Journal of Environmental Economics and Management 107 (2021) 102447

Contents lists available at ScienceDirect

Journal of Environmental Economics and Management

journal homepage: www.elsevier.com/locate/jeem

Booming gas – A theory of endogenous technological change in resource extraction☆

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ARTICLE INFO

Article history:
Received 30 March 2020
Revised 16 October 2020
Accepted 16 March 2021
Available online 1 April 2021

JEL classification:
D25
Q30
Q55

Keywords:
Shale gas
Endogenous technological change
Optimal order of extraction
Natural gas prices
Extraction costs
Renewable backstop
Optimal transition
Carbon tax

ABSTRACT

This paper introduces endogenous technological change in a Hotelling-Herfindahl model of natural resource use to study the recent developments in the U.S. natural gas industry. We consider optimal forward-looking technology investments, and study implications for the order of extraction of conventional and shale gas, and a backstop technology, and characterize the development of gas prices. We find that technology investments increase during the extraction of conventional gas. Once production shifts towards shale gas, investments decline. Consistent with current trends, our theory explains how gas prices can follow a U-shaped path. The calibrated model suggests that U.S. shale gas production continues to grow and prices continue to decrease until 2050. We analytically and numerically show that the introduction of a carbon tax would reduce technology investments, and thus could drastically change the temporal patterns of U.S. shale gas extraction. The forward-looking behaviour of firms is crucial for such an effect, which does not occur in models that treat the improvement in extraction technology as an unanticipated shock to the industry.

1. Introduction

Resource booms have become a reoccurring phenomenon across the world, including the natural gas and oil sector (Carter et al., 2011; Jacobsen and Parker, 2016). However, the Hotelling-Herfindahl workhorse model of natural resource economics is unable to explain this phenomenon of increasing resource use (Hotelling, 1931; Herfindahl, 1967). This paper studies endogenous technological change as an important driver of resource booms. We propose a simple, novel resource-economic theory that can explain the recent developments in the U.S. natural gas market.

☆ We thank Hassan Benchekroun and Till Requate, the audiences at WCNRM 2020, VfS-Conference 2019, WCERE 2018 and the participants at the Green-Econ Workshop 2018 for valuable comments. We gratefully acknowledge the support of the German Centre for Integrative Biodiversity Research (iDiv) Halle-Jena-Leipzig funded by the German Research Foundation (FZT 118), and funding from the German Research Foundation under grant agreement number RI 1833/4-1(CDRecon).

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https://doi.org/10.1016/j.jeem.2021.102447
0095-0696/

Within the last decade, the U.S. has experienced a major shift in its energy supply. From 2007 to 2018 the share of shale gas in total natural gas production grew from 8 to 69 percent (EIA, 2019b, 2014). At the same time, overall natural gas production increased by more than 50 percent and gas prices declined significantly (EIA, 2018b,a).

These recent developments are not due to the exploration of new reserves. The existence of abundant U.S. shale gas resources has been known for many years (Newell et al., 2019; Asche et al., 2012). In 1821, the first well was drilled in the Devonian Dunkirk Shale in Chautauqua County, New York. However, due to low permeability, the extraction from shale formations was regarded as technically difficult and not profitable. Consequently, production stayed on low levels for a long time (Wang et al., 2014).

The two main technologies that have allowed for the large scale production of shale gas in the early 21st century are hydraulic fracturing and horizontal drilling. Both technologies were developed by the natural gas and oil industry over decades. Hydraulic fracturing was first used in the 1940s but its application was quite limited. Horizontal drilling was common in the natural gas industry by the late 1970s. In the 1980s and 1990s, pioneering companies invested in the development of both techniques with the goal to make the production of shale formations profitable. It took until the turn of the century to reduce extraction costs sufficiently by facilitating a combination of hydraulic fracturing and horizontal drilling as well as new monitoring techniques using micro-seismic data (Aguilera and Radetzki, 2014; Joskow, 2013, 2015; Mason et al., 2015; Wang et al., 2014).[1] Although costs have strongly declined, production started when average extraction costs of shale gas were substantially higher than for conventional gas (Aguilera, 2014; IEA, 2013).[2]

The observation of increasing production and decreasing prices contradicts standard resource-economic theory. The classical model of Hotelling (1931) predicts monotonically increasing prices and decreasing production over time. For the case of multiple resource types, Herfindahl (1967) established the rule that deposits should optimally be exploited in ascending order with respect to their marginal extraction costs.

In this paper, we develop a novel variant of the Hotelling-Herfindahl model by endogenizing progress in extraction technology for one type of resource (shale gas) as a result of costly investments into research and development. We study the consequences for price development and order of extraction when another resource type (conventional gas) with mature extraction technology, or a renewable backstop can be used as well. Our theory is not restricted to the natural gas industry, but can also be applied to other exhaustible resources such as oil where similar technology developments are taking place.

The above mentioned stylized facts indicate that the development of shale extraction technology required continuous investments from natural gas and oil producers over decades. Yet, empirical studies often base their identification strategy on the assumption that the shale gas boom was an exogenous and unexpected shock to the industry (e.g. Arezki et al., 2017; Wakamatsu and Aruga, 2013). Our model puts this assumption in question, as it shows that the observed patterns of the shale gas boom are fully consistent with forward-looking behavior of firms investing in the development of extraction technology depending on expected future returns.

Our theory contributes to two strands of literature. First, we add to the literature that deals with the optimal order of resource extraction. This literature origins with Herfindahl (1967) who shows that under the assumption of constant marginal extraction costs, deposits with low costs should optimally be exploited first. Various authors have set up more generalized models by relaxing different assumptions of Herfindahl's model (Kemp and Long, 1980; Lewis, 1982; Amigues et al., 1998; Holland, 2003; Chakravorty et al., 2005, 2008; Chakravorty and Krulce, 1994; Gaudet et al., 2001; Gaudet and Lasserre, 2011; Gaudet and Salant, 2018). This literature has shown that Herfindahl's (1967) rule is part of a more general principle, according to which deposits should optimally be extracted in sequence of their full marginal cost, i.e. marginal extraction cost plus opportunity cost implied by the scarcity of the resource. This result has become known as the Generalized Herfindahl Principle (Gaudet et al., 2001). We find that a variant of the Generalized Herfindahl Principle also holds when taking endogenous technological change into account, but that the shadow values of both resources depend on the patterns of technological change.

Second, we contribute to the literature that tries to explain why prices of nonrenewable resources do not monotonically increase as implied by Hotelling (1931). Economic theory offers two explanations for this. For many nonrenewable resources, the discovery of additional deposits has exceeded extraction so that reserves have actually increased (Adelman, 1990). Pindyck (1978) develops a model in which he allows for the exploration of new reserves in the presence of stock dependent extraction costs. He shows that due to exploration, the price of a resource can follow a U-shaped path. Schwerhoff and Stuermer (2015) develop a growth model with constant marginal extraction costs, where the development of new extraction technologies allows the access to new resource deposits. Since there is no resource depleting effect, the model predicts stable resource prices over time. As U.S. shale gas resources have been known long before extraction started, our model assumes that resource stocks are known from the beginning.

The second explanation for declining prices is technological progress and its decreasing effect on extraction costs. Slade (1982) adds exogenous technological change to a Hotelling model and predicts a U-shaped relative price curve. Farzin (1995) looks at exogenous technological change where marginal extraction costs depend on cumulative production and current extraction rate. His model also shows the possibility of a U-shaped price path. Lin and Wagner (2007) account for exogenous tech-

[1] Shale formations contain natural gas and oil. The major technological innovations that have made production profitable are similar for both resources (Joskow, 2015).

[2] Other countries have not yet shifted to the extensive use of shale gas resources. China, for example, holds the world's largest shale gas reserves but still lacks in advanced extraction technologies (Lee and Sohn, 2014; Hu and Xu, 2013). China is expected to rise to the second largest shale gas producer by 2040 (EIA, 2017a).

nological change and stock dependent extraction costs. Their model allows for a steady state solution and thus can explain constant prices over time. Lin et al. (2009) studies how technological progress would have behaved in order to maintain a constant resource price over time. Rausser (1974) develops a model of endogenous technological progress via learning by doing in the extraction industry. As technological improvements come automatically with resource extraction, the implications of the model for resource prices are similar as the ones derived from the assumption of exogenous technological progress. Holland (2008) summarizes different models of U-shaped price paths and points out that modeling technological progress exogenously is insufficient as it leaves the main point of interest unexplained.

Our paper addresses this gap in the literature and offers insights on how technological progress develops over time. We show that technology investments increase during conventional gas extraction. Once production shifts completely towards shale gas, investments decline. The technology stock evolves in a S-shaped fashion, similar to the literature on technology diffusion (Davies, 1978; Jaffe and Stavins, 1994; Helm and Mier, 2019). Unlike Hotelling (1931), but consistent with current trends, our theory explains how gas prices can follow a U-shaped path. Further, we find that Herfindahl's least-cost-first principle does no longer apply. Endogenous technological progress allows for additional orders of resource extraction. In particular, even if firms have already shifted to the use of the renewable backstop, production can switch back to a nonrenewable resource for a period of time, when extraction technology has sufficiently advanced. In line with the literature (e.g. Heal and Schlenker, 2019), the introduction of a carbon tax shifts resource consumption from the present to the future. We analytically show that a carbon tax reduces optimal investment into shale gas extraction technology, and that the time-delaying effect of a carbon tax is even more pronounced when taking into account that technological change is endogeneous. Our calibrated model suggests that, without a carbon tax, U.S. shale gas production will continue to grow and prices will continue to decrease until 2050. We further find that the introduction of a carbon tax could have had a strong postponing effect on the U.S. shale gas boom. This time-delaying effect critically depends on the forward-looking behaviour of producers, and does not occur in models that threat the improvement in shale extraction technology as an exogenous and unanticipated shock to the industry. For policy analysis it thus makes a big difference whether technical progress is exogenous or endogenous.

