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by Tim Lohse, Julio Robledo, and Ulrich Schmidt

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JEL classification: G22, H41

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# 1 Introduction

For many of the risks in their daily life people can buy insurance in order to protect against possible losses. However, they may also take private action for risk reduction. In their seminal contribution, Ehrlich and Becker (1972) introduced the terms "self-insurance" (SI) for effort that reduces the size of the loss and "self-protection" (SP) for effort that reduces the probability of the loss. For instance, a homeowner may install an alarm device to reduce the probability of burglary. She may also buy a safe in order to reduce potential losses in case of burglary. Since in this example all costs and benefits only accrue to the individual homeowner, SI and SP are private goods.

However, SI and SP activities may also benefit many individuals in a non-rival way. In fact numerous public goods can be regarded as SI and SP activities since they mainly provide utility due to risk reduction. Good examples are lighthouses, dams, and national defense as pure public goods or police and fire departments[1] as local public goods. All these goods can be regarded as insurance devices against risks like shipwreck, flooding, etc. The present paper analyzes the insurance character of public goods in terms of SI and SP.

When analyzing SI and SP as public goods, a central question is whether risks are independent or correlated. If a dam on an island breaks, nearly all inhabitants of the island will be affected. On this example risks are highly correlated.[2]

By contrast, we consider the case where risks targeted by public goods are uncorrelated: lighthouses, police patrols, emergency medical aid, fire departments in rural areas, cancer research, etc. For instance, the risk of burglary affects all individuals in the population, but is not correlated among individuals.

Altogether, our paper combines the literature on private SI and SP with the public goods literature, the literature on private provision of public goods, and the insurance literature. First, we review the related literature in the next section. In the following section 3 we formulate our model of SI and SP as public goods. We characterize the efficient provision levels for given wealth, preferences, and size of a group of individuals,

[1] Orszag and Stiglitz (2002) have analyzed the efficient provision level of fire departments as public goods.

[2] For an analysis of SP as a public good under correlated risks, see Muermann and Kunreuther (2008).

and derive modified Samuelson conditions along with some comparative static results. If additionally market insurance is available, individuals may also increase their utility by buying this insurance. This case is also analyzed in section 3. Being protected by private market insurance makes individuals less sensitive to the possible loss and, therefore, changes the efficient provision levels of public SI and SP.

Section 4 is devoted to the private provision of public SI and SP. Here, individuals take the contributions to the public good by the other individuals as given and contribute to public SI and SP in a non-cooperative way as in Cornes and Sandler (1984) and Bergstrom et al. (1986). We highlight the theoretical similarities and differences between the standard model of private contributions to a public good and our model of private contributions to public SI and SP, where the role of income normality in the standard model is analogous to the role of risk aversion in the case of SI and SP as public goods. Again, we extend the analysis to cases where private market insurance is available and affects private contributions to public SI and SP. Section 5 summarizes our results and concludes.

## 2 Related literature

Since the seminal work of Ehrlich and Becker (1972), SI and SP continue to be the focus of many theoretical and empirical studies. Recent examples include Chiu (2000), Lee and Ligon (2001), Kelly and Kleffner (2003), Breuer (2005), Lakdawalla and Zanjani (2005), Yamauchi et al. (2009), and Kaplan and Violante (2009). An important aspect of SP is the fact that a higher degree of risk aversion does not imply higher expenditures for SP (Dionne and Eeckhoudt, 1985). A recent contribution by Dachraoui et al. (2004) introduces the concept of comparative mixed risk aversion and shows that a higher degree of comparative mixed risk aversion implies higher expenditures for SP as long as the probability of the loss is below 0.5.

In the last years, several papers have also analyzed SI and SP as public goods. In a related framework to ours, Kunreuther and coauthors analyze the case of correlated risks (Kunreuther and Heal (2003), Heal and Kunreuther (2005), and Muermann and Kunreuther, 2008). In these papers, SP efforts of one individual have positive externalities for other individuals. This leads to strategic complementarity of private insurance efforts and to Nash equilibria with underprovision compared to efficient investments. Our contribution employs a setting equivalent to that of Muermann and Kunreuther (2008), but focuses on risks that are uncorrelated across individuals. Also, besides SP, we consider SI, which, as showed by Ehrlich and Becker (1972), greatly differs in its effects from SP. We also analyze the interaction of private market insurance and private contributions to SI and SP as public goods.

In a setting of independent countries contributing to a defence alliance, Ihori and McGuire (2007) also focus only on the SP case and on how the risk diminishing technology and the size of the group affect the equilibrium outcome. In a forthcoming paper, Ihori and McGuire (2010, forthcoming) consider national investment efforts in security as private contributions to the public good "security of the alliance". Since the individual agents are sovereign countries, they assume non-linear, increasing costs of SI and also consider the situation when a government may spend effort on SI and SP at the same time. Compared to those papers, we analyze both SI and SP as public goods in separate models in order to concentrate on the specific characteristics of SI and SP in isolation. Moreover, we assume fair linear pricing when buying market insurance, since our individuals are not large, sovereign agents like nations. This setting allows to focus on the interaction of private market insurance and private contributions to SI and SP as public goods.

To summarize, our analysis is based on uncorrelated risks across consumers, complementing the papers by Kunreuther and coauthors, and analyzes both SI and SP, complementing the papers by Ihori and McGuire. We model the interaction between the expenditures on private insurance and the private contributions to the public goods, which to our knowledge has not been modeled in an explicit way in the literature, and show their strategic substitutability.

