The most beautiful variations on fair wages and the Phillips curve

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[1]The author would like to thank the editor Pok-sang Lam and two anonymous referees for insightful comments. The usual disclaimer applies.

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Abstract

The present paper explores the connection between inflation and unemployment in two different models with fair wages in both the short and the long runs. Under customary assumptions regarding the sign of the parameters of the effort function, more inflation lowers the unemployment rate, albeit to a declining extent. This is because firms respond to inflation - which spurs effort by decreasing the reference wage - by increasing employment, thus maintaining the effort level constant as implied by the Solow condition. A stronger short-run effect of inflation on unemployment is produced under varying as opposed to fixed capital, given that in the former case the boom produced by a monetary expansion is reinforced by an increase in investment. Therefore, we provide a new theoretical foundation for recent empirical contributions which find negative long- and short-run effects of inflation on unemployment.

Keywords: efficiency wages, money growth, long-run Phillips curve, trend inflation.

JEL classification codes: E3, E20, E40, E50.

1 Introduction

The economic literature has recently witnessed a flourishing of contributions which nest an efficiency wages framework in business cycle models. Earlier models were proposed within the real business cycle (RBC) realm. Danthine and Donaldson (1990), for instance, showed that efficiency wages within a RBC model can produce structural unemployment, but not wage stickiness over the economic cycle. Unlike Danthine and Donaldson (1990), who focused on a gift exchange model, Uhlig and Xu (1995) and Gomme (1999) adopted a shirking model. However, in a rather similar way, they found that wages tend to be too volatile, and employment not sufficiently so, over the cycle. In Kiley (1997) efficiency wages generate a-cyclical real wages, but not a greater endogenous price stickiness, because the a-cyclical real-wage requires countercyclical effort and hence a procyclical marginal cost.

Collard and de la Croix (2000) showed that, on including past compensations in the reference wage, an efficiency wages/RBC model can replicate wage acyclicality. Along similar lines, Danthine and Kurmann (2004) proposed a model combining efficiency wages of the gift exchange variety - also termed "fair wages" - with sticky prices, showing that it can account well for the low correlation between wages and employment, while also displaying a greater internal propagation of monetary shocks than do standard New Keynesian models. Danthine and Kurmann (2008), inspired by Rabin (1993), explicitly modelled the psychological benefits arising from gift exchanges between firms and workers in terms of remuneration and effort respectively. Danthine and Kurmann (2010) incorporated a reciprocity-based model of wage determination into a dynamic general equilibrium model, which was then estimated on U.S. data. They highlighted that wage setting is driven more by rent-sharing and past wages than by aggregate employment conditions.

Alexopoulos (2004, 2006, 2007) developed a model in which shirkers are not dismissed once detected. They instead forgo an increase in compensation. Under these assumptions, she showed that an efficiency wage model can well replicate empirical evidence on the response of the economic system to technological, fiscal and monetary shocks.

The present paper, instead, focuses on the long-run and short-run implications of efficiency wages for the connection between unemployment and inflation under trend money growth within a dynamic general equilibrium framework. In so doing, it extends a literature that to date has investigated the long-run and, to a lesser extent, the short-run effects of money growth by resorting only to models with wage/price stickiness. Pioneering contributions on this issue were King and Wolman (1996) and Ascari (1998). The former study considered a model with a shopping time technology, and it obtained a number of different results, among which that long-run inflation reduces firms' markup, boosting the level of output. Ascari (1998) instead showed that in wage-staggering models money can have considerable negative non-superneutralities when not considering simple utility and production functions. Deveraux and Yetman (2002) focused on a menu cost model. An analysis of dynamic general equilibrium models under different contract schemes in the presence of trend inflation was conducted by Ascari (2004). Graham and Snower (2004) examined the microeconomic mechanisms underlying this class of models. In the presence of Taylor wage staggering, in a monopolistically competitive labour market, they highlighted three channels through which inflation affects output: employment cycling, labour supply smoothing and time discounting. The first consists in firms continuously shifting labour demand from one cohort to the other according to their real wage. Given that different kinds of labour are imperfect substitutes for each other, this generates inefficiencies and tends to create a negative inflation-output nexus. The second channel is that households demand a higher wage in the presence of employment cycling, given that they would prefer smoother working time. This decreases labor supply and aggregate output. Finally under time discounting the contract wage depends more on the current (lower) level of prices than on the future (higher) level of prices, so that - over the contract period - the real wage will be lower, the greater the inflation rate, spurring labour demand and aggregate output. The time discounting effect dominates at lower inflation rates, while the other two effects do so at higher inflation rates, producing a hump-shaped long-run Phillips curve. The overall purpose of Graham and Snower (2004) is questioning the customary assumption to identify aggregate demand and supply shocks, namely that the former are temporary while the latter are not. As a consequence, also the concept of the NAIRU would be unsuitable for fruitful investigation of the dynamics of the unemployment rate.[2]

Proposed in what follows are two variations on the theme of efficiency wages and the Phillips curve. In the first, efficiency wages of the gift exchange variety are coupled with trend money growth, after specifying the reference wage as a function of the unemployment rate, the current individual real wage, the current aggregate real wage, and the current real value of the past aggregate wage. Since Becker (1996), this specification has been termed in the literature the "social norm case". Because here the reference wage is a function of the current real value of the past aggregate wage and not, as in Danthine and Kurmann (2004), of the past real wage, we can highlight the macroeconomic consequences of a peculiar gift exchange between firms and workers that was not investigated to date, although it is empirically relevant. Bewley (1999, pp. 160-161, 164-165, 208-209) repeatedly stresses that although firms dislike wage indexation, they are not insensitive to the damage caused by inflation to the purchasing power of wages. If workers perform well, pay managers will consider it fair to offset the negative effect of inflation on their standard of living.[3] This can be conceptualized as a

[2]Graham and Snower (2004) was extended in a number of different directions in Graham and Snower (2008), Vaona and Snower (2007, 2008) and Vaona (2012).

