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Modeling the Dynamics of EU Economic Sentiment Indicators: An Interaction-Based Approach*†

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Abstract

This paper estimates a simple univariate model of expectation or opinion formation in continuous time adapting a 'canonical' stochastic model of collective opinion dynamics (Weidlich and Haag, 1983; Lux, 1995, 2007). This framework is applied to a selected data set on survey-based expectations from the rich EU business and consumer survey database for twelve European countries. The model parameters are estimated through maximum likelihood and numerical solution of the transient probability density functions for the resulting stochastic process. The model's performance is assessed with respect to its out-of-sample forecasting capacity relative to univariate time series models of the ARMA(p, q) and ARFIMA(p, d, q) varieties. These tests speak for a slight superiority of the canonical opinion dynamics model over the alternatives in the majority of cases.

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I. Introduction

It is widely believed that expectations play a major role in determining macroeconomic outcomes. Unfortunately, there is no consensus about the appropriate modeling of *expectation formation*. Many theories and approaches have been suggested in the literature to formalize this important ingredient of economic models. Over the last decades, the rational expectations hypothesis has become the dominant paradigm of modern macroeconomic theory and survey data have been used to test for rational expectations of respondents, mostly with not too much support for rationality.[1] However, little has been done to test *alternative* theories of expectation formation using the vast amount of survey data on empirical expectations that are regularly published by private and academic institutes or governments in most developed countries.

Branch (2004), Carroll (2003) and Roberts (1998) are some of the rare examples that consider alternative theories of expectation formation that do not impose homogeneous rational expectations. While there is a scarcity of theoretical models of boundedly rational expectation formation, extant empirical research on survey-based expectations is quite rich. As an obvious research question a large number of papers investigates the predictive capacity of survey data for consumer spending or output (e.g., Lemmens et al., 2005; Taylor and McNabb, 2007; Gelper er al., 2007) or seeks for determinants of sentiment in macroeconomic or political data (Vuchelen, 1995) or even in compact measures of the generally optimistic or pessimistic disposition of a society (e.g. Zullow, 1991 who uses indices of positive and negative moods in pop songs and news articles). The later studies are close to our approach in so far as they presume some kind of propagation of a dominant mood via direct or indirect interaction. Popular culture and mass media might, then, both reflect and reinforce overall mass-psychological trends in a society. Our goal here is to contribute to such a behavioral theory of sentiment formation by moving from a purely statistical analysis to an explicit modeling of the interaction effects in consumer or business surveys. Such an attempt at modeling and testing alternative hypotheses of opinion and expectation formation is a relatively recent strand of literature. We follow closely the recent work by Lux (2007) and Franke (2007) who both estimate (with different econometric techniques) the parameters of a 'canonical' opinion dynamic model introduced below for a particular German business survey.

This study provides an empirical assessment of this opinion formation model on the base of social interaction using the rich EU business and consumer survey database for twelve European countries as collected and released by the European Commission Directorate-General For Economic

[1] e.g., Acemoglu and Scott, 1994; Delorme et al., 2001, and the survey by Nardo, 2003.

and Financial Affairs [henceforth, the Commission]. In particular, a simple univariate model of opinion or expectation formation in continuous time is postulated in the spirit of Weidlich and Haag (1983). Following the methodology of Lux (2007), based on previous contributions by Poulsen (1999) and Hurn *et al.* (2006), the model parameters are then estimated via approximate maximum likelihood. Since no closed-form solution of the transient density of this model is available, our ML algorithm will be based on the numerical solution of the relevant Fokker-Planck equation (the partial difference equation governing the dynamics of the pdf) using a finite difference approximation. The model's goodness-of-fit is checked with respect to its out-of-sample forecasting performance relative to standard univariate time series models of the ARMA(p, q) and ARFIMA(p, d, q) varieties. The results of these tests speak for the moderate superiority of the canonical continuous-time model over the alternatives, ARFIMA (10 successful cases out of 36), and ARMA (2/36), i. e. in approximately 67% of cases.

The paper proceeds as follows. Section II introduces briefly the content of the survey data under investigation. Section III sketches the theoretical framework suggested to model such data. Section IV provides the empirical analysis and checks the goodness-of-fit of the model against pure time series models. Section V considers briefly potential extensions of the canonical model. Section VI concludes and indicates further directions of research under the framework of this paper.

II. Overview of the EU Business and Consumer Survey Data

National institutes in the EU Member States and candidate countries regularly[2] conduct business and consumer surveys on behalf of the Joint Harmonised EU Programme of Business and Consumer Surveys [henceforth, BCS programme].[3] The collected data are compiled and published in the media by the Commission and are freely available. The purpose of the BCS programme is twofold. On the one hand, the database provides essential information for economic surveillance, short-term forecasting, economic research, and, in general, monitoring economic developments at the Member State, EU and euro-area level. On the other hand, with these data, the Commission builds composite indicators to track cyclical movements in a specific sector or in the economy as a whole with the aim of detecting turning points in the economic cycle.

The surveys are usually conducted in the following areas: manufacturing industry, construction, consumers, retail trade and services. The sample size of each survey varies across countries according to the heterogeneity of their economies, and is generally positively related to their respective population

[2]on monthly and quarterly bases.

[3]The programme was set up in 1961 and is currently managed by the Commission.

size.[4]

The way in which answers obtained from the surveys are compiled and released is worth mentioning since it is the aggregate information that is used in our estimation. The responses are aggregated in the form of "balances" or diffusion indices. Balances are constructed as the difference between the percentages of respondents giving positive and negative replies. 'Neutral' answers are ignored. For example, if among the total number of N^* respondents[5] (for some specific question) 'positive' ('negative') answers are given by N^+ (N^-) individuals, then the *balance*, B, is computed as follows

$$B = (N^+ - N^-)/N^*.$$

The *balance series*[6] constitute the major part of the output data of the BCS programme. These series are further used to build *composite indicators* like (a) various *confidence indicators* that provide information on economic developments in the different sectors; (b) the *Economic Sentiment Indicator* [ESI], whose purpose is to track GDP growth at Member State, EU and euro-area level; and (c) the factor model-based *Business Climate Indicator* [BCI], which uses the results of the industry survey and is designed to assess cyclical developments in the euro area.

While many studies use selected entries of the EU survey data for single countries, surprisingly little work exists on cross-sections of data. An exception is the paper by Lemmens et al. (2005), exploring the predictive content of production surveys and Clar et al. (2007) who compare the forecasts from a variety of simple time series models for out-of-sample survey data themselves. The later study is very close to our approach here: Like Clar et al., we are interested in forecasting sentiment itself on the base of past observations. We also use time series models (as a benchmark), but compare them to forecasts from a behavioral model of opinion dynamics that could, in principle, capture the intrinsic built-up of an optimistic or pessimistic mood in society (or in business).

From the vast amount of the available EU survey data the particular questions chosen for the analysis in this paper are the following that relate to future expectations:

- Industry Survey, Q5: *How do you expect your production to develop over the next 3 months? It will...*

  $+$ *increase* $=$ *remain unchanged* $-$ *decrease*

[4] About 125 000 firms and almost 40 000 consumers are currently surveyed every month across the EU. Sourse: `http://ec.europa.eu/economy_finance/indicators/business_consumer_surveys/userguide_en.pdf`.

[5] Note that N^* includes the number of 'neutral' agents, $N^\sim$, i.e. $N^* = N^+ + N^- + N^\sim$.

[6] Balance series are usually referred to as "opinion index", "climate index", or "diffusion" in the literature.

- Construction Survey, Q4: *How do you expect your firm's total employment to change over the next 3 months? It will...*

  $+$ *increase* $=$ *remain unchanged* $-$ *decrease*

- Retail Trade Survey, Q4: *How do you expect your business activity (sales) to change over the next 3 months? It (They) will...*

  $+$ *improve (increase)* $=$ *remain unchanged* $-$ *deteriorate (decrease)*

- Consumer Survey, Q4: *How do you expect the general economic situation in this country to develop over the next 3 months? It will...*[7]

$++$	*get a lot better*	$=$	*stay the same*	$-$	*get a little worse*
$+$	*get a little better*	$\mathbb{N}$	*don't know*	$--$	*get a lot worse*

Figure 1 below provides information on the evolution of the balance series for the relevant questions in the case of Germany. The dynamics of various composite indicators are also superimposed on these series for purely illustrative purposes. It should be noted that the individual balances and the composite series are not directly comparable.

The questions that the next sections attempt to answer are: How could we model expectation formation of agents faced with the above questions? Do agents independently form expectations or can we identify some sort of social interaction between respondents? Can we predict future expectations? How good are our forecasts? What could be done in order to improve predictions?

[7]Note that in the case of the last question the balance is calculated as

$$B = \left[(N^{++} + 1/2N^{+}) - (1/2N^{-} + N^{--}) \right] / N^{*}$$

with the intuitive notation of N^{++} (N^{--}) being the number of 'very optimistic' ('very pessimistic') respondents.

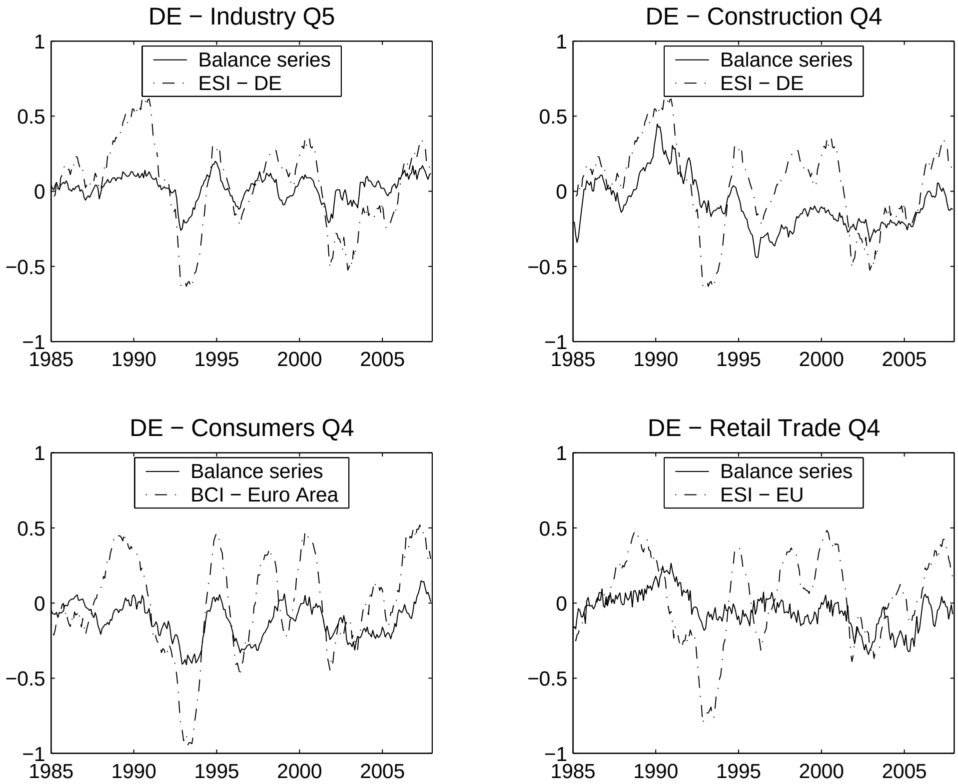

Figure 1: Balance series for Germany and composite indicators with appropriate scaling. These series are monthly observations over the 23 year period 01.1985 – 12.2007. Note that (a) ESI is calculated for the individual member countries as well as on the EU level, and (b) BCI is computed only for the Euro Area.

III. A Framework for Collective Opinion Formation

As a model of expectation formation we adopt a stochastic framework along the lines of Weidlich and Haag (1983) and Lux (1995). The model is stylized and is based on a set of mass-statistical regularities governing respondents' switches between two possible opinions.

We begin our exposition by assuming that (A1) the total number of respondents is constant and, without loss of generality, is given by $N^* = 2N$, and that (A2) the respondents are allowed to have only two relevant opinions or expectations, denoted by $+$ and $-$.

Let us denote by n_t^+ and n_t^- the numbers of agents holding positive and negative expectations at time t, respectively. We define next the *configuration*, n_t, as follows

$$n_t := (n_t^+ - n_t^-)/2, \tag{1}$$

$-N \leq n_t \leq N$, and introduce the notion of *aggregate* or *average expectations* as the ratio

$$x_t := n_t/N, \tag{2}$$

with $-1 \le x_t \le 1$. Since all agents have equal weight in the population, we can interpret the state $x_t = 0$ as representing the balance between overall optimism and pessimism, with the states $x_t > 0$ and $x_t < 0$ describing the cases of optimistic and pessimistic *majorities*, respectively. This opinion index is our proxy for the balance series (see remark below).

