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Abstract:
This paper characterizes long-run and short-run optimal fiscal policy in the labor selection framework. In a calibrated non-Ramsey decentralized equilibrium, labor market volatility is inefficient. Keeping fixed the structural parameters, the Ramsey government achieves efficient labor market volatility; doing so requires labor-income tax volatility that is orders of magnitude larger than the tax-smoothing results based on Walrasian labor markets, but a few times smaller than the results based on search and matching markets. We analytically characterize selection-modelconsistent wedges and inefficiencies in order to understand optimal tax volatility.

JEL Classification: E24; E32; E50; E62; E63; J20

Keywords: labor market frictions, hiring costs, efficiency, optimal taxation, labor wedge, zero intertemporal distortions

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1 Introduction

This paper characterizes long-run and short-run optimal fiscal policy in the labor selection framework, in which unemployed individuals are selected into new jobs based on idiosyncratic hiring costs. We start by calibrating structural parameters so that an *exogenous* policy economy displays empirically relevant business-cycle volatilities in the labor market. Keeping intact these parameters, the cyclical volatilities of these quantities are dramatically smaller under *optimal policy.* The reason is that wage determination (and hence surplus-sharing) in the calibrated non-Ramsey economy implies inefficient use of resources, which optimal tax policy corrects. The intuition is as follows: because societal returns from possible new jobs only depend on individuals' job search activities, in order for efficiency to be achieved in the decentralized economy, newly-hired workers should receive the full surplus. If the surplus is split between employers and firms, as it usually is, inefficient fluctuations ensue.

Our analysis focuses squarely on the selection phase of labor markets. In order to do so, we purposefully omit search and matching and hence its well-known notion of labor market tightness. Nonetheless, we analytically develop a highly-analogous concept of "tightness" in the pure selection model. Like in the matching model, the selection framework's tightness is a *primitive* of the economy. For the Ramsey government to keep tightness, and hence labor-market quantities, on its efficient path during business-cycle swings, optimal labor income tax volatility is orders of magnitude larger than in first-generation macro-Ramsey "tax smoothing" results, but is a few times smaller than the results based on search and matching labor markets in Arseneau and Chugh (2012).

In the selection model, the distribution of hiring costs (which could be interpreted as "match-quality" characteristics) is a technological primitive of the economy which leads to increasing marginal costs of hiring from the pool of searching individuals. Starting from the efficiency results of Chugh and Merkl (2015), we construct the selection framework's general-equilibrium transformation function and, in turn, its model-consistent marginal rates of transformation and labor-market wedges. Efficient allocations in the selection model require a particular *endogenous cost spread* between the average cost of hiring and the marginal cost of hiring.

To make our results more comparable to Arseneau and Chugh (2012) and, more broadly, the macro-Ramsey literature that began in the 1980s, we develop the selection model's appropriate *wedges* — see Chari and Kehoe (1999, p. 1674) for more on the importance of wedges in normative analysis — which in turn reveals the analytical concept of tightness in the labor selection framework. The definition of this new concept of tightness is of course different than in the matching literature, but it plays *exactly* the same crucial role as market tightness does in the search and matching model for both efficiency considerations and optimal policy.

In a decentralized economy with individualistic Nash bargaining, equilibrium wage dispersion arises, and the endogenous cost spread manifests itself as a *wage premium* between the *average* newly-hired employee relative to the *marginal* newly-hired employee. Our results show that optimal policy ensures that the wage premium (and thus the hiring-cost spread) is *identical* to its socially efficient value both in the long run and along the business cycle, independent of the assumed *exogenous* distribution of idiosyncratic hiring costs. The cost spread, upon which the wage premium depends, is what we define as "tightness" in the selection model; this new concept is a useful innovation for researchers working with the labor selection framework.

As mentioned, our analysis intentionally abstracts from search and matching in order to isolate the role of optimal fiscal policy in the selection model. Nonetheless, departures from efficient tightness create distortions in labor markets in both the selection model and the matching model. Despite the need by the Ramsey government to raise revenues via non-lump sum taxation, efficient labor market dynamics, not surprisingly, are a primary goal. Our quantitative results show that the Ramsey government achieves *perfect* wedge smoothing for all structural parameter constellations, which is different than the wedge-smoothing results in Arseneau and Chugh (2012). Referring to Arseneau and Chugh (2012, Table 5 on p. 957), there is only one parameter set for which *perfect* wedge smoothing is achieved in the search and matching model.

This comparison of wedge dynamics provides a normative perspective on how the *labor demand* mechanism operates differently in the selection model compared to the search and matching model. This difference is known in the growing labor selection literature, but not from the rigorous normative wedge perspective. Comparing the analytical results we discover to those discovered in Arseneau and Chugh (2012), we find that, apart from the newly-developed tightness concept, there is a deep connection between the selection model's normative results and the matching model's normative results once *endogenous labor supply* is included. Considering the two models separately, intuitively, optimal labor supply (more precisely, optimal labor-force participation) depends on the likelihood

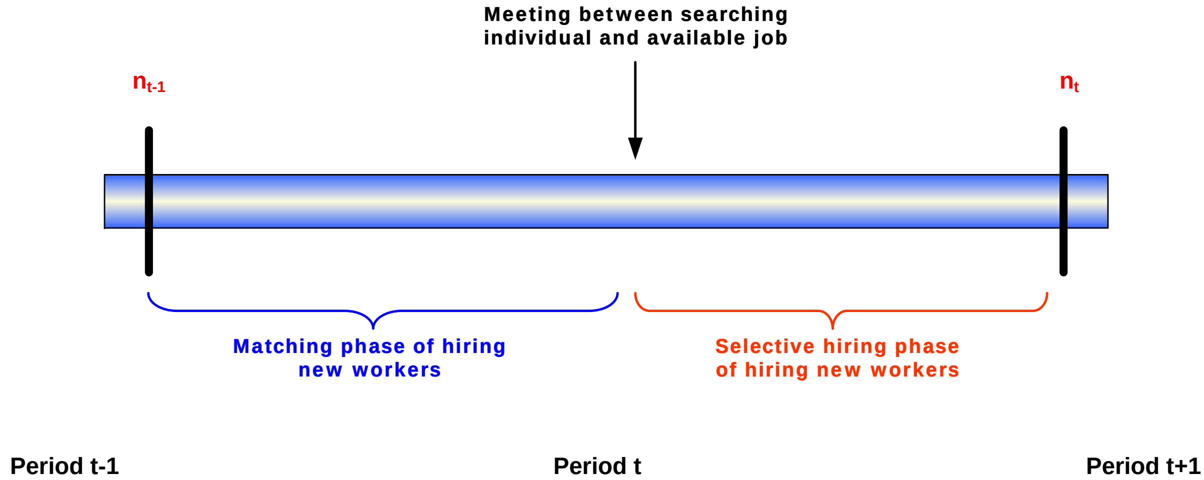

Figure 1: **Timing of Recruiting Costs.** Selective hiring costs occur after a meeting between an individual searching for a job and an open job opportunity, whereas vacancy posting costs (which are abstracted from in this paper) occur before a contact.

of obtaining the average wage in both models. With individual-specific Nash-bargained wages and constant-returns-to-scale production of goods, all new hires in the search and matching model earn the same wage, but *the average wage is larger than the marginal wage in the labor selection model.*

This difference in the average wage vs. the marginal wage arises from the different labor demand mechanisms across the two frameworks, which in turn depends on the *timing* of hiring costs. Recruiting costs include more than just vacancy posting costs, as documented at the establishment level by Davis, Faberman, and Haltiwanger (2013). Figure 1 fixes this idea: recruiting costs occur after a job match in the selection model, whereas they occur before a match in the search model.

Finally, in terms of our results, the large long-run hiring subsidy that emerges in the baseline calibration of the selection model provides some policy advice for current discussions and debates. Workers who have been out of the workforce or out of the labor force often require re-training for new jobs. Training has inherent costs, which would fall entirely upon the private sector without government intervention. Some portion of the costs of re-training workers is persistent, and some portion of the costs is more of a temporary nature. For the sake of computational tractability, our model is silent on the persistent component of training costs and, more broadly, costs of ingraining "permanent skills." Our model focuses on the temporary nature of (re-)training costs. The subsidy is indicative of how fiscal policy reforms can train displaced workers and ease labor-market distortions, as many policy makers have been advocating.[1]

Our paper contributes to a growing literature that studies optimal policy in "frictional" labor markets. "Frictional" labor markets are those that depart from Walrasian labor markets and allow for the possibility of unemployment, the availability of rents to be shared between newly-hired workers and their respective production units, and have technological labor-market primitives above and beyond those that are embedded in the standard aggregate consumption-goods resource constraint. A few recent papers along this line are Arseneau and Chugh (2012), Faia, Lechthaler, and Merkl (2014), Michaillat and Saez (2015), Cacciatore and Fiori (2016), and Jung and Kuester (2015).

Most of these recent papers concentrate on the search and matching framework. The idea of selection as an important margin of adjustment in firms' hiring decisions is a long-standing one, although it has not yet been much emphasized in the macro-labor literature. An early important empirical firm-level contribution was Barron, Bishop, and Dunkelberg (1985, p. 50), who adopted and found strong evidence for the view that "...most employment is the outcome of an employer selecting from a pool of job applicants..." due to cross-sectional heterogeneity in the pool of applicants. Davis, Faberman, and Haltiwanger (2013) add further evidence to the view that, in their terminology, "hiring standards" play an important role among the many margins of labor adjustment. Selection issues and their associated costs seemingly would be an important component of hiring standards. As shown by Lechthaler, Merkl, and Snower (2010), the selection model generates both empirically-relevant amplification in labor market measures and plausible correlations of key macroeconomic variables without extreme parameter assumptions.

The rest of the paper is organized as follows. Section 2 describes the economic environment. Section 3 calibrates and quantifies an exogenous-policy economy to match empirical volatilities in the job-finding rate, the unemployment rate, and in the labor-force participation rate. Using the structurally-calibrated model, Section 4 studies the Ramsey problem. Section 5 analytically develops the selection model's general-equilibrium concepts of static and intertemporal marginal rates of transformation and efficiency, and Section 6 shows which features of the decentralized economy disrupt efficiency. Section 7

[1] One prominent example is former Federal Reserve Chairman Ben Bernanke. In his memoir, *The Courage to Act* (2015), Bernanke states not just the importance of training programs but also the importance of *fiscal policy* stepping up and providing subsidies to such programs (for example, see p. 504 and p. 576).

discusses several aspects of the model and results, and also develops the analytics of the selection model's concept of labor-market "tightness." Section 8 concludes. A detailed set of Appendices proves the main results and provides many technical details of the model, along with many quantitative robustness checks, that will be useful for future research using the selection framework.

2 Model

This section presents the labor selection model, which is based on Lechthaler, Merkl, and Snower (2010) and its variant with endogenous labor force participation in Chugh and Merkl (2015). The model uses the "instantaneous hiring" view of transitions between unemployment and employment, in which new employees begin producing right away, rather than with a one-period delay.

2.1 Labor Market Accounting

Suppose that n_{t-1} individuals produced output in period $t-1$. At the beginning of period t, a fraction ρ of these individuals separate from their production opportunities. Some of these newly-separated individuals may immediately enter the period-t labor force, as may some individuals who were non-participants in period $t-1$; these two groups taken together constitute the measure s_t of individuals searching and available to begin work in period t. However, unlike models based on the Pissarides (1985) framework, there is no matching function that brings (with probability less than one) individuals available for work into contact with production opportunities. Rather, each individual available for work makes contact with ("matches" with) a production opportunity. For the sake of clarity of the optimal policy analysis, each individual available for work makes only one contact per period. However, Chugh and Merkl (2015) show that the number of per-period contacts can be generalized to $N > 1$ by embedding sequential search in the selection framework.

Unemployed individual i has idiosyncratic match-quality costs, denoted by ε_i, which is a draw from a cumulative distribution function $F(\varepsilon)$, with associated density $f(\varepsilon)$. It is only unemployed individuals that are heterogenous; individuals who have been employed for more than one period are identical in their characteristics. We refer interchangeably to an individual's ε as short-run "training costs," short-run "operating costs," or, most

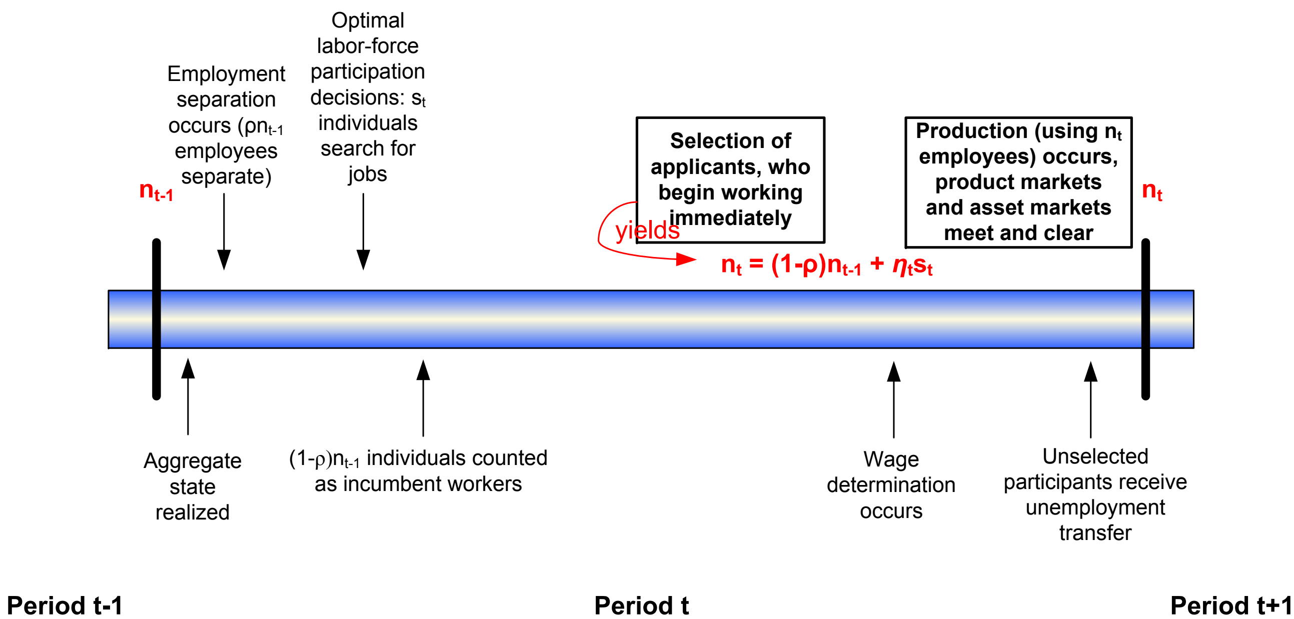

Figure 2: **Timing of events in decentralized economy.**

often, short-run "hiring" costs. Regardless of terminology, these costs of shadowing other workers or "apprenticeship" costs are measured in units of output, and marginal costs increase as more new employees are hired.

Of the s_t unemployed individuals, $(1-\eta_t)s_t$ individuals turn out to be unsuccessful in becoming employed, where η_t is the probability that an individual available for work is selected and begins producing. This probability is taken as given by individuals, but, as described below, it is endogenous to the environment. The measure

$$n_t = (1-\rho)n_{t-1} + s_t\eta_t \tag{1}$$

of individuals are thus employed and produce in period t. Each of the $(1-\eta_t)s_t$ individuals who does not find a job receives a government-provided unemployment benefit χ. With these definitions and timing of events, the measured labor force in period t is $lfp_t = n_t + (1-\eta_t)s_t$. By substituting (1), participation can alternatively be measured as $lfp_t = (1-\rho)n_{t-1} + s_t$. Figure 2 summarizes the timing of the model.

Because integrating a worker into production is costly, profit-maximization requires that only those individuals with sufficiently attractive characteristics are brought into the production process. Integrating an individual into the production process entails costs, which is interpreted as an *average hiring cost* that reflects training and other startup activities for each new worker. There is thus a threshold $\tilde{\varepsilon}_t$, which is a function of the state of the economy, for selection of unemployed individuals once screening has revealed their types. Because individuals' idiosyncratic characteristics are defined as a *cost*, only those individuals with $\varepsilon_{it} \leq \tilde{\varepsilon}_t$ are brought into the production process. The probability that an unemployed individual is hired is thus $\eta(\tilde{\varepsilon}_t)$ $(= F(\tilde{\varepsilon}_t))$, and the aggregate number of individuals selected in period t is $\eta(\tilde{\varepsilon}_t)s_t$.[2,3] The next two subsections describe the household's labor-force participation decision and the firm's selection (hiring) decision.

2.2 Households

There is a measure one of individuals in the economy. Each individual, whether employed, unemployed, or outside the labor force, has full consumption insurance, which is modeled by assuming that all individuals belong to a representative household that pools income and shares consumption. This "large household" assumption is a tractable way of modeling perfect consumption-risk insurance, and has been standard in the matching literature since Andolfatto (1996) and Merz (1995) (which in turn are based on the seminal Hansen (1985) and Rogerson (1988) results for RBC models.)

The subjective discount factor is β, the function $u(.)$ is a standard strictly-increasing and strictly-concave subutility function over consumption, and the function $h(.)$ is strictly increasing and strictly convex in the size of the labor force.[4] For intuition and because it facilitates analogy with both the RBC model and the basic search and matching matching model, it will be helpful to interpret the measure $1 - lfp_t$ of individuals outside the labor force as enjoying leisure. We thus use the terms leisure and non-participation interchangeably. Without any confusion between household-level variables and aggregate variables, the measure of participants s_t is understood in this section to be a consequence of the household's decisions, while the household takes as given the selection threshold $\tilde{\varepsilon}_t$ and thus any functions of it (in particular the hiring rate $\eta(.)$ and the average wage of a new hire $\omega_e(.)$).

[2]Depending on context, we sometimes emphasize the dependence of the hiring probability on $\tilde{\varepsilon}_t$ and sometimes simply write η_t to conserve on notation.

[3]Readers familiar with endogenous separations in a search and matching model can see the parallels between the endogenous mass of newly-*hired* workers who have idiosyncratic characteristics and the endogenous mass of newly-*separated* workers who have idiosyncratic characteristics.

[4]Given the definitions presented above, we sometimes will write $h(lfp_t)$.

The representative household maximizes expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h\left(n_t + (1-\eta_t)s_t\right)\right] \tag{2}$$

subject to the sequence of budget constraints

$$c_t + \sum_j \frac{1}{R_t^j} b_{t+1}^j = (1-\rho)n_{t-1}(1-\tau_t^n)w_t^I + \left(\frac{(1-\tau_t^n)\omega_{et}}{\eta_t}\right)\eta_t s_t + (1-\eta_t)s_t\chi + b_t + (1-\tau^{pr})\Pi_t \tag{3}$$

and perceived laws of motion for its employment level

$$n_t = (1-\rho)n_{t-1} + \eta_t s_t. \tag{4}$$

The rest of the notation is as follows. The wage each incumbent worker earns is w_t^I; ω_{et}/η_t is the *conditional* average wage paid to a new hire (conditional on being hired); τ_t^n is the tax rate on labor income for both incumbent and newly-hired individuals; χ is a government-provided unemployment benefit received by each unemployed (unselected) participant; end-of-period holdings of a complete set of state-contingent government bonds are denominated as b_{t+1}^j (j indexes the possible states in period $t+1$); and Π_t is aggregate operating profits of firms that are distributed in lump-sum manner to households and are taxed by the government at the rate τ^{pr}). As is well-understood in the Ramsey literature, flows of untaxed profits received by households in and of themselves affect optimal-policy prescriptions.[5] To make our results as comparable as possible to baseline models that prescribe labor-tax-rate smoothing, in which there are zero economic profits/dividends, our main analysis is conducted assuming $\tau^{pr} = 1$.

The formal analysis of the household's problem appears in Appendix A; here we simply describe the household's optimality conditions. Aside from the standard bond Euler conditions

$$u'(c_t) = \beta R_t^j u'(c_{t+1}^j), \ \forall j, \tag{5}$$

[5] See, for example, Stiglitz and Dasgupta (1971), Schmitt-Grohe and Uribe (2004), and Siu (2004) for examples in various contexts of this type of taxation incentive.

the household's *labor-force participation (LFP) condition* is

$$\frac{h'(lfp_t)}{u'(c_t)} = \eta_t \left[\frac{(1-\tau_t^n)\omega_{et}}{\eta_t} + (1-\rho)E_t \left\{ \Xi_{t+1|t} \left[(1-\tau_{t+1}^n) w_{t+1}^I + \frac{\mu_{ht+1}}{u'(c_{t+1})} \right] \right\} \right] + (1-\eta_t)\chi, \tag{6}$$

in which $\Xi_{t+1|t} = \frac{\beta u'(c_{t+1})}{u'(c_t)}$ is the one-period-ahead stochastic discount factor and μ_{t+1}^h is the shadow value at time $t+1$ of the household's beginning-of-period $t+1$ employment stock n_t. The LFP condition has straightforward interpretation: at the optimum, the household makes available for hiring a fraction of individuals such that the MRS between participation and consumption is equated to the expected payoff of participation. The payoff is either an unemployment benefit χ in the event a given individual is not selected (which happens with probability $1-\eta_t$) or, if a given individual is selected, an immediate after-tax (expected) wage (where the expectation is with respect to the possible realizations of worker characteristics) plus an expected discounted continuation value. The LFP condition is thus simply a free-entry condition on the part of households into the labor market, and it can be interpreted as the private economy's labor supply function.

2.3 Firms

The representative firm hires and makes wage payments to workers, some portion of which are incumbent workers and some portion of which are new employees. As indicated in Figure 2, wages are set after period-t selection has occurred. Mentioned in Section 2.2 was the conditional average wage paid to a newly-hired worker. More precisely, define

$$\omega_e(\tilde{\varepsilon}_t) \equiv \int^{\tilde{\varepsilon}_t} w(\varepsilon_{it}) f(\varepsilon_{it}) d\varepsilon_{it} \tag{7}$$

as the *average* wage paid to a new hire in period t. The notation $w(\varepsilon_{it})$ makes clear that in the period in which he is hired, a worker's wage may be conditioned on his idiosyncratic operating cost.

Letting $H(\tilde{\varepsilon}_t)/\eta(\tilde{\varepsilon}_t)$ denote the average *idiosyncratic* operating cost for each newly-selected worker — with $H(\tilde{\varepsilon}_t) \equiv \int_{-\infty}^{\tilde{\varepsilon}_t} \varepsilon f(\varepsilon) d\varepsilon$ — in period zero, the representative firm chooses state-contingent decision rules for its desired employment stock and the threshold operating cost $\tilde{\varepsilon}_t$ below which it is willing to hire in order to maximize discounted profits

$$E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left[z_t n_t - (1-\tau_t^h) \left(\frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} \right) \eta(\tilde{\varepsilon}_t) s_t - (1-\rho) n_{t-1} w_t^I - \left(\frac{\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} \right) \eta(\tilde{\varepsilon}_t) s_t \right]. \tag{8}$$

In (8), τ_t^h is a government-provided hiring subsidy, and $\Xi_{t|0}$ is the period-0 value to the representative household of period-t goods, which the firm uses to discount profit flows because households are the ultimate owners of firms. Profits are given by the level of production revenue net of hiring costs for newly-selected workers as well as wage payments to both incumbent and newly-selected workers.

Without any confusion between firm-level variables and aggregate variables, the hiring rate η_t is understood in this section to be a consequence of the firm's decisions, while the firm takes as given the number of job-seekers s_t as well as, as is standard in search and matching models, the wage-setting process. Because output is sold in a perfectly-competitive market, the firm's problem is to choose $\tilde{\varepsilon}_t$ and n_t, $\forall t$, to maximize (8) subject to a sequence of perceived laws of motion for its employment level,

$$n_t = (1-\rho)n_{t-1} + s_t\eta(\tilde{\varepsilon}_t). \tag{9}$$

The formal analysis of the firm's problem appears in Appendix B; here we simply intuitively describe the outcome. The firm's *hiring (selection) condition* is

$$(1-\tau_t^h)\cdot\tilde{\varepsilon}_t = z_t - w(\tilde{\varepsilon}_t) + (1-\rho)E_t\left\{\Xi_{t+1|t}\left((1-\tau_{t+1}^h)\cdot\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right\}. \tag{10}$$

At the optimum, the firm selects workers from the distribution of applicants until the post-subsidy cost of bringing an individual into its production activities, $\tilde{\varepsilon}_t$, is equated to the payoff of hiring, which is the net marginal revenue product $z_t - w(\tilde{\varepsilon}_t)$ plus, conditional on the individual working beyond the first period of his employment relationship, a continuation value. The continuation value is composed of the worker's (future, post-subsidy) replacement cost, $\tilde{\varepsilon}_{t+1}$, and the differential between his future wage as an incumbent and a marginal (future) replacement hire. The hiring condition is thus a free-entry condition on the part of firms into the labor market, and it can be interpreted as the private economy's labor demand function.

In symmetric equilibrium, firms' period-t operating profits that are returned lump-sum to households are $\Pi_t = z_t n_t - (1-\rho)n_{t-1}w_t^I - \omega_e(\tilde{\varepsilon}_t)s_t - (1-\tau_t^h)\left(\frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right)\eta(\tilde{\varepsilon}_t)s_t$.

2.4 Wage Determination

Labor-market models with frictions often assume generalized Nash bargaining over wages; for the sake of comparability with search-and-matching models, we maintain this assumption. It is important to note, however, that we have also computed the results with completely rigid real wages along the business cycle, with real wages held fixed at their long-run Nash-bargained values. This alternative equilibrium wage-determination mechanism changes *none* of the Ramsey dynamic allocations (e.g., $\tilde{\varepsilon}_t$, $\eta(\tilde{\varepsilon}_t)$, $H(\tilde{\varepsilon}_t)$, lfp_t, n_t, c_t), but does, naturally, change the dynamics of the fiscal instruments.

Each worker is assumed to bargain individually with the firm, and vice-versa. That is, each bilateral worker-firm negotiation takes outcomes in all other worker-firm negotiations as given; there are thus no strategic considerations in wage determination across employees. We assume that each worker's wage can be conditioned on his idiosyncratic characteristics ε_{it} and that all wages are re-bargained every period.[6] Finally, the bargaining power of each newly-selected worker is $\alpha^E \in [0,1]$, and the bargaining power of each incumbent worker is $\alpha^I \in [0,1]$. The solutions to the formal wage-bargaining problems for incumbent workers and new workers is presented in Appendix C. In what follows, we simply present the bargaining outcomes described in Definition 1.

Definition 1. *Individually-Bargained Nash Wages.* *Suppose each worker-firm pair Nash bargains over the real wage independently of every other worker-firm pair. If the Nash bargaining power of every newly-hired worker is α^E and if the Nash bargaining power of every incumbent worker is α^I, then the real wage earned by the marginal new hire is*

$$\begin{aligned} w(\tilde{\varepsilon}_t) = {} & \frac{\chi}{1-\tau_t^n} + \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \\ - {} & \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \left(\frac{\alpha^I}{1-\alpha^I}\right) (1-\tau_{t+1}^n) \left[(1-\tau_{t+1}^h) \cdot \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I \right] \right\}, \end{aligned} \tag{11}$$

the real wage earned by a new hire with idiosyncratic characteristics ε_{it} is

$$\begin{aligned} w(\varepsilon_{it}) = {} & \frac{\chi}{1-\tau_t^n} + \alpha^E (1-\tau_t^h)\left(\tilde{\varepsilon}_t - \varepsilon_{it}\right) + \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \\ - {} & \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \left(\frac{\alpha^I}{1-\alpha^I}\right) (1-\tau_{t+1}^n) \left((1-\tau_{t+1}^h) \cdot \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I \right) \right\}, \end{aligned} \tag{12}$$

the real wage earned by every incumbent worker is

$$w_t^I = \frac{\chi}{1-\tau_t^n} + \alpha^I (1-\tau_t^h) \cdot \tilde{\varepsilon}_t \tag{13}$$

[6] These assumptions are also standard in DSGE matching models.

$$
\begin{aligned}
&+ \quad \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \\
&- \quad \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \left(\frac{\alpha^I}{1-\alpha^I} \right) (1-\tau_{t+1}^n) \left((1-\tau_{t+1}^h) \cdot \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I \right) \right\},
\end{aligned}
$$

and (by integrating (12) over $\varepsilon_{it} \leq \tilde{\varepsilon}_t$) the average wage paid to a new hire is

$$
\begin{aligned}
\frac{\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} \quad = \quad & \frac{\chi}{1-\tau_t^n} + \alpha^E (1-\tau_t^h) \left(\tilde{\varepsilon}_t - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} \right) + \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \\
& - \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \left(\frac{\alpha^I}{1-\alpha^I} \right) (1-\tau_{t+1}^n) \left((1-\tau_{t+1}^h) \cdot \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I \right) \right\}.
\end{aligned} \tag{14}
$$

Several aspects of the wage functions (11), (12), (13), and (14) are useful to highlight. First, the net-of-tax unemployment benefit χ is the lower bound of all wages because it is the payoff an individual receives for sure if wage negotiations break down. Second, the continuation-value component of each wage function is identical because no matter a worker's type in period t, he will be a (homogenous) incumbent worker in period $t+1$ if he remains employed.[7] Third, both new hires with $\varepsilon_{it} < \tilde{\varepsilon}_t$ and incumbent workers receive a premium over a marginal new hire. These premia depend on their respective bargaining powers and the values they bring to the firm over and above that of a marginal new hire.

