiring the lower unemployment benefits decreases, which has a positive impact on aggregate income as in the previous simulation. However, compared to the previous simulation, we now have an additional positive effect on aggregate income. By leading to a fall in the number of people requiring unemployment benefits, subsidies generate a revenue for the government. Consequently, – compared to the situation without subsidies – the tax rates can be reduced, so the net wages and thereby aggregate income increase.[48]

Finally, also the size of complementarity with respect to aggregate income is higher. Again, the higher the impact of the training subsidy is, the higher is also the target group for hiring subsidies, which has a positive impact on the size of complementarity.

### 6.1.4 Model with a government budget constraint

In the next step, we analyze the impact of the government budget constraint. In contrast to the previous simulation, also subsidies have to be financed by taxes. According to eq. (4), the tax rates are now set in a way which ensures that the tax receipts of the government are equal to its total expenditures, i.e. the sum of unemployment benefits and subsidies. The impact of the subsidies on aggregate income and employment is shown in table (8). With respect to employment, the result is absolutely equal to the previous result for the same reason as before. But with respect to aggregate income, now, the effects of the three different policies are smaller than in the previous

[48]In the absence of taxes, the initial aggregate income is obviously higher as the tax burden is 0. However, by construction, subsidies are not able to reduce the tax burden, hence, the increase of aggregate income is lower.

| policy<br><br>increase of ... in % | $\sigma^{\eta_{S,T}} = 1,771$<br>$\sigma^{\eta_{T,N_m}} = 0$ | $\sigma^{\eta_{S,T}} = 0$<br>$\sigma^{\eta_{T,N_m}} = 5,000$ | $\sigma^{\eta_{S,T}} = 1,771$<br>$\sigma^{\eta_{T,N_m}} = 5,000$ | size of<br>complementarity<br>in % |
|---|---|---|---|---|
| income | 0.890373 | 0.580106 | 1.44726 | -1.57871 |
| employment | 0.549082 | 0.388578 | 0.947042 | 1.00055 |

Table 8: The size of complementarity in the presence of the government budget constraint.

simulation. Moreover, there are no longer complementarities with respect to aggregate income. Compared to the previous analysis, we now see a second effect of the subsidies on the tax rates. In the previous analysis subsidies induced a reduction in the tax rates, as they reduced unemployment and thereby the number of unemployment benefits recipients. Now, this positive effect is reduced by the negative effect. In the presence of the government budget constraint, also the subsidies have to be financed by taxes. Here, the reduction in the expenditures for unemployment benefits is partly compensated by the additional expenditures for the subsidies. Consequently, the tax rates are higher and the effect of the three policies on aggregate income is lower now compared to the previous simulation. In the previous simulation, the policies also created a government deficit as the tax receipts only had to cover the expenditures for the unemployment benefits. If the aggregate income in the previous simulation would have been adjusted by the government deficit, the effect of the three policies on aggregate income would be the same as the simulation being based on a government budget constraint.

So far, we have compared the results of the model including a government budget constraint with the results of a model without a government budget constraint in the presence of subsidies. Besides, it is interesting to analyze the results of the model with government budget constraint in more detail. Although the tax rate in the presence of the government budget constraint are higher than in the previous simulation, the effect of the subsidies on aggregate income is still positive. The result indicates that the subsidies are self-financing, i.e. the subsidy-induced reduction of the unemployment benefits overcompensates the expenditures for the unemployment benefits. The result not only holds for the joint analysis of training and hiring subsidies, it is also confirmed for each subsidy separately. Both, the implementation of hiring and training subsidies, causes a reduction of the unemployment benefits which overcompensates the additional expenditures induced by the subsidy. Hence, the subsidy induce net revenues for the government, the tax rates can be reduced and aggregate income increases.

So far, we have analyzed the impact of the three policies on aggregate income. Now, we have to analyze the effect on complementarity in the presence of the government budget constraint. In contrast to the absence of the government budget constraint, now, a complementarity no longer exists. The result is caused by the non-linearity of the subsidy-induced revenues for the government (see Figure (2)). The higher the hiring subsidy is, the lower is the marginal contribution of a further increase of the training subsidy, moreover, the latter can also be negative. Thus, the subsidy-induced revenues in the case of a simultaneous implementation of both subsidies are lower than the sum of the subsidy-induced revenues in the case of a separate implementation of each subsidy. Hence, the increase of the aggregate income in the case of a simultaneous income of the subsidies is lower than the sum of the increases in the case of a separate implementation of each subsidy.

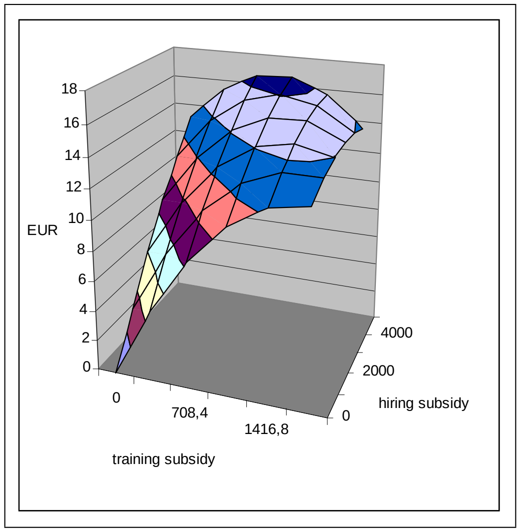


Figure 2: Subsidy-induced revenues for different annual subsidy levels.