We proceed as follows. In the next section, we include endogenous technological change in a standard Hotelling-Herfindahl model. Section 3 presents our propositions on technological progress, order of resource extraction, and price development. In section 4, we calibrate the model to the U.S. natural gas industry. Section 5 considers two variations of the calibrated baseline model. The last section concludes.

2. Resource economic model

We consider the extraction of a homogeneous resource (natural gas) from two types of deposits, S (shale gas) and C (conventional gas). The corresponding extraction quantities are denoted by $q^s(t) \geq 0$ and $q^c(t) \geq 0$ at any point in time, t. In the following, we omit the time dependency unless needed for clarity. Stocks S and C of the two resources thus evolve according to

$$\dot{S} = -q^s, \quad \text{with } S(0) > 0 \text{ given}, \tag{1}$$

$$\dot{C} = -q^c, \quad \text{with } C(0) > 0 \text{ given}. \tag{2}$$

Alternatively, consumers can use a renewable backstop at quantity $q^b(t) \geq 0$. All three are perfect substitutes, such that gross consumer benefit can be written as

$$U(q^s + q^c + q^b) = \int_0^{q^s+q^c+q^b} P(j)\, dj, \tag{3}$$

where $P(j)$ is the inverse demand for the resource with $P'(j) < 0$. Marginal cost for the backstop are constant and denoted by $k^b > 0$. Extraction cost for deposit type C, $K^c(q^c, C)$, are increasing and weakly convex in q^c. As deposit type C declines, extraction costs increase, $K^c_C(q^c, C) < 0$. The extraction technology for deposit type C is assumed to be mature and fixed. Extraction cost for deposit type S, by contrast, depend on the state of technology, $Z(t)$, and are denoted by $K^s(q^s, S, Z)$. Marginal extraction costs are positive and non-decreasing, $K^s_q > 0$, and $K^s_{qq} \geq 0$. As deposit type S diminishes, extraction costs increase, $K^s_S(q^s, S, Z) < 0$. Cost and marginal cost are decreasing with the state of technology, $K^s_Z < 0$, and $K^s_{qZ} < 0$. We consider extraction-biased technological change, i.e. $K^s_{SZ} = 0$ (Farzin, 1995). Extraction cost and the marginal cost reduction from an increase in the technology stock are zero, if there is no extraction from deposit type S, $K^s(0, S, Z) = 0$, and $K^s_Z(0, S, Z) = 0$.

The state Z of the extraction technology for deposit type S can be improved by technology investments, $w(t) \geq 0$. Choosing units of measurement, the technology stock evolves according to

$$\dot{Z} = w, \quad \text{with } Z(0) \geq 0 \text{ given}. \tag{4}$$

The cost of technology investments is given by $L(w)$ with positive and increasing marginal cost, $L'(w) > 0$, and $L''(w) > 0$. Competitive firms are assumed to maximize the present value of revenues minus extraction and investment cost,

$$\max_{\{q^s,q^c,q^b,w\}} \int_0^\infty \left[p\cdot(q^s+q^c+q^b) - K^s(q^s,S,Z) - K^c(q^c,C) - k^b\,q^b - L(w)\right] e^{-\delta\,t}\,dt \tag{5}$$

subject to (1), (2), and (4), and non-negativity constraints for all variables. The discount rate is denoted by δ.[3]

As in Pindyck (1984), market equilibrium requires that $P(q^s(t)+q^c(t)+q^b(t)) = p(t)$ at each point in time. To guarantee an interior solution to the market equilibrium conditions, we adopt the standard assumption that the difference between consumer benefit and costs, $W = U(q^s+q^c+q^b) - K^s(q^s,S,Z) - K^c(q^c,C) - k^b q^b - L(w)$, is weakly concave in its arguments. In addition to the above-stated assumptions on cost functions the weak concavity of W in q^s, S, and Z requires

$$\left(P'(q^s+q^c+q^b) - K^s_{qq}(q^s,S,Z)\right) K^s_{ZZ}(q^s,S,Z) + K^s_{qZ}(q^s,S,Z)^2 \le 0. \tag{6}$$

Note that Condition (6) is always fulfilled for standard specifications of the extraction cost function, e.g. for the specification $K^s(q^s,S,Z) = \left(k_0 + \frac{k_s}{S}\,q^s + \frac{k_z}{Z}\,q^s\right) q^s$.

3. Theoretical results

The shadow prices of the resource stocks are denoted by $\gamma(t)$ for deposit type S and by $\lambda(t)$ for deposit type C, and capture the current value of an additional unit of resource in situ. The shadow price of the technology stock is denoted by $\phi(t)$ and captures the current value of technological progress in the shale gas industry.

With this notation, the optimality conditions for resource production and technology investment are given by

$$P(q^s+q^c+q^b) = p \le K^s_q(q^s,S,Z) + \gamma, \tag{7a}$$

$$P(q^s+q^c+q^b) = p \le K^c_q(q^c,C) + \lambda, \tag{7b}$$

$$P(q^s+q^c+q^b) = p \le k^b, \tag{7c}$$

$$-L'(w) + \phi \le 0, \tag{7d}$$

where conditions hold with equality whenever the corresponding variable is positive and with inequality if the non-negativity constraint is binding. In (7) we have already inserted the market equilibrium condition that inverse demand $P(\cdot)$ is equal to the supply price p of the resource.

Equations (7a), (7b) and (7c) state that the resource price should equal the full marginal cost of the resource whenever a positive amount is used. The full marginal cost of each deposit type is characterized by marginal extraction cost plus opportunity cost associated with the scarcity of the resource stock. Following the literature, we define the sum of these costs as augmented marginal costs ($AMC^s(t)$ for deposit type S and $AMC^c(t)$ for deposit type C). Equation (7d) shows that investment in extraction technology is determined by the condition that marginal cost of technology development should equal the shadow price of technology.

The dynamic optimality conditions require that the shadow prices γ and λ develop according to

$$\dot{\gamma} = \delta\,\gamma + K^s_S(q^s,S,Z), \tag{8a}$$

$$\dot{\lambda} = \delta\,\lambda + K^c_C(q^c,C). \tag{8b}$$

If extraction costs are independent of the stock, shadow prices rise at the discount rate, δ. A stock effect on extraction costs reduces the growth rates of shadow-prices. Extraction from a resource stock ends when either the deposit type has been exhausted or its shadow value has turned to zero. Extraction from deposit type C stops before physical exhaustion if a stock size $\underline{C} > 0$ exists such that $k^b < K^c_q(0,\underline{C})$. Endogenous technology qualifies the corresponding condition for deposit type S. A sufficient condition for extraction from deposit type S to stop before physical exhaustion of the stock is that, for all technology levels $Z \ge Z_0$, there exists a stock size $\underline{S}(Z)$ such that $k^b < K^s_q(0,\underline{S}(Z),Z)$. The other way around, a sufficient condition such that deposit type S will eventually be physically exhausted is $k^b > K^s_q(0,0,Z_0)$. In all other cases, it depends on the costs of technology investments and the discount rate whether or not deposit type S will be physically exhausted. The optimal development for the shadow price of technology is determined by

$$\dot{\phi} = \delta\,\phi + K^s_Z(q^s,S,Z), \tag{8c}$$

with transversality condition

$$\lim_{t\to\infty} e^{-\delta\,t}\,\phi(t)\,Z(t) = 0. \tag{8d}$$

[3] We consider a deterministic setting with perfect foresight, and are interested in the long-term dynamics of aggregate resource extraction. On a shorter time scale, and with demand and supply shocks, production capacities of individual wells play an important role (Anderson et al., 2018).

Equation (8c) shows very different dynamics for the shadow price of technology for phases without extraction from deposit type S, i.e. when $q^s(t) = 0$ and thus $K^s_Z(0, S, Z) = 0$, and during phases with $q^s(t) > 0$ and thus $K^s_Z(q^s, S, Z) < 0$.

3.1. Technological progress

The dynamics of technological progress in the shale gas industry depend on how the marginal benefit of technology, $-K^s_Z(q^s, S, Z)$, develops over time:

$$\frac{d}{dt}\left(-K^s_Z(q^s, S, Z)\right) = -K^s_{ZZ}(q^s, S, Z)\, w - K^s_{qZ}(q^s, S, Z)\, \dot{q}^s \tag{9}$$

By assumption $K^s_Z(0, S, Z) = 0$ the marginal benefit of technology does not change over time if there is no resource extraction from deposit type S, and as we consider extraction-biased technological change, $K^s_{ZS}(q^s, S, Z) = 0$. If there is resource extraction exclusively from deposit type S, the marginal benefit of technology is decreasing over time.

Lemma 1. *The marginal benefit of technology, $-K^s_Z(q^s, S, Z)$, monotonically decreases over time during a period of resource extraction exclusively from deposit type S.*

proof. *See* Appendix A.

The decline in the marginal benefit of technology results from a combination of two effects captured by the two terms on the right-hand-side of equation (9): First, the technology stock may continue to grow if investment stays positive during resource extraction, and this decreases the marginal benefit of further technology improvements. Second, resource extraction may increase or decrease over time, which tends to increase or decrease the marginal benefit of technology. The change of resource scarcity over time drives the latter effect. Due to concavity of the objective function, the net effect is towards decreasing marginal benefit of technology, under optimal resource extraction, cf. Appendix A.

Lemma 1 allows us to characterize the dynamics of technological progress in the period without and in the period with resource extraction exclusively from deposit type S.

Proposition 1. *Investment in extraction technology (weakly) monotonically increases during a period without extraction from deposit type S, (weakly) monotonically decreases during a period with extraction exclusively from deposit type S, and then stops after deposit type S has been depleted.*

proof. *See* Appendix B.