# 3 Efficient provision

Consider an economy with $n$ identical individuals facing two possible states of the world. All individuals have the same probability $p$ of suffering a loss $L$, while with residual probability $1-p$ there is no loss. Each individual is endowed with wealth $\omega$ which she may spend on increasing the level of the public good $C$ with a non-negative contribution $c \geq 0$. For convenience and without loss of generality, we set the marginal cost of contributing to the public good to 1. The public good $C$ diminishes either the size of the loss (public SI) or the probability of the loss (public SP). The state contingent income level of an individual is denoted by $y$. All $n$ individuals have the same strictly monotonic and continuous von Neumann utility function $U$ with increasing and diminishing returns to state-contingent income, $U'(y) > 0, U''(y) < 0$.[3]

Individuals face the same risk $L$ with the same probability $p$. However, the event that individual $i$ suffers a loss is stochastically independent from the event that individual $j$, $j \neq i$, suffers the loss, i. e., the risks are not correlated across individuals.[4] Thus, it is not the case that the individuals are identical in the sense that they all end in the same state of the world, but that their probability distributions across states are identical. We believe this to be a sensible assumption for the analysis of local public goods like law and order.

In the following we will always present first the SI and then the SP version of our model. At a first glance, this separate treatment may seem a repetitive and unparsimonious approach. The concise alternative would be to specify a general framework containing both the SI and the SP elements. In practice it is sometimes difficult to classify an effort unambiguously as a SI or a SP effort. Yet from a theoretical perspective, SI and SP are very different. This may be the reason why, in the literature starting with Ehrlich and Becker (1972), both approaches have usually been treated separately and not in an integrated model.[5] SI (as the name implies) is similar to insurance in that wealth is

[3]In a more general setting, the individuals could have a different wealth endowment $\omega$, e. g., $\omega_i \neq \omega_j$ for $i \neq j$ if $i$ and $j$ denote different individuals. Following the advise of a referee, we assume that all individuals have an equal endowment. While we lose some generality, the heterogeneity of individuals is not the focus of our analysis and assuming equal endowments greatly simplifies notation and ensures interior solutions.

[4]Consider our example regarding law and order. The probability of being a victim of crime does not increase one neighbour's probability of being a victim, too. One may argue that the probability of being mugged is higher in, say, rough Johannesburg (South Africa) than in safe Stockholm (Sweden). But this simply means that the probability $p$ of a crime is higher in Johannesburg than in Stockholm.

[5]For an approach that considers the SI and SP in a joint model, see Ihori and McGuire (2010,

transferred from the good to the bad state of the world. In contrast, SP is in essence a moral hazard, hidden action situation. These differences will also become clear in the following, where SI and SP lead to different results. Therefore we present SI and SP efforts as two different approaches within the same general insurance framework model.

## 3.1 Samuelson conditions for SI and SP

In the SI case, for all individuals the size of the loss $L$ depends on the level of the public good $C$, $L(C)$, where $C$ is the sum of all private contributions to the public good; i. e., $C = nc$. The public good reduces the size of the loss with diminishing productivity: $L'(C) < 0$ and $L''(C) > 0$. We further assume that a loss always reduces the utility of the individual, independently of the SI level, i. e. $L > 0$ for all $C$.[6] Last, we assume that it is worthwhile to invest in loss reduction, i. e. $\lim_{C\to 0} L'(C) \to -\infty$, and that it does not pay to spend all wealth on SI effort, i. e. $\lim_{y\to 0} U'(y) \to \infty$.[7] The individual state contingent income levels in the situation where $C$ acts as a SI device are

$$\begin{aligned} y_1 &= \omega - c \\ y_2 &= \omega - c - L(C). \end{aligned}$$

The individual maximizes her expected utility given by

$$EU(c, C) = (1-p)U(\omega - c) + pU(\omega - c - L(C)) = (1-p)U_1 + pU_2. \quad (1)$$

The first-best, Pareto efficient outcome for $n > 1$ is found when the expected utility level of individual 1 is maximized, given the restrictions that individuals 2 to $n$ obtain given expected utility levels and that $C = nc$. Let $*$ as superscript denote the efficient level of the public good, the subscript $SI$ refer to the SI case and marginal expected utility $(1-p)U_1' + pU_2'$ be abbreviated as $EU'$. The Pareto efficient level of a public good $C$ which acts as a SI device satisfies the modified Samuelson condition

$$n\frac{-L'(C^*_{SI})pU_2'}{EU'} = 1. \quad (2)$$

forthcoming).

[6]This assumption precludes the reversal of the good and the bad state. We follow Ihori and McGuire (2010, forthcoming) who make the same realistic assumption.

[7]These Inada assumptions are innocuous, they just rule out corner solutions which are not the focus of our analysis.

The left hand side reflects the willingness to pay for the public good $C$: the marginal positive effect of an additional unit of $C$, measured in units of forgone income in both states of the world (marginal expected utility $EU'$). Since an additional unit of $C$ benefits all individuals, the left hand side is the sum of the marginal willingness to pay for public SI of *all* individuals which in a welfare maximum equals the marginal cost of an additional unit of $C$. The second-order conditions are fulfilled by the assumptions on $u$ and $L$.

