# Ramsey Monetary Policy With Financial Distortions and the Fisherian Theory of Debt Deflation

Ester Faia*
Universitat Pompeu Fabra

January 2006
First draft: May 2004

**Abstract**

I derive welfare maximizing policy for an economy characterized by three distortions: monopolistic competition, sticky prices and borrowing constraint on capital in the form of a premium on external finance. Under a zero inflation policy - i.e. which closes the gap - neither the monopolistic competition distortion - which act as a tax on labor - nor the external finance premium - which act as a tax on capital - can be offset. Both the product mark-up and the external finance premium act as countercyclical wedges that allow the policy maker to improve the flexible price allocation. Optimal monetary policy features a long run inflationary bias which even more pronounced under non-indexed loan contracts. In the latter case indeed inflation reduces the outstanding value of nominal debt and the services associated with it. Along the dynamic optimal policy is characterized by deviations from price stability - e.g. both under productivity and demand shocks - with an optimal inflation volatility which increases together with the increase in the external finance premium.

*JEL Classification Numbers:* E0, E4, E5, E6.

*Keywords:* optimal monetary policy, financial frictions, optimal degree of inflation.

## 1 Introduction

An ever standing question is whether optimal monetary policy should react to financial distress generated by distortions on the lending activity for investment. Such issues had received attention since the time of the Great Depression when Friedman-Schwartz and Fisher argued that the Federal Reserve should have increased the amount of liquidity available for investment in order to avoid

*I thank Ricardo Caballero, Thomas Cooley, Harris Dellas, Mark Gertler, Fabio Ghironi, Bob King, Timothy Kehoe, Ed Nelson, Albert Marcet, Fabrizio Perri, Vincenzo Quadrini and Alexander Wolman for useful discussions and suggestions and all participants to the IMOP conference in Dynamic Macroeconomic Theory, Hydra, the conference in Dynamic Macroeconomics held at the Copenhagen Institute of Economics and seminar participants at UPF. I gratefully acknowledge support from DSGE grant. All errors are mine. Correspondence to Ester Faia, Department of Economics, Universitat Pompeu Fabra, Ramon Trias Fargas, 08005, Barcelona, Spain. E-mail address: ester.faia@upf.edu. Homepage: www.econ.upf.edu/~faia.

the financial turmoil. Hence according to their argument reacting to an increase in asset prices by increasing the nominal interest rate and reducing disposable liquidity had welfare detrimental effects.

The question is timely even nowadays when most central banks have a mandate of price stability. In this context the question of whether the monetary authority should react to asset prices assumes a somehow different flavor. It is questioned indeed whether in a situation of financial distress it is optimal for the monetary authority to deviate from price stability goals and to target other financial indicators.

This paper examines this issue, namely whether in presence of financial distortion the monetary authority can improve upon the flexible price allocation obtained under a price stability rule. To this purpose I derive a welfare maximizing monetary policy for an economy characterized by three main distortions, namely monopolistic competition, sticky prices induced by an adjustment cost to inflation, and financial frictions in the form of a cost to external funds which distorts the margin between consumption at two different dates. Financial frictions are deliberately microfounded following the standard approach of introducing an asymmetric information problem between borrower and lender which is modelled using a costly state verification contract[1]. In addition the paper tests the implications of different loan contract arrangements in terms of indexation to expected inflation. The main goal is to derive principles of optimal policy in presence of a time-varying tax on capital represented by the external finance premium on investment. The paper asks first whether such a tax on capital can be offset completely when the monetary authority possesses only one instrument, namely the nominal interest rate. Secondly, it asks whether price stability objective is compatible with an economy where the monetary authority needs to trade-off the three mentioned distortions, namely monopolistic competition, sticky price distortion and an external finance premium on investment.

In the classic approach to the study of optimal policy in dynamic economies (Ramsey (1927), Atkinson and Stiglitz (1980), Lucas and Stokey (1983), Chari, Christiano and Kehoe (1992)) a social planner maximizes household's welfare subject to a resource constraint, to the constraints describing the equilibrium in the private sector economy. This implies that a benevolent planner should offset distortions present in the competitive economy. However in presence of multiple distortions a monetary authority endowed with a single instrument can only aim at trading-off the welfare costs associated with all the distortions but cannot achieve the unconstrained pareto optimum.

The analysis of this paper can be summarized in terms of two main contributions.

First, I show that a monetary authority following a price stability rule is not able to offset neither the mark-up distortion nor the time-varying external finance premium on investment. To

[1] See Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1998), Cooley and Nam (1998). Such models are appealing also since they have empirically founded implications.

achieve this conclusion I first show the equivalence under flexible prices between my model economy and an economy characterized by a constant tax on labor and a time-varying tax on capital. Indeed the mark-up wedge by distorting the margin between consumption and employment plays the role of a tax on labor while the external finance premium by distorting the margin between consumption today and tomorrow plays the role of a tax on capital investment. A monetary authority who decides to offset only the sticky price distortion by replicating the flexible price allocation - i.e. by closing the gap - will reach an equilibrium characterized by a constant mark-up and a time varying wedge on capital investment. Such an economy features inefficiently low levels of both employment and capital.

Secondly I show that the optimal allocation is characterized by a long run inflationary bias and short run deviations from price stability. The intuition for this result can be explained by looking at the interplay of the three distortions present in this economy. First, monopolistic competition in goods markets, which forces output below the socially optimal level hence calls for expansionary monetary policy. Second, (quadratic) adjustment costs in nominal goods prices, which entail a direct resource cost, as well as inefficient misalignment between the marginal utility of consumption and leisure due to time variations in the markup. The sole presence of adjustment costs in pricing would call for strong price stability target which closes the gaps[2]. Third, informational frictions in the form of agency costs, that characterize the relationship between borrowers and lenders in credit markets. In this context, the evolution of firms' net worth affects both the cost of access to credit and the price of capital. Yet in turn these developments feedback onto firms' financial position, further affecting investment and capital accumulation. Borrowing constraints have a twofold effect. In the long run they produce an inefficient low level of capital, hence output, since the economy suffers a deadweight loss associated with the monitoring activity of the intermediary. This coupled with the monopolistic competition gives rise to an inflationary bias.

Along the dynamics the presence of a time-varying external finance premium exacerbate the dynamics of investment and all financial variables beyond the one associated with an economy which simply features capital accumulation and/or adjustment costs to investment. Overall in presence of fundamental shocks the monetary authority has an incentive toward expansionary policy. This is because whenever the share of any component of aggregate demand varies across states it is optimal for the monetary authority to increase demand and deviate from price stability. In a business cycle model with capital the investment share in output reacts to technological shocks thereby allowing the monetary authority to improve upon the flexible price allocation. Those fluctuations are additionally exacerbated in presence of financial distortions thereby allowing for ampler deviations form price stability. Overall a pro-cyclical policy increases investment demand and the value of collateral thereby decreasing the external finance premium and relaxing the borrowing constraints.

[2] Gaps are defined as the difference in output and other variables dynamics between the flexible price and the sticky price allocation.

Under those circumstances firms are able to better exploit the benefits of an increase in investment opportunities.

It is worth noticing that the monetary authority does not have a direct leverage on the time-varying external finance premium since under indexed loan contract the latter does not depend upon expected inflation. The reduction of the investment wedge occurs only to the extent that an expansion of the economic activity boosts the real value of collateral. Alternatively under non-indexed loan contracts the external finance premium depends negatively upon future expected inflation. In this case surprise inflation reduces the outstanding value of debt thereby creating an additional incentive for the monetary authority to deviate from the price stability policy[3].

The rest of the paper is divided as follows. Section 2 presents the model economy. Section 3 shows the optimal monetary policy design. Section 4 discuss the implementability problem. Section 5 shows solution and simulation of the optimal long run policy. Section 5 shows solution and simulations of the optimal policy along the dynamic. Section 6 explores the alternative of non-neutral monetary policy stemming from the presence of a cash in advance constraint. Finally section 7 concludes.

# 2 The Model Economy

The structure of the economy borrows from similar models which introduce costly state verification contracts in dynamic general equilibrium analysis, such as Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1998) and Cooley and Nam (1998). More specifically I extend the dynamic general equilibrium model of Carlstrom and Fuerst (1997) to include nominal price rigidity and a role for monetary policy. In addition, as opposed to Bernanke, Gertler and Gilchrist (1998), I introduce sticky prices via adjustment costs a' la Rotemberg[4].

The economy is populated by two sets of agents, workers and entrepreneurs, that account for a total measure of one. Each type is simultaneously consumer and investor. The measure of each set of agents is exogenously given. Each agent in the economy has a probability $\eta$ of becoming a worker and a probability $(1-\eta)$ of becoming an entrepreneur. By the law of large of numbers the size of the worker population is given by $\eta$ and the size of the population of entrepreneurs is given by $(1-\eta)$.

There is a single production sector which is monopolistic competitive and produces different varieties of goods using capital and labor. The intermediate goods produced by this sector are then

[3] A non-explosive solution of inflation would still be possible due to the presence of welfare costs of inflation attached to the assumption of sticky prices. Hence a monetary authority acting optimally would have to trade off the inflation stability motive induced by adjustment costs on inflation with the possibility of increasing the real value of the leverage ratio, thereby reducing the external finance premium.

[4] This allows to simplify the production structure which in BGG contemplates two different sectors, one subject to random shocks to prices a' la Calvo while the other subject to idiosyncratic shocks to capital and agency costs. The two sources of heterogeneity in BGG do not allow aggregatioon within a single sector economy.

assembled by a final good production unit. To finance capital, firms asks for a loan to a competitive intermediary. Due to the presence of informational asymmetries between lender and borrower and of agency costs external funds are subject to the payment of a premium.

## 2.1 Workers (Lenders)

There is continuum of workers/savers, each indexed by $i \in (0, 1)$. They consume the final good, invest in safe bank deposits, supply labor and own shares of a monopolistic competitive sector that produces differentiated varieties of goods. The representative worker chooses the set of processes $\{C_t,\ N_t\}_{t=0}^{\infty}$ and one-period nominal deposits $\{D_t\}_{t=0}^{\infty}$ , taking as given the set of processes $\{P_t,$ $W_t,\ (1+R_t^n)\}_{t=0}^{\infty}$ and the initial condition $D_0$ to maximize:

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t U(C_t, N_t)\right\} \tag{1}$$

subject to the sequence of budget constraints:

$$P_tC_t + D_{t+1} \leq (1+R_t^n)D_t + W_tN_t + \Theta_t + T_t \tag{2}$$

where $C_t$ is workers' consumption of the final good, $W_t$ is the nominal wage, $N_t$ is total labor hours, $R_t^n$ is the nominal interest rate paid on deposits, $\Theta_t$ are the nominal profits that households receive from running production in the monopolistic sector and $T_t$ are lump sum taxes/transfers from the fiscal authority. The first order conditions of the above problem read as follows:

$$U_{c,t} = \beta(1+R_t^n)E_t\left\{U_{c,t+1}\frac{P_t}{P_{t+1}}\right\} \tag{3}$$

$$U_{c,t}\frac{W_t}{P_t} = -U_{n,t} \tag{4}$$

$$\lim_{j\to\infty}(1+R_{t+j}^n)^{-1}D_{t+j} = 0 \tag{5}$$

with the addition of (2) holding with equality. Let's define the real interest rate as:

$$(1+R_t) = (1+R_t^n)E_t\left\{\frac{P_t}{P_{t+1}}\right\} \tag{6}$$

## 2.2 Entrepreneurs (Borrowers)

The second set of agents in the economy are the entrepreneurs. These agents are risk neutral. They inelastically supply labor and use purchases of the final good for both consumption and investment in new capital. In each period they also rent owned capital to firms in the monopolistic competitive sector. To finance the purchase of new capital they employ internal funds but they also need to acquire an external loan from a financial intermediary. The relationship with the lender is subject to an agency cost problem, which forces the entrepreneur to pay a premium to service the loan. I will elaborate below on this point.

I follow Kiyotaki and Moore (1997) and Carlstrom and Fuerst (1998) and assume that the Entrepreneurs are finitely lived (with $\theta$ being the probability of dying in each period). This assumption assures that entrepreneurial consumption occurs to such an extent that self-financing never occurs and borrowing constraints on loans are always binding. Resorting to the law of large numbers and to the characteristics of the loan contract will allow aggregation for these agents. Each Entrepreneur chooses a sequence $\{C_t^e,\ I_t,\ K_{t+1},\ L_{t+1}\}_{t=0}^{\infty}$ to maximize

$$E_0 \sum_{t=0}^{\infty} (\theta\beta)^t C_t^e \tag{7}$$

subject to the following sequence of budget constraints (expressed in real terms):

$$Z_t K_t + L_{t+1} = C_t^e + I_t + (1 + R_t^L) L_t \tag{8}$$

and to a capital accumulation equation

$$K_{t+1} = (1-\delta)K_t + I_t - \frac{\phi_k}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t \tag{9}$$

Equation (8) is the Entrepreneurs' budget constraint. Wealth is derived from rental income $Z_t K_t$, and by acquiring a new loan $L_{t+1}$. Expenditure derives from final good consumption $C_t^e$, investment $I_t$ and from the service of the predetermined loan debt $R_t^L L_t$. Constraint (9) indicates that, when investing in capital, entrepreneurs face adjustment costs. The parameter $\phi_k$ yields the fraction of investment which is lost in production of new capital goods, where $\delta$ is the depreciation rate of capital.

Let's define $\{\lambda_t, Q_t\}_{t=0}^{\infty}$ as the sequence of Lagrange multipliers on the constraints (8) and (9) respectively. The first order conditions of the above problem read:

$$\lambda_t = 1 \tag{10}$$

$$\lambda_t = \theta\beta E_t \left\{(1 + R_{t+1}^L)\lambda_{t+1}\right\} \tag{11}$$

$$\left[1 - \phi_k\left(\frac{I_t}{K_t} - \delta\right)\right]^{-1} = \lambda_t Q_t \tag{12}$$

$$Q_t = \gamma E_t \left\{ Z_{t+1}\lambda_{t+1} + Q_{t+1} \left( 1 - \delta + \frac{\phi_k}{2} \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 - \phi_k \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} \right) \right\} \tag{13}$$

Equation (10) states that, due to risk neutrality, the marginal utility of additional real income is constant. Equation (11) is the Euler efficiency condition on the loan holding. Equations (12) and (13) are the efficiency conditions on capital investment. Notice that the lagrange multiplier $Q_t$ denotes the nominal shadow value of installing new capital and thus plays the role of the implicit price of capital (or asset price).