#### 6.1.5 Model with risk aversion

Now, the coefficient of relative risk aversion (CRRA, $\gamma$) is set to 1.5.[49] The value is relatively low, but taking into account the whole calibration, it can be justified. The period of analysis and thus the minimum duration of unemployment in the model is one year. Thus, the risk of unemployment is much bigger than in the real world, there agents could leave unemployment before the end of a year. In reality they therefore have a higher possibility to smooth income. Taking this into account, calibrating the utility function with a relatively low degree of risk aversion is justified as it compensates the higher risk in the model.

Given the construction of the model, a modified risk aversion would also imply modified elasticities. Recall that (*i*) the productivities are endogenously determined by setting the wages as well as the degree of risk aversion and (*ii*) given the productivities and the lower limits of the distribution with respect to the random operating costs, the elasticities are determined. In order to eliminate a possible effect over the elasticities, we reset the lower limits of the distribution function so that the initial values for the elasticities are achieved. Then, we calculate the impact of a modified risk aversion. The result is nearly equal to the result for $\gamma = 0$.[50]

### 6.2 Welfare Analysis

The critical reader might argue that the aggregate income given by eq. (22) is not an appropriate measure of welfare as it is not based on utility. Therefore, we also calculate the impact of the subsidies on welfare ($\Omega$). In this context, we use the concept of "consumption equivalents".[51] We

[49] A value of 1.5 is located within the reasonable scope. According to Rodepeter (1999) and Dohmen et al. (2006), the limit values for CRRA are 1 and 5.

[50] Only the adjustment of the lower limits of the random operating cost distribution imply a small difference to the results of the previous simulations, as the hiring rates as well as the training rates are functions of the lower limits.

[51] This is a common concept in the literature. See e.g. Conesa and Krueger (1999).

quantify the welfare change of a given policy reform for each state by asking by how much an individual´s consumption has to be increased in the old steady state so that her expected present value of utility equals that under a specific policy reform. Thus, for the benchmark case, the utility function becomes:

$$u_{i,t}(C) = \frac{1}{1-\gamma}[C_{i,t}(1+\lambda)]^{1-\gamma} \text{ with } i = l, m, h \tag{4a}$$

Given this, eq. (22) changes as follows in order to calculate welfare:

$$\Omega = \textstyle\sum_{i=m,l,h}[N_i \ \frac{1}{1-\gamma}(w_i(1-t_i)(1+\lambda))^{1-\gamma} + U_i \ \frac{1}{1-\gamma}(\beta_i \ w_i(1-t_i)(1+\lambda))^{1-\gamma}] \tag{22a}$$
$$+ T \ \frac{1}{1-\gamma}(w_v(1+\lambda))^{1-\gamma}$$

Here, $\lambda$, is the consumption equivalent, e.g. $\lambda = 0.1$ implies that if a certain subsidy is implemented, an individual in the population considered will experience an increase in welfare due to the subsidies equivalent to receiving 10% higher consumption in the initial steady state (in all future models of her event tree).

The impact of the subsidies on the consumption equivalent and finally on welfare is shown in table (9) for different degrees of risk aversion.[52] Again, to avoid an additional effect via the elasticities, we adjust the lower limits of the random operating cost distributions to guarantee constant elasticities.

| policy<br><br>increase of $\lambda$ in % for | $\sigma^{\eta_{S,T}} = 1,771$<br>$\sigma^{\eta_{T,N_m}} = 0$ | $\sigma^{\eta_{S,T}} = 0$<br>$\sigma^{\eta_{T,N_m}} = 5,000$ | $\sigma^{\eta_{S,T}} = 1,771$<br>$\sigma^{\eta_{T,N_m}} = 5,000$ | size of<br>complementarity<br>in % |
|---|---|---|---|---|
| $\gamma = 0$ | 0.890373 | 0.580106 | 1.44726 | -1.57871 |
| $\gamma = 1.5$ | 0.822169 | 0.532426 | 1.33907 | -1.14618 |
| $\gamma = 2$ | 0.618108 | 0.405705 | 1.00649 | -1.69225 |

Table 9: The size of complementarity with respect to welfare in the presence of taxes.

The higher the degree of risk aversion, the lower is the impact of the subsidies. Subsidies increase the proportion of the medium-skilled workforce being connected with a higher wage. However, the higher the degree of risk aversion, the lower is the weight of this state when calculating welfare. Thus, a higher degree of risk aversion reduces the effect of the subsidies on welfare. However, for a modified degree of risk aversion, the effect of the training rate and the effect of the hiring rate are affected differently. This may explain, why the degree of complementarity does not decrease monotonously.

## 6.3 Robustness Checks

Finally, we do some robustness checks with respect to the hiring elasticities, the training elasticities and the bargaining power.

[52]Recall that the results for $\gamma = 0$ correspond to the results with respect to aggregate income in the previous simulation.

### 6.3.1 The hiring elasticities

We start by varying the elasticity of the hiring rate, $\eta^1_{T,Nm}$, which can also be regarded as take over rate. The results for the effect of the three different policies on aggregate income and employment are illustrated in table (10). A (in absolute terms) higher hiring elasticity, $ela(\eta^1_{T,Nm})$, increases

| $ela(\eta^1_{T,Nm})$ | increase of ... in % | $\sigma^{\eta_{S,T}} = 1,771$ $\sigma^{\eta_{T,Nm}} = 0$ | $\sigma^{\eta_{S,T}} = 0$ $\sigma^{\eta_{T,Nm}} = 5,000$ | $\sigma^{\eta_{S,T}} = 1,771$ $\sigma^{\eta_{T,Nm}} = 5,000$ | size of complementarity in % |
|---|---|---|---|---|---|
| $-1.71$ | | 0.890373 | 0.580106 | 1.44726 | -1.57871 |
| $-3.00$ | income | 0.890373 | 0.864779 | 1.72843 | -1.52223 |
| $-4.00$ | | 0.890373 | 1.08675 | 1.94768 | -1.4888 |
| $-1.71$ | | 0.549082 | 0.388578 | 0.947042 | 1.00055 |
| $-3.00$ | employment | 0.549082 | 0.539893 | 1.10204 | 1.19945 |
| $-4.00$ | | 0.549082 | 0.658152 | 1.22317 | 1.31974 |

Table 10: The effect of subsidies for different level of the hiring elasticity

the effect of the hiring subsidy on aggregate income and employment. Moreover, with respect to employment, the size of complementarity increases. With respect to aggregate income the negative extent of complementarity decreases.