In the SP case, the size of the loss $L > 0$ is fixed and uniform for all individuals. Now the collective effort $C$ reduces the probability of the loss for all individuals which will be denoted by $p(C)$. The probability of a bad state can be reduced by contributing to the public good. For the relationship between the public good level and the probability of the bad state, we again assume realistically that increasing $C$ reduces its probability with diminishing returns: $p'(C) < 0$ and $p''(C) > 0$. We further assume Inada-like that it pays to invest in the reduction of the loss probability, i. e. $\lim_{C\to 0} p'(C) \to -\infty$, and that it does not pay to spend all wealth on SP effort, i. e. $\lim_{y\to 0} u'(y) \to \infty$. Additionally, in the SP case we assume that the probability $p(C)$ of the bad state of the world is sufficiently small (the loss is relatively seldom) in the following sense:

**Assumption 1**

*The slope of the line connecting the utility levels in the good and in the bad states of the world is larger than the average of the slopes at those utility levels, i. e., than the expected marginal utility, for all income levels:*

$$\frac{U_1 - U_2}{L} > EU' > 0. \tag{3}$$

*A similar condition applies to the slope of the line connecting the marginal utility levels in the good and in the bad states of the world, which is smaller than the average of the slopes at those marginal utility levels, for all income levels:*

$$-\frac{U_2' - U_1'}{L} < EU'' < 0. \tag{4}$$

The first part of Assumption 1 concerns the slopes of the utility function, while the second part concerns analogously the case of marginal utility function. Notice that for a concave utility function, both inequalities (3) and (4) always hold if $p \to 0$ and are do never hold if $p \to 1$.

The state contingent income levels in the SP case are given by

$$y_1 = \omega - c$$

$$y_2 = \omega - c - L,$$

where $C$ acts as a SP device by affecting the probabilities $1 - p(C)$ of the good and $p(C)$ of the bad state of the world. Note that SP does not involve the redistribution of income. Since the absolute size of the loss does not change, SP expenditures even increase the *relative* size of the loss.

The representative individual maximizes her expected utility given by[8]

$$EU(c, C) = (1 - p(C))U(\omega - c) + p(C)U(\omega - c - L) = (1 - p(C))U_1 + p(C)U_2. \quad (5)$$

Analogously to the SI case, the Pareto efficient outcome is found when the expected utility level of individual 1 is maximized given the restrictions that individuals 2 to $n$ obtain given expected utility levels and that $C = nc$. Using now the subscript $SP$ for SP, the Pareto efficient level of a public good $C$ which acts as a SP device is determined by the modified Samuelson condition

$$n \frac{-p'(C^*_{SP})(U_1 - U_2)}{EU'} = 1. \quad (6)$$

This condition resembles again the Samuelson condition. Since the reduction in the probability of the loss accrues to all individuals, the left hand side is the sum of the marginal willingness to pay of *all* individuals for this reduction. The marginal willingness to pay is the difference in utility between both states of the world, weighted with the marginal change in the probability of the loss and measured in units of forgone income as given by the marginal expected utility $EU'$ in the denominator. This sum of marginal benefits must equal the right hand side, which is the marginal cost of the public good.

As usual in the SP (and moral hazard) literature, under the assumptions made so far the second-order condition for the SP problem does not always hold.[9] In the following, we assume the Hessian matrix of the corresponding maximization program to be negative definite. Under this assumption, condition (6) describes the Pareto efficient outcome.

[8]Notice that, throughout the paper, we use the same notation $U_1$ and $U_2$ for the different settings SI and SP. Since it is always clear how the utility argument looks like, we will use this notation for the sake of a clear exposition with parsimonious notation.

[9]See, e. g., Ehrlich and Becker (1972) and Shavell (1979).

## 3.2 Comparative statics of increased risk

This section is devoted to the question how a change in the level of risk aversion affects the efficient provision levels of SI and SP when they are non-rival public goods. Individuals with utility function $V$ are more risk averse than those with utility $U$, if there exists a function $f$ satisfying $f'(\cdot) > 0$ and $f''(\cdot) < 0$ such that $V = f(U)$ (Pratt, 1964).

### Self-insurance

Considering first the case of SI, the following lemma shows that a change in risk has an unambiguous effect on the SI level.

**Lemma 1 (Effect of risk behavior on SI)**
*Increasing risk aversion as reflected by a concave transformation of the original utility function leads to a higher efficient level of public SI.*

Proof. Under the same endowed wealth and size of loss as in section 3.1, the appropriate first-order condition for the more risk averse society is given by

$$-npL'(\tilde{C}^*_{SI})\frac{f'(\tilde{U}_2)\tilde{U}'_2}{pf'(\tilde{U}_2)\tilde{U}'_2 + (1-p)f'(\tilde{U}_1)\tilde{U}'_1} = 1. \tag{7}$$

This condition characterizes the efficient level $\tilde{C}^*_{SI}$, where the tilde denotes the increased risk aversion. Note that a different level of public SI also changes the arguments of the utility terms. Now, we substitute the original $C^*_{SI}$ in the LHS of (7) and rearrange the fraction by dividing both numerator and denominator by $f'(U_2)$:

$$-npL'(C^*_{SI})\frac{U'_2}{pU'_2 + (1-p)\frac{f'(U_1)}{f'(U_2)}U'_1} \tag{8}$$

Since $U_1 > U_2$ and by the definition of the strictly concave transformation $f'' < 0$, it follows that $f'(U_1) < f'(U_2)$ and hence $f'(U_1)/f'(U_2) < 1$. Using the Pareto efficiency condition (2) it holds that (8) is greater than 1:

$$-npL'(C^*_{SI})\frac{U'_2}{pU'_2 + (1-p)\frac{f'(U_1)}{f'(U_2)}U'_1} > -npL'(C^*_{SI})\frac{U'_2}{pU'_2 + (1-p)U'_1} = 1. \tag{9}$$

