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JEL classification: G21, G01, E42

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Core-Periphery Structure in the Overnight Money Market: Evidence from the e-MID Trading Platform.[†]

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This version: March 2012

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1 Introduction and Existing Literature

Interbank markets allow banks to exchange central bank money in order to share liquidity risks.[1] At the macro level, however, a high number of bank connections could give rise to systemic risk.[2] Since it is well known that the structure of a network is important for its resilience,[3] policymakers need information on the actual topology of the interbank network.

The experiences of the last few years have made policymakers aware of the necessity of gathering information on the structure of the financial network in general and the interbank market in particular.[4] One reason for the previous scarcity of research on the connections between financial institutions is certainly the limitation of available data,[5] the other reason being the neglect of the internal structure of the financial system by the dominating paradigm in macroeconomics during the last quarter of a century.[6]

Recent research in the natural sciences has significantly advanced our understanding of the structure and functioning of complex networks. Network ideas have been applied to very diverse areas and data sets such as the internet, epidemiology, ecosystems, scientific collaboration and financial markets, to name a few.

Most previous studies on the topology of interbank markets have been conducted by physicists applying measures from the natural sciences to a network formed by interbank liabilities. Examples include Boss *et al.* (2004) for the Austrian interbank market, Inaoka *et al.* (2004) for the Japanese BOJ-Net, Soramäki *et al.* (2006) for the US Fedwire network, Bech and Atalay (2010) for the US Federal funds market, and De Masi *et al.* (2006) and Iori *et al.* (2008) for the Italian e-MID (electronic market for interbank deposit). Overall, the most important findings of this literature are: (1) interbank networks are sparse, i.e. their density is relatively low,[7] (2) degree distributions appear to be scale-free (with coefficients between 2-3),[8] (3)

[1]See Ho and Saunders (1985), Freixas *et al.* (2000) and Allen and Gale (2000).

[2]Systemic risk is closely related to financial contagion, see de Bandt and Hartmann (2000), and implies that an idiosyncratic shock causing the failure of one or few institutions may destabilize the entire system.

[3]See also Allen and Gale (2000).

[4]See Haldane (2009), Haldane and May (2011) and Trichet (2011).

[5]See Mistrulli (2007).

[6]See Colander *et al.* (2009) for a more general critique.

[7]The density of a network is simply the fraction of existing links, relative to the maximum possible number of links. Ignoring the diagonal elements, the density can be calculated as $M/(N^2 - N)$, with M being the number of observed links and N the number of active nodes (banks).

[8]The in-degree is the number of incoming links, while the out-degree is the number of

transaction volumes appear to follow scale-free distributions as well, (4) clustering coefficients are usually quite small, (5) interbank networks are close to 'small world' structures, and (6) the networks show disassortative mixing, i.e. high-degree nodes tend to trade with low-degree nodes, and vice versa.[9] This indicates that small banks tend to trade with large banks, but rarely among themselves. Thus, we might expect the interbank network to display some sort of hierarchical community structure.

In passing, many authors have indeed remarked that there seemed to be some kind of community structure in the interbank network they analyzed. For example, Boss *et al.* (2004) note that the Austrian interbank network shows a hierarchical community structure that mirrors the regional and sectoral organization of the Austrian banking system. Soramäki *et al.* (2006) show that the network includes a tightly connected core of money-center banks to which all other banks connect. Thus there is some form of tiering in the interbank market. The empirical findings of Cocco *et al.* (2009) also show that relationships between banks are important factors to explain differences in interest rates.

Identifying communities in networks is an important aspect and in this paper we are concerned with the identification of the set of arguably systemically important (core) banks. In order to do so, we estimate various versions of core-periphery models in the spirit of Borgatti and Everett (2000).[10] Similar to De Masi *et al.* (2006) and Iori *et al.* (2008) we use data from the Italian e-MID trading platform, which is a market for unsecured deposits virtually covering the entire domestic overnight deposit market in Italy. Core-periphery models have been applied in a number of interesting fields before, for example to identify the spreaders of sexually transmitted diseases (see Christley *et al.* (2005)), in protein interaction networks (see Luo *et al.* (2009)), and to identify opinion leaders in economic survey data (see Stolzenburg and Lux (2011)). To our knowledge, Craig and von Peter (2010) is the first and so far only contribution applying a core-periphery structure to an interbank market. Applying this core-periphery framework to a data set of credit relationships between German banks,[11] their results speak in favor of

outgoing links per bank.

[9] Quite interestingly, the conventional explanation of the scale-free degree distribution is that of preferential attachment. Note that this is rather the opposite of disassortative mixing.

[10] Another interesting approach in using network-based measures for financial regulation is presented in Markose et al. (2010). The authors construct a so-called super-spreader tax based on eigenvector centrality.

[11] The authors use comprehensive statistics from the so-called 'Gross- und Millionenkreditstatistik' (statistics on large loans and concentrated exposures) from the Deutsche Bundesbank. In Germany, financial institutions have to report (on a quarterly basis) their

a very stable set of core banks. Furthermore, they show that core membership can be predicted using bank-specific features such as balance sheet size.[12] In this paper we will apply the (unrestricted) discrete core-periphery model, the (restricted) tiering model due to Craig and von Peter (2010) as well as symmetric and asymmetric versions of a continuous core-periphery model (hitherto not applied to interbank data) to a different set of interbank market data. Using a detailed dataset containing all overnight interbank transactions in the Italian interbank market from January 1999 to December 2010, we find that a core-periphery structure provides a better fit for these interbank data than alternative network models. The identified core shows a high degree of persistence over time, consisting of roughly 28% of all banks before the global financial crisis and 23% afterwards. We can classify the majority of core banks as intermediaries, i.e. as banks both borrowing and lending money in the market. Furthermore, allowing for asymmetric 'coreness' with respect to lending and borrowing activity considerably improves the fit, and reveals more concentration in borrowing than lending activity of money center banks. We also shed light on the development during the financial crisis of 2008, finding that the reduction of interbank lending was mainly due to core banks' reducing their numbers of active outgoing links.

The remainder of this paper is structured as follows: section 2 gives a brief introduction into necessary terminology for the formalisation of (interbank) networks, section 3 introduces the Italian e-MID interbank data and highlights some of its important properties. Section 4 introduces different variants of the core-periphery model. Section 5 presents the results and different robustness checks. Section 6 discusses the findings and section 7 concludes. A set of appendices provides more technical details as well as further robustness checks.

2 Networks

A network consists of a set of N nodes that are connected by M edges (links). Taking each bank as a node and the interbank positions between them as links, the interbank network can be represented as a square matrix of dimension $N \times N$ (data matrix, denoted $\mathbf{D}$).[13] An element d_{ij} of this

total exposure to each counterparty to whom they have extended credit of at least 1.5 million Euros or 10% of their liable capital to the Bundesbank. These reports include outstanding claims of any maturity.

[12] We cannot carry out such an analysis since we do not observe bank IDs, see below.

[13] In the following, matrices will be written in bold, capital letters. Vectors and scalars will be written as lower-case letters.

matrix represents a gross interbank claim, the total value of credit extended by bank i to bank j within a certain period. The size of d_{ij} can thus be seen as a measure of link intensity. Row (column) i shows bank i's interbank claims (liabilities) towards all other banks. The diagonal elements d_{ii} are zero, since a bank will not trade with itself.[14] Off-diagonal elements are positive in the presence of a link and zero otherwise.

Interbank data usually give rise to directed, sparse and valued networks.[15] However, much of the extant network research ignores the last aspect by focusing on binary adjacency matrices only. An adjacency matrix $\mathbf{A}$ contains elements a_{ij} equal to 1, if there is a directed link from bank i to j and 0 otherwise. Since the network is directed, both $\mathbf{A}$ and $\mathbf{D}$ are asymmetric in general. In this paper, we also take into account valued information by using both the raw data matrix as well as a matrix containing the number of trades between banks, denoted as $\mathbf{T}$. In some cases it is also useful to work with the undirected version of the adjacency matrices, $\mathbf{A}^u$, where $a^u_{ij} = \max(a_{ij}, a_{ji})$.

As usual, some data aggregation is necessary to represent the system as a network. In the following, we use quarterly networks. The next section summarizes the most important properties of our data, more detailed information can be found in Finger *et al.* (2012).

3 Dataset

The Italian electronic market for interbank deposits (e-MID) is a screen-based platform for trading of unsecured money-market deposits in Euros, US-Dollars, Pound Sterling, and Zloty operating in Milan through e-MID SpA.[16] The market is fully centralized and very liquid; in 2006 e-MID accounted for 17% of total turnover in the unsecured money market in the Euro area. Average daily trading volumes were 24.2 bn Euro in 2006, 22.4 bn Euro in 2007 and only 14 bn Euro in 2008.

Available maturities range from overnight up to one year. Most of the transactions are overnight. While the fraction was roughly 80% of all trades in 1999, this figure has been continuously increasing over time with a value of

[14] This is of course only true when taking banks as consolidated entities. There are, however, important examples of self-referential networks: the typical node in a connection matrix of the brain represents a group of neurons; in citation networks authors cite articles appearing in the same journal. See Boyd *et al.* (2010) for a discussion.

[15] Directed means that $d_{i,j} \neq d_{j,i}$ in general. Sparse means that at any point in time the number of links is only a small fraction of the $N(N-1)$ possible links. Valued means that interbank claims are reported in monetary values as opposed to 1 or 0 in the presence or absence of a claim, respectively.

[16] The vast majority of trades (roughly 95%) is conducted in Euro.

more than 90% in 2010.[17] As of August 2011, e-MID had 192 members from EU countries and the US. Members were 29 central banks acting as market observers, 1 ministry of finance, 101 domestic banks and 61 international banks. We will see below that the composition of the active market participants has been changing substantially over time. Trades are bilateral and are executed within the limits of the credit lines agreed upon directly between participants. Contracts are automatically settled through the TARGET2 system.

The trading mechanism follows a quote-driven market and is similar to a limit-order-book in a stock market, but without consolidation. The market is transparent in the sense that the quoting banks' IDs are visible to all other banks. Quotes contain the market side (buy or sell money), the volume, the interest rate and the maturity. Trades are registered when a bank (aggressor) actively chooses a quoted order. The platform allows for credit line checking before a transaction will be carried out, so trades have to be confirmed by both counterparties. The market also allows direct bilateral trades between counterparties.

The minimum quote size is 1.5 million Euros, whereas the minimum trade size is only 50,000 Euros. Thus, aggressors do not have to trade the entire amount quoted.[18] Additional participant requirements, for example a certain amount of total assets, may pose an upward bias on the size of the participating banks. In any case, e-MID covers essentially the entire domestic overnight deposit market in Italy.[19]

We have access to all registered trades in Euro in the period from January 1999 to December 2010. For each trade we know the two banks' ID numbers (not the names), their relative position (aggressor and quoter), the maturity and the transaction type (buy or sell). As mentioned above, the majority of trades is conducted overnight and due to the global financial crisis (GFC) markets for longer maturities essentially dried up. We will focus on all overnight trades conducted on the platform, leaving a total number of 1,317,679 trades. The large sample size of 12 years allows us to analyze the network evolution over time. Here we focus on the quarterly aggregates, leaving us with 48 snapshots of the network.

[17] This development is driven by the fact that the market is unsecured. The recent financial crisis made unsecured loans in general less attractive, with stronger impact for longer maturities. See below. It should be noted, that there is also a market for secured loans called e-MIDER.

[18] The minimum quote size could pose an upward bias for participating banks. It would be interesting to check who are the quoting banks and who are the aggressors. Furthermore it would be interesting to look at quote data, as we only have access to actual trades.

[19] More details can be found on the e-MID website, see http://www.e-mid.it/.

The left panel of Figure 1 shows the development of the number of active banks over time. We see a clear downward trend in the number of active Italian banks over time (green line), whereas the additional large drop after the onset of the GFC is mainly due to the exit of foreign banks. The right panel shows that the decline of the number of active Italian banks went along with a relatively constant trading volume in this segment until 2008. This suggests that the decline of active Italian banks was mainly due to mergers and acquisitions within the Italian banking sector. The overall upward trend of trading volumes was due to the increase of active foreign banks until 2008, while their activities in this market virtually faded away after the onset of the crisis.

The data show a trivial community structure in that foreign banks tend to trade with each other preferentially, and so do Italian banks. Due to the limited extent of trading between both components, and the smaller number of foreign banks, we will focus on Italian banks only in our subsequent analysis. This leaves a total number of 1,215,759 trades for the analysis.

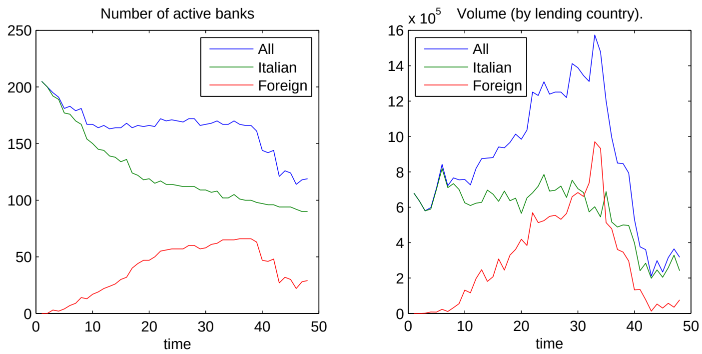

Figure 1: Number of active banks (left) and traded volume (right) over time. We also split the traded volume into money lent by Italian and foreign banks, respectively.

Other important findings are:

- The e-MID network has a relatively high density compared to other interbank networks investigated in the literature.[20] See Figures 1 and

[20]Note that the density in the German interbank network is smaller for two reasons: first, the number of active banks is much larger, so it is more likely to observe missing links. Second, in our analysis we focus on overnight trades only, while Craig and von Peter (2010) use aggregate credit volumes of all maturities (probably only with a small fraction of overnight trades). It seems plausible that the probability of observing a link between any two banks should be inversely related to the maturity of the loan.