[3]A notable example is the following sentence "When hiring someone, I pay them a salary equal to the value of their job. Inflation effectively reduces it, and fairness requires that I offset the reduction. I think that is the way it ought to be. If I hire people at a certain rate, I want to keep that level constant in terms of standard of living" (Bewley, 1999, p. 164).

Also Levine (1993) finds that companies tend to offer larger wage increases in the presence of higher inflation, though not in a one-to-one proportion.

gift exchange: workers elicit effort and firms maintain the purchasing power of their wages. We show that this mechanism can produce sizeable money non-superneutralities both in the short and long run. Our second variation extends the first one by considering varying instead of fixed capital[4].

Introducing capital accumulation at a later stage is not an unusual procedure in the New-Keynesian literature (see for instance Huang and Liu, 2002; Ascari, 2004; Danthine and Kurmann, 2010). Some contributions do not even consider capital accumulation (Ascari 1998; Graham and Snower, 2004, 2008; Danthine and Kurmann, 2008; Ascari and Ropele, 2009). This is for two main reasons. Firstly, as recalled by Ascari (2004), McCallum and Nelson (1999) argued that it is difficult to specify a capital demand function which is "both analytically tractable and empirically successful". Secondly - similarly to sticky wages/prices models (Ascari, 2004, Vaona, 2012) - the core of our model is the labour market and capital accumulation turns out to be only a superstructure, which does not induce any qualitative change in our results. Therefore, we believe our exposition strategy to be the one best suited to conveying the underlying intuition of our model.

Unlike Graham and Snower (2004, 2008), we provide not only a long-run analysis but also a short-run one, because we think that, even if it is not possible to identify demand and supply shocks on the basis of their

[4]We focus on models with the specific features described above because Vaona (2010) - an extended version of this paper - shows that the personal norm case and wage staggering can produce implausible results. On the other hand, Vaona (2010) also shows that shifting to an approach à la Danthine and Kurmann (2008, 2010) produces results similar to those set out below, although it entails a greater parametrization of the model.

transience, it will be interesting to investigate how the economic system reacts to temporary monetary shocks. In other words, transition dynamics do not lose interest.

Our results can offer a new theoretical foundation for the empirical findings obtained by various recent studies. As also discussed by Karanassou et al. (2010), Karanassou et al. (2003, 2005, 2008a, 2008b) apply a variety of econometric methods to data from different countries over various time periods. A common result of theirs is that inflation and unemployment are connected not only in the short-run but in the long-run as well. The long-run elasticity of inflation with respect to unemployment was estimated to be about -3.5, which was explained in terms of frictional growth, namely the interplay between frictions (lagged adjustments) and growth in economic variables. In the light of our models, this result can be also interpreted as the outcome of efficiency wages mechanisms as explained below.

The rest of this paper is structured as follows. The next section introduces the households' problem and the government budget constraint, which are common to both the models presented. Afterwards, we introduce the firms' problem for the social norm case with flexible wages, first, with fixed capital, and then with varying capital. In both cases, we show the impact of money growth on both the unemployment and the inflation rates in the short- as well as the long-run. The last section concludes.

2 The households' problem and the government budget constraint

We follow Danthine and Kurmann (2004, 2008, 2010) by supposing the economy to be populated by a continuum of households normalized to 1, each composed by a continuum of individuals also normalized to 1. We adopt a money-in-the-utility-function approach to preserve comparability with the trend inflation literature (Ascari 2004, Graham and Snower, 2004, 2008).[5] Households maximize their discounted utility

$$\max_{\{c_{t+i}(h),B_{t+i}(h),M_{t+i}(h),e_{t+i}(h)\}} \sum_{i=0}^{\infty} \beta^{t+i} E\left(U \left\{ \begin{array}{c} c_{t+i}(h), n_{t+i}(h) G\left[e_{t+i}(h)\right], \\ V\left[\frac{M_{t+i}(h)}{P_{t+i}}\right] \end{array} \right\} \right) \tag{1}$$

subject to a series of income constraints

$$\begin{aligned} c_{t+i}(h) &= \frac{W_{t+i}(h)}{P_{t+i}} n_{t+i}(h) + \frac{T_{t+i}(h)}{P_{t+i}} - \frac{M_{t+i}(h)}{P_{t+i}} + \\ &\quad + \frac{M_{t+i-1}(h)}{P_{t+i}} - \frac{B_{t+i}(h)}{P_{t+i}} + \frac{B_{t+i-1}(h)}{P_{t+i}} \iota_{t+i} + q_{t+i}(h) \end{aligned} \tag{2}$$

where β is the discount factor, E is the expectation operator, U is the utility function, $c_{t+i}(h)$ is consumption by household h at time $t+i$, $B_{t+i}(h)$ are the household's bond holdings, ι_{t+i} is the nominal interest rate, $n_{t+i}(h)$ is the

[5] Feenstra (1986) showed the functional equivalence of money-in-the-utility-function models and liquidity-costs ones.

fraction of employed individuals within the household, $G\left[e_{t+i}(h)\right]$ is the disutility of effort - $e_{t+i}(h)$ - of the typical working family member, $V\left[\frac{M_{t+i}(h)}{P_{t+i}}\right]$ is the utility arising from nominal money balances - $M_{t+i}(h)$ - over the price level - P_{t+i}. $W_{t+i}\left(h\right)$ and $T_{t+i}\left(h\right)$ are the household's nominal wage income and government transfers respectively. Finally, $q_{t+i}(h)$ are profits that households receive from firms.