The proof of Proposition 1 utilizes the connection between optimal investment and the shadow price of technology ϕ given by (7d). If this condition holds with equality, we can differentiate it to obtain $\frac{w\,L''(w)}{L'(w)}\,\frac{\dot{w}}{w} = \frac{\dot{\phi}}{\phi}$. As $L''(w) > 0$, there is a monotonic relationship between w and ϕ. The growth rate of w, multiplied by the elasticity of marginal investment cost, equals the growth rate of the shadow price of technology. Accordingly, the statements in Proposition 1 hold with strict monotonicity during a period where investment is positive.

During a period without shale gas extraction, (8c) implies $\dot{\phi} = \delta\,\phi$. The shadow price of technology increases over time at the rate of interest – provided it is positive to begin with. Due to the monotonic relationship between w and ϕ, investments increase over time as well and the technology stock grows in a convex fashion, $\ddot{Z} = \dot{w} > 0$. There are no revenues in the shale gas industry during this period. However, producers already have an investment incentive since the present value of investment is positive, $\phi w - L(w) = L'(w)w - L(w) > 0$.

During a period in which shale gas is the exclusive source of production, investments decline and the technology stock grows in a concave fashion, $\ddot{Z} = \dot{w} < 0$. As the shale gas stock approaches exhaustion, investments go to zero since technological progress becomes worthless.

Over the two periods, before shale gas is used, and after deposit type S is used exclusively, the technology stock develops over time in an S-shaped fashion, first convex and then concave. This is a pattern familiar from the literature on technology diffusion (Davies, 1978; Jaffe and Stavins, 1994; Helm and Mier, 2019).

3.2. Order of resource extraction

To obtain clear-cut statements about the temporal order of resource use, we assume constant marginal costs of extraction for both resources,[4]

$$\begin{aligned} K^c(q^c) &= k^c\, q^c, \\ K^s(q^s, Z) &= k(Z)\, q^s. \end{aligned} \tag{10}$$

[4] The effect of increasing marginal costs is to smooth out extraction of both resources over time, which blurs the effects of endogenous technology on patterns of resource use that we are interested in. The numerical analysis in section 4 shows that the qualitative results hold with increasing marginal costs as well.

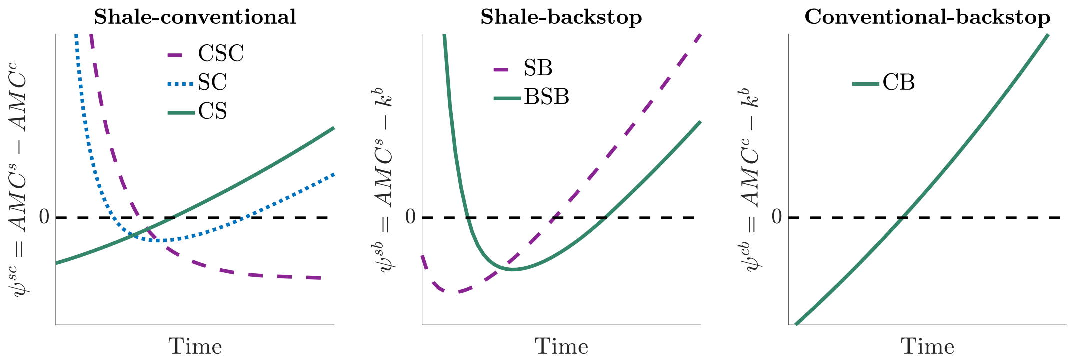

Fig. 1. Switch functions for each pair of resource types, and their corresponding production orders.

Without stock effects the dynamic optimality conditions require that the shadow prices γ and λ of the two deposit types increase exponentially at the discount rate (Hotelling, 1931). Using $\gamma(0) = \gamma_0$ and $\lambda(0) = \lambda_0$ to denote the – endogeneous – initial values of the two shadow prices, the shadow prices at time t thus are

$$\gamma(t) = \gamma_0 \, e^{\delta \, t}, \tag{11a}$$

$$\lambda(t) = \lambda_0 \, e^{\delta \, t}. \tag{11b}$$

If the initial shadow prices are positive, which we assume to be the case throughout the paper, both stocks will ultimately be exhausted.

Under the assumption of constant marginal costs, only one type of resource will be used at each point in time, as shown below. We return to the case of increasing marginal extraction costs in the quantitative analysis of a calibrated version of the model in Section 4. The general results on the order of resource extraction hold for the calibrated version as well, but are blurred by phases of simultaneous resource extraction.

We focus on the relevant case in which unit extraction cost for both nonrenewable resources (may) be low enough to make them competitive to the backstop technology, $k^c < k^b$ and there exists some $\underline{Z} \geq 0$ such that $k(Z) < k^b$ for all $Z \geq \underline{Z}$.

We further assume a linear marginal investment cost function, i.e. that the elasticity of marginal investment cost is equal to one,

$$\frac{w\,L''(w)}{L'(w)} = 1. \tag{12}$$

This assumption implies that the growth rate of investment is equal to the growth rate of the shadow price of technology.[5]

The optimality conditions (7a), (7b), and (7c), with assumption (10), imply that resources are used according to the Gerneralized Herfindahl Principle. Since the resources are perfect substitutes, and marginal costs are constant by assumption (10), optimality requires that only one resource will be used at a time. At each point in time, this is the resource with the smallest augmented marginal cost. For the backstop, the augmented marginal cost are equal to the constant marginal cost of production, $AMC^b = k^b$. For shale and conventional gas, augmented marginal cost is equal to the marginal cost of production plus the shadow price of the resource stock, $AMC^s = k(Z) + \gamma$ and $AMC^c = k^c + \lambda$. Thus, the decision whether a resource type is produced or not is not only governed by the change in shadow values over time but also by the dynamics of the technology stock.

To establish the optimal order of resource use we derive switch functions which, for each pair of resource types, is the difference between augmented marginal costs of the two resource types. A switch in production occurs when a switch function equals zero, i.e. the augmented marginal costs are equal between resource types. Fig. 1 shows the shape of all possible switch functions. The formal proof is in Appendix C.

The switch function for conventional gas and the backstop, ψ^{cb}, is monotonically increasing over time. Hence, there is only one possible extraction order. The switch functions ψ^{sc} and ψ^{sb} allow for more than one switch point. In Appendix C we show that $\ddot{\psi}^{sc} > 0$ ($\ddot{\psi}^{sb} > 0$) for any point in time where $\dot{\psi}^{sc} = 0$ ($\dot{\psi}^{sb} = 0$), which implies that $\dot{\psi}^{sc}$ ($\dot{\psi}^{sb}$) can change signs only once. Overall, there are at most two switch points between resource types. The switch functions for shale gas can follow a U-shaped path, but not an inverted U-shaped path.[6] Thus, the extraction order *SCS* cannot occur. Once investments into technology made extraction from deposit type *S* advantageous over the other resources and extraction from deposit type *S* starts, it is optimal

[5] We assume that marginal investment cost is sufficiently small so that deposit type *S* will be used. This is for example the case, when marginal cost is zero as investments are zero, $L'(0) = 0$. Here, optimality condition (7d) always holds with equality if the shadow price of technology is positive.

[6] Note that the switch function ψ^{sb} must become positive as $t \to \infty$ since shale gas will be depleted eventually, and the renewable backstop will be the exclusive source of production.

to fully reap the benefits of technology investments and continuously extract from deposit type S until it is exhausted. The following proposition summarizes our results on extraction orders.

Proposition 2. *The only possible orders of resource extraction are the following:*

(CSB) deposit type C → deposit type S → Backstop
(CBSB) deposit type C → Backstop → deposit type S → Backstop
(CSCB) deposit type C → deposit type S → deposit type C → Backstop
(SCB) deposit type S → deposit type C → Backstop

proof. *See* Appendix C.

In Herfindahl (1967) producers first extract the deposit type with the highest rent. In our model, this rule does not hold. Producers might start with the lower rent deposit type since the rent from the higher rent deposit type cannot yet be realized.

In Appendix C we show that extraction order *SCB* from Proposition 2 can only result, if $\lambda_0 < \gamma_0$. Since resource extraction follows the Generalized Herfindahl Principle, *SCB* implies that the initial technology stock is high and $k(Z(0)) < k^c$. As this seems not to be a sensible assumption for shale gas, in the following we focus on the first three extraction orders.

3.3. Price development

Investment decisions do not only affect the optimal order of extraction but also price development. Optimality requires that the resource price equals augmented marginal costs for the type of deposit extracted. Hence, during the extraction of deposit type C price monotonically increases over time as in Hotelling (1931),

$$\dot{p} = \delta \left(p - k^c\right) > 0 \quad \text{whenever } q^c > 0. \tag{13}$$

During periods where the backstop is used, price stays constant, $p = k^b$. In periods where deposit type S is produced, price behavior depends on the technology stock, and develops according to

$$\dot{p} = \delta \ (p - k(Z)) + k'(Z)\, w \quad \text{whenever } q^s > 0. \tag{14}$$

There are two opposite effects on price development during shale gas extraction. The first term represents the positive effect from the scarcity of the resource, just as it is the case for extraction from deposit type C. The second term shows the effect that technological progress reduces marginal extraction costs over time. This effect may dominate the effect of increasing scarcity. Endogenous technological progress in resource extraction may thus explain decreasing resource prices, and increasing resource extraction, especially if the discount rate is low enough.[7] We formally state this result in the following proposition.