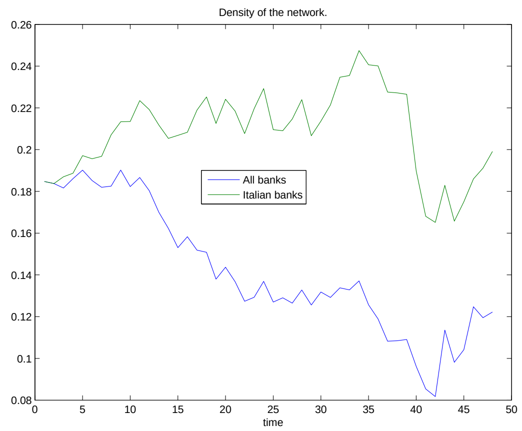

Figure 2: Density of the network over time, calculated as $M_t/(N_t^2 - N_t)$, with M_t being the number of observed links and N_t the number of active banks in the respective quarter. A Chow-test indicates that there is a structural break after quarter 39 at all sensible significance levels for the Italian banks. A CUSUM-test also indicates a structural break, however, the time series seems to revert towards its pre-GFC level.

2. For the density of the network formed by Italian banks, a Chow-test and a CUSUM-test both indicate that there is a structural break after quarter 39 (i.e. at the onset of the financial crisis). Later on, we will see that the core-periphery structure was also influenced by the GFC.

- The aggregation period is important for economic applications as the network structure is less volatile with longer aggregation periods. Since the network is sparse, short periods will only give an incomplete image of existing linkages, where many links between otherwise frequent trading partners may be dormant. In order to obtain a more comprehensive and less random picture of existing links, a larger aggregation period is required. We will, therefore, use quarterly data in the following (but results are robust to somewhat shorter or larger aggregation periods).

- There is very small (at times even negative) correlation between the banks' in- and out-degrees. Hence, the directed version of the network might contain important additional information.

- The underlying distributions of in- and out-degrees are apparently not scale-free at any aggregation level (including the daily level), cf. Finger *et al.* (2012). The same holds for transaction volumes.

- The network shows disassortative mixing patterns, so nodes with high overall degree (number of connections) tend to connect with low-degree nodes. We find similar assortativity coefficients for the relation between in- and out-degree, so high in-degree (out-degree) nodes tend to connect to low out-degree (in-degree) nodes.

In the next section, we will describe the different versions of the core-periphery model in detail.

4 Models

Core-periphery network models have been proposed first by Borgatti and Everett (2000). The basic idea is that a network can be divided into subgroups of core and periphery members. The discrete model partitions banks such that core (periphery) banks are maximally (minimally) connected to each other. The concept of discrete group membership can be extended by considering the core and periphery as opposite ends of a continuum. The continuous model overcomes the excessive simplicity of the discrete partitioning, by assigning a 'coreness' level to each bank. In the following we will first present the discrete model, with the tiering model proposed by Craig and von Peter (2010) as a special case, and then move on to the asymmetric continuous model for directed networks due to Boyd *et al.* (2010). Throughout the following we assume that a network cannot have more than one core.[21]

4.1 The Discrete Model

4.1.1 Formalisation

To identify the N_c core members among our sample of N banks, we aim at sorting the binary adjacency matrix such that we have the core-core region as a 1-block in the upper left part (of dimension $N_c \times N_c$) and the periphery-periphery region as a 0-block in the lower right part (of dimension $(N - N_c) \times (N - N_c)$). The idealized pattern matrix ($\mathbf{P}_I$) for a 'pure' core-periphery segmentation, then, looks as follows:[22]

$$\mathbf{P}_I = \begin{pmatrix} \mathbf{CC} & \mathbf{CP} \\ \mathbf{PC} & \mathbf{PP} \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{CP} \\ \mathbf{PC} & \mathbf{0} \end{pmatrix}, \tag{1}$$

where $\mathbf{1}$ and $\mathbf{0}$ denote submatrices of ones and zeros.

[21]Everett and Borgatti (2000) include the possibility of multiple cores.

[22]The diagonal elements will be ignored in all that follows, since the network is not self-referential.

The **CC**-block contains the top-tier banks, while the **PP**-block contains the periphery. Note that the off-diagonal blocks may be 1-blocks (each core member connected to all periphery-nodes), 0-blocks (no connection between core and periphery members) or something in between, depending on the problem. Borgatti and Everett (2000) claim that only the diagonal blocks are characteristic of CP structures and are thus the defining property. We will denote this version, without any restrictions on the off-diagonal blocks, as the discrete model.

In some cases however, the underlying model explicitly dictates requirements on the **CP** and **PC** blocks. For instance, Craig and von Peter (2010) propose a more strictly tiered interbank market than the benchmark discrete structure. In this model, a key characteristic of core banks (top tier) is that they intermediate between periphery banks. If at least a minimum level of intermediation activity is required of a 'core' bank, this means that **CP** and **PC** have to be row- and column-regular,[23] respectively, i.e. at least one entry has to be non-zero in each row of **CP** and in each column of **PC**.

4.1.2 Optimization Problem

The discrete core-periphery framework amounts to assigning to each bank the property of membership in the core or the periphery. This classification can be summarized in a vector c of zeros and ones of length N (the total number of banks). The usual approach to find the optimal coreness vector, c, referred to as the minimum residual (MINRES) approach, is to fit a pattern matrix $\mathbf{P} = cc'$, which should be as close as possible to the observed network matrix $\mathbf{A}$. This requires to identify the core banks, which are unknown a priori.

We start by defining a coreness vector, ordering the core banks first and writing the set of core members as $\mathcal{C} = \{1, \cdots, N_c\}$.[24] Then we can measure the 'fit' of the corresponding core-periphery structure as the total number of inconsistencies between the observed network and the idealized pattern matrix $\mathbf{P}_I$ of the same dimension. Depending on the problem, the distance involves certain restrictions on the off-diagonal blocks, **CP** and **PC**. The optimal partition $\mathcal{C}^*$ thus minimizes the residuals and gives the optimal set of core banks.

Residuals are obtained by simply counting the errors in each of the four blocks of Eq. (1) and aggregating over the blocks. The core-core block should

[23]See Doreian *et al.* (2005).

[24]Note that in order to have a core, N_c has to be ≥ 2. Also note the difference between $\mathcal{C}$ and c: $\mathcal{C}$ is the set of core banks and thus is a vector of dimension N_c, while c is a vector of zeros and ones. Of course, both $\mathcal{C}$ and c carry the same information.

be a complete **1**-block of dimension N_c, so any missing link represents an inconsistency (residual) with respect to the model.[25] Likewise any link between two periphery banks constitutes an error relative to the benchmark. Obviously, we can introduce any constraints on the off-diagonal blocks, so the tiering model can be easily implemented here as well: errors in the off-diagonal blocks penalize zero rows and columns, because these are inconsistent with row- and column-regularity, respectively. For example, a zero column could be penalized by as many errors as there are banks in the periphery $(N - N_c)$.

For the general version of the discrete model with arbitrary off-diagonal blocks, the aggregate errors in the individual blocks can be written as

$$\mathbf{E}(\mathcal{C}) = \begin{pmatrix} E_{CC} & E_{CP} \\ E_{PC} & E_{PP} \end{pmatrix} = \begin{pmatrix} N_c(N_c - 1) - \sum_{i,j \in \mathcal{C}} a_{ij} & 0 \\ 0 & \sum_{i,j \notin \mathcal{C}} a_{ij} \end{pmatrix}. \tag{2}$$

The total error score (e) then simply aggregates the errors across the relevant blocks, normalized by the total number of links in the network.[26] Formally this can be written as

$$e(\mathcal{C}) = \frac{E_{CC} + E_{CP} + E_{PC} + E_{PP}}{M} = \frac{E_{CC} + E_{PP}}{M}, \tag{3}$$

with $e(\cdot)$ being a function of $\mathcal{C}$ since every possible partition is associated with a particular value of e.

For the tiering model proposed by Craig and von Peter (2010), the aggregate errors in the off-diagonal blocks can be calculated as

$$E_{CP} = (N - N_c) \sum_{i \in \mathcal{C}} \max(0, 1 - \sum_{j \notin \mathcal{C}} a_{ij}) \tag{4}$$

and

$$E_{PC} = (N - N_c) \sum_{j \in \mathcal{C}} \max(0, 1 - \sum_{i \notin \mathcal{C}} a_{ij}), \tag{5}$$

respectively, leading to additional non-zero entries in $e(\mathcal{C})$.

The optimal partition $\mathcal{C}^*$ is the set of core banks producing the smallest distance to an idealized pattern matrix of the same dimension, i.e.

$$\mathcal{C}^* = \arg\min e(\mathcal{C}) = \{\mathcal{C} \in \Omega | e(\mathcal{C}) \leq e(C) \forall C \in \Omega\}, \tag{6}$$

where Ω denotes all strict and non-empty subsets of the population $\{1, \cdots, N\}$. It should be noted, however, that the discrete approach implicitly assumes

[25] The maximum number of possible inconsistencies in this block would be $N_c(N_c - 1)$ since the main diagonal is ignored. This upper bound is obviously never reached since otherwise there would be no core-periphery structure.

[26] Note that M is the maximum error possible in a network consisting only of a periphery.

symmetry of the underlying structure (or irrelevance of the direction of links). Therefore, in Section 4.2 we will turn to a continuous core-periphery model, which explicitly takes the directed nature of the network into account, characterizing coreness by two vectors rather than one.

4.1.3 Implementation

Fitting the discrete and the tiering model to a real-world network is a large scale problem in combinatorial optimization. Exhaustive search becomes impractical for large matrices, since the number of possible labeled bipartitions increases exponentially with the dimension of the matrix. More precisely, the number of nontrivial bipartitions (with both the core and the periphery having at least two members) is $2^N - 2N - 2$. The term 2^N corresponds to the number of all possible subsets, while the negative terms exclude partitions with only core or periphery banks. For example, with $N = 10$ banks there are 1002 nontrivial possible bipartitions. For a system with $N = 100$ banks there are already roughly 10^{30} partitions.

A number of algorithms have been applied to tackle such problems. We will use a Genetic Algorithm (GA) to fit both the discrete and the tiering model.[27] A GA uses operations similar to genetic processes of biological organisms to develop better solutions of an optimization problem from an existing population of (randomly initiated) candidate solutions. Typically the proposed solutions are encoded in strings (chromosomes) mostly using a binary alphabet, i.e. in our setting the strings have length N and consist of ones and zeros, depending on whether a bank is in the core or periphery. We use the rate of correct classifications (in terms of the error score) by a string l, $f_l = 1 - e(\mathcal{C}_l)$ as a fitness function that drives the evolutionary search. Details are explained in Appendix A.1.

4.2 The Continuous Model

4.2.1 Basic Structure

One limitation of the partition-based approach presented above is the excessive simplicity of defining just two homogeneous classes of nodes: core and periphery. Assuming that the network data consist of continuous values representing strengths or capacities of relationships (for banking data:

[27] We cross-checked the results using the sequential algorithm applied in Craig and von Peter (2010). Alternatives would be the Kerninghan-Lin Algorithm (Kernighan and Lin (1970)), see Boyd *et al.* (2006) for an application, and Branch-and-Bound Programming, see Brusco (2011).

credit volumes or number of transactions), it seems sensible to also consider a continuous model in which each node is assigned a measure of 'coreness'. Since a continuous measure of coreness allows for more flexibility in capturing the role of an institution, we apply this model to the valued matrix $\mathbf{D}$ of interbank liabilities rather than the binary adjacency matrix $\mathbf{A}$.

The usual approach in the symmetric continuous (SC) model is to find a coreness vector c, where $1 \geq c_i \geq 0 \; \forall i$, with pattern matrix $\mathbf{P} = cc'$ that approximates the observed data matrix as closely as possible. Similar to the presentation of the discrete model, the optimal coreness vector in the symmetric continuous (SC) model can be found using the MINRES approach.[28] Again however, this method imposes a symmetric pattern matrix, i.e. $p_{ij} = p_{ji} \; \forall i, j$. Thus, it is assumed that the strength of the relation from i to j is the same as that from j to i. To overcome this restriction, we also estimate an asymmetric continuous (AC) core-periphery model, as introduced by Boyd *et al.* (2010). This formulation involves two vectors, representing the degrees of outgoing and incoming centrality for each node. For networks of international trade, for example, the two vectors would correspond to exports and imports, respectively. In our setting, the two vectors correspond to out- and in-coreness. Note that both the SC and AC model can be applied to valued matrices, with binary adjacency matrices being just a special case. Thus the continuous models might allow us to extract important additional information from the directed, valued networks. However, a disadvantage of the continuous models is that restrictions, such as the tiering model, cannot be implemented. In the following, we will briefly introduce both model versions. More details on the AC model can be found in Appendix A.3.

4.2.2 The Symmetric Continuous (SC) Model

The SC model will again be estimated by minimization of residuals. MINRES seeks a column vector c such that the square matrix $\mathbf{D}$ is approximated by the pattern matrix $\mathbf{P} = cc'$. Ignoring the diagonal elements, this amounts to minimizing the sum of squared differences of the off-diagonal elements, or

$$\arg\min_{c} \sum_{i} \sum_{j \neq i} (d_{ij} - c_i c_j)^2. \tag{7}$$

In the same spirit as with our optimization algorithm in the discrete case, we use the proportional reduction of error (PRE) as our measure of fit. PRE

[28] An interesting alternative approach, based on the Kullback-Leiber distance, can be found in Muñiz and Carvajal (2006) and Muñiz *et al.* (2011).

is defined as

$$\mathrm{PRE}(cc'|\langle D\rangle) = 1 - \frac{SS(\mathbf{D} - cc')}{SS(\mathbf{D} - \langle D\rangle)}, \tag{8}$$

with $\langle D\rangle$ being the global average (across all elements, excluding the diagonal) of $\mathbf{D}$ and $SS(\cdot)$ is the sum of squared deviations of the off-diagonal elements of the input matrix. Thus, maximizing the PRE is equivalent to minimizing $SS(\mathbf{D} - cc')$. Boyd *et al.* (2010) argue that the continuous core/periphery model makes a reasonable contribution towards explaining empirical structures if the PRE significantly exceeds 0.5. Note that the reported coreness vectors in both the SC and AC model will be standardized by the Euclidean norm of the optimal solution vectors.