We continue by varying the elasticity of the hiring rate $\eta^2_{T,Nm}$, which determines the hiring of successful apprentices by firms which do not conduct vocational training. The results for the effect of the three different policies on aggregate income and employment are illustrated in table (11). Again, a higher hiring elasticity, $ela(\eta^2_{T,Nm})$, increases the effect of the hiring subsidy on aggregate

| $ela(\eta^2_{T,Nm})$ | increase of ... in % | $\sigma^{\eta_{S,T}} = 1,771$ $\sigma^{\eta_{T,Nm}} = 0$ | $\sigma^{\eta_{S,T}} = 0$ $\sigma^{\eta_{T,Nm}} = 5,000$ | $\sigma^{\eta_{S,T}} = 1,771$ $\sigma^{\eta_{T,Nm}} = 5,000$ | size of complementarity in % |
|---|---|---|---|---|---|
| $-1.76$ | | 0.890373 | 0.580106 | 1.44726 | -1.57871 |
| $-3.00$ | income | 0.890373 | 0.593915 | 1.46225 | -1.48466 |
| $-4.00$ | | 0.890373 | 0.605091 | 1.47438 | -1.4098 |
| $-1.76$ | | 0.549082 | 0.388578 | 0.947042 | 1.00055 |
| $-3.00$ | employment | 0.549082 | 0.405092 | 0.964964 | 1.13088 |
| $-4.00$ | | 0.549082 | 0.418458 | 0.979471 | 1.23312 |

Table 11: The effect of subsidies for different level of the hiring elasticity

income and employment. Moreover, with respect to employment, the size of complementarity increases. With respect to aggregate income the negative extent of complementarity decreases. However, compared to a variation of the hiring elasticity, $ela(\eta^1_{T,Nm})$, now the result are smaller. The difference can be explained by regarding to eq. (21). The training rate is affected also by the hiring rate $\eta^1_{T,Nm}$, but not by the hiring rate $\eta^2_{T,Nm}$. Thus, a higher hiring elasticity, $ela(\eta^1_{T,Nm})$ has an additional positive effect on employment and aggregate income as it also increases the training rate. The additional effect does not occur if the hiring elasticity, $ela(\eta^2_{T,Nm})$ is increased.[53]

[53] The fact that the initial value for $\eta^1_{T,Nm}$ (0.4) is higher than the initial value for $\eta^2_{T,Nm}$ (0.35) does not explain the difference.

### 6.3.2 The training elasticity

We continue by varying the elasticity of the training rate $\eta_{S,T}$. The results for the effect of the three different policies on aggregate income and employment are illustrated in table (12). A higher

| policy <br> $ela(\eta_{S,T})$ | increase of <br> ... in % | $\sigma^{\eta_{S,T}} = 1,771$ <br> $\sigma^{\eta_{T,Nm}} = 0$ | $\sigma^{\eta_{S,T}} = 0$ <br> $\sigma^{\eta_{T,Nm}} = 5,000$ | $\sigma^{\eta_{S,T}} = 1,771$ <br> $\sigma^{\eta_{T,Nm}} = 5,000$ | size of complementarity in % |
|---|---|---|---|---|---|
| -0.81 | | 0.890373 | 0.580106 | 1.44726 | -1.57871 |
| -1.25 | income | 1.49564 | 0.961113 | 2.42959 | -1.10556 |
| -0.81 | | 0.549082 | 0.388578 | 0.947042 | 1.00055 |
| -1.25 | employment | 0.850609 | 0.57528 | 1.44486 | 1.33011 |

Table 12: The effect of subsidies for different level of the hiring elasticity

training elasticity, $ela(\eta_{S,T})$, increases the effects of the training subsidy on aggregate income and employment. Moreover, with respect to employment, the size of complementarity increases. Compared to the robustness checks with respect to the hiring elasticities, we get one remarkable result. In the previous robustness checks, the effect of a higher hiring elasticity only occurred in the presence of hiring subsidies but not in the absence of them. Now, the effect of a higher training elasticity occurs independent of the presence of training subsidies. Even in the absence of training subsidies, a higher training elasticity cause an additional increase of employment and aggregate income. The result can be explained by again taking into account that hiring subsidies also increase the training rate according to eq. (21).[54] Now, for a higher training elasticity, the effect of hiring subsidies on the training rate increases. Hence, the number of people in vocational training and finally the number of people getting hiring subsidies increase.[55] Finally, for a higher training elasticity, even the effect of the hiring subsidies on employment and aggregate income increases.

### 6.3.3 The bargaining power

Finally, we assume, that in contrast to the benchmark calibration, the bargaining power of the medium-skilled, $\mu_m$, is not longer equal to the bargaining power of the low-skilled, $\mu_l$. In the following, we assume that $\mu_m = 1.2\,\mu_l$.[56] Again, to avoid an additional effect via the elasticities, we adjust the lower limits of the random operating cost distributions to allow for constant elasticities. The results for the effect of the three different policies on aggregate income and employment are illustrated in table (13).

