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Central Bank Learning and Monetary Policy

by

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We derive the optimal policy under active learning and compare it to two limiting cases---certainty equivalence policy and cautionary policy, in which learning takes place passively. Our novel result is that the two passive learning policies represent an upper and lower bound for the active learning policy, irrespective of the state of the economy.

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We analyze optimal monetary policy when a central bank has to learn about an unknown coefficient that determines the effect of surprise inflation on aggregate demand. We derive the optimal policy under active learning and compare it to two limiting cases—certainty equivalence policy and cautionary policy, in which learning takes place passively. Our novel result is that the two passive learning policies represent an upper and lower bound for the active learning policy, irrespective of the state of the economy.

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1 Introduction

A number of studies have analyzed the interaction of learning and control in optimal monetary policy (see e.g., Bertocchi and Spagat (1993), Balvers and Cosimano (1994), Wieland (2000a), Ellison and Valla (2001), Yetman (2003), Ellison (2006), Svensson and Williams (2007), and Tesfaselassie et al. (2006)). The linear process subject to control is typically assumed to be of the form[1]

$$x_t = \gamma_1 + \gamma_2 \pi_t + u_t \tag{1}$$

where π_t is the control variable and u_t is an unobserved iid shock. Of particular importance for optimal policy is the case where γ_2 is unknown to the decision maker. In that case, the choice of π_t not only affects y_t but also the sample estimate of γ_2.

In this paper we study the role of learning in a simple macro model

$$\begin{aligned} x_t &= \gamma_1 + \gamma_2(\pi_t - \pi_t^e) + u_t \\ \pi_{t+1}^e &= \pi_t^e + \gamma_0(\pi_t - \pi_t^e) \end{aligned} \tag{2}$$

where x_t is the output gap, π_t is the rate of inflation, π_t^e is the private sector's expectation of inflation, and $0 < \gamma_0 < 1$. Variants of this model can be found in papers dealing with optimal monetary policy (see e.g., Cogley and Colacito and Sargent (2005), Schaling (2003) and Tesfaselassie and Schaling (2008)). The particular form for inflation expectations may be motivated based on credibility (e.g., King (1996), Bomfim and Rudebusch (2000) and Yetman (2003))) or constant gain learning by the private sector (e.g. Orphanides and Williams (2004), Evans and Honkapohja (2001)). Unlike (1), what matters in (2) is the effect on x_t of the *deviation* of the control variable π_t from the state variable π_t^e.

Following the literature, we derive the active learning policy (ALP) when γ_2 is unknown to the central bank. We then compare the solution to two limiting cases—certainty equivalence policy (CEP) and cautionary policy (CP), which represent passive learning.

In our case, the ALP differs from the passive learning policies only when $\alpha > 0$. Similar results are commonly found in the literature (equation (1)). The new result in our paper is that the CEP and CP represent, respectively, an upper and a lower bound for the ALP. For any given state of the economy, the ALP never leaves the two bounds. By contrast, under equation (1), the ALP leaves the upper bound,

[1] Tesfaselassie et al. (2006) study a monetary model of the form $y_t = \gamma_1 y_{t-1} + \gamma_2 \pi_t + u_t$ and analyzed the case where γ_1 is unknown to the central bank.

implying a more aggressive response than the CEP, when the economy is near the steady state. This is because the decision maker needs to generate data in order to sharpen future inferences about γ_2. In our case, the dependence of π_t^e on the control π_t prevents the central bank from freely changing π_t.

Section 2 presents the policy problem and solves for optimal policy under active and passive policies. Section 3 discusses sensitivity of optimal policy to different parameter configurations. Finally, Section 4 gives concluding remarks.

2 The Policy Problem

The central bank's goal is to minimize, subject to (2),

$$E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} L(x_\tau, \pi_\tau) \tag{3}$$

where

$$L(\pi_t, x_t) = \frac{1}{2} x_t^2 + \frac{\alpha}{2} \pi_t^2 \tag{4}$$

E_t denotes expectations conditional on the central bank's information, α is the relative weight on inflation stabilization and δ is the discount factor.