#### 2.2.1 Asset Prices and Arbitrage Conditions

Using (11) and (10) in equation (13) we obtain an expression for the evolution of the asset price (in terms of price of new investment good relative to the final consumption good):

$$Q_t = E_t \left\{ Z_{t+1} + Q_{t+1} \left( 1 - \delta + \frac{\phi_k}{2} \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 - \phi_k \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} \right) \right\} \tag{14}$$

Let's define by $\mathcal{Y}_t^k \equiv Z_t + Q_t \left( 1 - \delta + \frac{\phi_k}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 - \phi_k \left( \frac{I_t}{K_t} - \delta \right) \frac{I_t}{K_t} \right)$ the *real income* from holding one unit of capital. Hence the *return* from holding a unit of capital between $t$ and $t+1$ reads:

$$(1 + R_{t+1}^k) \equiv \frac{\mathcal{Y}_{t+1}^k}{Q_t} \tag{15}$$

Notice that shocks (e.g., productivity) that raise the rental cost of capital $Z_t$ also increase capital income.

### 2.3 The Financial Contract between Lenders and Borrowers

The financial contract between the entrepreneurs and the intermediary assumes the form of an optimal debt contract à la Gale and Hellwig (1983). When the idiosyncratic shock to capital investment is above the cut-off value which determines the default states the entrepreneurs repay an amount $R_{t+1}^L$[5]. On the contrary, in the default states, the bank monitors the investment activity and repossesses the assets of the firm. Default occurs when the return from the investment activity $\omega_{t+1}^j \mathcal{Y}_{t+1}^k K_{t+1}^j$ falls short of the amount that needs to be repaid $R_{t+1}^L L_{t+1}^j$. Hence the *default space* is implicitly defined as the range for $\omega$ such that :

$$\omega_{t+1}^j < \varpi_{t+1}^j \equiv \frac{R_{t+1}^L L_{t+1}^j}{\mathcal{Y}_{t+1}^k K_{t+1}^j} \tag{16}$$

[5] In every period $t$ this amount must be independent from the idiosyncratic shock in order to satisfy incentive compatibility conditions.

where $\varpi^j_{t+1}$ is a cutoff value for the idiosyncratic productivity shock. Let's define by $\Gamma(\varpi^j) \equiv \int_0^{\varpi^j_{t+1}} \omega^j_{t+1} f(\omega)d\omega + \varpi^j_{t+1} \int_{\varpi_{t+1}}^{\infty} f(\omega)d\omega$ and $1 - \Gamma(\varpi^j)$ the fractions of net capital output received by the lender and the entrepreneur respectively. Expected bankruptcy costs are defined as $\mu M(\varpi^j_{t+1}) \equiv \mu \int_0^{\varpi^j_{t+1}} \omega^j_{t+1} f(\omega)d\omega$ with the *net share* accruing to the lender being $\Gamma(\varpi^j_{t+1}) - \mu M(\varpi^j_{t+1})$. The real return paid on deposits is given by the safe rate, $R_t$, which as such corresponds, for the lender, to the opportunity cost of financing capital. The *participation constraint* for the lender states that the expected return from the lending activity should not fall short of the opportunity cost of finance:

$$\mathcal{Y}^k_{t+1} K^j_{t+1} (\Gamma(\varpi^j_{t+1}) - \mu M(\varpi^j_{t+1})) \geq R_t (Q_t K^j_{t+1} - NW^j_{t+1}) \tag{17}$$

The contract specifies a pair $\left\{\varpi^j_{t+1}, K^j_{t+1}\right\}$ which solves the following maximization problem:

$$Max\ (1 - \Gamma(\varpi^j_{t+1}))\mathcal{Y}^k_{t+1} K^j_{t+1} \tag{18}$$

subject to the participation constraint (17). Two assumptions make aggregation feasible: 1) A constant fraction $\varsigma$ of entrepreneurs remain alive in every period. 2) The optimal contract linear relations. Using the first order conditions with respect $\left\{\varpi^j_{t+1}, K^j_{t+1}\right\}$ and aggregating yield a wedge between the return on capital and the safe return paid on deposits, $\rho(\varpi_{t+1}) = \left[\frac{(1-\Gamma(\varpi_{t+1}))(\Gamma'(\varpi_{t+1}) - \mu M'(\varpi_{t+1}))}{\Gamma'(\varpi_{t+1})} + (\Gamma(\varpi_{t+1}) - \mu M(\varpi_{t+1}))\right]^{-1}$, which is positively related to the default threshold. By defining $rp_t \equiv \frac{R^k_{t+1}}{R_t}$ as the *premium on external finance* and by combining (17) with the expression for $\rho(\varpi_{t+1})$ it is possible to write a relation between the ex-post external finance premium, $rp_t$, and the leverage ratio, $\frac{Q_t K_{t+1}}{NW_{t+1}}$:

$$\frac{R^k_{t+1}}{R_t} = rp_t(\frac{Q_t K_{t+1}}{NW_{t+1}}) \tag{19}$$

with $rp'_t(\frac{Q_t K_{t+1}}{NW_{t+1}}) > 0$. An increase in net worth or a decrease in the leverage ratio reduces the optimal cut-off value, as shown by equation (16). By reducing the size of the default space it also reduces the size of the bankruptcy costs and the external finance premium. The relation obtained in equation (19) can also be written in terms of borrowing limit as $L_{t+1} = NW_{t+1}(rp_t^{-1}(\frac{R^k_{t+1}}{R_t}) - 1)$ stating that the higher is the external finance premium the lower is the amount that can be borrowed. As it stands clear the quality of the financial system is determined by the size and the sensitivity to collateral (the net worth) of the external finance premium. In turn the external finance premium depends from the size of the bankruptcy costs and the volatility of the idiosyncratic shock (corporate risk) as from $\rho(\varpi_{t+1})$. In the calibration section the two countries are parametrized by assigning different values to those parameters.

Aggregate net wealth accumulation of the economy reads as follows:

$$NW_{t+1} = \varsigma[R_t^k Q_{t-1} K_t - (R_t + rp_{t-1}(\frac{Q_{t-1}K_t}{NW_t}))(Q_{t-1}K_t - NW_t) - \Sigma_t] \tag{20}$$

### 2.4 Production and Pricing of Intermediate Goods

Each domestic household owns an equal share of the intermediate-goods producing firms.[6] Each of these firms assembles labor (supplied by the workers) and entrepreneurial capital to operate a constant return to scale production function for the variety $i$ of the intermediate good:

$$Y_t(i) = A_t F(N_t(i), K_t(i), N_t^e(i)) \tag{21}$$

where $A_t$ is a productivity shifter common to all entrepreneurs. Each firm $i$ has monopolistic power in the production of its own variety and therefore has leverage in setting the price. In so doing it faces a quadratic cost equal to $\frac{\omega_p}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2$, where the parameter $\omega_p$ measures the degree of nominal price rigidity. The problem of each domestic monopolistic firm is the one of choosing the sequence $\{K_t(i), N_t(i), P_t(i)\}_{t=0}^{\infty}$ in order to maximize expected discounted real profits $\Theta_t \equiv P_t(i)Y_t(i) - (W_t N_t(i) + Z_t P_t K_t(i)) - \frac{\omega_p}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2$,

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U_{c,t} \frac{\Theta_t}{P_t} \right\} \tag{22}$$

subject to demand and production constraints. Let's denote by $\{mc_t\}_{t=0}^{\infty}$ the sequence of lagrange multipliers on the constraint (**??**) and by $\widetilde{p}_t \equiv \frac{P_t(i)}{P_t}$ the relative price of variety $i$. The first order conditions of the above problem read

$$\frac{W_t}{P_t} = mc_t A_t F_{n,t}; Z_t = mc_t A_t F_{k,t} \tag{23}$$

$$\begin{aligned} 0 \quad = \quad & Y_t \widetilde{p}_t^{-\vartheta} \left((1-\vartheta) + \vartheta mc_t\right) - \omega_p \left(\pi_t \frac{\widetilde{p}_t}{\widetilde{p}_{t-1}} - 1\right) \frac{\pi_t}{\widetilde{p}_{t-1}} \\ & + \gamma \omega_p \left(\pi_{t+1} \frac{\widetilde{p}_{t+1}}{\widetilde{p}_t} - 1\right) \pi_{t+1} \frac{\widetilde{p}_{t+1}}{\widetilde{p}_t^{\,2}} \end{aligned} \tag{24}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$is the gross inflation rate and where I have suppressed the superscript $i$ since all firms employ an identical capital/labor ratio in equilibrium.

[6] An alternative ownership structure could be explored, in which the entrepreneurs directly own the shares of the intermediate goods firms that employ capital in production. In this case monopolistic profits would be part of capital income $\mathcal{Y}_t^k$ , as in Cook (2002).

## 2.5 Final Good Sector

The aggregate final good $Y$ is produced by perfectly competitive firms. It requires assembling a continuum of intermediate goods, indexed by $i$, via the aggregate production function:

$$Y_t \equiv \left( \int_0^1 Y_t(i)^{\frac{\vartheta - 1}{\vartheta}} di \right)^{\frac{\vartheta}{\vartheta - 1}} \tag{25}$$

Maximization of profits yields typical demand functions:

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\vartheta} Y_t \tag{26}$$

for all $i$, where $P_t \equiv \left( \int_0^1 P_t(i)^{1-\vartheta} di \right)^{\frac{1}{1-\vartheta}}$ is the price index consistent with the final good producers earning zero profits.

### 2.5.1 Market Clearing Conditions and Competitive Equilibrium

Equilibrium in the final good market requires that the production of the final good be allocated to private consumption by households and entrepreneurs, investment and to resource costs that originate from the adjustment of prices as well as from the lender's monitoring of the investment activity:

$$Y_t = (1 - \eta) C_t + \eta C_t^e + I_t + \frac{\omega_p}{2} (\pi_t - 1)^2 + \mu M(\varpi_t) \mathcal{Y}_t^k K_t \tag{27}$$

Equilibrium in financial market implies that:

$$\frac{D_t}{P_t} = L_t \tag{28}$$

**Definition 1.** *A distorted competitive equilibrium for this economy is a sequence of allocation and prices* $\{C_t, C_t^e, N_t, K_{t+1}, I_t, L_t, D_t, NW_{t+1}, \varpi_t, mc_t, \lambda_t, \pi_t, Q_t, W_t, R_t, R_t^n, R_t^k\}_{t=0}^{\infty}$ *which, for given initial* $D_0$,$K_0$, $NW_0$, *satisfies equations*
*(3), (??),(5), (6), (9), (12),(14),(??),(??),(??),(??),(??),(15),(17), (28), (4) and (23).*

# 3 Deriving the Relevant Constraints

The optimal policy is determined by a monetary authority that maximizes the discounted sum of utilities of all agents given the constraints of the competitive economy. The next task is to select the relations that represent the relevant constraints in the planner's optimal policy problem. This amounts to describing the competitive equilibrium in terms of a minimal set of relations involving only real allocations, in the spirit of the primal approach described in Lucas and Stokey (1983).

There is a fundamental difference, though, between that classic approach and the one followed here, which stems from the impossibility, in the presence of sticky prices, of reducing the planner's problem to a maximization only subject to a single implementability constraint. Khan, King and Wolman (2003) adopt a similar structure to analyze optimal monetary policy in a closed economy with market power, price stickiness and monetary frictions, while Schmitt-Grohe and Uribe (2002) to analyze a problem of joint determination of optimal monetary and fiscal policy.

I start by to rearranging the optimality conditions for the production sector. This requires, at first, to express the real marginal cost and the real wage in terms of aggregate real quantities. Hence by combining (4) and (23) we can write

$$mc_t = -\frac{U_{n,t}}{U_{c,t}A_t} \tag{29}$$

This implies that the aggregate condition for optimal pricing (**??**) can be rewritten as

$$\begin{aligned} U_{c,t}\pi_t(\pi_t - 1) \quad &= \quad \beta U_{c,t+1} E_t \left\{ \pi_{t+1}(\pi_{t+1} - 1) \right\} + \\ &+ \frac{U_{c,t}\vartheta A_t F(.)}{\omega_p} \left( -\frac{U_{n,t}}{U_{c,t}F_{n,t}} - \frac{\vartheta - 1}{\vartheta} \right) \end{aligned} \tag{30}$$

Under the assumption of a Cobb-Douglas production function it is possible to write condition (30) as follows:

$$\begin{aligned} U_{c,t}\pi_t(\pi_t - 1) \quad &= \quad \beta U_{c,t+1} E_t \left\{ \pi_{t+1}(\pi_{t+1} - 1) \right\} - \\ &- \frac{\vartheta}{\omega_p} \frac{N_t U_{n,t}}{1 - \alpha} - U_{c,t} F_t(.) \frac{\vartheta - 1}{\omega_p} \end{aligned} \tag{31}$$

Next we need to merge all conditions which involve investment decisions and equilibrium in financial markets. Merging equations (14),(15),(**??**), (19) and (3) I get the following relation:

$$\begin{aligned} & Q_t U_{c,t} [rp_t(\frac{Q_t K_{t+1}}{NW_{t+1}})] \\ = \quad & E_t \{ \beta (Z_{t+1} + U_{c,t+1} Q_{t+1} \left( 1 - \delta + \frac{\phi_k}{2} \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 - \phi_k \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} \right) ) \} \end{aligned} \tag{32}$$

Using equations (**??**), (29), (12) and substituting for gross investment I obtain:

$$\begin{aligned} & \left[ 1 - \phi_k \left( \frac{K_{t+1}}{K_t} - 1 \right) \right]^{-1} U_{c,t}(1-\alpha)[rp(\frac{\left[ 1 - \phi_k \left( \frac{K_{t+1}}{K_t} - 1 \right) \right]^{-1} K_{t+1}}{NW_{t+1}})] = E_t \{ \beta ( \alpha \frac{N_{t+1}}{K_{t+1}} U_{n,t+1} \\ & + U_{c,t+1}(1-\alpha) \left[ 1 - \phi_k \left( \frac{K_{t+2}}{K_{t+1}} - 1 \right) \right]^{-1} \left( 1 - \delta + \frac{\phi_k}{2} \left( \frac{K_{t+2}}{K_{t+1}} - 1 \right)^2 - \phi_k \left( \frac{K_{t+2}}{K_{t+1}} - 1 \right) \frac{K_{t+2}}{K_{t+1}} \right) ) \} \end{aligned} \tag{33}$$

Finally we need to include among the constraints faced by monetary authority the feasibility condition which together with equation (9) becomes:

$$A_t F_t(.) = C_t + K_{t+1} - (1-\delta)K_t + \frac{\phi_k}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t + \frac{\omega_p}{2}(\pi_t - 1)^2 + \mu M(\varpi_t)\mathcal{Y}_t^k K_t \quad (34)$$

and the net worth accumulation:

$$NW_{t+1} = \theta\{\mathcal{Y}_t^k K_t - (\frac{U_{c,t-1}}{U_{c,t}}\beta^{-1} + rp_{t-1}(.))(\left[1 - \phi_k\left(\frac{K_{t+1}}{K_t} - 1\right)\right]^{-1} K_t - NW_t)\} \quad (35)$$

where $\mathcal{Y}_t^k \equiv \frac{\alpha}{1-\alpha}\frac{N_t}{K_t}U_{n,t} + \left[1 - \phi_k\left(\frac{K_{t+1}}{K_t} - 1\right)\right]^{-1}\left(1 - \delta + \frac{\phi_k}{2}\left(\frac{K_{t+2}}{K_{t+1}} - 1\right)^2 - \phi_k\left(\frac{K_{t+2}}{K_{t+1}} - 1\right)\frac{K_{t+2}}{K_{t+1}}\right)$.