As time passes individual agents may change their opinions about the relevant questions. Thus they might switch from being optimistic to becoming pessimistic and *vice versa*. These switches are theoretically governed by the next two assumptions: (A3) The respondents have the same individual probabilities of reactions and interactions in the expectation formation process, and (A4) the probability that more than one agent will change the opinion per (infinitesimal) unit time period is zero.

Let $p(n;t)$ denote the probability that at time t the configuration is equal to n. Obviously, the condition

$$\sum_{n=-N}^{N} p(n;t) = 1$$

holds for all t. Since changes of opinion of agents might happen at any point in time, we adopt a continuous-time framework for the dynamics of this opinion index. Let $\omega(j \to i)$ denote the *transition rate* per unit time for a change of the configuration defined in eq. (1) from state j to state i, for all $i, j \in [-N, N]$. Then the equation of motion for $p(n_t;t)$ is given by the following so called *Master equation*[8]:

$$\frac{dp(i;t)}{dt} = \sum_j [\omega(j \to i)p(j;t) - \omega(i \to j)p(i;t)]. \tag{3}$$

The first term of the right hand side of (3) describes the probability flux from all states j into state i and the second term describes the probability flux from state i into all states j.

Assumption (A4) ensures that it suffices to consider only the following transitions:

$$n \to (n+1) \text{ and } n \to (n-1).$$

We, therefore, define

$$\omega_\uparrow(n) := \omega(n \to n+1)$$
$$\omega_\downarrow(n) := \omega(n \to n-1)$$

[8]The Master equation represents the general and exact system of equations tracking the flow of probabilities between states, see Weidlich and Haag (1983) or Van Kampen (2007).

and set:

$$\omega(n \to n') = 0 \text{ for } n' \neq n \pm 1.$$

Under this new notation, Eq. (3) can be written as

$$\begin{aligned}\frac{dp(n;t)}{dt} = & \,\omega_\uparrow (n-1)\, p\,(n-1;t) - \omega_\uparrow (n)\, p\,(n;t) \\ & + \omega_\downarrow (n+1)\, p\,(n+1;t) - \omega_\downarrow (n)\, p\,(n;t)\,. \end{aligned} \tag{4}$$

It can be shown that an equivalent description can be given for the opinion index:

$$\begin{aligned}\frac{dP(x;t)}{dt} = & \, w_\uparrow \left(x - \frac{1}{N}\right) P\left(x - \frac{1}{N};t\right) - w_\uparrow (x)\, P\,(x;t) \\ & + w_\downarrow \left(x + \frac{1}{N}\right) P\left(x + \frac{1}{N};t\right) - w_\downarrow (x)\, P\,(x;t)\,, \end{aligned} \tag{5}$$

where $P(x,t)$ denotes the probability that at time t the configuration is equal to x.[9]

The official statistics provides us only with the following data: B_t and $N^* = 2N$. Note that the definitions of our opinion index and that of the diffusion indices of the BCS differ slightly due to the possibility of a 'neutral' opinion in the surveys. However, we can bridge this gap in a relatively straightforward way: Since in the theoretical model the following identity must hold,

$$n_t^+ + n_t^- = N^*,$$

we might use n_t^+ as a proxy for $N_t^+ + \frac{1}{2}N_t^\sim$ and, similarly, n_t^- as a proxy for $N_t^- + \frac{1}{2}N_t^\sim$. Under this assumption,

1. $n_t^+ + n_t^- = N_t^+ + \frac{1}{2}N_t^\sim + N_t^- + \frac{1}{2}N_t^\sim = N^*$ and

2. $x_t = \frac{n_t^+ - n_t^-}{N^*} = \frac{N_t^+ - N_t^-}{N^*} = B_t$.

Thus, if we assign the half of the 'neutral' agents to the 'optimistic' and 'pessimistic' groups, respectively, then the information given by the diffusion index B_t is exactly represented by the theoretical index x_t.[10]

[9] Note the notational change: $p(n,t) \to P(x,t)$ and $\omega(n) \to w(x)$.

[10] It might, however, be mentioned that we could also design a slightly modified framework allowing for a neutral disposition along with the "+" and "−" choices. We leave this for future research.

Behavioral Assumptions

By treating x as a continuous variable and using a Taylor series expansion we can approximate the Master equation of eq. (5) by the so-called *Fokker-Planck equation*[11]:

$$\frac{\partial P(x;t)}{\partial t} = -\frac{\partial}{\partial x}\left\{A(x)P(x;t)\right\} + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left\{D(x)P(x;t)\right\}. \tag{6}$$

where

$$\begin{aligned} A(x) &= \frac{1}{N}[w_\uparrow(x) - w_\downarrow(x)], \\ D(x) &= \frac{1}{N^2}[w_\uparrow(x) + w_\downarrow(x)]. \end{aligned}$$

A(x) and D(x) are the drift and diffusion terms that govern the dynamics od the first and second moment.

As a next step we specify the transition rates. Utilizing Poisson probabilities in continuous time to jump from the "+" to the "−" group or *vice versa* within the next instant, a simple stochastic process of individual moves between groups is obtained. Here we follow the earlier literature and assume the following 'canonical' representation (see Weidlich and Haag (1983), Lux (1995, 1997, 2007)):

$$\begin{aligned} w_\uparrow(x) &= \frac{n_-}{2N}\upsilon\exp\left\{U(x)\right\} = (1-x)\upsilon\exp\left\{U(x)\right\}, \\ w_\downarrow(x) &= \frac{n_+}{2N}\upsilon\exp\left\{-U(x)\right\} = (1+x)\upsilon\exp\left\{-U(x)\right\} \end{aligned} \tag{7}$$

with

$$U(x) = \alpha_0 + \alpha_1 x. \tag{8}$$

In eq. (7), $\frac{n_-}{2N}$ and $\frac{n_+}{2N}$ are the fractions of currently pessimistic or optimistic respondents who constitute the pool of those who could potentially switch to the"+" or"-" opinion, respectively. The remainder of the expression, $\upsilon\exp\{U(x)\}$ or $\upsilon\exp\{-U(x)\}$ determines the switching rate per individual. The function $U(\cdot)$ might be labeled the 'forcing function' for transitions.[12] We have the following model parameters: υ determines the frequency (time scale) of moves between groups, α_0 generates a *bias* towards the choice of "+" ("−") opinions if positive (negative), and α_1 formalizes

[11]See Weidlich and Haag (1983), Lux (1997, 2007), Gardiner (2004), and Van Kampen (2007) for more details.

[12]Our function $U(\cdot)$ resembles the utility function within a discrete choice framework (cf. Brock and Durlauf, 2001). However, there is no clear utility component to survey responses so that we prefer the notion of a 'forcing function'. The major difference of our framework to studies of discrete choice problems with social interaction (DSCI) is that we investigate a *dynamic* model of aggregate opinion formation while DSCI models are typically applied to cross-sections of micro data.

the *degree of group pressure* if it is positive (if negative, it would rather imply a tendency towards *non-conformity*).

Let θ denote the parameter vector, $\theta = (v, \alpha_0, \alpha_1)'$. Then, highlighting the θ dependence, the stochastic dynamics of our opinion model is finally specified by Eq. (6) with[13]

$$\begin{aligned} A(x;\theta) &= v(1-x)e^{\alpha_0+\alpha_1 x} - v(1+x)e^{-\alpha_0-\alpha_1 x}, \\ D(x;\theta) &= [v(1-x)e^{\alpha_0+\alpha_1 x} + v(1+x)e^{-\alpha_0-\alpha_1 x}]/N. \end{aligned}$$

The Fokker-Planck equation (6) corresponds to the representation of the opinion dynamics, x_t, as a solution to the stochastic differential equation [SDE]

$$dx_t = A(x_t;\theta)dt + \sqrt{D(x_t;\theta)}dW_t, \tag{9}$$

where W_t denotes the standard Wiener process.[14]

Model Properties

Since this is a stochastic model for the aggregate behavior of our pool of respondents, a characterization of the outcome of this process requires to track the temporal development of the density $P(x;t)$. Conditional on some initial value, the transient density follows the Fokker-Planck equation (6). Unfortunately, with the highly non-linear drift and diffusion terms of our system, no closed-form analytical solution to eq. (6) is available. We will therefore, rely on numerical approximations of the Fokker-Planck equation in our empirical application. However, it is easier to derive the equilibrium properties of this system. The stationary distribution can be obtained by setting the left hand side of eq. (6) equal to zero,

$$\frac{\partial P(x;t)}{\partial t} = 0. \tag{10}$$

We do not reproduce the closed-form solution of the stationary density here, but summarize its important properties: [15]

[13] Using the hyperbolic trigonometric functions, the drift and diffusion function can also be written as:

$$\begin{aligned} A(x;\theta) &= 2v\cosh(\alpha_0+\alpha_1 x)\{\tanh(\alpha_0+\alpha_1 x) - x\}, \\ D(x;\theta) &= 2v\cosh(\alpha_0+\alpha_1 x)\{1 - x\tanh(\alpha_0+\alpha_1 x)\}/N. \end{aligned}$$

[14] Note that this is only an approximation to our population dynamics in that the microscopic sources of randomness have been proxied by a macroscopic noise factor W_t. See Gardiner (2004, Ch. 3) for technical aspects of the diffusion approximation to Markov jump processes.

[15] cf. Weidlich and Haag (1983), Lux (2007).

1. For $\alpha_1 \leq 1$, the stationary distribution of the process x_t is characterized by a unique maximum (mode). If $\alpha_0 = 0$, this maximum is located at $x^* = 0$. It shifts to the right (left) for $\alpha_0 > 0$ (< 0).

2. For $\alpha_1 > 1$ and α_0 not too large, the stationary distribution has two maxima (two modes) $x_+ > 0$ and $x_- < 0$. If $\alpha_0 = 0$, the bimodal distribution is symmetric around 0. It becomes asymmetric if $\alpha_0 \neq 0$ with right-hand (left-hand) skewness and more concentration of probability mass in the right (left) maximum if $\alpha_0 > 0$ (< 0) holds.

3. If $|\alpha_0|$ becomes very large, the smaller mode vanishes and the stationary distribution becomes uni-modal again. This happens if $|\alpha_0|$ increases beyond the bifurcation value $\overline{\alpha_0}$ given by

$$\cosh^2(\overline{\alpha_0} - \sqrt{\alpha_1(\alpha_1 - 1)}) = \alpha_1 \tag{11}$$

where $\cosh(\cdot)$ denotes the hyperbolic cosine,

$$\cosh(x) = (\exp(x) + \exp(-x))/2.$$

These properties are illustrated in Figure 2 below. For more details we refer to Lux (2007).

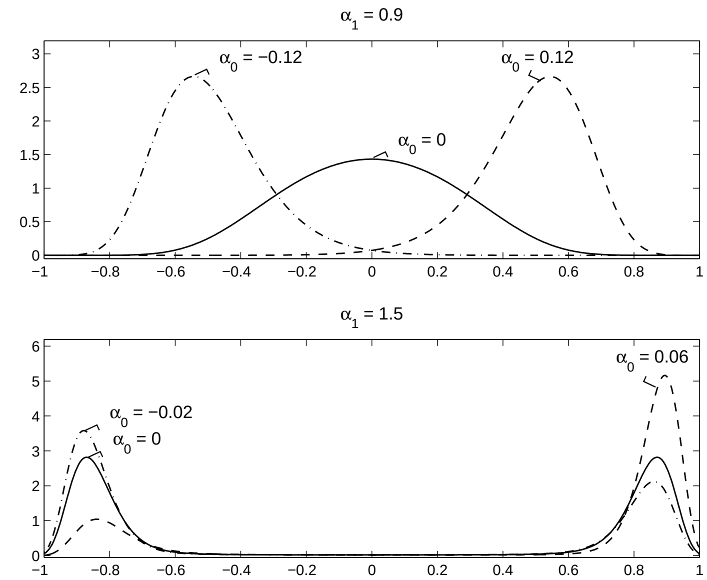

Figure 2: Equilibrium densities for various parameters.