However, the results are very robust with respect to a change of the bargaining power.

[54] In the previous robustness checks, there is no such kind of interaction as the hiring rates $\eta^1_{T,Nm}$ and $\eta^2_{T,Nm}$ are not affected by training subsidies. As the explaining factors of the hiring rates are constant, the effect of a higher elasticity is 0.

[55] The hiring rates $\eta^1_{T,Nm}$ and $\eta^2_{T,Nm}$ are not affected.

[56] Given the assumption that $\mu_h = \mu_m$, the bargaining power of the high-skilled changes correspondingly. In contrast to the robustness checks with respect to the analytical evaluation, we cannot modifiy the bargaining power for all three skill levels, as the degrees of freedom are not sufficient.

| policy<br><br>$\mu_m =$ | increase of<br>... in % | $\sigma^{\eta_{S,T}} = 1,771$<br>$\sigma^{\eta_{T,N_m}} = 0$ | $\sigma^{\eta_{S,T}} = 0$<br>$\sigma^{\eta_{T,N_m}} = 5,000$ | $\sigma^{\eta_{S,T}} = 1,771$<br>$\sigma^{\eta_{T,N_m}} = 5,000$ | size of<br>complementarity<br>in % |
|---|---|---|---|---|---|
| $\mu_l$ | | 0.890373 | 0.580106 | 1.44726 | -1.57871 |
| 1.2 $\mu_l$ | income | 0.890372 | 0.569692 | 1.43704 | -1.5769 |
| $\mu_l$ | | 0.549082 | 0.388578 | 0.947042 | 1.00055 |
| 1.2 $\mu_l$ | employment | 0.549081 | 0.383474 | 0.94186 | 0.997699 |

Table 13: The effect of subsidies for different level of the hiring elasticity

# 7 Conclusion

This paper has examined the interactions between employment policies and training policies. In particular, we have analyzed whether there are complementarities with respect to income and employment. Our definition of complementarity is straightforward: two policies are complementary, when the effect of each policy on, say, aggregate income is greater when it is implemented in conjunction with the other policy than in isolation. We define substitutability as a negative complementarity, i.e. the effect of one policy is smaller when implemented in conjunction with the other policy than in isolation. Ignoring the interaction could distort the evaluation of the performance of policies.

As an example for employment and training policies, we consider hiring subsidies and training subsidies, which play a prominent role within the active labor market policy in many OECD countries. In many countries, high unemployment is one of the biggest economic problems, especially, for the low-skilled people. In particular in Germany, the unemployment rate of this group is relatively high. However, in Germany, the unemployment rate of the youth relative to those of the unemployment of the prime-age people (25 - 54 years) is the lowest in the OECD, a fact which is often explained by the German dual system of vocational training. However, with respect to the vocational training, the situation has decreased in recent years. There has been an increasing gap between the declining demand for apprentices by firms and the supply, i.e. the number of school leavers wishing to enter the apprenticeship. Therefore it is often argued that the financial burden of firms providing vocational training should be reduced. A second problem for young people is the transition from vocational training to work. A significant fraction of apprentices who have successfully finished vocational training becomes unemployed.

In our model, we have analyzed the effects of subsidies which are expected to reduce the problems. Training subsidies are paid to employers in order to increase their incentive to provide vocational training. Hiring subsidies are meant to increase the transition from apprenticeship to work. They are provided for a limited period of time, in which they drive a wedge between the income, the worker receives, and the labor costs the employer is confronted with.

Our analysis has tackled, in particular, the assessment of interactions by presenting a macro model of the labor market that allows us to identify and qualify each effect being associated with the two subsidies when implemented in isolation and when implemented in conjunction. Taking into account the possibility of interactions helps to avoid distortions in the evaluation of labor market policies, policy makers are increasingly interested in. Moreover, to make our analysis expressly relevant to the decisions that policy makers commonly face in practice, we do not follow the mainstream practice of deriving policies as first-best responses to labor market failures. Instead, the model takes a variety of common labor market imperfections as given.

By evaluating the policies within a simplified model, we have shown that there are good theoretical reasons for these policies to be complementary. The simulation results reveal that there are significant interactions between hiring and training subsidies, however, complementarities between the two policies are quite weak over even absent (with respect to income). When comparing the results for different institutional and policy features of the economy, we can observe significant differences. In particular, the existence of complementarities with respect to aggregate income depends on the financial constraints of the government. In the absence of government-budget-constraint effects, there are complementarities, but the government budget constraint introduces substitutabilities. The net effect is small. Independent of the results for the specific policy examples, our analysis provides a methodology for examining policy interactions which may be useful well beyond the bounds of the hiring and training subsidies.

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# Appendix

## A1 Incentive for the firm to engage in general vocational training

When answering the question why firms have – in contrast to the initial theory (e.g. Becker, 1964) – an interest to pay for general training, two aspect are noteworthy with respect to the German labor market.

(*i*) High firing costs can create an incentive to invest in general training: Since an employer can decide not to take over an apprentice at the end of the training phase, but faces considerable costs when firing a regular employee, the firm may be willing to subsidize vocational training. Such a training may serve as an expensive employment test (i.e. the employer can screen potential future employees) for which employers are willing to pay (see Harhoff and Kane, 1993).