Proposition 3. *(i) After a transition from extraction of deposit type C to extraction of deposit type S, the resource price decreases if the discount rate is small enough. (ii) After a transition from backstop to extraction of deposit type S, the resource price decreases, irrespective of the discount rate.*

proof. (*i*) *At the switch from C to S,* $p - k(Z) < k^b$. *This is because price can never be higher than the marginal cost of the backstop. In the limit of zero discounting, from* (14) *it follows*

$$\lim_{\delta\to 0} \dot{p} < \lim_{\delta\to 0} \left(\delta\, k^b + k'(Z)\, w\right) = \lim_{\delta\to 0} k'(Z)\, w \le 0. \tag{15}$$

By continuity, *there exists an interval of positive discount rates where still* $\dot{p} < 0$ *at the switch point between conventional and shale gas.*

(*ii*) *At the transition from backstop to deposit type S, price must be decreasing, as otherwise it could not be optimal to switch away from the backstop.*

Denoting by $\underline{T}^s$ the time when extraction from deposit type S starts, by $\overline{T}^s$ the time when it ends, and by T^b the time of the switch from C to B, we can summarize the price dynamics for the first three extraction orders from Proposition 2 as follows:

$$CSB: \qquad p(t) = \begin{cases} AMC^c(t) = \lambda_0\, e^{\delta\, t} + k^c, & t \in [0, \underline{T}^s] \\ AMC^s(t) = \gamma_0\, e^{\delta\, t} + k(Z), & t \in [\underline{T}^s, \overline{T}^s] \\ AMC^b = k^b, & t \in [\overline{T}^s, \infty] \end{cases} \tag{16a}$$

[7] For a high discount rate the increasing shadow price of the resource can dominate the effect that technological progress decreases the extraction cost. In this case, augmented marginal cost monotonically increase during shale gas production, albeit at a rate less than the rate of interest.

$$CBSB: \qquad p(t) = \begin{cases} AMC^c(t) = \lambda_0\, e^{\delta\, t} + k^c, & t \in [0, T^b] \\ AMC^b = k^b, & t \in [T^b, \underline{T}^s] \\ AMC^s(t) = \gamma_0\, e^{\delta\, t} + k(Z), & t \in [\underline{T}^s, \overline{T}^s] \\ AMC^b = k^b, & t \in [\overline{T}^s, \infty] \end{cases} \tag{16b}$$

$$CSCB: \qquad p(t) = \begin{cases} AMC^c(t) = \lambda_0\, e^{\delta\, t} + k^c, & t \in [0, \underline{T}^s] \\ AMC^s(t) = \gamma_0\, e^{\delta\, t} + k(Z), & t \in [\underline{T}^s, \overline{T}^s] \\ AMC^c(t) = \lambda_0\, e^{\delta\, t} + k^c, & t \in [\overline{T}^s, T^b] \\ AMC^b = k^b, & t \in [T^b, \infty]. \end{cases} \tag{16c}$$

During the extraction of deposit type *C*, price equals augmented marginal costs of conventional gas and thus monotonically increases over time. Note that the initial shadow values of deposit types *C* and *S* are endogenous and in general different for extraction orders *CSB*, *CBSB* and *CSCB*. During the use of deposit type *C*, technology investments increase over time and shale extraction technology is build up at an increasing pace. In extraction order *CSB*, producers switch directly to shale gas after conventional gas is depleted, and price follows augmented marginal costs of shale gas (see (16a)). In extraction order *CBSB*, the conventional deposit type is depleted before shale gas becomes competitive and the market switches to the renewable first. During the period in which the backstop is used, price stays constant. Investments continue to increase and extraction costs of shale gas continue to decline. As augmented marginal costs become small enough, production finally switches to shale gas (see (16b)). In extraction order *CSCB*, the conventional deposit type is not depleted as shale gas becomes competitive. Producers switch back to deposit type *C* after deposit type *S* is used up (see (16c)).

During the extraction of shale gas, production can increase and price can decrease for a period of time, especially if the discount rate is low enough (see Proposition 3). Eventually, price must rise again since technological progress becomes worthless as investments go to zero and deposit type *S* approaches exhaustion. In the end, resource scarcity dominates the price behavior, giving rise to a U-shaped price path (for a schematic illustration see Fig. 2). Note that in extraction order *CBSB*, resource price will always follow a U-shaped path during shale gas production as otherwise the market would not switch away from the backstop. In the end, as all nonrenewable resources are used up, production switches (again) to the renewable and price stays constant at the marginal cost of the backstop.

3.4. Extension: carbon tax

We now study how a carbon tax on shale (τ^s) and conventional gas (τ^c), affects the dynamics of resource production. As we express the carbon tax in dollars per unit extracted, and emission factors may differ for the two deposits, tax rates may differ. Constant tax rates have the same effect as an increase in marginal extraction costs by a constant. Thus, the qualitative results on technological progress, extraction order and price dynamics from the previous sections remain valid under carbon taxation. Yet, the carbon tax may have important consequences especially for the use of shale gas, as it influences investment incentives in technology development.

With the carbon tax, the optimality conditions for resource production of conventional and shale gas become

$$P(q^s + q^c + q^b) = p \le k^c + \tau^c + \lambda, \tag{17a}$$

$$P(q^s + q^c + q^b) = p \le k(Z) + \tau^s + \gamma. \tag{17b}$$

As before, AMC_t^c and AMC_t^s are determined by the right hand side of equations (17a) and (17b). Besides the marginal production cost and the endogenous shadow price of the resource, the augmented marginal costs now additionally include the carbon taxes.

We first briefly review how the carbon tax alters extraction of conventional gas. First, at a tax rate beyond $\overline{\tau}^c = k^b - k^c$, the production of conventional gas turns uneconomic due to the availability of a backstop technology. A positive carbon tax below $\overline{\tau}^c$ has an effect on the time path of conventional gas production that is equivalent to an increase in the marginal extraction cost. Conventional production is lower in early points in time and higher in later points in time, compared to the case without the tax.

For shale gas, the effect of a carbon tax is more interesting. If the tax rate is very high, also shale gas will never be used. The critical value depends on the endogenous technical progress. An upper bound for this critical value is given by

$$\overline{\tau}^s = k^b - \lim_{Z\to\infty} k(Z). \tag{18}$$

If the tax rate is below $\overline{\tau}^s$, it may pay to sufficiently invest into technology development such that eventually extraction of shale gas becomes profitable even with the carbon tax in place. A long time of investing into technology development may be necessary, though, until *Z* is large enough and extraction costs have been pushed down sufficiently.

Generally, carbon taxation will postpone shale gas extraction. We show below that this would be true even for an exogenous technology development. As taxation reduces investment incentives, endogenous technical progress amplifies the effect that

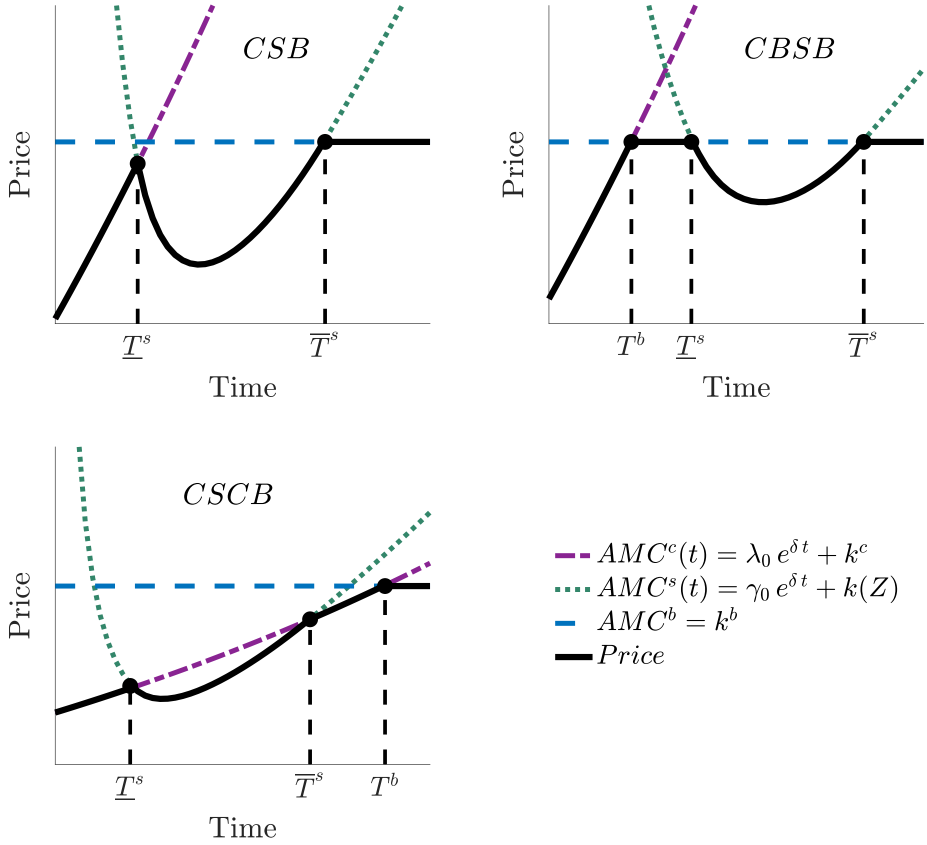

Fig. 2. Price and augmented marginal cost dynamics for extraction orders *CSB* (top left graph), *CBSB* (top right graph) and *CSCB* (bottom left graph). Price (in black) always follows the deposit with the smallest augmented marginal costs (shown in different colours). The graph assumes that the discount rate is sufficiently small (see Proposition 3). (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

carbon taxation postpones the period of time when shale gas is used. To develop the theoretical argument for both exogenous and endogenous technical change, we ignore conventional gas and focus on the case with only two resources, shale gas and the renewable backstop, and consider the case that the initial technology stock is so small that shale gas is not extracted initially.[8] Proposition 2 implies that, without the carbon tax, there will be a continuous time interval $[\underline{T}^s, \overline{T}^s]$ of shale gas extraction.