Notice that in this case we do not need to include the budget constraints of both type of consumers among the constraints imposed to the Ramsey planner. Indeed it is possible to show that the merging the aggregate budget constraint of workers and entrepreneurs leads exactly to the resource constraint described in equation (34). Hence the two are redundant. See Appendix A for the proof.

In what follows I formulate a proposition that establishes a mapping between the minimal form summarized by conditions (31), (33), (34) and (35) expressed above and the set of allocations describing the (imperfectly) competitive equilibrium.

**Proposition 1.** [Part A]*For given initial* $D_0$,$K_0$, $NW_0$, *any equilibrium allocation and prices* $\{C_t, C_t^e, N_t, K_{t+1}, I_t, L_t, D_t, NW_{t+1}, \varpi_t, mc_t, \pi_t, Q_t, W_t, R_t, R_t^n, R_t^k\}_{t=0}^{\infty}$ *satisfying equations (3), (5),(6), (9),(12),(14),(24),(20),(15),(17), (28), (4) and (23) also satisfies equations (31), (33), (34) and (35). [Part B] By reverse, using allocations* $\{C_t, N_t, K_{t+1}, \pi_t, NW_{t+1}\}_{t=0}^{\infty}$*that satisfy equations (31), (33), (34), (35) it is possible to construct all the remaining real allocations, nominal variables and policy instruments.*

*Proof.* *See Appendix B.*

## 3.1 Wedges and Equivalence with Tax Distortions

The economy so far described can be easily replicated by a representative agents economy which features a constant tax on labor and a time-varying tax on capital and government consumption. To prove the equivalence let's assume for simplicity that both adjustment costs on prices and investment are equal to zero, which means $\omega_p = 0$ and $\phi_k = 0$. Let's define a prototype economy where the Ramsey planner faces the following resource constraint:

$$C_t + I_t + G_t = F(K_t, L_t) \quad (36)$$

where $G_t$is government expenditure. Consumers in this economy choose consumption, capital investment and labor supply to solve the following maximization problem:

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t U(C_t, N_t)\right\} \quad (37)$$

subject to the sequence of budget constraints:

$$C_t + (1+\tau_{K,t})Q_t K_{t+1} = (1-\delta)Q_t K_t + Z_t K_t + (1-\tau_{N,t})\frac{W_t}{P_t}N_t \tag{38}$$

First order conditions to this problem read as follows:

$$(1+\tau_{K,t})Q_t U_{c,t} = \gamma E_t \left\{ Z_{t+1}U_{c,t+1} + Q_{t+1}(1-\delta) \right\} \tag{39}$$

$$U_{c,t}\frac{W_t}{P_t}(1-\tau_N) = -U_{n,t} \tag{40}$$

where $(1-\tau_N)$ and $(1+\tau_{K,t})$ represent respectively taxes on labor income and capital investment. Clearly higher is the tax lower is the incentive to invest in capital. The competitive production sector of this economy formulates the following optimal input demands:

$$Z_t = A_t F_{K,t}(.)$$

$$\frac{W_t}{P_t} = A_t F_{N,t}(.)$$

Equilibrium in the labor market implies:

$$U_{c,t} A_t F_{N,t}(.)(1-\tau_N) = -U_{n,t}$$

**Proposition 2.** *An economy where a fraction, $\eta$, of consumers is risk neutral and invest in capital facing an external finance premium on investment and a monopolistic competitive production sector is equivalent to a prototype economy with a representative agent who faces a constant tax on labor income and a time-varying tax on capital investment and where a certain fraction of aggregate demand is allocated to government expenditure as long as:*

$$(1-\tau_N) = \frac{\vartheta - 1}{\vartheta} \tag{41}$$

$$(1+\tau_{K,t}) = \rho(\varpi_{t+1}) \tag{42}$$

$$\eta C_t^e + \mu M(\varpi_t)(1+R_t^k)Q_{t-1}K_t = G_t \tag{43}$$

It is important to notice that the equivalence can be proved since consumers' heterogeneity is not a concern for policy makers. This is the case under the following assumptions. First, entrepreneurs are risk neutral so that their consumption demands only affect mean levels of welfare. Secondly, as it is shown in *Appendix A,* under the aggregation assumptions of the costly state verification contract the budget constraints of the two type of consumers - i.e. workers and entrepreneurs - can be merged together so that under strict equivalence they give rise to the following resource constraint:

$$Y_t - \eta C_t^e - (1-\eta)C_t - I_t - \mu M(\varpi_t)(1+R_t^k)Q_{t-1}K_t - \frac{\omega_p}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2 = 0 \qquad (44)$$

Once again under the assumption of zero adjustment cost on prices and using (43) it is possible to recover the equivalence between (44) and the resource constraint of the prototype economy,(36).

### 3.2 The Optimal Policy Problem Under Commitment

I now turn to the specification of a general set-up for the optimal policy conduct. In this section I take full advantage of the characterization of the equilibrium conditions in terms of a minimal set of relations involving only the choice of allocations for consumption, labor, capital and net worth along with the inflation instrument.

**Definition 2**. *Let $\{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}\}_{t=0}^{\infty}$ represent sequences of Lagrange multipliers on the constraints (31), (33), (34), (35) respectively. Let $NW_0, K_0$, be given. Then for given stochastic processes $\{A_t\}_{t=0}^{\infty}$, plans for the control variables $\{C_t, N_t, K_{t+1}, \pi_t, NW_{t+1}\}_{t=0}^{\infty}$ and for the co-state variables $\{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}\}_{t=0}^{\infty}$ represent a first best constrained allocation if they solve the following maximization problem:*

*Choose* $\Lambda_t^n \equiv \{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}\}_{t=0}^{\infty}$ and $\Xi_t^n \equiv \{C_t, N_t, K_{t+1}, \pi_t, NW_{t+1}\}_{t=0}^{\infty}$ to

$$Min_{\{\Lambda_t^n\}_{t=0}^{\infty}}\ Max_{\{\Xi_t^n\}_{t=0}^{\infty}}\ E_0\{\sum_{t=0}^{\infty} \beta^t E_t\{U(C_t, N_t) \qquad (45)$$

$$+\lambda_{1,t}\left[U_{c,t}\pi_t(\pi_t - 1) - \beta U_{c,t+1}E_t\left\{\pi_{t+1}(\pi_{t+1} - 1)\right\} + \frac{\vartheta}{\omega_p}\frac{N_t U_{n,t}}{1-\alpha} + U_{c,t}A_t F_t(.)\frac{\vartheta - 1}{\omega_p}\right] +$$
$$+\lambda_{2,t}[Q_t U_{c,t}(1-\alpha)rp_t(.) - \beta E_t\{\alpha\frac{N_{t+1}}{K_{t+1}}U_{n,t+1} + X_{t+1}(K_{t+2}, K_{t+1}, C_{t+1})\}] +$$
$$+\lambda_{3,t}\left[A_t F_t(.) - C_t - K_{t+1} + (1-\delta)K_t - \Phi_t - \frac{\omega_p}{2}(\pi_t - 1)^2 - \mu M(\varpi_t)\mathcal{Y}_t^k K_t\right]$$
$$+\lambda_{3,t}\left[NW_{t+1}U_{c,t} - \theta\mathcal{Y}_t^k K_t U_{c,t} - \theta(U_{c,t-1}\beta^{-1} + U_{c,t}rp_{t-1}(.))(Q_t K_t - NW_t)\right]\}\}$$

where:$Q_t = \left[1 - \phi_k\left(\frac{K_{t+1}}{K_t} - 1\right)\right]^{-1}$, $\Phi_t = \frac{\phi_k}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t$; $rp_t(.) = rp_t(\frac{\left[1-\phi_k\left(\frac{K_{t+1}}{K_t}-1\right)\right]^{-1}K_{t+1}}{NW_{t+1}})$.

$$\mathcal{Y}_t^k \equiv \frac{\alpha}{1-\alpha}\frac{N_t}{K_t}U_{n,t} + \left[1 - \phi_k\left(\frac{K_{t+1}}{K_t} - 1\right)\right]^{-1}\left(1 - \delta + \frac{\phi_k}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 - \phi_k\left(\frac{K_{t+1}}{K_t} - 1\right)^2\right)$$

and:

$$X_{t+1}(K_{t+2}, K_{t+1}, C_{t+1}) =$$
$$U_{c,t+1}(1-\alpha)\left[1-\phi_k\left(\frac{K_{t+2}}{K_{t+1}}-1\right)\right]^{-1}\left(1-\delta+\frac{\phi_k}{2}\left(\frac{K_{t+2}}{K_{t+1}}-1\right)^2-\phi_k\left(\frac{K_{t+2}}{K_{t+1}}-1\right)\frac{K_{t+2}}{K_{t+1}}\right))\}$$

#### 3.2.1 Non-recursivity and Initial Conditions

As a result of the constraint (31) and (33) exhibiting future expectations of control variables, the maximization problem as spelled out in (45) is intrinsically non-recursive.[7] As first emphasized in Kydland and Prescott (1980), and then developed by Marcet and Marimon (1999), a formal way to rewrite the same problem in a recursive stationary form is to enlarge the planner's state space with additional (pseudo) co-state variables. Such variables, that I denote $\chi_{1,t}$ and $\chi_{2,t}$ for (31) and (33) respectively, bear the crucial meaning of tracking, along the dynamics, the value to the planner of committing to the pre-announced policy plan. Another aspect concerns the specification of the law of motion of these lagrange multipliers. For in this case both constraints feature a simple one period expectation, the same co-state variables have to obey the laws of motion:[8]:

$$\begin{aligned}\chi_{1,t+1} &= \lambda_{1,t} \\ \chi_{2,t+1} &= \lambda_{2,t}\end{aligned} \tag{46}$$

A particularly important point concerns the definition of the appropriate initial conditions for $\chi_{1,t}$ and $\chi_{2,t}$. Marcet and Marimon (1999) show that for the modified (recursive) Lagrangian in (45) to generate a global optimum under time zero commitment it must hold:

$$\chi_1 = 0 = \chi_2 \tag{47}$$

The above condition states that there is no value to the policy planner, as of time zero, attached to prior commitments. Commitment, in this context, bears exactly the meaning that while it would be technically feasible for the planner to satisfy (47) for all $t > 0$, it would also be suboptimal to do so.

Using the new co-state variable so far described I amplify the state space of the Ramsey allocation to be $\{A_t, \chi_{1,t}, \chi_{2,t}\}_{t=0}^{\infty}$ and I define a new saddle point problem which is recursive in the new state space:

[7] See Kydland and Prescott (1977), Calvo (1978). As such the system does not satisfy per se the principle of optimality, according to which the optimal decision at time t is a time invariant function only of a small set of state variables.

[8] The laws of motion of the additional costate variables would take a more general form if the expectations horizon in the forward looking constraint(s) featured a more complicated structure, as, for instance, in the case of constraints in present value form. See Marcet and Marimon (1999).

**Definition 3**. *Let* $\{\lambda_{1,t},\lambda_{2,t},\lambda_{3,t},\lambda_{4,t}\}_{t=0}^{\infty}$ *represent sequences of Lagrange multipliers on the constraints (31), (33), (34), (35) respectively. Let* $NW_0, K_0$, *be given. Then given the state space* $\{A_t,\chi_{1,t},\chi_{2,t}\}_{t=0}^{\infty}$, *plans for the control variables* $\{C_t, N_t, K_{t+1}, \pi_t, NW_{t+1}\}_{t=0}^{\infty}$ *and for the co-state variables* $\{\lambda_{1,t},\lambda_{2,t},\lambda_{3,t},\lambda_{4,t}\}_{t=0}^{\infty}$ *represent a first best constrained allocation if they solve the following maximization problem:*

*Choose* $\Lambda_t^n \equiv \{\lambda_{1,t},\lambda_{2,t},\lambda_{3,t},\lambda_{4,t}\}_{t=0}^{\infty}$ and $\Xi_t^n \equiv \{C_t, N_t, K_{t+1}, \pi_t, NW_{t+1}\}_{t=0}^{\infty}$ to

$$Min_{\{\Lambda_t^n\}_{t=0}^{\infty}}\ Max_{\{\Xi_t^n\}_{t=0}^{\infty}}\ E_0\{\sum_{t=0}^{\infty}\beta^t E_t\{U(C_t,N_t) \tag{48}$$

$$\begin{aligned}
&+\lambda_{1,t}[U_{c,t}\pi_t(\pi_t-1)+\frac{\vartheta}{\omega_p}\frac{N_tU_{n,t}}{1-\alpha}+U_{c,t}F_t(.)\frac{\vartheta-1}{\omega_p}]-\chi_{1,t}U_{c,t}\pi_t(\pi_t-1)+\\
&+\lambda_{2,t}Q_tU_{c,t}(1-\alpha)rp_t(.)-\chi_{2,t}[\alpha\frac{N_t}{K_t}U_{n,t}+X_t(K_t,N_t,NW_t,C_t)]+\\
&+\lambda_{3,t}\left[A_tF_t(.)-C_t-K_{t+1}+(1-\delta)K_t-\Phi_t-\frac{\omega_p}{2}(\pi_t-1)^2-\mu M(\varpi_t)\mathcal{Y}_t^kK_t\right]\\
&+\lambda_{4,t}\left[NW_{t+1}U_{c,t}-\theta\mathcal{Y}_t^kK_tU_{c,t}+\theta(U_{c,t-1}\beta^{-1}+U_{c,t}rp_{t-1}(.))(Q_tK_t-NW_t)\right]
\end{aligned}$$