4.2.3 The Asymmetric Continuous (AC) Model

The idea of the asymmetric continuous (AC) model is to decompose overall 'coreness' into 'out-coreness' and 'in-coreness' (denoted by u_i and v_i in the following), respectively. Applying this distinction allows us to write the objective function for the AC model as

$$\arg\min_{u,v} \sum_i \sum_{j\neq i} (d_{ij} - u_i v_j)^2. \tag{9}$$

The optimal coreness vectors can be determined by finding the roots of the first-order conditions of Eq. (9).[29] The PRE of the AC model can be defined similarly as in Eq. (8) as

$$\mathrm{PRE}(cc'|\langle D\rangle) = 1 - \frac{SS(\mathbf{D} - uv')}{SS(\mathbf{D} - \langle D\rangle)}. \tag{10}$$

For both the SC and the AC models, we will, in order to adjust for the skewness of the network matrices, log-transform the data matrix in the form $\log(1 + \mathbf{D})$, where the factor 1 makes sure that zeros in the original matrix remain zeros in the transformed matrix.[30] Note that the split into in- and out-coreness is germane to a singular value decomposition of our matrix $\mathbf{D}$ of interbank liabilities. This similarity is exploited in the empirical estimation of the vectors u and v. Our numerical approach for estimating these two coreness vectors follows Boyd *et al.* (2010) and is detailed in Appendix A.3.

[29] This could be implemented by using standard algorithms for numerical optimization. Here we used a trust-region algorithm.

[30] We also tried to fit the core-periphery models to the raw network matrices, however, the high level of skewness in the data results in a very poor fit in general. These results are hardly comparable to those presented below, see Appendix A.9.

5 Results

This section presents and discusses the results from the different versions of the core-periphery framework. In the following, as noted above, we focus on the quarterly networks formed by Italian banks only. Robustness checks, using different aggregation periods and sample banks can be found in the Appendix.[31] Recall that the discrete and tiering model use the (binary) adjacency matrices $\mathbf{A}$, while the continuous model uses the valued matrix of transaction volumes $\mathbf{D}$, as defined in section 2.[32]

As a first step, we compare the coreness vectors between the different models. It will become clear that the discrete and tiering model are almost identical throughout. Later on, we show that the AC model contains important information from the asymmetric nature of the network, since the in- and out-coreness vectors are far from being perfectly correlated. Secondly, we investigate the properties of the core/periphery banks. We find that the core is large compared to the findings in Craig and von Peter (2010), but also very persistent over time. Due to the high network density, we find that the error scores are also much higher compared to the German market. In particular, the model fit deteriorates over time due to the GFC. Formal tests suggest a significant worsening of the fit of the core-periphery model after the GFC, pointing towards the breakdown some part of the core-periphery structure. As a last step, we investigate the significance of the results by comparing the identified cores and the corresponding error scores to the cores obtained from random and scale-free networks, calibrated to share similar properties as the observed ones along certain dimensions. Here we find that the identified cores are significant, i.e. the identified core-periphery structure is not a spurious network property.

5.1 Model Similarity

Table 1 presents selected correlations between the identified coreness vectors of the different model versions. For each combination, we compute the correlation between the (stacked) coreness vectors for the complete sample period. Note that the discrete and tiering coreness vectors contain only binary values, while the in- and out-coreness vectors contain real numbers. Obviously the correlation between the cores in the discrete and tiering model

[31] Appendix A.6 discusses the findings for other aggregation periods, most importantly for monthly and yearly networks. Appendix A.7 discusses the results when including foreign banks to the analysis.

[32] Appendix A.8 discusses the results for the continuous model using the matrix containing the number of transactions $\mathbf{T}$. Appendix A.9 discusses further robustness checks.

is very high with a value of around .95.[33] The same is true for the discrete core and the out-coreness with a value of .73, whereas the correlation between the discrete core and the in-coreness is much smaller with a value of .26.[34] Core banks from the discrete model are therefore more likely to be in the out-core of the continuous model as well, but not necessarily in the in-core. This result seems rather surprising at first, since for example the results from Cocco *et al.* (2009) suggest that small (periphery) banks are net lenders, which offer their excess liquidity to a preferred set of large (core) banks. Our analysis shows that at least in the present data set, the pattern of interbank linkages is more complex: again, periphery banks lend money to a relatively small set of selected core banks, but the core banks in turn tend to redistribute this liquidity not only among the other core banks, but also among a larger part of the periphery. Technically, we find that the density in the CP-block is on average three times higher than the density in the PC-block (see Figure 6 below), so for most core banks the out-degree clearly exceeds the in-degree.[35] Therefore, it is not surprising that the correlation between the discrete and the out-coreness is higher than the correlation with the in-coreness. This shows that there is a considerable amount of asymmetry in the network, also captured by the negative correlation of -.08 between the in- and out-coreness vectors, cf. Figure 3. We see that these relations are rather stable. Interestingly, the correlation between in- and out-coreness was always the smallest of these combinations, turning negative after 12 quarters and remaining so for the rest of the sample period. This hints towards the existence of different subgroups in the core.

In the following we present more detailed results for the discrete and tiering model, then moving on to the continuous model.

[33]Therefore, the correlations between the tiering core and the in-/out-coreness are not presented here since they are very similar to those from the general discrete model.

[34]Interestingly, we see that the correlation between the coreness vectors from the (symmetric) discrete and the SC model is only .7578. One might expect that this is partly driven by the fact that the input matrix is valued, rather than binary in the continuous case. Estimating the continuous model with binary network matrices, however, yields very similar results, see Appendix A.9, with a correlation of .7635. Thus, the main reason for the low correlation between the two vectors lies in the objective function: the continuous models approximate the complete matrix, while the discrete model focuses on the diagonal blocks.

[35]This also explains the small (at times even negative) correlation between individual banks' in- and out-degree.

Models		Correlation
Discrete	Tiering	.9526
Discrete	Out-coreness	.7267
Discrete	In-coreness	.2567
Discrete	Sym. coreness	.7578
In-coreness	Out-coreness	-.0809

Table 1: Correlations between individual coreness vectors of different models. For each model, we stack the coreness vectors over the entire sample period in a single vector. Then we compute the correlations between each combination. Note that the discrete and tiering coreness consists of binary values, while the in-, out-, and symmetric coreness vectors contain real numbers.

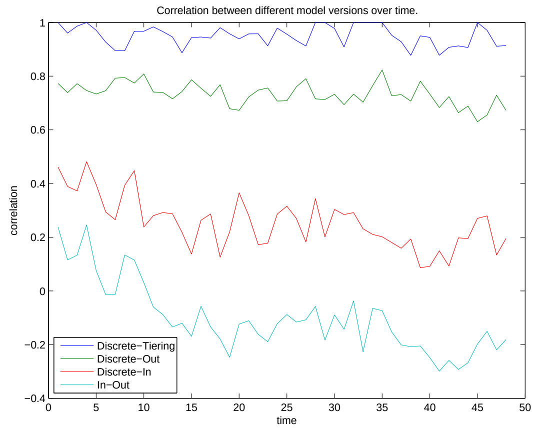

Figure 3: Time-varying correlations between different coreness vectors.

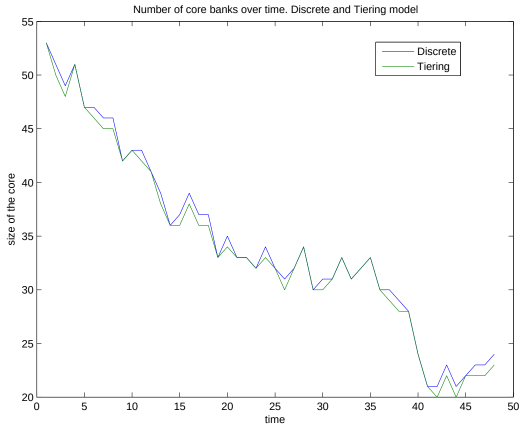

Figure 4: Absolute size of the core over time. A Chow-test indicates that there is a structural break for the detrended time series after quarter 10, while there is no evidence for a significant structural break after quarter 39. An additional CUSUM test indicates that this break is significant at all sensible confidence levels. We also note a significant level of autocorrelation in the detrended time series, while the first difference of the original time series is stationary.

5.2 Discrete and Tiering Model

5.2.1 The Size of the Core and Periphery

We saw that the identified cores in the discrete and tiering model are highly correlated. In fact, Figure 4 shows that the sizes of their cores are very similar over time. Note that the core in the discrete model is always at least as large as the core in the tiering model. The reason lies in the requirement that all core banks in the tiering model act as intermediaries, which is not necessarily true for the discrete model, even though again the vast majority of core banks acts as intermediaries in this case. Overall, the differences between the two model versions consist of a few borderline cases.[36]

Note also the negative trend in the absolute size of the cores over time. This is not surprising given that the number of active Italian banks has been decreasing over time. Interestingly, a Chow-test indicates the existence of a structural break in the (detrended) core sizes after quarter 10, with the trend going back towards its initial level in the post-GFC period.[37] The

[36]In cases where the row- and column-regularity constraints are binding, it may also happen that core banks from the discrete model are part of the periphery in the tiering model.

[37]Iori *et al.* (2007) also mention this structural break in the Italian interbank network in quarter 10, however, without conducting formal tests. They relate this breakpoint to

economic significance of this result is, however, questionable as we see in Figure 4 that a linear negative trend might fit the entire sample period quite well, and we know that the sharp drop after quarter 39 was due to the GFC. Given the overall trend in the number of active banks, it seems more interesting to consider the relative size of the core compared to the complete interbank network. Figure 5 shows that the relative size of the core is rather stable over time, fluctuating around 28% before the GFC, and around 23% afterwards. A Chow-test indicates that there is a structural break after quarter 39. However, under a CUSUM test this break is only marginally significant at the 5% level for the discrete model, and insignificant for the tiering model. Thus, there is some evidence that the GFC has led to a structural break in the formerly relatively stable structure of intermediation in the interbank market. However, we also see a positive trend in the core sizes for the last 3 quarters of the sample period, so that the relative core size seemed to revert to its pre-GFC level. Not surprisingly, the size of the core is highly correlated with the density of the network (cf. Figure 2). We should note that relative core sizes are very high compared to the value of 3% found for the German interbank market by Craig and von Peter (2010). This is driven by the very high overall network density of above 20%, compared to only 0.61% for the German market.[38]

The left panel of Figure 6 shows the densities of the complete network and the core-core and periphery-periphery subnetworks over time. Since results are virtually the same for both models, we only display those of the baseline discrete model[39] with rather stable values for the pre-GFC period, but again with a structural break after quarter 39 for all time series in the Figure. The density in the CC-block is at least 2.5 times that of the entire network and at least 6 times that of the PP-block. The right panel of Figure 6 shows the densities in the off-diagonal blocks. As already mentioned, the density in the CP-block is three times higher than the corresponding density in the PC-block. These values are very stable over time, and we do not find evidence for a structural break.

two events: (1) official and market interest rates changed their trend from positive to negative, (2) the ECB tried to support economic growth by increasing the amount of liquidity provided.

[38]Recall that the number of banks in the German market is roughly 1800, so the network is at least 10 times larger than the Italian network. Thus it is not surprising, that the density is much higher in the Italian case. Since the e-MID sample presumably contains mainly large banks, our core might be the core of the overall banking network. See Figure 21 in Appendix A.2 for a network illustration for one particular quarter.

[39]Results from the tiering model are available upon request. We checked that the results from the tiering model are statistically not distinguishable from the results of the discrete model.

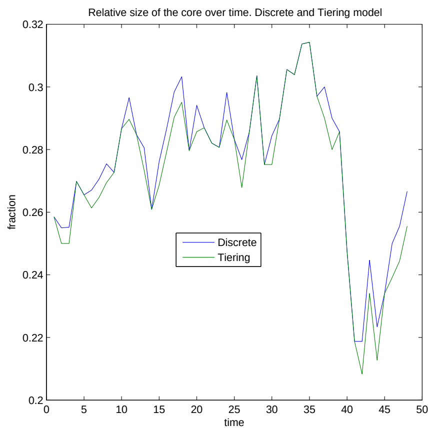

Figure 5: Relative size of the core over time. A Chow-test indicates that there is a structural break after quarter 39 at all sensible significance levels. An additional CUSUM test indicates that this break is marginally significant at the 5% level.

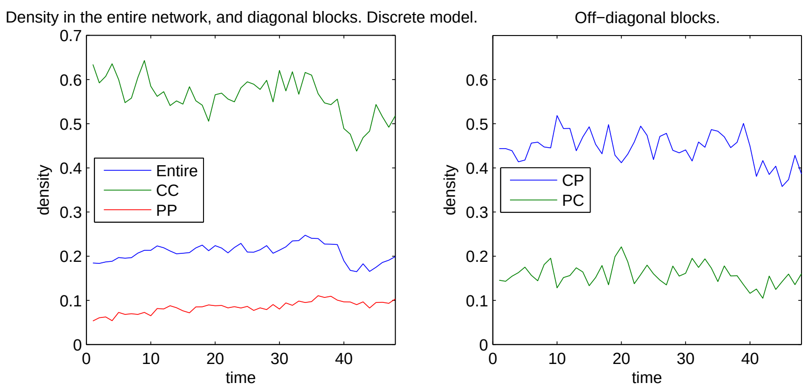

Figure 6: Density of the entire network, CC/PP blocks (left), and off-diagonal blocks (right). Individual Chow-tests indicate that there is a structural break in the time-series in the left panel after quarter 39 at all sensible significance levels (see also Figure 2). Additional CUSUM-tests indicate that the structural breaks are significant at all sensible significance levels, with the PP-density apparently containing an additional structural break around quarter 10. In contrast, we cannot reject the hypothesis of no structural break in the time-series of the right panel.

	I_t	L_t	B_t	E_t
I_{t-1}	.8845	.0752	.0190	.0214
L_{t-1}	.2971	.6508	.0009	.0513
B_{t-1}	.3661	.0164	.4590	.1585
E_{t-1}	.0049	.0038	.0028	.9885

Table 2: Transition matrix: trading strategies. I, L, B, and E denote intermediary, lender, borrower, and exit, respectively.