Indeed, the wage functions (11), (12), and (13) imply wage differentials that are intuitive to understand. A new hire with $\varepsilon_{it} < \tilde{\varepsilon}_t$ earns a premium over the marginal new hire

$$
w(\varepsilon_{it}) - w(\tilde{\varepsilon}_t) = \alpha^E (1-\tau_t^h) \left(\tilde{\varepsilon}_t - \varepsilon_{it} \right), \tag{15}
$$

which is the share of the operating cost *savings* he provides the firm that he is able to extract through his bargaining power. An incumbent worker earns a premium over the marginal new hire

$$
w_t^I - w(\tilde{\varepsilon}_t) = \alpha^I (1-\tau_t^h) \cdot \tilde{\varepsilon}_t, \tag{16}
$$

which is the share of the replacement cost *savings* (relative to a marginal new hire) he provides the firm that he is able to extract through his bargaining power. Finally, the

[7] The wage of the marginal new hire, (11), is simply the wage of an arbitrary new hire (12) evaluated at $\varepsilon_{it} = \tilde{\varepsilon}_t$.

premium that the average new hire earns over the marginal new hire is

$$\frac{\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} - w(\tilde{\varepsilon}_t) = \alpha^E(1-\tau_t^h)\left(\tilde{\varepsilon}_t - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right). \tag{17}$$

This wage premium crucially depends on the endogenous cost spread between the average cost of hiring and the marginal cost of hiring, $\tilde{\varepsilon}_t - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}$, which is important in understanding the results of the Ramsey analysis in Section 4 and its discussion in Section 7.

2.5 Government

The flow government budget constraint in period t is

$$\begin{aligned} &(1-\rho)n_{t-1}\tau_t^n w_t^I + \tau_t^n\left(\frac{\omega_{et}}{\eta_t}\right)\eta_t s_t + \tau^{pr}\cdot\Pi_t \\ +\ &\sum_j \frac{1}{R_t^j}b_{t+1}^j = g_t + b_t + (1-\eta_t)s_t\chi + \tau_t^h\left(\frac{H_t}{\eta_t}\right)\eta_t s_t, \end{aligned} \tag{18}$$

in which the left-hand side denotes period-t receipts and the right-hand side denotes period-t payments.

2.6 Private-Sector Equilibrium

A symmetric private-sector equilibrium is made up of endogenous state-contingent processes
$\{c_t, n_t, s_t, \tilde{\varepsilon}_t, w_t^I, w(\tilde{\varepsilon}_t), \omega_e(\tilde{\varepsilon}_t)\}_{t=0}^{\infty}$ that satisfy seven sequences of conditions: the representative household's LFP condition (6), the representative firm's selection condition (10), the three Nash-bargained wage conditions (11), (13), and (14), the aggregate law of motion for employment

$$n_t = (1-\rho)n_{t-1} + \eta(\tilde{\varepsilon}_t)s_t, \tag{19}$$

and the aggregate goods resource constraint

$$c_t + g_t + \left(\frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right)\eta(\tilde{\varepsilon}_t)s_t = z_t n_t. \tag{20}$$

	y_t	η_t	u_t	lfp_t	n_t	w_t	τ_t^n
Relative standard deviation	1	3.72	5.15	0.20	0.60	0.52	1.92
Autocorrelation	0.87	0.82	0.91	0.68	0.94	0.91	0.66
Correlation with y	1	0.79	-0.86	0.39	0.78	0.56	0.20
Long-run lfp	**0.74**						

Table 1: **Cyclical dynamics of U.S. labor markets.** Quarterly business-cycle statistics (1964:1-2005:1) for y, s, n, and w taken from Arseneau and Chugh (2012, Table 1). Business-cycle statistics for η_t taken from Chugh and Merkl (2015, Table 3).

The private sector takes as given stochastic processes $\{z_t, g_t, \tau_t^n, \tau_t^h\}_{t=0}^{\infty}$ and the fixed parameters $(\tau^{pr}, \chi, n_{-1})$.[8]

3 Exogenous Fiscal Policy

Before studying the model's implications for optimal tax policy, we study its cyclical properties under an exogenous fiscal policy. In this section, we set the hiring subsidy to $\tau_t^h = 0\ \forall t$ and allow the labor income tax rate to follow a stochastic process, while the government budget constraint is balanced in every period via a lump-sum tax levied on households. Given this, we calibrate the model so that it generates empirically-relevant business-cycle fluctuations, especially along important labor-market dimensions, when driven by empirically-relevant government spending and labor-income tax rate processes.

3.1 Data Targets

Table 1 presents empirical facts regarding U.S. labor markets.[9]

3.2 Calibration

The quarterly exogenous separation rate of employment is set to $\rho = 0.10$, in line with U.S. data. The rest of this section rationalizes the baseline values for the other structural parameters of the economy.

[8]The $\frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}$ term in the resource constraint denotes the average hiring cost per newly-hired employee $\eta(\tilde{\varepsilon}_t)s_t$.

[9]Arseneau and Chugh (2012) in turn use the business-cycle statistics for GDP, search unemployment, employment, and wages from Gertler and Trigari (2009, Table 2) and the long-run participation rate of 74 percent as well as the cyclical properties of participation from Veracierto (2008).

Distribution of Idiosyncratic Characteristics. The distribution of idiosyncratic characteristics plays an important role in the economy. In their analysis of efficiency in the labor selection model, Chugh and Merkl (2015) use several distributional assumptions: a normal distribution, a uniform distribution, and a log-normal distribution. Regardless of their distributional choice, a main message that emerges is that *efficient* labor market fluctuations in the selection model are larger than *efficient* fluctuations in the search and matching model. To get started on shedding economic intuition on Ramsey-optimal policy outcomes (the analysis of which is in Section 4) versus exogenous-policy outcomes, the quantitative analysis that follows uses the logistic distribution, with the mean parameter set to $\mu_\varepsilon = 0.30$ and the cross-sectional standard deviation σ_ε parameterized to obtain the unconditional volatility of the job-finding rate from the data.[10] We note that we have also used other distributional forms, and the analytical results presented in Sections 5, 6, and 7 are independent of distributional assumptions.

Bargaining Powers and Government-Provided Unemployment Benefits. The parameter setting for bargaining power α^E of new workers and the government-provided unemployment benefit χ are chosen to match a steady-state hiring rate of 62% and cyclical variation of the hiring rate reported in Table 1. The two values are $\alpha^E = 0.50$ and $\chi = 0.70$. Regarding the bargaining power of incumbent workers, α^I, the main results presented set $\alpha^I = 0.50$, but there are several robustness checks contained in Appendix I in which α^E and α^I are permitted to differ.

Utility. Standard balanced-growth preferences are used, $u(c_t) = \ln c_t$ and $h(lfp_t) = \left(\frac{\kappa}{1+1/\iota}\right) lfp_t^{1+1/\iota}$. The parameter ι is the elasticity of labor-force participation with respect to the conditional (conditional on being hired) average real wage of a newly-selected worker $\left(\frac{\omega_{et}}{\eta_t}\right)$, which we set to $\iota = 0.20$ in order to match the relative volatility of participation of 20 percent reported in Table 1. The scale parameter is set to $\kappa = 6.35$ to deliver a steady-state participation rate of 74 percent. The quarterly subjective discount factor is set to $\beta = 0.99$.

Exogenous Processes. The three exogenous processes are productivity, government spending, and the labor tax rate, each of which follows an AR(1) process in logs:

$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_t^z, \tag{21}$$

[10] Quarterly business-cycle statistics for the job-finding rate η_t were computed by the authors' research assistants.

$$\ln g_t = (1-\rho_g)\ln \bar{g} + \rho_g \ln g_{t-1} + \epsilon_t^g, \tag{22}$$

and

$$\ln \tau_t^n = (1-\rho_{\tau^n})\ln \bar{\tau}^n + \rho_{\tau^n} \ln \tau_{t-1}^n + \epsilon_t^{\tau^n}. \tag{23}$$

The innovations ϵ_t^z, ϵ_t^g, and $\epsilon_t^{\tau^n}$ are distributed $N(0,\sigma_{\epsilon^z}^2)$, $N(0,\sigma_{\epsilon^g}^2)$, and $N(0,\sigma_{\epsilon^{\tau^n}}^2)$ respectively, and are independent of each other. Matching the mean, persistence, and standard error for the empirical tax-rate series reported above requires setting $\bar{\tau}^n = 0.20$, $\rho_{\tau^n} = 0.66$, and $\sigma_{\tau^n} = 0.02$.

The steady-state level of government spending $\bar{g}$ is calibrated so that it constitutes 20 percent of steady-state output; the resulting value is $\bar{g} = 0.14$. It is important to note that g is spending *excluding* unemployment transfers. When including transfers, total government outlays are $\bar{g} + \chi(1-\eta)s$ in the steady state; given our calibrated values, we have $\frac{\bar{g}+\chi(1-\eta)s}{gdp} = 0.25$. We choose parameters $\rho_z = 0.95$, $\rho_g = 0.97$, $\sigma_{\epsilon^z} = 0.01$, and $\sigma_{\epsilon^g} = 0.027$, consistent with the RBC literature and Chari and Kehoe (1999). Also regarding policy, we assume that the steady-state government debt-to-GDP ratio (at an annual frequency) is roughly 0.5, in line with the calibrations of Schmitt-Grohe and Uribe (2004), Arseneau and Chugh (2012), and many others.

3.3 Results

Table 2 presents simulation results for the calibrated exogenous-policy model. The top panel presents dynamics when all three exogenous processes are active, and the lower panel presents results conditional only on shocks to TFP. Compared with the empirical evidence presented in Table 1, the small-scale model performs well. In particular, the volatilities of unemployment, the hiring rate, and the participation rate are all in line with the data.[11]

[11]If we were to use, in the spirit of Hagedorn and Manovskii (2008), "extreme" parameter values for new workers' Nash bargaining power ($\alpha^E = 0.03$) and the outside unemployment benefit ($\chi = 0.75$), volatilities for the unemployment rate and the job-finding rate are much larger than in the data. Comparing with the first row in Table 1, the relative standard deviations for this parametrization are, respectively: 1, 5.58, 21.36, 0.23, 0.49, 0.11, and 0.04.

	y_t	η_t	u_t	lfp_t	n_t	w_t^I	$\frac{\omega_{et}}{\eta_t}$	τ_t^n
			All shocks					
Relative standard deviation	1	3.49	8.64	0.22	0.42	0.60	0.20	0.98
Autocorrelation	0.96	0.94	0.97	0.96	0.97	0.95	0.70	0.66
Correlation with y	1	0.98	-0.97	-0.49	0.96	0.97	0.29	-0.07
			TFP shocks					
Relative standard deviation	1	3.48	8.63	0.14	0.38	0.61	0.08	-
Autocorrelation	0.96	0.95	0.97	0.97	0.97	0.95	0.94	-
Correlation with y	1	1	-1	-1	1	1	1	-

Table 2: **Baseline model under exogenous policy.** Long-run $lfp = 0.74$. Top panel: shocks to TFP, government purchases, and labor-income tax rate. Bottom panel: shocks only to TFP.

4 Optimal Fiscal Policy

With the baseline calibration established, we now discard the exogenous process (23) for the labor income tax rate and instead endogenize tax policy. As required for a Ramsey analysis, lump-sum taxes are set to zero. The Ramsey government uses a proportional income tax that is identical across all workers and a hiring subsidy. As shown in Section 7.6, this constitutes a complete system of taxes. While taxes are now optimally chosen by a Ramsey government, government purchases continue to follow the exogenous process (22).[12]

4.1 Ramsey Problem

A standard approach in Ramsey models based on neoclassical markets is to capture in a single, present-value implementability constraint (PVIC) all equilibrium conditions of the economy apart from the resource frontier. The PVIC is the key constraint in any Ramsey problem because it governs the welfare loss of using non-lump-sum taxes to finance government expenditures.[13] As is standard, we can construct a PVIC starting from

[12]Thus, we follow the standard convention in Ramsey analysis that spending is exogenous but the revenue side of fiscal policy is determined optimally.

[13]See, for example, Ljungqvist and Sargent (2012, p. 625) for more discussion. The PVIC is the household (equivalently, government) budget constraint expressed in intertemporal form with all prices and policies substituted out using equilibrium conditions. In relatively simple models, the PVIC encodes all the equilibrium conditions that must be respected by Ramsey allocations in addition to feasibility. In complicated environments that deviate substantially from neoclassical markets, however, such as Schmitt-Grohe and Uribe (2004) and Arseneau and Chugh (2008), it is not always possible to construct such a single constraint.

the household flow budget constraint (3) and using the household optimality conditions (5) and (6). However, because of the nature of the environment, it cannot capture all of the model's equilibrium conditions. As shown in Appendix F, the PVIC is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u'(c_t)c_t - h'(lfp_t)lfp_t - u'(c_t)(1-\tau^{pr})\Pi_t \right] = A_0, \tag{24}$$

in which the time-zero assets of the private sector are

$$A_0 \equiv u'(c_0)b_0 + (1-\rho)\left(\frac{1-\eta_0}{\eta_0}\right)\left[h'(lfp_0) - u'(c_0)\chi\right] n_{-1} + (1-\rho)u'(c_0)(1-\tau_0^n)\left[\frac{\omega_{e0}}{\eta_0} - w_0^I\right] n_{-1}. \tag{25}$$

Several observations about the PVIC are in order. First, because employment is a state variable, the household's "ownership" of the initial stock of employment relationships, n_{-1}, is part of its time-zero assets, as shown in A_0. Second, if labor markets were neoclassical, lfp_t would be interpreted as simply *labor* because there would be no notion of selection unemployment. Third, as mentioned above, a spot, neoclassical labor market can be interpreted as featuring $\rho = 1$ because there is no long-lived aspect to labor-market transactions. Fourth, with a constant-returns production technology in a neoclassical environment, $\Pi_t = 0 \; \forall t$. Imposing the last three of these conditions collapses the PVIC (24), as well as the initial assets A_0, to that in a standard Ramsey model based on neoclassical markets.[14]

However, unlike in a neoclassical model, the PVIC (24) does not capture all equilibrium conditions of the decentralized economy. In particular, Ramsey allocations must also respect the selection condition (10), the Nash wage outcomes (11), (13), and (14), and the law of motion for the aggregate employment stock (19). None of these restrictions is encoded in the PVIC (24).

The Ramsey problem is thus to choose state-contingent processes $\{c_t, n_t, s_t, \tilde{\varepsilon}_t, w_t^I, w(\tilde{\varepsilon}_t), \omega_e(\tilde{\varepsilon}_t), \tau_t^n, \tau_t^h\}_{t=0}^{\infty}$ to maximize (2) subject to the PVIC (24), the LFP condition (6), the selection condition (10), the Nash wage outcomes (11), (13), and (14), the law of motion for the aggregate employment stock (19), and the aggregate resource constraint (20), taking as given the stochastic processes $\{z_t, g_t\}_{t=0}^{\infty}$.[15] Finally, the

[14] In particular, we would have $E_0 \sum_{t=0}^{\infty} \beta^t \left[u'(c_t)c_t - h'(n_t)n_t\right] = u'(c_0)b_0$. This PVIC is identical to that in Chari and Kehoe (1999) for an environment without physical capital.

[15] The dividend tax rate τ^{pr} is omitted from the list of Ramsey choice variables for convenience. As described above, it is trivial to show that $\tau^{pr} = 1$ is optimal in every period because taxing households' receipts of lump-sum dividend payments is non-distortionary. Furthermore, χ is viewed as an institutional

Ramsey government has full commitment as of period $t = 0$ to its policy functions for periods $t > 0$. We emphasize that the statement of the Ramsey problem in and of itself does *not* depend on the particular functional form for the distribution of idiosyncratic characteristics (although, of course, the quantitative results will).

4.2 Computational Issues

The first-order conditions of the Ramsey problem are assumed to be necessary and sufficient, and all allocations are assumed to be interior. As in the exogenous-policy baseline, we use a nonlinear numerical solution algorithm to compute the deterministic Ramsey steady-state equilibrium. As is common in the Ramsey literature, when characterizing asymptotic policy dynamics (that is, the dynamics of the Ramsey equilibrium implied by the Ramsey $t > 0$ first-order conditions), we use only the $t > 0$ Ramsey first-order conditions. We then use the first-order accurate decision rules to simulate the Ramsey equilibrium in the face of productivity and government spending realizations. The productivity and government spending realizations used to conduct the Ramsey simulations are the same as those in the exogenous-policy experiments in Section 3, which means that any differences between the Ramsey equilibrium and exogenous-policy equilibrium are attributable entirely to the dynamics of tax policy.

4.3 Results

4.3.1 Ramsey Steady-State Results

Using the baseline parameter values described in Section 3.2, Table 3 presents steady-state allocations for the Ramsey economy (upper panel) and for the exogenous-policy economy (middle panel). The Ramsey government induces a higher long-run hiring probability η by heavily subsidizing the decentralized economy's hiring costs. The long-run subsidy is $\tau^h = 0.81$. Not reported in Table 3 is the wage premium between the average newly-hired employee relative to the marginal newly-hired employee. In the Ramsey steady state, the ratio of average wage $\omega_e(\tilde{\varepsilon}_t)/\eta(\tilde{\varepsilon}_t)$ to the marginal wage $w(\tilde{\varepsilon}_t)$ is roughly 10%, which is about half the value of 19% in the exogenous policy economy. Hence the Ramsey government shrinks the wage gap for new hires to ensure efficient labor markets. The reason we don't spotlight the numerical values for the wage premium is that our calibration

parameter and hence not chosen during the course of "normal" fiscal policy.

u	lfp	$\tilde{\varepsilon}\eta(\tilde{\varepsilon}) - H(\tilde{\varepsilon})$	n	y	τ^n	τ^h	labor supply wedge	labor demand wedge
				Ramsey Policy.				
0.0005	0.734	**0.9674**	0.733	0.712	0.216	0.810	0.797	1
				Exogenous Policy.				
0.040	0.740	**0.1867**	0.700	0.690	0.200	0	4.140	-3.271
				Efficiency.				
0.0005	0.760	**0.9674**	0.760	0.740	0	0	1	1

Table 3: **Steady-state allocations.** Upper panel: Ramsey-policy allocations. Middle panel: Exogenous-policy allocations. Lower panel: Efficient allocations. Definition of $y = c + g$.

procedure was not designed to measure empirically-relevant wage premia for newly-hired employees, but this is an interesting point for further research.

We defer an in-depth discussion of model-consistent distortions and wedges to Section 5 and Section 6, but it is worth pointing out that the inefficiencies in both labor demand and labor supply shrink in the Ramsey-policy allocations compared to the exogenous-policy allocations. As the last two columns in Table 3 indicate, both wedges are sharply smaller in the Ramsey allocations compared to the exogenous-policy allocations (efficiency on each margin requires the model-consistent wedge to equal one, as defined in Sections 5 and 6). An intratemporal wedge (along the labor supply dimension) arises in the long-run Ramsey allocation because of the need to raise revenue through proportional taxes. But the intertemporal margin (the labor demand dimension) is efficient, as can be seen by comparing the top panel of Table 3 with the bottom panel, which display socially-efficient allocations. Another important point to note is that the term $\tilde{\varepsilon}\eta(\tilde{\varepsilon}) - H(\tilde{\varepsilon})$ is identical in the Ramsey allocations and efficient allocations. As discussed further in Section 7, $\tilde{\varepsilon}\eta(\tilde{\varepsilon}) - H(\tilde{\varepsilon})$ is the selective hiring model's analogue of market tightness in the search and matching model.

In terms of long-run welfare, measured as the percentage of consumption compensation the representative household would need to be induced to remain in the exogenous-policy environment, the gain is 3%.

Given the complete set of tax instruments included in our framework (which is dis-

Ramsey multipliers with respect to	λ_1 RC	λ_2 LOM	λ_3 $PVIC$	λ_4 SEL	λ_5 LFP	λ_6 $w(\tilde{\varepsilon}_t)$	λ_7 $\omega_e(\tilde{\varepsilon}_t)$	λ_8 w_t^I
	1.754	0.524	0.042	0	0	0	0	0

Table 4: **Ramsey steady-state endogenous multipliers.** The Ramsey multiplier λ_6 is with respect to the real wage earned by the marginal new hire (11); the Ramsey multiplier λ_7 is with respect to the average real wage paid to a new hire (14); and the Ramsey multiplier λ_8 is with respect to the real wage earned by incumbents (13).

cussed further in Section 7.6), the natural hypothesis is that the endogenous Ramsey steady-state multipliers other than the ones on the primitive technology of the model (here, the aggregate goods resource constraint and the aggregate law of motion of employment) and the long-run budget constraint (the PVIC) should be zero. In other words, all of the decentralized equilibrium conditions aside from the long-run budget and the primitive technology of the economy should *not* bind at the Ramsey equilibrium. Completely in line with this long-running macro-Ramsey result, Table 4 shows that this indeed arises.

4.3.2 Ramsey Business-Cycle Results

Table 5 displays second moments for the Ramsey economy, using shocks to both government spending and TFP (upper panel) or to TFP alone (lower panel). Focusing on TFP shocks, the Ramsey-equilibrium volatilities of the hiring rate, the participation rate, and employment itself are an order of magnitude smaller than in the comparable exogenous-policy equilibrium in the lower panel of Table 2. Regarding optimal labor-income tax rate volatility (second-to-last column in Table 5), it empirically resembles that in the U.S. data (see Table 1), even though the calibration was not designed to do so. Perhaps more importantly, from a theoretical perspective, tax-rate volatility is orders of magnitude larger than the conventional “tax-smoothing” wisdom. Comparison with the optimal tax volatility in Arseneau and Chugh (2012, Table 4), though, shows that it’s smaller than in the search and matching framework (the volatility of the labor tax rate is roughly one-sixth and the volatility of the hiring subsidy is roughly one-seventh).

Whether it’s low volatility or high volatility of tax rates that emerges, what is crucial is the volatility of model-appropriate wedges. Sections 5 and 6 provide the analytics

	y_t	η_t	u_t	lfp_t	n_t	w_t^I	$\frac{\omega_{et}}{\eta_t}$	τ_t^n	τ_t^h
					All shocks				
Relative standard deviation	1	0.03	5.29	0.23	0.23	0.80	0.54	0.97	2.28
Autocorrelation	0.95	0.95	0.93	0.97	0.95	0.95	0.95	0.95	0.95
Correlation with y	1	0.97	-0.96	0.06	0.08	0.97	0.98	-0.97	-0.97
					TFP shocks				
Relative standard deviation	1	0.04	5.39	0.04	0.03	0.83	0.56	0.99	2.34
Autocorrelation	0.95	0.95	0.94	0.95	0.95	0.95	0.95	0.95	0.95
Correlation with y	1	1	-1	-1	-1	1	1	-1	-1

Table 5: **Optimal policy.** Long-run $lfp = 0.73$. Top panel: shocks to TFP and government purchases. Bottom panel: shocks only to TFP.

of efficiency and decentralized wedges, and Section 7 develops the analytical concept of market tightness in the selection model and discusses various parts of the framework.

5 Efficient Allocations

The main focus in this section is on the nature of efficient allocations in this environment. Chugh and Merkl (2015) have stated and solved the model-consistent efficiency problem, but, because their analysis did not require it, did not define the model-consistent transformation function. We extend the Chugh and Merkl (2015) efficiency results by characterizing the transformation frontier for the selective hiring model.

Efficient allocations $\{c_t, s_t, \tilde{\varepsilon}_t, n_t\}_{t=0}^{\infty}$ are characterized by four (sequences of) conditions:

$$\frac{h'(lfp_t)}{u'(c_t)} = \eta(\tilde{\varepsilon}_t)\left(\tilde{\varepsilon}_t - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right), \tag{26}$$

$$\tilde{\varepsilon}_t = z_t + (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[\tilde{\varepsilon}_{t+1} - \eta(\tilde{\varepsilon}_{t+1})\left(\tilde{\varepsilon}_{t+1} - \frac{H(\tilde{\varepsilon}_{t+1})}{\eta(\tilde{\varepsilon}_{t+1})}\right)\right]\right\}, \tag{27}$$

$$c_t + g_t + \left(\frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right)\eta(\tilde{\varepsilon}_t)s_t = z_t n_t, \tag{28}$$

and

$$n_t = (1-\rho)n_{t-1} + s_t\eta(\tilde{\varepsilon}_t). \tag{29}$$

The efficiency conditions (26) and (27) are obtained by maximizing household welfare (2) subject to the technological frontier defined by the sequence of goods resource constraints (28) and laws of motion for employment (29). The formal analysis of this problem appears in Appendix D.

Condition (26) is a static dimension of efficiency and is analogous to static consumption-leisure efficiency in the RBC model. Condition (27) is an intertemporal dimension of efficiency, and it corresponds to the matching model's efficient job-creation condition; it also corresponds to the RBC model's Euler equation for efficient capital accumulation. Even though the model does not have physical capital in the strict RBC sense, the creation of an employer-employee relationship is an investment activity that yields a long-lasting asset. Because of frictions, employment thus inherently has both static and intertemporal dimensions in a selection framework, just as it does in a matching framework. Together, conditions (26) and (27) define the two "zero-wedge" conditions for the model, both of which are statements about labor markets.

To highlight this "zero-wedges" view, Proposition 1 restates efficiency in terms of marginal rates of substitution (MRS) and corresponding marginal rates of transformation (MRT). For the intertemporal condition, this restatement is most straightforward for the non-stochastic case, which allows an informative disentangling of the preference and technology terms inside the $E_t(.)$ operator in (27).

Proposition 1. ***Efficient Allocations.*** *The MRS and MRT for the pairs (c_t, lfp_t) and (c_t, c_{t+1}) are defined by*

$$MRS_{c_t,lfp_t} \equiv \frac{h'(lfp_t)}{u'(c_t)} \qquad MRT_{c_t,lfp_t} \equiv \eta(\tilde{\varepsilon}_t)\left(\tilde{\varepsilon}_t - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right)$$

$$IMRS_{c_t,c_{t+1}} \equiv \frac{u'(c_t)}{\beta u'(c_{t+1})} \qquad IMRT_{c_t,c_{t+1}} \equiv \frac{(1-\rho)\left[\tilde{\varepsilon}_{t+1} - \eta(\tilde{\varepsilon}_{t+1})\left(\tilde{\varepsilon}_{t+1} - \frac{H(\tilde{\varepsilon}_{t+1})}{\eta(\tilde{\varepsilon}_{t+1})}\right)\right]}{\tilde{\varepsilon}_t - z_t}.$$

Static efficiency (26) is characterized by $MRS_{c_t,lfp_t} = MRT_{c_t,lfp_t}$, and (for the non-stochastic case) intertemporal efficiency (27) is characterized by $IMRS_{c_t,c_{t+1}} = IMRT_{c_t,c_{t+1}}$.

Proof. See Appendix D. □

Each MRS in Proposition 1 has the standard interpretation as a ratio of relevant marginal utilities. By analogy, each MRT has the interpretation as a ratio of the marginal products of an appropriately-defined transformation frontier.[16] As per economic theory,

[16]We have in mind a very general notion of transformation frontier as in Mas-Colell, Whinston, and Green (1995, p. 129), in which every object in the economy can be viewed as either an input to or an output of the technology to which it is associated. Appendix D provides formal details.

then, efficient allocations are characterized by an MRS = MRT condition along each optimization margin, implying zero distortion on each margin. These efficiency conditions are the welfare-relevant ones in this environment and hence must be the basis for any normative analysis. However, rather than take the efficiency conditions as prima facie justification that the expressions in Proposition 1 *are* properly to be understood as MRTs, each can be derived from primitives, independent of the characterization of efficiency. Formal details of the following mostly intuitive discussion appear in Appendix D.

5.1 Static MRT

To understand the static MRT, MRT_{c_t,lfp_t}, in Proposition 1, consider how the economy can transform a unit of *non*-participation (leisure) in period t into a unit of output, and hence consumption, in period t. By construction, this within-period transformation holds fixed all allocations beyond period t. The transformation is described in terms of leisure because leisure is a good (and hence gives positive utility), while participation is a bad (and gives disutility); we proceed by describing transformation as occurring between goods.

A one-unit reduction in leisure allows a one-unit increase in s_t, which leads to a sequence of further transformations. With probability $\eta(\tilde{\varepsilon}_t)$, the individual's revealed operating cost is below the cutoff $\tilde{\varepsilon}_t$ and he is selected to join a production opportunity; integrating the individual into production entails a random operating cost. Newly-selected individuals with idiosyncratic traits $\tilde{\varepsilon}_t$ generate zero value to the firm because they incur larger training costs than those newly-selected individuals whose training costs are $\varepsilon_{it} < \tilde{\varepsilon}_t$. This latter group with traits $\varepsilon_{it} < \tilde{\varepsilon}_t$ generate positive value in the period in which they are hired due to the *savings* of training costs compared to those with traits $\tilde{\varepsilon}_t$.