In this framework, households, and not individuals, make all the decisions regarding consumption, bond holdings, real money balances and effort.[6] Individuals are identical ex-ante, but not ex-post, given that some of them are employed - being randomly and costlessly matched with firms independently from time - and some others are unemployed. The fraction of the unemployed is the same across all families, so that their ex-post homogeneity is preserved.

Note that in our model no utility arises from leisure. Therefore individual agents inelastically supply one unit of time for either work - or unemployment - related activities. Furthermore, following Akerlof (1982), workers, though disliking effort, will be ready to exert it as a gift to the firm if they receive some other gift in exchange, such as a real compensation above some reference level.

[6] This modelling device is not only common to efficiency wages models (Danthine and Kurmann, 2004, 2008, 2010), it is also used in neo-keynesian models with search frictions in the labour market (Blanchard and Galí, 2010 drawing on Merz, 1995). Its underlying assumption is full risk sharing, and its ultimate goal is to preserve a representative agent setup. Alexopoulos (2004) justifies a similar framework assuming that households can observe individuals' behavior and that they can punish workers declining job offers by withdrawing income insurance. It would also be possible to think that workers and not households decide how much effort to elicit. However, since all workers within a household are symmetrical, it would not change our results.

Similarly to Danthine and Kurmann (2004) and focusing on the social norm case, on the basis of the empirical evidence furnished by Bewley (1998), we specify the effort function, $G\left[e_{t+i}(h)\right]$ as follows[7]

$$G\left[e_{t+i}(h)\right]=\left\{e_{t+i}(h)-\left[\begin{array}{c}\phi_0+\phi_1\log\frac{W_{t+i}(h)}{P_{t+i}}+\phi_2\log u_{t+i}(h)+\\+\phi_3\log\frac{W_{t+i}}{P_{t+i}}+\phi_4\log\frac{W_{t+i-1}}{P_{t+i}}\end{array}\right]\right\}^2 \tag{3}$$

where W_{t+i} is the aggregate nominal wage and $u_{t+i}(h)=1-n_{t+i}(h)$ is the unemployment rate. Note that, differently to Danthine and Kurmann (2004), the nominal (either individual or aggregate) wage at time $t+i-1$ is assessed at the prices of time $t+i$. This assumption does not entail any money illusion. On the contrary, its underlying intuition is that households are aware of the damage that inflation can cause to their living standards. They are consequently ready to exchange more effort for a pay policy that allows nominal wages to keep up with inflation. More briefly, a higher inflation rate reduces the reference wage.

Throughout the paper, similarly to Danthine and Kurmann (2004), we assume $\phi_1,\phi_2>0$ and $\phi_3,\phi_4<0$. In words, a higher household's real wage and a higher unemployment rate induce more effort. On the other hand, a higher reference wage - due to either a higher aggregate wage or a higher real

[7] An alternative approach to the effort function is the one taken by Campbell (2006, 2008a and 2008b), which entails a more general functional specification to be linearized at a later stage. However, calibration is less straightforward in this context and economic theorizing is usually followed by a number of numerical exercises where parameters and results display a somewhat large variation. For this reason we prefer to follow Danthine and Kurmann (2004), on the basis of whose empirical evidence, we also do not take the approach of Collard and de la Croix (2000).

value of past compensation - depresses effort.

Note that, under the hypothesis of an additively separable utility function, utility maximization implies that

$$G'\left[e_{t+i}(h)\right] = 0 \tag{4}$$

and, therefore

$$e_{t+i}(h) = \phi_0 + \phi_1 \log \frac{W_{t+i}(h)}{P_{t+i}} + \phi_2 \log u_{t+i}(h) + \phi_3 \log \frac{W_{t+i}}{P_{t+i}} + \phi_4 \log \frac{W_{t+i-1}}{P_{t+i}} \tag{5}$$

Similarly to Danthine and Kurmann (2004), we assume that $c_{t+i}\,(h)$ and $\frac{M_{t+i}(h)}{P_{t+i}}$ enter (1) in logs

$$U\left(\cdot\right) = \log c_{t+i}\,(h) - n_{t+i}\,(h)\, G\left[e_{t+i}(h)\right] + b \log \left[\frac{M_{t+i}(h)}{P_{t+i}}\right] \tag{6}$$

Utility maximization implies

$$\frac{1}{c_{t+i}\,(h)} = E\left[\frac{\iota_{t+i}}{\pi_{t+i+1}} \frac{1}{c_{t+i+1}\,(h)} \beta\right] \tag{7}$$

$$\left(\frac{\mu_{t+i}}{\pi_{t+i}}\right)^{-1} = \frac{c_{t+i-1}\,(h)}{c_{t+i}\,(h)} \left(1 - \frac{1}{\iota_{t+i}}\right) / \left(1 - \frac{1}{\iota_{t+i-1}}\right) \tag{8}$$

where μ_{t+i} is the money growth rate and π_{t+i} is the inflation rate. The government rebates its seigniorage proceeds to households by means of lump-sum transfers, $T_t\,(h)$:

$$\int_{0}^{1} \frac{T_{t+i}(h)}{P_{t+i}} dh = \int_{0}^{1} \frac{M_{t+i}(h)}{P_{t+i}} dh - \int_{0}^{1} \frac{M_{t+i-1}(h)}{P_{t+i}} dh \tag{9}$$

3 First variation: the social norm case with fixed capital

3.1 The long-run

To produce their output firms in the perfectly competitive product market hire individuals belonging to all the households. Firms maximize their profits - $P_{t+i}y_{t+i} - \int_{h=0}^{1} W_{t+i}(h)n_{t+i}(h)dh$, where y_{t+i} is output - subject to their production function - $y_{t+i} = \left[\int_0^1 e_{t+i}(h)^{\frac{\theta_n-1}{\theta_n}} n_{t+i}(h)^{\frac{\theta_n-1}{\theta_n}} dh\right]^{\frac{\theta_n}{\theta_n-1}}$, where θ_n is the elasticity of substitution among different labour kinds - and to (5), by choosing $n_{t+i}(h)$ and $W_{t+i}(h)$. Note that the production function displays decreasing marginal returns to each labour type and constant returns to scale.