5.2.2 The Structure of the Core and Periphery

To gain more insights into the structure of the network, Figure 7 shows the fraction of intermediaries, lenders and borrowers in the complete network over time. Here we define borrowers as banks with an out-degree of zero but positive in-degree in a given quarter, whereas the reverse holds for lenders. The remaining banks, with both positive in- and out-degree, are thus intermediaries. We see that these fractions are relatively stable over time: most of the banks (roughly 75%) act as intermediaries, a smaller fraction acts as lenders (20%) and the remainder consists of sole borrowers. Interestingly, the fraction of sole borrowers seems to increase significantly after the GFC, since we find a structural break after quarter 39. This may hint towards the entry of banks who use the market only to attract funds. In contrast, there is no significant structural break for the fraction of intermediaries and lenders. Table 2 shows the transition probabilities for each strategy, with I_t denoting that a bank is an intermediary in t. L, B and E stand for lending, borrowing and exit, respectively. The matrix shows, for example, that with a probability of 88.45% an intermediating bank in $t-1$ will also be an intermediary in t. Note that the diagonal elements are largest, even though the borrowing strategy is less persistent over time compared to the other strategies. This is in line with the observation of a more intense entry of sole borrowers during and after the GFC.

Figure 8 shows the fractions of intermediaries, lenders, and borrowers in the core and periphery of the general discrete model. Again these results are very similar to those of the tiering model: the fraction of intermediaries in the core is highest, while the fraction of intermediaries in the periphery is second highest. As expected, only very small fractions of borrowers and lenders are found in the core (none in the tiering model), while banks that appear only as borrowers are a significant fraction (about 30 percent) of the periphery.

To elucidate the stability of these structural properties, consider Table

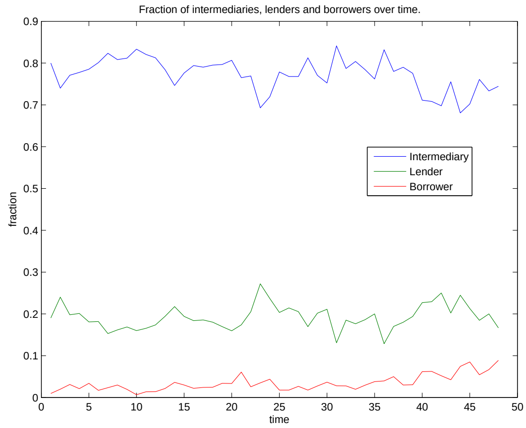

Figure 7: Fraction of intermediaries, lenders and borrowers over time. Individual Chow-tests point towards the existence of a structural break after quarter 39 in all time-series. Additional CUSUM-tests, however, indicate that this structural break is only significant for the fraction of borrowers.

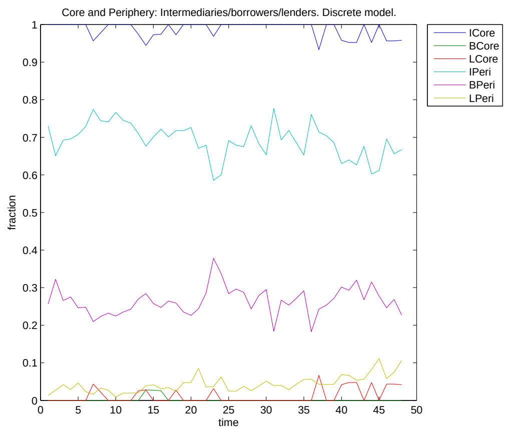

Figure 8: Structure of the core and periphery in the discrete model. Fractions of intermediaries, borrowers and lenders, in the core and periphery, respectively. *Note:* ICore=intermediaries in the core, BCore=borrowers in the core, LCore=lenders in the core. Similarly for the periphery.

	C_t	P_t	E_t
C_{t-1}	.8324	.1565	.0110
P_{t-1}	.0555	.9055	.0391
E_{t-1}	.0012	.0104	.9885

Table 3: Transition matrix: discrete model. C, P and E stand for core, periphery and exit, respectively.

3 containing transition probabilities of the state of a bank for the discrete model. For example, the first rows show the probabilities of a core bank in $t-1$ (C_{t-1}) being a core member in t, switching to the periphery (P_t) or exiting the market (E_t). There is some asymmetry in the Table, for example, the probability of switching from the core to the periphery is roughly 15.6%, while the reverse probability is only 5.5%. In particular, the diagonal entries are very high with values above 80%, such that there is significant persistence (autocorrelation) in the group memberships.[40]

The above transition probabilities are aggregate values over the entire sample period. To investigate the inherent structural stability, the values in this matrix should be roughly constant over time. Figure 9 shows the time evolution of these values for the discrete model. We see that the elements on the main diagonal are quite stable over time and very large in general. However $P(C|C)$ becomes smaller due to the GFC simply because a number of core banks become part of the periphery, which can be seen by the increase in $P(P|C)$ to more than 20%. Again we emphasize that we do not observe banks' names, so we are unable to track for example bank mergers and acquisitions.

Besides the overall structural stability, one might also be interested in the stability of the system at the micro-level of bilateral connections. In order to assess the stability of the link structure in the different blocks, we use the so-called Jaccard Index. This is defined as

$$J = \frac{M_{11}}{M_{01} + M_{10} + M_{11}}, \tag{11}$$

where M_{xy} is the number of relations with status x in period $t-1$ and with status y in the next period. It thus measures the similarity between subsequent graphs, taking only links into account which were present in at least one period. Social networks are usually considered to be sufficiently stable for values of J larger than .3, in which case the network is likely to

[40]Note that the structure is very stable despite the existence of a structural break due to the GFC after quarter 39, cf. section 5.5.

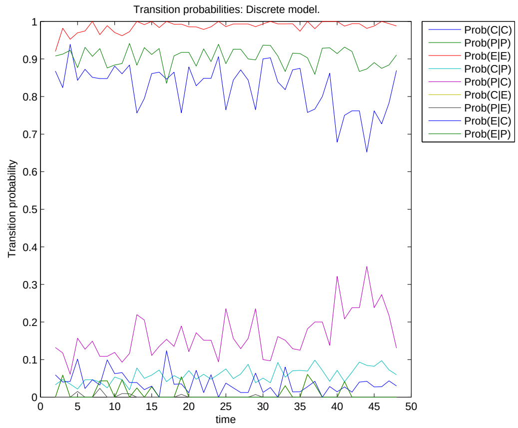

Figure 9: Transition probabilities over time, discrete model. $P(y|x)$ is the probability of going from state x to state y.

display recognizable structure.[41] For the complete Italian interbank network we observe an average Jaccard Index of .5302 (std. dev.: .0368).[42] When calculating the Jaccard Index for the different blocks, we restrict ourselves to those banks having the same status of being a core/periphery bank in the two adjacent quarters.[43] Figure 10 shows the results: the Jaccard Index is largest for the CC- and the CP-blocks with average values of .6273 and .6565 (std. devs: .0366 and .0380), respectively, i.e. two thirds of all links are maintained over adjacent quarters. These values are roughly 1.5 times larger than those in the PC- and PP-blocks, with average values of .4261 and .4241 (std. devs.: .0471 and .0622), respectively. Interestingly, we do not find significant evidence of a structural break due to the GFC in any of the time series. Even though the values dropped for most of the time-series after quarter 39 (except for the CP-block), the values tend to stabilize later around the pre-GFC levels. This might indicate that many interbank relationships tended to survive through the GFC.[44]

Overall, our calculations show that the outgoing links of core banks are highly persistent, both with respect to the core and the periphery. Outgoing links from the periphery are persistent as well, but to a significantly lower degree. Given that the Jaccard Index is independent of the density of the

[41] See Snijders *et al.* (2009).

[42] See Finger *et al.* (2012).

[43] Given that the coreness vectors are highly autocorrelated, this is not a very restrictive assumption, but it is likely to reduce some noise in the calculated numbers.

[44] Cf. Affinito (2011) and Braeuning (2011) for related evidence on the robustness of lending relationships over the crisis.

network (non-existing links are ignored), these findings indicate that core banks generally lend towards a large set of core and periphery banks. In contrast, periphery banks are not only reluctant to create links among themselves, but also, given the relatively small density in the PC-block, trade with a small set of core banks, which is not necessarily the same set in each quarter. This finding is interesting, since the persistence in the PC-block should be much larger, if periphery banks would have a preferred partner among the core banks. These findings may, however, be driven to some extent by the relatively small trading volumes of periphery banks (see below). In any case, the asymmetry between the CP- and PC-blocks is remarkable and will be discussed in more detail below.

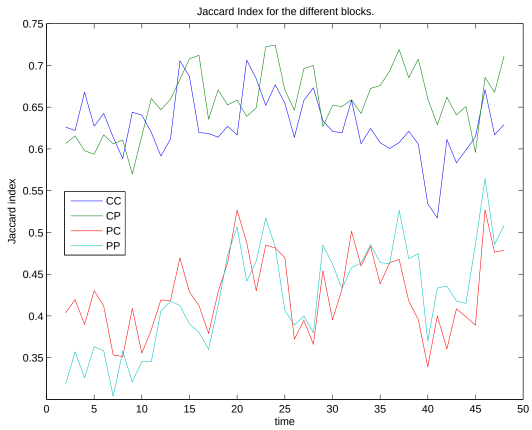

Figure 10: Jaccard Index for the CC-, CP-, PC-, and PP-blocks over time. Coreness is taken from the discrete model. Individual Chow- and CUSUM-tests show no evidence of a structural break due to the GFC in any of the time series. However, the CP- and PP-blocks appear to contain a structural break after quarter 10.

5.2.3 Model Fit and Significance

In this section we turn to a quantitative analysis of the error scores and their significance. When investigating the significance of our results, we compare the core sizes and error scores of the empirical networks with those of network structures sharing similar properties along certain dimensions. This analysis helps us evaluating whether the core-periphery structure offers a meaningful characterization of our data or whether the data rather generate a 'spurious' core by chance.

The left panel of Figure 11 shows that the error scores (fractions of residuals) are on average roughly 42%, which is rather high compared to the

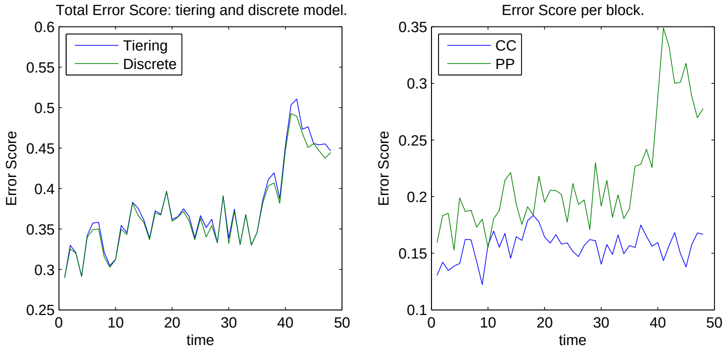

Figure 11: Left: Error score in tiering (blue) and discrete (green) model over time. A Chow-test indicates that there is a structural break after quarter 39 at all sensible significance levels. The results from an additional CUSUM-test are also in favor of the existence of a structural break. Right: Error score for the CC- and the PP-block in the discrete model. For the CC-block there is a significant structural break after quarter 10, while the PP-block contains a significant structural break after quarter 39.

maximum value of 12% for the German interbank market reported by Craig and von Peter (2010). These values are, however, way below unity, so the core-periphery model is indeed a better explanation of the data than an unstructured alternative consisting only of a periphery. We also see that the GFC made the fit somewhat worse, yielding an error score that is roughly 1.3 times the average score before the GFC, albeit with a declining trend. A Chow-test and a CUSUM-test again indicate the existence of a structural break after quarter 39 at all sensible significance levels. The right panel of Figure 11 shows that this structural break is mainly due to the increase in the error score in the PP-block. In contrast, we find no evidence for a structural break in the error score of the CC-block after quarter 39, but after quarter 10. Given that the relative core size has been significantly smaller, the overall picture is thus that some previous core banks have reduced their interbank activities so strongly that they are assigned to the periphery after quarter 39. We will investigate the effect of the GFC in more detail in section 5.5.

In order to shed light on the significance of the observed error scores, we compute the average core size and error scores by generating 100 random samples of particular network structures (see below) and compare the results to our findings above. The analyzed networks are:

- Erdös-Renyi (ER) random graphs, where a link is formed with probability p. The value of p will be set equal to the observed density of the network. This network is completely random and we do not expect to find a convincing core-periphery structure in this case. The error scores should be relatively high, since identified cores would be completely spurious. Note that this is tantamount to a bootstrap test for the significance of our identified core-periphery structures, as the random graphs could also be generated by random resampling of the empirical links (with replacement). If the error scores of the core-periphery model are below a certain percentage boundary of those obtained for the sample of random networks, we could exclude with a significance level equal to the inverse of that probability, that our results are spuriously obtained from a completely random system of interbank liabilities. As it turns out, all error scores are always way below the minimum obtained for the random networks.

- Scale-free random graphs, with scaling parameter 2.3.[45] Even though we found the degree distribution not to be scale-free, see Finger *et al.* (2012), most interbank markets appear to have a certain resemblance of their degree distributions to a scale-free distribution. Reported scaling parameters vary between 2 and 3, but are roughly similar for in- and out-degree. We generate these networks using the approach of Goh *et al.* (2001). Note that scale-free networks are assortative by definition, since high-degree nodes tend to connect with each other. Therefore we expect the scale-free network to have a much tighter core and significantly lower error scores.[46]

In the following we only discuss results from the discrete model to save space.[47] Most of the results were expected: all models show a structural break due to the GFC. The actual error scores lie between those from a completely random network (ER) and those of a scale-free network as can be seen from the left panel of Figure 12, where we plot the actual error score and the average error scores of the ER and scale-free networks (including plus and minus one standard error for the simulated models). Not surprisingly, the actual network is closer to a scale-free network even though the distance seems to increase with the GFC.

The right panel of Figure 12 shows the core sizes for the actual and random networks (again including one standard error for the simulated models).