Parameter Uncertainty and Belief Updating

The central bank's belief about γ_2, before setting π_t, can be characterized by a prior mean $c_t = E(\gamma_2 | \Omega_t)$ and prior variance $p_t = E(\gamma_2 - c_t)^2$ where $\Omega_t = \{..., x_{t-2}, x_{t-1}\}$.[2] After π_t is chosen and u_t realizes, determining x_t, the central bank updates its belief to (c_{t+1},p_{t+1}). Belief updating follows a standard recursive form (see e.g., Beck and Wieland (2002))

$$c_{t+1} = c_t + p_t F_t^{-1} (\pi_t - \pi_t^e)(x_t - c_t(\pi_t - \pi_t^e)) \tag{5}$$

$$p_{t+1} = p_t - p_t^2 (\pi_t - \pi_t^e)^2 F_t^{-1} \tag{6}$$

where $F_t \equiv p_t(\pi_t - \pi_t^e)^2 + \sigma_u^2$ is the conditional variance of x_t. The filtering process maps the sequence of output gap prediction errors into a sequence of revisions of

[2]This bounded rationality assumption follows, among others, Evans and Honkapohja (2001) in that the central bank acts like an econometrician.

beliefs.

Optimal Policy with Active Learning

The full-fledged model has three state variables—π_t^e, c_t and p_t. The dynamic control problem is

$$\min_{\{\pi_\tau\}_{\tau=t}^{\infty}} E\Big[\sum_{\tau=t}^{\infty} \delta^{\tau-t} L(z_\tau, \pi_\tau)|(\pi_t^e, c_t, p_t)\Big] \tag{7}$$

subject to the linear Phillips curve (2) and the non-linear updating equations (5) and (6). The Bellman equation associated with the dynamic programming problem (7) is

$$\begin{aligned} V(c_t, p_t, \pi_t^e) &= \min_{\pi_t} \Big\{ L(z_t, \pi_t) + \delta E_t V(c_{t+1}, p_{t+1}, \pi_{t+1}^e) \Big\} \\ &= \min_{\pi_t} \Big\{ \frac{1}{2} E_t x_t^2 + \frac{\alpha}{2} \pi_t^2 \\ &+ \delta \int V(c_{t+1}, p_{t+1}, \pi_{t+1}^e) f(x_t | c_t, p_t, \pi_t^e, \pi_t) dx_t \Big\} \end{aligned} \tag{8}$$

where $E_t x_t^2 = (c_t^2 + p_t)(\pi_t - \pi_t^e)^2 + \sigma_u^2$. The right hand side of (8) shows a tradeoff between current control and future estimation. The larger is the deviation of π_t from π_t^e, the larger is the current expected loss due to variability in x_t, but the smaller are future expected losses due to better information about γ (smaller p_{t+1}). Given $p_t > 0$, $\pi_t - \pi_t^e \neq 0$ implies $p_{t+1} < p_t$. Therefore, by internalizing the effect of π_t on p_{t+1}, active learning implies a more aggressive response of π_t to π_t^e (thus, larger disinflation) than than the CP.

As the updating equations are non-linear, the optimal policy under active learning can be solved only numerically. Easley and Kiefer (1988) and Kiefer and Nyarko (1989) have shown that a stationary optimal feedback rule exists, and that the value function is continuous and satisfies the Bellman equation. Thus, policy and value functions can be obtained numerically using an iterative algorithm on the Bellman equation (see e.g., Wieland (2000a)).

Below we compare the ALP with the CP and CEP. The CP is derived by setting $c_{t+1} = c_t$ and $p_{t+1} = p_t$, while the CEP is a limiting case where $c_{t+1} = c_t$ and $p_{t+1} = p_t = 0$.

First we show the results for a set of baseline parameters ($\alpha = 0.5, \sigma_u^2 = 1, \delta = 0.95$ and $\gamma_0 = 0.02$).