Estimation

Note that we use discrete observations in order to estimate the parameters of a continuous-time process of opinion formation. For a sample of observations $x_0, ..., x_T$ we can estimate these parameters most efficiently via maximum likelihood. The log likelihood of our sample of observations amounts to

$$\log P_0(x_0|\theta) + \sum_{s=0}^{T-1} \log P(x_{s+1}|x_s, \theta)$$

Note that conditional probabilities $P(x_{s+1}|x_s, \theta)$ can be obtained by numerical iteration of the Fokker-Planck equation over a unit time interval taking x_s as the initial condition. Since we do not have a previous observation for x_0, we have to use the unconditional probability $P_0(x_0|\theta)$ to evaluate this component (however since its influence is negligible, we will simply skip this observation in our empirical applications).

For the numerical approximation of the Fokker-Planck equation we follow the methodology developed by Lux (2007) and suggested earlier by Poulsen (1999) and Hurn *et al.* (2006) in a different context. First, the Fokker-Planck equation (6) is solved numerically via a Crank-Nicolson finite difference scheme. Then, the log-likelihood function is evaluated for the 192 in-sample observations and is numerically maximized with respect to the unknown parameters. More details on the numerical aspects can be found in Lux (2007). Computations have been performed in GAUSS. The results are summarized in Tables 1 – 12.

IV. Empirical Results

The main objective of this paper is to estimate the parameters of our behavioral model for the selected balance series and assess the performance of this model as a hypothesized data-generating process for the BCS sentiment data. Since the EU Business and Consumer Survey Data is huge we have chosen only the four questions described in Section II for only those twelve countries for which the series were available from 1985. Thus, for each single question-country pair we have a sample of 276 monthly observations. In order to test the forecasting power of the model we have chosen the first 192 observations as our in-sample, covering the 16 year period 01.1985 – 12.2000. The number of out-of-sample observations is 84, covering the next 7 year period 01.2001 – 12.2007. These data series are seasonally adjusted by the provider.

Previous experience indicates the need to consider different versions of the basic opinion dynamics model (9) (see Lux (2007)). Therefore the following set of four models have been estimated:

M1: The parameter vector to be estimated is $\theta = (v, \alpha_0, \alpha_1)'$, with the number of respondents fixed at the 'official' number given in the documentation of the BCS programms.[16] Model 1 is exactly the full model specified above as Eq. (9) and will henceforth be referred to as the *canonical model.*

M2: N is fixed and is given as in Model 1. The parameter vector to be estimated is $\theta = (v, \alpha_1)'$. Here we neglect the bias parameter α_0. The reason is that for relatively weak interaction (α_1 small), approximate collinearity between α_0 and α_1 could impede our estimation.

M3: Under Model 3 N is no longer fixed. The parameter vector to be estimated is thus $\theta = (v, \alpha_0, \alpha_1, N)'$. Here we let the in-sample data provide the information about the implied 'effective' number of respondents.[17]

M4: As in Model 3, N is also not fixed here. The parameter vector to be estimated is $\theta = (v, \alpha_1, N)'$. We neglect the effect of the bias parameter α_0.

The total number of models that we estimate thus amounts to 144. Below in sec. V we will also consider a slight extension of our models M1 to M4.

Goodness-of-fit

The goodness-of-fit of all four models is checked with respect to their out-of-sample forecasting performance relative to a benchmark. In particular, one month out-of-sample forecasts are constructed for all models and two types of forecasting errors are computed: *root mean-squared errors* [RMSE] and *absolute mean errors* [AME]. The same quantities are calculated for univariate time series models such as ARMA(p, q) and ARFIMA (p, d, q), which serve as our benchmarks.

Forecasting

The notion of a prediction derived from a model like ours needs special attention. Taking the mathematical expectation of x_{t+i} conditional on time t information would certainly be an obvious choice for a uni-modal distribution, but it appears quite questionable in the multi-modal case (e.g. in the

[16]This information can be obtained from the EU Business and Consumer Surveys database. It actually varies widely across countries and sections.

[17]The idea is that despite the inclusion of a social interaction term our model might not capture all correlation between respondents. For example, there might be groups that always switch simultaneously which would, indeed, reduce the number of effectively independent agents. Of course, the officially reported number should be an upper boundary to the 'effective' number.

lower panel of Fig. 2). Note that for a bi-modal symmetric density like the ones in Fig. 2 the point estimate corresponding to the mean would coincide with the least likely value (the minimum of the density). The most likely values are the two modes that are quite different from the mean prediction. Taking these considerations into account, we also use as an alternative predictor besides the mean the value of the mode nearest to the last observation at time t. This choice is determined by the time dependency of our stochastic process: Because of the inertia of the opinion dynamics, the process will remain within its current mode for some time before stochastic fluctuations will trigger a switch to the alternative mode. This paper,therefore, considers two different one-month-ahead forecasts for the models M1–M4: *expected value* and *nearest maximum* of the predictive density function. The term 'expected' in the figures and tables below stands for the expected value of the opinion index x_{t+1} at one-month horizon conditional on its value one month earlier, x_t. The needed predictive density is again obtained via numerical solution of the Fokker-Planck equation with the previously estimated parameter vector $\hat{\theta}$. A similar procedure applies to the computation of the 'nearest' forecast, which is the nearest maximum of the predictive density, and therefore represents the most likely mode of the opinion index at some future date. These 'expected' and 'nearest' forecasts are calculated for the out-of-the-sample data of respective balance series.

In order to set benchmarks, we have also estimated the best ARMA(p, q) and ARFIMA(p, d, q) in-sample. For ARMA we have set $p, q \leq 5$, for ARFIMA, $p, q \leq 1$ (as the longer lags should be captured by the parameter of fractional differentiation). From the range of the ARMA and ARFIMA models within this set, the one that minimizes the Akaike information criterium is chosen for forecasting. Then, based on the fitted models, out-of-sample one-month-ahead forecasts have been computed.

Empirical results show the following regularities:

1. ARMA forecasting accuracy is usually outperformed by both, the predictive power of ARFIMA and that of models M1–M4. The exceptions from this pattern are the cases of Irish and French industries (see Tables 4 and 6). For example, both Model 1 and the ARFIMA model outperformed ARMA with respect to RMSE and AMSE in 94.4% of all cases. In 24 cases out of a total of 36 the canonical continuous-time model was slightly superior to the ARFIMA (which dominated in 10 out of 36 cases) and ARMA (2 out of 36) models, i. e. in approximately 67% of cases. For M2 to M4 the results are very similar.

2. Considering the full range of our interaction-based models M1–M4 we find better fits of at least one specification with respect to RMSE and AMSE than the best ARFIMA performance in the majority of cases which correspond to 75% of our experiments (27 cases out of 36).

3. Predictive accuracy within the family of interaction-based models is usually increasing and only sometimes slightly decreasing when going from M1 to M4, whereas the estimated values for corresponding log-likelihoods do mostly not improve essentially. This is surprising since allowing for N, the 'effective' number of participants as a free parameter, provided for a crucial improvement of goodness-of-fit in the case of a German sentiment index (Lux, 2007).

4. The Diebold-Mariano test could not reject the null hypothesis of equal predictive accuracy at the 5% level between (a) the expected value and ARFIMA forecasts in 90.3% of cases, (b) the nearest value and ARFIMA forecasts in 88.9% cases, and (c) the expected value and the nearest value forecasts in 88.2% of cases, when the total number of 144 experiments is taken into account.[18]

5. For the parameter of the opinion model, we typically find the crucial entry for the intensity of interaction, α_1, to be in the close vicinity of its bifurcation value 1 (at which the system behavior switches from unimodal to bi-modal) for Model 1. However, allowing for endogeneity of N, this value mostly turns out to be lower. Similar findings have been reported in Lux (2007). It appears that there is a trade-off between the number of independent agents and their interaction intensity: If we insist on N in accordance with the official numbers, the model can only reproduce the fluctuations of the index with α_1 close to its crucial value. If we allow for a lower number of "effectively independent" agents, lower interaction intensity will be sufficient in the best fit to our model.[19]

An Example. Next we visualize the out-of-sample forecasting performance of competing models. The data come from the German industry survey Q5 for the (out-of-sample) period 01.2001 – 12.2007. On the one hand, we consider the canonical model 1 with two potential one-month-ahead best forecasts: *expected value* and *nearest maximum* of the predictive density function. On the other hand, the best ARMA and ARFIMA forecasts are presented in Figure 4. For this case the preferred time series models were ARMA(2,1) and ARFIMA(1,d,1) models with $d = 0.7202$.

This example highlights the overall impression that ARMA models perform poorly for the balance series, whereas both the canonical model and the ARFIMA model track the dynamics of the future values in this case

[18] We considered only one-month-ahead forecasts.

[19] Note that using the large official numbers of respondents would lead to very low predicted volatility due to the law of large numbers. This can to a certain degree be overcome by high sensitivity of the system to changes. This is what characterizes the vicinity of α_1 while moving away from this benchmark in both directions leads to more persistent macroscopic dynamics.

quite well. Both, expected and nearest forecasts show better performance than ARFIMA with respect to RMSE and AMSE (see details in Table 3).

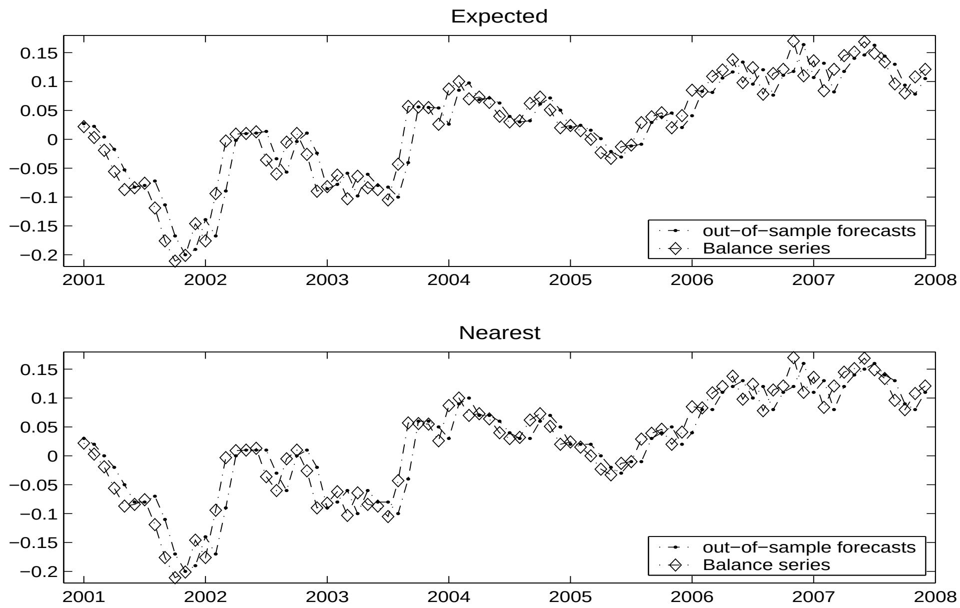

Figure 3: One-month-ahead forecasting performance of the canonical model for the German industry survey, question 5. 'Expected' represents the best forecast computed as a conditionally expected value of the predictive density. 'Nearest' corresponds to the best forecast calculated as the nearest maximum of the predictive density.