Consider first the case that the technology development is fixed at the pattern without the carbon tax. For such a fixed technology development, the carbon tax decreases the initial shadow value of the shale gas deposit type, and increases the augmented marginal cost of shale gas at the point in time $\underline{T}^s$ where shale gas extraction starts without the carbon tax. We prove this formally in Appendix D. As a consequence, it is not profitable to extract shale gas at $\underline{T}^s$, but possibly at a later point in time, when technology has advanced sufficiently such that the augmented marginal costs of shale gas extraction, including the carbon tax, have fallen below k^b. This is illustrated in Fig. 3.

The changing pattern of shale gas extraction due to the carbon tax has repercussions on the shadow value of technology. We show in Appendix D that it initially (i.e., before and at $\underline{T}^s$) decreases with fixed technology development and even more so with endogenous technology development, as illustrated in Fig. 3. As a consequence, the endogenous technology stock is lower at $\underline{T}^s$ with the carbon tax than without and even lower when taking into account endogenous technical progress. This further increases the augmented marginal costs of shale gas extraction at $\underline{T}^s$. Overall, carbon taxation delays shale gas extraction with endogenous technology development.

4. Calibration to the U.S. gas industry

To verify that the rather simple model developed here is able to reproduce patterns of actual resource use, we specify and calibrate the model for the U.S. natural gas industry. The transition from conventional to shale gas in the U.S. shows a period of simultaneous extraction. To capture this, we allow for increasing marginal extraction costs for both types of deposits. For

[8] A sufficient condition for this is $k(Z(0)) > k^b$.

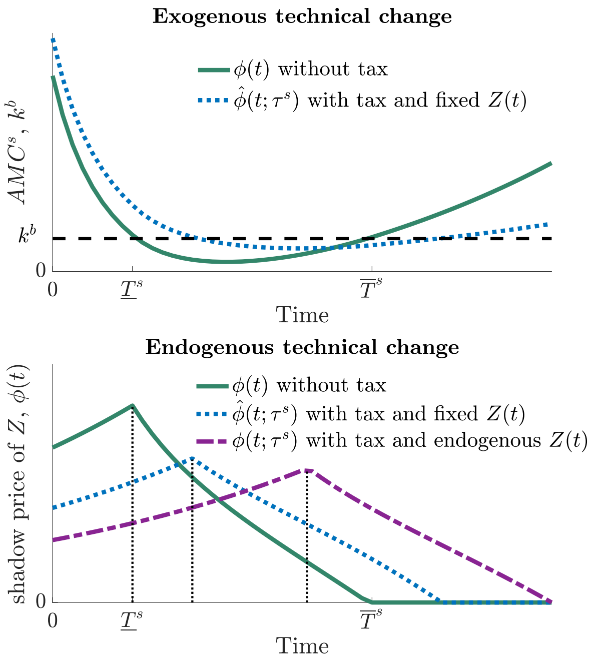

Fig. 3. Effect of a carbon tax in models with exogenous (top graph) and endogenous (bottom graph) technical change.

conventional gas we assume an extraction cost function of the form

$$K^c(q^c) = \left(k_0 + k^c \cdot (q^c)^\beta\right) \cdot q^c, \tag{19}$$

with (positive) parameters k_0, k^c, and β to be calibrated. We use the same type of cost function for the extraction of shale gas, but the non-constant marginal cost term depends on the state of technology, i.e.

$$K^s(Z, q^s) = \left(k_0 + \frac{k^s}{Z} \cdot (q^s)^\beta\right) \cdot q^s, \tag{20}$$

where k^s is a further parameter to be calibrated. This calibration assures that the marginal extraction cost for shale gas will not be below the lower bound of extraction cost for conventional gas. This seems a sensible assumption for shale and conventional gas, since shale gas extraction requires the extra effort of horizontal drilling and hydraulic fracturing which comes at a cost even with the most advanced technology.

The marginal cost of the renewable backstop is constant and given by k^b. As the backstop is not scarce it is used at a constant quantity whenever its production cost is lower than the augmented marginal cost of conventional gas and shale gas.[9] For investments in shale gas extraction technology we assume a quadratic cost function with cost parameter $l > 0$ to be calibrated,

$$L(w) = \frac{l}{2} w^2. \tag{21}$$

We assume a linear inverse demand function for natural gas

$$P(q^s + q^c + q^b) = d_0 - d_1 \left(q^s + q^c + q^b\right), \tag{22}$$

where d_0, with $d_0 > k^b > 0$, is a choke price for natural gas use, and $d_1 > 0$ is the slope of the inverse demand function.

We calibrate this model to yearly production data of shale and conventional gas in the U.S. natural gas industry from 1997 to 2017. The data is publicly available on the EIA (2017b) website.

Our calibration procedure is to specify some starting parameter set, and numerically optimize investment in shale gas extraction technology, and optimal extraction of both types of natural gas. The dynamic optimization model is numerically implemented in AMPL with Knitro, which implements state of the art interior-point and active-set algorithms for large-scale nonlinear programming problems (Byrd et al., 1999, 2006).[10] We then adjust the parameter set to minimize the distance of extraction quantities derived from the model and the data. The resulting calibration of the model's parameter values is given in Table 1.

[9] Marginal costs of renewable may be increasing as well, with repercussions on their expansion (Lancker and Quaas, 2019). As our focus is on the patterns of use for non-renewable resources, we use the standard model for the backstop technology, which assumes constant marginal costs.

[10] Programming codes are available from the authors upon request.

Table 1
Calibrated parameter values.

Parameters	Values	Units
k_0	0.3	bn USD/Tcf
k^c	3	bn USD/Tcf
k^s	30	bn USD/Tcf
β	0.23	
k^b	8	bn USD/Tcf
l	55	bn USD
d_0	15	bn USD/Tcf
d_1	0.4	bn USD/Tcf
δ	0.1	100%
$S(1997)$	1700	Tcf
$C(1997)$	340	Tcf
$Z(1997)$	0.1	

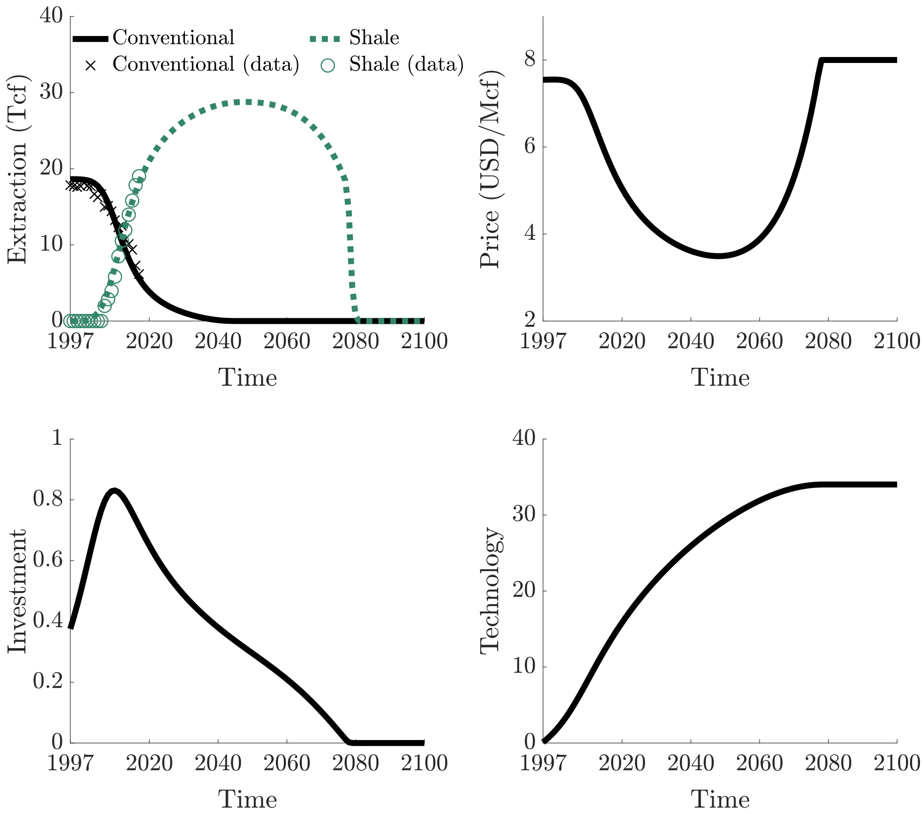

Fig. 4. Development of conventional and shale gas extraction (top left graph), gas price (top right), investment into shale gas extraction technology (bottom left) and technology stock (bottom right) for the calibrated model. The crosses and dots in the top left graph show the EIA (2017b) data on which the model is calibrated.

Overall, the calibrated parameter values seem to be in a plausible range. The calibration suggests that the overall reserves of shale gas, at the beginning of the model period in 1997, have been considerably larger than the reserves of conventional gas. This seems to be in line with expectations. Total technically recoverable dry natural gas resources in the U.S. are estimated at around 2200 Tcf, more than half of it from unconventional sources (EIA, 2012). To capture the rather strong effect of increased gas supply in a model that does not include any exogenous influences on the gas price, we obtain a rather large value for d_1, the slope of the inverse demand function. Also the discount rate is rather high, although not outrageous for an industry that is operating under various risks. In the calibration, the rather high discount rate allows to reproduce the fast dynamics of transition from conventional to shale gas.

Fig. 4 shows the output of the calibrated model, along with the extraction data on which the model is calibrated. The calibrated model reproduces the observed decrease in conventional gas extraction and the strong increase in shale gas extraction after 2007. The strong increase in shale gas extraction more than compensates the decrease in conventional gas production. Overall natural gas production increases and gas prices decline, also in line with observations. From 1997 to 2017, the U.S. natural gas price varied between 2 and 9 USD/Mcf (EIA, 2018a). Thus, the outcome on gas price development seems to be within a reasonable range. Our calibrated model suggests that the decrease in the gas price will continue for the next three decades, before the price starts to monotonically increase up to the marginal cost of the backstop.