The first order condition with respect to $n_{t+i}(h)$ equates the marginal cost of labour to its marginal product. All households are symmetrical. Hence we can drop the h index and write[8]

$$\frac{W_{t+i}}{P_{t+i}} = \frac{y_{t+i}}{n_{t+i}} \tag{10}$$

The first order condition with respect to $W_{t+i}(h)$ instead equates the

[8]Equation (10) implies that $q_t(h) = 0$.

marginal cost of raising the real wage to the benefit that this induces by increasing effort

$$\frac{W_{t+i}}{P_{t+i}}\frac{n_{t+i}}{y_{t+i}} = \frac{\phi_1}{e_{t+i}} \tag{11}$$

By substituting (10) into (11) , one obtains the well known Solow condition

$$e_{t+i} = \phi_1 \tag{12}$$

Firms, maximizing their profits, therefore, demand the same effort from all households, across time and independently from the rate of inflation. Furthermore, (12) and the production function, under the condition of households' symmetry, imply

$$\frac{W_{t+i}}{P_{t+i}} = \frac{y_{t+i}}{n_{t+i}} = \phi_1 \tag{13}$$

Substituting (12) and (13) into (5) and considering that trend inflation is equal to steady state money growth, μ, yields

$$\log u = \frac{\phi_1 - \phi_0}{\phi_2} - \frac{(\phi_1 + \phi_3 + \phi_4)}{\phi_2}\log\phi_1 + \frac{\phi_4}{\phi_2}\log\mu \tag{14}$$

which, together with our standard assumptions on the sign of ϕ_4 and ϕ_2 implies that the elasticity of the unemployment rate with respect to inflation is negative

$$\frac{d\log u}{d\log\mu} = \frac{\phi_4}{\phi_2} < 0 \tag{15}$$

The intuition underlying this result is the following. An increase in inflation produces a decrease in the reference wage by reducing the current real value of the past compensation. This would spur effort, but the firms' optimal level of effort does not depend on inflation. As a consequence firms increase employment (and decrease unemployment) in order to keep the level of effort constant. Following the results by Karanassou et al. (2005, 2008a, 2008b), one may calibrate $\frac{\phi_4}{\phi_2} \approx -0.29$.

Note that this mechanism does not imply that hyperinflation will produce large decreases in unemployment. In order to understand this point we focus on the semielasticity of the unemployment rate with respect to the money growth rate. In our context, the advantage of the semi-elasticity versus the elasticity is that it is a measure of the reactiveness of the unemployment rate to absolute, and not percentage, changes in the money growth rate, mirroring, in this respect, the results provided by, among others, Ascari (1998, 2004) and Graham and Snower (2004, 2008). The semielasticity of the unemployment rate with respect to money growth is

$$\frac{d \log u}{d\mu} = \frac{\phi_4}{\phi_2} \frac{1}{\mu} < 0 \tag{16}$$

which is still negative, given that $\mu \geq 1$, but $\lim_{\mu \to \infty} \frac{d \log u}{d\mu} = 0$.

3.2 The short-run

In order to analyze the short run dynamics of the present economic model, consider first that the only steady state condition we imposed to obtain (14) was the equality of money growth and inflation. Out of steady state one can write (14) as $\log u_{t+i} = \frac{\phi_1 - \phi_0}{\phi_2} - \frac{(\phi_1+\phi_3+\phi_4)}{\phi_2} \log \phi_1 + \frac{\phi_4}{\phi_2} \log \pi_{t+i}$. The other equations of the system are (7) , (8) , the aggregate resource constraint, $y_{t+i} = c_{t+i}$, the production function and the condition $n_{t+i} = 1 - u_{t+i}$. The unknowns of this model are enclosed in the sequence $\{\pi_{t+i}, \iota_{t+i}, u_{t+i}, n_{t+i}, y_{t+i}, c_{t+i}\}$.

This system of equations, after log-linearization around the steady state, can be expressed as a second order difference equation in inflation, which in its turn can be re-arranged to obtain the following system of first order difference equations

$$E\left(\hat{x}_{t+i+1}\right) = \left[\iota_{ss} + \frac{\frac{u_{ss}}{n_{ss}}\phi_\pi}{\left(1 + \frac{u_{ss}}{n_{ss}}\phi_\pi\right)}\right]\hat{x}_{t+i} - \frac{\frac{u_{ss}}{n_{ss}}\phi_\pi \iota_{ss}}{\left(1 + \frac{u_{ss}}{n_{ss}}\phi_\pi\right)}\hat{\pi}_{t+i} \tag{17}$$

$$E\left(\hat{\pi}_{t+i+1}\right) = \hat{x}_{t+i} \tag{18}$$

In the equations above, hats denote deviations from steady state, $\phi_\pi \equiv -\frac{\phi_4}{\phi_2}$ and i_{ss}, u_{ss} and n_{ss} are the steady state values of the nominal interest rate, of the unemployment rate, and of the employment rate respectively. In order to investigate the stability of (17)-(18) we need to calibrate not only $\frac{\phi_4}{\phi_2}$ as above, but also i_{ss}, u_{ss} and n_{ss}. For this purpose we take as reference the averages of the post-second-world-war US time series and we set $u_{ss} = 0.056$, $n_{ss} = 1 - u_{ss}$ and $i_{ss} = 1.02 * (1 + \mu)$. We compute the roots of (17)-(18) for various values of trend inflation, and the results are shown in Figure 1, where it will be seen that the system is always saddle-path stable, given that one root is outside the unit circle and the other one within it.