In order to illustrate what the potential added explanatory power of our opinion model could be, we exhibit some more details in the case of German consumers. This is particularly interesting because the fitted canonical model displays the possibilities for *phase transitions*. In particular, both M1 and M2 have α_1 parameters higher than unity. This setting corresponds to the bi-modal equilibrium distribution of consumer opinions, switching from optimistic to pessimistic long-run equilibria and *vice versa*. In Figure 5 below we have graphed in- and out-of-sample observations for the series. We have also superimposed two standard deviation bounds on the evolution of the predictive density conditional on the very last in-sample observation, $x_0 = -0.03$. As parameters we have chosen the simple averages of the two models, M1 and M2: $v = .5475$, $\alpha_0 = -0.0006$, $\alpha_1 = 1.0109$, $N = 1000$. As can be seen from the graph, we are able to track the global maximum of the predictive density. This is represented by the dashed black curve in the out-of-sample interval starting in 2001. The evolution of the mean is given by the green line and it stays closely in the neighborhood of the initial condition, $x_0 = -0.03$. The evolution of the global maximum, on the contrary, diverges from the mean downwards a strongly negative configura-

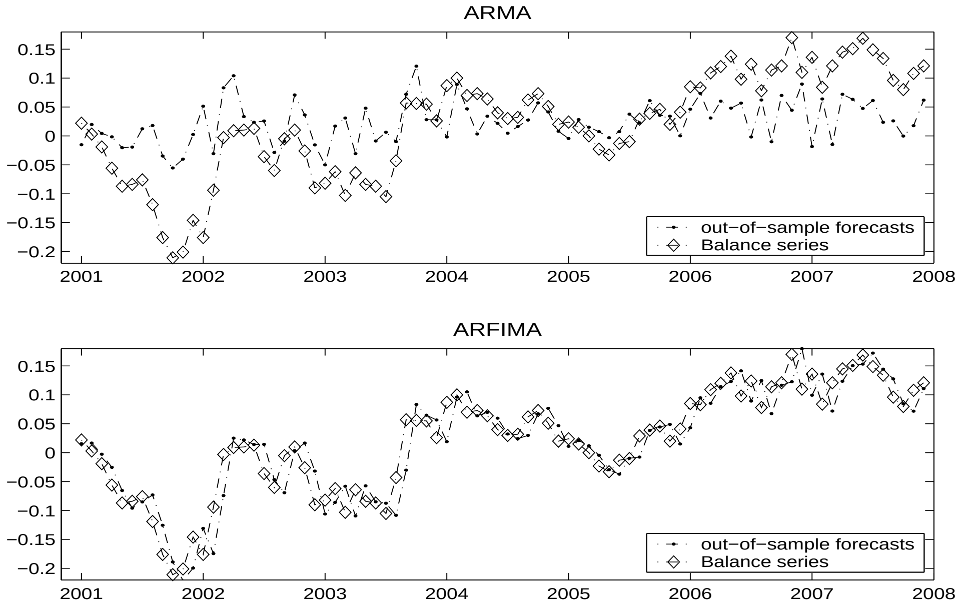

Figure 4: One-month-ahead forecasting performance of the ARMA(2,1) and ARFIMA(1,d,1) models, $d = 0.7202$ for the German industry survey, Q5.

tion. Why is the global maximum not capable of pulling the mean with it? What is the force that keeps the dynamics of the mean almost unchanged? The answers to these questions are provided by Figure 6. It displays the complete evolution of the predictive density. We see how a second local maximum develops in the positive quadrant. This is exactly the reason for the observed dynamics of the mean that roughly corresponds to the average between both modes.

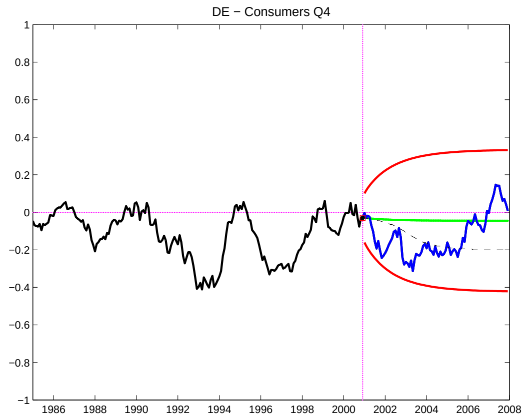

Figure 5: The evolution of the moments of the predictive density conditional on the last in-sample observation, $x_0 = -0.03$, see Table 3, Consumers. The parameters are the simple averages of the two models, M1 and M2: $v = .5475$, $\alpha_0 = -0.0006$, $\alpha_1 = 1.0109$, $N = 1000$. The red bands represent the two standard deviation bounds. The green line is the (conditional) mean process. The dashed black line denotes the evolution of the global maximum.

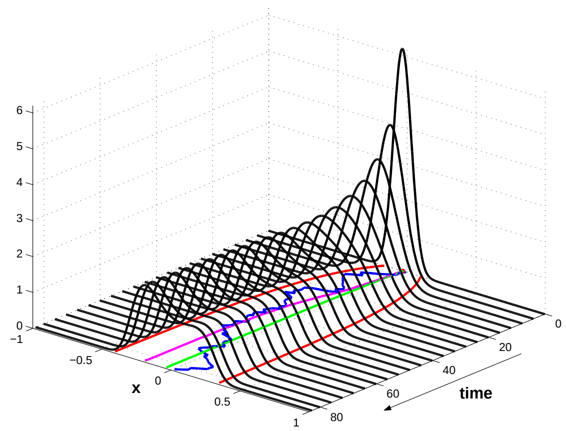

Figure 6: The evolution of the predictive density conditional on the last in-sample observation, $x_0 = -0.03$, see Table 3, Consumers. The parameters are the simple averages of the two models, M1 and M2: $v = .5475$, $\alpha_0 = -0.0006$, $\alpha_1 = 1.0109$, $N = 1000$.

V. An Extension

The framework of the interaction-based model for expectation formation can be easily extended to incorporate the effects of important exogenous macroeconomic variables. In order to allow for additional determinants in the opinion process, one could simply expand the forcing function:

$$U(x_t) = \alpha_0 + \alpha_1 x_t + \alpha_2 Y_t, \tag{12}$$

where α_2 is an m-dimensional vector of coefficients and Y represents the m-dimensional vector of relevant macro variables.

The vector of determinants Y_t could cover any set of socioeconomic variables that could possibly have an influence on agents' opinion formation. Lux (2007) reports that in the case of a German business survey, inclusion of macroeconomic data as well as political proxies only lead to minor improvements of the likelihood function. However, he also finds that taking into account a 'momentum' effect in the self-referential part of the opinion process leads to a relatively large gain in explanatory power. We also allow for this effect introducing the following variations of the opinion model:

M5: $U(x_t) = \alpha_0 + \alpha_1 x_t + \alpha_2 \Delta x_t$. Here Δx_t denotes the difference between the time t and the time $(t-1)$ observations of the index which stays fixed over the time interval $[t, t+1)$. The idea is that respondents might react not only to the net influence of their environment but also be particularly sensitive to changes of the index itself.

M6: $U(x_t) = \alpha_1 x_t + \alpha_2 \Delta x_t$. This specification discards the influence of a constant bias in the expectation formation process.[20]

One could easily imagine that respondents' attention is captured more by pronounced changes of a sentiment index than by its raw numbers. Large positive or negative movements could, therefore, triggers avalanches of subsequent changes of mood of other participants. However, nothing needs to be said *a priori* about the signs of the coefficient α_2 in our framework — both a positive as well as negative feedback (if any) could be allowed for. In the following we provide the empirical results for this extension of our baseline model, cf. Tables 13 – 16.

Somewhat surprisingly, the momentum effects turns out to be negative in the majority of cases (i.e., it is of a contrarian nature). In contrast to the case of German business climate (Lux, 2007), its contribution to in-sample fit and out-of-sample performance is also relatively modest in the various EU sentiment indices. Typically, there are no considerable but still slight

[20] Note that in the specification $U(x_t) = \alpha_1 x_t + \alpha_2 \Delta y_t$, where y_t tracks some changes in the fundamentals, the term $\alpha_2 \Delta y_t$ can be interpreted as the *time varying trend* or *bias* fixed over the period $[t, t+1)$.

improvements. In particular, M5 does outperform M6 in general. Third, M5 does show better predictive accuracy than ARFIMA in 25 cases out of 36 with respect to RMSE, i.e. 69.4%, and in 23 cases out of 36 with respect to AME, i.e. 63.8%. Overall, however, the differences in goodness-of-fit and forecasting performances are minor between models M1 to M4 and the new specifications M5/M6.

VI. Conclusions

This paper has explored the explanatory and predictive power of a non-rational model of opinion formation among interacting agents for European business and consumer sentiment data. Applying the canonical model of opinion formation by Weidlich and Haag (1983) to four selected indices across 12 core countries of the European Union, we found the following:

1. In contrast to our pilot application of the present estimation methodology in Lux (2007), different specifications of the model make little difference to its in-sample and out-of-sample fit for the survey data of the BCS data base. In particular, we found little improvement through adding a 'momentum' effect to the opinion dynamics.

2. With respect to its forecasting performance out-of-sample, the endogenous opinion model typically did better that an ARMA model. Compared to the more persistent ARFIMA class, predictive power was mostly not significantly different for single series (as judged by the Diebold-Mariano test). However, for the cross-section of data as a whole, we find a dominance of the opinion model in about two thirds of all cases (although its advantage over ARFIMA might be small).

It is worthwhile to note that it is not really clear whether we could expect more predictive power if this model were the 'true' data generating process. On the one hand, the model output is characterized by stochastic switches between two maxima of its probability density in the case of strong interaction. Although our model could help in understanding such transitions between prevailing optimism and pessimism, the stochasticity of these swings would prevent successful prediction of changes of the public's mood. On the other hand, if interaction is relatively weak ($\alpha_1 < 1$), the built-in persistency of the stochastic ARFIMA model might be a good approximation to the behavioral persistency of the opinion model.[21] Both aspects need to be explored in order to get an idea of the potential forecasting performance of such models.

[21] Alfarano and Lux (2007) demonstrate that a closely related model mimics the long-term dependency that is the defining feature of ARFIMA models. Lux (2008) shows that both a behavioral opinion model and a parsimonious diffusion process provide nearly equivalent fits to a financial sentiment index.

A certain deficit of our present approach is the uni-variate nature of our models. Of course, the opinion dynamics will not be decoupled from other economic data and might be influenced by exogenous news about economic and possibly political conditions. In order to get a handle on such factors, we could let them enter the formalization of transition rates (as we did for momentum in sec. V)[22] or we could combine our opinion dynamics with additional dynamic components formalizing the time development of, for example, GDP, interest rates etc. One would, then, hope to disentangle the influence of objective factors from the intrinsic propagation of moods among the population of respondents. This daunting task is left for our future research.

Remark on Tables 1–16: The symbol $\mathfrak{L}$ stands for the logarithmic maximum likelihood. AIC and BIC represent the Akaike and Bayesian information criteria, respectively. The minimal forecast errors within the columns of single questions are given in bold numbers. The global minimal values within the questions are emphasized by stars. In a few cases, standard errors could not be obtained which is indicated by the sign '–'. Information on BIC is absent from the Table 13 due to space considerations.

[22]Lux (2007) considered various macroeconomic factors in the analysis of a German business climate index but found surprisingly little value added compared to the 'canonical' model.

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Table 1: Belgium [BE]

	Parameter Estimates and Standard Errors						
	$\hat{v}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{N}$	$\mathfrak{L}$	AIC	BIC
Ind.Q5							
M1	0.6880 (0.0733)	-0.0016 (0.0023)	0.9683 (0.0195)	775.0	-536.9	1079.8	1084.3
M2	0.6858 (0.0728)		0.9720 (0.0187)	775.0	-537.1	1078.3	1084.8
M3	0.0209 (0.0629)	-0.0312 (0.0702)	-0.5213 (4.3190)	23.8 (69.5)	-536.5	1080.9	1083.4
M4	0.0143 (0.0165)		-1.0567 (2.0992)	16.6 (18.2)	-536.5	1079.1	1083.6
Cst.Q4							
M1	0.3266 (0.0347)	0.0007 (0.0045)	0.9256 (0.0357)	440.0	-519.6	1045.2	1049.6
M2	0.3270 (0.0347)		0.9241 (0.0342)	440.0	-519.6	1043.2	1049.7
M3	0.0656 –	0.0043 –	0.5685 –	88.6 –	-519.6	1047.1	1049.6
M4	0.0533 –		0.4492 –	71.9 –	-519.6	1045.1	1049.6
Cns.Q4							
M1	0.9323 (0.0987)	-0.0005 (0.0025)	1.0073 (0.0141)	800.0	-569.2	1144.4	1148.9
M2	0.9303 (0.0979)		1.0091 (0.0107)	800.0	-569.2	1142.5	1148.9
M3	0.0230 –	-0.0564 –	-0.1585 –	19.5 –	-566.7	1141.4	1143.9
M4	0.0836 –		0.7582 –	70.2 –	-566.9	1139.9	1144.5
RTr.Q4							
M1	1.9951 (0.2618)	0.0070 (0.0018)	0.8702 (0.0197)	575.0	-624.4	1254.9	1259.4
M2	1.8437 (0.2242)		0.9093 (0.0170)	575.0	-631.8	1267.6	1274.1
M3	0.2140 –	0.0660 –	-0.2613 –	62.4 –	-624.8	1257.7	1260.1
M4	0.3746 (0.1301)		0.6287 (0.1212)	117.5 (37.6)	-632.3	1270.6	1275.1

Comparison of the out-of-sample one month forecasting errors:

	Nearest Mode		**Expected Value**		**ARMA**		**ARFIMA**	
	RMSE	AME	RMSE	AME	RMSE	AME	RMSE	AME
Ind.Q5								
M1	0.03308	0.02676	0.03265	0.02660	0.08242	0.06719	0.03343	0.02764
M2	0.03296	0.02702	0.03268	0.02676				
M3	0.03249	**0.02638***	**0.03254**	**0.02646**				
M4	**0.03247***	0.02655	0.03257	0.02657				
Cst.Q4								
M1	0.03025	0.02363	0.03013	0.02322	0.17509	0.15137	**0.02925***	**0.02262***
M2	0.03019	**0.02339**	0.03014	0.02322				
M3	0.03062	0.02387	**0.03011**	0.02319				
M4	**0.03010**	0.02339	0.03012	**0.02318**				
Cns.Q4								
M1	0.05661	0.04392	0.05589	0.04331	0.14855	0.12578	0.05648	0.04366
M2	0.05652	0.04387	0.05604	0.04347				
M3	**0.05513**	**0.04294**	**0.05498***	**0.04234***				
M4	0.05555	0.04325	0.05543	0.04279				
RTr.Q4								
M1	0.06277	0.04783	0.06292	**0.04784**	0.07086	0.05266	0.06184	**0.04645***
M2	0.06048	0.04981	**0.06048**	0.04963				
M3	0.06298	**0.04783**	0.06297	0.04784				
M4	**0.06046***	0.04986	0.06056	0.05009				

Table 2: Denmark [DK]

	Parameter Estimates and Standard Errors						
	$\hat{v}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{N}$	$\mathfrak{L}$	AIC	BIC
Ind.Q5							
M1	0.2675 (0.0320)	0.0309 (0.0098)	0.6984 (0.0783)	250.0	-547.9	1101.8	1106.2
M2	0.2520 (0.0288)		0.8878 (0.0514)	250.0	-552.6	1109.2	1115.7
M3	0.0670 –	0.1326 –	-0.3066 –	63.3 –	-547.5	1102.9	1105.4
M4	0.0808 –		0.6132 –	80.4 –	-552.8	1111.6	1116.0
Cst.Q4							
M1	1.1220 (0.1327)	-0.0029 (0.0026)	0.9333 (0.0196)	375.0	-643.3	1292.7	1297.2
M2	1.1138 (0.1309)		0.9390 (0.0190)	375.0	-643.9	1291.9	1298.4
M3	0.1994 –	-0.0169 –	0.5479 –	67.0 –	-643.4	1294.9	1297.4
M4	0.1804 –		0.5365 –	61.1 –	-644.1	1294.2	1298.7
Cns.Q4							
M1	0.8919 (0.1059)	-0.0046 (0.0023)	0.9190 (0.0230)	750.0	-562.2	1130.4	1134.9
M2	0.8690 (0.1008)		0.9423 (0.0200)	750.0	-564.1	1132.2	1138.7
M3	0.0055 (0.0044)	-0.7620 (0.3699)	-7.1684 (2.3278)	6.1 (4.0)	-560.0	1128.1	1130.5
M4	0.0509 (0.0572)		-0.1166 (1.2107)	44.2 (48.9)	-564.3	1134.5	1138.9

Comparison of the out-of-sample one month forecasting errors:

	Nearest Mode		Expected Value		ARMA		ARFIMA	
	RMSE	AME	RMSE	AME	RMSE	AME	RMSE	AME
Ind.Q5								
M1	0.04854	0.03737	0.04814	0.03717	0.06842	0.05192	0.05169	0.04039
M2	0.05035	0.03851	0.05043	0.03875				
M3	**0.04811**	**0.03689***	**0.04801***	**0.03709**				
M4	0.05038	0.03839	0.05045	0.03873				
Cst.Q4								
M1	0.03790	0.03110	0.03788	0.03129	0.47826	0.40125	0.04067	0.03129
M2	**0.03712***	**0.03055***	**0.03741**	**0.03086**				
M3	0.03861	0.03195	0.03825	0.03164				
M4	0.03763	0.03102	0.03777	0.03122				
Cns.Q4								
M1	**0.05798**	0.03645	0.05790	0.03625	0.15902	0.13965	**0.05350***	**0.03321***
M2	0.05851	0.03469	0.05798	**0.03435**				
M3	0.05810	0.04074	**0.05726**	0.03682				
M4	0.05857	**0.03464**	0.05811	0.03437				

Table 3: Germany [DE]

	Parameter Estimates and Standard Errors						
	$\hat{v}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{N}$	$\mathfrak{L}$	AIC	BIC
Ind.Q5							
M1	0.5587 (0.0611)	0.0010 (0.0017)	0.9703 (0.0187)	1800.0	-440.3	886.7	891.7
M2	0.5579 (0.0609)		0.9733 (0.0180)	1800.0	-440.5	885.0	891.5
M3	0.0275 (0.1929)	0.0269 (0.2322)	0.1979 (5.5973)	88.9 (619.0)	-440.2	888.3	890.8
M4	0.0202 –		0.0135 –	65.4 –	-440.4	886.9	891.4
Cst.Q4							
M1	0.4772 (0.0487)	0.0000 (0.0041)	1.0309 (0.0168)	700.0	-521.9	1049.8	1054.2
M2	0.4772 (0.0487)		1.0310 (0.0117)	700.0	-521.9	1047.8	1054.2
M3	0.0619 –	-0.0326 –	0.7503 –	86.9 –	-517.1	1042.2	1044.6
M4	0.0531 –		0.8164 –	75.0 –	-517.7	1041.1	1045.6
Cns.Q4							
M1	0.5482 (0.0570)	-0.0012 (0.0030)	1.0085 (0.0176)	1000.0	-495.6	997.1	1001.6
M2	0.5469 (0.0567)		1.0132 (0.0129)	1000.0	-495.6	995.3	1001.8
M3	0.0512 –	-0.0278 –	0.7101 –	92.0 –	-493.9	995.8	998.3
M4	0.0229 –		0.8610 –	40.6 –	-494.7	995.6	999.9
RTr.Q4							
M1	0.5346 (0.0592)	-0.0001 (0.0035)	0.8558 (0.0377)	405.0	-565.4	1136.8	1141.3
M2	0.5346 (0.0592)		0.8559 (0.0376)	405.0	-565.4	1134.8	1141.3
M3	0.6038 (2.8489)	-0.0001 (0.0033)	0.8733 (0.6385)	457.4 (2157.5)	-565.4	1138.8	1141.3
M4	0.0745 (0.0898)		-0.0676 (1.2590)	56.6 (67.4)	-565.5	1137.0	1141.5

Comparison of the out-of-sample one month forecasting errors:

	Nearest Mode		Expected Value		ARMA		ARFIMA	
	RMSE	AME	RMSE	AME	RMSE	AME	RMSE	AME
Ind.Q5								
M1	0.03299	0.02580	**0.03323**	**0.02621**	0.07729	0.06118	0.03399	0.02696
M2	**0.03241***	**0.02544***	0.03328	0.02627				
M3	0.03303	0.02585	0.03325	0.02625				
M4	0.03283	0.02582	0.03333	0.02636				
Cst.Q4								
M1	0.03245	0.02361	0.03218	0.02351	0.50231	0.47217	0.03338	0.02497
M2	0.03245	0.02361	**0.03218***	**0.02351**				
M3	**0.03239**	**0.02351***	0.03278	0.02416				
M4	0.03276	0.02377	0.03280	0.02419				
Cns.Q4								
M1	0.03394	0.02820	0.03355	0.02810	0.08820	0.07452	0.03400	**0.02787***
M2	0.03386	0.02820	0.03360	0.02819				
M3	0.03409	**0.02811**	**0.03354***	**0.02795**				
M4	**0.03376**	0.02820	0.03375	0.02833				
RTr.Q4								
M1	0.05461	0.04405	0.05511	0.04447	0.13057	0.10435	0.05830	0.04828
M2	0.05463	0.04417	0.05516	0.04450				
M3	0.05480	0.04417	0.05520	0.04456				
M4	**0.05437***	**0.04381***	**0.05458**	**0.04392**				

Table 4: Ireland [IE]

	Parameter Estimates and Standard Errors						
	$\hat{v}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{N}$	$\mathfrak{L}$	AIC	BIC
Ind.Q5							
M1	2.5303 (0.3167)	0.0093 (0.0021)	0.9309 (0.0142)	550.0	-659.1	1324.2	1328.6
M2	2.2776 (0.2574)		0.9775 (0.0100)	550.0	-669.0	1342.0	1348.5
M3	1.3496 (2.2901)	0.0181 (0.0320)	0.8515 (0.2888)	292.3 (495.5)	-659.0	1325.9	1328.4
M4	4.2625 (2.2347)		0.9961 (0.0122)	1029.9 (534.1)	-668.6	1343.2	1347.7
Cst.Q4							
M1	2.2200 (0.2635)	0.0040 (0.0023)	1.0272 (0.0080)	250.0	-728.3	1462.7	1467.2
M2	2.1975 (0.2582)		1.0304 (0.0079)	250.0	-729.9	1463.7	1470.2
M3	3.2175 (0.7754)	0.0032 (0.0016)	1.0341 (0.0064)	362.3 (78.9)	-727.3	1462.6	1465.0
M4	2.9496 (0.7402)		1.0355 (0.0068)	335.4 (76.6)	-729.2	1464.4	1468.9
Cns.Q4							
M1	0.7146 (0.0769)	-0.0005 0.0024	0.9828 (0.0150)	650.0	-556.0	1118.1	1122.6
M2	0.7137 (0.0766)		0.9834 (0.0148)	650.0	-556.1	1116.1	1122.6
M3	0.0660 (0.3682)	0.0016 (0.0311)	0.6496 (2.0290)	60.9 (337.8)	-556.9	1121.8	1124.2
M4	0.0691 (0.1753)		0.6641 (0.8980)	63.6 (160.7)	-556.9	1119.7	1124.2

Comparison of the out-of-sample one month forecasting errors:

	Nearest Mode		**Expected Value**		**ARMA**		**ARFIMA**	
	RMSE	AME	RMSE	AME	RMSE	AME	RMSE	AME
Ind.Q5								
M1	0.10111	0.08199	0.09976	0.08094	**0.09770***	**0.07931***	0.09786	0.08026
M2	0.10698	0.08673	0.10489	0.08527				
M3	**0.10008**	**0.08151**	**0.09936**	**0.08053**				
M4	0.10773	0.08758	0.10570	0.08590				
Cst.Q4								
M1	**0.20797**	**0.16905**	**0.20356**	0.16777	0.20172	0.17083	**0.19222***	**0.15244***
M2	0.20852	0.17014	0.20424	0.16918				
M3	0.20867	0.16905	0.20374	**0.16769**				
M4	0.20904	0.16955	0.20438	0.16903				
Cns.Q4								
M1	**0.05228***	**0.04273***	**0.05266**	**0.04297**	0.06989	0.05719	0.05407	0.04462
M2	0.05230	0.04273	0.05278	0.04305				
M3	0.05268	0.04296	0.05294	0.04317				
M4	0.05268	0.04296	0.05290	0.04314				

Table 5: Greece [EL]

	Parameter Estimates and Standard Errors										
	$\hat{v}$		$\hat{\alpha}_0$		$\hat{\alpha}_1$		$\hat{N}$		$\mathfrak{L}$	AIC	BIC
Ind.Q5											
M1	1.1746	(0.1360)	0.0168	(0.0057)	0.9571	(0.0227)	850.0		-559.8	1125.7	1130.2
M2	1.1182	(0.1241)			1.0215	(0.0068)	850.0		-564.1	1132.2	1138.7
M3	0.1330	–	0.2217	–	0.1122	–	96.2	–	-559.9	1127.4	1129.8
M4	0.1220	–			0.9644	–	98.0	–	-569.0	1144.0	1148.5
Cst.Q4											
M1	1.5145	(0.1680)	0.0039	(0.0030)	1.0068	(0.0126)	220.0		-726.0	1457.9	1462.4
M2	1.5011	(0.1653)			1.0116	(0.0121)	220.0		-726.8	1457.6	1464.1
M3	0.7291	(0.5179)	0.0075	(0.0079)	0.9589	(0.0698)	104.0	(73.2)	-724.8	1457.5	1460.0
M4	0.6752	(0.5166)			0.9620	(0.0731)	97.0	(73.6)	-725.5	1456.9	1461.5
Cns.Q4											
M1	0.7327	(0.0812)	-0.0076	(0.0034)	0.9558	(0.0204)	750.0		-541.6	1089.2	1093.7
M2	0.7161	(0.0778)			0.9909	(0.0135)	750.0		-544.1	1092.3	1098.8
M3	0.0779	–	-0.0750	–	0.4387	–	80.9	–	-542.6	1093.1	1095.6
M4	0.0776	(0.3034)			0.7892	(0.8767)	82.5	(321.3)	-545.4	1096.7	1101.2

Comparison of the out-of-sample one month forecasting errors:

	Nearest Mode		**Expected Value**		**ARMA**		**ARFIMA**	
	RMSE	AME	RMSE	AME	RMSE	AME	RMSE	AME
Ind.Q5								
M1	0.03549	**0.02854***	**0.03533***	**0.02856**	0.10063	0.08716	0.03666	0.02928
M2	0.03647	0.02920	0.03625	0.02906				
M3	**0.03543**	0.02877	0.03546	0.02861				
M4	0.03866	0.03108	0.03749	0.02992				
Cst.Q4								
M1	0.10630	**0.08255**	0.10639	**0.08246***	0.11216	0.09168	0.11301	0.08833
M2	**0.10517***	0.08279	**0.10618**	0.08364				
M3	0.10693	0.08288	0.10754	0.08325				
M4	0.10765	0.08474	0.10775	0.08478				
Cns.Q4								
M1	0.04704	0.03517	**0.04662***	**0.03486**	0.04722	0.03523	0.04685	**0.03453***
M2	0.04691	0.03517	0.04739	0.03564				
M3	**0.04678**	**0.03469**	0.04693	0.03505				
M4	0.04802	0.03593	0.04794	0.03596				

Table 6: France [FR]

	Parameter Estimates and Standard Errors						
	$\hat{v}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{N}$	$\mathfrak{L}$	AIC	BIC
Ind.Q5							
M1	1.7768 (0.1942)	0.0013 (0.0010)	0.9960 (0.0076)	2000.0	-536.8	1079.5	1083.9
M2	1.7705 (0.1927)		1.0004 (0.0068)	2000.0	-537.6	1079.3	1085.8
M3	0.0690 –	0.0385 –	0.5681 –	78.2 –	-537.1	1082.2	1084.7
M4	0.0632 (0.1927)		0.6761 (1.0165)	71.3 (214.6)	-538.3	1082.7	1087.1
Cst.Q4							
M1	0.7898 (0.0821)	0.0023 (0.0019)	1.0458 (0.0078)	1500.0	-512.7	1031.4	1035.9
M2	0.7989 (0.0828)		1.0402 (0.0063)	1500.0	-513.5	1030.9	1037.4
M3	0.0016 (0.0014)	0.2988 (0.4187)	-1.9889 (1.3183)	4.0 (3.2)	-500.3	1008.5	1011.0
M4	0.0005 (0.0013)		-2.9117 (0.8679)	1.3 (3.2)	-500.8	1007.7	1012.2
Cns.Q4							
M1	1.3952 (0.1557)	-0.0009 (0.0022)	1.0106 (0.0108)	1650.0	-533.5	1073.1	1077.6
M2	1.3912 (0.1548)		1.0144 (0.0054)	1650.0	-533.6	1071.2	1077.7
M3	0.0631 (0.1275)	-0.0932 (0.1945)	0.3581 (1.3685)	73.6 (147.9)	-533.2	1074.4	1076.8
M4	0.0628 –		0.7619 –	75.7 –	-534.9	1075.9	1080.5
RTr.Q4							
M1	1.0343 (0.0056)	-0.0026 (0.0015)	0.9964 (0.0028)	1875.0	-1172.7	2351.4	2355.9
M2	1.0031 (0.0042)		1.0002 (0.0002)	1875.0	-1184.1	2372.2	2378.7
M3	0.1108 (0.1589)	-0.0405 (0.0785)	0.8644 (0.3726)	38.9 (55.6)	-643.9	1295.9	1298.4
M4	0.1133 (0.1789)		0.9643 (0.2183)	40.2 (63.0)	-644.5	1295.1	1299.6

Comparison of the out-of-sample one month forecasting errors:

	Nearest Mode		Expected Value		ARMA		ARFIMA	
	RMSE	AME	RMSE	AME	RMSE	AME	RMSE	AME
Ind.Q5								
M1	0.04003	0.03068	0.03990	0.03050	**0.03833***	**0.02939***	0.04042	0.03113
M2	0.03996	0.03049	0.04011	0.03049				
M3	**0.03949**	**0.03042**	**0.03964**	**0.03044**				
M4	0.04021	0.03104	0.03999	0.03065				
Cst.Q4								
M1	0.02903	0.02386	0.02897	0.02372	0.29539	0.27448	0.02672	**0.02192***
M2	0.02789	0.02271	0.02781	0.02259				
M3	0.02681	0.02219	**0.02671**	**0.02209**				
M4	**0.02641***	**0.02210**	0.02677	0.02208				
Cns.Q4								
M1	0.05028	0.03668	0.04973	0.03613	0.09868	0.07829	0.05068	0.03860
M2	0.05035	0.03704	0.05002	0.03649				
M3	**0.04948**	**0.03596**	**0.04905***	**0.03534***				
M4	0.05070	0.03749	0.05049	0.03731				
RTr.Q4								
M1	0.05095	0.03974	0.05077	0.03956	0.22687	0.19629	0.05223	**0.03830***
M2	**0.05080**	0.03888	**0.05066***	0.03887				
M3	0.05191	0.04050	0.05152	0.04020				
M4	0.05107	**0.03871**	0.05098	**0.03867**				

Table 7: Italy [IT]

	Parameter Estimates and Standard Errors						
	$\hat{\upsilon}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{N}$	$\mathfrak{L}$	AIC	BIC
Ind.Q5							
M1	1.4947 (0.1607)	0.0027 (0.0016)	0.9979 (0.0096)	2050.0	-515.1	1036.2	1040.7
M2	1.4773 (0.1572)		1.0110 (0.0057)	2050.0	-516.5	1037.1	1043.5
M3	1.0080 (1.0467)	0.0045 (0.0062)	0.9853 (0.0422)	1379.4 (1427.3)	-515.0	1038.0	1040.5
M4	0.0456 (0.1167)		0.7950 (0.5654)	64.3 (164.1)	-518.2	1042.4	1046.9
Cst.Q4							
M1	1.8635 (0.2075)	-0.0057 (0.0028)	0.9671 (0.0153)	250.0	-730.2	1466.4	1470.9
M2	1.8086 (0.1953)		0.9831 (0.0132)	250.0	-732.2	1468.4	1474.9
M3	0.0235 (0.0138)	-0.3857 (0.1915)	-3.5683 (1.4569)	3.8 (1.8)	-720.9	1449.9	1452.3
M4	0.0253 (0.0135)		-2.3553 (1.0157)	4.1 (1.8)	-724.1	1454.3	1458.8
Cns.Q4							
M1	1.2686 (0.1366)	-0.0026 (0.0017)	0.9778 (0.0137)	1000.0	-572.1	1150.3	1154.8
M2	1.2522 (0.1333)		0.9890 (0.0116)	1000.0	-573.3	1150.6	1157.1
M3	0.1287 –	-0.0299 –	0.5444 –	98.6 –	-569.3	1146.6	1149.0
M4	0.0911 –		0.5386 –	71.1 –	-570.7	1147.4	1151.8

Comparison of the out-of-sample one month forecasting errors:

	Nearest Mode		Expected Value		ARMA		ARFIMA	
	RMSE	AME	RMSE	AME	RMSE	AME	RMSE	AME
Ind.Q5								
M1	**0.03771***	**0.02865**	0.03789	0.02856	0.04521	0.03583	0.03830	0.02947
M2	0.03851	0.02968	0.03845	0.02918				
M3	0.03803	0.02920	**0.03787**	**0.02855***				
M4	0.03917	0.02980	0.03889	0.02945				
Cst.Q4								
M1	0.06481	0.05183	0.06689	0.05406	0.13318	0.11536	**0.05618***	**0.04518***
M2	**0.05776**	**0.04614**	**0.05793**	**0.04683**				
M3	0.08885	0.07236	0.07331	0.05919				
M4	0.06283	0.05186	0.05952	0.04867				
Cns.Q4								
M1	**0.03122**	**0.02427***	**0.03111***	**0.02436**	0.03790	0.03030	0.03148	0.02494
M2	0.03272	0.02620	0.03187	0.02552				
M3	0.03148	0.02520	0.03163	0.02525				
M4	0.03243	0.02663	0.03274	0.02665				

Table 8: Luxembourg [LU]

	Parameter Estimates and Standard Errors						
	$\hat{v}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{N}$	$\mathfrak{L}$	AIC	BIC
Ind.Q5							
M1	0.2511 (0.0284)	-0.0119 (0.0140)	0.6984 (0.0845)	55.0	-683.9	1373.8	1378.3
M2	0.2501 (0.0281)		0.7119 (0.0831)	55.0	-684.3	1372.5	1379.0
M3	0.1168 (0.1723)	-0.0225 (0.0411)	0.3174 (1.0478)	25.7 (37.4)	-683.8	1375.6	1378.1
M4	0.3322 (0.8854)		0.7924 (0.6561)	73.1 (195.5)	-684.4	1374.7	1379.2
Cst.Q4							
M1	0.1232 (0.0132)	-0.0506 (0.0408)	0.7473 (0.1309)	20.0	-713.9	1433.9	1438.4
M2	0.1217 (0.0128)		0.8386 (0.1090)	20.0	-714.7	1433.4	1439.9
M3	0.0997 (0.1283)	-0.0619 (0.0883)	0.6643 (0.5723)	16.2 (20.5)	-713.9	1435.9	1438.4
M4	0.0840 (0.1898)		0.7185 (0.8798)	13.9 (30.7)	-714.7	1435.4	1439.9

Comparison of the out-of-sample one month forecasting errors:

	Nearest Mode		**Expected Value**		**ARMA**		**ARFIMA**	
	RMSE	AME	RMSE	AME	RMSE	AME	RMSE	AME
Ind.Q5								
M1	0.08809	0.07317	0.08796	0.07296	0.09219	0.07562	0.08859	0.07497
M2	0.08784	**0.07281**	0.08772	0.07270				
M3	0.08791	0.07305	0.08794	0.07295				
M4	**0.08775**	0.07281	**0.08771***	**0.07268***				
Cstr.Q4								
M1	**0.07550**	**0.06063**	**0.07528***	0.06042	0.13168	0.10302	0.07770	0.06180
M2	0.07731	0.06137	0.07699	0.06130				
M3	0.07567	0.06063	0.07533	**0.06041***				
M4	0.07708	0.06163	0.07713	0.06139				

Table 9: Netherlands [NL]

	Parameter Estimates and Standard Errors								$\mathfrak{L}$	AIC	BIC
	$\hat{\upsilon}$		$\hat{\alpha}_0$		$\hat{\alpha}_1$		$\hat{N}$				
Ind.Q5											
M1	0.5087	(0.0630)	0.0258	(0.0048)	0.6224	(0.0628)	850.0		-469.9	945.8	950.3
M2	0.4392	(0.0479)			0.9147	(0.0348)	850.0		-483.6	971.3	977.8
M3	11.7242	–	0.0010	–	0.9887	–	20115.4	–	-469.0	946.0	948.4
M4	0.0371	(0.0243)			-0.0736	(0.5117)	72.0	(46.1)	-483.9	973.7	978.2
Cst.Q4											
M1	0.2535	(0.0279)	0.0121	(0.0064)	0.8630	(0.0624)	300.0		-532.0	1070.1	1074.5
M2	0.2523	(0.0276)			0.9092	(0.0578)	300.0		-533.8	1071.6	1078.1
M3	0.0761	–	0.0400	–	0.5119	–	90.1	–	-531.8	1071.6	1074.1
M4	0.0653	–			0.6068	–	77.7	–	-533.5	1073.1	1077.6
Cns.Q4											
M1	0.9296	(0.1000)	-0.0011	(0.0020)	0.9981	(0.0123)	750.0		-571.3	1148.7	1153.2
M2	0.9270	(0.0995)			0.9991	(0.0122)	750.0		-571.5	1147.0	1153.5
M3	0.0603	–	-0.0024	–	0.6071	–	48.6	–	-570.9	1149.9	1152.4
M4	0.0032	(0.0025)			-3.9698	(1.1338)	3.5	(2.5)	-566.5	1139.1	1143.6

Comparison of the out-of-sample one month forecasting errors:

	Nearest Mode		Expected Value		ARMA		ARFIMA	
	RMSE	AME	RMSE	AME	RMSE	AME	RMSE	AME
Ind.Q5								
M1	0.03186	0.02556	**0.03085**	0.02486	0.03560	0.02854	**0.02949***	**0.02396**
M2	0.03154	**0.02496**	0.03155	**0.02480**				
M3	**0.03088**	0.02527	0.03122	0.02545				
M4	0.03157	0.02496	0.03156	0.02480				
Cst.Q4								
M1	0.04184	0.03455	0.04197	0.03463	0.10610	0.08656	0.04527	0.03697
M2	**0.04161***	**0.03383***	**0.04167**	**0.03388**				
M3	0.04184	0.03455	0.04211	0.03467				
M4	0.04161	0.03383	0.04184	0.03394				
Cns.Q4								
M1	**0.05691**	**0.04508**	**0.05678***	**0.04489***	0.16798	0.13644	0.05834	0.04628
M2	0.05713	0.04527	0.05696	0.04497				
M3	0.05739	0.04532	0.05719	0.04524				
M4	0.05702	0.04518	0.05718	0.04516				