First order conditions[9] for time $t\geq 1$ for the choice of $C_t$, $N_t, K_{t+1}, \pi_t$ and $NW_{t+1}$ imply respectively:

- $C_t$ :

$$\begin{aligned}
0 \;=\;& U_{c,t}+U_{cc,t}\ \pi_t(\pi_t-1)\ (\lambda_{1,t}-\chi_{1,t})+\frac{\lambda_{1,t}(\vartheta-1)}{\omega_p}A_tF_t(.)U_{cc,t}\ + \\
&+\lambda_{2,t}\ (1-\alpha)U_{cc,t}\ Q_trp_t(.)-\chi_{2,t}\frac{\partial X_t(.)}{\partial C_t}-\lambda_{3,t}+\\
&+\lambda_{4,t}NW_{t+1}U_{cc,t}-\lambda_{4,t}\theta K_tU_{cc,t}\mathcal{Y}_t^k-\\
&-(\lambda_{4,t+1}-\lambda_{4,t}rp_t(.))\theta U_{cc,t}(Q_tK_t-NW_t)
\end{aligned} \tag{49}$$

- $N_t$ :

$$\begin{aligned}
0 \;=\;& U_{n,t}+\frac{\lambda_{1,t}\vartheta}{\omega_p}(U_{n,t}+N_tU_{nn,t})\frac{1}{1-\alpha}+\frac{\lambda_{1,t}(\vartheta-1)}{\omega_p}A_t\frac{\partial F_t(.)}{\partial N_t}U_{c,t}-\\
&-\chi_{2,t}\alpha(K_t^{-1})(U_{n,t}+N_tU_{nn,t})+\lambda_{3,t}A_t\frac{\partial F_t(.)}{\partial N_t}-\lambda_{4,t}\theta K_t\frac{\partial \mathcal{Y}_t^k}{\partial N_t}U_{c,t}
\end{aligned} \tag{50}$$

[9]When taking first order conditions I neglect the term $\mu M(\varpi_t)\mathcal{Y}_t^kK_t$ appearing in the feasibility constraint. This simplifies derivatives without affecting significantly the quantitative results. Indeed the value for $\mu$ is simply 0.03 of the steady state value for output, hence the all term has a negligible effect on output dynamics.

- $K_{t+1}$ :

$$
\begin{aligned}
0 \quad = \quad & \lambda_{1,t+1}U_{c,t+1}\frac{(\vartheta-1)}{\omega_p}A_t\frac{\partial F_{t+1}(.)}{\partial K_{t+1}}+\lambda_{2,t}(1-\alpha)U_{c,t}\frac{\partial Q_t}{\partial K_{t+1}}rp_t(.) \qquad (51)\\
&+\lambda_{2,t+1}(1-\alpha)U_{c,t+1}[\frac{\partial Q_{t+1}}{\partial K_{t+1}}rp_{t+1}(.)-\frac{\partial rp_{t+1}(.)}{\partial K_{t+1}}Q_{t+1}(.)]\beta-\lambda_{2,t}\beta\alpha N_{t+1}U_{n,t+1}(-K_{t+1}^{-2})-\\
&-\beta\lambda_{2,t}\frac{\partial X_{t+1}}{\partial K_{t+1}}-\chi_{2,t}\frac{\partial X_t}{\partial K_{t+1}}+\lambda_{3,t+1}\beta A_t\frac{\partial F_{t+1}(.)}{\partial K_{t+1}}-\lambda_{3,t}+\lambda_{3,t+1}\beta(1-\delta)-\\
&-\lambda_{3,t}\frac{\partial \Phi_t}{\partial K_{t+1}}-\lambda_{3,t+1}\beta\frac{\partial \Phi_{t+1}}{\partial K_{t+1}}-\lambda_{4,t}\theta K_tU_{c,t}\frac{\partial \mathcal{Y}_t^k}{\partial K_{t+1}}-\lambda_{4,t+1}\theta U_{c,t+1}(\frac{\partial \mathcal{Y}_{t+1}^k}{\partial K_{t+1}}K_{t+1}+\mathcal{Y}_{t+1}^k)-\\
&-\lambda_{4,t}\theta\beta^{-1}U_{c,t}K_t(\frac{\partial Q_t}{\partial K_{t+1}})+\lambda_{4,t+1}\theta U_{c,t+1}[\frac{\partial Q_{t+1}}{\partial K_{t+1}}K_{t+1}+Q_{t+1}]-\\
&-\lambda_{4,t}\theta U_{c,t}[(\frac{\partial rp_t}{\partial K_{t+1}}Q_tK_{t+1}+\frac{\partial Q_t}{\partial K_{t+1}}rp_tK_{t+1}+Q_trp_t]-\lambda_{4,t+1}\theta U_{c,t+1}[(\frac{\partial rp_{t+1}}{\partial K_{t+1}}Q_{t+1}K_{t+2}\\
&+\frac{\partial Q_{t+1}}{\partial K_{t+1}}rp_{t+1}K_{t+2}]+\lambda_{4,t+1}\theta U_{c,t+1}NW_{t+1}\frac{\partial rp_{t+1}}{\partial K_{t+1}}
\end{aligned}
$$

- $\pi_t$ :

$$U_{c,t}(2\pi_t-1)\left(\lambda_{1,t}-\chi_{1,t}\right)-\theta(\pi_t-1)\lambda_{2,t}=0 \qquad (52)$$

- $NW_{t+1}$ :

$$
\begin{aligned}
0 \quad = \quad & \lambda_{3,t+1}U_{c,t+1}(1-\alpha)\frac{\partial rp_t}{\partial NW_{t+1}}+\lambda_{4,t}U_{c,t}-\lambda_{4,t+1}\theta U_{c,t+1} \qquad (53)\\
&-\lambda_{4,t+1}\theta\beta U_{c,t+1}rp_t-\lambda_{4,t+1}\theta\beta U_{c,t+1}[\frac{\partial rp_{t+1}}{\partial NW_{t+1}}NW_{t+1}+rp_{t+1}]
\end{aligned}
$$

The system of efficiency conditions is completed by the law of motion (46), the initial condition (47) and by the constraints *(31), (33), (34) and* (35) holding with equality.

**Definition 4** *(Optimal policy under commitment) The set of processes* $\Lambda_t^n \equiv \{\lambda_{1,t},\lambda_{2,t},\lambda_{3,t},\lambda_{4,t}\}_{t=0}^{\infty}$, $\Xi_t^n \equiv \{C_t,\ N_t, K_{t+1}, \pi_t, NW_{t+1}\}_{t=0}^{\infty}$*fully describes a constraint pareto optimal equilibrium under commitment if they solve the system of equations (49), (50), (51), (52) and (53) together with equations (46), (31), (33), (34) and (35) holding with equality and with initial conditions (47).*

# 4 Non-Optimality of The Zero Inflation Policy along the Dynamic

I now obtain the main result concerning the non-optimality of the zero inflation policy. To accomplish this task I follow two directions. First, I prove sufficient conditions such that deviating from the zero inflation policy allows to improve upon a flexible price allocation. Second using the first order conditions of the optimal policy problem I prove that deviating from zero inflation policies provides a necessary and sufficient condition for optimality.

Clearly one possible choice for the policy maker is to follow a zero inflation policy which replicates the flexible price allocation. This policy which closes the gaps - i.e. defined as the difference between the flexible price allocation and the sticky price one - is often described as neutral policy. Under this regime, real activity fluctuates in a manner which is identical to how it would behave if prices were completely flexible. Hence to characterize the zero inflation policy we must characterize the set of implementable allocation under flexible prices

**Proposition 3.** *Under flexible prices the economy is distorted by both the mark-up and the external finance premium distortion.*

**Proof.** Under flexible price the set of implementable allocation for the monetary authority is characterized by equations *(33), (34) and (35).* Finally equation *(31)* must be substituted with the following:

$$\frac{-U_{n,t}}{U_{c,t}A_t F_{n,t}} = \frac{\vartheta - 1}{\vartheta} \tag{54}$$

Since neither of those equations depends on inflation it is not possible for the monetary authority to undo any of the distortions Q.E. D..

To the extent that under the sticky prices allocation the monetary authority chooses to depart from the neutral outcomes, it is because it is responding to the distortions present in the economy. Notice that it is relatively easy to forecast in which direction there would be departure in our context. Indeed both our distortions, mark-up and external finance premium, imply an inefficiently low level of output due to misallocations of both labor and capital and call for an expansionary policy. Hence a trade-off exists only between the sticky price distortion and either of the two remaining distortions.

The presence of the mark-up on sales acting as a tax on labor would per se call for expansionary policy and deviations toward positive inflation. This has been noted in the literature by several other authors - i.e. see Schmitt-Grohe and Uribe (2003) and Khan, King and Wolman (2002) among others - however no one had found deviations from the zero inflation policy which were quantitatively significant with the sole distortion stemming from monopolistic competition. The presence of an external finance premium which acts as a tax on capital does nothing but simply amplifies this bias toward positive inflation. In summary what we expect is that the sticky price allocation by endowing the monetary authority with a margin of control over demand can improve upon the flexible price allocation.

**Proposition 4.** *The set of implementable allocations under sticky prices contains the corresponding set under flexible prices. Thus, the optimal allocation under sticky prices makes the households at least as well off as under flexible prices.*

**Proof.** Under sticky prices it is always possible to replicate the flexible price allocation with a zero inflation policy. Under zero inflation constraint *(31)* becomes equal to *(54)* hence the two allocations coincide Q.E. D..

The fact that a zero inflation policy is possible but not necessarily optimal can also be simply proved by looking at the first order condition with respect to inflation of the Ramsey plan, i.e. equation *(52).* Indeed the condition is satisfied even when the gross rate of inflation is different than one.

To asses whether deviation from zero inflation policy is actually a necessary and sufficient condition for optimality I now analyze the optimality conditions of the Ramsey planner.

**Proposition 5.** *A zero inflation policy does not satisfies the optimality conditions of the Ramsey planner when all the three distortions operate along the dynamic.*

**Proof.** Optimality requires that equation *(31)* - i.e. the first order condition with respect to the lagrange multiplier, $\lambda_{1,t}$, holds with strict equality. Under the zero inflation policy this implies that the ratio $\frac{-U_{n,t}}{U_{c,t}A_tF_{n,t}}$ must be constant across states and times and must be equal to $\frac{\vartheta-1}{\vartheta}$. We need then to analyze the behavior of the mentioned ratio. It is instructive to consider the general class of separable preferences which are described by monotonic transformation of additively separable and constant elasticity of preferences:

$$u = \frac{C_t^{1-\sigma}}{1-\sigma} - \zeta\frac{N_t^{1-\gamma}}{1-\gamma}$$

For such class indeed we can impose a constant elasticity of labor across states and times. In this case to prove that the ratio $\frac{-U_{n,t}}{U_{c,t}A_tF_{n,t}}$ is constant across states and times amounts at proving that the demand components are also constant across states and times. Under the mentioned assumption on preferences and using the feasibility constraint and the Cobb-Douglas technology the following is true:

$$\frac{-N_t^{\gamma}}{(F_t(K_{t-1},N_t)-I_t)^{-\sigma}A_tF_{n,t}} = \frac{\vartheta-1}{\vartheta} \tag{55}$$

Even when labor is constant because of the constant elasticity assumption, investment would always respond to productivity shock even under a zero inflation policy. Hence $\frac{-N_t^{\gamma}}{(F_t(K_{t-1},N_t)-I_t)^{-\sigma}A_tF_{n,t}}$ can never be constant Q.E.D..

Two considerations are worth notice at this point. First, it must be clear at this point that deviations from price stability occur only to the extent that the monetary authority faces an incentive to increase final good demand in order to increase the demand for labor and capital. Output and input demand are indeed inefficiently low due to the distortionary effect exerted by the mark-up and the external finance premium. In particular it is the variation in investment demand in response to productivity shocks which induces an inflationary motive in view of the optimal management of distortions. This also implies that the sole mark-up distortion coupled with variable endogenous capital accumulation would generate deviation from price stability[10]. However it is also true that higher fluctuations in investment demand call for higher deviations from price stability. Hence the presence of financial frictions by simultaneously reducing the long run level of capital and exacerbating business cycle fluctuations of investment demand amplify deviations from price stability.

Secondly, it must be noticed that in view of the optimal management of all the distortions present in the economy the monetary authority does want to reduce the investment wedge too. However in this context and since inflation does not enter any financial relation, the monetary authority can exert only an indirect impact on the investment distortion through non-neutral effects on final good demand. Nevertheless it is possible to depict situations where the monetary authority by driving future expectation of inflation can have a direct impact on the external finance premium. This is the case for instance under non-indexed loan contracts.

**Non-indexed loan contracts.** So far I have explored the effects of borrowing constraints by assuming that the contract between the lender and the borrower was set in real terms. Alternatively one could view the contract so far designed as involving nominal variables but debt indexation. In this context indeed normalizing variables with future expected inflation simply implies a translation of the contract in real terms. Different is the situation that arises when the services on nominal debt are not indexed on future inflation. In this case a price stability policy which implies long run average deflation might increase the real value of debt thereby inducing disruptive effect on the financial side of the economy. Those ideas which were pioneered in the fisherian theory of debt deflation (1933) remain relevant today as well. If the services of debt are directly affected by future expected inflation and capital investment is inefficiently low due to credit constraints a monetary authority concerned with the financial distortion faces a incentive to set positive inflation rate in order to decrease the real value of debt. This inflationary motive goes beyond the one featured with non-neutral effects on final goods demand.

To explore more formally this idea I start by deriving the external finance premium from a contract expressed in nominal terms and with debt services set in terms of current inflation. In this case it is possible to show that the external finance premium assumes the following form[11]:

$$\frac{(1+R^k_{t+1})}{(1+R_t)} = rp_t(\frac{Q_t K_{t+1}}{NW_{t+1}}\frac{1}{\pi_{t+1}}) \tag{56}$$

Future expected inflation increases the real value of collateral by reducing the real value of

[10] In a companion paper called "Ramsey Monetary Policy Under Nominal Rigidities and Endogenous Capital Accumulation " I explore exactly this point.