[45]In actual interbank networks, the observed scaling parameters vary between 2 and 3. Here we take the value found by De Masi *et al.* (2006).

[46]Interestingly, Craig and von Peter (2010) found that the error scores for the German interbank market are significantly smaller than for SF networks.

[47]Again we note that the results are almost identical to those from the tiering model.

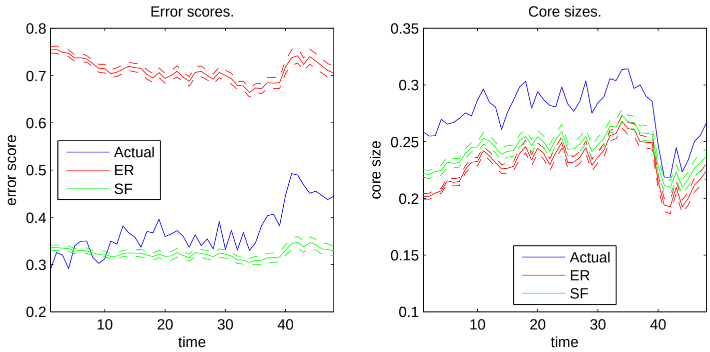

Figure 12: Error scores (left) and core sizes (right) in discrete model. Actual and random graphs. For the SF networks we used a scaling parameter of $\alpha = 2.3$.

We see that the observed core is significantly larger than both the core of the scale-free and ER networks. For the ER network, the core is spurious, while it would capture the most highly connected nodes in the scale-free network (although the core-periphery model would be a misspecification of the overall structure of such a network).

Overall, we find that we can reject the hypothesis that our findings are just artifacts of applying the core-periphery algorithms to random data, and we also find that the popular scale-free model could not have generated our particular sets of identified cores and fit of the model (error scores).

5.3 Continuous Model

We now move to the results from the continuous framework, mostly concentrating on the added explanatory power of the asymmetric version. We have seen in Table 1 and Figure 3 that the in- and out coreness vectors are mostly negatively correlated. Figure 13 shows a scatter-plot of the two variables, explicitly linking the findings to the results of the discrete model.[48] Obviously, core banks have on average a higher in- and out-coreness. Indeed, we see a relatively sharp distinction between core and periphery banks. Core banks (red) are typically characterized by a sum of their in- and out-coreness above .2, while this sum is lower for banks assigned to the periphery. For both categories, there might be a dominance of lending and borrowing or a more balanced composition of their transactions. The systemic importance of a bank, in terms of its in- and out-coreness, is therefore not identical in

[48] Recall that the coreness values from the continuous model are standardized values.

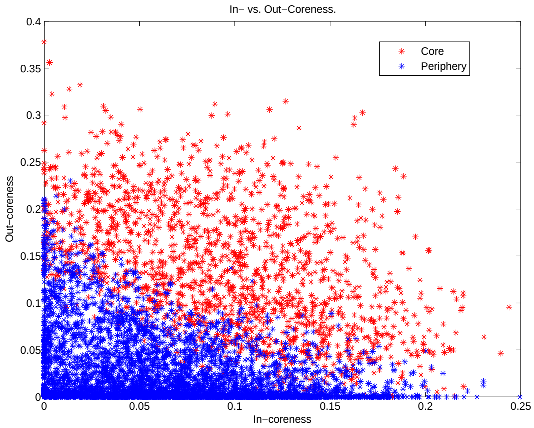

Figure 13: In-coreness vs. Out-coreness for all observations, by core and periphery, as indicated by the discrete model.

general.[49]

In Figure 14, we show the time-varying autocorrelations of the two coreness vectors. The autocorrelations were calculated as the correlation between two subsequent coreness vectors, using only banks that were active in both periods. We see that both the in- and out-coreness vectors are highly autocorrelated (average values: .8474 and .9186, respectively). We also calculated cross-correlations between the two vectors, where In-Out (Out-In) is the correlation between in-coreness in $t-1$ (t) and out-coreness in t ($t-1$). These cross-correlations are significantly lower with slightly negative average values of -.0698 and -.0764, respectively. Thus, lagged values of one coreness vector are not very informative for the expected value of the other coreness vector in the next period.

An important question is by how much the fit of the model improves by using the AC model rather than the SC model. As a rule of thumb, Boyd *et al.* (2010) argue that the PRE of the SC model should be at least .5 in order to have a superior fit to an unstructured distribution of activity. Here we find values around .2 for the SC model, but higher values of around .58 for the AC model (cf. Figure 15).[50] Similar to the discrete and tiering models above, the fit of the model deteriorates somewhat with the GFC, with lower average values afterwards. In line with the previous findings for the discrete and tiering model, the PRE of the AC model displays a structural break after

[49]An example of a fitted network matrix is shown in Appendix A.4.

[50]Obviously the fit has to be better in the AC model, since we have twice as many parameters. Interestingly, the fit is mostly more than twice as good as the fit of the SC model.

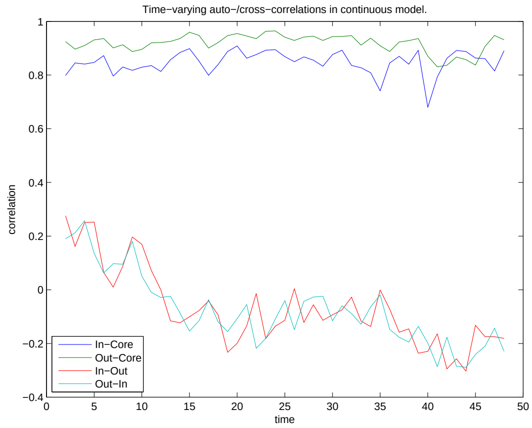

Figure 14: Persistence of coreness vectors. The plot shows the autocorrelations and cross-correlations of the two vectors over time. The autocorrelation is simply the correlation of the coreness vector in t with the one in $t-1$, using only the banks active in both periods. The cross-correlations are the correlations between in-coreness in $t-1$ and out-coreness in t (In-Out), and vice versa (Out-In).

quarter 39 (based on a Chow-test and a CUSUM test), but not in the SC model.

In order to check the significance of the PREs, we use a similar approach as in the previous section on the discrete and tiering model, however, here we use the (valued) network of interbank liabilities.[51] Figure 15 compares the PREs of the actual networks with the mean values from 100 realizations of random ER and SF networks (again with scaling parameter 2.3) minus and plus one standard deviation. As expected, the actual PREs of the SC and AC models significantly exceed those from the ER networks, which are very low in general. In contrast, for the SF networks, the PREs of the SC model are close to the actual ones, while this is not true for the AC model.[52] This finding underscores the observed asymmetries in the network, which are absent from scale-free networks, where in- and out-degrees of individual

[51] In this approach, we generated random ER and SF networks as explained above. Then, we randomly assigned observed transaction volumes from the actual networks (log-transformed) to the random ones. The results are essentially identical with and without replacement. Here we present the results without replacement.

[52] Note that the PREs of the AC model are always larger than those from the SC model, both for the actual and the random networks (even though for the random networks not always significantly). This is driven by the higher number of parameters (degrees of freedom) in the AC model.

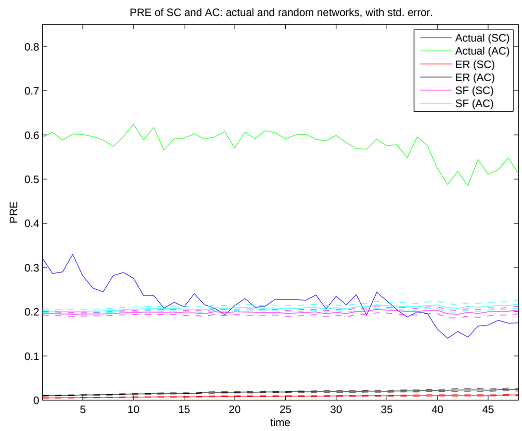

Figure 15: PRE for the SC and the AC model, actual and random graphs. A Chow-test indicates that there is a structural break after quarter 39 at all sensible significance levels for the PRE of the AC model. The results from an additional CUSUM-test are also in favor of the existence of a structural break. The PRE of the SC model appears to display an additional structural break after quarter 10. For the SF networks we used a scaling parameter of $\alpha = 2.3$.

banks are highly correlated by construction.

In comparison with the closeness of the error scores of the empirical data and their scale-free resamples in Figure 12, the consideration of the asymmetries of the concentration of incoming and outgoing links shows the limitations of the scale-free networks. While it appears reasonably similar to a symmetric core-periphery framework, it falls back behind the asymmetric continuous CP model at all levels of significance. Since the fit of the two-dimensional continuous approach (AC) is way better than that of the one-dimensional continuous approach (SC), we conclude that the directed version of the model contains important information about the structure of the interbank market.

5.4 What Defines a Core Bank?

In the following we will focus on the results from the discrete model.[53] As a first step, we calculate the correlations between the coreness vectors and different observable variables (degree, size, and trading activity).[54] It

[53] Again the results for the tiering model are very similar and available upon request.

[54] It would be interesting to analyze the interest rates charged in the different blocks in more detail. This is, however, beyond the scope of this paper. Here we just note that the

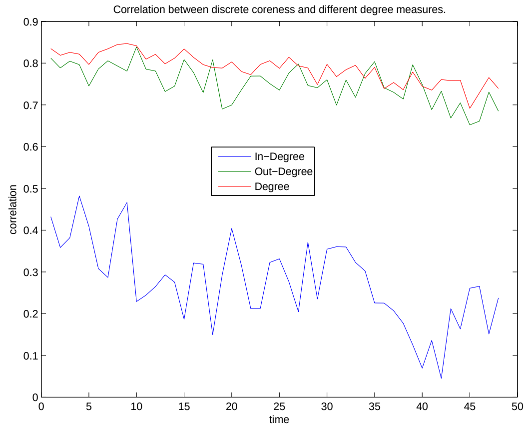

Figure 16: Time-varying correlation between discrete coreness and in-degree/out-degree/total degree. Total degree is the degree we would obtain from transforming the directed network into an undirected network.

would also be very interesting to forecast the coreness vectors based on non-network-related observable variables, e.g. balance sheet size. Due to the anonymity of the data, such an analysis is, unfortunately, not possible.

Figure 16 shows the correlation between the discrete coreness vectors and the in-, out-, and total degree (total degree is the degree from the undirected version of the network), respectively. The correlation is far higher for out-degree compared to in-degree, and the former has practically the same correlation with coreness as the total degree. Hence, it is the distribution of liquidity rather than its absorption, that identifies the core banks in our sample.

We constructed similar measures for the individual sizes and the number of transactions per bank, see Figures 17 and 18, respectively. We proxy the bank size by the transaction volumes in a particular quarter.[55] Here, in-size contains the total volume of borrowing transactions (per quarter and per bank), out-size the total volume of lending transactions and total size the

(volume-weighted) average interest rate charged in the CC-block exceeds that of all other blocks (average value of 2.72% for the complete sample). Thus, it seems that core banks price in the systemic importance of other core banks, while giving more favorable prices to periphery banks (average value of 2.70%). Furthermore, the average interest rate charged between periphery banks tends to be quite small as well (average value of 2.71%). Thus, two periphery banks may grant each other more favorable prices as soon as they trade with each other on a regular basis. Note that, due to the non-stationarity of the interest rates, we checked the significance by comparing the first differences.

[55]See De Masi *et al.* (2006).

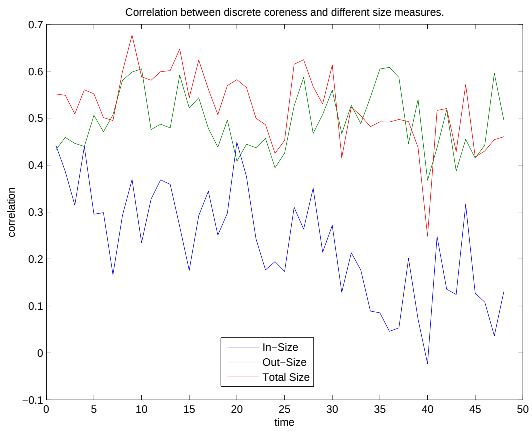

Figure 17: Time-varying correlation between discrete coreness and in-size/out-size/total size as defined in the text.

sum of in-size and out-size. Similarly, the number of in-transactions (out-transactions) is the number of borrowing (lending) transactions per bank. The total number of transactions is the sum of the two. We find that the core banks are significantly larger and more active than periphery banks (unreported). However, the size measure appears to be a less reliable indicator than the simple number of transactions, since it is far more volatile. Both measures, however, confirm again the dominant aspect of the out-direction (lending activity) for the core membership of a bank.

We also constructed the same figures for the continuous model, see Appendix A.5. As expected, the two coreness vectors can be better explained based on the directed version of the network. Most importantly, the correlation with the total degree is smaller compared to the correlation of in-coreness with in-degree and out-coreness with out-degree, respectively. Again, the correlations with the size measures are highly volatile.

We conclude that all measures point towards the lending activity as the more relevant aspect of core banks' participation in the market. The much lower relevance of their borrowing activity, then, explains why in- and out-coreness vectors in the asymmetric model are virtually uncorrelated.

5.5 What Happened During the GFC?

In this section, we provide a more detailed analysis of the effects of a major shock to the interbank network, namely the collapse of Lehman Brothers in quarter 39. So far, our analysis shows that the GFC indeed had a substantial impact on the network along many dimensions, in particular in terms of the

Figure 18: Time-varying correlation between discrete coreness and number of in- and out transactions. Total number of transactions is the sum of the two.

goodness-of-fit of the core-periphery models. To investigate the effects of the structural break in more detail, we split our sample into a short pre-crisis period (quarters 37 and 38) and a post-crisis period (40 and 41).[56] Interestingly, despite the clear negative trend in the number of active banks during the complete sample period (cf. Figure 1), the actual number during the analyzed subperiod is relatively stable with an average value of 98 banks. Thus the network sizes during this particular period are comparable, which allows to compare different network-related measures. As a first step, we will investigate network-related variables from a macro perspective. Then we take a closer look at the behavior of one particular exemplary core bank around the breakpoint.