The aggregate expected savings on training costs for those new individuals hired is thus $\int_{-\infty}^{\tilde{\varepsilon}_t} [\tilde{\varepsilon}_t - \varepsilon_{it}] f(\varepsilon_{it}) d\varepsilon_{it} = \tilde{\varepsilon}_t \eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)$, where the equality follows from the definitions of $\eta(\tilde{\varepsilon}_t)$ and $H(\tilde{\varepsilon}_t)$. This *savings* on period-t training costs allows an *increase* in period-t consumption. The overall marginal transformation between leisure and consumption described thus far is $\eta(\tilde{\varepsilon}_t) \left[\frac{\tilde{\varepsilon}_t \eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right] = \tilde{\varepsilon}_t \eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)$.

However, this is not *ceteris paribus* because the larger stock of employment in period t has intertemporal consequences — as per the aggregate law of motion (29), n_{t+1} would be larger. Measuring only within-period effects requires controlling for this intertemporal effect. We show in Appendix D that the appropriate adjustment leads to the lifetime social asset value of a match. Hence, the overall within-period MRT between leisure and consumption is $\tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)$, as shown in Proposition 1. As mentioned earlier and as will be discussed further in Section 7, $\tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)$ is an important object in the selective hiring model.

5.2 Intertemporal MRT

Now consider the intertemporal MRT (IMRT) in Proposition 1. The IMRT measures how many additional units of c_{t+1} the economy can achieve if one unit of c_t is foregone. By construction, this transformation across periods t and $t+1$ holds fixed all allocations *beyond* period $t+1$.

A one-unit reduction in c_t frees up resources that can be devoted to selection of individuals. As (28) shows, resources devoted to "investment" in selection of individuals can be increased by $\frac{1}{H'(\tilde{\varepsilon}_t)s_t}$ units, by selecting some marginally worse (in terms of higher idiosyncratic operating costs) individuals who otherwise would not have been selected. Relaxing the selection criteria increases period-t aggregate employment by $\frac{\eta'(\tilde{\varepsilon}_t)s_t}{H'(\tilde{\varepsilon}_t)s_t} = \frac{1}{\tilde{\varepsilon}_t}$ units.[17]

The addition of $\frac{1}{\tilde{\varepsilon}_t}$ individuals to period-t employment has two effects. Because workers become productive in the period in which they are selected, period-t output, and hence period-t consumption, *rises* by $\frac{z_t}{\tilde{\varepsilon}_t}$ units. This *rise* in period-t consumption must be netted from the one-unit *reduction* in period-t consumption that started the thought experiment. Thus, we can now view all effects on period-$t+1$ consumption as arising from a (net) reduction of c_t by $1 - \frac{z_t}{\tilde{\varepsilon}_t} = \frac{\tilde{\varepsilon}_t - z_t}{\tilde{\varepsilon}_t}$ (< 1) units.

The second effect of the additional $\frac{1}{\tilde{\varepsilon}_t}$ units of period-t employment is that, in period $t+1$, there are $\frac{1-\rho}{\tilde{\varepsilon}_t}$ additional units of employment — that is, n_{t+1} rises by $\frac{1-\rho}{\tilde{\varepsilon}_t}$ units. Each of these additional units of employment produces z_{t+1} units of output, and hence consumption. The overall addition to period $t+1$ consumption (starting, recall from immediately above, from a reduction of period-t consumption by $\frac{\tilde{\varepsilon}_t - z_t}{\tilde{\varepsilon}_t}$ units) described thus far is $\frac{z_{t+1}(1-\rho)}{\tilde{\varepsilon}_t}$ units.

However, this transformation is not *ceteris paribus* because the the larger stock of employment in period $t+1$ has intertemporal consequences *beyond* period $t+1$; measuring the marginal transformation across only period t and period $t+1$ requires controlling

[17] The simplification follows from using the Fundamental Theorem of Calculus to compute the derivatives of the functions $\eta(\tilde{\varepsilon}_t)$ and $H(\tilde{\varepsilon}_t)$. As the goods resource constraint (28) shows, resources devoted to hiring are denominated in consumption goods units.

for this additional intertemporal effect. We show using the implicit function theorem in Appendix D that the appropriate adjustment factor is $\frac{H(\tilde{\varepsilon}_{t+1})-\tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1})+\tilde{\varepsilon}_{t+1}}{z_{t+1}}$, which measures the one-period-ahead social asset value of a match in period $t+1$ (that is, valued from the perspective of period t). The z_{t+1} term in this asset value serves only to convert units of labor into units of consumption goods, so focus on the numerator. The social cost of selecting a worker in period $t+1$ to replace a worker selected in period t is $\tilde{\varepsilon}_{t+1}$. Furthermore, due to uncertainty about individuals' idiosyncratic characteristics, a replacement worker selected in period $t+1$ entails an expected operating cost $H(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) = \int_{-\infty}^{\tilde{\varepsilon}_{t+1}} [\varepsilon - \tilde{\varepsilon}_{t+1}] f(\varepsilon) d\varepsilon$. These total *costs* in period $t+1$ of selecting a replacement individual thus define the *value* of an individual selected in period t.

Putting together this logic leads to the IMRT shown in Proposition 1. The fully stochastic intertemporal efficiency condition can thus be represented as

$$1 = E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left[\frac{(1-\rho)\left(H(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) + \tilde{\varepsilon}_{t+1}\right)}{\tilde{\varepsilon}_t - z_t} \right] \right\} = E_t \left\{ \frac{IMRT_{c_t,c_{t+1}}}{IMRS_{c_t,c_{t+1}}} \right\}. \quad (30)$$

5.3 Nesting the RBC Model

These selection-based static and intertemporal MRTs apply basic economic theory to a general equilibrium labor selection model. They compactly describe the *two* technologies — the selection technology embodied by the costly hiring process, and the production technology $z_t n_t$ — that must operate for the within-period transformation of leisure into consumption and the transformation of consumption across time. Due to the participation decision and the investment nature of costly labor selection, employment inherently features both static and intertemporal dimensions.

To see how the efficiency concepts developed here nest the standard Walrasian notion of consumption-leisure efficiency, suppose that $\rho = 1$, which makes employment a one-period, though not a frictionless, phenomenon. With one-period employment outcomes, the intertemporal condition (27) simplifies to

$$\tilde{\varepsilon}_t = z_t. \quad (31)$$

This can be combined with the efficiency condition (26), so that overall efficiency in the case of one-period employment is characterized by the single within-period condition,

$$\begin{aligned}\frac{h'(lfp_t)}{u'(c_t)} &= \eta(\tilde{\varepsilon}_t)\tilde{\varepsilon}_t - H(\tilde{\varepsilon}_t) \\ &= \eta(\tilde{\varepsilon}_t)z_t - H(\tilde{\varepsilon}_t) \\ &= \eta(\tilde{\varepsilon}_t)\left(z_t - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right). \end{aligned} \tag{32}$$

Viewed as a primitive, the "frictions" captured by the hiring costs are formally part of the MRT of the economy, even though a neoclassical "labor wedge accounting" exercise would regard them as wedges between the MRS and the marginal product z_t of the production technology. The selection framework's full-turnover efficiency condition (32) is compared with the matching model's analogue in Section 7.4.2.

Moving all the way to the RBC model also requires discarding costly hiring. The RBC model can be easily viewed as featuring $H(\tilde{\varepsilon}_t) = 0$ and $\eta(\tilde{\varepsilon}_t) = 1\ \forall t$ (in addition to $\rho = 1$). The one-period efficiency condition (32) then reduces to the familiar $\frac{h'(n_t)}{u'(c_t)} = z_t$, with "participation" now interchangeably interpretable as "employment" because there is no hiring-cost friction between the two.

6 Decentralized Equilibrium Wedges

With the model-appropriate characterizations of static and intertemporal efficiency just developed, equilibrium wedges are defined as the deviations of MRS from MRT that arise in the decentralized economy. These wedges measure inefficiencies, and, because the inefficiencies all relate to the allocation of labor, it may be informative to think of them jointly as a "labor wedge." Understanding the determinants and consequences of these inefficiencies provides the foundation for understanding optimal policy.

6.1 Static Distortion — "Labor Supply Wedge"

In the decentralized economy with Nash bargaining and selective hiring, the within-period (static) equilibrium margin can be expressed as

$$\begin{aligned}\frac{h'(lfp_t)}{u'(c_t)} &= \eta(\tilde{\varepsilon}_t)\left(\tilde{\varepsilon}_t - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right)\left[\frac{\chi}{\tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)} + \alpha^E(1-\tau_t^n)(1-\tau_t^h)\right] \qquad (33) \\ &= \chi + (1-\tau_t^n)(1-\tau_t^h)\alpha^E\eta(\tilde{\varepsilon}_t)\left(\tilde{\varepsilon}_t - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right),\end{aligned}$$

in which the term in brackets in the first line measures the static, or labor supply, distortion.

Comparison of either the first line or the second line in (33) with the static efficiency condition (26) makes clear that sufficient conditions for the decentralized economy with Nash bargaining and selective hiring to achieve efficiency are: the decentralized economy features $\alpha^E = 1$; the unemployment transfer is $\chi = 0$; proportional labor income taxation is $\tau_t^n = 0$; and the proportional hiring subsidy is $\tau_t^h = 0$. These conditions are not necessary, however, because for any arbitrary $(\alpha^E < \xi, \chi \neq 0)$, an appropriate setting for policy (τ_t^n, τ_t^h) achieves efficiency.

To obtain the second line of (33), substitute the wage premium of the average new hire relative to the marginal new hire ($\frac{\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} - w(\tilde{\varepsilon}_t)$ in (17)) into the decentralized economy's participation condition (6); a few steps of algebra then yields the second line in (33). The second expression in (33) highlights that the difference between the *cost* of hiring the marginal new employee relative to the *cost* of hiring the average new employee (which, as stated in Proposition 1, is the MRT_{c_t, lfp_t}) can equivalently be considered in terms of the *wage premium* $\frac{\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} - w(\tilde{\varepsilon}_t)$ of the average new hire over the marginal new hire. Our definition of the selection-model's tightness is this cost spread, or, equivalently, this wage premium.

6.2 Intertemporal Distortion — "Labor Demand Wedge"

In the decentralized economy with Nash bargaining and selective hiring, the intertemporal equilibrium margin can be expressed as

$$1 = E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left[\frac{\left(\frac{1-\rho}{1-\tau_t^n}\right) \left[\left(\alpha^I (1-\tau_{t+1}^n) + (1-\alpha^I)(1-\tau_t^n) \right) (1-\tau_{t+1}^h) \tilde{\varepsilon}_{t+1} - \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right]}{(1-\tau_t^h)\tilde{\varepsilon}_t - \left[z_t - \frac{\chi}{1-\tau_t^n} \right]} \right] \right\}. \tag{34}$$

Comparing the term in square brackets with the term in square brackets in the intertemporal efficiency condition (30) implicitly defines the intertemporal distortion.

6.3 Efficiency in Decentralized Equilibrium Without Taxes

Suppose the decentralized economy features zero taxes at all dates ($\tau_t^n = \tau_t^h = 0 \; \forall t$). In this case, expressions (33) and (34) make clear that efficient allocations require zero unemployment benefits ($\chi = 0$) and full Nash bargaining power for newly-hired workers

($\alpha^E = 1$). This latter result is highly analogous to the efficient bargaining condition in matching models that was originally developed by Mortensen (1982) and Hosios (1990). Section 7 describes further the intuition behind this result, as well as several other issues regarding efficiency and optimal policy.

7 Analysis and Discussion of Optimal Taxation

Based on the welfare-relevant concepts of efficiency and wedges developed in Sections 5 and 6, it is now straightforward to explain the optimal policy results through the lens of basic Ramsey theory. In doing so, we also quantify the role of the two key features of the decentralized bargaining economy that disrupt efficiency, as well as briefly discuss a few other aspects of the model and results.

7.1 Wedge Smoothing...

A basic result in dynamic Ramsey analysis is that the least distortionary way for a government to collect a present value of revenue through proportional taxes is to maintain *low volatility of distortions* — "wedge smoothing" — across time periods. Keeping distortions constant (or nearly constant) over time is the basic insight behind Barro's (1979) partial-equilibrium tax-smoothing result, which carries over to quantitative general equilibrium models, as first shown by Chari, Christiano, and Kehoe (1991) and many others over the past two decades.

This basic Ramsey insight also applies to our model. As Table 6 shows, *optimal policy keeps both static distortions and intertemporal distortions completely stable over the business cycle* regardless of either the quantity of unemployment benefits or the bargaining power of new employees. In the the baseline calibration of the exogenous-policy equilibrium, volatility of the static wedge relative to that of GDP is 4.8 and the volatility of the intertemporal wedge relative to that of GDP is 5.2. These quantitative results make it quite clear that the basic Ramsey principle of smoothing static distortions carries over to the selective-hiring model.

Albanesi and Armenter (2012) recently showed that for a wide class of optimal-policy models, achieving zero intertemporal distortions is the primary goal. Their results generalize the well-known zero-capital-taxation results of Chamley (1986) and Judd (1985). Existing zero intertemporal distortions results apply only to the steady state, however;

Parameter Set	SD(%) of static wedge		SD(%) of intertemp. wedge		Opt. tax dynamics	
	Exog. policy	Opt. policy	Exog. policy	Opt. policy	Vol. of τ_t^n	Vol. of τ_t^h
Baseline	4.78	0	5.22	0	0.97	2.28
$\alpha^E = 1$	5.05	0	3.93	0	0.31	1.06
$\chi = 0$	0.73	0	0	0	0.51	0.13
$\alpha^E = 1$, $\chi = 0$	0	0	0	0	0	0

Table 6: **Volatility results.** Volatility of static and intertemporal wedges in exogenous-policy equilibria and Ramsey equilibria, and volatility of taxes in Ramsey equilibria. Volatility of labor income tax reported as coefficient of variation relative to that of GDP, and volatility of hiring subsidy reported as absolute level of around the long-run subsidy. Shocks are to TFP and government purchases. In all experiments, the bargaining power of incumbent workers α^I is set at its baseline value.

the result here (as well as in the matching model of Arseneau and Chugh (2012)) is that intertemporal efficiency is achieved not only in the long run, but also along the business cycle.[18] Our model does not include physical capital in the strict sense, but intertemporal efficiency is nonetheless a primary concern of policy due to the asset nature of employment. For the overall economy, employment is a form of capital; as Proposition 1 implies, long-lasting employer-employee relationships are in fact the means by which consumption is transformed across time and hence the means by which the economy "saves." The intertemporal efficiency insight of Ramsey analysis is thus not limited to a narrow notion of "physical capital," but instead applies to any accumulation decision.[19]

7.2 ...Supports Efficient Labor-Market Fluctuations...

If static wedges are constant over time and intertemporal wedges are always zero, the decentralized economy achieves efficient fluctuations. To see this, first recall the characterization of efficient allocations in Proposition 1. Considering the deterministic case for clarity, the period t and period $t+1$ static efficiency conditions can be written in

[18]This difference arises from the fact that newly-matched employees begin working and producing output immediately in our model, which implies a static component in forward-looking match-creation activities; whereas the standard assumption in RBC models is that, due to time-to-build lags, newly-created physical capital does not yield any contemporaneous output.

[19]Another recent example in which intertemporal efficiency is a central goal of policy, despite the absence of "physical capital," is the model of dynamic product creation and destruction in which Chugh and Ghironi (2015) study optimal policy.

intertemporal form as

$$\begin{aligned}
\frac{u'(c_t)}{\beta u'(c_{t+1})} &= \frac{h'(lfp_t)}{\beta h'(lfp_{t+1})} \cdot \frac{\eta(\tilde{\varepsilon}_{t+1})}{\eta(\tilde{\varepsilon}_t)} \cdot \left(\frac{\frac{\tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) - H(\tilde{\varepsilon}_{t+1})}{\eta(\tilde{\varepsilon}_{t+1})}}{\frac{\tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}} \right) \\
&= \frac{h'(lfp_t)}{\beta h'(lfp_{t+1})} \cdot \left(\frac{\tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) - H(\tilde{\varepsilon}_{t+1})}{\tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)} \right). \qquad (35)
\end{aligned}$$

Together with the intertemporal efficiency condition (27), the goods resource constraint (28), and the law of motion (29), this expression describes the *efficient* fluctuation of the economy between periods t and $t+1$.

Now, in the decentralized economy, suppose that the static wedge, even if not zero, is constant across periods t and $t+1$. Condition (33) shows that static wedge (the term in brackets in (33)) smoothing implies

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{h'(lfp_t)}{\beta h'(lfp_{t+1})} \cdot \left(\frac{\tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) - H(\tilde{\varepsilon}_{t+1})}{\tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)} \right), \qquad (36)$$

which is identical to the implication (35) of period-by-period static efficiency. Thus, *fluctuations in the decentralized economy are efficient if static wedges are constant over time and intertemporal wedges are always zero.*

The goods-denominated term $\tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)$ is crucial in the selective hiring framework.[20] As Table 3 shows, the efficient long-run value of $\tilde{\varepsilon}\eta(\tilde{\varepsilon}) - H(\tilde{\varepsilon})$ (lower panel) is *identical* to that of the Ramsey planner (upper panel).

Table 6 showed that optimal policy achieves complete stabilization of both intertemporal wedges and static wedges. The implication of wedge smoothing, then, is that Ramsey equilibria display efficient fluctuations. Figure 3 illustrates this through impulse responses to a TFP shock.[21] Comparing this analysis to the analysis in Arseneau and Chugh (2012, Section VII.C), $\tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)$ is the selective hiring framework's analogue of labor-market tightness θ_t.

[20] The units of $\tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)$ are in consumption goods, which can be inferred from, for example, the static efficiency condition (26), in which the MRS $\frac{h'(lfp_t)}{u'(c_t)}$ is (as usual) denominated in consumption goods.

[21] The efficient and Ramsey impulse responses in all panels of Figure 3 are identical.

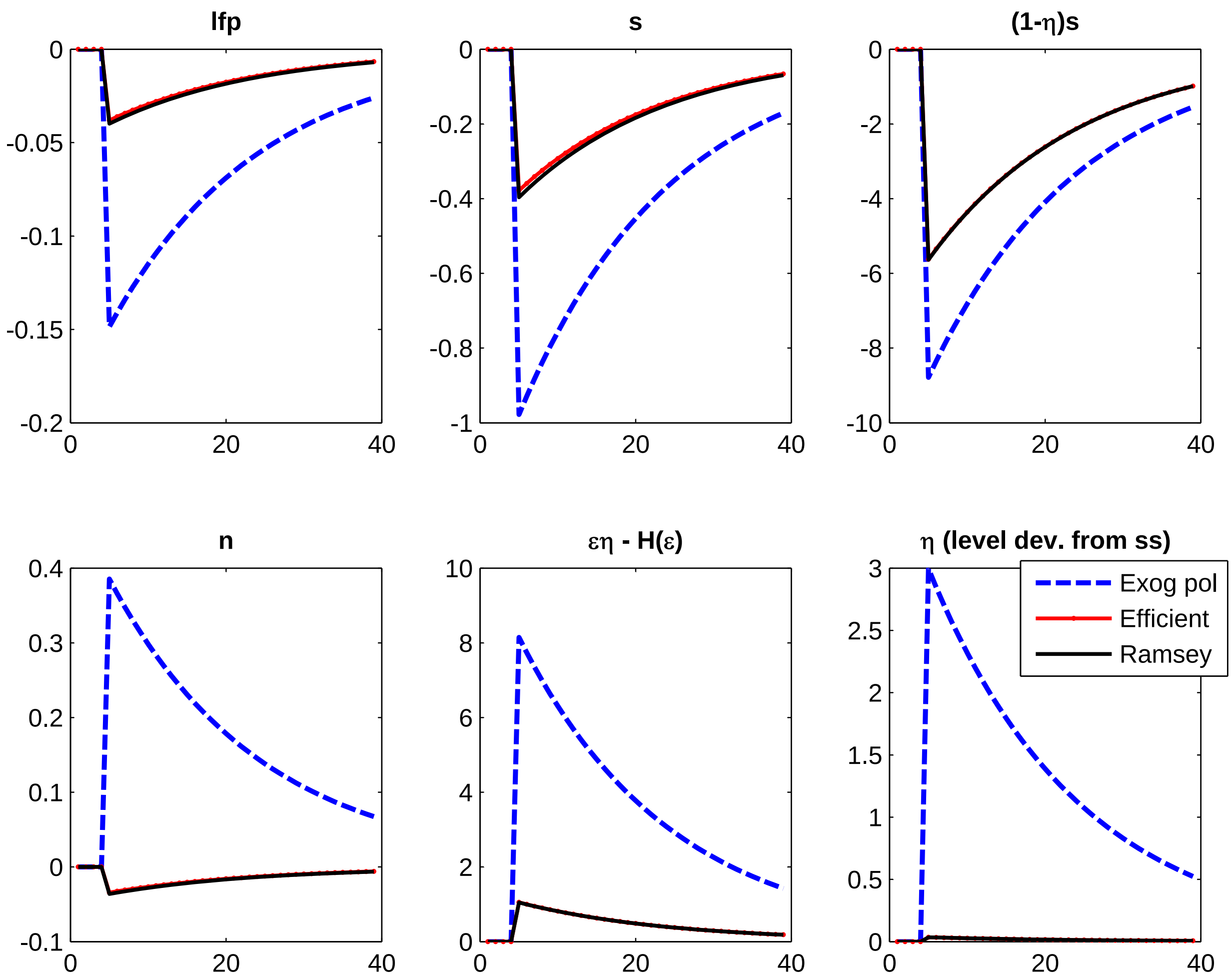

Figure 3: **Labor market responses to one-percent positive shock to z_t in three different equilibria.** The efficient equilibrium in dotted lines, the baseline exogenous-policy equilibrium in dashed lines, and Ramsey equilibrium in solid lines. The efficient responses and the Ramsey responses are identical in each panel. Unless otherwise noted, vertical axes plot percentage deviation from respective long-run equilibrium.

7.3 ...Which Requires Tax Volatility

The final step, then, is to describe how efficient fluctuations in tightness are decentralized using tax policy. In baseline Ramsey models, the mapping from the Ramsey-optimal intertemporal and static wedges to the *set* of available taxes is straightforward, and the mappings along each margin are almost always independent of each other: the Ramsey-optimal intertemporal wedge pins down the intertemporal tax independently of the static wedge, and the Ramsey-optimal static wedge pins down the labor (or consumption) tax independently of the intertemporal wedge.[22]

In our model, the mapping from a given period-t allocation to the pair of taxes (τ_t^n, τ_t^h) is defined by the wedge conditions (33) and (34). These two conditions *jointly* determine the period-t tax policy (τ_t^n, τ_t^h) that supports the period-t Ramsey allocation, rather than each wedge condition pinning down a single tax instrument in isolation.[23] Except for the special case of $(\alpha^E = 1, \chi = 0)$ mentioned at the end of Section 6, the mapping from allocations to taxes is a complicated endogenous object that can only be approximated quantitatively, and it is apparent that *wedge* smoothing does not immediately imply *tax* smoothing, as it typically does in Walrasian-based Ramsey analysis.

Intuitively, it is useful to think of the mapping from fluctuations in wedges to fluctuations in taxes in the following way. Intertemporal efficiency is the paramount concern, which can be thought of as requiring the efficient fluctuation in tightness

$$\frac{\tilde{\varepsilon}_t \eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)}{\tilde{\varepsilon}_{t-1} \eta(\tilde{\varepsilon}_{t-1}) - H(\tilde{\varepsilon}_{t-1})} \tag{37}$$

in every period t. Given the period-t state of the economy and, loosely speaking, expectations of period-$t+1$ allocations, an appropriate subsidy τ_t^h induces firms to hire a quantity of new employees that, for a given level of unemployment, induces the efficient $\tilde{\varepsilon}_t \eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)$ and thus a zero intertemporal wedge. This argument is based on the discussion in Section 7.2. Depending on parameter values and the size of realized shocks, the hiring subsidy τ_t^h may differ substantially from τ_{t-1}^h; Table 6 shows that fluctuations in τ^h are indeed large for the baseline parameters.

[22] A caveat to this simple decentralization is if an *incomplete* tax system is in place; this point is discussed below.

[23] Aruoba and Chugh (2010) present another environment in which frictions (affecting monetary exchange) imply joint mappings from wedges to taxes, and Arseneau, Chahrour, Chugh, and Finkelstein-Shapiro (2015) present a customer search-and-matching framework that also implies joint mappings from wedges to taxes.

The effect just described takes as given the measure of searching unemployed individuals and, by implication, the labor-force participation rate. Now viewing hiring costs instead as given, variation in τ^h between period $t-1$ and t causes an inefficient fluctuation of search activity and, in turn, $\tilde{\varepsilon}_t \eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)$. Given the period-$t$ state of the economy, an appropriate labor tax rate τ_t^n induces a rate of participation that, for a given quantity of potential job opportunities, induces the efficient $\tilde{\varepsilon}_t \eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)$ and thus a static wedge unchanged from period $t-1$. This argument is based on the static wedge condition (33), which is usefully thought of as the equilibrium version of the LFP condition, along with the accompanying discussion of the wage premium in Section 6.1. Again depending on parameter values and the size of realized shocks, the tax rate τ_t^n may differ sharply from τ_{t-1}^n.

An appropriate combination of time-varying labor taxes and hiring subsidies thus jointly achieves zero intertemporal distortions and static wedge smoothing, which is tantamount to stabilizing the two-dimensional notion of the "labor wedge." More generally, the mapping from wedge smoothing to the dynamics of taxes in the model depends on whether or not the non-tax components of the wedges fluctuate efficiently. If they do not, then tax variability offsets inefficient fluctuations in the wedge; if they do, then tax variability is unnecessary.[24,25]

7.4 Comparison with Matching Models

This subsection describes important parallels between the efficiency results we have derived for labor-selection models and those that exist for matching models.

7.4.1 Matching Function

It appears that there is no matching function in the selection model. However, suppose there were a "trivial" matching function. Denoting by v_t the number of job vacancies, we could suppose that there is a "matching function" defined over active job seekers and

[24] We emphasize that this is an efficiency-based motivation for tax volatility, unlike the results in Aiyagari, Marcet, Marimon, and Sargent (2002), Schmitt-Grohe and Uribe (2005), or Chugh (2006), in which the inability (or undesirability) of the government to make debt repayments fully state contingent leads to large fluctuations in tax rates in order to meet budget shocks; such a channel does not exist in our model because government debt payments are fully state contingent.

[25] Although not formalized by Arseneau and Chugh (2008), the result that possibly-time-varying policy can achieve efficiency in a search-and-bargaining economy is the idea of the "dynamic bargaining power effect" in their monetary policy study.

open job vacancies in any period t that takes the Cobb-Douglas form $s_t^{\xi} v_t^{1-\xi}$, with $\xi = 1$. With this trivial specification, the analogy with the Hosios condition for matching models, which requires that the Nash bargaining power of workers equal the elasticity of aggregate matches with respect to the number of active job seekers, is apparent: $\alpha^E = \xi = 1$. Furthermore, it is only new workers to whom this Hosios-like condition applies because incumbent workers are not available for work in the labor market at large; they are already joined to their current employer.[26]

7.4.2 "Labor-Market Tightness"

Figure 1 shows a crucial difference in the matching model vs. the selection model, which is the *timing* of labor demand. Allowing for both endogenous LFP and instantaneous production in the selection model helps us easily compare and contrast the selection model's concept of "cost spread" tightness with the matching model's concept of tightness. There are both similarities and differences across the two frameworks.

The most important similarity is that tightness in the selection model and in the search and matching model is crucial for generating both "first-best" (efficient) allocations and "second-best" Ramsey allocations. Mortensen (1982), Hosios (1990), Moen (1997), and others showed the importance of market tightness for efficient allocations in matching markets, and Arseneau and Chugh (2012) analogously showed its importance for "second-best" Ramsey allocations.[27] Our results regarding $\tilde{\varepsilon}\eta(\tilde{\varepsilon}) - H(\tilde{\varepsilon})$ portray that the same idea is true for selection markets.

The most important difference is that, in the matching phase (which is the first portion of period t in Figure 1), tightness appears *contemporaneously* in *both* the efficient LFP condition *and* the efficient vacancy-creation condition *before* a match occurs. By contrast, in the selection model, *contemporaneous* tightness — the cost spread — appears *only* in the efficient LFP condition (26). In the efficient selection condition (27), tightness only shows up *after* a match has occurred (which is the second portion of period t in Figure 1) and thus in *future* discounted-value terms.

With full turnover of the workforce ($\rho = 1$), efficient selection is independent of the *contemporaneous* cost spread, but efficient labor supply, described in Section 5.3 and

[26] Leaving aside on-the-job search, which one could presumably incorporate into selection models as tractably as can be incorporated in matching models.

[27] See Stiglitz (2014) for further discussion on the fiscal-policy terminology "second-best," "third-best," and so.

repeated here for convenience,

$$\frac{h'(lfp_t)}{u'(c_t)} = \eta(\tilde{\varepsilon}_t)\left(z_t - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right), \tag{38}$$

remains a function of the contemporaneous cost spread. In the matching model, though, full-turnover of the workforce causes — as shown in Arseneau and Chugh (2012, Section V) — *neither* period-t efficient labor supply *nor* period-t efficient labor demand to be independent of period-t tightness, which spotlights the fundamental difference in labor demand across the two frameworks.