One might wonder what the effects of trend inflation are on the stable arm of the system. The answer to this question is shown in Figure 2 where, following Shone (2001), different trajectories along the stable arm are projected on the $\{\pi_t, \pi_{t+1}\}$ plane for trend inflation rates equal to 2%, 20% and 80%. The higher trend inflation is, the flatter is the stable arm. In other words, the higher the trend inflation, the sharper inflation reductions should be in order to achieve stability.

4 Second variation: the social norm case with varying capital

On considering varying capital within the model, we assume the existence of capital adjustment costs after Bernanke et al. (1999) and Gertler (2002). The households' budget constraint changes to

$$\begin{aligned} c_{t+i}(h) &= \frac{W_{t+i}(h)}{P_{t+i}} n_{t+i}(h) + \frac{T_{t+i}(h)}{P_{t+i}} - \frac{M_{t+i}(h)}{P_{t+i}} + \frac{M_{t+i-1}(h)}{P_{t+i}} - \frac{B_{t+i}(h)}{P_{t+i}} + \\ &+ \frac{B_{t+i-1}(h)}{P_{t+i}} \iota_{t+i-1} + \frac{R_{t+i}}{P_{t+i}} K_{t+i}(h) - \frac{Q_{t+i}}{P_{t+i}} \left[K_{t+i}(h) - (1-\delta) K_{t+i-1}(h) \right] + q_{t+i}(h) \end{aligned} \tag{19}$$

where $K_{t+i}(h)$ is the capital held by household h, δ is the capital depreciation rate, R_{t+i} is the capital rental rate, and Q_{t+i} is the nominal Tobin's q. Furthermore, households maximize utility with respect to capital as well. Interacting the first order conditions for capital and consumption leads, under households' symmetry, to the following equation

$$E(c_{t+i+1}\frac{Q_{t+i}}{P_{t+i}}) = \frac{R_{t+i}}{P_{t+i}}E(c_{t+i+1}) + c_{t+i}\beta\left(1-\delta\right)\frac{Q_{t+i+1}}{P_{t+i+1}} \tag{20}$$

As in the New-Keynesian tradition, we assume the existence of an intermediate labour market, where labour intermediaries hire households' horizontally differentiated labour inputs to produce homogeneous labour to be sold to firms operating on the final product market. In the intermediate labour market we assume productivity to depend on effort. The profit maximization problem of labour intermediaries is

$$\max_{\{n_{t+i}(h),W_{t+i}(h)\}} W_{t+i}n_{t+i} - \int_0^1 W_{t+i}(h)n_{t+i}(h)dh \tag{21}$$
$$s.t.\ n_{t+i} = \left[\int_0^1 e_{t+j}(h)^{\frac{\theta_n-1}{\theta_n}} n_{t+j}\left(h\right)^{\frac{\theta_n-1}{\theta_n}} dh\right]^{\frac{\theta_n}{\theta_n-1}}$$

The solution of this problem and households' symmetry imply

$$\frac{W_{t+i}(h)}{W_{t+i}} = \frac{n_{t+i}}{n_{t+i}(h)} = \phi_1 = e_{t+i} = 1 \tag{22}$$

Firms in the final product market maximize profits by hiring labour and capital and adopting a Cobb-Douglas production function. The solution of their problems leads to two customary demand functions for labour and capital

$$(1-\alpha)\frac{y_{t+i}}{\frac{W_{t+i}}{P_{t+i}}} = n_{t+i} \tag{23}$$

$$\alpha\frac{y_{t+i}}{\frac{R_{t+i}}{P_{t+i}}} = K_{t+i} \tag{24}$$

Substituting these two equations into the production function yields

$$\frac{W_{t+i}}{P_{t+i}} = \left(\frac{\frac{R_{t+i}}{P_{t+i}}}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}(1-\alpha) \tag{25}$$

Finally, capital producer j has the following production function

$$Y_{t+i}^{k}(j) = \phi\left[\frac{I_{t+i}(j)}{K_{t+i-1}(j)}\right]K_{t+i}(j) \tag{26}$$

where $Y_{t+i}^{k}(j)$ is new capital, $I_{t+i}(j)$ is raw output used as material input at time $t+i$ and $\phi'(\cdot)>0$, $\phi''(\cdot)<0$, $\phi(0)=0$ and $\phi\left(\frac{I}{K}\right)=\frac{I}{K}$, with $\frac{I}{K}$ being the steady state investment-capital ratio. $K_{t+i}(j)$ is capital rented after it has been used to produce final output within the period. The profits of the j-th capital producer can be written as $\frac{Q_{t+i}}{P_{t+i}}\phi\left[\frac{I_{t+i}(j)}{K_{t+i-1}(j)}\right]K_{t+i}(j)-I_{t+i}(j)-Z_{t+i}^{k}K_{t+i}(j)$ where Z_{t+i}^{k} is the rental price of capital used to produce new capital. The first order condition for $I_{t+i}(j)$ is, under a symmetry condition:

$$\frac{Q_{t+i}}{P_{t+i}}\phi'\left(\frac{I_{t+i}}{K_{t+i-1}}\right)-1=0 \tag{27}$$

where $I_{t+i}=\int_0^1 I_{t+i}(j)\,dj$ and $K_{t+i-1}=\int_0^1 K_{t+i-1}(j)\,dj$. One can show that the first order condition with respect to $K_{t+i}(j)$, $\phi\left(\frac{I}{K}\right)=\frac{I}{K}$ and (27) imply that Z_{t+i}^{k} is approximately zero near the steady state and can therefore be ignored.