(*ii*) Another reason for the willingness of the firm to pay for general training can be given by the wage structure. A compressed wage structure, caused by labor market frictions can create an incentive for the firm to invest in general training. This aspect is analyzed in detail by Acemoglu and Pischke (1999). As there is a reason to believe that this is an essential aspect in Germany, the theoretical background is shown in the following.

Assume that the amount of training is a continuous variable, $\Theta$. The product of the worker $a(\Theta)$ as well as the wage $w(\Theta)$ is a function of the amount of training. The worker gets a wage which corresponds to the outside option $o(\Theta)$. Given that there are no frictions, there is: $w(\Theta) = o(\Theta) = a(\Theta)$. The profit of the employer is $\pi = a(\Theta) - w(\Theta) = 0$. The employer has no incentive to invest in general training. A higher amount of general human capital would cause an increase of the productivity and an increase of the wage in the same extent, thus the profit is not affected. Now, assume that there are labor market frictions. The outside option of the worker is $o(\Theta) = a(\Theta) - \Delta(\Theta)$ with $\Delta'(\Theta) > 0$. Again, the worker gets a wage corresponding to the outside option: $w(\Theta) = o(\Theta) = a(\Theta) - \Delta(\Theta)$. But now, due to $\Delta(\Theta)$, the wage structure is compressed, which is illustrated by the fact that $\frac{\partial w(\Theta)}{\partial \Theta} < \frac{\partial a(\Theta)}{\partial \Theta}$. Now, the profit of the firm is given by $\pi(\Theta) = a(\Theta) - w(\Theta) = \Delta(\Theta)$. As the profit increases with $\Theta$, the firm has an interest to invest in general training.

So far, there was the implicit assumption that the wage can be set by the firm, thus the bargaining power, $\mu$, of the employee is 0. Now, we assume $\mu > 0$. In a first step, moreover, we assume, that there are no firing costs, i.e. $c_\varphi = 0$. The wage is calculated as follows:

$$w(\Theta) = \frac{\mu}{1 - \beta + \beta\mu}\, a(\Theta)$$

where $\beta$ is the replacement rate. Thus, the profit of the firm, $\pi(\Theta)$, can be calculated as follows:

$$\pi(\Theta) = a(\Theta) - w(\Theta) = a(\Theta)(1 - \frac{\mu}{1 - \beta + \beta\mu})$$

Thus, as $\frac{\partial \pi(\Theta)}{\partial \Theta} > 0$, an increase in the amount of training, $\Theta$, has a positive impact on the profit of the firm. This implies that $(1 - \frac{\mu}{1-\beta+\beta\mu}) > 0 \Leftrightarrow$ (1.) $\mu < 1$ and (2.) $\beta < 1$. Both conditions have to be satisfied. The first condition, $\mu < 1$, implies that the firm must be able to capture a fraction of the profit in the bargaining process. The second condition again stresses what has already been mentioned. Here, $\beta$ is the replacement rate, but in a more general interpretation it can also be seen as the ratio of the outside option relative to the wage. $\beta < 1$ implies that the worker with the higher wage faces a worse outside option. This causes a compression of the wage structure, which is a necessary condition for the firm to invest in general training. Taking also firing costs into account (i.e. $c_\varphi > 0$), the conditions to be fulfilled is $\frac{\mu}{1-\beta+\beta\mu-\mu c_\varphi} < 1$. For our calibration (with: $\mu = 0.257$, $c_\varphi = 0.6$ and $\beta = 0.6825$), the condition is satisfied!

Now, we deliver a short numerical exercise showing, that firms in the model framework of the paper have an incentive to invest in general training. During the training phase, the firm realize a loss: $(7,730 - 8,269 - 5,716)\sum_{c=1}^{3}\delta^{c-1}(1-\rho-\theta)^{c-1} = -17021$ EUR. Given that the successful apprentice will stay in the firm which conducted the vocational training (with a probability: 0.40 $(1-\rho-\theta)^3 = 0.33$) and generate an expected profit of 154.900 EUR (according to eq. (9) which has to be discounted (by the factor $\delta^p = 0.89$), the overall surplus is $23,411$ EUR. Thus, the firm (on average) has an incentive to engage in general vocational training.

However, one might argue, that a positive overall surplus is not enough because the opportunity costs have to be taken into account. In our model, the opportunity costs are 0 as we assume two exogenously given types of firms: firms which are engaged in vocational training and firms which are not. In contrast to our model one might argue, that firms have to decide between between the employment of a low-skilled employee and the training of the person. In this case, the opportunity costs are given by the expected profit generated by a low-skilled person. In our model, the corresponding profit is 15,393 EUR, which is lower than the expected overall surplus in the case of training. Hence, also when taking possible opportunity costs into account, the firm has an interest to engage in vocational training.

## A2 A supply-driven transition from school to work

In the model it is assumed, that the transition from school ($S$) to training ($T$) is demand-driven, so that the transition rate $\eta_{S,T}$ can be considered as a hiring rate. However, one might argue, that the transition is supply-driven (see for a model with a supply-driven transition Oskamp and Snower, 2008). In this case, the people leaving school decide whether they start an apprenticeship or try to find a job as low-skilled. In the following, we model the transition as supply-driven. People leaving school compare the difference between the expected return of entering into training and the expected return of entering the low-skilled labor market with their education costs. The education costs are induced e.g. by a skill adverse environment or mobility costs. We assume that the individuals are heterogenous with respect to their education costs. For a fraction of the people leaving school, the costs are so high that an apprenticeship is not worthwhile. This may explain why 15,387 vocational training vacancies have not been filled in 2006 (Bundesministerium für Bildung und Forschung, 2007). In the remainder, the training rate $\eta_{S,T}$ (i.e. the proportions of people leaving school who enter vocational training) is modelled as function of the differences in the expected utilities being associated with the two alternative labor market states. For the marginal individual leaving school the sum of the expected life-time utility in the case of joining training, $V^T$, and the education costs ($e^*$) is equal to the expected life time utility in the case of joining the low-skilled labor force.