[11] This can simply be done by writing the costly state verification contract in nominal terms. Hence by normalizing the contract in terms of future prices and assuming that the debt services are valued in terms of current prices allows to obtain an expression for the leverage ratio which depends on future expected inflation.

debt, $L_t$. This in turn implies a decrease in the external finance premium[12]. In this context the margin between consumption today and tomorrow is clearly directly affected by future expectations of inflation:

$$
\begin{aligned}
& Q_t U_{c,t}[rp_t(\frac{Q_t K_{t+1}}{NW_{t+1}}\frac{1}{\pi_{t+1}})] = \qquad (57)\\
= \quad & E_t\{\beta(Z_{t+1}+U_{c,t+1}Q_{t+1}\left(1-\delta+\frac{\phi_k}{2}\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)^2-\phi_k\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)\frac{I_{t+1}}{K_{t+1}}\right))\}
\end{aligned}
$$

In absence of welfare costs of inflation such as the ones associated with sticky prices, it is optimal to set a level of inflation such that the external finance premium is reduced to zero. In this case indeed the monetary authority would be able to replicate the same margin between consumption today and tomorrow which stems from a neoclassical economy with no distortions[13]. It is worth noticing that in this context an incentive toward positive inflation survives also in absence of other elements which typically induce monetary non-neutrality - i.e. such as sticky prices. I will explore this point further in the next section.

# 5 Long Run Optimal Policy

When looking at the optimal monetary policy design in the long-run a distinction between the *constrained* and the *unconstrained* optimal inflation rate is warranted. The former is the inflation rate that maximizes households' instantaneous utility under the constraint that the steady state conditions are imposed ex-ante. In analogy with the terminology of the neoclassical growth .model, and as in King and Wolman (1997), I define this as the policy maker's *golden rule*. However it is important to recall that in dynamic economies with discounted utility the golden rule does not in general coincide with the unconstrained optimal long-run rate of inflation, which is the one to which the planner would like the economy to converge to if allowed to undertake its optimization unconditionally. This second equilibrium concept is indeed obtained imposing steady state conditions ex-post on the first order conditions of the Ramsey planner.

## 5.1 Golden Rule

There are three distortion and the task of the monetary authority is to trade-off among those three using one single instrument, inflation. Before turning to the characterization of the golden rule policy I will disentangle how the trade-off among the three distortions works.

[12] Notice that equation (56) can also be written in terms of real value of the loans:

$$\frac{(1+R^k_{t+1})}{(1+R_t)} = rp_t((\frac{L_{t+1}}{NW_{t+1}}-1)\frac{1}{\pi_{t+1}})$$

[13] This is clear if one also assumes absence of adjustment costs to capital.

The price stickiness distortion, summarized by the quadratic term in inflation in the resource constraint, is obviously minimized at zero net inflation (i.e., $\pi = 1$). On the other hand, the market power distortion, stemming from the level of activity being inefficiently low, calls for a higher level of output and consumption and hence for a *positive* rate of inflation. King and Wolman (1997), in the context of a closed economy, show that once the tension between these two distortions is taken into account the welfare maximizing steady state inflation rate must necessarily be positive.[14] In our context the presence of restrictions on loanable funds reduce investment demand, hence capital. This additional deadweight loss which further reduces aggregate output amplifies the incentive of the monetary authority toward positive inflation.

To exemplify this idea let's derive the function $\mu(\pi, N, K)$ from the steady-state version of (30) as:

$$\mu(\pi, N, K) = \frac{\vartheta F(K, N)}{\omega_p \pi(\pi - 1)(1 - \beta) + (\vartheta - 1)F(K, N)} \tag{58}$$

It is immediate to see that when the cost of inflation, $\omega_p$, is equal to zero the mark-up becomes equal to the one that would prevail under the flexible price allocation, $\mu = \frac{\vartheta}{(\vartheta - 1)}$. Under costly adjustment an increase in inflation reduces the mark-up distortion. A zero inflation policy under sticky prices would reproduce the flexible price allocation but would not undo the mark-up distortion.

Let's now derive the function $rp(\mu, K, N)$ from the steady state version of *(33):*

$$rp(\mu, K, N) = \beta[\frac{F_k}{\mu} + (1 - \delta)] \tag{59}$$

Two things stand clear. First, when both distortions, mark-up and external finance premium are zero, the productivity of capital is equal to the one which characterize the neoclassical efficient equilibrium. Second, when the external finance premium in the steady state is equal to zero the mark-up is exactly equal to the inverse of the marginal cost to production. Hence we conclude that the external finance premium reduces the marginal productivity of capital as it would be the case under distortionary taxation. Now by merging together (58) and (59) it is possible to find a relation between inflation and the external finance premium which reads as follows:

$$rp(\pi, K, N) = \beta[\frac{\omega_p \pi(\pi - 1)(1 - \beta) + (\vartheta - 1)F(K, N)}{\vartheta F(K, N)} F_k + (1 - \delta)] \tag{60}$$

One should view equation (60) as an iso-efficiency condition. A higher external finance premium calls for higher inflation. The intuition is simple. When the adjustment cost on prices is

[14] This argument is correct, though, to the extent that a money distortion associated to the presence of transaction frictions, which would drive incentives towards the Friedman rule and hence a negative steady state inflation rate, is ignored.

equal to zero - i.e. under flexible prices - both employment and investment are inefficiently low due to the joint presence of the mark-up and the external finance premium which increase the marginal cost. However when prices are sticky the monetary authority can increase demand and reduce the marginal cost to production thereby increasing output and welfare. It is important to mention once again that by increasing inflation the policy maker does not have a *direct* impact on the external finance premium. As mentioned before this would be the case only under non-indexed loan contracts. However the monetary authority is able to increase investment demand and output above the inefficiently low level generated by the presence of an external finance premium due exactly to the presence of a countercyclical mark-up.

Let's now formalize the optimal golden rule policy:

$$\{\pi, C, N, K\}^{gr} \equiv \arg\max\{U(C,N)\} \tag{61}$$

subject to a (steady state) pricing-implementability condition

$$\pi(\pi-1)(1-\beta) \leq \frac{\vartheta K^{\alpha} N^{1-\alpha}}{\omega_p}\left(\frac{-U_n(N)}{U_c(C)K^{\alpha}N^{-\alpha}(1-\alpha)} - \frac{\vartheta-1}{\vartheta}\right) \tag{62}$$

to a (steady state) financial-implementability condition:

$$rp \leq \beta\frac{\alpha}{1-\alpha}\frac{U_n}{U_c}\frac{N}{K} - \beta\frac{\alpha}{1-\alpha}(1-\delta) \tag{63}$$

and to a (steady state) feasibility constraint

$$K^{\alpha}N^{1-\alpha} \leq C + K\delta + \frac{\omega_p}{2}(\pi-1)^2 \tag{64}$$

First order conditions of this problem are reported in *Appendix D.*

**Result 1** *(Golden rule inflationary bias)***.** *In an economy with price adjustment costs, monopolistic competition and an investment wedge in the form of an external finance premium, the inflation rate that maximizes steady-state utility increases monotonically with the increase in the investment wedge.*

*Figure 1* plots inflation, employment, investment and consumption as a functions of the steady state value of the external finance premium. Inflation increases monotonically which is consistent with the expansionary motive explored so far. Output, employment and capital in a model with labor and investment wedges are inefficiently low hence the monetary authority is tempted to loosen inflation. Nevertheless capital, employment and consumption decrease monotonically as the investment wedge increases. This is because the monetary authority which sets inflation does not have a direct leverage on the external finance premium hence it cannot offset completely the distortion.

**Result 2.** (*Impossibility of offsetting financial distortion under superneutrality of money). When the discount factor $\beta$ is equal to one the monetary authority does not have any leverage on the economic activity hence capital and output must remain inefficiently low.*

It is clear from equation (60) that when $\beta = 1$ capital and output must remain inefficiently low since there is nothing the monetary authority can do to increase the productivity of labor and capital.

**Result 3.** *(Non indexed loan contracts) Under non indexed contract the long run rate of inflation is higher than the one obtained under indexed contracts. Furthermore it is monotonically increasing with respect to the wedge on investment.*

*Figure 2* plots inflation, employment, investment and consumption as functions of the steady state value of the external finance premium in the case in which the cost of the loan depends negatively on inflation. For all values of the external finance premium inflation lies above the one obtained in the case of indexed contracts. Furthermore once again inflation increases monotonically with respect to the external finance premium.

As discussed in the previous section in presence of non-indexed contracts the cost of debt depends negatively on inflation. Hence the monetary authority features an incentive to inflate the economy which goes beyond the one featured by an economy with the sole monetary non-neutrality on final good demand.

# 6 Dynamic Properties of the Optimal Plan

I here evaluate the dynamic properties of the optimal plan based on impulse response functions, optimal volatilities and welfare costs. A quantitative assessment is indeed necessary in order to evaluate a series of things among which the importance of deviations from price stability.

## 6.1 Computation and Calibration

To characterize the optimal dynamic path of variable I resort on perturbation methods which compute second order approximation of the policy and the transition function around a non-stochastic steady state - i.e. see *Appendix E* for a discussion of the method. The non-stochastic steady state around which I approximate the dynamic economy is represented by the unconstrained pareto optimal allocation - i.e. see a discussion in the next paragraph. Perturbation methods are particularly suitable in this context since they allow to account for the effect of the volatility of variables on mean levels. This might be crucial when computing optimal policy plans for economies which shows a high degree of non-linearities as it is in this case. The following calibration will be used for simulating both the optimal policy in the long run and along the dynamics.

**Preferences**. I set the discount factor $\beta = 0.99$, so that the annual interest rate is equal to 4 percent. Utility is chosen separable in consumption and labor:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma}$$

with parameters $\sigma = 2$ and $\gamma = 3$. Sensitivity analysis is done to assess the robustness of the results.

**Technology.** I set the share of capital in the production functions equal to $\alpha = 0.3$, the quarterly depreciation rate $\delta = 0.025$, the steady state mark-up value to $\frac{\vartheta}{\vartheta-1}1.1$ which corresponds to a value for the elasticity of demand, $\vartheta = 6$. The adjustment cost parameter, $\omega_p$, is varied between 17.5 and 50[15]. The elasticity of the price of capital with respect to investment output ratio $\phi_k = 0.5$.

**Financial frictions parameters:** The financial frictions parameters are obtained by solving the steady state version of the competitive economy under the optimal contracting problem. Some primitive parameters are set so as to match values for industrialized countries. I assume a uniform distribution for the idiosyncratic shock. I set the monitoring cost for the bank, $\mu$, equal to 3% of output and the survival rate of firms, $\theta = 0.973$. Of the two primitive parameters the first affects the contracting problem directly whereas the second affects net wealth directly and the contracting problem indirectly. From the solution to the steady state of the competitive economy which contains the optimality conditions for the contract I get the relation between the external finance premium and the collateral, $\rho(\bullet)$, the steady state leverage ratio, $\frac{QK}{NW}$, the steady state external finance premium, $\rho$, the optimal cut-off value, $\varpi$, and the functions $\Gamma, \Gamma', M$ and $M'$.

**Shocks.** I simulate the model under productivity shocks and demand shocks which follow $AR(1)$ processes. Persistence and volatility are calibrated on data for industrialized countries[16].

### 6.2 Steady State Ramsey Policy

A deterministic Ramsey steady state is a set of allocations $\{C, N, K_t, \pi, NW_t, \lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ that solves the steady-state version of the efficiency conditions associated to the program under (48). In *Appendix F* I characterize such system of equations. As expected the unconstrained long run optimal inflation policy is associated with price stability. this can be easily seen from the first order condition with respect to inflation (in Appendix F) which is verified only for $\pi = 1$. The reason for this being that any optimal policy problem is asymptotically characterized by monetary neutrality.

*Table 1* compares the value of selected variables under the Ramsey long run unconstrained policy and the solution to the distorted competitive economy. Clearly under the optimal policy the economy features a higher level of consumption, investment, output and welfare and a lower level of employment.

[15] See Schmitt-Grohe and Uribe (2002) and Ireland (1998).

[16] The persistence of the productivity shock is set to 0.95 while the one for government expenditure shock is set to 0.9. The volatility of the productivity shock is set to $(0.0056^2)$. Finally government expenditure in steady state is set equal to 0.25 of steady state output.

The value of the deterministic steady state are then used to compute second order approximations.

### 6.3 Dynamic Responses to Shocks

*Figure 3* shows impulse responses[17] of optimal policy to a one percent standard deviation shock in productivity. Output, capital and net worth increase do to the improved investment opportunities. Inflation shows a significant deviation from price stability. *Figure 6* also shows that the degree of inflation volatility is increasing with respect to the external finance premium. An increase in the external finance premium has indeed a twofold effect. On the one side it decreases the long run level of capital investment, while on the other it exacerbates investment fluctuations. Both those elements induce the monetary authority to loosen nominal interest rates. The presence of an inefficient low level of capital calls for an expansionary policy which increases final good demands, thereby increasing capital and labor input demands. Secondly as seen from equation (55):

$$\frac{-N_t^{\gamma}}{(F_t(K_{t-1}, N_t) - I_t)^{-\sigma} A_t F_{n,t}} = \frac{\vartheta - 1}{\vartheta} \tag{65}$$

changes in investment demand call for deviation from constant mark-up. This implies that large fluctuations of investment demand call for larger deviations form price stability.

It is interesting to notice also that the response of inflation features overshooting before converging toward the steady state. This is due to the fact that under commitment the monetary authority can affect future expectations of inflation in a way that renders convergence toward the steady state faster. The desire to affect future expectations of inflation is typical of situations in which the monetary authority faces binding trade-offs. Such overshoot of inflation can indeed be found also in the analysis of Woodford (2002) however in that context they are associated only with cost-push shocks. Indeed in absence of other distortions cost push shocks are the only element that can create trade-offs between output and inflation stabilization. On the contrary in the present context overshoots are also associated with productivity shocks since the existence of distortions create trade-offs along the dynamic.

*Figure 4* shows on the other side impulse responses to a one percent standard deviation shock in productivity under price stability[18]. In this case output, capital and net worth are lower since

[17] To calculate impulse response functions for the model I simplified calculations for the first order conditions by avoiding the term $\frac{\phi_k}{2}\left(\frac{I_{t+1}}{K_{t+1}} - \delta\right)^2 - \phi_k\left(\frac{I_{t+1}}{K_{t+1}} - \delta\right)\frac{I_{t+1}}{K_{t+1}}$ which appears in the return of capital. Such an omission however should not affect the results since under linear approximation that specific term has a small quantitative effect. Moreover if any by adding that term deviations from price stability should be amplified since investment demand fluctuations are amplified as well.