As we have seen (cf. Figure 6), the GFC affected the block-structures of the discrete and the tiering model: Core banks trade significantly less with each other (density in the CC-block smaller), and so do periphery banks (density in the PP-block smaller). In contrast, there is no evidence for a significant structural break in the densities of the off-diagonal blocks. Core banks also tend to lend less money to the periphery (density in the CP-block smaller), while there is no clear trend in the amount that peripheral banks lend to the core, thus the periphery tends to maintain their links to the core during and after the crisis. Given that the GFC, and the resulting tensions in money markets, can be seen as the result of a crisis of confidence, it comes as

[56]Of course one could argue that the pre-GFC period should be further away from the breakpoint, however, here we are particularly interested in the network changes right at the phase transition.

no surprise that core banks tend to reduce their risk exposure by cutting down the number of links going both to core and periphery banks.[57] Concerning the market activity, we find that the total trading volumes (and also the total number of trades) in the CC- and the CP-blocks dropped substantially during the crisis, while it actually increased in the PP-block immediately after the GFC but then dropped substantially. In contrast, after a sharp drop of market activity in the PC-block right before the GFC, the total amount of credit flowing from the periphery to the core actually increased after the GFC.[58] Thus it seems, that the crisis mainly affected the behavior of core banks, which rather hoarded their liquidity than providing it to a large number of other counterparties.[59] In contrast, periphery banks tend to keep (at times even expand) the number of outgoing links with core banks, while reducing the exposure to the periphery. The findings on the Jaccard Index of the PC-block (cf. Figure 10), however, indicate that periphery banks do not necessarily lend money to the same core banks over time. Overall, from the relatively stable Jaccard indices it appears that no major disruption of the network pattern occurred (cf. section 5.2.2), but that the aggregate volume of lending by core banks has declined substantially. Hence, most of the network structure remained intact, but continued its operations at a much lower level of activity. This finding speaks in factor of a positive effect of relationship lending that helped to prevent a complete collapse of the interbank market after the onset of the financial crisis (as suggested by Affinito (2011), and Braeuning (2011)).

To illustrate the generally observed tendencies, we picked the (core) bank with the highest aggregate trading volume.[60] During this period, the particular bank had an average in-degree of 30, while its average out-degree was substantially higher with 64.[61] These mean values, however, hide the dynamic development, since there was a sharp drop in the banks' out-degree during the GFC (the maximum level pre-GFC was 80, the minimum level at the end of the period is merely 32), while the in-degree actually increased during the crisis (the minimum level pre-GFC was 15, the maximum level at

[57]Interestingly, the number of reciprocal links, i.e. the fraction of links pointing in both directions, goes down due to the GFC. This is somewhat surprising, since we would expect that bilateral relationship become closer in crisis times.

[58]The increase in the number of trades in the PC-block after the GFC is even more impressive, ending up above the pre-GFC level.

[59]Interestingly, core banks lend more money than they borrow from the periphery, thus the core is a net lender to the periphery.

[60]In fact, this bank (ID number 'IT0278') was in the core during the complete sample period.

[61]These numbers just underline the observed asymmetry between the CP- and PC-blocks.

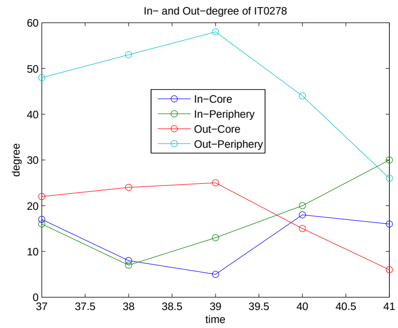

Figure 19: In- and out-degrees of bank IT0278, by core and periphery. In-core gives the number of incoming links from other core banks, In-periphery the number of incoming links from periphery banks. Out-core gives the number of outgoing links to other core banks, Out-periphery the number of outgoing links to periphery banks.

the end of the period is 45). In Figure 19, we split up the bank's links into outgoing links to core and periphery banks, respectively, and the same for the incoming links during the period under study. We see that the bank had reduced the number of outgoing links, both with core and periphery banks, but it had increased the number of incoming links, in line with the overall tendencies.[62] Interestingly, while the bank was a net-lender during most of the sample period, we see that the bank actually reversed its strategy during the GFC, since it became a net-borrower afterwards (see Figure 20). Thus, the bank tried to attract liquidity, mainly from periphery banks, since core banks became reluctant to trade with other core banks.

Summing up, we conclude that the GFC both affected the behavior of core and periphery banks: Periphery banks seem to have increased their lending to the core, both in terms of the number of links and trading volumes. In contrast, core banks have reduced their lending, not only to other core banks, but also to the periphery. The decline in goodness-of-fit of the core-periphery structure is therefore mostly due to a loosening of the core. Core banks activated a smaller number of their previous outgoing links. Hence they started to hoard liquidity rather than distributing it in the system. Therefore, it seems that core banks tend to rely on the liquidity of periphery banks during times of distress, while in 'normal' times they would more freely redistribute liquidity in the complete system.

[62]It would be interesting to see the quote data, rather than the transaction data. We suspect, that many quotes are simply never executed during the GFC.

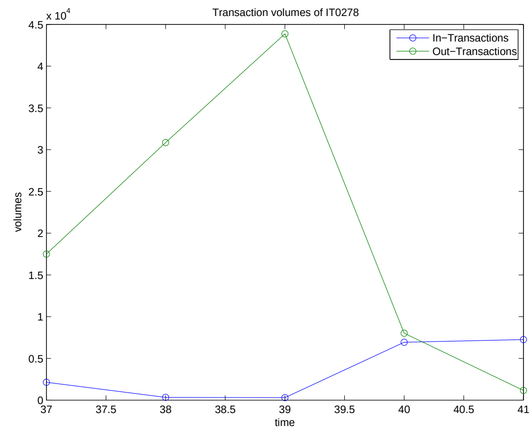

Figure 20: Transaction volumes of bank IT0278. In-transactions gives the total amount of credit borrowed by the bank, while Out-transactions gives the total amount of credit lend by the bank to other banks.

6 Discussion

The majority of studies on the structure of interbank networks has hitherto concentrated on the distribution of degrees. Many authors mention the finding of some form of community structure in the interbank market, suggesting a tightly connected core of money-center banks.[63] The finding of a core-periphery structure in the Italian interbank market can be seen as a special case of community structure,[64] where the core is a tightly connected part of the network, and the periphery is the loosely connected component.[65] Even though we only know of only one other study in this regard, it may well be that the finding of a core-periphery structure could be seen as a new 'stylized fact' of modern banking systems. As far as data are available, it would be important to test this hypothesis in other interbank networks.

An important question is of course why we find a core-periphery structure in the interbank market. In the literature on social network analysis, two main explanations for the emergence of a core-periphery structure exist: (1)

[63]See Iori *et al.* (2006) and Soramäki *et al.* (2006).

[64]Note that communities are usually defined as very dense subgraphs, with few connections between them. The periphery is thus more of an anti-community.

[65]We also checked several standard community detecting algorithms for the Italian interbank network. The main finding is that, for the entire market, we find two separate communities consisting of foreign and Italian banks, respectively. Interestingly, it is impossible to split these communities further into smaller subcommunities. Thus it seems even more remarkable that we find a core-periphery structure in this market.

'Superior' core members possess an intrinsic advantage over the 'inferior' periphery members, such that the core exerts power over the periphery.[66] In order for a core-periphery structure to emerge, the advantage of the core members must be reflected in attributes affecting the linking behavior of all agents.[67] Then core agents would be able to translate their advantage into a positional advantage in the social network.[68] Transferring this idea to banking networks, one encounters several problems. First, it is not clear a priori which attributes might make core banks 'superior' to the periphery. We would also need to come up with an explanation why core banks share attributes that periphery banks do not have. Note also that this definition implies that it should be preferable for all banks to be part of the core, which is not very plausible. For example, a small bank (in terms of its balance sheet) would find it hard to intermediate between other core banks, simply because it does not command a sufficient amount of funds to do so. Therefore, this bank will always prefer being in the 'inferior' periphery, where it still might intermediate between other small banks. Furthermore, the general finding of disassortative mixing patterns in banking networks[69] is not in line with the power-based explanation, since core banks would then be reluctant to create links with periphery agents. Nevertheless, if we define power as the ability of influencing the market, it may well be that it is an important driver for the emergence of a core-periphery structure in the banking network.

(2) Core members have a comparative advantage in gathering (and spreading) information about other members of the network.[70] Thus, information costs are higher for periphery-periphery relationships compared to core-periphery relationships (in both directions). For the banking network, this would mean that periphery banks have an incentive in cutting down the number of links to other periphery banks, maintaining only a few links to core banks. Core banks on the other hand connect among themselves and to periphery banks.[71] This explanation would not only be in line with the disas-

[66]See Persitz (2009).

[67]For example, in a scientific network, the core agents are the highly productive agents being cited by many others. See Mullins *et al.* (1977).

[68]Persitz (2009) provides a formal model for a power-based core-periphery network. The basic idea is that linking preferences are such that all agents prefer establishing links to 'superior' agents relative to 'inferior' agents.

[69]See Finger *et al.* (2012).

[70]For banks, the comparative advantage may stem from economies of scope and scale, but also from very frequent interactions on the market which small periphery banks usually do not have.

[71]Note that, despite the overall disassortative mixing patterns, the core-periphery structure indicates that we should actually differentiate between these patterns in the core and the periphery: the periphery mostly shows disassortative mixing within itself, while the

sortative mixing patterns, but also with the evidence in Cocco *et al.* (2009): small banks, with limited access to international capital markets and possibly limited investment/financing opportunities due to their more locally oriented business model, tend to rely on preferential relationships with (large) core banks. Thus, core banks act as intermediaries between different parts of the periphery of the domestic banking system, resulting in indirect relationships between peripheral banks. Note that this explanation is also in line with the observed asymmetry between the densities in the CP- and the PC-blocks, since they imply that periphery banks cut down their credit risk by focusing on a few selected core banks, while they are prepared to borrow money from a larger set of core banks.

Finally, we would like to focus on the potential implications of our findings for regulators. It is well known that the structure of a network is important for its resilience, hence policymakers should be interested in the actual topology of the interbank network. For stress-testing exercises, it would, therefore, be crucial to use a topological description of the connections within the banking sector that is both realistic and computationally tractable. Most stress-testing scenarios have actually adapted an entropy-maximization approach for filling the unknown matrices of interbank liabilities.[72] This means, that given some overall statistics for the whole system, interbank credit is spread as evenly as possible across the system[73] An idealized core-periphery structure amounts to pretty much the opposite in terms of concentration of interbank liabilities. If the data were closer to the latter type of structure, the entropy-based approach could give misleading results for the expected aftereffects to shocks affecting single institutions. If, as we believe, the core-periphery structure turns out to be a stylized fact of the interbank market, stress-tests should take this particular topology into account. Unknown amounts of interbank liabilities could then be calibrated along the structural features of typical core-periphery models for available data (like those of the present paper and Craig and von Peter, 2011). As our results show, it might also be important to take into account asymmetries in the borrowing and lending attitudes of core banks. Even when comparing the effects of shocks between different network models with some tendency of concentration of links, important differences might exist. For example, networks with scale-free degree distributions are known to be robust with respect to random failures, but fragile with respect to targeted attacks on the most central

core shows more of assortative mixing among its members, since core banks tend to connect among themselves.

[72]See Sheldon and Maurer (1998), and Upper and Worms (2004).

[73]Note that this is equivalent to the benchmark against which the error reduction by the continuous core-periphery model is measured.

nodes.[74] The usual mechanism to construct scale-free degree distributions is that of preferential attachment, see Barabasi and Albert (1999), so high-degree nodes tend to attract more links than low-degree nodes over time. As described above, we did not find evidence for scale-free degree distributions in the e-MID data and also find disassortative rather than assortative mixing patterns. Comparing assortative to disassortative networks, Newman (2002) shows that, for the same degree distribution, assortative ones are more robust to targeted attacks compared to disassortative ones. Since an assortative network possesses a whole set of nodes with large in- and out-degrees, i.e. many connections across the entire network, the system is characterized by a certain degree of redundancy that makes it more robust under attacks on single highly connected nodes. In contrast, the disassortative network is more susceptible to removal of high-degree nodes, which are not as tightly connected as in the assortative case. Thus, removing high-degree nodes allows to attack different parts of the network.

In a somewhat related strand of research, Brede and de Vries (2009) show that core-periphery structures might emerge from an evolutionary process as a compromise between resilience (concentration makes the network more vulnerable) and efficiency of a network (concentration creates short average path lengths).[75] From an economic point of view, the question would be whether the self-organization of the interbank network into a core-periphery structure creates important externalities so that policymakers should attempt to shift the balance towards higher resilience and somewhat lower efficiency.

Of course, regulators should also be interested in the dynamics of a network, when the breakdown of one node has knock-on effects on other nodes. This contagion effect is for example investigated by Caccioli *et al.* (2011) for different network structures. The authors analyze the extent of contagion in artificial banking systems after the random failure of individual institutions. Their main finding is that the likelihood of contagion, i.e. the breakdown of the entire system, is smaller for disassortative networks. Since in the latter, high-degree nodes tend to connect with low-degree nodes, the failure of a random node is unlikely to spread through the entire system. Conversely, the random breakdown of a high-degree node will severely affect other high-degree nodes in assortative networks. Note that this is different from the aspect of vulnerability under targeted attacks. As a consequence, a disassortative core-periphery framework might be more robust in 'normal' times, but more fragile under exceptional circumstances when key nodes are under

[74] See Albert *et al.* (2000).

[75] Note that the highest efficiency is realized in star-like configurations, while the highest resilience is related to the avoidance of short loops and degree homogeneity. See also Netotea and Pongor (2006).

stress or withdraw from the market. Hence, the 'coreness' translates to a certain extent into 'systemic relevance' of certain institutions.[76] The GFC seems to have been a major shock to the interbank network, as tests for structural breaks indicate. The observation that the fit of the core-periphery models significantly worsened with the GFC, might provide important information per se on the endogenous reaction of the system to stress which could be incorporated in stress-test scenarios. Furthermore the goodness-of-fit of the core-periphery framework might be seen as an indicator of tensions in the interbank market, so that various statistics based upon such a framework could be used as early warning signals of impending crises.