The system of equations is therefore composed of (7), (8), the aggregate resource constraint $y_{t+i}=c_{t+i}+I_{t+i}$, the law of motion of capital $K_{t+i}=\phi_K\left(\frac{I_{t+i}}{K_{t+i-1}}\right)K_{t+i}-(1-\delta)K_{t+i-1}$, the definition of the unemployment rate $n_t=1-u_t$, the generating process for money growth

$$\mu_{t+i}=\mu^{1-\zeta}\mu_{t+i-1}^{\zeta}\exp(\epsilon_{t+i}) \tag{28}$$

the equations (20), (23), (24), (25), (27), and (5), which imposing (22) and after rearranging becomes

$$\log u_{t+i}=\frac{\phi_1-\phi_0}{\phi_2}-\frac{\phi_4}{\phi_2}\log\pi_{t+i}-\frac{(\phi_1+\phi_3)}{\phi_2}\log\frac{W_{t+i}}{P_{t+i}}+\frac{\phi_4}{\phi_2}\log\frac{W_{t+i-1}}{P_{t+i-1}} \tag{29}$$

The unknowns of the model are enclosed in the sequence $\left\{\frac{R_{t+i}}{P_{t+i}},\frac{W_{t+i}}{P_{t+i}},y_{t+i},n_{t+i},K_{t+i},c_{t+i},\right.$ $\left.u_{t+i},\mu_{t+i},\pi_{t+i},\iota_{t+i},I_{t+i},\frac{Q_{t+i}}{P_{t+i}}\right\}$.

Regarding the long-run, to be noted is that in steady state the real Tobin's q is equal to one and therefore that $\frac{R}{P}$ and $\frac{W}{P}$ are pinned down by (20) and (25) independently from money growth. On the basis of (29) and of the steady state equality of inflation and money growth, this entails that (15) and (16) also hold for the present model.

Regarding the short-run, we log-linearized the system around a steady state with $u_{ss} = 0.056$ on the basis of the US post-WWII experience. We calibrated the system parameters as customary in the New-Keynesian literature (see for instance Ascari, 2004): $\beta = 1.04^{-1}$, $\mu = 1.02$, $\theta_n = 5$, $\frac{\phi_4}{\phi_2} = -0.29$, $\zeta = 0.57$. In order to attach a value to $\frac{(\phi_1+\phi_3)}{\phi_2}$ we note that it can be considered as the inverse of the elasticity of households' wages with respect to the unemployment rate, and so we set it to 0.07^{-1}after Nijkamp and Poot (2005). Furthermore, as customary, $\alpha = 0.33$, $\delta = 0.08$ and, after Bernanke et al. (1999), $\eta = -\frac{\phi''\left[\frac{I}{K}\right]\frac{I}{K}}{\phi'\left[\frac{I}{K}\right]} = 0.5$.

Figure 3 plots the percentage deviations from steady state of the inflation rate against those of the unemployment rate. In other words, we plot the impulse response function of the inflation rate against that of the unemployment rate in order to show the unemployment-inflation trade-off in a more direct way. The result that higher inflation goes hand in hand with a lower unemployment rate, whose intuition was discussed when commenting on equation (15), is confirmed also for the present model. As it is possible to see, varying capital implies a flatter short run Phillips curve than under fixed capital, given that the boom following a monetary expansion is reinforced by an increase of investments, which rise upon impact by 0.08%. Changing η would have only negligible effects on the Phillips curve.[9] It is worth noting that our model does not produce a persistent reaction of either the unemployment or the inflation rate after a monetary shock. This accords well with the empirical evidence produced by the inflation persistence network,

[9]Further results are available from the author on request.

whose main result is that, on allowing for structural breaks in the mean of the inflation time series, inflation has low persistence (Altissimo et al., 2007). Empirical evidence of a fast adjustment of unemployment after a monetary shock was produced by Karanassou et al. (2007, p. 346) where the unemployment rate took just two periods to hit its new long-run level after a permanent monetary shock. However, this low persistence is not a property of efficiency wages themselves. Danthine and Kurmann (2004, 2010) showed that, once efficiency wages are coupled with price rigidities, it is possible to produce persistent impulse response functions.

Figure 3 also shows that increasing trend inflation decreases the responsiveness of both the inflation and unemployment rates to a 1% monetary shock. This is consistent with our above results that increasing trend inflation flattens the stable arm of the economic system without capital, and it can be explained by keeping two facts in mind. First, households smooth consumption and, second, an increase in trend inflation decreases the elasticity of the money demand function to the nominal interest rate.[10] If households smooth consumption, they will tend to smooth real money holdings as well - see equation (8). This, in the presence of a smaller reactiveness of money demand to the nominal interest rate, can happen only thanks to a larger reaction in the latter one (Figure 4). In other words, households achieve a stable path for consumption and real money holdings in face of a monetary shock

[10]Loglinearizing (8) , one can show that this elasticity is $\frac{1}{i_{ss}-1}$ where i_{ss} is equal to trend inflation over the discount factor.

with higher trend inflation by letting the interest rate react more, which stabilizes the whole economy and implies a smaller change in inflation as well. A smaller change in the inflation rate translates into a smaller change in the unemployment rate via the Phillips curve (29).