$$V^T - e^* = \eta_l V_l^{N_l} + (1-\eta_l) V_l^{U_l} \tag{A2.1}$$

with:[57]

$$V_{l,t}^N = \frac{1}{1-\gamma}(w_l)^{1-\gamma} + \delta((1-\rho-\varphi_l)V_{l,t+1}^N + \varphi_l V_{l,t+1}^U) \tag{A2.2}$$

$$V_l^U = \frac{1}{1-\gamma}(b_l)^{1-\gamma} + \delta((1-\rho-\eta_l)V_{l,t+1}^U + \eta_l V_{l,t+1}^N) \tag{A2.3}$$

and

$$\begin{aligned} V_t^T &= \frac{1}{1-\gamma}(w_v + \sigma^{\eta_{S,T}})^{1-\gamma} \textstyle\sum_{c=1}^{p3} \delta^{c-1}(1-\rho-\theta)^{c-1} \\ &\quad + \delta^p (1-\rho-\theta)^p [\eta_{T,Nm} V_m^N + (1-\eta_{T,Nm}) V_m^U] \end{aligned} \tag{A2.4}$$

In contrast to the benchmark model, now, the training subsidies are not paid to the firm but to the apprentice. Hence, training subsidies are supposed to have a positive effect on the decision to enter training. Given also the equations for $V_m^N$ and $V_m^U$ we have a system of 7 equation and 7

[57] In the following, we do not take into account, that also a low-skilled person can decide to enter training (which is illustrated by the rate $\tau^N$ for the low-skilled employed and by the rate $\tau^U$ for the low-skilled unemployed). This is justified by the idea that the decision-making at state $S$ is the motivation of this calculation. We assume that a person being in state $S$ does not take into account the possiblity to enter training at a later point in time. Moreover, taking this effect into account, would significantly increase the complexity of the model.

unknown variables which can be solved so that we get the steady state expression for the value being associated with each labor market state. Substituting $V^T$, $V_{l,t}^N$ and $V_l^U$ in eq. (A2.1) by their corresponding steady state expressions, we obtain an equation for $e^*$, the education cost of the marginal school leaver.

As we are interested in the proportion of the people in $S$ who enter training (i.e. the training rate $\eta_{S,T}$ ), we have to model the relationship between $e^*$ and $\eta_{S,T}$ . The higher the marginal costs, $e^*$, are, the higher is $\eta_{S,T}$ Ordering the workers in terms of their individual disutility $e$, from the lowest to the highest, we let the cumulative distribution of the costs be approximated by a continuum given by the function $e^*(\eta_{S,T}\ )$, $(\partial e^*/\partial\ \eta_{S,T}\ ) > 0$. As we are interested in $\eta_{S,T}$ , we use the inverse function in the remainder: $\eta_{S,T}\ (\ e^*)$, with $(\partial\ \eta_{S,T}\ /\partial e^*) > 0$. For simplicity we assume: $\eta_{S,T}\ = x\ (e^*)^\alpha$ with $\alpha, x > 0$, where $\alpha$ is the elasticity of the training rate with respect to the income differential. Finally, we get:

$$\eta_{S,T}\ /x = (V^T - (\eta_l V_l^{N_l} + (1-\eta_l)V_l^{U_l}))^\alpha \tag{A2.5}$$

The intuition is straightforward. The implementation of training subsidies ( $\sigma^{\eta_{S,T}}$) increases the incentive of the people leaving school to enter vocational training. According to eq. (A2.4), the expected utility in the case of entering training increases. Hence, the value on the right-hand-side in eq. (A2.5) and finally $\eta_{S,T}$ increases. The higher the expected utility being associated with entering training, the higher is the education cost of the marginal worker, $e^*$ , and the higher is the fraction of the people with $e < e^*$, i.e. the fraction of people for which entering training is worthwhile.

In order to analyze the effects of the subsidies on complementarities, we replace eq. (21) in the model by eq. (A2.5). Doing the same policy simulation, the qualitative results do not change.

## A3 A simple model with government budget constraint

In contrast to the main model, the following one is a 2-period model. This and some other properties allow to get an analytical solution even in the presence of the government budget constraint. The worker´s possible labor market states are illustrated in Figure (3): training $T$, medium-skilled employment $N_m$, medium-skilled unemployment $U_m$, low-skilled employment in period 1 and 2, $N_l^1$ and $N_l^2$, and finally low-skilled unemployment in period 1 and 2, $U_l^1$ and $U_l^2$. At the end of the second period, all people die. We assume, that the number of deaths is equal to the number of people leaving school, so the labor force is constant. A person leaving school is hired with a probability $\eta_{S,T}$ as apprentice and with a probability $(1-\eta_{S,T})\eta_l$ as low-skilled employee. The rest remains unemployed. A trained person becomes a medium-skilled employee in the $2^{nd}$ period with a probability $\eta_{T,N}$. For the sake of simplicity, we assume that low-skilled people do not change their labor market state in the second period. Given all this, we get the following expressions for the steady state:

$$N_l^1 = (1-\eta_{S,T})\eta_l\ (\ N_m + U_m + N_l^2 + U_l^2) \tag{A3.1}$$

$$U_l^1 = (1-\eta_{S,T})(1-\eta_l)\ (\ N_m + U_m + N_l^2 + U_l^2) \tag{A3.2}$$

$$N_m = \eta_{T,N}\ T \tag{A3.3}$$

$$U_m = (1-\eta_{T,N})\ T \tag{A3.4}$$

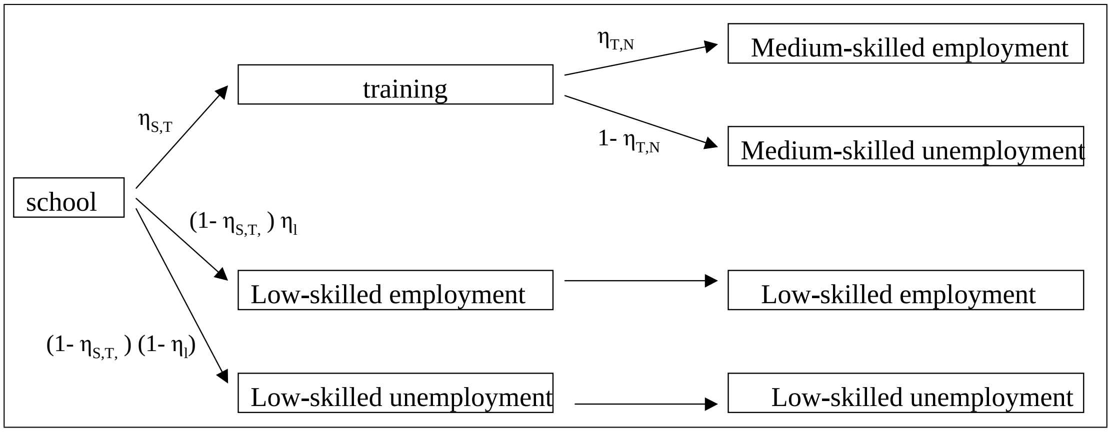


Figure 3: The dynamic structure of the simple model.

$$N_l^2 = N_l^1 \tag{A3.5}$$

$$U_l^2 = U_l^1 \tag{A3.6}$$

Moreover, we normalize the total labor force to 1

$$1 = T + N_m + U_m + N_l^1 + U_l^1 + N_l^2 + U_l^2 \tag{A3.7}$$

Given these equations, we can calculate the steady state expressions for each labor market state as a function of the transition rates: $N_m = 0.5\ \eta_{S,T}\ \eta_{N,T}$, $U_m = 0.5(1-\eta_{T,N})\eta_{S,T}$, $T = \frac{\eta_{S,T}}{2}$, $N_l^{1,2} = 0.5\ (1-\eta_{S,T})\eta_l$ and finally $U_l^{1,2} = 0.5(1-\eta_{S,T})(1-\eta_l)$. Assuming a uniform distribution of the random operating cost $\epsilon$ with a lower limit of 0, the hiring rate $\eta_{T,N}$ and the training rate $\eta_{S,T}$ which are affected by the subsidies are expressed as:

$$\eta_{T,N} = \frac{a_m - w_m + \sigma^{\eta_{T,N_m}}}{\epsilon^+_{\eta_{T,N_m}}} \tag{A3.8}$$

and:

$$\eta_{S,T} = \frac{a_v - w_v + \ \sigma^{\eta_{S,T}} + \eta_{T,N}\ (a_m - w_m + \sigma^{\eta_{T,N_m}})}{\epsilon^+_{\eta_{S,T}}} \tag{A3.9}$$

where $w_m = a_m\ \mu$ which is given by eq. (14) for $\gamma = 0$, $\beta_m = 0$ and $c_\varphi = 0$. Given these expression, the labor markets states can be written as function of the subsidies. In order to determine aggregate income, we have to connect each labor market state with a value. In the case of training the value corresponds to the wage $w_v$ and in the case of employment, the value corresponds to the net wage $w_i(1-t)$ with $i = l, m$. With respect to unemployment, we assume the absence of unemployment benefits, so that the value associated with unemployment is 0.

In the absence of taxes, aggregate income is calculated as follows:

$$\Phi = T\ w_v + N_m\ w_m + (N_l^1 + N_l^2)\ w_l \tag{A3.10}$$

Substituting the variables presenting the labor market states in eq. (A3.10) by their steady state expressions gives an expression of aggregate income as a function of the two subsidies. We can calculate the cross derivative of aggregate income w.r.t. to the two subsidies:

$$\frac{\partial^2 W}{\partial\ \sigma^{\eta_{T,N_m}}\ \partial\ \sigma^{\eta_{S,T}}} = \frac{a_m\ \mu}{2\ \epsilon^{+}_{\eta_{T,N_m}} \epsilon^{+}_{\eta_{S,T}}} \tag{A3.11}$$

The cross derivative is unambiguously positive. Hence, the two subsidies are complementary w.r.t. aggregate income.