[18] This case is obtained simply by inserting equation $\pi_t = 1$ in place of the first order condition with respect to inflation.

in this case the economy benefits less of the technological improvement. As expected variations in inflation are negligible and close to zero[19].

Finally *figure 5* shows impulse responses of optimal policy to a one percent standard deviation shock in demand in the form government expenditure. Even in this case there are significant variations from price stability however they are small than under productivity shocks. Under zero inflation and in presence of government expenditure the implementability condition *(31)* becomes:

$$\frac{-N_t^{\gamma}}{(1-\frac{G_t}{Y_t}-\frac{I_t}{Y_t})^{-\sigma}A_tF_{n,t}}=\frac{\vartheta-1}{\vartheta} \tag{66}$$

which cannot be verified whenever the share of government expenditure over output is not constant. This implies that even under government expenditure shocks a constant mark-up and zero inflation cannot be part of the optimal policy. The reason for which in this second case variations from price stability are lower than the one featured under productivity shock is related to the fact that government expenditure shock do not impact directly investment demand. Hence additional variations in investment demand are here much lower than the ones which occur under technology shocks

# 7 Conclusions

This paper analyzes the design of optimal monetary policy in presence of financial frictions. The economy considered features three main distortions: monopolistic competition, sticky prices and borrowing constraints on investment which stems from the presence of an external finance premium on loanable funds. Two main conclusions stem from the analysis. Optimal policy requires deviations from price stability. The mark-up on sales and the external finance premium act respectively as taxes on labor and capital thereby inducing an inefficiently low level of investment and output. Under flexible prices and assuming indexed loan contracts the two distortions cannot be offset since neither of the two depend upon inflation, which is the target of the monetary authority. Under sticky prices the monetary non-neutrality allows the authority to improve upon the flexible price allocations by setting expansionary policies which imply deviations from price stability both under the long run constrained optimal allocation and along the dynamic. The paper also shows that a more pronounced inflationary bias occurs under nominal non-indexed loan contracts.

Even tough the results of this paper depend upon the presence of monetary non-neutrality in the form of sticky prices, they can survive under alternative monetary transmission mechanisms. For instance Carlstrom and Fuerst (2000) also show by means of a simple model with borrowing

[19]Notice that even in this case inflation variations cannot be exactly equal to zero. This is because as shown in proposition 5 a price stability rule which implies a constant mark-up violates one of the implementability conditions, namely the one on the optimal pricing.

constraints a' la Kiyotaki and Moore (1998) and cash in advance constraints on liquidity that optimal monetary policy should be pro-cyclical. They argue that if positive productivity shocks occur the monetary authority should loosen nominal interest rate - i.e. hence inflation - to allow the entrepreneurs to benefit of the technological improvement which would not occur otherwise because of the borrowing constraints on loanable funds.

A number of fruitful extensions can be done to the analysis. First, one must explore the effects of alternative specifications for the utility functions. Indeed as shown in Adao, Correia and Teles (2000) non-separable utility generate per se deviations from price stability of the optimal policy even in model economies without capital. Hence allowing for such specifications in model economies with capital accumulation might amplify the inflationary motive. Secondly, it might be useful to explore the optimal policy response to other type of shocks which are also relevant for the financial sector, such as net worth shocks. Third, it might be interesting to reproduce this analysis in the context of discretionary monetary policies.

# 8 Appendix

## 8.1 Appendix A. Relation Between the Budget Constraints and the Resource Constraint

In what follows I prove that by merging the aggregate budget constraint for the workers and the entrepreneurs result in the resource constraint. This also implies that if the resource constraint is included in the set of conditions which summarize the competitive equilibrium the two aggregate budget constraints are redundant.

Let's start by writing the aggregate budget constraint for the workers in real terms. In absence of government expenditure aggregate transfers from the government are zero. Hence rearranging we obtain:

$$\frac{D_{t+1}}{P_t} - (1+R_t)\frac{D_t}{P_t} \leq \frac{W_t}{P_t}N_t + \frac{\Theta_t}{P_t} - C_t \tag{67}$$

Substituting the market clearing condition in the loan market, (28), we can rewrite the budget constraint for the entrepreneur as follows:

$$Z_t K_t + \frac{D_{t+1}}{P_t} = C_t^e + I_t + (1+R_t^L)\frac{D_t}{P_t} \tag{68}$$

Using the first order conditions of the maximization problem of the entrepreneur and after imposing arbitrage we get:

$$(1+R_t^L) = (1+R_{t+1}^k) \tag{69}$$

In addition from the optimality condition of the contract we know that:

$$(1+R_{t+1}^k) = (1+R_t)rp_t(.) \tag{70}$$

and that:

$$rp_t(.) = (1 + \frac{\mu M(\varpi_t)(1+R_t^k)Q_{t-1}K_t}{Q_{t-1}K_t - NW_t}) \tag{71}$$

Substituting in the budget constraint of the entrepreneur and rearranging I get:

$$\frac{D_{t+1}}{P_t} - \frac{D_t}{P_t} = C_t^e + I_t - Z_tK_t + \frac{\mu M(\varpi_t)(1+R_t^k)Q_{t-1}K_t}{Q_{t-1}K_t - NW_t}\frac{D_t}{P_t} \tag{72}$$

which after substituting again for (28) in the right end side becomes:

$$\frac{D_{t+1}}{P_t} - \frac{D_t}{P_t} = C_t^e + I_t - Z_tK_t + \mu M(\varpi_t)(1+R_t^k)Q_{t-1}K_t \tag{73}$$

The left end side of equation (73) can be equated to the left end side of equation (67) to give:

$$C_t^e + I_t - Z_tK_t + \mu M(\varpi_t)(1+R_t^k)Q_{t-1}K_t = \frac{W_t}{P_t}N_t + \frac{\Theta_t}{P_t} - C_t \tag{74}$$

Rearranging and substituting the aggregate expression for the monopolistic profits, $\Theta_t$, and rearranging we get:

$$A_tK_t^{\alpha}N_t^{1-\alpha} - C_t^e - C_t - I_t - \mu M(\varpi_t)(1+R_t^k)Q_{t-1}K_t - \frac{\omega_p}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2 = 0 \tag{75}$$

Finally substituting for investment and production we get the following resource constraint:

$$Y_t - C_t^e - C_t - I_t - \mu M(\varpi_t)(1+R_t^k)Q_{t-1}K_t - \frac{\omega_p}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2 = 0 \tag{76}$$

## 8.2 Appendix B. Proof of Proposition 1

The proof of Part A just follows from the substitutions done in section 3. As for Part B we need to show that starting from the optimal allocation for $\{C_t, N_t, K_{t+1}, \pi_t\}_{t=0}^{\infty}$ it is possible to construct all the remaining real allocations, nominal variables and policy instruments using competitive equilibrium conditions.

Using the optimal value for $C_t$ we get $U_{c,t}$and using the values for the marginal utility we recover the real interest rate:

$$U_{c,t} = \beta(1+R_t)E_t\{U_{c,t+1}\} \tag{77}$$

Using the value for the real interest rate and the optimal value for inflation we get the nominal interest rate as:

$$(1+R_t) = (1+R_t^n)E_t\left\{\frac{P_t}{P_{t+1}}\right\} \tag{78}$$

Using the optimal value for employment we get $U_{n,t}$ which together with $U_{c,t}$ gives the real wage using *(4)*. From the real wage it is possible to recover the value for the marginal cost using *(23)*.

From the value of $K_{t+1}$ we get the optimal value of $I_t$ using the law of motion for capital, *(9)*.

Next, substituting the equation the optimal value for investment in equation *(12)* we get $Q_t$, and using the value for the asset price and equation *(14)* we get $R_t^k$.

Finally using $R_t^k, R_t$ it is possible to obtain the external finance premium which after inverting gives the leverage ratio $\frac{Q_t K_{t+1}}{NW_{t+1}}$. The leverage ratio together with some of the previous variables can then be used to recover $C_t^e, L_t, D_t, NW_{t+1}, \varpi_t$.

### 8.3 Appendix C. The Stationary Policy problem

Here I derive the stationary form of the policy problem. Let's consider the optimal plan as formulated in equation (45) in the text. By applying the law of iterated expectations and by grouping expectations and multipliers that share the same date one obtains:

$$Min_{\{\Lambda_t\}_{t=0}^{\infty}}\ Max_{\{\Xi_t\}_{t=0}^{\infty}} E_0\{\mathcal{U}(C_0, N_0, \pi_0, \Omega)$$

$$\begin{aligned}
&+\lambda_{1,0}\left[U_{c,0}\pi_0(\pi_0-1)+\frac{\vartheta}{\omega_p}\frac{N_0 U_{n,0}}{1-\alpha}+U_{c,0}F_0(.)\frac{\vartheta-1}{\omega_p}\right]\\
&+\lambda_{2,0}\left[1-\phi_k\left(\frac{K_1}{K_0}-\delta\right)\right]^{-1}U_{c,0}(1-\alpha)rp_0(.)\\
&+\lambda_{3,0}\left[A_0F_0(.)-C_0-K_1+(1-\delta)K_0-\frac{\phi_k}{2}\left(\frac{K_1}{K_0}-1\right)^2K_0-\frac{\omega_p}{2}\left(\pi_0-1\right)^2-\mu M(\varpi_0)\mathcal{Y}_0^k K_0\right]\\
&+\lambda_{4,t}\left[NW_1U_{c,0}-\theta\mathcal{Y}_0^kK_0U_{c,0}-\theta(U_{c,-1}\beta^{-1}+U_{c,0}rp_{-1}(.))(Q_0K_0-NW_0)\right]\\
&+\beta\{U(C_1,N_1)+(\lambda_{1,1}-\beta\lambda_{1,0})(U_{c,1}\pi_1(\pi_1-1))+\lambda_{1,1}\left(\frac{\vartheta}{\omega_p}\frac{N_1U_{n,1}}{1-\alpha}+U_{c,1}F_1(.)\frac{\vartheta-1}{\omega_p}\right)\\
&+\lambda_{2,1}\left[1-\phi_k\left(\frac{K_2}{K_1}-\delta\right)\right]^{-1}U_{c,1}(1-\alpha)rp_1(.)-\beta[\alpha\frac{N_0}{K_0}U_{n,0}+\lambda_{2,0}X(K_2,N_1,NW_1,U_{c,1})]+\\
&+\lambda_{3,1}\left[A_1N_1-C_1-K_2+(1-\delta)K_1-\frac{\phi_k}{2}\left(\frac{K_2}{K_1}-1\right)^2K_1-\frac{\omega_p}{2}\left(\pi_1-1\right)^2-\mu M(\varpi_1)\mathcal{Y}_1^kK_1\right]\\
&+\lambda_{4,1}\left[NW_2U_{c,1}-\theta\mathcal{Y}_1^kK_1U_{c,1}-\theta(U_{c,0}\beta^{-1}+U_{c,1}rp_0(.))(Q_1K_1-NW_1)\right]...\}\}
\end{aligned}$$

Notice that this problem is not time-invariant due to the fact that the constraints at time zero lack the terms $-\beta\lambda_{1,-1}(U_{c,0}\pi_0(\pi_0-1))$ and -$\beta[\alpha\frac{N_{-1}}{K_{-1}}U_{n,-1}+\lambda_{2,-1}X(K_1,N_0,NW_0,U_{c,0})]$. For this reason I amplify the state space to introduce a new (pseudo) co-state variables $\chi_{1,t}$ and $\chi_{2,t}$ to define a new policy functional $\mathcal{W}(C_t, N_t\Omega) \equiv U(C_t,N_t,\Omega) - \chi_{1,t}(U_{c,t}\pi_t(\pi_t-1)) - \beta\chi_{2,t}X(K_{t+1},N_t,NW_t,U_{c,t})$. I can then rewrite the optimal policy plan as in (48).

## 8.4 Appendix D. First Order Conditions of Golden Rule

Let $\lambda_1$,$\lambda_2$ and $\lambda_3$ be the Lagrange multipliers associated to the steady-state constraints (62), (63) and (64) respectively. Hence one can set up the Lagrangian:

$$\begin{aligned}\mathcal{L} &= U(C,N)+\lambda_1\left\{U_c\pi(\pi-1)(1-\beta)+\frac{U_c\vartheta}{\omega_p}\left(\frac{U_nN}{U_c(1-\alpha)}+K^\alpha N^{1-\alpha}\frac{\vartheta-1}{\vartheta}\right)\right\} \\ &+\lambda_2\left\{rp(1-\alpha)U_c-\beta\alpha U_nN(K^{-1})-\beta(1-\alpha)U_c\right\} \\ &+\lambda_3\left\{K^\alpha N^{1-\alpha}-C-K\delta-\frac{\omega_p}{2}\left(\pi-1\right)^2\right\}\end{aligned}$$

First order necessary conditions for this problem read as follows

- $(C)$

$$\begin{aligned}0 &= U_c+\lambda_1U_{cc}\pi(\pi-1)(1-\beta)+\lambda_1U_{cc}\frac{\vartheta-1}{\omega_p}K^\alpha N^{1-\alpha}+\lambda_2(1-\alpha)U_{cc}- \\ &-\lambda_2\alpha(1-\delta)U_{cc}-\lambda_3\end{aligned} \tag{79}$$

- $(N)$

$$\begin{aligned}0 &= U_n-\lambda_1\frac{\vartheta}{\omega_p}\frac{1}{1-\alpha}(U_n+NU_{nn})-\lambda_2\beta\alpha(K^{-1})(U_n+NU_{nn})+ \\ &+\lambda_3K^\alpha(1-\alpha)N^{-\alpha}(1-\mu)+\lambda_1\frac{\vartheta-1}{\omega_p}K^\alpha(1-\alpha)N^{-\alpha}U_c\end{aligned} \tag{80}$$

- $(\pi)$

$$U_c\lambda_1(1-\beta)(2\pi-1)-\lambda_3\omega_p(\pi-1)=0 \tag{81}$$

- $(K)$ :

$$-\lambda_2\beta\alpha U_n(-K^{-2})N+\lambda_3K^{\alpha-1}\alpha N^{1-\alpha}-\lambda_3\delta+\lambda_1U_c\frac{\vartheta-1}{\omega_p}\alpha N^{1-\alpha}=0 \tag{82}$$

In order to define the set of conditions that maximize steady state utility one should add the constraints (62), (63) and (64) holding with equality. A similar set of conditions define the optimal steady state policy of Foreign.