Table 10: Austria [AT]

	Parameter Estimates and Standard Errors						
	$\hat{v}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{N}$	$\mathfrak{L}$	AIC	BIC
Ind.Q5							
M1	0.1591 (0.0185)	0.0078 (0.0069)	0.8518 (0.0733)	405.0	-466.5	939.1	943.6
M2	0.1584 (0.0183)		0.8841 (0.0678)	405.0	-467.2	938.3	944.8
M3	0.0161 (0.0482)	0.0871 (0.3178)	-0.4479 (3.9818)	41.4 (121.4)	-466.7	941.3	943.8
M4	0.0284 (0.0410)		0.3505 (0.9232)	72.7 (104.4)	-467.3	940.6	945.1

Comparison of the out-of-sample one month forecasting errors:

	Nearest Mode		**Expected Value**		**ARMA**		**ARFIMA**	
	RMSE	AME	RMSE	AME	RMSE	AME	RMSE	AME
Ind.Q5								
M1	0.03377	0.02696	**0.03347**	0.02674	0.07542	0.06324	0.03765	0.02998
M2	**0.03335***	**0.02642***	0.03372	0.02682				
M3	0.03373	0.02696	0.03347	**0.02672**				
M4	0.03343	0.02654	0.03374	0.02681				

Table 11: Finland [FI]

	Parameter Estimates and Standard Errors						
	$\hat{v}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{N}$	$\mathfrak{L}$	AIC	BIC
Ind.Q5							
M1	1.6466 (0.1936)	0.0077 (0.0026)	0.9538 (0.0153)	425.0	-659.4	1324.7	1329.2
M2	1.5751 (0.1777)		0.9837 (0.0118)	425.0	-663.8	1331.6	1338.1
M3	2.6658 (1.5102)	0.0046 (0.0033)	0.9803 (0.0256)	687.3 (382.9)	-659.1	1326.3	1328.8
M4	0.2960 –		0.8251 –	81.2 –	-665.4	1336.7	1341.2
Cst.Q4							
M1	0.4292 (0.0423)	-0.0182 (0.0108)	1.1490 (0.0326)	60.0	-780.2	1566.4	1570.9
M2	0.4248 (0.0417)		1.1551 (0.0327)	60.0	-781.6	1567.3	1573.8
M3	0.0208 (0.0251)	-0.1356 (0.2676)	0.1930 (0.7405)	2.5 (2.8)	-745.6	1499.3	1501.8
M4	0.0258 (0.0225)		0.3916 (0.4781)	3.1 (2.5)	-745.8	1497.7	1502.2

Comparison of the out-of-sample one month forecasting errors:

	Nearest Mode		**Expected Value**		**ARMA**		**ARFIMA**	
	RMSE	AME	RMSE	AME	RMSE	AME	RMSE	AME
Ind.Q5								
M1	**0.10058***	**0.07720***	**0.10088**	**0.07815**	0.10803	0.08418	0.10227	0.07911
M2	0.10355	0.07965	0.10377	0.08033				
M3	0.10092	0.07777	0.10152	0.07896				
M4	0.10399	0.08011	0.10397	0.08040				
Cst.Q4								
M1	0.13602	0.10326	0.13352	0.10197	0.39744	0.35258	0.12240	0.09561
M2	0.13674	0.10438	0.13390	0.10256				
M3	**0.12511**	0.09755	0.12494	**0.09723**				
M4	0.12511	**0.09729**	**0.12490**	0.09727				

Table 12: United Kingdom [UK]

	Parameter Estimates and Standard Errors							
	$\hat{v}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{N}$		$\mathfrak{L}$	AIC	BIC
Ind.Q5								
M1	1.6677 (0.1807)	0.0029 (0.0018)	0.9947 (0.0101)	750.0		-617.4	1240.9	1245.4
M2	1.6480 (0.1766)		1.0041 (0.0082)	750.0		-618.7	1241.4	1247.8
M3	0.1676 (0.4734)	0.0339 (0.0997)	0.7176 (0.8708)	75.9	(213.1)	-617.9	1243.9	1246.4
M4	0.1774 (0.6034)		0.8407 (0.6247)	81.7	(276.6)	-619.7	1245.5	1249.9
Cst.Q4								
M1	0.4952 (0.0505)	-0.0006 (0.0037)	1.0602 (0.0154)	400.0		-595.8	1197.6	1202.1
M2	0.4949 (0.0504)		1.0604 (0.0154)	400.0		-595.8	1195.7	1202.2
M3	0.0022 (0.0021)	0.2287 (0.1974)	-2.5212 (0.7171)	2.4	(2.1)	-576.0	1160.1	1162.5
M4	0.0011 (0.0020)		-2.8015 (0.7066)	1.2	(2.2)	-576.9	1159.7	1164.2
Cns.Q4								
M1	1.5160 (0.1714)	-0.0038 (0.0017)	0.9594 (0.0139)	1000.0		-579.2	1164.4	1168.9
M2	1.4689 (0.1614)		0.9788 (0.0108)	1000.0		-581.6	1167.1	1173.6
M3	0.3173 (1.5598)	-0.0190 (0.0953)	0.7478 (1.3170)	208.7	(1025.1)	-578.9	1165.9	1168.4
M4	0.1127 –		0.5551 –	77.0	–	-581.6	1169.2	1173.7
RTr.Q4								
M1	0.3899 (0.0434)	0.0122 (0.0076)	0.9580 (0.0323)	250.0		-589.1	1184.2	1188.7
M2	0.3847 (0.0423)		0.9954 (0.0226)	250.0		-590.4	1184.8	1191.3
M3	0.1437 (0.3951)	0.0347 (0.1007)	0.8169 (0.6175)	92.8	(253.3)	-589.7	1187.4	1189.9
M4	0.8553 (0.4518)		1.0190 (0.0143)	553.3	(287.2)	-589.9	1185.7	1190.2

Comparison of the out-of-sample one month forecasting errors:

	Nearest Mode		Expected Value		ARMA		ARFIMA	
	RMSE	AME	RMSE	AME	RMSE	AME	RMSE	AME
Ind.Q5								
M1	0.06214	0.05171	0.06229	0.05148	0.23264	0.17157	**0.06057***	**0.04841***
M2	0.06325	0.05193	0.06274	0.05105				
M3	**0.06173**	0.05048	**0.06167**	0.05058				
M4	0.06242	**0.05033**	0.06223	**0.05025**				
Cst.Q4								
M1	0.03732	0.03112	0.03738	0.03093	0.12580	0.11568	0.03707	0.03078
M2	0.03698	0.03057	0.03744	0.03098				
M3	**0.03519***	**0.02919***	**0.03598**	**0.02980**				
M4	0.03643	0.03033	0.03611	0.02991				
Cns.Q4								
M1	**0.03789***	**0.02921**	**0.03800**	**0.02886***	0.03908	0.02955	0.03866	0.02892
M2	0.03931	0.02948	0.03942	0.02961				
M3	0.03873	0.02948	0.03839	0.02906				
M4	0.04051	0.03064	0.04016	0.03007				
RTr.Q4								
M1	0.07965	0.06712	0.07982	0.06723	0.07971	0.06709	0.08056	0.06767
M2	0.08032	0.06736	0.08038	0.06747				
M3	**0.07945***	**0.06700***	**0.07965**	**0.06708**				
M4	0.08098	0.06760	0.08077	0.06773				

Table 13: 'Momentum effects' for selected EU countries: M5

M5	Parameter Estimates and Standard Errors $\hat{\upsilon}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	N	$\mathfrak{L}$	AIC
Ind.							
BE	0.6817 (0.0727)	-0.0014 (0.0023)	0.9714 (0.0198)	-0.0528 (0.0548)	775	-536.4	1080.9
DK	0.2606 (0.0314)	0.0413 (0.0105)	0.5964 (0.0853)	0.5266 (0.1529)	250	-541.6	1091.2
DE	0.5274 (0.0574)	0.0015 (0.0017)	0.9545 (0.0198)	0.2541 (0.0700)	1800	-433.3	874.5
IE	2.1391 (0.2673)	0.0080 (0.0023)	0.9440 (0.0160)	-0.0671 (0.0217)	550	-653.4	1314.7
EL	1.1737 (0.1373)	0.0167 (0.0059)	0.9574 (0.0237)	-0.0015 (0.0330)	850	-559.8	1127.7
FR	1.7515 (0.1917)	0.0012 (0.0010)	0.9976 (0.0078)	-0.0244 (0.0215)	2000	-536.1	1080.2
IT	1.4211 (0.1528)	0.0022 (0.0016)	1.0020 (0.0100)	-0.0705 (0.0273)	2050	-511.6	1031.2
LU	0.2292 (0.0261)	-0.0103 (0.0147)	0.7502 (0.0902)	-0.5178 (0.1766)	55	-679.1	1366.2
NL	0.4279 (0.0531)	0.0225 (0.0053)	0.6790 (0.0709)	-0.3466 (0.1053)	850	-463.4	934.8
AT	0.1582 (0.0184)	0.0085 (0.0069)	0.8311 (0.0748)	0.3365 (0.2294)	405	-465.4	938.9
FI	1.4424 (0.1703)	0.0065 (0.0028)	0.9662 (0.0168)	-0.0945 (0.0295)	425	-653.4	1314.9
UK	1.5688 (0.1701)	0.0023 (0.0019)	0.9993 (0.0105)	-0.0661 (0.0252)	750	-613.8	1235.5
Cst.							
BE	0.3190 (0.0339)	0.0015 (0.0046)	0.9325 (0.0364)	-0.2105 (0.1182)	440	-518.0	1043.9
DK	0.9931 (0.1179)	-0.0024 (0.0028)	0.9484 (0.0214)	-0.1324 (0.0401)	375	-637.1	1282.1
DE	0.4629 (0.0473)	-0.0011 (0.0041)	1.0265 (0.0172)	0.2085 (0.0799)	700	-518.3	1044.7
IE	1.8214 (0.2157)	0.0041 (0.0025)	1.0317 (0.0089)	-0.0995 (0.0247)	250	-718.3	1444.5
EL	1.5226 (0.1698)	0.0042 (0.0030)	1.0025 (0.0130)	0.0354 (0.0260)	220	-725.0	1458.1
FR	0.7903 (0.0823)	0.0022 (0.0019)	1.0457 (0.0080)	0.0067 (0.0657)	1500	-512.7	1033.4
IT	1.4801 (0.1652)	-0.0033 (0.0032)	0.9864 (0.0177)	-0.1320 (0.0295)	250	-717.2	1442.4
LU	0.1189 (0.0128)	-0.0391 (0.0411)	0.7883 (0.1344)	-0.6554 (0.3398)	20	-712.0	1432.1
NL	0.2501 (0.0276)	0.0120 (0.0064)	0.8756 (0.0641)	-0.1603 (0.1558)	300	-531.5	1071.0
FI	0.4235 (0.0417)	-0.0176 (0.0108)	1.1558 (0.0331)	-0.1427 (0.0870)	60	-778.8	1565.7
UK	0.4933 (0.0504)	-0.0007 (0.0037)	1.0577 (0.0156)	0.1017 (0.0709)	400	-594.8	1197.6
Cns.							
BE	0.9316 (0.0988)	-0.0004 (0.0025)	1.0076 (0.0144)	-0.0049 (0.0435)	800	-569.2	1146.4
DK	0.7819 (0.0922)	-0.0035 (0.0025)	0.9385 (0.0252)	-0.1755 (0.0480)	750	-554.3	1116.7
DE	0.5345 (0.0555)	-0.0019 (0.0030)	1.0018 (0.0180)	0.1721 (0.0707)	1000	-492.5	993.0
IE	0.7124 (0.0767)	-0.0005 (0.0024)	0.9836 (0.0151)	-0.0253 (0.0504)	650	-555.9	1119.8
EL	0.7335 (0.0814)	-0.0077 (0.0034)	0.9550 (0.0207)	0.0123 (0.0478)	750	-541.6	1091.1
FR	1.3900 (0.1556)	-0.0007 (0.0022)	1.0113 (0.0110)	-0.0105 (0.0282)	1650	-533.5	1074.9
IT	1.2028 (0.1299)	-0.0022 (0.0018)	0.9844 (0.0143)	-0.0778 (0.0317)	1000	-568.9	1145.9
NL	0.9306 (0.1002)	-0.0011 (0.0020)	0.9975 (0.0125)	0.0133 (0.0377)	750	-571.3	1150.6
UK	1.4908 (0.1697)	-0.0035 (0.0018)	0.9624 (0.0144)	-0.0236 (0.0261)	1000	-578.8	1165.6
RTr.							
BE	1.6187 (0.2108)	0.0062 (0.0020)	0.8887 (0.0224)	-0.0958 (0.0291)	575	-617.7	1243.5
DE	0.4327 (0.0476)	0.0007 (0.0039)	0.8862 (0.0425)	-0.4608 (0.0951)	405	-550.4	1108.9
FR	1.0351 (0.0060)	-0.0026 (0.0015)	0.9963 (0.0029)	0.0016 (0.0028)	1875	-1172.5	2353.1
UK	0.3894 (0.0434)	0.0120 (0.0076)	0.9592 (0.0327)	-0.0242 (0.1070)	250	-589.1	1186.2