7.5 Restoring the Optimality of Tax Smoothing

The non-tax components of the wedges that make tax variability optimal are inefficiently-low worker bargaining power and the existence of positive unemployment transfers, as the preceding analysis and discussions make clear. The second, third, and fourth rows of Table 6 document the volatility of Ramsey-optimal wedges and taxes when, respectively, efficient selection ($\alpha^E = 1$) is restored, unemployment transfers are assumed to be zero ($\chi = 0$), or both. Each of these experiments is conducted keeping all other parameters fixed at their baseline settings; the aim of these experiments is thus not to preserve empirical relevance of the exogenous-policy model, but rather to shed light on the quantitative importance of these two structural features in determining the dynamics of optimal taxes.

Raising new workers' bargaining power α^E to unity by itself or reducing unemployment transfers to zero by itself leads to an order-of-magnitude reduction in tax variability. What raising α^E and lowering χ have in common is that each shifts surplus-sharing through Nash bargaining towards efficient surplus-sharing: the former because of the results we showed in Section 5 and Section 6, the latter because, given the primitives of the model, χ has no role in determining efficient allocations because it represents neither preferences nor technology. Indeed, α^E (and, for that matter, α^I) also has no role in determining efficient allocations because bargaining is *only* a feature of decentralization.

The first three rows of Table 6 show that it is really the combination of unemployment transfers and low worker bargaining power for new employees that is important in driving the tax volatility in the baseline model. If both structural parameters are simultaneously set to their efficient values (the fourth row of Table 6), then both static and intertemporal wedges are completely stabilized across time. The mapping from wedges to taxes in this case is easy. Comparing (34) with (30) shows that intertemporal efficiency is achieved with $\tau_t^h = 0\ \forall t$. In turn, condition (33) shows that static wedge smoothing implies labor tax smoothing. Moreover, the dynamics of *all Ramsey allocations are identical* regardless of the (α^E, χ) pair in the decentralized economy.

7.6 Optimal Taxation Issues: Completeness of Tax System

An important issue in models of optimal taxation is whether or not the available tax instruments constitute a *complete* tax system. The tax system is complete in our model. Establishing this is important for two reasons. First, at a technical level, proving completeness reaffirms that the Ramsey problem as formulated in Section 4 is indeed correct. As shown by Chari and Kehoe (1999, p. 1680), Correia (1996), Armenter (2008), and many others, incompleteness of the tax system requires imposing additional constraints that reflect the incompleteness. Second, it is well-understood in Ramsey theory that incomplete tax systems can lead to a wide range of "unnatural" policy prescriptions in which the use of some instruments (in either the short run or the long run) proxy for other, perhaps more natural, instruments.[28] Demonstrating completeness therefore establishes that none of our results is due to any policy instrument serving as imperfect proxies for other, unavailable, instruments.

As Chari and Kehoe (1999, pp. 1679-1680) explain, an incomplete tax system is in place if, for at least one pair of goods in the economy, the government has *no* policy instrument that, in the decentralized economy, uniquely creates a wedge between MRS of those goods and the corresponding MRT. Based on the model-appropriate concepts of MRTs and wedges developed in Sections 5 and 6, it is trivial to show that the pair of instruments (τ_t^n, τ_t^h) constitutes a complete tax system.

The argument is as follows: Proposition 1 proved that there are two margins of adjustment in the economy. Completeness thus requires two policy instruments whose *joint* setting induces a unique wedge in each of the two margins. The two instruments τ_t^n and τ_t^h do exactly this. Even though *both* instruments appear in *both* the static wedge (33) *and* the intertemporal wedge (34), they appear in different relation to each other in the two different wedges. A policy pair (τ_t^n, τ_t^h) thus determines each wedge uniquely.

A consequence of completeness of the tax system is that the introduction of any

[28] Stiglitz (2014) provides a high-level summary of the optimal taxation literature dating all the way back to Ramsey (1927).

additional tax instruments into the environment necessarily implies indeterminacy of the decentralization of Ramsey allocations. Some of the resulting new policy decentralizations would feature constant labor income tax rates along the business cycle. If one were to prefer this way of "restoring the optimality of tax-smoothing," it must be driven by considerations outside the scope of the model. The model does not provide any basis for preferring one decentralization over another, which is a well-understood point in Ramsey models. Hence, loading redundant policy instruments onto the static and intertemporal wedges would be an uninteresting way of restoring labor tax smoothing.

8 Conclusion

As stated in the conclusion of Arseneau and Chugh (2012), it's not just inefficient matching markets that can generate large volatility in optimal tax rates. Instead, it's *any* model that departs from Walrasian labor markets that generates surpluses to be shared *and* in which decentralized surpluses are split in an inefficient manner that has the potential to generate large volatility in fiscal policy instruments. Our results show that this occurs in the selection model, which, at both face value and in terms of the model's primitives, is seemingly quite different from the conventional search and matching model. More importantly, it's inefficiencies in labor markets that are of utmost concern for reactive policy, be it optimal fiscal policy, optimal monetary policy, or optimal regulatory policy. Both policy makers and researchers in policy-related areas must be aware of the market primitives and its implied distortions in an economy before deciding upon best policy paths.

Our paper has analytically developed the appropriate wedges for the selective-hiring environment and a selection-model-consistent notion of market tightness. Taking together the growing literature on optimal policy in "frictional" markets, one natural next step is to study both positive and normative issues when labor markets feature both search and matching and selective hiring.

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A Household Optimization (Online Appendix)

The representative household maximizes expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h\left((1-\eta_t)s_t + n_t\right)\right] \tag{39}$$

subject to the sequence of budget constraints

$$c_t + \sum_j \frac{1}{R_t^j} b_{t+1}^j = (1-\rho)n_{t-1}(1-\tau_t^n)w_t^I + \eta_t \frac{(1-\tau_t^n)\omega_{et}}{\eta_t} s_t + (1-\eta_t)s_t\chi + b_t + (1-\tau^{pr})\Pi_t, \tag{40}$$

and perceived laws of motion for its employment level

$$n_t = (1-\rho)n_{t-1} + \eta_t s_t. \tag{41}$$

Let $\beta^t \phi_t$ denote the Lagrange multiplier on the period-t budget constraint, and $\beta^t \mu_{ht}$ denote the Lagrange multiplier on the household's period-t perceived law of motion. The first-order conditions with respect to c_t, s_t, n_t, and b_{t+1}^j are

$$u'(c_t) - \phi_t = 0, \tag{42}$$

$$-(1-\eta_t)h'((1-\eta_t)s_t + n_t) + \phi_t\left((1-\tau_t^n)\omega_{et} + (1-\eta_t)\chi\right) + \mu_{ht}\eta_t = 0, \tag{43}$$

$$-\mu_{ht} - h'((1-\eta_t)s_t + n_t) + (1-\rho)\beta E_t\left\{\phi_{t+1}(1-\tau_{t+1}^n)w_{t+1}^I + \mu_{ht+1}\right\} = 0, \tag{44}$$

and

$$u'(c_t) = \beta R_t^j u'(c_{t+1}^j), \ \forall j. \tag{45}$$

With first-order conditions now computed, switch to the notation $lfp_t = (1-\eta_t)s_t + n_t$, which follows from the accounting identities of the model.

From (43), we can isolate

$$\mu_{ht} = \frac{(1-\eta_t)h'(lfp_t) - u'(c_t)\left((1-\tau_t^n)\omega_{et} + (1-\eta_t)\chi\right)}{\eta_t}. \tag{46}$$

Substituting this into (44),

$$\frac{(1-\eta_t)h'(lfp_t) - u'(c_t)\left((1-\tau_t^n)\omega_{et} + (1-\eta_t)\chi\right)}{\eta_t} = -h'(lfp_t)$$

$$+ \quad (1-\rho)\beta E_t \left\{ u'(c_{t+1})(1-\tau^n_{t+1})w^I_{t+1} + \left(\frac{(1-\eta_{t+1})h'(lfp_{t+1}) - u'(c_{t+1})\left((1-\tau^n_{t+1})\omega_{et+1} + (1-\eta_{t+1})\chi\right.}{\eta_{t+1}} \right.\right.$$

Dividing by $u'(c_t)$ and using the notation $\Xi_{t+1|t} \equiv \beta u'(c_{t+1})/u'(c_t)$,

$$\frac{(1-\eta_t)h'(lfp_t) - u'(c_t)\left((1-\tau^n_t)\omega_{et} + (1-\eta_t)\chi\right)}{\eta_t u'(c_t)} = -\frac{h'(lfp_t)}{u'(c_t)}$$

$$+ \quad (1-\rho)E_t \left\{ \Xi_{t+1|t} \left[(1-\tau^n_{t+1})w^I_{t+1} + \left(\frac{(1-\eta_{t+1})h'(lfp_{t+1}) - u'(c_{t+1})\left((1-\tau^n_{t+1})\omega_{et+1} + (1-\eta_{t+1})\chi\right)}{\eta_{t+1}u'(c_{t+1})} \right.\right.\right.$$

which is a representation of the LFP condition that is useful for the Nash bargaining problem in Appendix C because it is recursive in the term $\frac{(1-\eta_t)h'(lfp_t)-u'(c_t)\left((1-\tau^n_t)\omega_{et}+(1-\eta_t)\chi\right)}{\eta_t u'(c_t)}$.

To obtain the representation that appears in the main text, first recognize that the second additive term under the expectation operator in the previous expression can be written compactly as $\frac{\mu_{ht+1}}{u'(c_{t+1})}$, so that

$$\frac{(1-\eta_t)h'(lfp_t) - u'(c_t)\left((1-\tau^n_t)\omega_{et} + (1-\eta_t)\chi\right)}{\eta_t u'(c_t)} = -\frac{h'(lfp_t)}{u'(c_t)} + (1-\rho)E_t \left\{ \Xi_{t+1|t} \left[(1-\tau^n_{t+1})w^I_{t+1} + \frac{\mu_{ht+1}}{u'(c_{t+1}} \right.\right. \tag{49}$$

Rearranging,

$$\frac{(1-\eta_t)h'(lfp_t)}{\eta_t u'(c_t)} = \frac{(1-\tau^n_t)\omega_{et} + (1-\eta_t)\chi}{\eta_t} - \frac{h'(lfp_t)}{u'(c_t)} + (1-\rho)E_t \left\{ \Xi_{t+1|t} \left[(1-\tau^n_{t+1})w^I_{t+1} + \frac{\mu_{ht+1}}{u'(c_{t+1})} \right] \right\}. \tag{50}$$

Expanding the terms on the left-hand side,

$$\frac{h'(lfp_t)}{\eta_t u'(c_t)} - \frac{h'(lfp_t)}{u'(c_t)} = \frac{(1-\tau^n_t)\omega_{et} + (1-\eta_t)\chi}{\eta_t} - \frac{h'(lfp_t)}{u'(c_t)} + (1-\rho)E_t \left\{ \Xi_{t+1|t} \left[(1-\tau^n_{t+1})w^I_{t+1} + \frac{\mu_{ht+1}}{u'(c_{t+1})} \right] \right\}, \tag{51}$$

which allows canceling a couple of terms, to give

$$\frac{h'(lfp_t)}{\eta_t u'(c_t)} = \frac{(1-\tau^n_t)\omega_{et} + (1-\eta_t)\chi}{\eta_t} + (1-\rho)E_t \left\{ \Xi_{t+1|t} \left[(1-\tau^n_{t+1})w^I_{t+1} + \frac{\mu_{ht+1}}{u'(c_{t+1})} \right] \right\}. \tag{52}$$

Multiplying by η_t gives

$$\frac{h'(lfp_t)}{u'(c_t)} = \eta_t \left[\frac{(1-\tau^n_t)\omega_{et}}{\eta_t} + (1-\rho)E_t \left\{ \Xi_{t+1|t} \left[(1-\tau^n_{t+1})w^I_{t+1} + \frac{\mu_{ht+1}}{u'(c_{t+1})} \right] \right\} \right] + (1-\eta_t)\chi, \tag{53}$$

which is the representation of the LFP condition that appears as condition (6) in the main text. It is also useful to express this as

$$\frac{h'(lfp_t)}{u'(c_t)} - \chi = \eta_t \left[\frac{(1-\tau_t^n)\omega_{et}}{\eta_t} - \chi + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left[(1-\tau_{t+1}^n) w_{t+1}^I + \frac{\mu_{ht+1}}{u'(c_{t+1})} \right] \right\} \right] \tag{54}$$

because the term on the left-hand side of this latter representation turns out to appear in the continuation values of all of the Nash wage equations derived in Appendix C. Furthermore, this form of the LFP condition also allows for expression in terms of the value equations derived in Appendix C, as shown below.

For the Nash bargaining problem in Appendix C, define the value function associated with the household problem as $\mathbf{V}(n_{t-1})$. The associated envelope condition is thus

$$\begin{aligned} \mathbf{V}'(n_{t-1}) &= (1-\rho) \left[\phi_t (1-\tau_t^n) w_t^I + \mu_{ht} \right] \\ &= (1-\rho) \left[u'(c_t)(1-\tau_t^n) w_t^I + \frac{(1-\eta_t) h'(lfp_t) - u'(c_t)\left((1-\tau_t^n)\omega_{et} + (1-\eta_t)\chi\right)}{\eta_t} \right] \end{aligned} \tag{55}$$

where the second line follows from (46). Finally, for use in Appendix C, the period $t+1$ envelope condition can be expressed in discounted terms as

$$\begin{aligned} \frac{\beta u'(c_{t+1})}{u'(c_t)} \frac{\mathbf{V}'(n_t)}{u'(c_{t+1})} &= (1-\rho) \frac{\beta u'(c_{t+1})}{u'(c_t)} \left[(1-\tau_{t+1}^n) w_{t+1}^I + \frac{\mu_{ht+1}}{u'(c_{t+1})} \right] \\ &= (1-\rho) \frac{\beta u'(c_{t+1})}{u'(c_t)} (1-\tau_{t+1}^n) w_{t+1}^I \\ &\quad + (1-\rho) \frac{\beta u'(c_{t+1})}{u'(c_t)} \left[\frac{(1-\eta_{t+1}) h'(lfp_{t+1}) - u'(c_{t+1}) \left((1-\tau_{t+1}^n)\omega_{et+1} + (1-\eta_{t+1})\chi \right)}{\eta_{t+1} u'(c_{t+1})} \right] \end{aligned} \tag{56}$$

B Firm Optimization (Online Appendix)

The representative firm chooses state-contingent processes $\{\tilde{\varepsilon}_t, n_t\}_{t=0}^{\infty}$ to maximize the present value of discounted profits

$$E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left[z_t n_t - (1-\tau_t^h) \left(\frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} \right) \eta(\tilde{\varepsilon}_t) s_t - \frac{\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} \eta(\tilde{\varepsilon}_t) s_t - (1-\rho) n_{t-1} w_t^I \right] \tag{57}$$

subject to the sequence of perceived laws of motion for its employment stock

$$n_t = (1-\rho) n_{t-1} + s_t \eta(\tilde{\varepsilon}_t). \tag{58}$$

Letting $\beta^t \mu_{ft}$ denote the Lagrange multiplier on the period-t law of motion (58), the first-order conditions with respect to $\tilde{\varepsilon}_t$ and n_t are

$$\mu_{ft} \eta'(\tilde{\varepsilon}_t) s_t - \omega_e'(\tilde{\varepsilon}_t) s_t - (1-\tau_t^h) \left(\frac{H'(\tilde{\varepsilon}_t)}{\eta'(\tilde{\varepsilon}_t)} \right) \eta'(\tilde{\varepsilon}_t) s_t = 0 \tag{59}$$

and

$$z_t - \mu_{ft} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left(\mu_{ft+1} - w_{t+1}^I \right) \right\} = 0, \tag{60}$$

in which $\Xi_{t+1|t} \equiv \Xi_{t+1|0} / \Xi_{t|0}$ is the one-period stochastic discount factor. From (59), the value to the firm of an employee can be measured as

$$\begin{aligned} \mu_{ft} &= \frac{\omega_e'(\tilde{\varepsilon}_t) + (1-\tau_t^h) \left(\frac{H'(\tilde{\varepsilon}_t)}{\eta'(\tilde{\varepsilon}_t)} \right)}{\eta'(\tilde{\varepsilon}_t)} \\ &= \frac{w(\tilde{\varepsilon}_t) f(\tilde{\varepsilon}_t) + (1-\tau_t^h) \cdot \tilde{\varepsilon}_t f(\tilde{\varepsilon}_t)}{f(\tilde{\varepsilon}_t)} \\ &= w(\tilde{\varepsilon}_t) + (1-\tau_t^h)(\tilde{\varepsilon}_t), \end{aligned} \tag{61}$$

where the second line follows from the Fundamental Theorem of Calculus.

Substituting (61) into (60),

$$(1-\tau_t^h)\tilde{\varepsilon}_t = z_t - w(\tilde{\varepsilon}_t) + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left[(1-\tau_{t+1}^h) \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I \right] \right\}, \tag{62}$$

which is the firm's hiring (selection) condition that appears as expression (10) in the main text.

Define the value function associated with the firm problem as $\mathbf{F}(n_{t-1})$. The envelope condition is thus

$$\begin{aligned}\mathbf{F}'(n_{t-1}) &= (1-\rho)\left[\mu_{ft} - w_t^I\right] \\ &= (1-\rho)\left[(1-\tau_t^h)\tilde{\varepsilon}_t + w(\tilde{\varepsilon}_t) - w_t^I\right], \qquad (63)\end{aligned}$$

where the second line makes use of (61). For use in the analysis of the Nash bargaining problems in Appendix C, the period $t+1$ envelope condition can be expressed in discounted terms as

$$\begin{aligned}\frac{\beta u'(c_{t+1})}{u'(c_t)}\mathbf{F}'(n_t) &= (1-\rho)\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[(1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right] \\ &= (1-\rho)\Xi_{t+1|t}\left[(1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right]. \qquad (64)\end{aligned}$$

C Nash-Bargained Wages (Online Appendix)

This section presents the details of the derivation of the Nash wage equations given in Proposition 1. This requires first defining the values to the both household and the firm of a newly-hired worker with idiosyncratic characteristics ε_{it} and any given incumbent worker *at the time bargaining occurs.* Because of the timing of events in the model (see Figure 2), these values are properly defined in the "second subperiod" of period t, immediately after worker selection has taken place and thus each individual's measured labor market status for period t is known. In contrast, household-level decisions (in particular, the participation decision of how many individuals to send to look for jobs) occurs in the "first subperiod" of period t, *before* selection has taken place. The temporal separation of events in the model requires that we construct the bargaining-relevant value equations by simply accounting for the payoffs (viewed from the perspectives of the household and the firm).

A further observation is in order, one that applies to firms and households. Regardless of whether a given individual is a newly-hired or an incumbent worker in period t, he will be an incumbent worker in period $t+1$ if he remains employed. From the perspective of the household (or the firm), the continuation value of any worker is thus identical to that of any other worker (because it is only in the first period of employment that workers are heterogeneous) and is measured by the envelope condition of the household (or firm) problem. It is thus already apparent that the envelope conditions derived above measure the values of an incumbent worker, although this is verified below.

C.1 Value Equations for Household

A labor-market participant who either was not selected in period t or was selected (or was an incumbent) but fails to successfully complete wage negotiations is classified as "unemployed" and receives a transfer from the government, and thus has value (measured in goods) to the household

$$\mathbf{U}_t = \chi. \tag{65}$$

There is zero continuation payoff to the household of an unemployed individual because the household re-optimizes participation at the start of period $t+1$, and unemployment is *not* a state variable for the household at the start of period $t+1$. Note that, because the solution to the Nash bargaining problem will yield an interior solution, in equilibrium it is only individuals that were looking for work but were not selected that receive the unemployment transfer (which justifies including only unemployment transfers for this group of individuals in the household budget constraint (40)).

C.1.1 Incumbent Workers

An incumbent worker in period t has value (measured in period-t goods) to the household

$$\begin{aligned}
\mathbf{W}_{It} &= (1-\tau_t^n)w_t^I + E_t\left\{\Xi_{t+1|t}\frac{\mathbf{V}'(n_t)}{u'(c_{t+1})}\right\} \\
&= (1-\tau_t^n)w_t^I + (1-\rho)E_t\Bigg\{\Xi_{t+1|t}\Bigg[(1-\tau_{t+1}^n)w_{t+1}^I + \frac{(1-\eta_{t+1})h'(lfp_{t+1}) - u'(c_{t+1})\Big((1-\tau_{t+1}^n)\omega_{et+}}{\eta_{t+1}u'(c_{t+1})}
\end{aligned}$$

in which the first line follows from the discussion above, and the second line makes use of the expression for the household's envelope condition (56).

The surplus earned by the household from having an incumbent worker successfully complete wage negotiations is thus

$$\begin{aligned}
\mathbf{W}_{It} - \mathbf{U}_t = (1-\tau_t^n)w_t^I - \chi \\
+ \quad (1-\rho)E_t\Bigg\{\Xi_{t+1|t}\Bigg[(1-\tau_{t+1}^n)w_{t+1}^I + \frac{(1-\eta_{t+1})h'(lfp_{t+1}) - u'(c_{t+1})\Big((1-\tau_{t+1}^n)\omega_{et+1} + (1-\eta_{t+1})\chi\Big)}{\eta_{t+1}u'(c_{t+1})}\Bigg]
\end{aligned} \tag{67}$$

The goal of the next several steps is to rewrite this expression in a form convenient for the Nash bargaining problem in Appendix C.

Comparing expression (67) with the LFP condition (48) allows for expressing the surplus $\mathbf{W}_{It} - \mathbf{U}_t$ as

$$\mathbf{W}_{It} - \mathbf{U}_t = (1-\tau_t^n)w_t^I - \chi + \frac{(1-\eta_t)h'(lfp_t) - u'(c_t)\left((1-\tau_t^n)\omega_{et} + (1-\eta_t)\chi\right)}{\eta_t u'(c_t)} + \frac{h'(lfp_t)}{u'(c_t)}. \tag{68}$$

Rearranging,

$$(1-\tau_t^n)w_t^I + \frac{(1-\eta_t)h'(lfp_t) - u'(c_t)\left((1-\tau_t^n)\omega_{et} + (1-\eta_t)\chi\right)}{\eta_t u'(c_t)} = \mathbf{W}_{It} - \mathbf{U}_t + \chi - \frac{h'(lfp_t)}{u'(c_t)}, \tag{69}$$

which is the period-t counterpart of the term inside expectations in expression (67). Making this substitution,

$$
\begin{aligned}
\mathbf{W}_{It} - \mathbf{U}_t &= (1-\tau_t^n)w_t^I - \chi + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1} - \mathbf{U}_{t+1} + \chi - \frac{h'(lfp_{t+1})}{u'(c_{t+1})}\right]\right\} \\
&= (1-\tau_t^n)w_t^I - \chi + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1} - \mathbf{U}_{t+1}\right]\right\} + (1-\rho)\chi E_t\Xi_{t+1|t} \\
&\quad -(1-\rho)E_t\left\{\Xi_{t+1|t}\frac{h'(lfp_{t+1})}{u'(c_{t+1})}\right\},
\end{aligned}
\tag{70}
$$

C.1.2 Newly-Hired Workers

A newly-hired worker with idiosyncratic characteristics ε_{it} in period t has value (measured in period-t goods) to the household

$$
\mathbf{W}_E(\varepsilon_{it}) = (1-\tau_t^n)w(\varepsilon_{it}) + E_t\left\{\Xi_{t+1|t}\frac{\mathbf{V}'(n_t)}{u'(c_{t+1})}\right\},
\tag{71}
$$

in which the first line again follows from the discussion above, and the second line makes use of the expression for the household's envelope condition (56). Note that the wage payment to a newly-hired worker $w(\varepsilon_{it})$ can be conditioned on his idiosyncratic characteristics.

The surplus earned by the household from having a newly-selected individual successfully complete wage negotiations is thus

$$
\begin{aligned}
\mathbf{W}_E(\varepsilon_{it}) - \mathbf{U}_t &= (1-\tau_t^n)w(\varepsilon_{it}) - \chi + E_t\left\{\Xi_{t+1|t}\frac{\mathbf{V}'(n_t)}{u'(c_{t+1})}\right\} \\
&= (1-\tau_t^n)w(\varepsilon_{it}) - \chi + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[(1-\tau_{t+1}^n)w_{t+1}^I + \frac{(1-\eta_{t+1})h'(lfp_{t+1}) - u'(c_{t+}}{\eta_{t+}}\right.\right. \\
&= (1-\tau_t^n)w(\varepsilon_{it}) - \chi + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1} - \mathbf{U}_{t+1} + \chi - \frac{h'(lfp_{t+1})}{u'(c_{t+1})}\right]\right\},
\end{aligned}
$$

in which the second line makes use of (56), and the third line uses expression (69) from the derivation of the surplus expression $\mathbf{W}_{It} - \mathbf{U}_t$ above. Breaking apart the terms inside the expectation, we have

$$
\begin{aligned}
&\mathbf{W}_E(\varepsilon_{it}) - \mathbf{U}_t = (1-\tau_t^n)w(\varepsilon_{it}) - \chi \\
&\quad +(1-\rho)E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1} - \mathbf{U}_{t+1}\right]\right\} + (1-\rho)\chi E_t\Xi_{t+1|t} - (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{h'(lfp_{t+1})}{u'(c_{t+1})}\right\},
\end{aligned}
$$

(74)

the form of the household's surplus from a new employment relationship used in the derivation of the Nash wage function below. Before proceeding, however we note that integrating the surplus $\mathbf{W}_E(\varepsilon_{it}) - \mathbf{U}_t$ expressed as in the first line above gives

$$\begin{aligned}
\int_{-\infty}^{\tilde{\varepsilon}_t} \mathbf{W}_E(\varepsilon_{it}) f(\varepsilon_{it}) d\varepsilon_{it} - \mathbf{U}_t &= (1-\tau_t^n) \int_{-\infty}^{\tilde{\varepsilon}_t} w(\varepsilon_{it}) f(\varepsilon_{it}) d\varepsilon_{it} - \chi \int_{-\infty}^{\tilde{\varepsilon}_t} f(\varepsilon_{it}) d\varepsilon_{it} + E_t \left\{ \Xi_{t+1|t} \frac{\mathbf{V}'(n_t)}{u'(c_{t+1})} \right\} \int_{-\infty}^{\tilde{\varepsilon}} \\
&= (1-\tau_t^n)\omega_e(\tilde{\varepsilon}_t) - \chi\eta(\tilde{\varepsilon}_t) + \eta(\tilde{\varepsilon}_t) E_t \left\{ \Xi_{t+1|t} \frac{\mathbf{V}'(n_t)}{u'(c_{t+1})} \right\} \\
&= \eta(\tilde{\varepsilon}_t) \left[\frac{(1-\tau_t^n)\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} + E_t \left\{ \Xi_{t+1|t} \frac{\mathbf{V}'(n_t)}{u'(c_{t+1})} \right\} \right] - \eta(\tilde{\varepsilon}_t)\chi.
\end{aligned}$$

With this expression for the *expected* surplus to the household of having one of its unemployed members selected for work, the LFP condition (54) derived in Appendix A can be expressed as

$$\begin{aligned}
\frac{h'(lfp_t)}{u'(c_t)} - \chi &= \eta(\tilde{\varepsilon}_t) \left[\frac{(1-\tau_t^n)\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} - \chi + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left[(1-\tau_{t+1}^n) w_{t+1}^I + \frac{\mu_{ht+1}}{u'(c_{t+1})} \right] \right\} \right] \\
&= \eta(\tilde{\varepsilon}_t) \left[\frac{(1-\tau_t^n)\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left[(1-\tau_{t+1}^n) w_{t+1}^I + \frac{\mu_{ht+1}}{u'(c_{t+1})} \right] \right\} \right] - \eta(\tilde{\varepsilon}_t)\chi \\
&= \eta(\tilde{\varepsilon}_t) \left[\frac{(1-\tau_t^n)\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} + E_t \left\{ \Xi_{t+1|t} \frac{\mathbf{V}'(n_t)}{u'(c_{t+1})} \right\} \right] - \eta(\tilde{\varepsilon}_t)\chi \\
&= \int_{-\infty}^{\tilde{\varepsilon}_t} \mathbf{W}_E(\varepsilon_{it}) f(\varepsilon_{it}) d\varepsilon_{it} - \eta(\tilde{\varepsilon}_t)\mathbf{U}_t \\
&= \eta(\tilde{\varepsilon}_t) \left(\frac{\int_{-\infty}^{\tilde{\varepsilon}_t} \mathbf{W}_E(\varepsilon_{it}) f(\varepsilon_{it}) d\varepsilon_{it}}{\eta(\tilde{\varepsilon}_t)} \right) - \eta(\tilde{\varepsilon}_t)\mathbf{U}_t, \qquad (76)
\end{aligned}$$

in which the third line uses the household-level envelope condition derived above and the fourth line uses the definition of $\mathbf{U}_t$. This expression states that optimal participation equates the (net) marginal utility cost (denominated in goods) to the household of participation to the expected surplus from having an unemployed individual selected for work. The expectation is taken over both the probability of being selected as well as an individual's idiosyncratic characteristics, which are unknown at the time participation decisions are made.

C.2 Value Equations for Firm

C.2.1 Incumbent Workers

An incumbent worker in period t has value (measured in period-t goods) to the firm

$$\begin{aligned} \mathbf{J}_{It} &= z_t - w_t^I + E_t\left\{\Xi_{t+1|t}\mathbf{F}'(n_t)\right\} \\ &= z_t - w_t^I + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[(1-\tau_{t+1}^h)\cdot\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right]\right\}, \end{aligned} \tag{77}$$

in which the first line follows from the discussion above, and the second line makes use of the expression for the firm's envelope condition (64).