Combining (7) and (9) yields

$$-npL'(C^*_{SI})\frac{U'_2}{pU'_2 + (1-p)\frac{f'(U_1)}{f'(U_2)}U'_1} > 1 = -npL'(\tilde{C}^*_{SI})\frac{f'(\tilde{U}_2)\tilde{U}'_2}{pf'(\tilde{U}_2)\tilde{U}'_2 + (1-p)f'(\tilde{U}_1)\tilde{U}'_1}. \tag{10}$$

Since the first-order condition is a decreasing function of $C_{SI}$ by the concavity of the objective function, $\tilde{C}^*_{SI} > C^*_{SI}$ follows from (10). QED.
The intuition of (8) is straightforward. The current level of the public SI is $C^*_{SI}$, and the cost of an additional unit of $C$ is 1. But as the society has become more risk averse, the sum of the marginal willingness to pay for public SI of all individuals exceeds the additional cost. Hence, the efficient level of the provision of the public good must be higher than $C^*_{SI}$. Notice that this SI result also means that when the individuals become less risk averse, the efficient provision level of public SI decreases. This interpretation of Lemma 1 will be used later.

**Self-protection**

For an increase in risk aversion in the SP case, consider again a concave transformation as described above. The resulting first-order condition is

$$n \cdot \frac{-p'(\tilde{C}^*_{SP})(f(U_1) - f(U_2))}{pf'(U_2)U_2' + (1-p)f'(U_1)U_1'} = 1 \tag{11}$$

and gives $\tilde{C}^*_{SP}$. Now, substitute $C^*_{SP}$ in (11) to obtain

$$n \cdot \frac{-p'(C^*_{SP})(f(U_1) - f(U_2))}{pf'(U_2)U_2' + (1-p)f'(U_1)U_1'}. \tag{12}$$

However, the term (12) can be positive or negative, so we cannot establish unambiguously whether $\tilde{C}^*_{SP}$ is greater or smaller than $C^*_{SP}$. This ambiguous result about increased risk and SP does not follow from having more than one individuals, but is already the case for $n = 1$ as analyzed by Dionne and Eeckhoudt (1985) and McGuire et al. (1991).

## 3.3 Efficient provision with market insurance

Even in places where public SI (e. g., emergency medical aid) or public SP (e. g., police patrols) are present, individuals may want to additionally buy private market insurance to cover the residual risk. The price of this insurance depends, of course, on the provided level of public SI and public SP. Suppose an individual can buy coverage $s \in [0, L]$ at a uniform price $\pi$, and can contribute to the public device $C$ at the marginal cost of 1. For coverage $s$, a premium of $\pi s$ has to be paid.

Since we want to focus on the relationship between public insurance through the public good and private market insurance, we assume that market insurance is fair; i. e., the expected payoff of the insurance is zero and its price equals the probability of a loss. Thus, a risk averse individual will always choose to buy full insurance (Mossin, 1968). The assumption of a risk neutral private insurance company that helps consolidate individuals' risks is a reasonable one, given that our risks are stochastically independent across individuals.[10] Thus, after having bought fair full insurance, the individual behaves as a risk neutral maximizer of her expected income, which is given by $\omega - c - pL(C)$ and $\omega - c - p(C)L$ in the SI and SP cases, respectively.

**Self-insurance**

In the case of SI, fair private insurance means $\pi = p$. The resulting utility level of an individual is

$$U(E(y)) = U(\omega - c - pL(C)). \tag{13}$$

The efficient level of SI as a public good when private market insurance is available is found by maximizing the utility of individual 1 for fixed utility of the other individuals and taking into account the public SI restriction $C = n \cdot c$. Solving this problem and using a hat to denote the public good level that is obtained in the presence of private market insurance, the Pareto efficient level of SI is now given by

$$n \cdot p(-L'(\hat{C}^*_{SI})) = 1. \tag{14}$$

The left hand side of condition (14) is the expected marginal benefit of an additional unit of SI, while the right hand side is its marginal cost. Since $C$ is a public good, the probability weighted marginal benefit $p(-L'(\hat{C}^*_{SI}))$ accrues to all $n$ individuals and thus has to be multiplied by $n$.

The effect of the availability of market insurance on the efficient provision level of SI as a public good can be determined by comparing the public good levels $C^*_{SI}$ and $\hat{C}^*_{SI}$ given by conditions (2) and (14). On both right hand sides of the conditions (2) and (14) we

[10] In reality there will be some administrative costs and positive profits at insurance companies that should cause a small but positive loading factor. However, our results carry over with only quantitative changes if we assume a positive loading factor.

have 1, the marginal cost of an additional unit of public SI. The left hand side of (2) can be rewritten as

$$n \cdot p(-L'(C^*_{SI}))\frac{U'_2}{EU'}. \tag{15}$$

Since income is lower in state 2, marginal utility $U'_2$ is greater than expected marginal utility, which is the probability average of both marginal utilities. Thus, the fraction is greater than 1. Compared to condition (14) above, we obtain

$$-L'(C^*_{SI}) < -L'(\hat{C}^*_{SI}) \quad \Longleftrightarrow \quad C^*_{SI} > \hat{C}^*_{SI} \tag{16}$$

Hence, the availability of private insurance decreases the efficient provision level of public SI. Independently of the degree of risk aversion, the efficient provision level is given by that amount of public SI which maximizes expected income. The individuals behave as if they were risk neutral and, according to Lemma 1, this decreases the efficient provision level compared to the case were private insurance is not available. Therefore, we can conclude that market insurance and public SI are strategic substitutes.