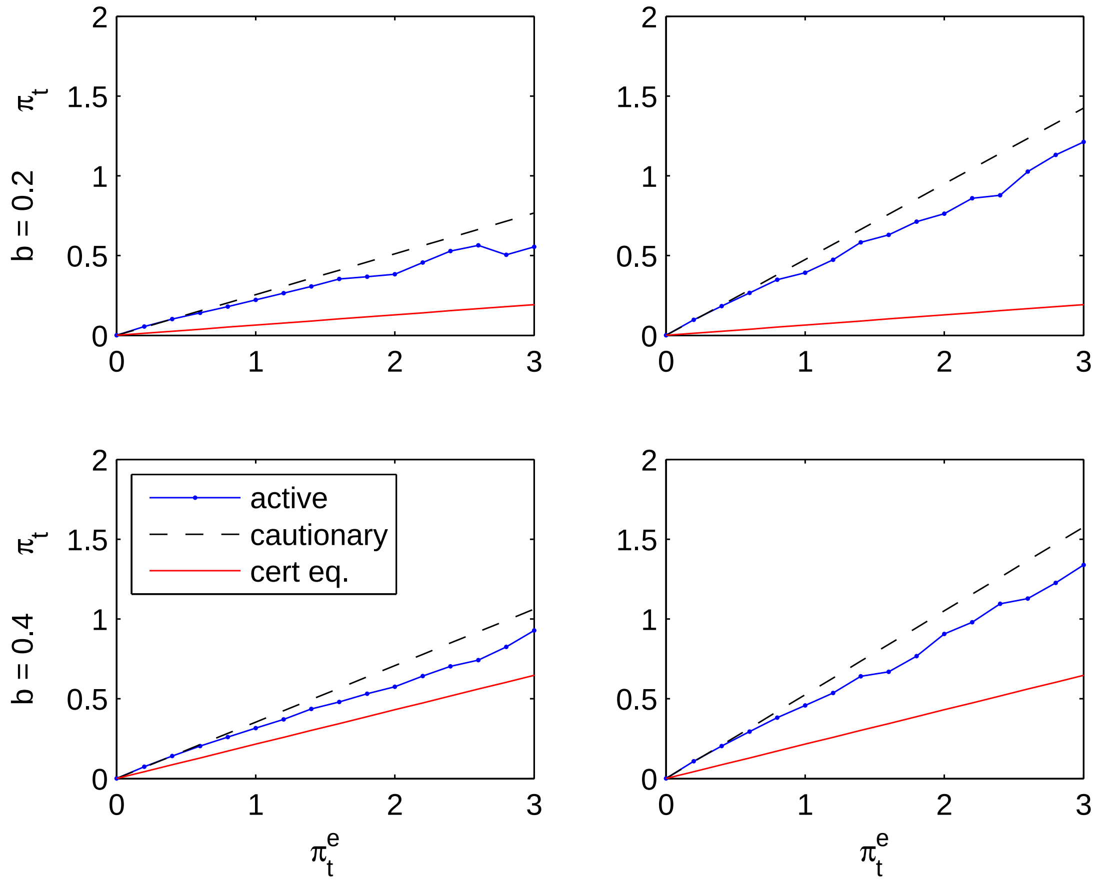

Figure 1: CEP, CP, and ALP ($\alpha = 0.5, \gamma_0 = 0.01, \sigma^2 = 1$).

Figure 1 plots our first result—the three policy rules as functions of π_t^e, for alternative initial beliefs (c_t, p_t) about the γ_2. As is shown in the figure, irrespective of the level of π_t^e, the ALP is less aggressive than the CEP but more aggressive than the CP. Note also that when π_t^e is close to zero (the inflation target), the ALP and the CP are almost identical. For larger values of π_t^e, the ALP deviates from the CP, implying a role for experimentation. Importantly, we see that the ALP stays between the CP and the CEP. This result is in contrast to other studies that model parameter uncertainty based on equation (1). In those studies, the ALP may be more aggressive than the CEP when the economy is close to the steady state (see e.g., Beck and Wieland (2002)).

3 Sensitivity Analysis

Policy leverage over expected inflation

In the model, the parameter γ_0 captures the degree of leverage of monetary policy over inflation expectations. For instance, the larger γ_0 is, the larger the effect of actual inflation on expected inflation, thus implying a higher degree of leverage of monetary policy over the economy.

Note that, the three types of policies are all functions of γ_0. For example, a larger value of γ_0 induces all policies to respond less aggressively to expected inflation, so as to compensate the increase in policy leverage over the economy. What matters is, therefore, how the *difference* between the ALP and the CP (i.e., the degree of experimentation) behaves when γ_0 changes (see Figure 2).

It turns out that the degree of experimentation is inversely related to the size of γ_0; i.e., the larger γ_0 is the smaller the degree of experimentation, and vice versa. This shows that the ALP is more sensitive to changes in γ_0 than the CP.