Next, note from the hiring condition (62) that the last term on the right-hand side of (77) is $(1-\tau_t^h)\tilde{\varepsilon}_t + w(\tilde{\varepsilon}_t) - z_t$; substituting this in (77) gives

$$\mathbf{J}_{It} = (1-\tau_t^h)\tilde{\varepsilon}_t + w(\tilde{\varepsilon}_t) - w_t^I. \tag{78}$$

Comparing this expression with (77), we see that the value of an incumbent worker to the firm can be expressed recursively,

$$\mathbf{J}_{It} = z_t - w_t^I + (1-\rho)E_t\left\{\Xi_{t+1|t}\mathbf{J}_{It+1}\right\}, \tag{79}$$

and, furthermore, the relationship between the value to the firm of an incumbent worker and the firm's envelope condition is $\mathbf{J}_{It} = (1-\rho)\mathbf{F}'(n_{t-1})$.

C.2.2 Newly-Hired Workers

Similarly, a newly-hired individual with idiosyncratic characteristics ε_{it} in period t has value (measured in period-t goods) to the firm

$$\begin{aligned}
\mathbf{J}_E(\varepsilon_{it}) &= z_t - w(\varepsilon_{it}) + E_t\left\{\Xi_{t+1|t}\mathbf{F}'(n_t)\right\} \\
&= z_t - (1-\tau_t^h)\varepsilon_{it} - w(\varepsilon_{it}) + (1-\rho)E_t\left\{\Xi_{t+1|t}\mathbf{J}_{It+1}\right\} \\
&= z_t - (1-\tau_t^h)\varepsilon_{it} - w(\varepsilon_{it}) + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[(1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right]\right\}.
\end{aligned} \tag{80}$$

The first line again follows from the discussion above, the second line uses the relationship proven above between the envelope condition and the value to the firm of an incumbent worker, and the third line substitutes the expression for the firm's envelope condition (64).

Once again note that the wage payment $w(\varepsilon_{it})$ made to a newly-hired worker can be conditioned on his idiosyncratic characteristics. An important implication of individual-specific bargaining is that the "hiring condition" (62) does not hold with equality for *only* the marginal new hire, but instead for *every* new hire whose $\varepsilon_{it} \leq \tilde{\varepsilon}_t$. That is,

$$(1-\tau_t^h)\varepsilon_{it} = z_t - w(\varepsilon_{it}) + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[(1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right]\right\}, \ \forall \varepsilon_{it} \leq \tilde{\varepsilon}_t. \tag{81}$$

Again noting from the hiring condition (62) that the last term on the right-hand side of (80) is $(1-\tau_t^h)\cdot\tilde{\varepsilon}_t + w(\tilde{\varepsilon}_t) - z_t$, the value of a newly-hired worker with idiosyncratic characteristics ε_{it} can be expressed as

$$\mathbf{J}_E(\varepsilon_{it}) = (1-\tau_t^h)(\tilde{\varepsilon}_t - \varepsilon_{it}) + w(\tilde{\varepsilon}_t) - w(\varepsilon_{it}). \tag{82}$$

Clearly, the value of a new hire with the threshold idiosyncratic characteristics $\tilde{\varepsilon}_t$ has value

$$\mathbf{J}_E(\tilde{\varepsilon}_t) = 0, \tag{83}$$

that is, and as is intuitive, the firm earns zero value from a new worker who was exactly on the selection margin.

C.3 Nash Bargaining

The firm bargains individually with each of its workers, whether an incumbent or a new hire with idiosyncratic characteristics $\varepsilon_{it} < \tilde{\varepsilon}_t$, in every period. For every worker, the firm and the worker choose the real wage that maximizes the generalized Nash product

$$(\mathbf{W}_t - \mathbf{U}_t)^{\alpha^K} \mathbf{J}_t^{1-\alpha^K}, \tag{84}$$

in which $\alpha^K \in [0,1]$, $K \in \{E, I\}$, measures the bargaining power of the worker (α^I is the bargaining power of an incumbent worker, α^E is the bargaining power of a newly-hired worker). For the different types of workers, $\mathbf{W}_t$ is replaced by either $\mathbf{W}_{It}$ or $\mathbf{W}_E(\varepsilon_{it})$, $\mathbf{J}_t$ is replaced by either $\mathbf{J}_{It}$ or $\mathbf{J}_E(\varepsilon_{it})$, and α^K is replaced by either α^I or α^E.

Using the generic notation $\mathbf{W}_t$, $\mathbf{J}_t$, and α^K, the first-order condition of (84) with respect to the period-t real wage (which in the various cases below is either w_t^I or $w(\varepsilon_{it})$ — here, denote it simply w_t) is

$$\alpha^K (\mathbf{W}_t - \mathbf{U}_t)^{\alpha^K - 1} \mathbf{J}_t^{1-\alpha^K} \left(\frac{\partial \mathbf{W}_t}{\partial w_t} - \frac{\partial \mathbf{U}_t}{\partial w_t} \right) + (1 - \alpha^K) (\mathbf{W}_t - \mathbf{U}_t)^{\alpha^K} \mathbf{J}_t^{-\alpha^K} \frac{\partial \mathbf{J}_t}{\partial w_t} = 0. \tag{85}$$

To simplify, multiply by $\mathbf{J}_t^{\alpha^K}$, and also multiply by $(\mathbf{W}_t - \mathbf{U}_t)^{1-\alpha^K}$, which gives

$$\alpha^K \mathbf{J}_t \left(\frac{\partial \mathbf{W}_t}{\partial w_t} - \frac{\partial \mathbf{U}_t}{\partial w_t} \right) + (1 - \alpha^K) (\mathbf{W}_t - \mathbf{U}_t) \frac{\partial \mathbf{J}_t}{\partial w_t} = 0. \tag{86}$$

It is clear from the value equations above that, no matter the type of worker, the marginals are $\frac{\partial \mathbf{J}_t}{\partial w_t} = -1$, $\frac{\partial \mathbf{U}_t}{\partial w_t} = 0$, $\frac{\partial \mathbf{W}_{It}}{\partial w_t^I} = 1 - \tau_t^n$, and $\frac{\partial \mathbf{W}_E(\varepsilon_{it})}{\partial w(\varepsilon_{it})} = 1 - \tau_t^n$. Substituting these, the first-order condition simplifies, for incumbent workers and newly-hired workers, respectively, to

$$\frac{\mathbf{W}_{It} - \mathbf{U}_t}{1 - \tau_t^n} = \frac{\alpha^I}{1 - \alpha^I} \mathbf{J}_{It} \tag{87}$$

and

$$\frac{\mathbf{W}_E(\varepsilon_{it}) - \mathbf{U}_t}{1 - \tau_t^n} = \frac{\alpha^E}{1 - \alpha^E} \mathbf{J}_E(\varepsilon_{it}). \tag{88}$$

C.3.1 Incumbent Workers

To obtain an expression for the period-t bargained wage of an incumbent, begin with the sharing rule

$$\frac{\mathbf{W}_{It} - \mathbf{U}_t}{1-\tau_t^n} = \frac{\alpha^I}{1-\alpha^I}\mathbf{J}_{It}, \tag{89}$$

and substitute (70). This gives

$$\begin{aligned} & w_t^I - \frac{\chi}{1-\tau_t^n} + \frac{1-\rho}{1-\tau_t^n} E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1} - \mathbf{U}_{t+1}\right]\right\} \\ & + \frac{(1-\rho)\chi E_t \Xi_{t+1|t}}{1-\tau_t^n} - \frac{(1-\rho)E_t\left\{\Xi_{t+1|t}\frac{h'(lfp_{t+1})}{u'(c_{t+1})}\right\}}{1-\tau_t^n} = \frac{\alpha^I}{1-\alpha^I}\mathbf{J}_{It}. \end{aligned}$$

Then, substitute the time-$(t+1)$ sharing rule (89) in the third term on the left-hand side, which gives

$$\begin{aligned} & w_t^I - \frac{\chi}{1-\tau_t^n} + \frac{1-\rho}{1-\tau_t^n}\left(\frac{\alpha^I}{1-\alpha^I}\right) E_t\left\{\Xi_{t+1|t}(1-\tau_{t+1}^n)\mathbf{J}_{It+1}\right\} \\ & + \frac{(1-\rho)\chi E_t \Xi_{t+1|t}}{1-\tau_t^n} - \frac{(1-\rho)E_t\left\{\Xi_{t+1|t}\frac{h'(lfp_{t+1})}{u'(c_{t+1})}\right\}}{1-\tau_t^n} = \frac{\alpha^I}{1-\alpha^I}\mathbf{J}_{It}. \end{aligned}$$

Next, substitute using (78) for both $\mathbf{J}_{It}$ and $\mathbf{J}_{It+1}$, which gives

$$\begin{aligned} & w_t^I - \frac{\chi}{1-\tau_t^n} + \frac{1-\rho}{1-\tau_t^n}\left(\frac{\alpha^I}{1-\alpha^I}\right) E_t\left\{\Xi_{t+1|t}(1-\tau_{t+1}^n)\left((1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right\} \\ & + \frac{(1-\rho)\chi E_t \Xi_{t+1|t}}{1-\tau_t^n} - \frac{(1-\rho)E_t\left\{\Xi_{t+1|t}\frac{h'(lfp_{t+1})}{u'(c_{t+1})}\right\}}{1-\tau_t^n} = \frac{\alpha^I}{1-\alpha^I}\left((1-\tau_t^h)\tilde{\varepsilon}_t + w(\tilde{\varepsilon}_t) - w_t^I\right). \end{aligned}$$

Rearranging,

$$\begin{aligned} & w_t^I\left[1 + \frac{\alpha^I}{1-\alpha^I}\right] = \frac{\chi}{1-\tau_t^n} + \frac{\alpha^I}{1-\alpha^I}\left((1-\tau_t^h)\tilde{\varepsilon}_t + w(\tilde{\varepsilon}_t)\right) \\ & + \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t\left\{\Xi_{t+1|t}\left[\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi - \left(\frac{\alpha^I}{1-\alpha^I}\right)(1-\tau_{t+1}^n)\left((1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right]\right\}. \end{aligned}$$

Multiplying by $(1-\alpha^I)$,

$$\begin{aligned} & w_t^I = \left(\frac{1-\alpha^I}{1-\tau_t^n}\right)\chi + \alpha^I\left((1-\tau_t^h)\tilde{\varepsilon}_t + w(\tilde{\varepsilon}_t)\right) \\ & + \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t\left\{\Xi_{t+1|t}\left[(1-\alpha^I)\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right) - \alpha^I(1-\tau_{t+1}^n)\left((1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right]\right\} \end{aligned}$$

Finally, because it will be useful in further manipulations below, multiply and divide the last term on the right-hand side by $(1-\alpha^I)$, which gives

$$
\begin{aligned}
w_t^I = & \left(\frac{1-\alpha^I}{1-\tau_t^n}\right)\chi + \alpha^I\left((1-\tau_t^h)\tilde{\varepsilon}_t + w(\tilde{\varepsilon}_t)\right) \\
+ & \left(\frac{1-\rho}{1-\tau_t^n}\right)(1-\alpha^I)E_t\left\{\Xi_{t+1|t}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})}-\chi\right)\right\} \\
- & \left(\frac{1-\rho}{1-\tau_t^n}\right)(1-\alpha^I)E_t\left\{\Xi_{t+1|t}\left(\frac{\alpha^I}{1-\alpha^I}\right)(1-\tau_{t+1}^n)\left((1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right\}.
\end{aligned}
\tag{90}
$$

Note that this expression is not a closed-form expression for w_t^I (even taking the continuation value as given) because the endogenous wage $w(\tilde{\varepsilon}_t)$ also appears. A closed-form expression requires also solving for a new hire's wage, which is done below.

C.3.2 Newly-Hired Workers

To obtain an expression for the period-t bargained wage of a new hire, begin with the sharing rule

$$
\frac{\mathbf{W}_E(\varepsilon_{it}) - \mathbf{U}_t}{1-\tau_t^n} = \frac{\alpha^E}{1-\alpha^E}\mathbf{J}_E(\varepsilon_{it}),
\tag{91}
$$

and substitute (74). This gives

$$
\begin{aligned}
& w(\varepsilon_{it}) - \frac{\chi}{1-\tau_t^n} + \left(\frac{1-\rho}{1-\tau_t^n}\right)E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1} - \mathbf{U}_{t+1}\right]\right\} \\
+ & \left(\frac{1-\rho}{1-\tau_t^n}\right)\chi E_t\Xi_{t+1|t} - \left(\frac{1-\rho}{1-\tau_t^n}\right)E_t\left\{\Xi_{t+1|t}\frac{h'(lfp_{t+1})}{u'(c_{t+1})}\right\} = \frac{\alpha^E}{1-\alpha^E}\mathbf{J}_E(\varepsilon_{it}).
\end{aligned}
\tag{92}
$$

Then, substitute the time-$(t+1)$ sharing rule (89) *for incumbents* in the third term on the left-hand side, which gives

$$
\begin{aligned}
& w(\varepsilon_{it}) - \frac{\chi}{1-\tau_t^n} + \left(\frac{1-\rho}{1-\tau_t^n}\right)\left(\frac{\alpha^I}{1-\alpha^I}\right)E_t\left\{\Xi_{t+1|t}(1-\tau_{t+1}^n)\mathbf{J}_{It+1}\right\} \\
+ & \left(\frac{1-\rho}{1-\tau_t^n}\right)\chi E_t\Xi_{t+1|t} - \left(\frac{1-\rho}{1-\tau_t^n}\right)E_t\left\{\Xi_{t+1|t}\frac{h'(lfp_{t+1})}{u'(c_{t+1})}\right\} = \frac{\alpha^E}{1-\alpha^E}\mathbf{J}_E(\varepsilon_{it}).
\end{aligned}
\tag{93}
$$

Next, substitute using (78) for $\mathbf{J}_{It+1}$ and using (82) for $\mathbf{J}_E(\varepsilon_{it})$, which gives

$$
w(\varepsilon_{it}) - \frac{\chi}{1-\tau_t^n} + \left(\frac{1-\rho}{1-\tau_t^n}\right)\left(\frac{\alpha^I}{1-\alpha^I}\right)E_t\left\{\Xi_{t+1|t}(1-\tau_{t+1}^n)\left((1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right\}
$$

$$+ \left(\frac{1-\rho}{1-\tau_t^n}\right) \chi E_t \Xi_{t+1|t} - \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \frac{h'(lfp_{t+1})}{u'(c_{t+1})} \right\} = \frac{\alpha^E}{1-\alpha^E} \left[(1-\tau_t^h)(\tilde{\varepsilon}_t - \varepsilon_{it}) + w(\tilde{\varepsilon}_t) - w(\varepsilon_{it} \right. \quad (9$$

Rearranging to isolate $w(\varepsilon_{it})$ gives us

$$w(\varepsilon_{it}) \left[1 + \frac{\alpha^E}{1-\alpha^E} \right] = \frac{\chi}{1-\tau_t^n} + \frac{\alpha^E}{1-\alpha^E} \left[(1-\tau_t^h)(\tilde{\varepsilon}_t - \varepsilon_{it}) + w(\tilde{\varepsilon}_t) \right]$$
$$+ \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \left[\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi - \left(\frac{\alpha^I}{1-\alpha^I}\right) (1-\tau_{t+1}^n) \left((1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I \right) \right] \right\}.$$

Next, multiplying by $(1-\alpha^E)$, we have

$$w(\varepsilon_{it}) = \left(\frac{1-\alpha^E}{1-\tau_t^n}\right) \chi + \alpha^E \left[(1-\tau_t^h)(\tilde{\varepsilon}_t - \varepsilon_{it}) + w(\tilde{\varepsilon}_t) \right] \quad (95)$$
$$+ \left(\frac{1-\rho}{1-\tau_t^n}\right) (1-\alpha^E) E_t \left\{ \Xi_{t+1|t} \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\}$$
$$- \left(\frac{1-\rho}{1-\tau_t^n}\right) (1-\alpha^E) E_t \left\{ \Xi_{t+1|t} \left(\frac{\alpha^I}{1-\alpha^I}\right) (1-\tau_{t+1}^n) \left((1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I \right) \right\},$$

which is the bargained wage for a newly-hired worker with idiosyncratic characteristics ε_{it}. This expression is not a closed-form solution for $w(\varepsilon_{it})$ because the endogenous wage $w(\tilde{\varepsilon}_t)$ also appears. However, at this stage of the analysis, it is not difficult to obtain closed-form solutions.

C.3.3 Closed-Form Solutions for Bargained Wages

There are three more steps required to obtain closed-form expressions for bargained wages, which completes the proof of Proposition 1. First, construct expressions for each of the three period-t wages in which no other *contemporaneous* wage appears. Second, compute wage differentials. Third, substitute wage differentials into the *continuation value* components of each wage expression.

Using τ_t^n in (95) and evaluating it at $\varepsilon_{it} = \tilde{\varepsilon}_t$ gives the bargained wage of the threshold new hire,

$$\begin{aligned} w(\tilde{\varepsilon}_t) = & \frac{\chi}{1-\tau_t^n} + \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \\ - & \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \left(\frac{\alpha^I}{1-\alpha^I} \right) (1-\tau_{t+1}^n) \left[(1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I \right] \right\}. \end{aligned} \tag{96}$$

Then, substitute (96) in (95), which gives the bargained wage for a new hire with idiosyncratic characteristics ε_{it},

$$\begin{aligned} w(\varepsilon_{it}) = & \frac{\chi}{1-\tau_t^n} + \alpha^E (1-\tau_t^h)\left(\tilde{\varepsilon}_t - \varepsilon_{it}\right) + \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \\ - & \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \left(\frac{\alpha^I}{1-\alpha^I} \right) (1-\tau_{t+1}^n) \left((1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I \right) \right\}. \end{aligned} \tag{97}$$

Next, substitute (96) in (90), which gives the bargained wage for an incumbent worker,

$$\begin{aligned} w_t^I = & \frac{\chi}{1-\tau_t^n} + \alpha^I (1-\tau_t^h)\tilde{\varepsilon}_t \\ + & \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \\ - & \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \left(\frac{\alpha^I}{1-\alpha^I} \right) (1-\tau_{t+1}^n) \left((1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I \right) \right\}. \end{aligned} \tag{98}$$

Note that in all three wages (96), (97), and (98), the continuation value (which is $E_t \left\{ \Xi_{t+1|t} \left[\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi - \left(\frac{\alpha^I}{1-\alpha^I} \right) (1-\tau_{t+1}^n) \left((1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I \right) \right] \right\}$) is identical because no matter what type a worker is in period t, he will be a (homogenous) incumbent worker in period $t+1$ if he remains employed. Moreover, the period-$(t+1)$ wage differential $w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I$ appears in all three.

These expressions allow us to explicitly compute wage differentials between the different types of workers, which are intuitive to understand. A new hire with $\varepsilon_{it} < \tilde{\varepsilon}_t$ earns a premium over the threshold new hire

$$w(\varepsilon_{it}) - w(\tilde{\varepsilon}_t) = \alpha^E(1-\tau_t^h)\left(\tilde{\varepsilon}_t - \varepsilon_{it}\right), \tag{99}$$

which is the share of the operating cost *savings* he provides the firm that he is able to earn through his bargaining power. An incumbent worker earns a premium over the threshold new hire

$$w_t^I - w(\tilde{\varepsilon}_t) = \alpha^I(1-\tau_t^h)\cdot\tilde{\varepsilon}_t, \tag{100}$$

which, similarly, is the share of the replacement cost *savings* (relative to a marginal new hire) he provides the firm that he is able to earn through his bargaining power.

Substitute $w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I = -\alpha^I(1-\tau_{t+1}^h)\cdot\tilde{\varepsilon}_{t+1}$ into (96) to obtain

$$\begin{aligned} w(\tilde{\varepsilon}_t) = {} & \frac{\chi}{1-\tau_t^n} + \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t\left\{\Xi_{t+1|t}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right\} \\ - & \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t\left\{\Xi_{t+1|t}\left(\frac{\alpha^I}{1-\alpha^I}\right)(1-\tau_{t+1}^n)\left[(1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} - \alpha^I\left((1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1}\right)\right]\right\}, \end{aligned}$$

which, in two steps can be simplified to, first,

$$\begin{aligned} w(\tilde{\varepsilon}_t) = {} & \frac{\chi}{1-\tau_t^n} + \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t\left\{\Xi_{t+1|t}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right\} \\ - & \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t\left\{\Xi_{t+1|t}\left(\frac{\alpha^I}{1-\alpha^I}\right)(1-\alpha^I)(1-\tau_{t+1}^n)(1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1}\right\}, \end{aligned}$$

and then to

$$w(\tilde{\varepsilon}_t) = \frac{\chi}{1-\tau_t^n} + \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t\left\{\Xi_{t+1|t}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi - \alpha^I(1-\tau_{t+1}^n)(1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1}\right)\right\}. \tag{101}$$

Next, substitute $w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I = -\alpha^I(1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1}$ into (97), which gives

$$\begin{aligned} w(\varepsilon_{it}) = {} & \frac{\chi}{1-\tau_t^n} + \alpha^E(1-\tau_t^h)\left(\tilde{\varepsilon}_t - \varepsilon_{it}\right) \\ + & \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t\left\{\Xi_{t+1|t}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi - \alpha^I(1-\tau_{t+1}^n)(1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1}\right)\right\}. \end{aligned} \tag{102}$$

Finally, substitute $w(\tilde{\varepsilon}_{t+1}) - w^I_{t+1} = -\alpha^I(1-\tau^h_{t+1})(\tilde{\varepsilon}_{t+1})$ into (98), which gives

$$
\begin{aligned}
w^I_t &= \frac{\chi}{1-\tau^n_t} + \alpha^I(1-\tau^h_t)(\tilde{\varepsilon}_t) \qquad (103)\\
&+ \left(\frac{1-\rho}{1-\tau^n_t}\right) E_t\left\{\Xi_{t+1|t}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi - \alpha^I(1-\tau^n_{t+1})(1-\tau^h_{t+1})\cdot\tilde{\varepsilon}_{t+1}\right)\right\}.
\end{aligned}
$$

Expressions (101), (102), (103) are the forms of the wage solutions that appear in Proposition 1.

Integrating (102) gives the average wage paid to a new hire

$$
\begin{aligned}
\omega_e(\tilde{\varepsilon}_t) &= \int_{-\infty}^{\tilde{\varepsilon}_t} w(\varepsilon_{it}) f(\varepsilon_{it}) d\varepsilon_{it}\\
&= \left(\frac{\chi}{1-\tau^n_t}\right)\int_{-\infty}^{\tilde{\varepsilon}_t} f(\varepsilon_{it})d\varepsilon_{it} + \alpha^E(1-\tau^h_t)\int_{-\infty}^{\tilde{\varepsilon}_t}(\tilde{\varepsilon}_t-\varepsilon_{it}) f(\varepsilon_{it})d\varepsilon_{it}\\
&\quad + \left(\frac{1-\rho}{1-\tau^n_t}\right) E_t\left\{\Xi_{t+1|t}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi - \alpha^I(1-\tau^n_{t+1})(1-\tau^h_{t+1})\tilde{\varepsilon}_{t+1}\right)\right\}\int_{-\infty}^{\tilde{\varepsilon}_t} f(\varepsilon_{it})d\varepsilon_{it}\\
&= \left(\frac{\chi}{1-\tau^n_t}\right)\eta(\tilde{\varepsilon}_t) + \alpha^E(1-\tau^h_t)\cdot\eta(\tilde{\varepsilon}_t)\cdot\left(\tilde{\varepsilon}_t - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right)\\
&\quad + \left(\frac{1-\rho}{1-\tau^n_t}\right) E_t\left\{\Xi_{t+1|t}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi - \alpha^I(1-\tau^n_{t+1})(1-\tau^h_{t+1})\tilde{\varepsilon}_{t+1}\right)\right\}\eta(\tilde{\varepsilon}_t),
\end{aligned}
$$

in which the third line follows from the definition $H(\tilde{\varepsilon}_t) \equiv \int_{-\infty}^{\tilde{\varepsilon}_t}\varepsilon_{it} f(\varepsilon_{it})d\varepsilon_{it}$. Dividing by $\eta(\tilde{\varepsilon}_t)$ gives

$$
\begin{aligned}
\frac{\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} &= \left(\frac{\chi}{1-\tau^n_t}\right) + \alpha^E(1-\tau^h_t)\left(\tilde{\varepsilon}_t - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right) \qquad (104)\\
&\quad + \left(\frac{1-\rho}{1-\tau^n_t}\right) E_t\left\{\Xi_{t+1|t}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi - \alpha^I(1-\tau^n_{t+1})(1-\tau^h_{t+1})\tilde{\varepsilon}_{t+1}\right)\right\}.
\end{aligned}
$$

Finally, using the definition in (76), the continuation value that appears on the right-hand side of all the wage functions (101), (102), (103), and (105) can be expressed as

$$
\begin{aligned}
&\left(\frac{1-\rho}{1-\tau^n_t}\right) E_t\left\{\Xi_{t+1|t}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi - \alpha^I(1-\tau^n_{t+1})(1-\tau^h_{t+1})(\tilde{\varepsilon}_{t+1})\right)\right\}\\
&= \left(\frac{1-\rho}{1-\tau^n_t}\right) E_t\left\{\Xi_{t+1|t}\left(\int_{-\infty}^{\tilde{\varepsilon}_{t+1}}\mathbf{W}_E(\varepsilon_{it+1})f(\varepsilon_{it+1})d\varepsilon_{it+1} - \eta(\tilde{\varepsilon}_{t+1})\mathbf{U}_{t+1} - \alpha^I(1-\tau^n_{t+1})(1-\tau^h_{t+1})\tilde{\varepsilon}_{t+1}\right)\right\}.
\end{aligned}
$$

Integrating (97) gives the average wage paid to a new hire

$$\begin{aligned}
\omega_e(\tilde{\varepsilon}_t) &= \int_{-\infty}^{\tilde{\varepsilon}_t} w(\varepsilon_{it}) f(\varepsilon_{it}) d\varepsilon_{it} \\
&= \left(\frac{\chi}{1-\tau_t^n}\right) \int_{-\infty}^{\tilde{\varepsilon}_t} f(\varepsilon_{it}) d\varepsilon_{it} + \alpha^E (1-\tau_t^h) \int_{-\infty}^{\tilde{\varepsilon}_t} \left(\tilde{\varepsilon}_t - \varepsilon_{it}\right) f(\varepsilon_{it}) d\varepsilon_{it} \\
&\quad + \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \int_{-\infty}^{\tilde{\varepsilon}_t} f(\varepsilon_{it}) d\varepsilon_{it} \\
&\quad - \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \left(\frac{\alpha^I}{1-\alpha^I}\right) (1-\tau_{t+1}^n) \left((1-\tau_{t+1}^h) \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I \right) \right\} \int_{-\infty}^{\tilde{\varepsilon}_t} f(\varepsilon_{it}) d\varepsilon_{it}. \\
&= \left(\frac{\chi}{1-\tau_t^n}\right) \eta(\tilde{\varepsilon}_t) + \alpha^E (1-\tau_t^h) \eta(\tilde{\varepsilon}_t) \left(\tilde{\varepsilon}_t - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} \right) + \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \eta(\\
&\quad - \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \left(\frac{\alpha^I}{1-\alpha^I}\right) (1-\tau_{t+1}^n) \left((1-\tau_{t+1}^h) \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I \right) \right\} \eta(\tilde{\varepsilon}_t),
\end{aligned}$$

in which the third line follows from the definition $H(\tilde{\varepsilon}_t) \equiv \int_{-\infty}^{\tilde{\varepsilon}_t} \varepsilon_{it} f(\varepsilon_{it}) d\varepsilon_{it}$. Dividing by $\eta(\tilde{\varepsilon}_t)$ gives

$$\begin{aligned}
\frac{\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} &= \left(\frac{\chi}{1-\tau_t^n}\right) + \alpha^E (1-\tau_t^h) \left(\tilde{\varepsilon}_t - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} \right) + \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \\
&\quad - \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \Xi_{t+1|t} \left(\frac{\alpha^I}{1-\alpha^I}\right) (1-\tau_{t+1}^n) \left((1-\tau_{t+1}^h) \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I \right) \right\}.
\end{aligned} \tag{105}$$

D Efficient Allocations (Online Appendix)

A social planner in this economy optimally allocates the measure one of individuals in the representative household to leisure, unemployment, and employment. There are several representations of the planning problem available: suppose that c_t, lfp_t, n_t, and $\tilde{\varepsilon}_t$ are the formal objects of choice. Given the accounting identities of the model, the measure of individuals available for work can thus be expressed $s_t = lfp_t - (1-\rho)n_{t-1}$.