Note that the provision level $\hat{C}^*_{SI}$ leads to higher welfare, even though the provision level of public SI is lower than the Samuelson provision level $C^*_{SI}$. This is the case because through insurance the individuals have a second instrument at hand. With the public good acting as a SI device, the individuals jointly reduce the loss as much as efficiently possible. In a second step, they cover this residual risk by buying full insurance.

**Self-protection**

In the SP case, fair insurance implies that the premium depends on the level of SP, i.e. $\pi(C) = p(C)$ for all $C$. Again, individuals choose to buy full insurance which leads to utility

$$U(E(y)) = U(\omega - c - p(C)L) \tag{17}$$

The efficient level of SP in the presence of private market insurance is found by maximizing the utility of individual 1 for fixed utility of the other individuals and taking into account the public SP restriction $C = c \cdot n$. The condition describing the Pareto efficient level of SP is

$$n \cdot (-p'(\hat{C}^*_{SP}))L = 1. \tag{18}$$

The left hand side of (18) is the probability weighted marginal benefit of an additional unit of the SP public good to the $n$ individuals, while the right hand side is its marginal cost.

To compare the efficient public good levels $C^*_{SP}$ and $\hat{C}^*_{SP}$ without and with market insurance, we analyze conditions (6) and (18), respectively. Rearranging condition (6) and using Assumption 1 yields

$$1 = -p'(C^*_{SP})n \underbrace{\frac{(U_1 - U_2)}{EU'}}_{>L} > -p'(C^*_{SP})nL = n \cdot (-p'(C^*_{SP}))L \tag{19}$$

Combining (18) and (19) leads to

$$-p'(C^*_{SP})L < -p'(\hat{C}^*_{SP})L \qquad \Longleftrightarrow \qquad C^*_{SP} > \hat{C}^*_{SP}. \tag{20}$$

The provision level $\hat{C}^*_{SP}$ leads to higher welfare with a lower public good level of SP. Through the public SP device, the individuals jointly reduce the probability of the loss as much as efficiently possible. In a second step, they cover this residual loss by buying full private insurance. Again, switching resources from the public good SP (law and order) to private market insurance makes the individuals better off. Owing to public SP, private market insurance has become cheaper. But compared to the situation without market insurance, the efficient public SP effort level decreases, because insuring the residual risk with social risk consolidation is more efficient than via SP effort.

# 4 Private provision of SI and SP

Suppose now that there is no coordinating institution able or willing to provide the efficient provision level of the insurance public good $C$. Therefore it is the $n > 1$ individuals who contribute privately to public SI effort (e. g. local emergency medical aid) and to public SP effort (e. g. local police patrols). As usual in situations of private contributions to a public good, we assume best response behavior as introduced by Cornes and Sandler (1984) and Bergstrom et al. (1986). We will denote the resulting equilibrium levels with the superscript $N$ for Nash. If all individuals have equal endowments $\omega$, there are no pure free-riders and all individuals belong to the set of contributors. In equilibrium, all individuals are at an inner solution and we can disregard corner solutions. This outcome also excludes the anomaly of overprovision of a public good for normal preferences (see Buchholz and Peters, 2001).

## 4.1 Private provision equilibria

**Self-insurance**

In the SI situation, each individual maximizes her expected utility $EU$ by her choice of $c$, taking the contributions of the other $n-1$ individuals, which already reduce the size of the loss, as given. $C_{-i}^{SI} = \sum_{j=1, j\neq i}^{n} c_j^{SI} = C_{SI}^N - c$ is the sum of the contributions of all other individuals but individual $i$. The condition determining the equilibrium level of $C$ is:

$$\frac{-pL'(C_{SI}^N)U_2'}{EU'} = 1. \tag{21}$$

The left hand side represents the marginal willingness to pay for additional SI in units of foregone utility, while the right hand side denotes the marginal costs of such an effort. To express the marginal benefit and the marginal cost with respect to the public good the condition can be rearranged to

$$-pL'(C_{SI}^N)U_2' = (1-p)U_1' + pU_2'. \tag{22}$$

Each individual contributes until the marginal benefit of an additional investment in the public good to reduce the size of the loss (left hand side) equals the marginal cost of this additional spending on the public good, which accrues in both states of the world (right hand side). From condition (21) we can calculate the slope of the reaction function for a representative individual:

$$\frac{dc_i}{dC_{-i}^{SI}} = -\frac{pU_2''(-L'(C_{SI}^N))(-1-L'(C_{SI}^N)) + pU_2'(-L''(C_{SI}^N))}{pU_2''(-1-L'(C_{SI}^N))^2 + pU_2'(-L''(C_{SI}^N)) + (1-p)U_1''}. \tag{23}$$

The slope (23) of the reaction function is negative, which means that $C_{-i}^{SI}$ and one's own contribution $c_i$ are substitutes. It obtains because both numerator and denominator in (23) are negative (the Inada assumptions imply that $-1-L'(C_{SI}^N) > 0$). Whether the slope is larger or smaller than -1 (i. e., whether one under- or overcompensates the contributions of the other individuals) depends as follows on the measure of absolute risk aversion. The difference between denominator and numerator is

$$-pU_2''(-1-L'(C_{SI}^N)) + (1-p)U_1''. \tag{24}$$

For the slope (23) to lie between -1 and 0, this difference must be negative, i. e., the denominator must be larger than the numerator in absolute terms, which using the FOC (21) is equivalent to

$$\begin{aligned}
& (1-p)U_1'' &<& \; pU_2''(-1 - L'(C_{SI}^N)) \\
\iff & (1-p)U_1'' &<& \; pU_2''\frac{1-p}{p}\frac{U_1'}{U_2'} \\
\iff & \frac{U_1''}{U_1'} &<& \; \frac{U_2''}{U_2'} \\
\iff & A_1 := A(y_1) &>& \; A(y_2) =: A_2,
\end{aligned}$$

which establishes the following

**Lemma 2 (Privately provided SI)**
*The slope of the reaction function in a setting of private provision of SI depends on how the Arrow-Pratt measure of absolute risk aversion $A$ changes with wealth. The slope*