5 Conclusions

The paper has explored the relationship between inflation and unemployment in different models with fair wages. It has been shown that, under customary assumptions regarding the parameters of the effort function, they have negative long- and short-run nexuses. This is due to the fact that firms respond to inflation - which spurs effort via a decrease in the reference wage - by increasing employment in order to maintain the effort level constant, as implied by the Solow condition. This effect is stronger in the short-run on considering varying instead of fixed capital as booms generated by monetary expansions are reinforced by greater investment.

Our results can offer new theoretical insights into the evidence produced by recent empirical contributions finding a negative long-run relationship between unemployment and inflation.

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Figure 1 – The roots of the system for different trend inflation rates

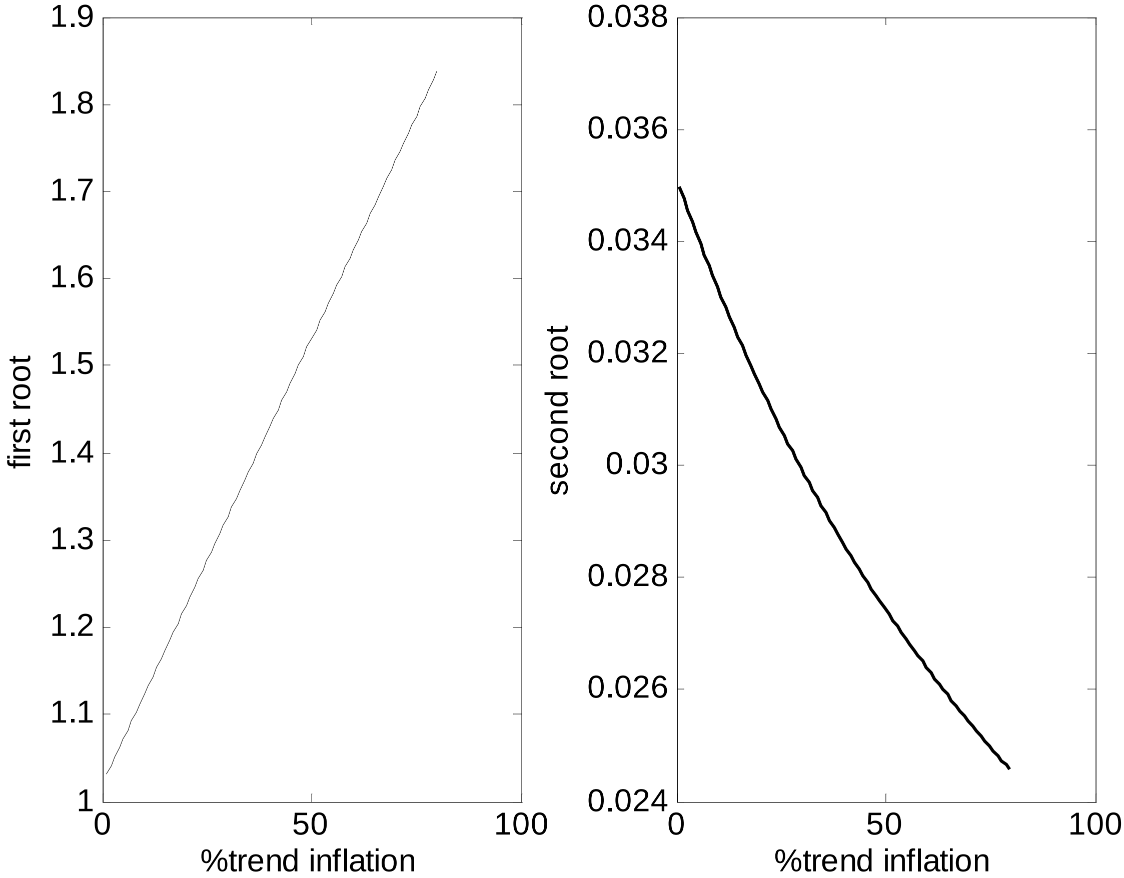

Figure 2 - The stable arm for different trend inflation rates

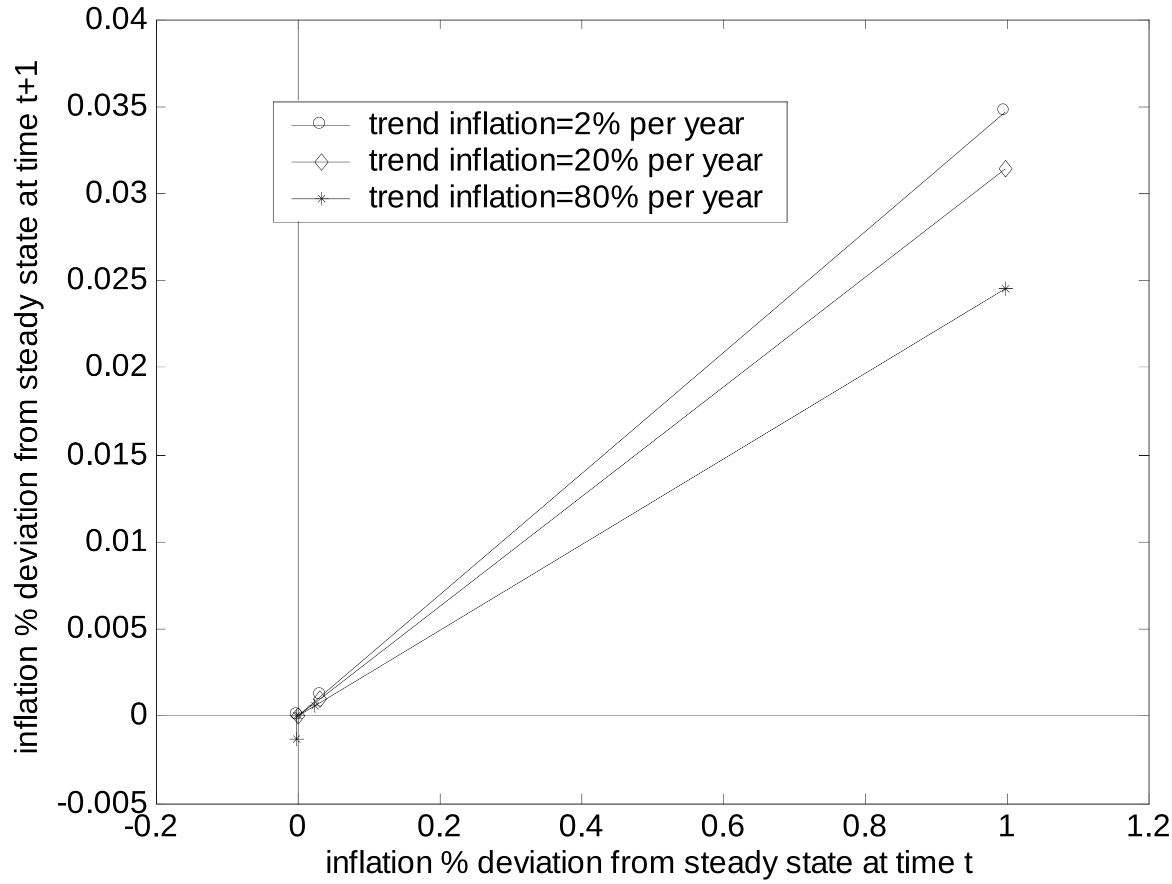

Figure 3 – The short-run Phillips curve with fixed and varying capital

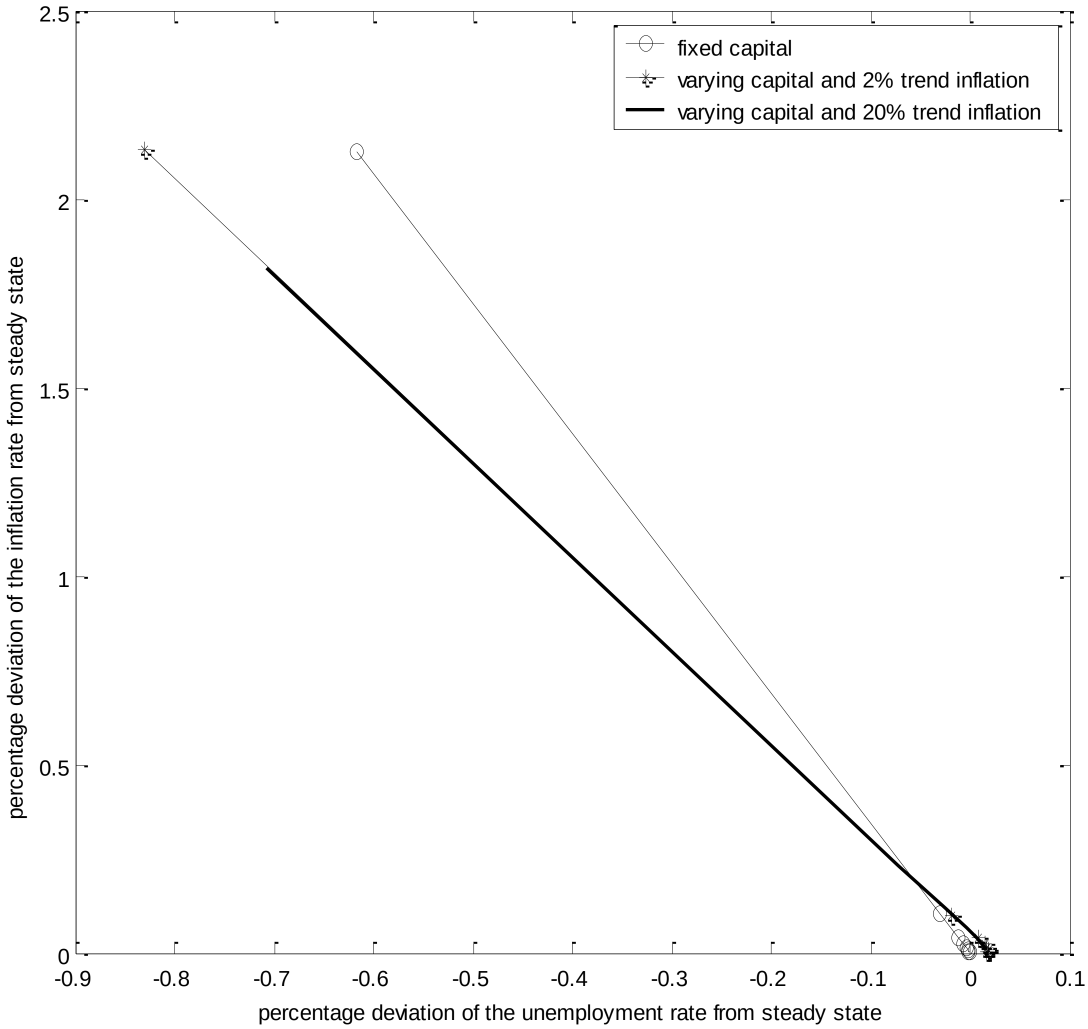

Figure 4 – Impulse response functions of the nominal interest rate after a 1% monetary shock under different trend inflation rates