The social planner problem is to maximize lifetime expected utility of the representative household

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h\left(lfp_t\right)\right] \tag{106}$$

subject to the sequence of goods resource constraints

$$c_t = z_t n_t - H(\tilde{\varepsilon}_t)[lfp_t - (1-\rho)n_{t-1}], \tag{107}$$

and laws of motion for the employment stock

$$n_t = (1-\rho)n_{t-1} + [lfp_t - (1-\rho)n_{t-1}]\eta(\tilde{\varepsilon}_t). \tag{108}$$

The social planner takes into account the dependence of the hiring rate and the average operating cost of a newly-selected worker on the threshold $\tilde{\varepsilon}_t$, which is made explicit in the notation here. Recalling that $\eta(\tilde{\varepsilon}_t) \equiv \int_{-\infty}^{\tilde{\varepsilon}_t} f(\varepsilon)d\varepsilon$ and $H(\tilde{\varepsilon}_t) \equiv \int_{-\infty}^{\tilde{\varepsilon}_t} \varepsilon f(\varepsilon)d\varepsilon$, we have $\eta'(\tilde{\varepsilon}_t) = f(\tilde{\varepsilon}_t)$ and $H'(\tilde{\varepsilon}_t) = \tilde{\varepsilon}_t f(\tilde{\varepsilon}_t)$, by the Fundamental Theorem of Calculus.

Let $\beta^t \lambda_t$ be the Lagrange multiplier on the period-t goods resource constraint, and $\beta^t \mu_t$ be the Lagrange multiplier on the period-t law of motion for employment. The first-order conditions of the social planner problem with respect to c_t, lfp_t, n_t, and $\tilde{\varepsilon}_t$ are, respectively,

$$u'(c_t) - \lambda_t = 0, \tag{109}$$

$$-h'(lfp_t) - \lambda_t H(\tilde{\varepsilon}_t) + \mu_t \eta(\tilde{\varepsilon}_t) = 0, \tag{110}$$

$$\lambda_t z_t - \mu_t + (1-\rho)\beta E_t \left\{\mu_{t+1}\left[1 - \eta(\tilde{\varepsilon}_{t+1})\right] + \lambda_{t+1} H(\tilde{\varepsilon}_{t+1})\right\} = 0, \tag{111}$$

and

$$-\lambda_t s_t H'(\tilde{\varepsilon}_t) + \mu_t s_t \eta'(\tilde{\varepsilon}_t) = 0. \tag{112}$$

D.1 Static Efficiency (Participation)

Isolating the multiplier μ_t from (112),

$$\begin{aligned} \mu_t &= \frac{u'(c_t) \cdot H'(\tilde{\varepsilon}_t)}{\eta'(\tilde{\varepsilon}_t)} \\ &= u'(c_t) \cdot \tilde{\varepsilon}_t, \end{aligned} \tag{113}$$

in which we have substituted (109). Substituting this expression for μ_t in (110) gives

$$\begin{aligned} \frac{h'(lfp_t)}{u'(c_t)} &= \tilde{\varepsilon}_t \eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t) \\ &= \int_{-\infty}^{\tilde{\varepsilon}_t} \left[\tilde{\varepsilon}_t - \varepsilon\right] f(\varepsilon) d\varepsilon, \end{aligned} \tag{114}$$

in which the second line substitutes the definitions of $H(\tilde{\varepsilon}_t)$ and $\eta(\tilde{\varepsilon}_t)$. The term in square brackets in the integral is unambiguously positive. Expression (114) is the static efficiency condition that appears as condition (26) in the main text.

D.2 Intertemporal Efficiency (Hiring)

Next, substituting expression (113) for μ_t (and its time $t+1$ counterpart) in (111), we have

$$\begin{aligned} u'(c_t) \cdot \tilde{\varepsilon}_t &= u'(c_t) z_t + (1-\rho)\beta E_t \left\{ u'(c_{t+1}) H(\tilde{\varepsilon}_{t+1}) + u'(c_{t+1}) \cdot \tilde{\varepsilon}_{t+1} \left(1 - \eta(\tilde{\varepsilon}_{t+1})\right) \right\} \\ &= u'(c_t) z_t + (1-\rho)\beta E_t \left\{ u'(c_{t+1}) \left[H(\tilde{\varepsilon}_{t+1}) + \tilde{\varepsilon}_{t+1}(1 - \eta(\tilde{\varepsilon}_{t+1})) \right] \right\} \\ &= u'(c_t) z_t + (1-\rho)\beta E_t \left\{ u'(c_{t+1}) \left[H(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1} \eta(\tilde{\varepsilon}_{t+1}) + \tilde{\varepsilon}_{t+1} \right] \right\}. \end{aligned} \tag{115}$$

Dividing by $u'(c_t)$,

$$\tilde{\varepsilon}_t = z_t + (1-\rho) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left[H(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1} \eta(\tilde{\varepsilon}_{t+1}) + \tilde{\varepsilon}_{t+1} \right] \right\}, \tag{116}$$

which is the representation of efficiency along the intertemporal margin that appears as condition (27) in the main text.

D.3 MRS-MRT Representation of Efficiency

The efficiency conditions (114) and (116) can be described in terms of appropriately-defined concepts of marginal rates of substitution (MRS) and corresponding marginal rates of transformation (MRT). Defining MRS and MRT in a model-appropriate way allows us to describe efficiency in terms of the basic principle that efficient allocations are characterized by MRS = MRT conditions along all optimization margins.

Consider the static efficiency condition (114). The left-hand side is clearly the within-period MRS between consumption and participation in any period t. We claim that the right-hand side is the corresponding MRT between consumption and participation. Rather than take the efficiency condition (114) as prima facie evidence that the right-hand side *must be* the static MRT, however, this MRT can be derived from the primitives of the environment (i.e., independent of the context of any optimization).

First, though, define MRS and MRT relevant for intertemporal efficiency. To do so, first restrict attention to the non-stochastic case because it makes especially clear the separation of components of preferences from components of technology (due to endogenous covariance terms inherent in the $E_t(.)$ operator). The non-stochastic intertemporal efficiency condition can be expressed as

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{(1-\rho)\left(H(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) + \tilde{\varepsilon}_{t+1}\right)}{\tilde{\varepsilon}_t - z_t}. \tag{117}$$

The left-hand side of (117) is clearly the intertemporal MRS (hereafter abbreviated IMRS) between c_t and c_{t+1}. We claim that the right-hand side is the corresponding intertemporal MRT (hereafter abbreviated IMRT). Applying this definition to the fully stochastic condition (116), we can thus express intertemporal efficiency as

$$1 = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[\frac{(1-\rho)\left(H(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) + \tilde{\varepsilon}_{t+1}\right)}{\tilde{\varepsilon}_t - z_t}\right]\right\} = E_t\left\{\frac{IMRT_{c_t,c_{t+1}}}{IMRS_{c_t,c_{t+1}}}\right\}. \tag{118}$$

Rather than take the efficiency condition (117) as prima facie evidence that the right-hand side *must be* the IMRT, however, the IMRT can be derived from the primitives of the environment (i.e., independent of the context of any optimization), which is shown next.

D.4 Proof of Proposition 1: Transformation Frontier and Derivation of MRTs

Based only on the primitives of the environment — that is, independent of the context of any optimization — we now prove that the right-hand sides of (114) and (117) are, respectively, the model-appropriate concepts of the static MRT and deterministic IMRT. Doing so thus proves Proposition 1 in the main text. This requires defining the transformation frontier of the economy, a joint description of the goods resource constraint and the law of motion for employment.

In order to define the within-period MRT between c_t and lfp_t, the within-period transformation frontier needs to be viewed in the space (c_t, lfp_t). In principle, this requires eliminating the variable $\tilde{\varepsilon}_t$ between the period-t goods resource constraint (107) and the period-t law of motion (108) to express them as a single condition. However, this cannot be done explicitly. The within-period transformation is thus implicitly defined by the pair of functions

$$\Psi^{RC}(c_t, lfp_t, \tilde{\varepsilon}_t; .) \equiv z_t n_t - c_t - H(\tilde{\varepsilon}_t)\left[lfp_t - (1-\rho)n_{t-1}\right] = 0, \tag{119}$$

which is condition (107), and

$$\Psi^{LOM}(lfp_t, \tilde{\varepsilon}_t; .) \equiv n_t - (1-\rho)n_{t-1} - \left[lfp_t - (1-\rho)n_{t-1}\right]\eta(\tilde{\varepsilon}_t) = 0, \tag{120}$$

which is condition (108). The within-period transformation frontier is implicitly defined by the pair of functions (119) and (120).

Computing the MRT between c_t and lfp_t requires total differentiation of (119) and (120), due to the fact that $\tilde{\varepsilon}_t$ is the variable that we would like to, but cannot, eliminate between the two expressions. Total differentiation gives:

$$\begin{aligned} MRT_{c_t, lfp_t} &= -\frac{\Psi^{RC}_{lfp_t}}{\Psi^{RC}_{c_t}} + \frac{\Psi^{LOM}_{lfp_t}}{\Psi^{LOM}_{\tilde{\varepsilon}_t}}\frac{\Psi^{RC}_{\tilde{\varepsilon}_t}}{\Psi^{RC}_{c_t}} \\ &= -\frac{\Psi^{RC}_{lfp_t}}{\Psi^{RC}_{\tilde{\varepsilon}_t}}\frac{\Psi^{RC}_{\tilde{\varepsilon}_t}}{\Psi^{RC}_{c_t}} + \frac{\Psi^{LOM}_{lfp_t}}{\Psi^{LOM}_{\tilde{\varepsilon}_t}}\frac{\Psi^{RC}_{\tilde{\varepsilon}_t}}{\Psi^{RC}_{c_t}} \\ &= -\frac{\Psi^{RC}_{\tilde{\varepsilon}_t}}{\Psi^{RC}_{c_t}}\left[\frac{\Psi^{RC}_{lfp_t}}{\Psi^{RC}_{\tilde{\varepsilon}_t}} - \frac{\Psi^{LOM}_{lfp_t}}{\Psi^{LOM}_{\tilde{\varepsilon}_t}}\right]. \end{aligned} \tag{121}$$

The first term on the right-hand side of the first line is, by the implicit function theorem, the slope $\frac{\partial c_t}{\partial lfp_t}$ embodied *directly* in the period-t goods resource constraint. The second term on the right-hand side of the first line is, by the implicit function theorem, the slope $\frac{\partial c_t}{\partial lfp_t}$ computed *through the marginal effect of a change in* lfp_t *on* $\tilde{\varepsilon}_t$ *embodied in the period-t law of motion* — hence the need for total differentiation.

Based on this computation and using the functions (119) and (120), the MRT is

$$\begin{aligned}
MRT_{c_t,lfp_t} &= -\left[\frac{-H'(\tilde{\varepsilon}_t)s_t}{-1}\right]\left[\frac{-H(\tilde{\varepsilon}_t)}{-H'(\tilde{\varepsilon}_t)s_t} - \frac{-\eta(\tilde{\varepsilon}_t)}{-\eta'(\tilde{\varepsilon}_t)s_t}\right] \\
&= -H'(\tilde{\varepsilon}_t)\left[\frac{H(\tilde{\varepsilon}_t)}{H'(\tilde{\varepsilon}_t)} - \frac{\eta(\tilde{\varepsilon}_t)}{\eta'(\tilde{\varepsilon}_t)}\right] \\
&= -H(\tilde{\varepsilon}_t) + \frac{\eta(\tilde{\varepsilon}_t)H'(\tilde{\varepsilon}_t)}{\eta'(\tilde{\varepsilon}_t)} \\
&= -H(\tilde{\varepsilon}_t) + \frac{\eta(\tilde{\varepsilon}_t)\tilde{\varepsilon}_t f(\tilde{\varepsilon}_t)}{f(\tilde{\varepsilon}_t)} \\
&= -H(\tilde{\varepsilon}_t) + \tilde{\varepsilon}_t\eta(\tilde{\varepsilon}) \\
&= \tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t) \qquad (122) \\
&= \int_{-\infty}^{\tilde{\varepsilon}_t} [\tilde{\varepsilon}_t - \varepsilon]\, f(\varepsilon)d\varepsilon, \qquad (123)
\end{aligned}$$

which formalizes, independent of the solution to the social planning problem, the notion of the static MRT on the right-hand side of the efficiency condition (114).

Before computing the IMRT, note that the implicit function theorem allows us to also compute

$$\begin{aligned}
\frac{\partial c_t}{\partial n_t} &= -\frac{\Psi^{RC}_{n_t}}{\Psi^{RC}_{c_t}} + \frac{\Psi^{RC}_{\tilde{\varepsilon}_t}}{\Psi^{RC}_{c_t}}\frac{\Psi^{LOM}_{n_t}}{\Psi^{LOM}_{\tilde{\varepsilon}_t}} \\
&= -\frac{z_t}{-1} + \frac{-s_t H'(\tilde{\varepsilon}_t)}{-1}\frac{1}{-s_t\eta(\tilde{\varepsilon}_t)} \\
&= z_t - \frac{H'(\tilde{\varepsilon}_t)}{\eta'(\tilde{\varepsilon}_t)} \\
&= z_t - \frac{\tilde{\varepsilon}_t f(\tilde{\varepsilon}_t)}{f(\tilde{\varepsilon}_t)} \\
&= z_t - \tilde{\varepsilon}_t, \qquad (124)
\end{aligned}$$

which measures the marginal effect on period-t consumption of a change in period-t employment. This effect has intertemporal consequences because n_t is the stock of employment entering period $t+1$. In constructing this, note that the first term on the right-hand side of the first line is, by the implicit function theorem, the slope $\frac{\partial c_t}{\partial n_t}$ embodied *directly* in the period-t goods resource constraint; and the second term on the right-hand side of the first line is, by the implicit function theorem, the slope $\frac{\partial c_t}{\partial n_t}$ computed *through the marginal effect of a change in* n_t *on* $\tilde{\varepsilon}_t$ *embodied in the period-t law of motion* — hence, as above, the need for total differentiation. The second line computes the necessary partials of the functions (119) and (120), and the fourth line uses the Fundamental Theorem of Calculus to compute the derivatives of $\eta(\tilde{\varepsilon}_t)$ and $H(\tilde{\varepsilon}_t)$.

Next, define the period $t+1$ analogs of the functions (119) and (120):

$$G^{RC}(c_{t+1}, lfp_{t+1}, \tilde{\varepsilon}_{t+1}, c_t; .) \equiv z_{t+1}n_{t+1} - c_{t+1} - H(\tilde{\varepsilon}_{t+1})\left[lfp_{t+1} - (1-\rho)n_t\right] = 0 \quad (125)$$

and

$$G^{LOM}(lfp_{t+1}, \tilde{\varepsilon}_{t+1}, c_t; .) \equiv n_{t+1} - (1-\rho)n_t - \left[lfp_{t+1} - (1-\rho)n_t\right]\eta(\tilde{\varepsilon}_{t+1}). \quad (126)$$

The functions $G^{RC}(.)$ and $G^{LOM}(.)$ clearly have the same form as (119) and (120), but, for the purpose of computing the IMRT, it is useful to view them as generalizations in that $G^{RC}(.)$ and $G^{LOM}(.)$ are viewed as functions of *both* period-t *and* period $t+1$ allocations. This generalization is emphasized by using the notation $G(.)$, rather than $\Psi(.)$, and by highlighting both c_{t+1} and c_t as arguments. The two-period (across period t and $t+1$) transformation frontier is implicitly defined by the pair of functions (125) and (126).

Computing the IMRT between c_t and c_{t+1} thus requires computing the total derivative

$$\begin{aligned}
\underbrace{\frac{\partial c_{t+1}}{\partial c_t}} + \underbrace{\frac{\partial c_{t+1}}{\partial c_t}} &= \frac{\partial c_{t+1}}{\partial n_t \frac{\partial c_t}{\partial n_t}} + \frac{\partial c_{t+1}}{\partial \tilde{\varepsilon}_{t+1}} \frac{\partial \tilde{\varepsilon}_{t+1}}{\partial n_t \frac{\partial c_t}{\partial n_t}} \\
&= \left\{ \frac{\partial c_{t+1}}{\partial n_t} + \frac{\partial c_{t+1}}{\partial \tilde{\varepsilon}_{t+1}} \frac{\partial \tilde{\varepsilon}_{t+1}}{\partial n_t} \right\} \frac{1}{\partial c_t / \partial n_t} \\
&= \left\{ -\frac{G^{RC}_{n_t}}{G^{RC}_{c_{t+1}}} + \frac{G^{RC}_{\tilde{\varepsilon}_{t+1}}}{G^{RC}_{c_{t+1}}} \frac{G^{LOM}_{n_t}}{G^{LOM}_{\tilde{\varepsilon}_{t+1}}} \right\} \frac{1}{\partial c_t / \partial n_t}.
\end{aligned}$$

The right-hand side of the first line highlights that the effect of c_t on c_{t+1} occurs through its effect on n_t (which is why we computed $\frac{\partial c_t}{\partial n_t}$), and the third line uses the implicit function theorem. Using the functions (125) and (126), the IMRT is

$$\begin{aligned}
&IMRT \\
&= -\left[-\frac{(1-\rho)H(\tilde{\varepsilon}_{t+1})}{-1} + \frac{-s_{t+1}H'(\tilde{\varepsilon}_{t+1})}{-1} \frac{-(1-\rho)(1-\eta(\tilde{\varepsilon}_{t+1}))}{-s_{t+1}\eta'(\tilde{\varepsilon}_{t+1})} \right] \frac{1}{\partial c_t / \partial n_t}
\end{aligned}$$

$$
\begin{aligned}
&= -(1-\rho)\left[H(\tilde{\varepsilon}_{t+1}) + (1-\eta(\tilde{\varepsilon}_{t+1}))\left(\frac{H'(\tilde{\varepsilon}_{t+1})}{\eta'(\tilde{\varepsilon}_{t+1})}\right)\right]\frac{1}{\partial c_t/\partial n_t} \\
&= -(1-\rho)\left[H(\tilde{\varepsilon}_{t+1}) + (1-\eta(\tilde{\varepsilon}_{t+1}))\left(\frac{\tilde{\varepsilon}_{t+1}f(\tilde{\varepsilon}_{t+1})}{f(\tilde{\varepsilon}_{t+1})}\right)\right]\frac{1}{\partial c_t/\partial n_t} \\
&= -(1-\rho)\left[H(\tilde{\varepsilon}_{t+1}) + (1-\eta(\tilde{\varepsilon}_{t+1}))\cdot\tilde{\varepsilon}_{t+1}\right]\frac{1}{\partial c_t/\partial n_t} \\
&= -(1-\rho)\left[H(\tilde{\varepsilon}_{t+1}) + \tilde{\varepsilon}_{t+1} - \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1})\right]\frac{1}{\partial c_t/\partial n_t} \\
&= -(1-\rho)\left[H(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) + \tilde{\varepsilon}_{t+1}\right]\frac{1}{\partial c_t/\partial n_t} \\
&= \frac{(1-\rho)\left(H(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) + \tilde{\varepsilon}_{t+1}\right)}{\tilde{\varepsilon}_t - z_t}, \qquad (127)
\end{aligned}
$$

in which the third line makes use of the Fundamental Theorem of Calculus to compute the derivatives of $\eta(\tilde{\varepsilon}_t)$ and $H(\tilde{\varepsilon}_t)$, and the last line makes use of the slope (124). The sign convention is that the IMRT is the negative of the slope of the two-period transformation frontier. This derivation formalizes, independent of the social planning problem, the notion of the IMRT on the right-hand side of the (deterministic) efficiency condition (116).

E Equilibrium Wedges (Online Appendix)

To present the algebra behind the wedges defined, the following equilibrium conditions are needed: the household's LFP condition

$$\frac{(1-\eta_t)h'(lfp_t) - u'(c_t)\left((1-\tau_t^n)\omega_{et} + (1-\eta_t)\chi\right)}{\eta_t u'(c_t)} = -\frac{h'(lfp_t)}{u'(c_t)}$$
$$+ \quad (1-\rho)E_t\left\{\Xi_{t+1|t}\left[(1-\tau_{t+1}^n)w_{t+1}^I + \left(\frac{(1-\eta_{t+1})h'(lfp_{t+1}) - u'(c_{t+1})\left((1-\tau_{t+1}^n)\omega_{et+1} + (1-\eta_{t+1})\chi\right)}{\eta_{t+1}u'(c_{t+1})}\right.\right.\right.$$

the bargained wage of the threshold new hire

$$w(\tilde{\varepsilon}_t) = \frac{\chi}{1-\tau_t^n} + \left(\frac{1-\rho}{1-\tau_t^n}\right)E_t\left\{\Xi_{t+1|t}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right\} \tag{129}$$
$$- \quad \left(\frac{1-\rho}{1-\tau_t^n}\right)E_t\left\{\Xi_{t+1|t}\left(\frac{\alpha^I}{1-\alpha^I}\right)(1-\tau_{t+1}^n)\left[(1-\tau_{t+1}^h)(\tilde{\varepsilon}_{t+1}) + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right]\right\},$$

the average wage paid to a new hire

$$\frac{\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} = \left(\frac{\chi}{1-\tau_t^n}\right) + \alpha^E(1-\tau_t^h)\left(\tilde{\varepsilon}_t - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right) + \left(\frac{1-\rho}{1-\tau_t^n}\right)E_t\left\{\Xi_{t+1|t}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right\}$$
$$-\left(\frac{1-\rho}{1-\tau_t^n}\right)E_t\left\{\Xi_{t+1|t}\left(\frac{\alpha^I}{1-\alpha^I}\right)(1-\tau_{t+1}^n)\left((1-\tau_{t+1}^h)(\tilde{\varepsilon}_{t+1}) + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right\}, \tag{130}$$

and the selection condition

$$(1-\tau_t^h)\tilde{\varepsilon}_t = z_t - w(\tilde{\varepsilon}_t) + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[(1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right]\right\}. \tag{131}$$

E.1 Definition of Static Wedge

To obtain the decentralized economy's static wedge, substitute (130) into the LFP condition (128), which, after a few steps of algebra, gives

$$\frac{h'(lfp_t) - u'(c_t)\chi}{\eta(\tilde{\varepsilon}_t)u'(c_t)} = \alpha^E(1-\tau_t^n)(1-\tau_t^h)\left(\tilde{\varepsilon}_t - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right)$$
$$+ \quad (1-\rho)E_t\left\{\Xi_{t+1|t}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi - \left(\frac{\alpha^I}{1-\alpha^I}\right)(1-\tau_{t+1}^n)\left((1-\tau_{t+1}^h)(\tilde{\varepsilon}_{t+1}) + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right)\right\}$$

$$+ \quad (1-\rho)E_t\left\{\Xi_{t+1|t}\left[(1-\tau^n_{t+1})w^I_{t+1} + \left(\frac{(1-\eta_{t+1})h'(lfp_{t+1}) - u'(c_{t+1})\left((1-\tau^n_{t+1})\omega_{et+1} + (1-\eta_{t+1})\chi\right)}{\eta_{t+1}u'(c_{t+1})}\right.\right.\right.$$

Using the period-$(t+1)$ value (78), the second line above can be re-written as

$$\frac{h'(lfp_t) - u'(c_t)\chi}{\eta(\tilde{\varepsilon}_t)u'(c_t)} = \alpha^E(1-\tau^n_t)(1-\tau^h_t)\left(\tilde{\varepsilon}_t - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right)$$
$$+ \quad \left(\frac{1-\rho}{1-\tau^n_t}\right)E_t\left\{\Xi_{t+1|t}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi - \left(\frac{\alpha^I}{1-\alpha^I}\right)(1-\tau^n_{t+1})\mathbf{J}_{It+1}\right)\right\}$$
$$+ \quad (1-\rho)E_t\left\{\Xi_{t+1|t}\left[(1-\tau^n_{t+1})w^I_{t+1} + \left(\frac{(1-\eta_{t+1})h'(lfp_{t+1}) - u'(c_{t+1})\left((1-\tau^n_{t+1})\omega_{et+1} + (1-\eta_{t+1})\chi\right)}{\eta_{t+1}u'(c_{t+1})}\right.\right.\right.$$

Next, the Nash surplus sharing rule (89) for incumbent workers allows us to rewrite the second line yet again,

$$\frac{h'(lfp_t) - u'(c_t)\chi}{\eta_t u'(c_t)} = \alpha^E(1-\tau^n_t)(1-\tau^h_t)\left(\tilde{\varepsilon}_t - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right)$$
$$+ \quad (1-\rho)E_t\left\{\Xi_{t+1|t}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi - (\mathbf{W}_{It+1} - \mathbf{U}_{t+1})\right)\right\}$$
$$+ \quad (1-\rho)E_t\left\{\Xi_{t+1|t}(1-\tau^n_{t+1})w^I_{t+1}\right\}. \qquad (134)$$
$$+ \quad (1-\rho)E_t\left\{\Xi_{t+1|t}\left(\frac{(1-\eta_{t+1})h'(lfp_{t+1}) - u'(c_{t+1})\left((1-\tau^n_{t+1})\omega_{et+1} + (1-\eta_{t+1})\chi\right)}{\eta_{t+1}u'(c_{t+1})}\right)\right\}.$$

Then, noting from (67) that the third line can be rewritten in period-t terms, we have

$$\frac{h'(lfp_t) - u'(c_t)\chi}{\eta_t u'(c_t)} = \alpha^E(1-\tau^n_t)(1-\tau^h_t)\left(\tilde{\varepsilon}_t - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right)$$
$$+ \quad (1-\rho)E_t\left\{\Xi_{t+1|t}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi - (\mathbf{W}_{It+1} - \mathbf{U}_{t+1})\right)\right\}$$
$$+ \quad \mathbf{W}_{It} - \mathbf{U}_t - \left[(1-\tau^n_t)w^I_t - \chi\right].$$

Finally, based on the recursion in $\mathbf{W}_{It} - \mathbf{U}_t$ stated in (70), the second and third lines immediately above cancel. Rearranging, we have that in the decentralized Nash bargaining economy with taxes and unemployment benefits,

$$\frac{h'(lfp_t) - u'(c_t)\chi}{u'(c_t)} = \alpha^E (1-\tau_t^n)(1-\tau_t^h) \cdot (\tilde{\varepsilon}_t \eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)), \tag{135}$$

in which, recall, α^E is the Nash bargaining power of each newly-hired worker. To represent this in terms of a wedge in the static efficiency condition (114), we have

$$\frac{h'(lfp_t)}{u'(c_t)} = (\tilde{\varepsilon}_t \eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)) \left[\frac{\chi}{\eta(\tilde{\varepsilon}_t)\left(\tilde{\varepsilon}_t - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right)} + \alpha^E (1-\tau_t^n)(1-\tau_t^h) \right], \tag{136}$$

in which the term in brackets is the wedge between MRS and MRT. Four conditions are sufficient for the decentralized economy to achieve static efficiency: the decentralized economy features $\alpha^E = 1$; the unemployment transfer is $\chi = 0$; proportional labor income taxation is $\tau_t^n = 0$; and the proportional hiring subsidy is $\tau_t^h = 0$. These conditions are not necessary, however, because for any arbitrary $(\alpha^E < 1, \chi \neq 0)$, an appropriate setting for policy (τ_t^n, τ_t^h) achieves efficiency.

E.2 Definition of Intertemporal Wedge

To obtain the decentralized economy's intertemporal wedge, first substitute the bargained wage of the threshold new hire (129) into the selection condition (131), which gives

$$
\begin{aligned}
(1-\tau_t^h)\tilde{\varepsilon}_t = {} & z_t - \frac{\chi}{1-\tau_t^n} \\
- & \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi - \left(\frac{\alpha^I}{1-\alpha^I}\right)(1-\tau_{t+1}^n)\left[(1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right]\right)\right. \\
+ & (1-\rho)E_t\left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left[(1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right]\right\}.
\end{aligned}
$$

Slightly re-arranging terms,

$$
\begin{aligned}
(1-\tau_t^h)\tilde{\varepsilon}_t = {} & z_t - \frac{\chi}{1-\tau_t^n} \\
- & (1-\rho)E_t\left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left(\frac{1}{1-\tau_t^n}\right)\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right\} \\
+ & (1-\rho)\left(\frac{\alpha^I}{1-\alpha^I}\right) E_t\left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left(\frac{1-\tau_{t+1}^n}{1-\tau_t^n}\right)\left[(1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right]\right\} \\
+ & (1-\rho)E_t\left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left[(1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right]\right\}.
\end{aligned}
$$

Next, combining the square-bracketed terms on the third and fourth lines yields

$$
\begin{aligned}
(1-\tau_t^h)\tilde{\varepsilon}_t = {} & z_t - \frac{\chi}{1-\tau_t^n} \\
- & (1-\rho)E_t\left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left(\frac{1}{1-\tau_t^n}\right)\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right\} \\
+ & (1-\rho)E_t\left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left[1 + \left(\frac{\alpha^I}{1-\alpha^I}\right)\left(\frac{1-\tau_{t+1}^n}{1-\tau_t^n}\right)\right]\left[(1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right]\right\}.
\end{aligned}
$$

Slightly re-writing terms in the third line gives

$$
\begin{aligned}
(1-\tau_t^h)\tilde{\varepsilon}_t = {} & z_t - \frac{\chi}{1-\tau_t^n} \\
- & (1-\rho)E_t\left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left(\frac{1}{1-\tau_t^n}\right)\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right\} \\
+ & (1-\rho)E_t\left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left[\frac{\alpha^I(1-\tau_{t+1}^n) + (1-\alpha^I)(1-\tau_t^n)}{(1-\alpha^I)(1-\tau_t^n)}\right]\left[(1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right]\right\}.
\end{aligned}
$$

Then, use the period-$(t+1)$ version of the wage differential (100) to rewrite once again the third line, which gives

$$
\begin{aligned}
(1-\tau_t^h)\tilde{\varepsilon}_t &= z_t - \frac{\chi}{1-\tau_t^n} \\
&- \quad (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(\frac{1}{1-\tau_t^n}\right)\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})}-\chi\right)\right\} \\
&+ \quad (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[\frac{\alpha^I(1-\tau_{t+1}^n)+(1-\alpha^I)(1-\tau_t^n)}{(1-\alpha^I)(1-\tau_t^n)}\right](1-\alpha^I)(1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1}\right\}.
\end{aligned}
$$

Cancelling the $(1-\alpha^I)$ terms and re-arranging then gives

$$
\begin{aligned}
(1-\tau_t^h)\tilde{\varepsilon}_t &= z_t - \frac{\chi}{1-\tau_t^n} \\
&- \quad \left(\frac{1-\rho}{1-\tau_t^n}\right)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})}-\chi\right)\right\} \\
&+ \quad \left(\frac{1-\rho}{1-\tau_t^n}\right)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[\alpha^I(1-\tau_{t+1}^n)+(1-\alpha^I)(1-\tau_t^n)\right](1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1}\right\}.
\end{aligned}
$$

In non-stochastic terms,

$$
\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{\left(\frac{1-\rho}{1-\tau_t^n}\right)\left[\left(\alpha^I(1-\tau_{t+1}^n)+(1-\alpha^I)(1-\tau_t^n)\right)(1-\tau_{t+1}^h)\tilde{\varepsilon}_{t+1}-\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})}-\chi\right)\right]}{(1-\tau_t^h)\tilde{\varepsilon}_t-\left[z_t-\frac{\chi}{1-\tau_t^n}\right]} \tag{137}
$$

The sufficient conditions for intertemporal efficiency in the decentralized equilibrium are the same as those that achieve static efficiency: $\alpha^E = 1$; $\chi = 0$; $\tau_t^n = 0\ \forall t$; and $\tau_t^h = 0\ \forall t$.