7 Conclusions

The main findings of our paper are the following: we find a significant core-periphery structure in the Italian interbank network for a sample period from January 1999 to December 2010. The identified core is quite persistent over time, consisting of roughly 28% of sample banks before the GFC and 23% afterwards (discrete model). Given the substantial differences in the German and Italian interbank market data investigated by Craig and von Peter (2010) and the present paper, e.g. in the underlying region and the maturity structure of the credit relationships, the finding of a core-periphery structure is unlikely to be a coincidence. We expect that other interbank markets display a similar hierarchical structure, which might be classified as a new 'stylized fact' of modern interbank networks and actually concretizes on a system level the role of money center banks. Going beyond the analysis of Craig and von Peter (2010), we also investigate the continuous and asymmetric versions of core-periphery models and find evidence for strong asymmetries. In particular, overall coreness is mainly driven by the function of provision of liquidity to large parts of the banking system by the core members. Overall coreness is, therefore, largely identical to out-coreness, while its connection to in-coreness is very weak. Regulators should be aware of the fact that a bank which is part of the in-core but not of the out-core, may play a completely different role in the system than a bank with the reverse strategy.

Formal tests favor the existence of a structural break in the last quarter of 2008, the time when Lehman Brothers collapsed. We investigated this time period in more detail and found that the deteriorating fit of the core-periphery structure in the post-GFC period is mainly due to the loosening of connections in the core, particularly on the lending side. Furthermore,

[76] See also Markose et al. (2010).

it seems that during times of distress, core banks tend to rely on periphery banks as an important source of funding, since other core banks are reluctant to provide as much liquidity to other banks as in normal times.

Our findings provide some support for the view that the network structure is non-random due to the existence of preferential lending relationships. This is in line with the results of Cocco *et al.* (2009), Affinito (2011), and Braeuning (2011). Further evidence in this regard is provided by Finger and Lux (2011), who analyze the evolution of the banking network using the actor-oriented approach by Snijders (1996, 2001). The general conclusion is that preferential lending relationships at the micro-level lead to hierarchal structure at the macro-level. An open question is why the interbank network shows such a hierarchy. We argue that the comparative advantage of core banks in gathering and distributing information about their counterparties is likely to be a crucial factor.

In the future we plan to apply the model to other interbank data, in order to evaluate whether the core-periphery structure is indeed a new stylized fact of banking systems. Furthermore, it would be interesting to relate the results to bank-specific variables, such as individual balance-sheet data. In any case, this approach can be seen as a contribution to identifying the systemically important banks in a quantitative way. We also believe that the methods presented here could be an important tool for regulators since they allow to reduce the complexity of large-scale network data, and to represent the salient structural features of the complicated web of dispersed activity in the interbank market in a compact way.

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A Appendix

A.1 Genetic Algorithm

The GA maintains a population of L solution candidates and evaluates the quality of each solution candidate according to a problem-specific fitness function, which defines the environment for the evolution. New solution candidates are created by selecting relatively fit strategies which are then recombined through various genetic operators:

1. **Reproduction:** From the pool of current solutions, L copies are selected (with replacement) randomly with probabilities depending on a solution's relative fitness, i.e. prob(reproduce solution j) $= f_j / \sum_i f_i$ with f being the fitness function as defined below (tournament selection).

2. **Crossover:** Copies from the reproduction step are randomly paired in a mating process with each couple producing two offspring via exchange of genetic material. The simplest way is to select two copies randomly and swap bits between both of them. Here we randomly select an integer in the range of $[1, L-1]$ and construct offsprings by combining the left of this position from parent one with that from the right-hand part of parent two and vice versa. The cross-over operation is carried out with probability π_{cross}, while with probability $(1 - \pi_{\text{cross}})$ the offspring are unchanged copies of their parents.

3. **Mutation:** After exchanging genetic material, slightly different solutions are formed by altering each position within a string with probability π_{mut} to the other value of the binary alphabet.

4. **Election:** The election operator avoids an overall decrease of fitness in the population by allowing only offspring with a higher fitness than their parents to the new generation.

We also add an operator that we call 'genetic engineering' (GE) to accelerate the convergence of the algorithm.[77] GE selects the binary string with highest fitness value of the current population so far and tries to improve its fitness value as follows: we search the core member with the lowest connection to the core in this partition (i) and the periphery member with the highest connection to the core (j).[78] Then we compare the fitness of the

[77]See also Stolzenburg and Lux (2011).

[78]This is done by computing the average connections with all core members.

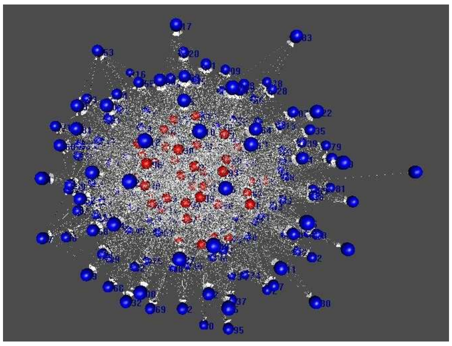

Figure 21: Example for the core-periphery structure of the Italian interbank network, 1999Q1. Red dots show core, blue dots periphery banks. Dotted lines show directed edges from one bank to another.

current partition with three alternatives: (1) flip i's core membership from 1 to 0, and j's core membership from 0 to 1, (2) only flip i's core membership, (3) only flip j's core membership. Using the best of these alternatives, we put the resulting string back into the evolving population. By using this additional operation we can manipulate our population target-oriented and do not have to rely solely on blind exploration of the search space by the random evolutionary steps of 'mutation' and 'crossover'.

Summing up, the binary-coded GA has only two parameters π_{cross} and π_{mut}. Obviously, the fitness function is the crucial element of the GA. In our setting, the fitness of each solution depends on the corresponding error score e. Since the optimal solution would have zero errors, we simply take the rate of 'correct' classifications as the fitness function, i.e.

$$f_l = 1 - e(\mathcal{C}), \tag{12}$$

with the error score defined in Eq. (3).

A.2 Discrete Model: Illustration

Figure 21 illustrates the outcome of the estimation of the discrete core-periphery model for the first quarter of 1999. We clearly see that the core banks (red dots) form the cluster of the most central nodes, with periphery banks (blue dots) connecting to parts of this cluster.[79]

[79] Note the high network density and the existence of only a single network component. See also footnote 65.

A.3 Empirical Estimation of the Asymmetric Continuous (AC) Model

This section summarizes Boyd *et al.*'s (2010) approach for estimation of the AC model.

A.3.1 Problem Formulation

We first note the similarity between the optimization problem of Eq. 9 in the main text, and a Singular Value Decomposition (SVD) of a matrix. SVD allows to decompose a matrix $\mathbf{D}_{\{M\times N\}}$ of rank r into

$$\mathbf{D} = \mathbf{USV}', \tag{13}$$

where $\mathbf{U}_{\{M\times r\}}$, $\mathbf{V}_{\{N\times r\}}$ are real matrices with orthonormal columns, with the columns corresponding to the singular vectors, and $\mathbf{S}_{\{r\times r\}}$ is a diagonal matrix containing the singular values (ordered) on the main diagonal.[80]

For any $k \leq r$, SVD gives the best (least-squares) rank-k approximation of $\mathbf{D}$, i.e.

$$\mathbf{D}_{(k)} = \mathbf{U}_{(k)}\mathbf{S}_{(k)}\mathbf{V}'_{(k)}, \tag{14}$$

with $\mathbf{U}_{(k)}$ and $\mathbf{V}_{(k)}$ as the first k columns of $\mathbf{U}$ and $\mathbf{V}$ respectively, and $\mathbf{S}_{(k)}$ as the diagonal matrix formed by the first k singular values.

Singular Value Decomposition (SVD) is defined for rectangular matrices, for which symmetry is not an issue, but it can also handle square matrices, whether symmetric or asymmetric, as a special case.[81] However, it does require the presence of the diagonal elements of a matrix. Thus, by definition, SVD can handle asymmetric data matrices.

Estimation of the AC model is again performed via minimization of residuals (MINRES) taking stock of the proximity of the problem to a SVD. The basic idea of MINRES/SVD is to use a rank-1 approximation of $\mathbf{D}$, using the first singular value s and the first singular vectors u and v: $\mathbf{D}_{(1)} = usv'$.[82] Since SVD requires diagonal elements, we use a SVD of rank 1 and, similar to the MINRES approach, exclude the diagonal elements in the analysis. Thus, our objective function for the AC model looks as follows

$$\arg\min_{u,v} \sum_i \sum_{j\neq i} (d_{ij} - u_i s v_j)^2. \tag{15}$$

[80] $\mathbf{U}$ contains the eigenvectors of $\mathbf{D}\ \mathbf{D}'$ while $\mathbf{V}$ contains the eigenvectors of $\mathbf{D}'\mathbf{D}$. The diagonal elements of $\mathbf{S}$ are the square roots of the non-zero eigenvalues of $\mathbf{D}\ \mathbf{D}'$.

[81] See Stewart (1993) for an overview.

[82] In the future it would be interesting to look at higher dimensional approximations. This would allow splitting up the core and periphery even further.

The normality constraints on u and v can be eliminated, such that we can neglect the singular value s by absorbing it into the unconstrained vectors u and v. Obviously, the solution is not unique, but without s the model is even simpler, since we approximate $\mathbf{D}$ using only uv', leaving us with the objective function

$$\arg\min_{u,v} \sum_i \sum_{j\neq i} (d_{ij} - u_i v_j)^2. \tag{16}$$

The optimal vectors can be determined by finding the roots of the first-order conditions of Eq. (16). The original u, s and v can be obtained by defining $s = ||u||||v||$, with $||u|| = \sqrt{\sum_i u_i^2}$ being the Euclidian norm of u, and then dividing u and v by their norms. The reported coreness vectors are normalized.

A.3.2 Optimization Problem for MINRES/SVD

We could solve the problem of finding the vectors $u_{\{N\times1\}}$ and $v_{\{N\times1\}}$ numerically by using standard optimization procedures. However, following Boyd *et al.* (2010), the problem can be solved easier by setting the first derivative of Eq. (9) with respect to u_i and v_j equal to zero and solving the resulting equations numerically.
More formally, this amounts to

$$\frac{\partial L}{\partial u_i} = \sum_{j\neq i}^{N} (d_{ij} v_j - u_i v_j^2) = 0. \tag{17}$$

Remembering that the diagonal elements in $\mathbf{A}$ equal zero, we can write this as

$$\sum_{j=1}^{N} d_{ij} v_j = -u_i v_i^2 + \sum_{j=1}^{N} u_i v_j^2. \tag{18}$$

For each row i this equation has to hold, so we can write the set of equations in matrix notation as

$$\mathbf{D}v = u.(-v.^2 + v'v), \tag{19}$$

where a dot indicates elementwise multiplication. The other set of equations can be calculated in a similar fashion

$$\frac{\partial L}{\partial v_j} = \sum_{i\neq j}^{N} (d_{ij} u_i - u_i^2 v_j) = 0, \tag{20}$$

leading to

$$\sum_{i=1}^{N} d_{ij} u_i = -u_j^2 v_j + \sum_{j=1}^{N} u_i^2 v_j. \tag{21}$$

Now, this equation has to hold for each column j, which can be written in compact form as

$$u'\mathbf{D} = v'.(-u.^2 + u'u)'. \tag{22}$$

Consequently, the optimal vectors u and v can be obtained by solving Eqs. (19) and (22) simultaneously. Using an appropriate set of initial values (see below), the optimization is much faster than solving Eq. (15) directly.[83]

We should also note that we checked an alternative approach proposed by Boyd *et al.* (2010), where we impute values on the diagonal and apply the usual one-dimensional SVD to this matrix. The results from this approach cannot be distinguished from those presented in the following, so this approach is likely to be more efficient when working with very large networks.

A.3.3 Initial values for MINRES/SVD

The choice of the initial values is important in many numerical problems, most importantly with respect to computation time. Here we follow the approach in Boyd *et al.* (2010) and impute diagonals to the data matrix first, then using the first step in the reciprocal averaging method for computing the SVD. The algorithm will then work on the original data matrix, without diagonal elements.

Let c_i, r_j and t be the column, row and total sums of the matrix $\mathbf{D}$, respectively, excluding the diagonal elements.[84] A single missing value at the position d_{ij} could then be imputed by assuming independence. This leads us to $(r_i + d_{ij})(c_j + d_{ij}) = d_{ij}(t + d_{ij})$ or solving for the missing entry

$$d_{ij} = \frac{r_i c_j}{t - r_i - c_j}. \tag{23}$$

If all of the diagonal elements were missing, one could use this formula to estimate each of the diagonal elements. However, this neglects the contribution of the other $N-1$ diagonal elements to the total sum t. So a better approximation would be to estimate the sum of all the matrix elements by adding to t an estimate for the other $N-1$ diagonal elements, the average value of the off-diagonal elements, $t/(N^2 - N)$. After canceling the factor $N-1$, the independence model for estimating the diagonal elements appears as $(r_k + d_{kk})(c_k + d_{kk}) = d_{kk}(t + t/N + d_{kk})$ for each element d_{kk}.[85] This leads

[83] We have used a very similar approach for the SC model, where we can also speed up the estimation by taking the first derivative of Eq. (7) with respect to c and solving the resulting system of equations numerically. See Boyd *et al.* (2010).

[84] For simplicity, if these were zero the sums would remain unaffected.