## 8.5 AppendixF. Ramsey First Order Conditions in Steady State

- $C$ :

$$\begin{aligned}0 &= U_c+\frac{\lambda_1(\vartheta-1)}{\omega_p}F(.)U_{cc}\ +\lambda_2\ (1-\alpha)U_{cc}rp-\lambda_2U_{cc}(1-\alpha)(1-\delta) \\ &-\lambda_3+\lambda_4NWU_{cc}-\lambda_4\theta KU_{cc}\mathcal{Y}^k-(\lambda_4(1-rp)\theta U_{cc}(K-NW)\end{aligned} \tag{83}$$

- $N$ :

$$0 = U_n + \frac{\lambda_{1,t}\vartheta}{\omega_p}(U_n + NU_{nn})\frac{1}{1-\alpha} + \frac{\lambda_1(\vartheta-1)}{\omega_p}\frac{\partial F(.)}{\partial N}U_c - \tag{84}$$
$$-\lambda_2\alpha(K^{-1})(U_n + NU_{nn}) + \lambda_3\frac{\partial F(.)}{\partial N} - \lambda_4\theta K_t\frac{\partial \mathcal{Y}^k}{\partial N}U_c$$

- $K$ :

$$0 = \lambda_1 U_c\frac{(\vartheta-1)}{\omega_p}\frac{\partial F(.)}{\partial K} - \lambda_2(1-\alpha)U_{c,t}\frac{\phi}{K}rp + \lambda_{2,}(1-\alpha)U_c[-\frac{\phi}{K}rp]\beta - \lambda_2\beta\alpha NU_n(-K^{-2}) \tag{85}$$
$$+\lambda_3\beta\frac{\partial F(.)}{\partial K} - \lambda_3 + \lambda_3\beta(1-\delta) - \lambda_4\theta KU_c\frac{\partial \mathcal{Y}^k}{\partial K} - \lambda_4\theta U_c(\frac{\partial \mathcal{Y}^k}{\partial K}K + \mathcal{Y}^k) -$$
$$-\lambda_4\theta\beta^{-1}U_{c,t}K_t\frac{\phi}{K} + \lambda_4\theta U_c[\frac{\phi}{K}K + 1] - \lambda_4\theta U_c[\frac{\phi}{K}rpK + rp] - \lambda_4\theta U_c[\frac{\phi}{K}rpK]$$

- $\pi$ :

$$-\theta(\pi-1)\lambda_2 = 0 \tag{86}$$

- $NW$ :

$$NW_{t+1} - 0.48 * K_{t+1} = 0 \tag{87}$$

# References


[1] Adao, B., Correia, I., P. Teles (2000), "Gaps and Triangles", *CEPR w.p. No. 2668.*

[2] Akerlof, G. (1970). "The Market for Lemons: Quality Uncertainty and The Market Mechanism", *Quarterly Journal of Economics,* 85, August, 488-500.

[3] Benigno, P. and G. Benigno (2000), "Price Stability as a Nash Equilibrium in Monetary Open-Economy Models", mimeo, Bank of England and New York University.

[4] Bernanke, B., M. Gertler and S. Gilchrist (1999). "The Financial Accelerator in a Quantitative Business Cycle Framework", in J.B. Taylor, and M. Woodford, eds., *Handbook of Macroeconomics*, Amsterdam: North-Holland.

[5] Carlstrom, C. and T. Fuerst (1997). "Agency Costs, Net Worth and business Fluctuations: A Computable General Equilibrium Analysis", *American Economic Review,* 87, 893-910.

[6] Chamley, C. (1986), "Optimal Taxation of Capital Income In General Equilibrium With Infinite Lives", *Econometrica*, 54:607 622.

[7] Chari, V.V., L. J. Christiano and P.J. Kehoe (1991), "Optimal Fiscal and Monetary Policy: Some Recent Results", *Journal of Money, Credit And Banking,* 23:519 539.

[8] Chari V. V. , L. J. Christiano and P.J. Kehoe (1994), "Optimal Fiscal Policy in A Business Cycle Model", *Journal of Political Economy*, 102:617 652.

[9] Chari, V.V. and P.J. Kehoe (1998), "Optimal Fiscal and Monetary Policy", *Federal Reserve Bank of Minneapolis Research Department Staff Report* 251.

[10] Cole, H. L. and N. Kocherlacota (1998), "Zero Nominal Interest Rate: Why They're Good and How to Get Them", *Federal Reserve Bank of Minneapolis Quarterly Review*, 22(2), 2-10.

[11] Cooley T. F. and G.D. Hansen (1989), "The Inflation Tax in A Real Business Cycle Model", *American Economic Review,* 79:733 748.

[12] Cooley, T. and V. Quadrini (1999), "Monetary Policy and the Financial Decision of Firms", manuscript, New York University.

[13] Fisher, I. (1896), "Appreciation and Interest", in *Publications of the American Economic Association*, volume XI. New York: Macmillan.

[14] Fisher, I. (1930), "The Theory of Interest", New York, Macmillian.

[15] Friedman, M. (1959), "The Optimum Quantity of Money", in *The Optimum Quantity of Money and Other Essays*, Aldine Publishing Company, Chicago.

[16] Gale, D. and M. Hellwig (1985),"Incentive-Compatible Debt Contracts: The One-Period Problem", *Review of Economic Studies,* 52, 647-663.

[17] Goodfriend, M. and R. King (2000), "The Case for Price Stability", CEPR w.p..

[18] Ireland, P. N. (1996), "The Role of Countercyclical. Monetary Policy", *Journal of Political Economy,* 104, 704-723.

[19] Kehoe, P. J. and F. Perri (2000), "International Business Cycles With Endogenous Incomplete Markets", NBER w.p. 7870.

[20] Khan, A., R. King and A.L. Wolman (2000), "Optimal Monetary Policy", *Federal Reserve of Richmond* w.p. No. 00-10.

[21] King, R. and A. L. Wolman (1999), "What Should the Monetary Authority Do When Prices Are Sticky", in Taylor, J. B., ed., Monetary Policy Rules, Chicago: university of Chicago Press, 349-398.

[22] Kiyotaki, N. and J. Moore (1997), "Credit Cycles", *Journal of Political Economy*, 105, April , 211-48.

[23] Kydland, F. and E. C. Prescott (1980), "Dynamic Optimal Taxation, Rational Expectations and Optimal Control", *Journal of Economic Dynamics and Control,* 2:79-91.

[24] Lucas, R. E. (2000), "Inflation and Welfare", *Econometrica,* 68(2), 247-274.

[25] Lucas, R. E. and N. Stokey (1983), "Optimal Fiscal and Monetary Policy in an Economy Without Capital", *Journal of Monetary Economics*, 12:55-93.

[26] Marcet, A. and R. Marimon (1999), "Recursive Contracts", mimeo, Universitat Pompeu Fabra and European University Institute.

[27] Klein, P. and J.V. Rios-Rull (1999), "Time-Consistent Optimal Fiscal Policy", mimeo, Institute for International Economic Studies and University of Pennsylvania.

[28] Ramsey, F. P. (1927), "A contribution to the Theory of Taxation", *Economic Journal,* 37:47-61.

[29] Rotemberg, J. and M. Woodford (1997), "An Optimizing Based Econometric Framework for the Evaluation of Monetary Policy", in B. Bernanke and J. Rotemberg, eds., *NBER Macroeconomics Annual*, Cambridge, MA, MIT Press.

[30] Schmitt-Grohe, S. and M. Uribe (2001), "Optimal Fiscal and Monetary Policy Under Sticky Prices", *Journal of Economic Theory.*

[31] Schmitt-Grohe, S. and M. Uribe (2001), "Optimal Fiscal and Monetary Policy Under Imperfect Competition", mimeo, Rutgers University and University of Pennsylvania.

[32] Woodford, M. (1999), "Inflation Stabilization and Welfare", manuscript, Princeton University.

[33] Yun, T. (1996), "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycle", *Journal of Monetary Economics,* 37: 345-370.

Table 1: **Steady state values of Ramsey allocation versus competitive economy**

| | $C$ | $N$ | $I$ | $K$ | $NW$ | $Y$ |
|---|---|---|---|---|---|---|
| Ramsey Allocation | 0.60 | 0.36 | 0.09 | 1.80 | 0.86 | 0.69 |
| Competitive Economy | 0.36 | 0.23 | 0.05 | 1.10 | 0.42 | 0.52 |

Table 2:

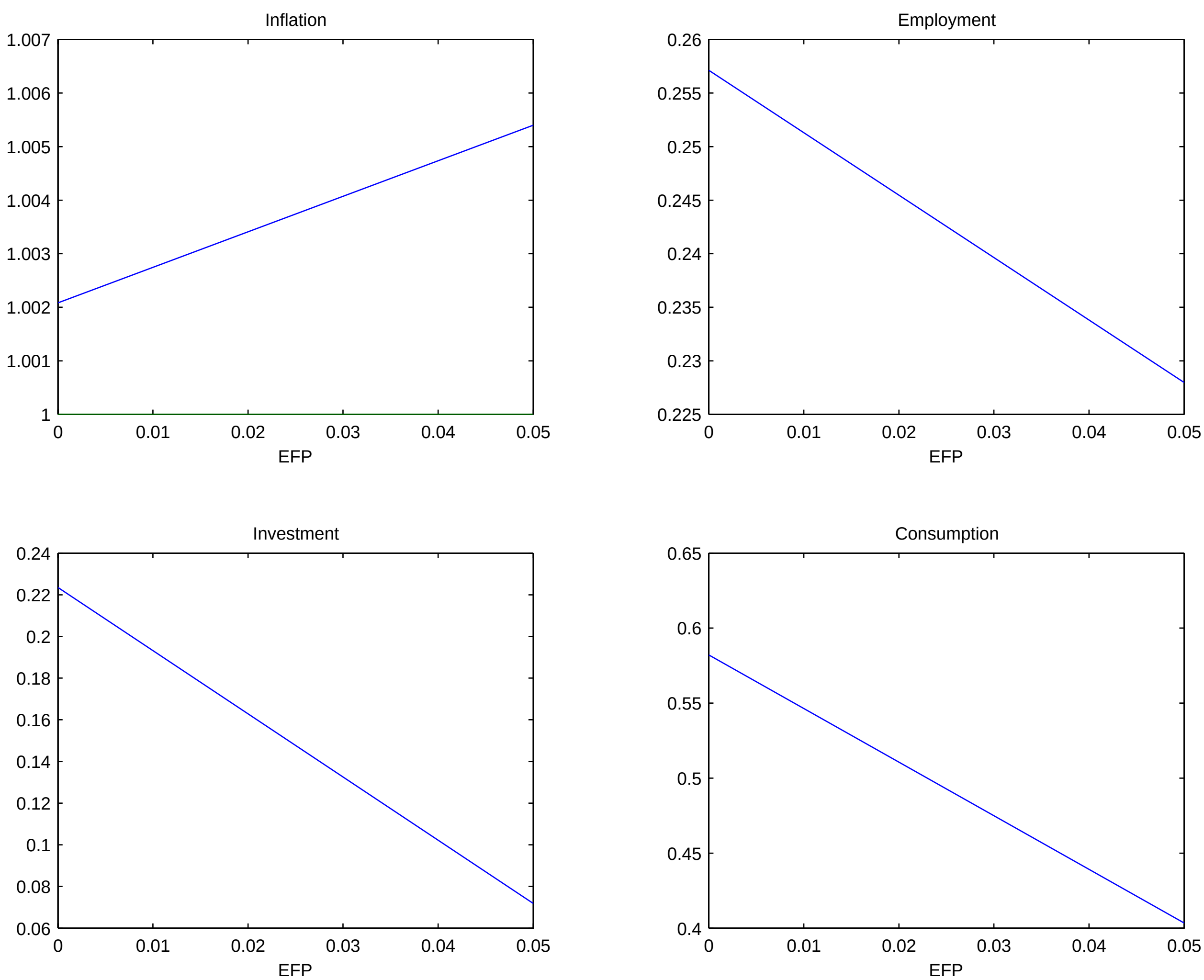



Figure 1: **Golden rule level of inflation, employment, investment and consumption in response to changes in the external finance premium. Indexed contracts.**

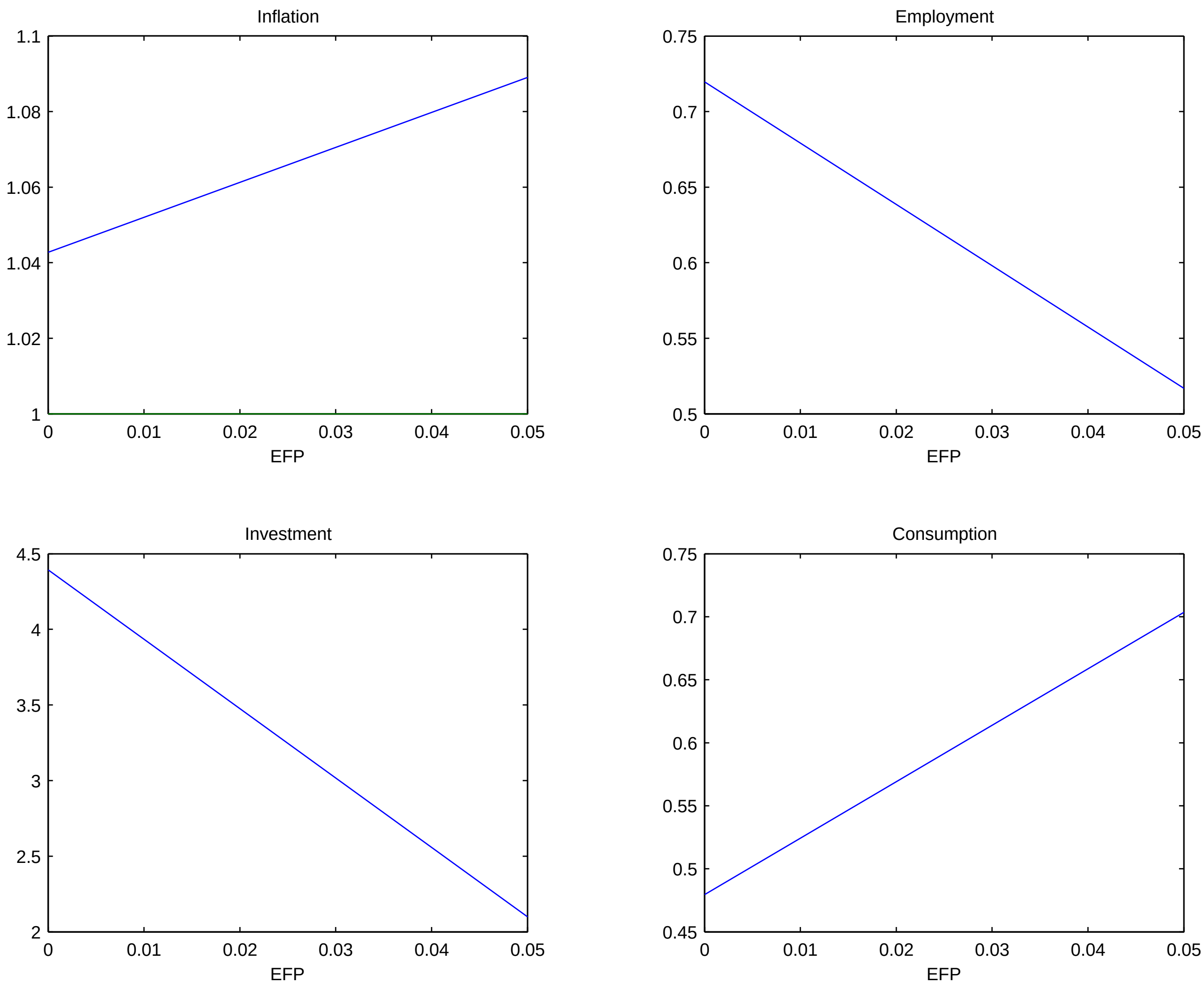



Figure 2: **Golden rule level of inflation, employment, investment and consumption in response to changes in the external finance premium. Non indexed contracts.**