F Derivation of Implementability Constraint (Online Appendix)

The derivation of the present-value implementability constraint (PVIC) follows that laid out in Lucas and Stokey (1983) and Chari and Kehoe (1999). For notational convenience, in what follows we use the definition $ue_t = (1-\eta_t)s_t$ where possible and also use $lfp_t = ue_t + n_t$ to conserve on notation. In deriving the implementability constraint, the sequence of the household's perceived (based on expectations dated E_0) laws of motion for employment,

$$n_t = (1-\rho)n_{t-1} + \eta_t s_t, \tag{138}$$

the sequence of labor-force-participation conditions (based on expectations dated E_0)

$$\begin{aligned} u'(c_t)\left(\frac{(1-\tau_t^n)\omega_{et}}{\eta_t}\right) &= \left(\frac{1-\eta_t}{\eta_t}\right)\left[h'(lfp_t) - u'(c_t)\chi\right] + h'(lfp_t) && (139)\\ &- (1-\rho)\beta u'(c_{t+1})(1-\tau_{t+1}^n)w_{t+1}^I \\ &- (1-\rho)\beta u'(c_{t+1})\left(\frac{1-\eta_{t+1}}{\eta_{t+1}}\right)\left[h'(lfp_{t+1}) - u'(c_{t+1})\chi\right] \\ &+ (1-\rho)\beta u'(c_{t+1})\left(\frac{(1-\tau_{t+1}^n)\omega_{et+1}}{\eta_{t+1}}\right) && (140) \end{aligned}$$

will be useful. Note that the mechanism by which wages are determined in the decentralized economy are *irrelevant* in constructing the PVIC.

Start with the household flow budget constraint in equilibrium:

$$c_t + \sum_j \frac{1}{R_t^j} b_{t+1}^j = (1-\rho)n_{t-1}(1-\tau_t^n)w_t^I + \eta_t \frac{(1-\tau_t^n)\omega_{et}}{\eta_t} s_t + (1-\eta_t)s_t\chi + b_t + (1-\tau^{pr})\Pi_t. \tag{141}$$

Multiply by $\beta^t u'(c_t)$ and, conditional on the information set at time zero, sum the sequence of budget constraints over dates and states starting from $t=0$ to arrive at the present-value budget constraint:

$$\begin{aligned} &E_0\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t + E_0\sum_{t=0}^{\infty}\sum_j \beta^t u'(c_t)\frac{1}{R_t^j}b_{t+1}^j && (142)\\ &= E_0\sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau_t^n)w_t^I(1-\rho)n_{t-1} + E_0\sum_{t=0}^{\infty}\beta^t u'(c_t)\left(\frac{(1-\tau_t^n)\omega_{et}}{\eta_t}\right)\eta_t s_t \\ &+ E_0\sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\eta_t)s_t\chi + E_0\sum_{t=0}^{\infty}\beta^t u'(c_t)b_t + E_0\sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau^{pr})\Pi_t. \end{aligned}$$

Now begin to impose equilibrium conditions on this present-value budget constraint. For ease of notation, drop the E_0 term, but it is understood that all terms are conditional on the information set at time zero.

First, in the second summation on the left-hand side, substitute the sequence of state-contingent bond Euler equations, $u'(c_s) = \beta R_s^j u'(c_{s+1}^j)$, $\forall j, s$:

$$\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t u'(c_t) c_t + \sum_{t=0}^{\infty} \sum_j \beta^{t+1} u'(c_{t+1}^j) b_{t+1}^j \\
&\quad = \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\tau_t^n) w_t^I (1-\rho) n_{t-1} + \sum_{t=0}^{\infty} \beta^t u'(c_t) \left(\frac{(1-\tau_t^n)\omega_{et}}{\eta_t} \right) \eta_t s_t \\
&\quad + \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\eta_t) s_t \chi + \sum_{t=0}^{\infty} \beta^t u'(c_t) b_t + \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\tau^{pr}) \Pi_t.
\end{aligned} \tag{143}$$

The term $\sum_j u'(c_{t+1}^j) b_{t+1}^j$ can be expressed as the payoff of a synthetic risk-free bond, $u'(c_{t+1}) b_{t+1}$, which then allows canceling terms in the second summation on the left-hand side with their counterpart terms in the next-to-last summation on the right-hand side, leaving only the time-zero bond-return term:

$$\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t u'(c_t) c_t = \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\tau_t^n) w_t^I (1-\rho) n_{t-1} \\
&\quad + \sum_{t=0}^{\infty} \beta^t u'(c_t) \left(\frac{(1-\tau_t^n)\omega_{et}}{\eta_t} \right) \eta_t s_t + \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\eta_t) s_t \chi + \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\tau^{pr}) \Pi_t + u'(c_0) b_0.
\end{aligned} \tag{144}$$

Second, in the first term on the second line, substitute $\eta_t s_t = n_t - (1-\rho) n_{t-1}$ (i.e., using the law of motion (138)) to get

$$\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t u'(c_t) c_t = \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\tau_t^n) w_t^I (1-\rho) n_{t-1} \\
&\quad + \sum_{t=0}^{\infty} \beta^t u'(c_t) \left(\frac{(1-\tau_t^n)\omega_{et}}{\eta_t} \right) n_t - \sum_{t=0}^{\infty} \beta^t u'(c_t) \left(\frac{(1-\tau_t^n)\omega_{et}}{\eta_t} \right) (1-\rho) n_{t-1} \\
&\quad + \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\eta_t) s_t \chi + \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\tau^{pr}) \Pi_t + u'(c_0) b_0.
\end{aligned} \tag{145}$$

Collecting the terms on the right-hand side of the first line and the right-hand side of the second line gives

$$\sum_{t=0}^{\infty} \beta^t u'(c_t) c_t = \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\tau_t^n) \left[w_t^I - \frac{\omega_{et}}{\eta_t} \right] (1-\rho) n_{t-1} \tag{146}$$

$$+ \sum_{t=0}^{\infty} \beta^t u'(c_t) \left(\frac{(1-\tau_t^n)\omega_{et}}{\eta_t} \right) n_t$$
$$+ \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\eta_t) s_t \chi + \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\tau^{pr})\Pi_t + u'(c_0)b_0.$$

The square-bracketed expression in the first line is completely independent of the protocol by which wages are determined.

Next, use the sequence of LFP conditions (139) to substitute for the sequence of terms $u'(c_t)\left(\frac{(1-\tau_t^n)\omega_{et}}{\eta_t}\right)$ in the summation in the second line, which gives

$$\sum_{t=0}^{\infty} \beta^t u'(c_t) c_t = \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\tau_t^n) \left[w_t^I - \frac{\omega_{et}}{\eta_t} \right] (1-\rho) n_{t-1} \tag{147}$$
$$+ \sum_{t=0}^{\infty} \beta^t \left(\frac{1-\eta_t}{\eta_t} \right) \left[h'(lfp_t) - u'(c_t)\chi \right] n_t + \sum_{t=0}^{\infty} \beta^t h'(lfp_t) n_t$$
$$- (1-\rho) \sum_{t=0}^{\infty} \beta^{t+1} u'(c_{t+1})(1-\tau_{t+1}^n) w_{t+1}^I n_t$$
$$- (1-\rho) \sum_{t=0}^{\infty} \beta^{t+1} \left(\frac{1-\eta_{t+1}}{\eta_{t+1}} \right) \left[h'(lfp_{t+1}) - u'(c_{t+1})\chi \right] n_t$$
$$+ (1-\rho) \sum_{t=0}^{\infty} \beta^{t+1} u'(c_{t+1}) \left(\frac{(1-\tau_{t+1}^n)\omega_{et+1}}{\eta_{t+1}} \right) n_t$$
$$+ \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\eta_t) s_t \chi + \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\tau^{pr})\Pi_t + u'(c_0)b_0.$$

Proceeding slowly: distributing the terms on the right-hand side in the first line above and slightly re-arranging the third and fifth lines yields

$$\sum_{t=0}^{\infty} \beta^t u'(c_t) c_t = \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\tau_t^n) w_t^I (1-\rho) n_{t-1} \tag{148}$$
$$- \sum_{t=0}^{\infty} \beta^t u'(c_t) \left(\frac{(1-\tau_t^n)\omega_{et}}{\eta_t} \right) (1-\rho) n_{t-1}$$
$$+ \sum_{t=0}^{\infty} \beta^t \left(\frac{1-\eta_t}{\eta_t} \right) \left[h'(lfp_t) - u'(c_t)\chi \right] n_t + \sum_{t=0}^{\infty} \beta^t h'(lfp_t) n_t$$
$$- \sum_{t=0}^{\infty} \beta^{t+1} u'(c_{t+1})(1-\tau_{t+1}^n) w_{t+1}^I (1-\rho) n_t$$
$$- (1-\rho) \sum_{t=0}^{\infty} \beta^{t+1} \left(\frac{1-\eta_{t+1}}{\eta_{t+1}} \right) \left[h'(lfp_{t+1}) - u'(c_{t+1})\chi \right] n_t$$
$$+ \sum_{t=0}^{\infty} \beta^{t+1} u'(c_{t+1}) \left(\frac{(1-\tau_{t+1}^n)\omega_{et+1}}{\eta_{t+1}} \right) (1-\rho) n_t$$

$$+ \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\eta_t)s_t\chi + \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\tau^{pr})\Pi_t + u'(c_0)b_0.$$

The summation on the right-hand side of the first line cancels with the summation on the fourth line, leaving only the time-zero term; similarly, except for the time-zero term, the summation in the second line cancels with the summation on the sixth line, which gives

$$\begin{aligned}
\sum_{t=0}^{\infty} \beta^t u'(c_t)c_t &= \sum_{t=0}^{\infty} \beta^t \left(\frac{1-\eta_t}{\eta_t}\right) [h'(lfp_t) - u'(c_t)\chi]\, n_t + \sum_{t=0}^{\infty} \beta^t h'(lfp_t) n_t \\
&- (1-\rho)\sum_{t=0}^{\infty} \beta^{t+1} \left(\frac{1-\eta_{t+1}}{\eta_{t+1}}\right) [h'(lfp_{t+1}) - u'(c_{t+1})\chi]\, n_t \\
&+ \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\eta_t)s_t\chi + \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\tau^{pr})\Pi_t + u'(c_0)b_0 \\
&- (1-\rho)u'(c_0)(1-\tau_0^n)\left[\frac{\omega_e(\tilde{\varepsilon}_0)}{\eta(\tilde{\varepsilon}_0)} - w_0^I\right] n_{-1},
\end{aligned} \tag{149}$$

in which the period-zero value of the pre-determined stock of employment (stated in period-zero marginal utility terms) appears on the last line. Note that in the period-zero term, we have included the functional dependence of $\omega_e(\tilde{\varepsilon}_0)$ and $\eta(\tilde{\varepsilon}_0)$ on $\tilde{\varepsilon}_0$; we will maintain this notation in the subsequent steps.

The remainder of the derivation of the PVIC follows that in Arseneau and Chugh (2012, Appendix E). Use the law of motion $n_t = (1-\rho)n_{t-1} + \eta_t s_t$ to substitute for the sequence of n_t terms that appears in the first summation on the right-hand side, which gives

$$\begin{aligned}
\sum_{t=0}^{\infty} \beta^t u'(c_t)c_t &= (1-\rho)\sum_{t=0}^{\infty} \beta^t \left(\frac{1-\eta_t}{\eta_t}\right) [h'(lfp_t) - u'(c_t)\chi]\, n_{t-1} \\
&+ \sum_{t=0}^{\infty} \beta^t \left(\frac{1-\eta_t}{\eta_t}\right) [h'(lfp_t) - u'(c_t)\chi]\, \eta_t s_t + \sum_{t=0}^{\infty} \beta^t h'(lfp_t) n_t \\
&- (1-\rho)\sum_{t=0}^{\infty} \beta^{t+1} \left(\frac{1-\eta_{t+1}}{\eta_{t+1}}\right) [h'(lfp_{t+1}) - u'(c_{t+1})\chi]\, n_t \\
&+ \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\eta_t)s_t\chi + \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\tau^{pr})\Pi_t + u'(c_0)b_0 \\
&- (1-\rho)u'(c_0)(1-\tau_0^n)\left[\frac{\omega_e(\tilde{\varepsilon}_0)}{\eta(\tilde{\varepsilon}_0)} - w_0^I\right] n_{-1}.
\end{aligned} \tag{150}$$

Next, the first summation on the right-hand side cancels with the fourth summation on the right-hand side, leaving only the time-zero term:

$$
\begin{aligned}
\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t &= \sum_{t=0}^{\infty}\beta^t\left(\frac{1-\eta_t}{\eta_t}\right)\left[h'(lfp_t)-u'(c_t)\chi\right]\eta_t s_t + \sum_{t=0}^{\infty}\beta^t h'(lfp_t)n_t \\
&+ \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\eta_t)s_t\chi + \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau^{pr})\Pi_t + u'(c_0)b_0 \\
&+ (1-\rho)\left(\frac{1-\eta(\tilde{\varepsilon}_0)}{\eta(\tilde{\varepsilon}_0)}\right)\left[h'(lfp_0)-u'(c_0)\chi\right]n_{-1} \\
&- (1-\rho)u'(c_0)(1-\tau_0^n)\left[\frac{\omega_e(\tilde{\varepsilon}_0)}{\eta(\tilde{\varepsilon}_0)}-w_0^I\right]n_{-1}.
\end{aligned} \tag{151}
$$

Expanding and re-arranging the first summation on the right-hand side gives

$$
\begin{aligned}
\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t &= \sum_{t=0}^{\infty}\beta^t h'(lfp_t)\left(1-\eta_t\right)s_t + \sum_{t=0}^{\infty}\beta^t h'(lfp_t)n_t \\
&- \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\eta_t)s_t\chi \\
&+ \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\eta_t)s_t\chi + \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau^{pr})\Pi_t + u'(c_0)b_0 \\
&+ (1-\rho)\left(\frac{1-\eta(\tilde{\varepsilon}_0)}{\eta(\tilde{\varepsilon}_0)}\right)\left[h'(lfp_0)-u'(c_0)\chi\right]n_{-1} \\
&- (1-\rho)u'(c_0)(1-\tau_0^n)\left[\frac{\omega_e(\tilde{\varepsilon}_0)}{\eta(\tilde{\varepsilon}_0)}-w_0^I\right]n_{-1}.
\end{aligned} \tag{152}
$$

Cancelling the summations involving $(1-\eta_t)s_t\chi$ gives

$$
\begin{aligned}
\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t &= \sum_{t=0}^{\infty}\beta^t h'(lfp_t)\left(1-\eta_t\right)s_t + \sum_{t=0}^{\infty}\beta^t h'(lfp_t)n_t \\
&+ \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau^{pr})\Pi_t + u'(c_0)b_0 \\
&+ (1-\rho)\left(\frac{1-\eta(\tilde{\varepsilon}_0)}{\eta(\tilde{\varepsilon}_0)}\right)\left[h'(lfp_0)-u'(c_0)\chi\right]n_{-1} \\
&- (1-\rho)u'(c_0)(1-\tau_0^n)\left[\frac{\omega_e(\tilde{\varepsilon}_0)}{\eta(\tilde{\varepsilon}_0)}-w_0^I\right]n_{-1}.
\end{aligned} \tag{153}
$$

Finally, collecting terms, using the identity $lfp_t = (1-\eta_t)s_t + n_t$, and reintroducing the conditional expectation E_0, we have the present-value implementability constraint

$$
E_0\sum_{t=0}^{\infty}\beta^t\left[u'(c_t)c_t - h'(lfp_t)lfp_t - u'(c_t)(1-\tau^{pr})\Pi_t\right] = A_0, \tag{154}
$$

where

$$A_0 \equiv u'(c_0)b_0+(1-\rho)\left(\frac{1-\eta(\tilde{\varepsilon}_0)}{\eta(\tilde{\varepsilon}_0)}\right)[h'(lfp_0)-u'(c_0)\chi]\,n_{-1}-(1-\rho)u'(c_0)(1-\tau_0^n)\left[\frac{\omega_e(\tilde{\varepsilon}_0)}{\eta(\tilde{\varepsilon}_0)}-w_0^I\right]n_{-1}. \tag{155}$$

An important note is that the wages (and hence the wage differential) contained in A_0 have nothing to do with how wages are determined. That is, they have nothing to do with whether wages are determined via individual-specific generalized Nash bargaining, collective bargaining, a rigid wage norm, and so on.

G Ramsey Problem (Online Appendix)

As stated in Section 4.1, the Ramsey government's problem conditional on $t = 0$ for its policy functions for $t > 0$ is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h\left(n_t + (1 - \eta(\tilde{\varepsilon}_t)) s_t\right) \right] \tag{156}$$

subject to the sequence of goods resource constraints

$$c_t + g_t + \left(\frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} \right) \eta(\tilde{\varepsilon}_t) s_t = z_t n_t, \tag{157}$$

the sequence of laws of motion for the aggregate employment stock

$$n_t = (1 - \rho) n_{t-1} + \eta(\tilde{\varepsilon}_t) s_t, \tag{158}$$

the PVIC

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u'(c_t) c_t - h'(lfp_t) lfp_t - u'(c_t)(1 - \tau^{pr}) \Pi_t \right] = A_0 \tag{159}$$

(in which

$$A_0 \equiv u'(c_0) b_0 + (1-\rho) \left(\frac{1 - \eta(\tilde{\varepsilon}_0)}{\eta(\tilde{\varepsilon}_0)} \right) \left[h'(lfp_0) - u'(c_0)\chi \right] n_{-1} + (1-\rho) u'(c_0)(1-\tau_0^n) \left[\frac{\omega_e(\tilde{\varepsilon}_0)}{\eta(\tilde{\varepsilon}_0)} - w_0^I \right] n_{-1} \tag{160}$$

is the time-zero assets of the private sector), the sequence of selection conditions

$$(1 - \tau_t^h) \cdot \tilde{\varepsilon}_t = z_t - w(\tilde{\varepsilon}_t) + (1 - \rho) E_t \left\{ \Xi_{t+1|t} \left((1 - \tau_{t+1}^h) \cdot \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I \right) \right\}, \tag{161}$$

the sequence of LFP conditions

$$\frac{h'(lfp_t)}{u'(c_t)} = \eta(\tilde{\varepsilon}_t) \left[\frac{(1 - \tau_t^n)\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} + (1 - \rho) E_t \left\{ \Xi_{t+1|t} \left[(1 - \tau_{t+1}^n) w_{t+1}^I + \frac{\mu_{ht+1}}{u'(c_{t+1})} \right] \right\} \right] + (1 - \eta(\tilde{\varepsilon}_t))\chi, \tag{162}$$

the sequence of Nash wage outcomes for marginal new hires

$$\begin{aligned} w(\tilde{\varepsilon}_t) = & \frac{\chi}{1 - \tau_t^n} + \left(\frac{1 - \rho}{1 - \tau_t^n} \right) E_t \left\{ \Xi_{t+1|t} \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \\ & - \left(\frac{1 - \rho}{1 - \tau_t^n} \right) E_t \left\{ \Xi_{t+1|t} \left(\frac{\alpha^I}{1 - \alpha^I} \right) (1 - \tau_{t+1}^n) \left[(1 - \tau_{t+1}^h)(\tilde{\varepsilon}_{t+1}) + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I \right] \right\}, \end{aligned} \tag{163}$$

the sequence of the average wage paid to a new hire

$$\frac{\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} = \frac{\chi}{1-\tau_t^n} + \alpha^E(1-\tau_t^h)\left(\tilde{\varepsilon}_t - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right) + \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t\left\{\Xi_{t+1|t}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right\} - \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t\left\{\Xi_{t+1|t}\left(\frac{\alpha^I}{1-\alpha^I}\right)(1-\tau_{t+1}^n)\left((1-\tau_{t+1}^h)(\tilde{\varepsilon}_{t+1}) + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right\}, \tag{164}$$

and the sequence of Nash wage outcomes for incumbent workers

$$w_t^I = \frac{\chi}{1-\tau_t^n} + \alpha^I(1-\tau_t^h)(\tilde{\varepsilon}_t) + \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t\left\{\Xi_{t+1|t}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right\} - \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t\left\{\Xi_{t+1|t}\left(\frac{\alpha^I}{1-\alpha^I}\right)(1-\tau_{t+1}^n)\left((1-\tau_{t+1}^h)(\tilde{\varepsilon}_{t+1}) + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right\}. \tag{165}$$

The Ramsey choice variables are c_t, n_t, s_t, $\tilde{\varepsilon}_t$, w_t^I, $w(\tilde{\varepsilon}_t)$, $\omega_e(\tilde{\varepsilon}_t)$, τ_t^n, and τ_t^h for $t > 0$. For ease of notation, the sequences of constraints contain the sequences of terms lfp_t; when we compute the Ramsey first-order conditions below, we will use the model's definition $lfp_t = n_t + (1-\eta(\tilde{\varepsilon}_t))s_t$.

Associate the sequence of multipliers $\lambda_{1,t}$ with the first sequence of constraints, the sequence of multipliers $\lambda_{2,t}$ with the second sequence of constraints, the multiplier λ_3 with the PVIC, the sequences of multipliers $\lambda_{4,t}$, $\lambda_{5,t}$, $\lambda_{6,t}$, $\lambda_{7,t}$, and $\lambda_{8,t}$ with the final four sequences of constraints, respectively.

For further ease of notation in writing the Ramsey first-order conditions, define

$$\Phi_t^{RC}(\cdot) \equiv z_t n_t - \left(c_t + g_t + \left(\frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right)\eta(\tilde{\varepsilon}_t)s_t\right), \tag{166}$$

$$\Phi_t^{LOM}(\cdot) \equiv (1-\rho)n_{t-1} + \eta(\tilde{\varepsilon}_t)s_t - n_t, \tag{167}$$

$$\Phi_t^{SEL}(\cdot) \equiv z_t - w(\tilde{\varepsilon}_t) + (1-\rho)E_t\left\{\Xi_{t+1|t}\left((1-\tau_{t+1}^h)\cdot\tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right\} - (1-\tau_t^h)\cdot\tilde{\varepsilon}_t, \tag{168}$$

$$\Phi_t^{LFP}(\cdot) \equiv \eta(\tilde{\varepsilon}_t)\left[\frac{(1-\tau_t^n)\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[(1-\tau_{t+1}^n)w_{t+1}^I + \frac{\mu_{ht+1}}{u'(c_{t+1})}\right]\right\}\right]$$

$$
+ \quad (1-\eta(\tilde{\varepsilon}_t))\chi - \frac{h'(lfp_t)}{u'(c_t)}, \tag{169}
$$

$$
\begin{aligned}
\Phi_t^{w(\tilde{\varepsilon})}(\cdot) \quad \equiv & \quad \frac{\chi}{1-\tau_t^n} + \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t\left\{\Xi_{t+1|t}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right\} \\
- & \quad \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t\left\{\Xi_{t+1|t}\left(\frac{\alpha^I}{1-\alpha^I}\right)(1-\tau_{t+1}^n)\left[(1-\tau_{t+1}^h)(\tilde{\varepsilon}_{t+1}) + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right]\right\} \\
- & \quad w(\tilde{\varepsilon}_t),
\end{aligned} \tag{170}
$$

$$
\begin{aligned}
\Phi_t^{\omega_e(\tilde{\varepsilon})}(\cdot) \quad \equiv & \quad \frac{\chi}{1-\tau_t^n} + \alpha^E(1-\tau_t^h)\left(\tilde{\varepsilon}_t - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right) + \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t\left\{\Xi_{t+1|t}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right\} \\
- & \quad \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t\left\{\Xi_{t+1|t}\left(\frac{\alpha^I}{1-\alpha^I}\right)(1-\tau_{t+1}^n)\left[(1-\tau_{t+1}^h)(\tilde{\varepsilon}_{t+1}) + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right]\right\} \\
- & \quad \frac{\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)},
\end{aligned} \tag{171}
$$

and

$$
\begin{aligned}
\Phi_t^{w^I}(\cdot) \quad \equiv & \quad \frac{\chi}{1-\tau_t^n} + \alpha^I(1-\tau_t^h)(\tilde{\varepsilon}_t) \\
+ & \quad \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t\left\{\Xi_{t+1|t}\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right\} \\
- & \quad \left(\frac{1-\rho}{1-\tau_t^n}\right) E_t\left\{\Xi_{t+1|t}\left(\frac{\alpha^I}{1-\alpha^I}\right)(1-\tau_{t+1}^n)\left((1-\tau_{t+1}^h)(\tilde{\varepsilon}_{t+1}) + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right\} \\
- & \quad w_t^I.
\end{aligned} \tag{172}
$$

Thus, the formal statement of the Ramsey problem is to maximize

$$
\begin{aligned}
& E_0\sum_{t=0}^{\infty}\beta^t\left[u(c_t) - h\left(n_t + (1-\eta(\tilde{\varepsilon}_t))s_t\right)\right] + E_0\sum_{t=0}^{\infty}\beta^t\lambda_{1,t}\Phi_t^{RC}(\cdot) + E_0\sum_{t=0}^{\infty}\beta^t\lambda_{2,t}\Phi_t^{LOM}(\cdot) \\
& + \quad \lambda_3\cdot E_0\sum_{t=0}^{\infty}\beta^t\left[u'(c_t)c_t - h'(lfp_t)lfp_t - u'(c_t)(1-\tau^{pr})\Pi_t\right] \\
& \qquad -\lambda_3\left(u'(c_0)b_0 + (1-\rho)\left(\frac{1-\eta(\tilde{\varepsilon}_0)}{\eta(\tilde{\varepsilon}_0)}\right)\left[h'(lfp_0) - u'(c_0)\chi\right]n_{-1} + (1-\rho)u'(c_0)(1-\tau_0^n)\left[\frac{\omega_e(\tilde{\varepsilon}_0)}{\eta(\tilde{\varepsilon}_0)} - w_0^I\right] n\right. \\
& + \quad E_0\sum_{t=0}^{\infty}\beta^t\lambda_{4,t}\Phi_t^{SEL}(\cdot) + E_0\sum_{t=0}^{\infty}\beta^t\lambda_{5,t}\Phi_t^{LFP}(\cdot) \\
& + \quad E_0\sum_{t=0}^{\infty}\beta^t\lambda_{6,t}\Phi_t^{w(\tilde{\varepsilon})}(\cdot) + E_0\sum_{t=0}^{\infty}\beta^t\lambda_{7,t}\Phi_t^{\omega_e(\tilde{\varepsilon})}(\cdot) + E_0\sum_{t=0}^{\infty}\beta^t\lambda_{8,t}\Phi_t^{w^I}(\cdot).
\end{aligned}
$$