Variance of shocks

The second important parameter in the model is the variance of the shock to output gap σ^2. Since the CEP and the CP are independent of σ^2, in Figure 3 we only plot the ALP for two alternative values of σ^2. We see that the relation between σ^2 and the ALP is not independent of the state variable π_t^e. In particular, for small to moderate values of π_t^e, the ALP becomes (marginally) more aggressive for $\sigma^2 = 0.25$ than for $\sigma^2 = 1$. On the other hand, for large values of π_t^e, the ALP becomes less aggressive. This effect is more visible for larger values of p_t.

To get an intuition for this result, note that the output gap x_t is less volatile under $\sigma^2 = 0.25$ than under $\sigma^2 = 1$. Given π_t^e, the updating equation for c_{t+1} implies that the forecast error $x_t - c_t(\pi_t - \pi_t^e)$ becomes more informative the smaller the variance of x_t due to σ^2 (alternatively, the larger the variance of x_t due to estimation errors). Thus, in general the more stable is the system, the smaller is the incentive for experimentation. However, when π_t^e is closer to the steady state of zero, there is more incentive for experimentation, the more so the smaller is σ^2 (i.e., when economy is inherently more stable).

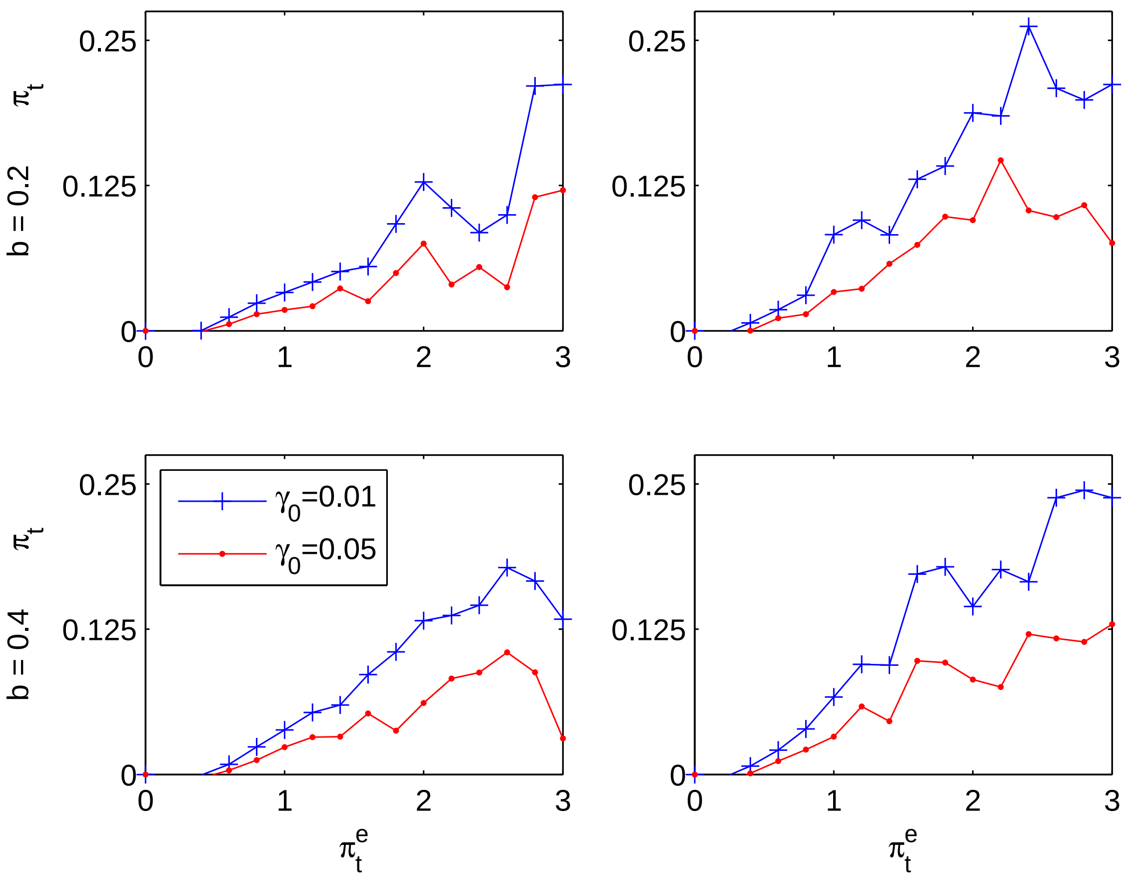

Figure 2: ALP relative to CP($\gamma_0 = 0.01$ vs. $\gamma_0 = 0.05$).