5. Quantitative analysis

In this section, we use the calibrated model for three quantitative analyses. First, we compare the outcomes of the model with endogenous, forward-looking technological change to an alternative model specification where the improvement in extraction technology comes as an unanticipated shock, as often assumed in the empirical literature. Second, we use the model with endogenous technological change to study how the extraction of natural gas would have been influenced by the introduction of a carbon tax. Last, we also include a carbon tax in the alternative model specification and compare the results of both model types.

5.1. Endogenous technological change vs. technology shock

We now contrast the outcome of the calibrated model presented in the previous section to the outcome under the alternative assumption that the improvement in shale extraction technology (and the decrease in extraction costs) comes from an exogenous and unanticipated shock rather than from the forward-looking behaviour of firms that invest in research and development to maximize profits. We assume that in 2007 technology improved from the initial to the final level of the technology stock in our calibrated model with endogenous technological change:

$$Z(t) = \begin{cases} Z(0) \ = 0.1, & t \in [1997, 2006], \\ Z(\overline{T}^s) = 34, & t \in [2007, \infty]. \end{cases} \tag{23}$$

Fig. 5 shows how natural gas extraction and price dynamics change due to this modification. Whereas at first glance both models seem to generate similar patterns of resource extraction, there are pronounced differences, clearly visible especially in the price development. In the model with exogenous technology shock, the price is continuously increasing, except for the discontinuous drop in the year where the technology shock hits. In particular, the price path follows the Hotelling rule of exponentially increasing prices before shale gas becomes available, and again after shale gas extraction starts. Moreover, shale gas extraction continuously decreases after the technology shock. All these patterns are inconsistent with observations. By contrast, the model with endogenous, forward-looking technological change shows a continuous increase of shale gas extraction, before it peaks a few decades after shale gas extraction started. Accordingly, the price continuously decreases for a period of several years, and only eventually starts to increase again.

5.2. Effects of a carbon tax on natural gas extraction

We now turn back to our model with endogenous technological change and include a carbon tax (τ) for producers. Assuming a carbon content of natural gas of 117 pounds of CO_2 per million British thermal units (EIA, 2019a), we convert the tax from USD/tCO_2 into USD/Tcf. Similar as in Section 3.4, we include the tax in the production cost functions for shale and conventional gas in the following way:

$$K^c(q^c) = \left(k_0 + k^c \cdot (q^c)^\beta + \tau\right) \cdot q^c \tag{24}$$

$$K^s(Z, q^s) = \left(k_0 + \frac{k^s}{Z} \cdot (q^s)^\beta + \tau\right) \cdot q^s \tag{25}$$

Fig. 6 shows how a carbon tax affects the point in time when shale gas production starts and the point in time when shale gas production peaks. We define the start year as the year in which shale gas production exceeds 3 Tcf for the first time. This implies that in our baseline model, without a carbon tax, shale gas production starts in 2008 and peaks in 2048. If the carbon tax is small, the shale gas boom is only delayed by a couple of years. However, as the carbon tax is scaled up, the effect gets more pronounced. For a carbon tax of 45 USD/tCO_2, the start of the shale gas boom is pushed far into the future. Production starts in the year 2061 and peaks in 2112. Considering an even higher carbon tax of 46 USD/tCO_2 pushes the start year and peak year outside the here considered time horizon of 2120.

The quantitative analysis exemplifies our analytical results from Section 3.4. The carbon tax decreases technology investments, and thus marginal extraction costs of shale gas decrease at a lower pace, leading to a time delaying-effect for the extraction of shale gas. Fig. 6 suggests that the time-delaying effect is more pronounced with a more stringent climate policy.

Fig. 7 shows the sensitivity of the start year 2008 and 2050 with respect to a change in the carbon tax and the discount rate. We can observe that both, an increase in the carbon tax, but also an increase in the discount rate, induces a shift of technology investments into the future. Due to this, the production of shale gas is postponed. Considering for example the year 2050, the introduction of a carbon tax of 45 USD/tCO_2 is equivalent to a increase in the discount rate of about 8 percent. The sensitivity of the timing of the shale gas extraction to the discount rate underscores the importance of forward-looking behavior in technology development.

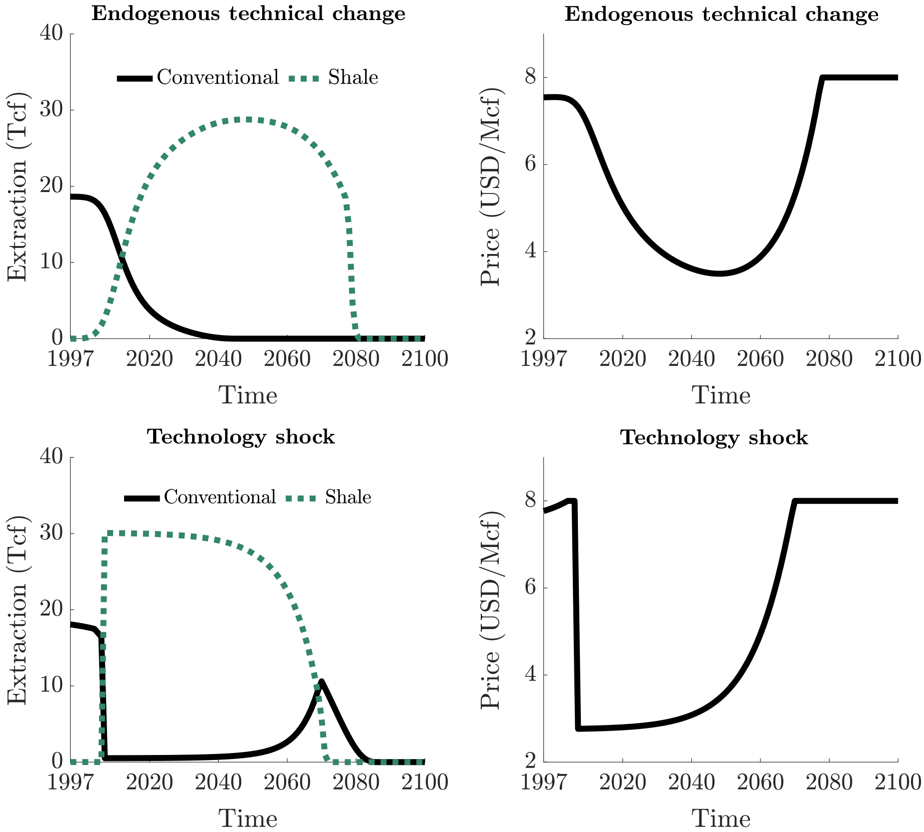

Fig. 5. Natural gas extraction and price dynamics in the model with endogenous technological change (top graphs) and under the alternative assumption of an exogenous shock to shale extraction technology in the year 2007 (bottom graphs).

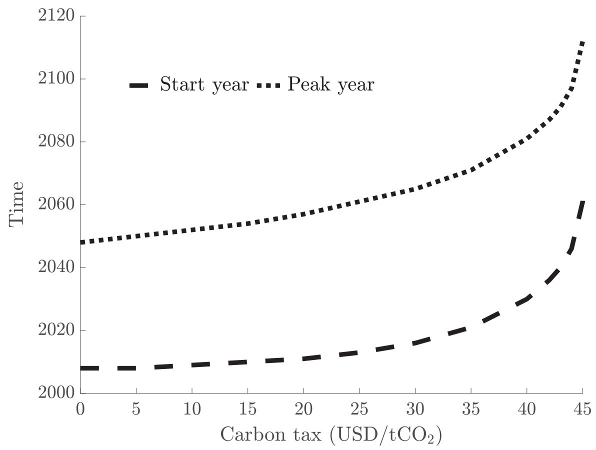

Fig. 6. The graph shows the year in which shale gas production starts (dashed line) and the year in which shale gas production peaks (dotted line) as a function of the carbon tax. The start year is defined as the year in which shale gas production exceeds 3 Tcf for the first time. For a carbon tax higher than 45 USD/tCO_2, the year of the start and peak lie outside the here considered time horizon.

5.3. Effects of a carbon tax in models with endogenous technological change vs. technology shock

Last, we also include a carbon tax in our alternative model specification from section 5.1 where the improvement in extraction technology comes as an unanticipated shock to the industry. Afterwards, we compare the results from this model to the results from the model with endogenous technological change from section 5.2. Fig. 8 shows how a carbon tax of 45 USD/tCO_2 would have affected natural gas extraction and price development for both model types.

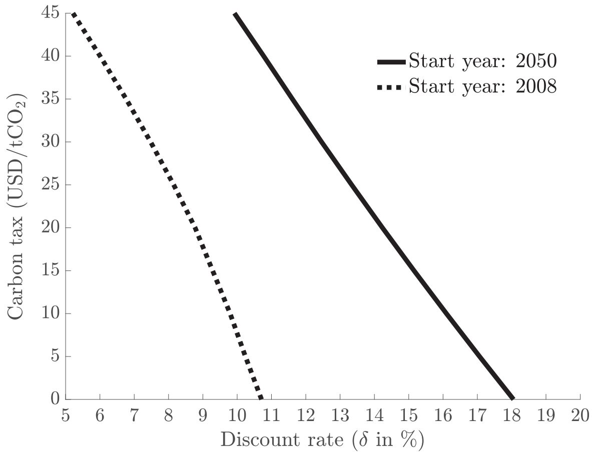

Fig. 7. Sensitivity of the start year 2008 (dotted line) and 2050 (solid line) to a change in the carbon tax and the discount rate.