1. *is smaller than -1 for decreasing absolute risk aversion (DARA): $A_1 < A_2$.*

2. *is equal to -1 for constant absolute risk aversion (CARA): $A_1 = A_2$.*

3. *lies between -1 and 0 for increasing absolute risk aversion (IARA): $A_1 > A_2$.*

In effect, the absolute risk aversion takes the role of normality in determining the reaction to increased provision of the public good. In the standard model of private contributions to a public good (where the goods are consumption goods and not insurance devices), all goods are usually assumed to be (strict) normal in consumption. This results in (strictly) decreasing reaction functions with slope between 0 and 1 in absolute value. When some individual increases his contribution to the public good, the increased provision of the public good amounts to an income increase in units of the public good. By normality of all goods, the individual distributes this income increase between all consumption goods. If the slope of the reaction function lies between 0 and 1 in absolute value, following some increase in the contribution to the public good, the other individual reduce their commitment underproportionally, such that the original increase in contributions leads to an overall increase in provision, but by less than the original increase. This is termed a “normal” reaction.

As we interpret the public good as a SI device, the reaction to an increase of provision by the other players depends on how the absolute risk aversion changes with wealth and the effect of absolute risk aversion leads to a different effect. Suppose some player increases her contribution to the SI device, which leads to a smaller loss $L$. This amounts to a wealth transfer, and this wealth increase changes the absolute risk aversion of the players. For the empirically relevant case of decreasing absolute risk aversion, the slope of the reaction function is greater than 1 in absolute value. A higher income means less demand for insurance, and crucially, this reaction is overproportional, i. e., an original *increase* in the contribution to public SI may lead, after the reactions of the other individuals who reduce their commitment, to a *decrease* in the total provision level of public SI.

**Proposition 1 (Equilibrium of privately provided SI)**
*For all risk attitudes, the private provision Nash equilibrium $C_{SI}^N$ of SI contributions exists, but is, in general, not unique. For decreasing absolute risk aversion we may obtain multiple Nash equilibria. For increasing absolute risk aversion, the private provision Nash equilibrium is unique.*

Proof. The existence of an equilibrium is assured, since preferences are strictly monotonic and continuous (see Bergstrom et al., 1986). By Lemma 2, for decreasing absolute risk aversion, the slope of the reaction function (23) is smaller than -1. Thus, the reaction of individual $i$ to a change in the sum of the contributions of the other individuals $C_{-i}$ is not normal in the sense that an increase in the contribution by some player may lead to a more than proportional reduction of the other players, resulting in a lower provision level. These non-normal preferences may lead to multiple equilibria and to overprovision of the public good when the good is provided privately and the preferences and non-normal (see Buchholz and Peters, 2001). For increasing absolute risk aversion the slope of the reaction function (23) lies between -1 and 0, which corresponds to “normal” preferences in the standard consumer theory leading to a unique private provision level (Cornes et al., 1999). QED.

The usual theoretical and empirical assumption of the literature regarding absolute risk aversion is that it is decreasing in wealth (DARA). This means that an individual who gets richer is willing to take higher absolute risks. In our insurance setting, DARA leads the individual to *reduce* his commitment to the SI device. The other individuals react by *increasing* their contributions in an overproportional way, such that the resulting provision level is higher than before. This is the mechanism by which "non-normal" voluntary contributions may lead to overprovision of the public good (see Kerschbamer and Puppe, 1998). Below, we will be put an upper bound to this overprovision anomaly using an intermediate result from a situation where the individuals can buy additional private insurance.

**Self-protection**

In the SP case each individual maximizes her expected utility $EU$ by her choice of $c$, taking the contributions of the other $n-1$ individuals, which also reduce the probability of the loss, as given. $C_{-i}^{SP} = \sum_{j=1, j\neq i}^{n} c_j^{SP} = C_{SP}^N - c$ is the sum of the contributions of all other individuals but individual $i$. The first-order condition is:

$$\frac{-p'(C_{SP}^N)(U_1 - U_2)}{EU'} = 1. \tag{25}$$

To express the marginal benefit and the marginal cost with respect to the public good this condition can be rearranged to

$$-p'(C_{SP}^N)(U_1 - U_2) = (1 - p(C_{SP}^N))U_1' + p(C_{SP}^N)U_2'. \tag{26}$$

Each individual contributes until the marginal benefit of an additional investment in the public good to reduce the probability of the loss (left hand side) equals the marginal cost of this additional spending on the public good, which accrues in both states of the world (right hand side). From (25) we can calculate the slope of the reaction function for a representative individual:

$$\frac{dc_i}{dC_{-i}^{SP}} = -\frac{-p''(C_{SP}^N)(U_1 - U_2) - p'(C_{SP}^N)(U_2' - U_1')}{-p''(C_{SP}^N)(U_1 - U_2) - 2p'(C_{SP}^N)(U_2' - U_1') + (1 - p(C_{SP}^N))U_{i1}'' + p(C_{SP}^N))U_{i2}''}. \tag{27}$$