4 Concluding Remarks

The paper derives optimal monetary policy under parameter uncertainty for a linear process that differs from previous literature. In particular, the central bank has to learn about an unknown coefficient that determines the effect of surprise inflation on aggregate demand.

We derive the optimal policy under active learning and compare it to two limiting cases—CEP and CP, in which learning takes place passively. It turns out that the two passive learning policies represent an upper and lower bound for the ALP. In particular, the ALP implies a less (more) aggressive degree of disinflation (measured by the difference between actual inflation and expected inflation) compared to the

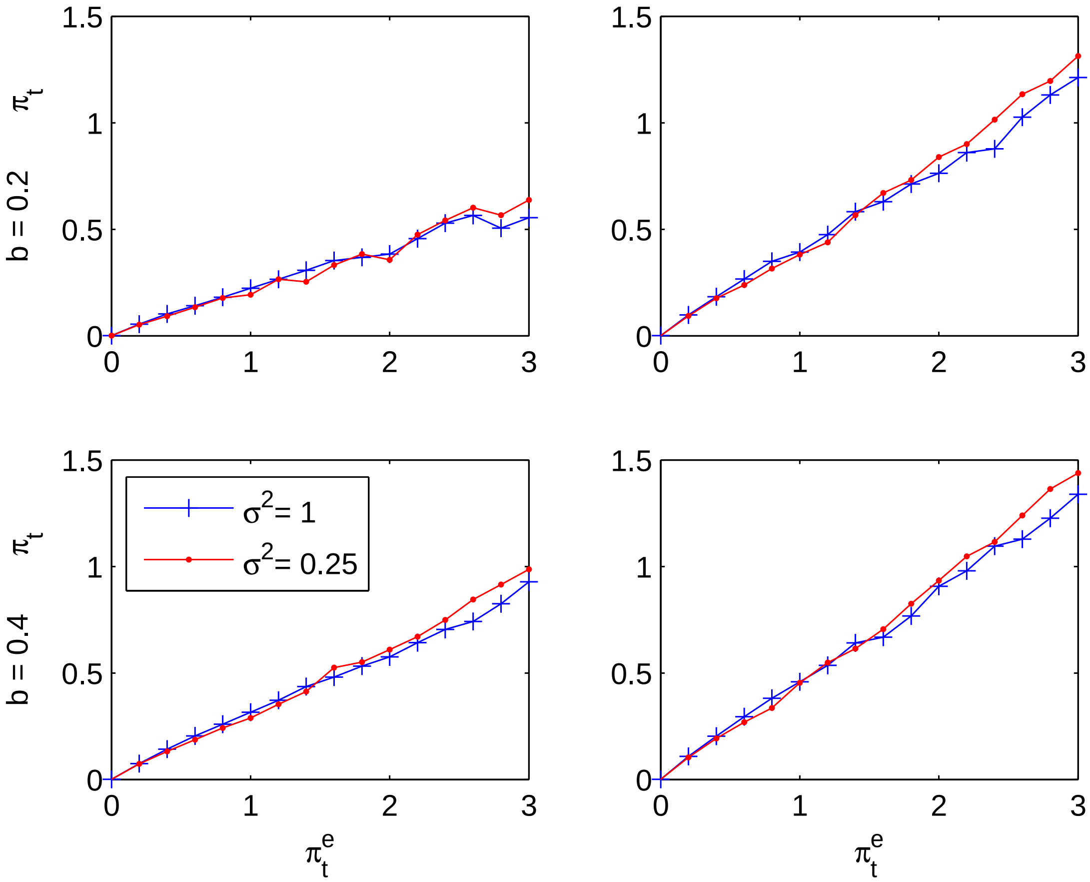

Figure 3: ALP ($\sigma^2 = 1$ vs $\sigma^2 = 0.25$).

certainty equivalence (cautionary) policy. The novelty is that the above result holds irrespective of the state of the economy.

One possible extension of the analysis is to let γ_2 be time-dependent, for example a random walk $\gamma_{2,t} = \gamma_{2,t-1} + \eta_t$ as in Beck and Wieland (2002) or an autoregressive process $\gamma_{2,t} = \rho\gamma_{2,t-1} + \eta_t$, $0 < \rho < 1$ as in Balvers and Cosimano (1994). It is easy to conjecture that in this case, learning is perpetual, as the underlying parameter is time-varying. This reduces the incentive for experimentation.

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