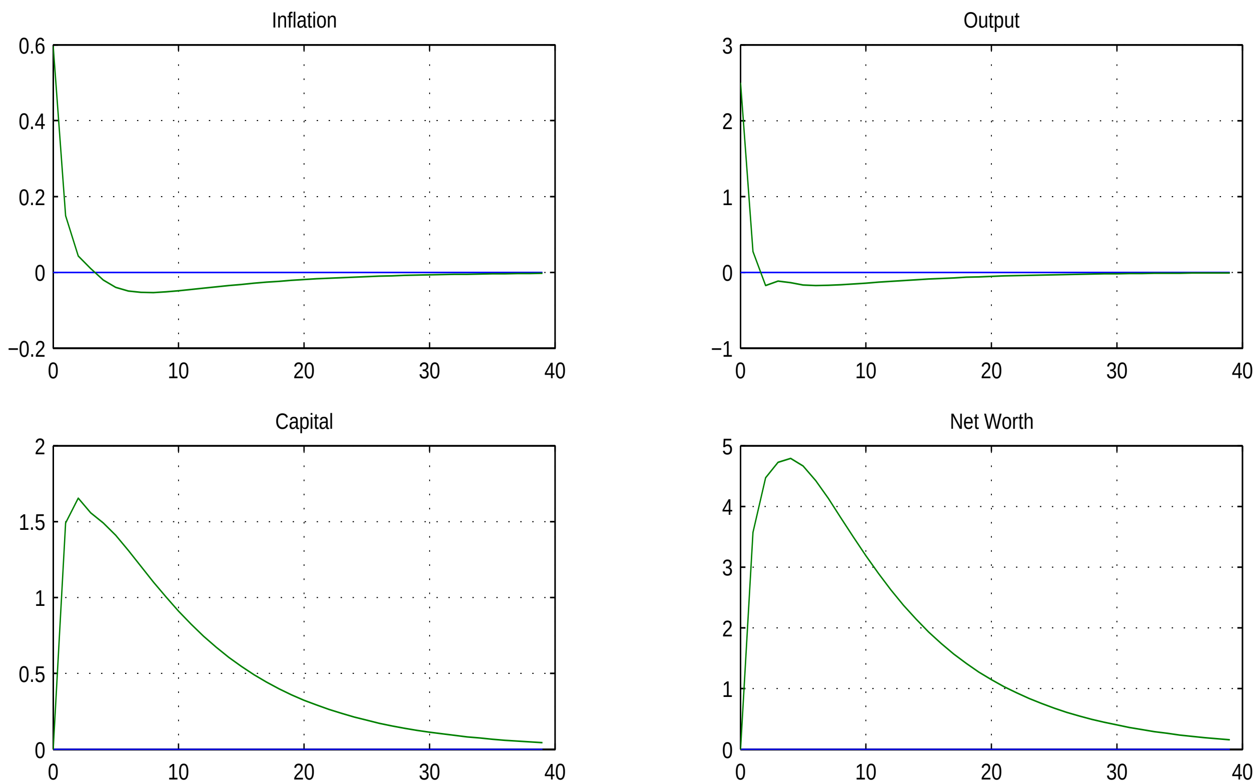
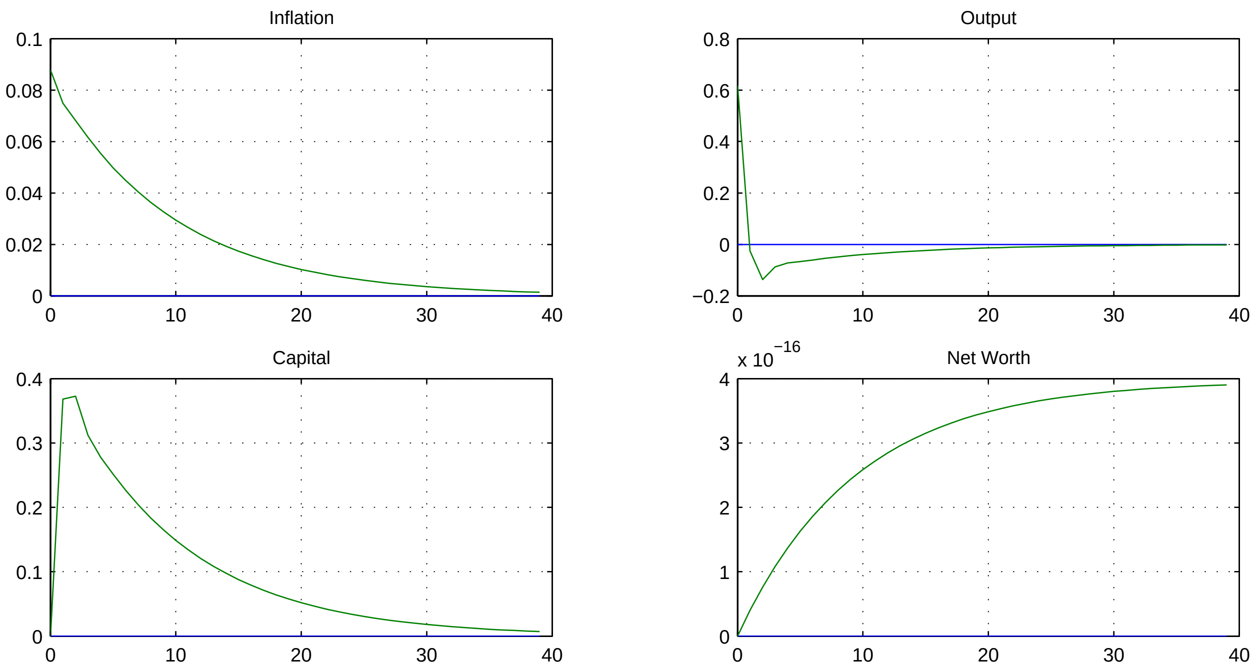



Figure 3: **Optimal response to productivity shocks.**




Impulse responses under price stability

Figure 4: **Impulse response under price stability.**

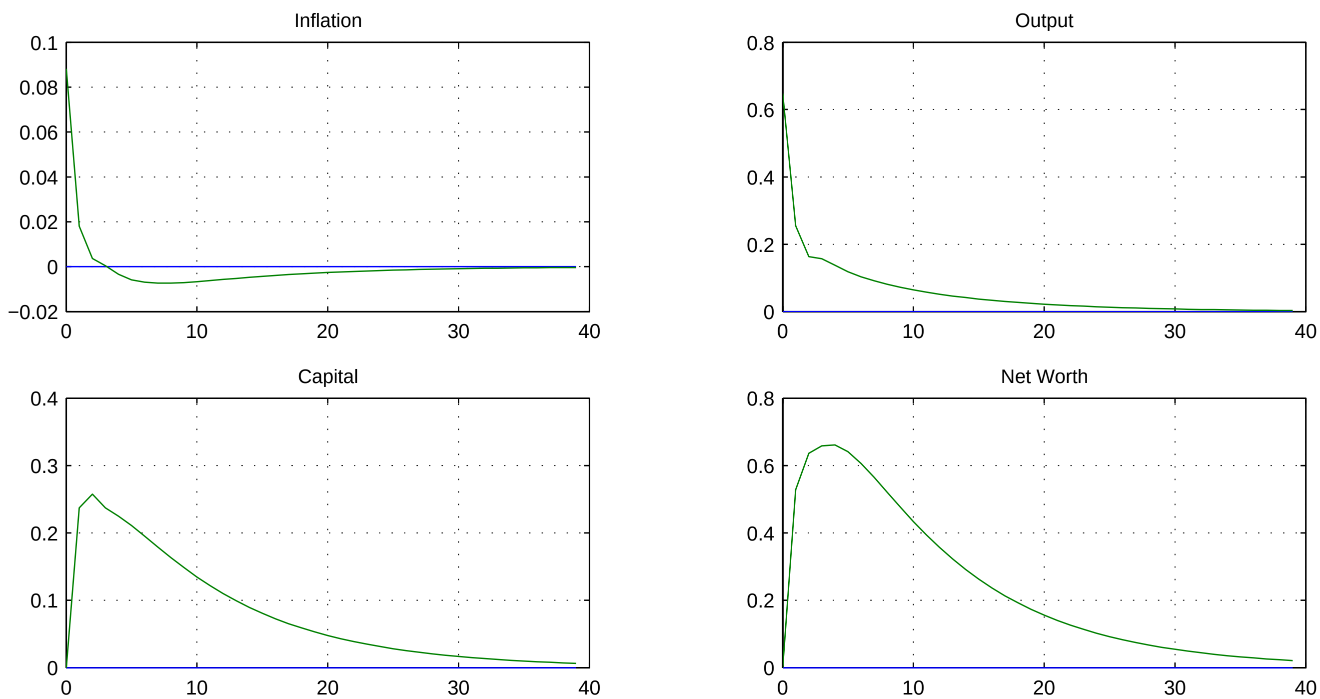



Figure 5: **Optimal response to demand shocks.**

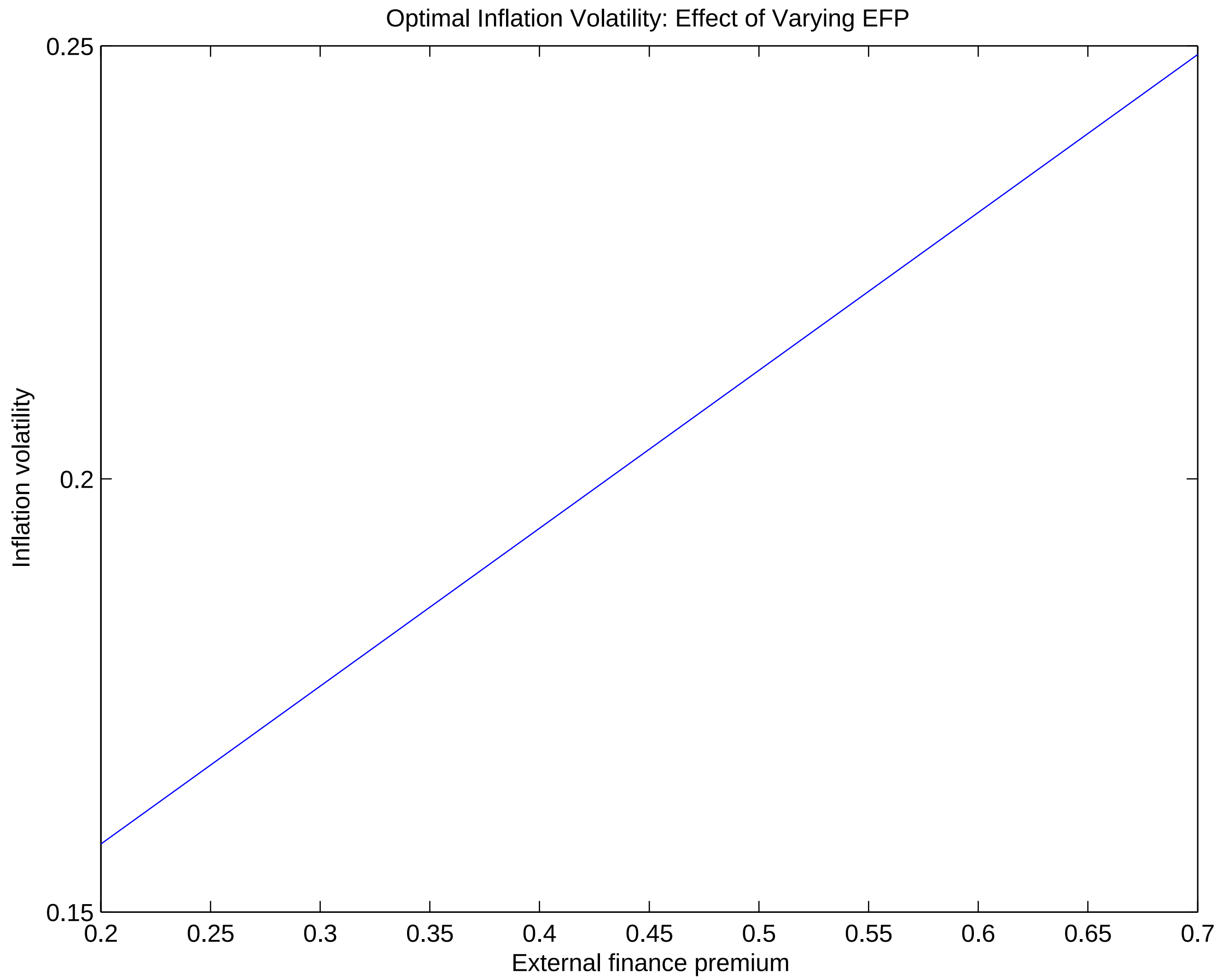



Figure 6: **Optimal volatility of inflation with respect to changes in the size of the external finance premium.**