The denominator is negative because the second order condition holds. The numerator is also negative and larger than the denominator in absolute terms by Assumption 1:

$$\begin{aligned} & -p'(C_{SP}^N)\underbrace{(U_2' - U_1')}_{>-L\cdot EU''} + EU'' > -p'(C_{SP}^N)(-L \cdot EU'') + EU'' \\ = \; & EU''[\underbrace{(-p'(C_{SP}^N)(-L) + 1)}_{<0}] > 0, \end{aligned}$$

Thus, the slope (27) of the reaction function is negative, which again means that $C_{-i}^{SP}$ and one's own contribution $c_i$ are substitutes and, remarkably, the slope (27) is smaller than -1:

**Lemma 3 (Privately provided SP)**
*The slope of the reaction function in a setting of private provision of SP is smaller than -1 if the second order condition and Assumption 1 hold.*

In the SP case, it is Assumption 1 that describes and determines the reaction of an individuals risk response to a change in wealth. Lemma 3 means that the reaction of the individual is, as in the SI case, overproportional and that there are multiple SP effort equilibria. This result parallels the results of Ihori and McGuire (2007).

**Proposition 2 (Equilibrium of privately provided SP)**
*If Assumption 1 is met, the private provision Nash equilibrium $C_{SP}^N$ of SP contributions exists but is, in general, not unique.*

Proof. The proof proceeds along the same lines as the proof of Proposition 1, where the slope of the reaction function (27) is smaller than -1 by Lemma 3. QED.

Thus, the reaction of individual $i$ to a change in the sum of the contributions of the other individuals $C_{-i}$ is overproportional which may lead to the overprovision anomaly because the wealth increase through the contributions to public SP changes the risk incentives of the individuals. Again, below we will put an upper bound on the overprovision anomaly.

## 4.2 Interaction of private provision with market insurance

In the following, we analyze the interaction between a public good that is privately provided and private market insurance and especially whether it is individually optimal to contribute to a public good which acts as an insurance device when private market insurance is available.

### Self-insurance

For SI, the representative individual maximizes her expected utility

$$EU(c, C, s) = pU(\omega - c - L(C) + (1 - \pi)s) + (1 - p)U(\omega - c - \pi s). \qquad (28)$$

We write $\hat{C}_{SI}^{N}$ for the Nash equilibrium level of the public good in the SI case with market insurance. The maximization of (28) with respect to $c$ and $s$ leads to

$$\frac{\pi}{1-\pi} = \frac{1}{-1-L'(\hat{C}_{SI}^{N})}. \tag{29}$$

The optimum is reached when the shadow price of SI, as given by the right hand side, is equal to the market price of insurance (left hand side). In other words, the individual is indifferent whether to spend an additional unit of wealth in private SI or private market insurance. If the price for market insurance is fair, $\pi = p$, condition (29) can be rearranged to implicitly determine the privately provided efficient level of a SI public good $C$ in the presence of market insurance:

$$\frac{1}{-1-L'(\hat{C}_{SI}^{N})} = \frac{p}{1-p} \qquad \Longleftrightarrow \qquad p \cdot (-L'(\hat{C}_{SI}^{N})) = 1. \tag{30}$$

Condition (30) is also the condition that maximizes expected income. However, in contrast to the efficient provision, expected income is maximized at the individual and not at the social level. The provision level $\hat{C}_{SI}^{N}$ of condition (30) can be compared with the privately provided provision level $C_{SI}^{N}$ without market insurance as given by equation (21):

$$-L'(\hat{C}_{SI}^{N}) = \frac{1}{p} > \frac{1}{p}\frac{EU'}{U'_{12}} = -L'(C_{SI}^{N}). \tag{31}$$

Since the marginal utility in the loss state 2 is larger than in non-loss state 1, $U'_{12} > U'_{11}$,

$$\frac{EU'}{U'_2} = \frac{(1-p)U'_1 + pU'_2}{U'_2} < 1, \tag{32}$$

such that

$$-L'(\hat{C}_{SI}^{N}) > -L'(C_{SI}^{N}) \qquad \Longleftrightarrow \qquad \hat{C}_{SI}^{N} < C_{SI}^{N}. \tag{33}$$

The possibility of buying market insurance decreases the privately provided level of the public good further. Thus, market insurance and SI are strategic substitutes in the sense that a market price increase in market insurance decreases the demand for market insurance and increases the demand for SI, which has become relatively cheaper.

To compare the efficient and the private provision level of SI when market insurance is available, we use conditions (14) and (30). Since the efficiency condition (14) contains the size $n$ of the population that benefits from public SI and the private provision condition

(30) does not reflect the positive external effect of the public good,

$$\hat{C}_{SI}^{N} < \hat{C}_{SI}^{*}. \tag{34}$$

In order to compare the provision levels in the SI situation, it remains to compare the first best situation $\hat{C}_{SI}^{*}$ (Samuelson level with private market insurance) vs. the situation with private provision $C_{SI}^{N}$ (Nash equilibrium level with no private market insurance). Intuitively, the provision level under the second situation should be smaller, the larger the number $n$ of individuals. Comparing conditions (14) and (21) leads to the inequality

$$-L'(\hat{C}_{SI}^{*}) = \frac{1}{n \cdot p} < \frac{1}{p}\frac{EU'}{U_2'} = -L'(C_{SI}^{N}), \tag{35}$$

for a big enough $n$, since $EU'/U_2' < 1$ and has a fixed value. It follows that

$$-L'(\hat{C}_{SI}^{*}) < -L'(C_{SI}^{N}) \qquad \Longleftrightarrow \qquad \hat{C}_{SI}^{*} > C_{SI}^{N}. \tag{36}$$