For $t > 0$ (and supposing that Ramsey allocations are interior), the Ramsey first-order conditions with respect to c_t, n_t, s_t, $\tilde{\varepsilon}_t$, w_t^I, $w(\tilde{\varepsilon}_t)$, $\omega_e(\tilde{\varepsilon}_t)$, τ_t^n, and τ_t^h are, respectively:

$$\begin{aligned} & u'(c_t) + \lambda_{1,t}\frac{\partial \Phi_t^{RC}(\cdot)}{\partial c_t} + \frac{1}{\beta}\lambda_{4,t-1}\frac{\partial \Phi_{t-1}^{SEL}(\cdot)}{\partial c_t} + \lambda_{4,t}\frac{\partial \Phi_t^{SEL}(\cdot)}{\partial c_t} \\ + \quad & \frac{1}{\beta}\lambda_{5,t-1}\frac{\partial \Phi_{t-1}^{LFP}(\cdot)}{\partial c_t} + \lambda_{5,t}\frac{\partial \Phi_t^{LFP}(\cdot)}{\partial c_t} + \frac{1}{\beta}\lambda_{6,t-1}\frac{\partial \Phi_{t-1}^{w(\tilde{\varepsilon})}(\cdot)}{\partial c_t} + \lambda_{6,t}\frac{\partial \Phi_t^{w(\tilde{\varepsilon})}(\cdot)}{\partial c_t} \\ + \quad & \frac{1}{\beta}\lambda_{7,t-1}\frac{\partial \Phi_{t-1}^{\omega_e(\tilde{\varepsilon})}(\cdot)}{\partial c_t} + \lambda_{7,t}\frac{\partial \Phi_t^{\omega_e(\tilde{\varepsilon})}(\cdot)}{\partial c_t} + \frac{1}{\beta}\lambda_{8,t-1}\frac{\partial \Phi_{t-1}^{w^I}(\cdot)}{\partial c_t} + \lambda_{8,t}\frac{\partial \Phi_t^{w^I}(\cdot)}{\partial c_t} \\ + \quad & \lambda_3\left[u''(c_t)\cdot c_t + u'(c_t) + u''(c_t)(1-\tau^{pr})\Pi_t\right] = 0, \end{aligned} \tag{174}$$

$$\begin{aligned} & -h'(lfp_t) + \lambda_{1,t}\frac{\partial \Phi_t^{RC}(\cdot)}{\partial n_t} + \frac{1}{\beta}\lambda_{2,t-1}\frac{\partial \Phi_{t-1}^{LOM}(\cdot)}{\partial n_t} + \lambda_{2,t}\frac{\partial \Phi_t^{LOM}(\cdot)}{\partial n_t} \\ + \quad & \frac{1}{\beta}\lambda_{5,t-1}\frac{\partial \Phi_{t-1}^{LFP}(\cdot)}{\partial n_t} + \lambda_{5,t}\frac{\partial \Phi_t^{LFP}(\cdot)}{\partial n_t} + \frac{1}{\beta}\lambda_{6,t-1}\frac{\partial \Phi_{t-1}^{w(\tilde{\varepsilon})}(\cdot)}{\partial n_t} + \frac{1}{\beta}\lambda_{7,t-1}\frac{\partial \Phi_{t-1}^{\omega_e(\tilde{\varepsilon})}(\cdot)}{\partial n_t} + \frac{1}{\beta}\lambda_{8,t-1}\frac{\partial \Phi_{t-1}^{w^I}(\cdot)}{\partial n_t} \\ + \quad & \lambda_3\left[h''(lfp_t)\cdot lfp_t + h'(lfp_t)\right] = 0, \end{aligned} \tag{175}$$

$$\begin{aligned} & -(1-\eta(\tilde{\varepsilon}_t)\cdot h'(lfp_t) + \lambda_{1,t}\frac{\partial \Phi_t^{RC}(\cdot)}{\partial s_t} + \lambda_{2,t}\frac{\partial \Phi_t^{LOM}(\cdot)}{\partial s_t} \\ + \quad & \frac{1}{\beta}\lambda_{5,t-1}\frac{\partial \Phi_{t-1}^{LFP}(\cdot)}{\partial s_t} + \lambda_{5,t}\frac{\partial \Phi_t^{LFP}(\cdot)}{\partial s_t} + \frac{1}{\beta}\lambda_{6,t-1}\frac{\partial \Phi_{t-1}^{w(\tilde{\varepsilon})}(\cdot)}{\partial s_t} + \frac{1}{\beta}\lambda_{7,t-1}\frac{\partial \Phi_{t-1}^{\omega_e(\tilde{\varepsilon})}(\cdot)}{\partial s_t} + \frac{1}{\beta}\lambda_{8,t-1}\frac{\partial \Phi_{t-1}^{w^I}(\cdot)}{\partial s_t} \\ + \quad & \lambda_3\left[(1-\eta(\tilde{\varepsilon}_t))\cdot h''(lfp_t)\cdot lfp_t + h'(lfp_t)\cdot(1-\eta(\tilde{\varepsilon}_t))\right] = 0, \end{aligned} \tag{176}$$

$$\begin{aligned} & h'(lfp_t)\cdot\eta'(\tilde{\varepsilon}_t)\cdot s_t + \lambda_{1,t}\frac{\partial \Phi_t^{RC}(\cdot)}{\partial \tilde{\varepsilon}_t} + \lambda_{2,t}\frac{\partial \Phi_t^{LOM}(\cdot)}{\partial \tilde{\varepsilon}_t} \\ + \quad & \frac{1}{\beta}\lambda_{4,t-1}\frac{\partial \Phi_{t-1}^{SEL}(\cdot)}{\partial \tilde{\varepsilon}_t} + \lambda_{4,t}\frac{\partial \Phi_t^{SEL}(\cdot)}{\partial \tilde{\varepsilon}_t} + \frac{1}{\beta}\lambda_{5,t-1}\frac{\partial \Phi_{t-1}^{LFP}(\cdot)}{\partial \tilde{\varepsilon}_t} + \lambda_{5,t}\frac{\partial \Phi_t^{LFP}(\cdot)}{\partial \tilde{\varepsilon}_t} \\ + \quad & \frac{1}{\beta}\lambda_{6,t-1}\frac{\partial \Phi_{t-1}^{w(\tilde{\varepsilon})}(\cdot)}{\partial \tilde{\varepsilon}_t} + \lambda_{6,t}\frac{\partial \Phi_t^{w(\tilde{\varepsilon})}(\cdot)}{\partial \tilde{\varepsilon}_t} + \frac{1}{\beta}\lambda_{7,t-1}\frac{\partial \Phi_{t-1}^{\omega_e(\tilde{\varepsilon})}(\cdot)}{\partial \tilde{\varepsilon}_t} + \lambda_{7,t}\frac{\partial \Phi_t^{\omega_e(\tilde{\varepsilon})}(\cdot)}{\partial \tilde{\varepsilon}_t} \\ + \quad & \frac{1}{\beta}\lambda_{8,t-1}\frac{\partial \Phi_{t-1}^{w^I}(\cdot)}{\partial \tilde{\varepsilon}_t} + \lambda_{8,t}\frac{\partial \Phi_t^{w^I}(\cdot)}{\partial \tilde{\varepsilon}_t} \\ - \quad & \lambda_3\left[\eta'(\tilde{\varepsilon}_t)\cdot s_t\cdot h''(lfp_t)\cdot lfp_t + h'(lfp_t)\cdot\eta'(\tilde{\varepsilon}_t)\cdot s_t\right] = 0, \end{aligned} \tag{177}$$

$$\frac{1}{\beta}\lambda_{4,t-1}\frac{\partial \Phi_{t-1}^{SEL}(\cdot)}{\partial w_t^I} + \frac{1}{\beta}\lambda_{5,t-1}\frac{\partial \Phi_{t-1}^{LFP}(\cdot)}{\partial w_t^I} + \frac{1}{\beta}\lambda_{6,t-1}\frac{\partial \Phi_{t-1}^{w(\tilde{\varepsilon})}(\cdot)}{\partial w_t^I}$$

$$+ \quad \frac{1}{\beta}\lambda_{7,t-1}\frac{\partial \Phi_{t-1}^{\omega_e(\tilde{\varepsilon})}(\cdot)}{\partial w_t^I} + \frac{1}{\beta}\lambda_{8,t-1}\frac{\partial \Phi_{t-1}^{w^I}(\cdot)}{\partial w_t^I} + \lambda_{8,t}\frac{\partial \Phi_t^{w^I}(\cdot)}{\partial w_t^I} = 0, \tag{178}$$

$$\frac{1}{\beta}\lambda_{4,t-1}\frac{\partial \Phi_{t-1}^{SEL}(\cdot)}{\partial w(\tilde{\varepsilon}_t)} + \lambda_{4,t}\frac{\partial \Phi_t^{SEL}(\cdot)}{\partial w(\tilde{\varepsilon}_t)}$$
$$+ \quad \frac{1}{\beta}\lambda_{6,t-1}\frac{\partial \Phi_{t-1}^{w(\tilde{\varepsilon})}(\cdot)}{\partial w(\tilde{\varepsilon}_t)} + \lambda_{6,t}\frac{\partial \Phi_t^{w(\tilde{\varepsilon})}(\cdot)}{\partial w(\tilde{\varepsilon}_t)} + \frac{1}{\beta}\lambda_{7,t-1}\frac{\partial \Phi_{t-1}^{\omega_e(\tilde{\varepsilon})}(\cdot)}{\partial w(\tilde{\varepsilon}_t)} + \frac{1}{\beta}\lambda_{8,t-1}\frac{\partial \Phi_{t-1}^{w^I}(\cdot)}{\partial w(\tilde{\varepsilon}_t)} = 0, \tag{179}$$

$$\frac{1}{\beta}\lambda_{5,t-1}\frac{\partial \Phi_{t-1}^{LFP}(\cdot)}{\partial w_t^I} + \lambda_{5,t}\frac{\partial \Phi_t^{LFP}(\cdot)}{\partial w_t^I} + \lambda_{7,t}\frac{\partial \Phi_t^{\omega_e(\tilde{\varepsilon})}(\cdot)}{\partial w_t^I} = 0, \tag{180}$$

$$\frac{1}{\beta}\lambda_{5,t-1}\frac{\partial \Phi_{t-1}^{LFP}(\cdot)}{\partial \tau_t^n} + \lambda_{5,t}\frac{\partial \Phi_t^{LFP}(\cdot)}{\partial \tau_t^n} + \frac{1}{\beta}\lambda_{6,t-1}\frac{\partial \Phi_{t-1}^{w(\tilde{\varepsilon})}(\cdot)}{\partial \tau_t^n} + \lambda_{6,t}\frac{\partial \Phi_t^{w(\tilde{\varepsilon})}(\cdot)}{\partial \tau_t^n}$$
$$+ \quad \frac{1}{\beta}\lambda_{7,t-1}\frac{\partial \Phi_{t-1}^{\omega_e(\tilde{\varepsilon})}(\cdot)}{\partial \tau_t^n} + \lambda_{7,t}\frac{\partial \Phi_t^{\omega_e(\tilde{\varepsilon})}(\cdot)}{\partial \tau_t^n} + \frac{1}{\beta}\lambda_{8,t-1}\frac{\partial \Phi_{t-1}^{w^I}(\cdot)}{\partial \tau_t^n} + \lambda_{8,t}\frac{\partial \Phi_t^{w^I}(\cdot)}{\partial \tau_t^n} = 0, \tag{181}$$

and

$$\frac{1}{\beta}\lambda_{4,t-1}\frac{\partial \Phi_{t-1}^{SEL}(\cdot)}{\partial \tau_t^h} + \lambda_{4,t}\frac{\partial \Phi_t^{SEL}(\cdot)}{\partial \tau_t^h} + \frac{1}{\beta}\lambda_{6,t-1}\frac{\partial \Phi_{t-1}^{w(\tilde{\varepsilon})}(\cdot)}{\partial \tau_t^h}$$
$$+ \quad \frac{1}{\beta}\lambda_{7,t-1}\frac{\partial \Phi_{t-1}^{\omega_e(\tilde{\varepsilon})}(\cdot)}{\partial \tau_t^h} + \lambda_{7,t}\frac{\partial \Phi_t^{\omega_e(\tilde{\varepsilon})}(\cdot)}{\partial \tau_t^h} + \frac{1}{\beta}\lambda_{8,t-1}\frac{\partial \Phi_{t-1}^{w^I}(\cdot)}{\partial \tau_t^h} + \lambda_{8,t}\frac{\partial \Phi_t^{w^I}(\cdot)}{\partial \tau_t^h} = 0. \tag{182}$$

Ramsey period-zero policy and its relationship with the timeless perspective. The Ramsey policy functions that emerge from the first-order conditions are decided upon in $t = 0$, and the Ramsey government is, as is standard, committed to implementing them for $t > 0$. Now consider the case of $t = 0$. In $t = 0$ (and provided the $t = 0$ present-value of lifetime government spending is sufficiently large), the Ramsey government would want to confiscate the entire inelastic stock of n_{-1}.

If we allowed full confiscation, which is an extreme case, of n_{-1} from the private sector, then the business-cycle results we present for Ramsey allocations will not be from a timeless perspective (because $n_{-1} = 0 \neq n_s^{RAM} s$, where $n_s^{RAM} s$ is the eventual Ramsey steady state.

If we instead take the other extreme case of completely forbidding the Ramsey government from confiscating n_{-1}, then (and only then) would the business-cycle results we present for Ramsey allocations be from a timeless perspective.

None of the main Ramsey business-cycle results of this paper, though, depend on whether we allow for no confiscation, partial confiscation, or full confiscation of n_{-1}, because there are no "wedges" created by confiscation of the initial assets of the economy. In fact, full confiscation of n_{-1} (provided that the $t = 0$ present-value of lifetime government spending is sufficiently large) would be Ramsey-optimal (in the sense that it would lead to the highest possible steady-state welfare for the economy).

Finally, if one wanted to permit an *incomplete* set of instruments (in our model, dropping either the labor income tax or the proportional hiring subsidy, and/or *fixing* either one of them to some particular numerical value which generically would not be the Ramsey government's policy setting), additional constraints on the Ramsey planning problem beyond those listed above would be required — see Chari and Kehoe (1999, p. 1679 - 1680, 1683) for more.

H Ramsey Taxation and Pigouvian Taxation (Online Appendix)

As Tables 7 and 8 show, part of the Ramsey optimal taxes are Pigouvian in nature. Holding the structural parameters fixed at their values stated in Table **??** and permitting lump-sum taxes (which zeros out the Lagrange multiplier λ_3 on the present-value government budget constraint), the long-run Pigouvian corrective labor-income tax rate is τ^n is 8.3%, the corrective hiring subsidy τ^h is 55.5%, and, as implied by the corrective nature of Pigouvian analysis, there are zero wedges along both the labor-supply and the labor-demand margins.

Table 9 displays Ramsey steady-state allocations for several parameters; in each case, the long-run value of $\tilde{\varepsilon}\eta(\tilde{\varepsilon}) - H(\tilde{\varepsilon})$ are *identical* to each other (machine-level precision $1e^{-16}$). Figures 4 and 5 show this in further detail by allowing, respectively, the unemployment benefit χ and newly-hired employees' Nash bargaining power to vary across a wide range of values. Focusing on the first two panels of each Figure, the Ramsey steady-state values for $\tilde{\varepsilon}\eta(\tilde{\varepsilon}) - H(\tilde{\varepsilon})$ and $\eta(\tilde{\varepsilon})$ are identical to the socially-efficient values. Figure 6 slightly extends Figure 5 by allowing for both $\alpha^I = 0.70$ and $\alpha^I = 1$.

η	u	lfp	$\tilde{\varepsilon}\eta(\tilde{\varepsilon}) - H(\tilde{\varepsilon})$	n	y	τ^n	τ^h	labor supply wedge	labor demand wedge
			Ramsey Policy.						
0.993	0.0005	0.734	**0.967**	0.733	0.733	0.216	0.810	0.797	1
			Exogenous Policy.						
0.620	0.040	0.740	0.187	0.697	0.697	0.200	0	4.144	-3.271
			Efficiency.						
0.993	0.0005	0.761	**0.967**	0.761	0.761	0	0	1	1
			Pigouvian.						
0.993	0.0005	0.761	**0.967**	0.761	0.761	-0.015	0.453	1	1

Table 7: **Steady-state allocations.** Upper panel: Ramsey-policy allocations. Second panel: Exogenous-policy allocations. Third panel: Efficient allocations. Lower panel: Pigouvian allocations.

Multipliers with respect to	λ_1 RC	λ_2 LOM	λ_3 $PVIC$	λ_4 SEL	λ_5 LFP	λ_6 $w(\tilde{\varepsilon}_t)$	λ_7 $\omega_e(\tilde{\varepsilon}_t)$	λ_8 w_t^I
			Ramsey Policy.					
	1.750	0.524	0.042	0	0	0	0	0
			Pigovian Policy.					
	1.677	0.501	0	0	0	0	0	0

Table 8: **Steady-state endogenous multipliers.** The multiplier λ_6 is with respect to the real wage earned by the marginal new hire (11); the multiplier λ_7 is with respect to the average real wage paid to a new hire (14); and the multiplier λ_8 is with respect to the real wage earned by incumbents (13).

I Ramsey Steady State for Alternative Parameter Sets (Online Appendix)

η	u	lfp	$\tilde{\varepsilon}\eta(\tilde{\varepsilon}) - H(\tilde{\varepsilon})$	n	y	τ^n	τ^h	labor supply wedge	labor demand wedge
			Ramsey Policy.						
0.993	0.0005	0.754	**0.967**	0.753	0.753	0.207	0.795	0.805	1
			Ramsey Policy ($\chi = 0$).						
0.993	0.0005	0.753	**0.967**	0.753	0.753	0.086	-0.756	0.803	1
			Ramsey Policy ($\alpha^E = 1$).						
0.993	0.0005	0.753	**0.967**	0.753	0.753	0.218	0.895	0.805	1
			Ramsey Policy ($\chi = 0$, $\alpha^E = 1$).						
0.993	0.0005	0.753	**0.967**	0.753	0.753	0.192	0	0.808	1

Table 9: **Steady-state allocations.** Ramsey allocations for several structural calibrations. In each panel, all other parameter values are set as in the baseline calibration described in Section 3.2.

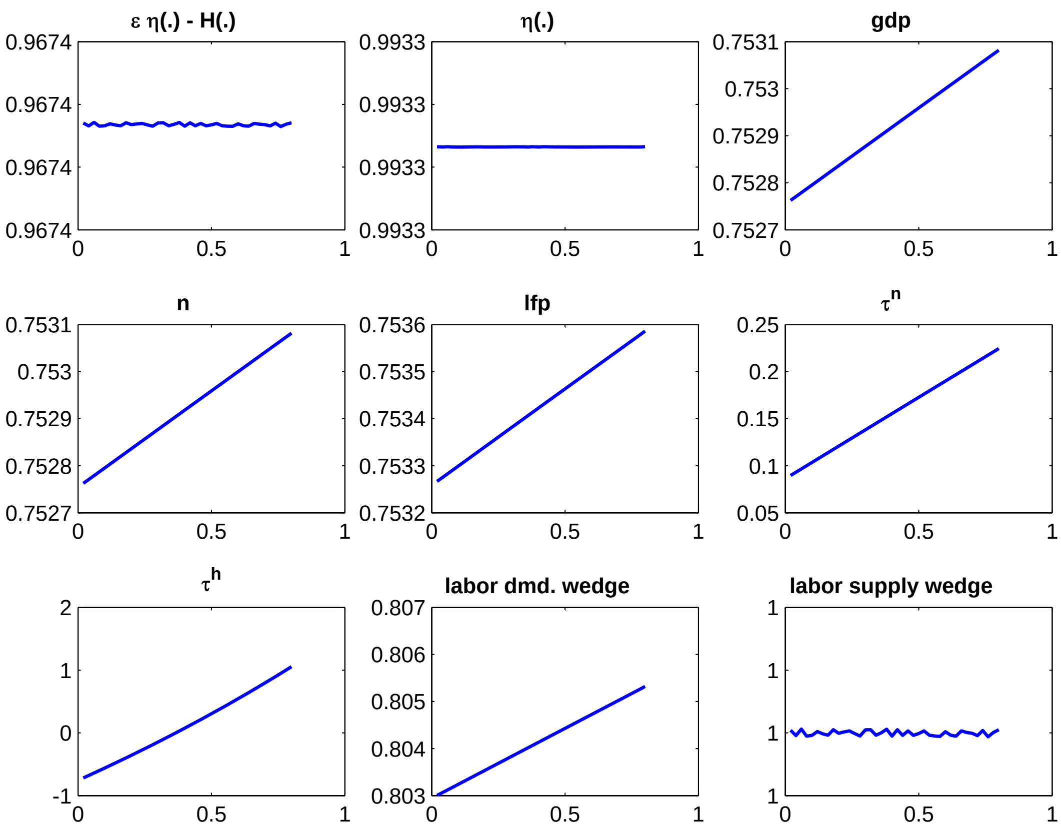

Figure 4: **Ramsey steady-state as a function of χ.** Except for the government provided unemployment benefit χ, all other parameter values are set as in the baseline calibration described in Section 3.2.

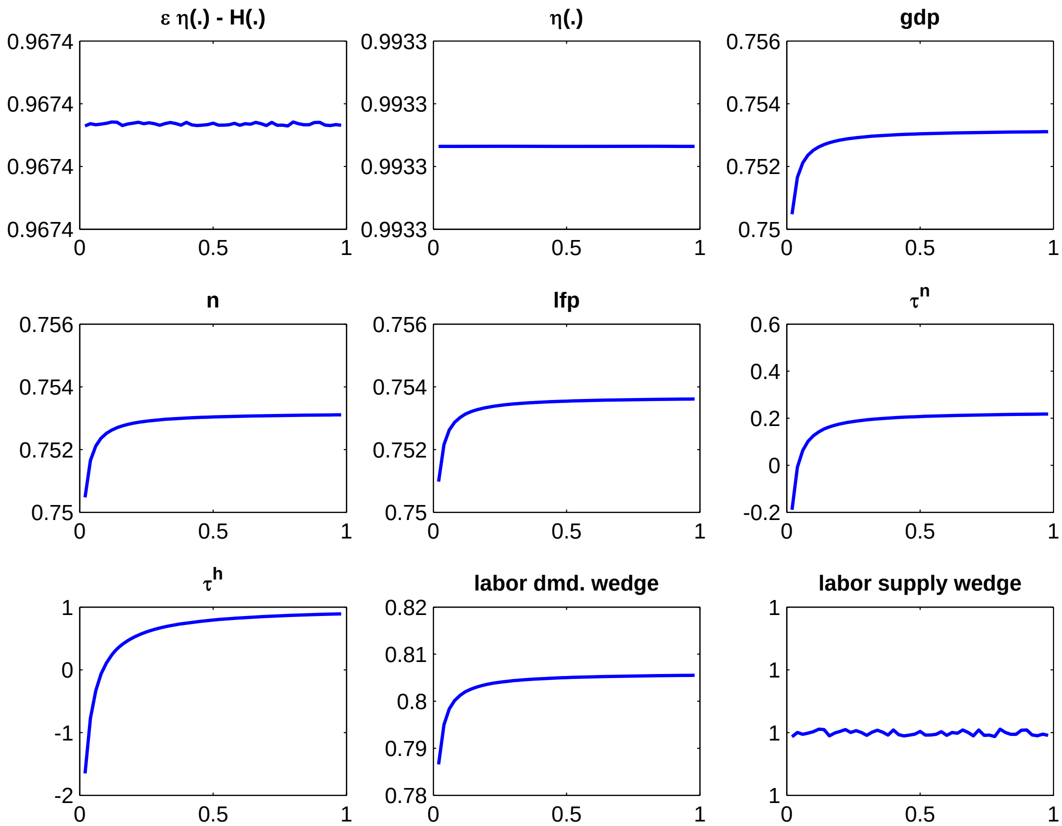

Figure 5: **Ramsey steady-state as a function of α^E.** Except for the newly-hired employees' bargaining power α^E, all other parameter values are set as in the baseline calibration described in Section 3.2.

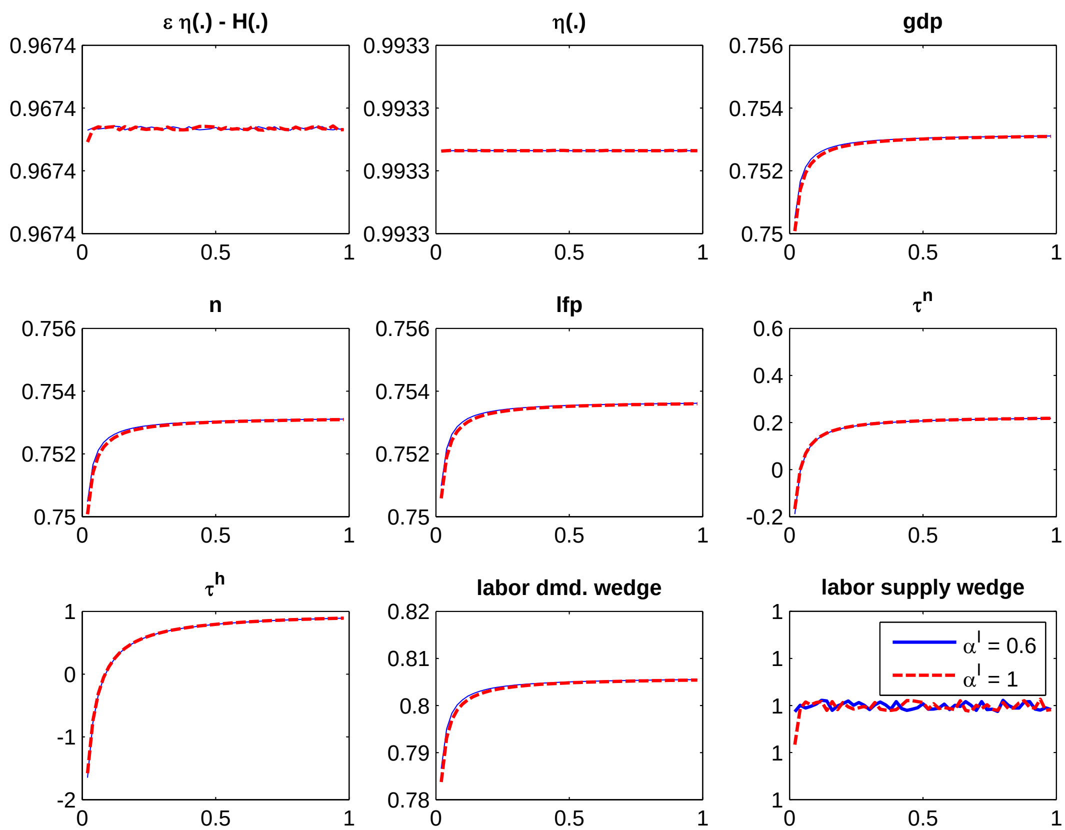

Figure 6: **Ramsey steady-state as a function of α^E.** Except for the newly-hired employees' bargaining power α^E and the two values of α^I, all other parameter values are set as in the baseline calibration described in Section 3.2.

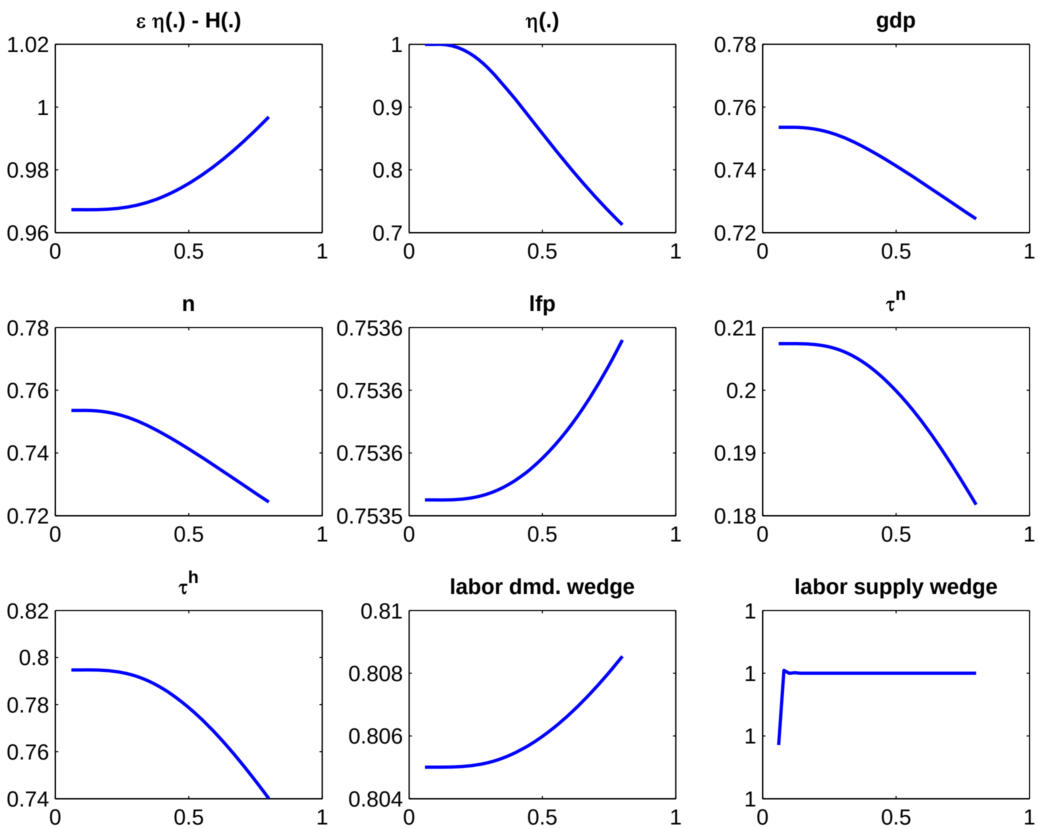

Figure 7: **Ramsey steady-state as a function of σ_ε.** Except for the cross-sectional dispersion σ_ε of idiosyncratic characteristics, all other parameter values are set as in the baseline calibration described in Section 3.2.