In the presence of the government budget constraint, aggregate income is calculated as follows:

$$W = T\ w_v + N_m\ w_m(1-t) + (N_l^1 + N_l^2)\ w_l\ (1-t) \tag{A3.12}$$

The government budget constraint is given by:

$$t\ N_m w_m + t\ (N_l^1 + N_l^2)\ w_l = T\ \sigma^{\eta_{S,T}} + N_m\ \sigma^{\eta_{T,N_m}} \tag{A3.13}$$

Solving for $t$ yields:

$$t = (T\ \sigma^{k_v} + N_m \sigma^{\eta,A,N_m})/(N_m w_m +\ (N_l^1 + N_l^2)\ w_l) \tag{A3.13a}$$

Substituting the variables presenting the labor market stated in eq. (A3.12) and in eq. (A3.13a) by their steady state expressions and then substituting $t$ in eq. (A3.12) by the expression in eq. (A3.13a) gives an expression of aggregate income as a function of the two subsidies. Then, we can calculate the cross derivative of aggregate income w.r.t. to the two subsidies:

$$\begin{aligned} \frac{\partial^2 W}{\partial \sigma^{\eta,A,N_m} \partial \sigma^{k_v}} &= \frac{3\ a_m\mu - 2a_m - \eta_{T,N} - 2\ \sigma^{\eta_{T,N_m}}}{2\ \epsilon^{+}_{\eta_{T,N_m}} \epsilon^{+}_{\eta_{S,T}}} \\ &= \frac{a_m\mu}{2\ \epsilon^{+}_{\eta_{T,N_m}} \epsilon^{+}_{\eta_{S,T}}} + \frac{2\ a_m(\mu-1) - \eta_{T,N} - 2\ \sigma^{\eta_{T,N_m}}}{2\ \epsilon^{+}_{\eta_{T,N_m}} \epsilon^{+}_{\eta_{S,T}}} \end{aligned} \tag{A3.14}$$

This equation shows the contrast to the cross derivative in the absence of the government budget constraint, which is given by the last term on the right-hand side. The additional term is unambiguously negative. For a plausible parameter value ($\mu < 0.75$) the total term is negative, so in the presence of the government budget constraint, there are no complementarities with respect to income.

## A4 The derivation of the cross derivative

To derive the cross derivative of the income with respect to the subsidies (eq. (23)), we have to rewrite eq. (22). This is done in four steps. First, we substitute $N_i$ and $U_i$ for $i = l, m, h$ and $T$ in eq. (22) by the corresponding steady state expression. They can be derived by solving eq. (3) for $Z = MT * Z$. Second, we substitute the training rate $\eta_{S,T}$ according to eq. (21) and the hiring rates $\eta^1_{T,Nm}$ and $\eta^2_{T,Nm}$ according to eq. (17) for $\eta_{T,Nm} = \eta^1_{T,Nm}$ and eq. (18), respectively. Third, we substitute the wages $w_i$ according to eq. (14) for $i = l, m, h$. Finally, we set $(i)$ $t_i = 0$ for $i = l, m, h$, $(ii)$ $\theta = 0$, $(iii)$ $p = 1$ and $(iv)$ $\gamma = 0$. Hence, we get an expression for the aggregate income, $\Phi$, being an explicit function not only of the subsidies, $\sigma^{\eta_{S,T}}$ and $\sigma^{\eta_{T,N_m}}$, but also of the limits of the different uniform distribution and the bargaining power, $\mu_i$, with $i = l, m, h$.

## A5 The labor market states

To quantify the labor market states for the initial steady state, we use the "Labor Force Statistics by educational attainment by sex and age - indicators" and "Labor Force Statistics by educational attainment by sex and age - composition" delivered by the OECD (1999–2005) for the years 1997 – 2003 (new data are not available). As underlying labor force, we choose the group of people between 15 and 64 years. In a first step, we calculate the averages for the variables "Employment/population ratio", "Unemployment Rate", Share of the Labour Force" and "Share of Unemployment". Moreover, we calculate the annual employment rates.

In a second step, we calculate the annual values for employment and unemployment for four skill categories ("below upper secondary", "upper secondary, general", "upper secondary, vocational" and "at least post secondary". The results are given in table (14).

| | employment | unemployment | sum |
|---|---|---|---|
| below upper secondary | 14.3 | 2.4 | 16.7 |
| upper secondary, general | 2.8 | 0.3 | 3.1 |
| upper secondary, vocational | 46.3 | 4.6 | 50.9 |
| at least post secondary | 27.7 | 1.6 | 29.3 |
| sum | 91.1 | 8.9 | 100 |

Table 14: Value for the initial Labor Market States

However, according to these data, the group of employed people with "below upper secondary" education also contains the apprentices. Indeed, for the purpose of the analysis, we have to distinguish explicitly between those who are regularly working and the apprentices. Based on data from the Statistisches Bundesamt (2006, 2008), we calculate the fraction of apprentices in the total labor force (4%). Given this, we can quantify the level of apprentices and adjust the level of employed people with "below upper secondary" education, correspondingly.

In the analysis, people with "below upper secondary" education are classified as low-skilled, people with "upper secondary vocational" education are classified as "medium-skilled", people with "at least post secondary" education are classified as "high-skilled". The latter group as well as people with "upper secondary general" education are not part of the labor force being relevant for the analysis of the transition rate. The results are given in table (15).

| | employment ($N_i$) | unemployment ($U_i$) |
|---|---|---|
| low-skilled labor ($i = l$) | 10.3 | 2.4 |
| medium-skilled labor ($i = m$) | 46.3 | 4.6 |
| training ($T$) | 4.0 | – |

Table 15: Values for the Labor Market States including Training

Given all these data, now, we can calculate simultaneously, the number of people leaving, $S$, the firing rate of the medium-skilled employees, $\varphi_m$ (= 0.05), the number of people in the first period of training $T^1$ and the death rate, $\rho$ $(= 0.023)$. This is done by using the equations $1, 2$ and $6$ of the matrix $MT$ and the fact, that the number of people being in vocational training is given by $T_1 \sum_{c=1}^{p} (1 - \rho - \theta)^{c-1}$. After this, we normalize all relevant values ($N_m$, $U_m$, $N_l$, $U_l$, $T_1$, $S$), so that the sum of the relevant population is 1.