Combining (36) with (16) and (33), we obtain

**Proposition 3 (Comparison of SI provision levels)**
*If $n$ is big enough such that condition (35) is satisfied, the efficient SI provision levels with and without market insurance $\hat{C}_{SI}^{*}$ and $C_{SI}^{*}$, respectively, and the Nash provision levels with and without market insurance $\hat{C}_{SI}^{N}$ and $C_{SI}^{N}$, respectively, are ranked in the following order:*

$$\hat{C}_{SI}^{N} < C_{SI}^{N} < \hat{C}_{SI}^{*} < C_{SI}^{*}. \tag{37}$$

Notice that this result indirectly bounds the overprovision anomaly that can occur at the Nash equilibrium of private provision. If $n$ is big enough, (36) shows that the efficient Samuelson provision level when market insurance is available is greater than the Nash equilibrium level. Since the availability of market insurance reduces the efficient provision level, a large enough population rules out the overprovision anomaly which may arise because of the overproportional best-response reactions of the contributors.

**Self-protection**

Let us now focus on the SP situation. The fair price for market insurance is then given by $\pi = p(C)$. Hence, the public good does not only - to some extent - protect individuals, but decreases also the price of the insurance. As insurance is assumed to be fair, individuals always fully insure. The representative individual maximizes her expected utility

$$EU(c, C, s) = p(C)U(\omega - c - L + (1 - p(C))s) + (1 - p(C))U(\omega - c - p(C)s). \quad (38)$$

In this situation, the individual equals the marginal utilities (and thus income levels) in both states of the world. Since insurance is fair, the individuals choose full cover $s = L$ independently of the additional SP effort. Since $U_1 = U_2$, we can write the condition describing implicitly the privately provided efficient level of a public SP good $C$ in the presence of market insurance as

$$-p'(\hat{C}_{SP}^N)L = 1. \quad (39)$$

We can compare the Nash private provision equilibrium without market insurance as defined by (25) with the corresponding private provision equilibrium when market insurance is available as described by (39):

$$\frac{-p'(C_{SP}^N)(U_1 - U_2)}{EU'} = -p'(\hat{C}_{SP}^N)L \quad (40)$$

Using Assumption 1, $(U_1 - U_2)/(EU') > L$, we find

$$-p'(C_{SP}^N) < -p'(\hat{C}_{SP}^N) \quad \Longleftrightarrow \quad C_{SP}^N > \hat{C}_{SP}^N \quad (41)$$

then market insurance reduces further the private provision level of the public good. To compare the efficient and the private provision level of SP when market insurance is available, conditions (18) and (39) are relevant. As in the case of SI, the efficiency condition (18) contains the size $n$ of the population that benefits from public SI and the private provision condition (39) does not reflect the positive external effect of the public good, the private provision level is inefficiently small:

$$\hat{C}_{SP}^N < \hat{C}_{SP}^*. \quad (42)$$

In order to compare the provision levels in the SP situation, it remains to compare the first best situation $\hat{C}_{SP}^*$ (Samuelson level with private market insurance) vs. the situation with private provision $C_{SP}^N$ (Nash equilibrium level with no private market insurance).

Intuitively, the provision level under the second situation should be smaller, the larger the number $n$ of individuals. Comparing conditions (18) and (25) leads to the inequality

$$-p'(\hat{C}^*_{SP}) = \frac{n}{L} < \frac{EU'}{U_1 - U_2} = (-p'(C^N_{SP})), \tag{43}$$

again assuming $n$ is big enough and remembering that $(EU')/(U_1 - U_2) > 1/L$ by Assumption 1. We thus obtain

$$-p'(\hat{C}^*_{SP}) > -p'(C^N_{SP}) \qquad \Longleftrightarrow \qquad \hat{C}^*_{SP} > C^N_{SP}. \tag{44}$$

Combining (44) with (20) and (41)we get

**Proposition 4 (Comparison of SP provision levels)**
*If $n$ is big enough such that condition (43) is satisfied, the efficient SP provision levels with and without market insurance $\hat{C}^*_{SP}$ and $C^*_{SP}$ and the Nash provision levels with and without market insurance $\hat{C}^N_{SP}$ and $C^N_{SP}$ are ranked in the following order:*

$$\hat{C}^N_{SP} < C^N_{SP} < \hat{C}^*_{SP} < C^*_{SP}. \tag{45}$$

For both SI and SP, market insurance is a strategic substitute and decreases the provision level of the public good. Yet the decrease caused by Nash behavior is even greater. Thus, the provision level of the public good under a Nash private provision equilibrium is smaller than the Samuelson equilibrium level reduced by the availability of market insurance.

# 5 Conclusion

Many public goods provide utility to the society only due to an insurance effect of reducing the size or the probability of possible uncorrelated losses. Our paper analyzes such public goods and thereby extends and combines three strands of the literature: the public goods literature, the literature on private provision of public goods and the private SI and SP literature.

Combining all those elements in one model, we study how more risk averse societies prefer higher levels of public SI and public SP. We show how the “normality” concept of the public goods literature can be interpreted in our risk model as decreasing absolute risk aversion (in the SI case) and as a condition of the probability of the loss (in the SP case). These conditions highlight the theoretical similarities and differences that our model brings out.

An interesting aspect of regarding public goods as insurance devices is the interaction with market insurance. The presence of market insurance decreases the efficient provision level of the public good, since fully insured individuals behave as if they were risk neutral. The private provision of public goods is also reduced by the availability of market insurance. Since the publicly provided level of the public good will, in general, be observable by insurers in the case of SP, public goods may be superior to private SP activities if moral hazard problems are involved. This means that the moral hazard problem may not occur in the case of public SP, which is an advantage compared to private SP expenditures.

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