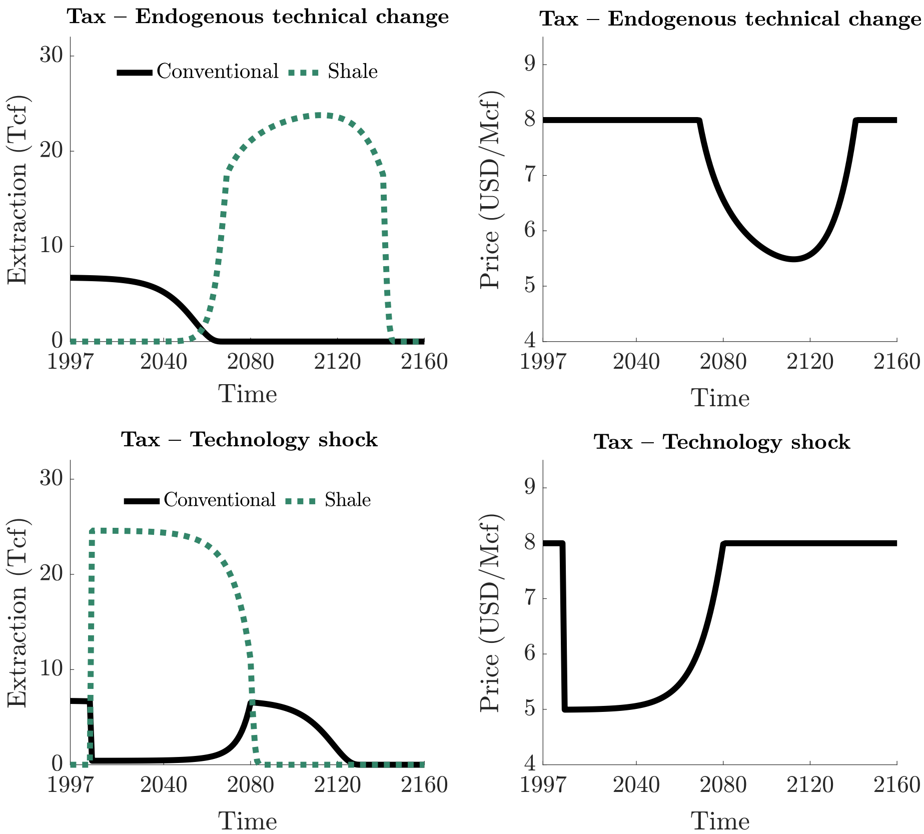

Fig. 8. Natural gas extraction and price development with a carbon tax of 45 USD/tCO_2 in the calibrated model with endogenous technological change (top graphs) and under the alternative assumption of an exogenous shock to shale extraction technology in the year 2007 (bottom graphs).

The results are strikingly different. As already discussed in the previous section, in the model with endogenous technological change, the year of the start and peak of shale gas extraction is sensitive to the level of the carbon tax. Due to the tax of 45 USD/tCO_2, the shale gas boom is pushed into the future. Extraction starts in 2061 and peaks in 2112. This strong time-delaying effect is a specific result of the forward-looking behaviour of firms and stands in sharp contrast to the other model specification where the improvement in extraction technology comes as an unanticipated shock. Under this alternative assumption, the start year of shale gas extraction is completely insensitive to the carbon tax. Similar to the model without a tax, extraction starts and price drops in 2007 when the shock hits. Afterwards, extraction continuously decreases and price continuously increases until shale and conventional gas are depleted. This demonstrates that for policy analysis it makes a big difference to endogenize technological change in the model.

6. Summary and conclusions

This paper presents a novel resource-economic theory that describes the main technological developments in the U.S. natural gas industry over the last two decades as a result of endogenous decisions on technology investments.

We find that investments in extraction technology for shale gas start slowly and are increasing before shale gas extraction begins. As production shifts completely towards shale gas, our theory predicts that investments will start to decline. Similar to the literature on technology diffusion, the technology stock develops over time in a S-shaped fashion, first convex and then concave. Our theory also provides a simple explanation of the recent decrease in natural gas prices and increase in gas production. According to the model, resource price can follow a U-shaped path after shale gas production starts. This finding is in contrast to Herfindahl (1967) where price always increases after producers switch from a low to a high cost deposit type.

In our setting, we abstract from technological change in the renewable backstop technology (e.g. Tsur and Zemel, 2003; Cook and Lin Lawell, 2020), which would have a decreasing effect on the value of shale gas. In the extreme case, when cost reductions from technological change in the backstop technology are sufficiently large and cheap, the shale gas resource would not be used at all. Producers would shift their investments away from the shale gas industry, and directly invest into the renewable resource. Increasing energy demand due to economic growth has the opposite effect and would increase the value of shale gas.

The calibrated model reproduces well the past development of U.S. shale gas production. The model suggests that natural gas production will continue to grow up to 30 Tcf and prices continue to decrease until 2050. We have also shown that for policy analysis it makes a big difference whether technical change is the endogenous result of forward-looking investment decisions or assumed to be an exogenous shock. The model with endogenous technical change suggests that a carbon tax could have postponed the shale gas boom considerably. This time-delaying effect is specific to the model with forward-looking investment behaviour, and does not occur in models that treat the improvement in extraction technology as an exogenous shock.

Our theory is not restricted to the natural gas industry but can also be applied to other exhaustible resources such as oil where similar technology developments are taking place. For all these resources, our model offers theoretical insights in the order of resource extraction. We find that by endogenizing technological investments in the Hotelling-Herfindahl model, production decisions do not only depend on initial shadow values but also on the state of extraction technology. This allows for additional orders of resource extraction. In particular, even if firms have already shifted to the use of the renewable backstop, production can switch back to a nonrenewable resource for a period of time, when extraction technology has sufficiently advanced.

Declaration of competing interest

None.

Appendix

A. Proof of Lemma 1

Given extraction-biased technical progress, $K^s_{ZS} = 0$, the marginal benefit of the technology develops according to equation (9). During the phase of shale gas extraction, (7a) holds with equality. Differentiating that condition with respect to time, and using (4) yields

$$(P'(q^s + q^c + q^b) - K^s_{qq}(q^s, S, Z))\,\dot{q}^s = K^s_{qZ}(q^s, S, Z)\,w - K^s_{qS}(q^s, S, Z)\,q^s + \delta\,(P(q^s + q^c + q^b) - K^s_q(q^s, S, Z)). \tag{26}$$

Using this in (9) and collecting terms delivers

$$\begin{aligned}
&-\left(P'(q^s + q^c + q^b) - K^s_{qq}(q^s, S, Z)\right) \frac{d}{dt}\left(-K^s_Z(q^s, S, Z)\right)\\
&= -\left(P'(q^s + q^c + q^b) - K^s_{qq}(q^s, S, Z)\right)\left(-K^s_{ZZ}(q^s, S, Z)\,w + K^s_{ZS}(q^s, S, Z)\,q^s - K^s_{qZ}(q^s, S, Z)\,\dot{q}^s\right)\\
&= \left(\left(P'(q^s + q^c + q^b) - K^s_{qq}(q^s, S, Z)\right) K^s_{ZZ}(q^s, S, Z) + K^s_{qZ}(q^s, S, Z)^2\right) w\\
&- K^s_{qZ}(q^s, S, Z)\,K^s_{qS}(q^s, S, Z)\,q^s + \delta\,K^s_{qZ}(q^s, S, Z)\left(P(q^s + q^c + q^b) - K^s_q(q^s, S, Z)\right).
\end{aligned} \tag{27}$$

The first term on the left-hand-side of this equation is positive, as the inverse demand function is downward-sloping and the marginal extraction costs are non-decreasing. The first term on the right-hand-side of the equation is non-positive by the concavity assumption, cf. (6). The second term, including the minus sign in front, is non-positive since, by assumption, $P' < 0$, $K^s_{qZ} < 0$, and $K^s_{qS} \leq 0$. The third term on the right-hand side of the equation is negative, as $K^s_{qZ} < 0$ and $P - K_q = \gamma > 0$.

B. Proof of Proposition 1

By (7d), the shadow price of technology is non-negative. Whenever investment is positive, such that (7d) holds with equality, the temporal change of investment is monotonic in the change in the shadow price of technology, as $L''(w)\,\dot{w} = \dot{\phi}$ and $L''(w) > 0$ by assumption. In a corner solution $w = 0$, $\dot{w} = 0$ by definition.

Whenever $q^s(t) = 0$, we have $K^s_Z(q^s, Z) = 0$ and thus (8c) implies $\dot{\phi} = \delta\,\phi$. The shadow price of technology, and, hence, investment, monotonically increases over time – provided it is positive to begin with.

Consider the time interval $t \in [\underline{T}^s, \overline{T}^s]$ with extraction from deposit type S, $q^s(t) > 0$, where $\overline{T}^s$ is the point in time where shale gas extraction ends. It must be $\phi(t) = 0$ for all $t \geq \overline{T}^s$, as otherwise $\phi(t)$ would grow at the discount rate after $\overline{T}^s$, whereas $Z(t) > 0$, which would violate the transversality condition (8d).