[85] There is a slight error in the version by Boyd *et al.* (2010), since they missed the d_{kk}^2 term on the right-hand side of the Equation.

to

$$d_{kk} = \frac{r_k c_k}{t + t/N - r_k - c_k}. \tag{24}$$

The reciprocal averaging method is analogous to the power method for computing eigenvectors. It works as follows: choose initial vectors x_0 and y_0 to be $x_0(i) = y_0(i) = 1, i = 1, \cdots, N$. Then the iterative formulas

$$\begin{aligned} \tilde{x}_k &= \mathbf{D} y_{k-1},\ \ x_k = \tilde{x}_k / ||\tilde{x}_k|| \\ \tilde{y}_k &= x_{k-1} \mathbf{D},\ \ y_k = \tilde{y}_k / ||\tilde{y}_k||. \end{aligned} \tag{25}$$

give a sequence of vectors such that x_k and y_k converge to the first singular vectors u and d, respectively. A good approximation for usv' would be $x_2 \mathbf{D} y_2'$. However, we do not specify the singular value but absorb it into the vectors u and d, which are now not normalized. Thus the initial vectors u_0 and v_0 are $\sqrt{s} x_2$ and $\sqrt{s} y_2$, where a good approximation for d is

$$d_0 = \frac{r}{||r||} \mathbf{D} \frac{c'}{||c||}. \tag{26}$$

A.4 Model Fit: AC Model

Figure 22 shows an example of a fitted network matrix, where the matrix has been sorted according to the individual sums of the in- and out-coreness values. The Figure illustrates the typical asymmetry of the in- and out-cores, since most of the core banks lend money to the periphery (out-core), while there are fewer connections from the periphery to the core.

A.5 What Defines a Core Bank (in the Continuous Model)?

Figures 23-25 show the results for the continuous model. Obviously, the two coreness vectors can be better explained based on the directed version of the network. Most importantly, the correlation with the total degree is smaller compared to the correlation of in-coreness with in-degree and out-coreness with out-degree, respectively. Again, the correlations with the size measures are highly volatile.

These results suggest that, in contrast to the findings of Craig and von Peter (2010), total balance sheet size may not be as informative for explaining banks' coreness as we might expect. When splitting the banking network into in-core, out-core and periphery, we should rather focus on asymmetric figures. Examples might be loans granted on the asset side of the balance sheet and the size of the debt on the liability side. It would be interesting to investigate this in other interbank markets in the future.

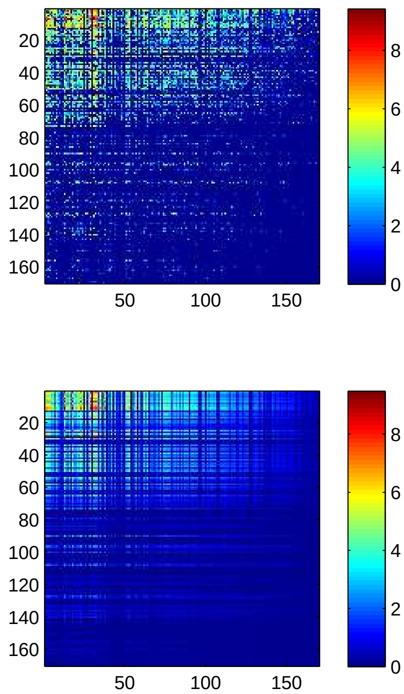

Figure 22: Example: Data matrix and approximation based on the AC model for 2000 Q3, with warm colors indicating high transaction values between individual banks. The upper panel shows the log-transformed data matrix $\mathbf{D}$ after applying the MINRES/SVD approach and sorting the network according to the coreness vectors. The lower panel shows the MINRES/SVD approximation based on the AC model, with the same ordering of the network nodes.

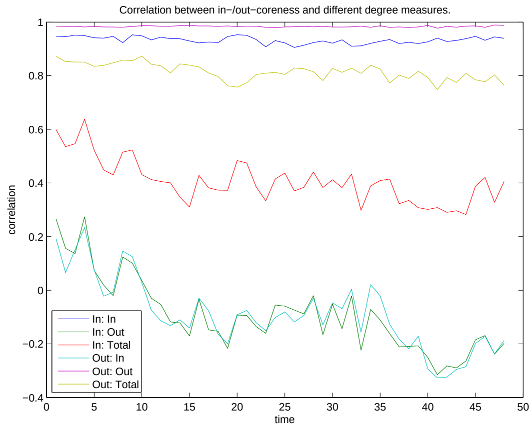

Figure 23: Time-varying correlation between in-/out-coreness and in-degree/out-degree/total degree. Total degree is the degree we would obtain from transforming the directed network into an undirected network.

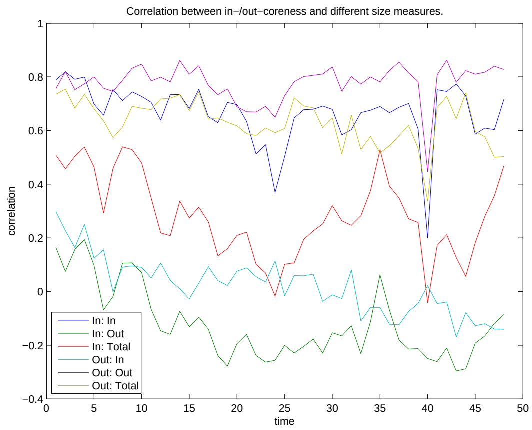

Figure 24: Time-varying correlation between in-/out-coreness and in-size/out-size/total size as defined in the text.

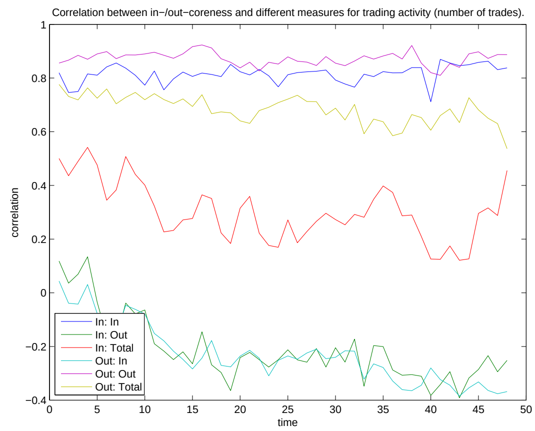

Figure 25: Time-varying correlation between in-/out-coreness and number of in- and out transactions. Total number of transactions is the sum of the two.

A.6 Changing the Aggregation Period

Here we briefly discuss the results for other than quarterly aggregation periods, by focusing on the results from monthly and annual networks. To be precise, the typical element in $\mathbf{A}$ is $a_{ij} = 1$ if there was at least one transaction from bank i to j during the respective month/year.[86] In both cases, we expect comparable results as for the quarterly networks, however, with certain differences. For example, given the fact that the activity structure of banks is less stable for monthly compared to quarterly networks, we expect the coreness vectors to be less stable over time. Furthermore, due to the aggregation, the total number of active banks in one quarter will be at least as big as the number of active banks in any of the 3 months in this particular quarter. Therefore, we expect the relative size of the core to be somewhat smaller for monthly data. For annual data, it is a priori not clear what might happen to the coreness vectors. While we expect a larger core as compared to the quarterly case (due to the reduction of noise and the relatively large number of active banks per year), the volatility in the coreness vectors (over time) could in fact be higher, given that annual aggregation makes it more likely that two banks being active in different quarters are being put together in a network where they in fact never could have interacted. Again, the anonymity of the data set makes it impossible for us to disentangle these effects.

[86]We do not expect significant changes, if we used a higher threshold here, e.g. banks might have to trade at least twice within a particular period to establish a link.

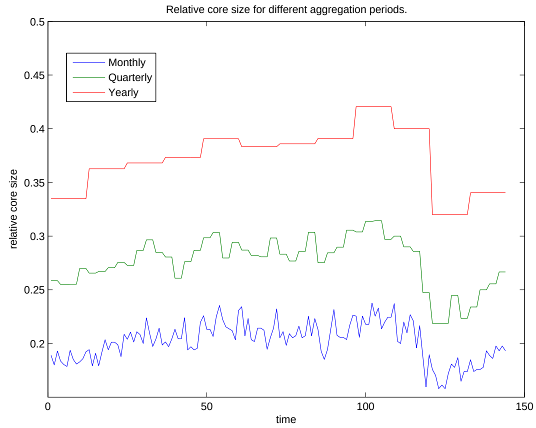

Figure 26: Comparison of the time-varying relative core sizes for different aggregation periods.

Figure 26 shows the time-varying relative core sizes (discrete model) of different aggregation periods for the entire sample period of 144 months. As expected, the relative size of the core depends positively on the length of the aggregation period, so the core is largest for yearly networks, consisting of roughly 36% of sample banks before the GFC and close to 30% afterwards. The structural break due to the GFC is clearly visible for yearly and quarterly data. This is not so much true for monthly data, where pre- and post-GFC average core sizes are 20% and 17% respectively. The average core size appears to be relatively stable for monthly data, however with wilder fluctuations as compared to longer frequencies. Thus, there appears to be substantial noise in the monthly networks, backing up our use of quarterly data in the baseline scenario. Similar remarks apply to the error scores in Figure 27. As expected, the high level of noise at higher frequencies deteriorates the model fit: the fit is worst for monthly data, with the highest error scores. Interestingly, at the monthly level, the GFC seems less like a big shock as compared to quarterly and yearly data, since the error score increases already before the GFC. In contrast, the fit of the longer aggregation periods drops with the GFC, which is most clearly visible for the yearly data. In any case, it seems that the Italian interbank market shows a core-periphery structure at all frequencies under study, even though for monthly networks the noise level is rather high so that it is much harder to identify the core at such high frequencies.

Concerning the correlation between individual coreness vectors from different models, we just note here that the correlation between the discrete and tiering model is roughly .8750 for monthly and .9918 for yearly net-

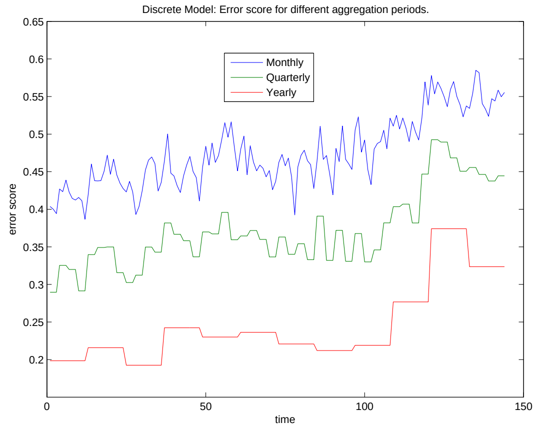

Figure 27: Comparison of the error-scores for different aggregation periods.

works. Thus, the results from the discrete and tiering model are very similar in general, but for monthly data there may be some differences. While the correlation between out-coreness and the discrete model is comparable for all frequencies (around .73), the correlation with in-coreness depends positively on the aggregation period.[87] Thus, it seems that network becomes more symmetric for longer aggregation periods. This is confirmed by the fact that the correlation between in- and out-coreness is -.2126 for monthly data and .1383 for yearly data. Thus, at the yearly level, in- and out-coreness appear to be positively related. In any case, the correlation between the two vectors is rather small in absolute terms, so the asymmetric MINRES/SVD approach captures the inherent asymmetry of the network at all frequencies.

A.7 Including Foreign Banks

Analyzing the network formed by foreign banks only, we see average error scores around 45% before the GFC and values close to 90% afterwards. Given that foreign banks' activity is rather unstable, due to the simple fact that they have access to other sources of funding, it comes as no surprise that foreign banks form a structurally different separate subnetwork.

We also analyzed the complete network, with Italian and foreign banks. Interestingly, most of our findings from the baseline scenario with Italian banks only, remain unaffected. However, there is a clear upward trend in the error score over time which is driven by the rather unstable nature of foreign

[87]For monthly data the correlation between the discrete and in-coreness vector is roughly .1578, while the value is .3861 for yearly data.

banks' activity. Thus it seems justified to exclude foreign banks from the analysis.

A.8 Continuous Model Using the Number of Transactions

As another robustness check, we ran the continuous model using the quarterly matrices containing the number of (directed) transactions between individual banks $\mathbf{T}$, instead of the total transaction volumes $\mathbf{D}$.[88] We find very similar coreness vectors in both settings. For the SC model the correlation between the vectors is .9618. For the AC model, the correlations are .9594 and .9844 for in- and out-coreness, respectively.[89] Quite interestingly, it seems that the fit (in terms of PRE) of the MINRES model in this case is even worse compared to the values presented in the main text, while the fit of the MINRES/SVD model is slightly better than before. It not not quite clear, why this is the case, but we should stress that all of the results here indicate that the model is quite robust. Most importantly, it seems that the conclusions from above also hold for alternative valued matrices, that measure the intensity of bilateral relations in a meaningful way.

A.9 Further Robustness Checks

We performed additional robustness checks for the discrete and continuous models.

- We estimated the discrete models using the correlation-based approach of Stolzenburg and Lux (2011); there we used the correlation between the observed data matrix and the pattern matrix constructed from the coreness vectors as the objective function. Note that the correlation-based approach can only be used for the discrete model in the case of arbitrary off-diagonal blocks. We found very similar results compared to the baseline scenario. In this regard, we also ran the discrete model with valued networks (transaction volumes and number of transactions, both log-transformed) and found very similar results compared to the baseline scenario.

- We used the binary networks, rather than the valued ones, as input matrices in the continuous models. The coreness vectors are very similar

[88] Again we log-transform the data matrix to reduce the level of skewness, cf. section 4.2.

[89] We also find similar values for monthly and annual data.

to the baseline case (correlation of above .93), however, with constantly smaller PREs.

- We estimated the continuous models without log-transforming the data. Due to the high level of skewness in the data (driven by many zeros in the matrices), it is not possible to identify a sensible core for the transaction volumes. The coreness vectors are hardly comparable, with a correlation of .4928 between the out- and .3829 between the in-coreness vectors compared to the baseline scenario, respectively. Furthermore the PREs are highly volatile over time. In contrast, using the number of transactions the correlations are .8510 and .7668 for the out- and in-coreness vectors compared to the baseline scenario, respectively. Also the PREs are comparable to the baseline scenario, however, with extreme values around quarter 10 and 39, i.e. the two candidates for structural breaks.

- We also estimated the continuous models using the correlation-based approach. We find identical results in both cases, however, the approach presented in the paper is preferable, since the computation is much faster. We should note that the correlation between the pattern and the observed matrices is always above .70 in the AC model, while it may be as low as .37 for the SC model after the GFC.