Thus, during the phase of shale gas extraction,

$$\phi(t) = -\int_t^{\overline{T}^s} e^{-\delta\,(\tau-t)}\,K^s_Z(q^s(\tau), Z(\tau))\,d\tau. \tag{28}$$

By Lemma 1 we have

$$\delta\,\phi(t) < -K^s_Z(q^s(t), Z(t)) \int_t^{\overline{T}^s} \delta\,e^{-\delta\,(\tau-t)}\,d\tau \tag{29}$$

$$= -K^s_Z(q^s(t), Z(t)) \left(1 - e^{-\delta\left(\overline{T}^s - t\right)}\right) < -K^s_Z(q^s(t), Z(t)). \tag{30}$$

Thus,

$$\dot{\phi} = \delta\,\phi + K^s_Z(q^s(t), Z(t)) < 0. \tag{31}$$

C. Proof of Proposition 2

A switch in production can occur when augmented marginal costs of two resource types are equal. Thus, we have three switch functions.

i) Switch points between shale and conventional gas are determined by the condition that the difference of AMC^s and AMC^c is zero. The switch points are given as the roots of the switch function

$$\psi^{sc}(t) = AMC^s(t) - AMC^c(t) = k(Z(t)) - k^c - (\lambda(0) - \gamma(0))\,e^{\delta\,t}. \tag{32}$$

Note that at a switch point from conventional to shale gas, investments in the technology stock are positive. The derivative of the switch function with respect to time is

$$\dot{\psi}^{sc} = k'(Z)\,w - \delta\,(\lambda(0) - \gamma(0))\,e^{\delta\,t}. \tag{33}$$

The first term is negative. The second term is negative as well if $\lambda(0) - \gamma(0) > 0$, whereas it is positive for $\lambda(0) - \gamma(0) < 0$. In case $\lambda(0) - \gamma(0) \geq 0$, the switch function ψ^{sc}_t is monotonically decreasing at any switch point. As the switch function is continuous, there is at most one switch point between regimes. Furthermore, the switch must be from conventional to shale gas. The only possibility is first deposit type C, then deposit type S. In case $\lambda(0) - \gamma(0) < 0$, there may be more than one switch point. As the switch function is continuous, it must assume a maximum or minimum in between any two switch points. At this maximum or minimum, the curvature of the switch function is

$$\ddot{\psi}^{sc} = k''(Z)\,w^2 + k'(Z)\,\dot{w} + \delta^2\,(\gamma(0) - \lambda(0))\,e^{\delta\,t} \tag{34a}$$

$$= k''(Z)\,w^2 + k'(Z)\,\frac{w}{\phi}\,(\delta\,\phi + k'(Z)\,q^s) + \delta^2\,(\gamma(0) - \lambda(0))\,e^{\delta\,t} \tag{34b}$$

$$= \underbrace{k''(Z)\,w^2 + (k'(Z))^2\,\frac{w}{\phi}\,q^s}_{>0} + \delta\,\underbrace{(k'(Z)\,w + \delta\,(\gamma(0) - \lambda(0))\,e^{\delta\,t})}_{=\dot{\psi}^{sc}} > 0. \tag{34c}$$

Thus, $\ddot{\psi}^{sc} > 0$ for any point in time where $\dot{\psi}^{sc} = 0$, which implies that $\dot{\psi}^{sc}$ can change signs only once. Overall, there are at most two switch points between the regimes. The only possible orders of production are SC, CSC, and CS.

ii) Switch points between conventional gas and the backstop are determined by

$$\psi^{cb} = k^c + \lambda(0)\,e^{\delta\,t} - k^b = 0. \tag{35}$$

We have $k^c - k^b < 0$ is constant. Thus, if there is a switch point at all, the switch function is monotonically increasing at this point. As the switch function is continuous, there is exactly one switch point and the switch must be from deposit type C to the backstop.

iii) Switch points between shale gas and the backstop are determined by

$$\psi^{sb} = k(Z) + \gamma(0)\, e^{\delta\, t} - k^b = 0. \tag{36}$$

The derivative of the switch function with respect to time is

$$\dot{\psi}^{sb} = k'(Z)\, w + \delta\, \gamma(0)\, e^{\delta\, t}. \tag{37}$$

The first term is negative and the second term is positive, i.e. $\dot{\psi}^{sb}$ may switch signs, and thus $\psi^{sb} = 0$ may have more than one solution. Consider the curvature of ψ^{sb} at the minimum or maximum, i.e. when $\dot{\psi}^{sb} = 0$:

$$\ddot{\psi}^{sb} = k''(Z)\, w^2 + k'(Z)\, \dot{w} + \delta^2\, \gamma(0)\, e^{\delta\, t} \tag{38a}$$

$$= k''(Z)\, w^2 + k'(Z)\, \frac{w}{\phi}\, (\delta\, \phi + k'(Z)\, q^s) + \delta^2\, \gamma(0)\, e^{\delta\, t} \tag{38b}$$

$$= \underbrace{k''(Z)\, w^2 + (k'(Z))^2\, \frac{w}{\phi}\, q^s}_{>0} + \delta\, \underbrace{(k'(Z)\, w + \delta\, \gamma(0)\, e^{\delta\, t})}_{=\dot{\psi}^{sb}} > 0. \tag{38c}$$

Thus, $\ddot{\psi}^{sb} > 0$ for any point in time where $\dot{\psi}^{sb} = 0$, which implies that $\dot{\psi}^{sb}$ can change signs only once. Overall, there are at most two switch points between regimes. The only possible orders of production are *BSB*, and *SB*. Note that *BS* is not possible since w goes to zero as the shale deposits approaches depletion. Hence, in the end ψ^{sb} must be positive.
Looking at all combinations of the switch functions yields the four extraction orders from Proposition 2. Note that $\psi^{sc}(0) > \psi^{sb}(0)$ since $k^b > k^c + \lambda(0)$. Hence, *BSB* can never occur in combination with *SC*.

D. Proof that a carbon tax postpones shale gas extraction

We first consider a setting of exogenous technology development, i.e. the case where the technology development is fixed at the pattern without the carbon tax. We denote the variables with a carbon tax τ^s and fixed technology development by a hat on top. With this notation, fixed technology development means $\widehat{Z}(t;\tau^s) \equiv Z(t;0)$, where the first argument is the point in time, the second one the carbon tax rate, and $Z(t;0)$ denotes the technology development without the carbon tax.
The claim thus is

$$\widehat{\gamma}_0(\tau^s) < \gamma_0(0) \quad \text{and} \tag{39}$$

$$(\widehat{Z}(\underline{T}^s;\tau^s)) + \tau^s + \widehat{\gamma}_0(\tau^s)\, e^{\delta\, \underline{T}^s} > k(\widehat{Z}(\underline{T}^s;\tau^s)) + \gamma_0(0)\, e^{\delta\, \underline{T}^s} = k^b. \tag{40}$$

This is shown by ruling out the alternative possibilities. First, $\widehat{\gamma}_0(\tau) \geq \gamma_0(0)$ is ruled out, as this would imply $k(\widehat{Z}(t;\tau^s)) + \tau^s + \widehat{\gamma}_0(\tau^s)\, e^{\delta\, t} > k(\widehat{Z}(t;\tau^s)) + \gamma_0(0)\, e^{\delta\, t}$ for all t, and thus less shale gas would be extracted with the tax $\tau^s > 0$ than without at any point in time. The shale gas deposit would not be depleted in the end, violating the transversality condition for deposit type *S*. Second, $k(\widehat{Z}(0;\tau^s)) + \tau^s + \widehat{\gamma}_0(\tau^s) \leq k(\widehat{Z}(0;\tau^s)) + \gamma_0(0)$ and $\widehat{\gamma}_0(\tau^s) < \gamma_0(0)$ is also ruled out, as this would imply $k(\widehat{Z}(t;\tau^s)) + \tau^s + \widehat{\gamma}_0(\tau^s)\, e^{\delta\, t} < k(\widehat{Z}(t;\tau^s)) + \tau^s + \gamma_0(0)\, e^{\delta\, t}$ for all t, and thus more shale gas would be extracted than available.
Inequality (40) implies that with the tax rate it is not (yet) profitable to use shale gas at $\underline{T}^s$. For fixed technology development, carbon taxation postpones the point in time where shale gas extraction starts to $\widehat{T}^s(\tau) > \underline{T}^s$.
The changing pattern of shale gas extraction due to carbon taxation has repercussions on the shadow price of technology. The shadow price for fixed technology development is given by

$$\widehat{\phi}(t;\tau^s) = -\int_t^\infty e^{-\delta\,(\vartheta - t)}\, k'(\widehat{Z}(\vartheta;\tau^s))\, \widehat{q}^s(\vartheta;\tau^s)\, d\vartheta \quad \text{for } t \geq \widehat{T}^s, \tag{41}$$

$$\widehat{\phi}(t;\tau^s) = e^{\delta\,(t - \widehat{T}^s)}\, \widehat{\phi}(\widehat{T}^s;\tau^s) \quad \text{for } t < \widehat{T}^s. \tag{42}$$

As carbon taxation postpones extraction of shale gas, we have

$$\widehat{\phi}(\underline{T}^s;\tau^s) = e^{\delta\,\left(\underline{T}^s - \widehat{T}^s\right)}\, \widehat{\phi}(\widehat{T}^s;\tau^s) < -\int_t^\infty e^{-\delta\,(\vartheta - t)}\, k'(\widehat{Z}(\vartheta;\tau^s))\, q^s(\vartheta;0)\, d\vartheta = \phi(\underline{T}^s, 0), \tag{43}$$

and thus also $\widehat{\phi}(t,\tau^s) < \phi(t,0)$ for all $t < \underline{T}^s$.
Carbon taxation decreases the incentive to invest in shale gas extraction technology development. Endogenous technology development thus further postpones shale gas extraction. The same argument as for fixed technology development shows that the shadow price of the technology stock with endogenous investments is even below $\widehat{\phi}(t,\tau^s)$ at or before $\underline{T}^s$,

$$\phi(t,\tau^s) < \hat{\phi}(t,\tau^s) < \phi(t,0) \quad \text{for all } t < \underline{T}^s. \tag{44}$$

As a consequence, the endogenous technology stock at $\underline{T}^s$ is below the corresponding level without a carbon tax, $Z(\underline{T}^s,\tau^s) < \hat{Z}(\underline{T}^s,\tau^s) = Z(\underline{T}^s,0)$. The reduced incentives to invest in extraction technology further postpone shale gas extraction with a carbon tax in place.

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