Table 14: 'Momentum effects' for selected EU countries: M6

M6	Parameter Estimates and Standard Errors $\hat{v}$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	N	$\mathfrak{L}$	AIC	BIC
Ind.							
BE	0.6794 (0.0722)	0.9749 (0.0190)	-0.0550 (0.0554)	775	-536.6	1079.3	1083.7
DK	0.2454 (0.0281)	0.8608 (0.0531)	0.3847 (0.1515)	250	-549.2	1104.5	1109.0
DE	0.5274 (0.0573)	0.9598 (0.0189)	0.2507 (0.0700)	1800	-433.6	873.3	877.8
IE	1.9224 (0.2160)	0.9865 (0.0111)	-0.0895 (0.0222)	550	-659.1	1324.2	1328.7
EL	1.1055 (0.1228)	1.0221 (0.0068)	-0.0340 (0.0355)	850	-563.7	1133.3	1137.8
FR	1.7444 (0.1902)	1.0020 (0.0070)	-0.0257 (0.0218)	2000	-536.9	1079.9	1084.3
IT	1.4042 (0.1494)	1.0129 (0.0059)	-0.0754 (0.0273)	2050	-512.5	1030.9	1035.4
LU	0.2282 (0.0259)	0.7624 (0.0886)	-0.5239 (0.1764)	55	-679.3	1364.7	1369.2
NL	0.3729 (0.0407)	0.9430 (0.0384)	-0.4882 (0.1095)	850	-471.5	949.0	953.5
AT	0.1574 (0.0182)	0.8669 (0.0691)	0.3199 (0.2290)	405	-466.2	938.4	942.9
FI	1.3788 (0.1562)	0.9928 (0.0129)	-0.1080 (0.0298)	425	-656.1	1318.3	1322.8
UK	1.5505 (0.1661)	1.0071 (0.0085)	-0.0700 (0.0252)	750	-614.5	1235.0	1239.5
Cst.							
BE	0.3199 (0.0340)	0.9291 (0.0347)	-0.2067 (0.1177)	440	-518.0	1042.0	1046.5
DK	0.9857 (0.1164)	0.9535 (0.0207)	-0.1349 (0.0402)	375	-637.4	1280.9	1285.4
DE	0.4629 (0.0473)	1.0297 (0.0120)	0.2065 (0.0795)	700	-518.4	1042.7	1047.2
IE	1.8062 (0.2121)	1.0350 (0.0087)	-0.1007 (0.0248)	250	-719.6	1445.2	1449.6
EL	1.5080 (0.1668)	1.0080 (0.0124)	0.0327 (0.0262)	220	-726.0	1458.1	1462.5
FR	0.7996 (0.0829)	1.0401 (0.0063)	0.0156 (0.0455)	1500	-513.4	1032.8	1037.3
IT	1.4501 (0.1573)	0.9963 (0.0150)	-0.1382 (0.0292)	250	-717.7	1441.4	1445.9
LU	0.1175 (0.0125)	0.8605 (0.1115)	-0.7078 (0.3381)	20	-712.5	1430.9	1435.4
NL	0.2488 (0.0273)	0.9222 (0.0595)	-0.1701 (0.1554)	300	-533.2	1072.4	1076.9
FI	0.4192 (0.0411)	1.1620 (0.0332)	-0.1487 (0.0870)	60	-780.2	1566.3	1570.8
UK	0.4929 (0.0503)	1.0579 (0.0155)	0.1014 (0.0712)	400	-594.8	1195.6	1200.1
Cns.							
BE	0.9298 (0.0979)	1.0093 (0.0107)	-0.0061 (0.0405)	800	-569.2	1144.4	1148.9
DK	0.7633 (0.0879)	0.9574 (0.0217)	-0.1853 (0.0479)	750	-555.3	1116.7	1121.1
DE	0.5328 (0.0552)	1.0096 (0.0132)	0.1679 (0.0707)	1000	-492.7	991.4	995.9
IE	0.7116 (0.0765)	0.9842 (0.0149)	-0.0258 (0.0523)	650	-555.9	1117.9	1122.4
EL	0.7160 (0.0778)	0.9910 (0.0137)	-0.0027 (0.0606)	750	-544.1	1094.3	1098.7
FR	1.3861 (0.1544)	1.0144 (0.0054)	-0.0121 (0.0269)	1650	-533.5	1073.0	1077.5
IT	1.1876 (0.1268)	0.9942 (0.0121)	-0.0824 (0.0317)	1000	-569.7	1145.4	1149.9
NL	0.9279 (0.0997)	0.9985 (0.0123)	0.0128 (0.0386)	750	-571.4	1148.9	1153.4
UK	1.4408 (0.1588)	0.9812 (0.0111)	-0.0333 (0.0262)	1000	-580.7	1167.5	1172.0
RTr.							
BE	1.4929 (0.1797)	0.9275 (0.0195)	-0.1158 (0.0298)	575	-622.3	1250.6	1255.1
DE	0.4331 (0.0477)	0.8856 (0.0423)	-0.4599 (0.0950)	405	-550.5	1106.9	1111.4
FR	1.0115 (0.0044)	1.0010 (0.0003)	-0.0089 (0.0026)	1875	-1180.8	2367.6	2372.0
UK	0.3840 (0.0422)	0.9962 (0.0227)	-0.0444 (0.0924)	250	-590.3	1186.6	1191.1

Table 15: Predictive power of M5.

M5	Nearest Mode RMSE	Nearest Mode AME	Expected Value RMSE	Expected Value AME	ARMA RMSE	ARMA AME	ARFIMA RMSE	ARFIMA AME
Ind.Q5								
BE	0.03287	0.02688	**0.03277**	**0.02686**	0.08242	0.06719	0.03343	0.02764
DK	0.05207	**0.04037**	0.05204	0.04062	0.06842	0.05192	**0.05169**	0.04039
DE	0.03347	**0.02618**	**0.03343**	0.02633	0.07729	0.06118	0.03399	0.02696
IE	0.09937	0.08313	0.09882	0.08226	**0.09770**	**0.07931**	0.09786	0.08026
EL	0.03580	0.02877	**0.03531**	**0.02855**	0.10063	0.08716	0.03666	0.02928
FR	0.03912	0.03020	0.03936	0.03024	**0.03833**	**0.02939**	0.04042	0.03113
IT	0.03866	0.02994	**0.03816**	**0.02934**	0.04521	0.03583	0.03830	0.02947
LU	0.08690	0.07381	**0.08639**	**0.07353**	0.09219	0.07562	0.08859	0.07497
NL	0.03048	0.02475	0.03063	0.02491	0.03560	0.02854	**0.02949**	**0.02396**
AT	**0.03420**	**0.02723**	0.03424	0.02742	0.07542	0.06324	0.03765	0.02998
FI	**0.10074**	**0.07861**	0.10149	0.07927	0.10803	0.08418	0.10227	0.07911
UK	**0.06027**	0.04886	**0.06027**	0.04883	0.23264	0.17157	0.06057	**0.04841**
Cst.Q4								
BE	0.02920	0.02244	**0.02886**	**0.02219**	0.17509	0.15137	0.02925	0.02262
DK	**0.03981**	0.03138	0.04027	0.03219	0.47826	0.40125	0.04067	**0.03129**
DE	0.03280	0.02437	**0.03240**	**0.02404**	0.50231	0.47217	0.03338	0.02497
IE	0.19317	0.15129	**0.19003**	**0.15074**	0.20172	0.17083	0.19222	0.15244
EL	**0.10479**	**0.08229**	0.10527	0.08302	0.11216	0.09168	0.11301	0.08833
FR	0.02914	0.02386	0.02898	0.02374	0.29539	0.27448	**0.02672**	**0.02192**
IT	0.05757	**0.04405**	0.05836	0.04511	0.13318	0.11536	**0.05618**	0.04518
LU	**0.07633**	0.06308	0.07652	0.06276	0.13168	0.10302	0.07770	**0.06180**
NL	0.04200	0.03452	**0.04174**	**0.03455**	0.10610	0.08656	0.04527	0.03697
FI	0.12956	0.09779	0.12736	0.09623	0.39744	0.35258	**0.12240**	**0.09561**
UK	0.03925	0.03274	0.03887	0.03243	0.12580	0.11568	**0.03707**	**0.03078**
Cns.Q4								
BE	0.05637	0.04380	**0.05589**	**0.04329**	0.14855	0.12578	0.05648	0.04366
DK	0.05527	0.03493	0.05451	0.03455	0.15902	0.13965	**0.05350**	**0.03321**
DE	0.03367	0.02789	**0.03336**	0.02788	0.08820	0.07452	0.03400	**0.02787**
IE	**0.05214**	**0.04251**	0.05267	0.04297	0.06989	0.05719	0.05407	0.04462
EL	0.04693	0.03474	**0.04685**	0.03503	0.04722	0.03523	**0.04685**	**0.03453**
FR	0.05022	0.03699	**0.04964**	**0.03623**	0.09868	0.07829	0.05068	0.03860
IT	**0.03082**	**0.02427**	0.03114	0.02461	0.03790	0.03030	0.03148	0.02494
NL	0.05690	0.04496	**0.05677**	**0.04492**	0.16798	0.13644	0.05834	0.04628
UK	**0.03818**	0.02926	0.03832	0.02914	0.03908	0.02955	0.03866	**0.02892**
RTr.Q4								
BE	0.06119	**0.04593**	**0.06107**	0.04641	0.07086	0.05266	0.06184	0.04645
DE	**0.05566**	0.04588	0.05573	**0.04570**	0.13057	0.10435	0.05830	0.04828
FR	0.05095	0.03979	**0.05074**	0.03956	0.22687	0.19629	0.05223	**0.03830**
UK	0.07977	**0.06714**	**0.07970**	0.06716	0.07971	0.06709	0.08056	0.06767

Table 16: Predictive power of M6.

	Nearest Mode		Expected Value		ARMA		ARFIMA	
M6	RMSE	AME	RMSE	AME	RMSE	AME	RMSE	AME
Ind.Q5								
BE	0.03313	**0.02700**	**0.03282**	0.02707	0.08242	0.06719	0.03343	0.02764
DK	0.05369	0.04151	0.05384	0.04163	0.06842	0.05192	**0.05169**	**0.04039**
DE	0.03363	**0.02625**	**0.03350**	0.02633	0.07729	0.06118	0.03399	0.02696
IE	0.10317	0.08525	0.10191	0.08394	**0.09770**	**0.07931**	0.09786	0.08026
EL	**0.03565**	**0.02873**	0.03581	0.02880	0.10063	0.08716	0.03666	0.02928
FR	0.04024	0.03135	0.03949	0.03024	**0.03833**	**0.02939**	0.04042	0.03113
IT	0.03913	0.03027	0.03853	0.02972	0.04521	0.03583	**0.03830**	**0.02947**
LU	0.08632	0.07321	**0.08618**	**0.07307**	0.09219	0.07562	0.08859	0.07497
NL	0.03092	0.02499	0.03126	0.02498	0.03560	0.02854	**0.02949**	**0.02396**
AT	0.03495	0.02787	**0.03449**	**0.02743**	0.07542	0.06324	0.03765	0.02998
FI	0.10313	0.08037	0.10347	0.08077	0.10803	0.08418	**0.10227**	**0.07911**
UK	0.06071	0.04857	**0.06052**	**0.04821**	0.23264	0.17157	0.06057	0.04841
Cst.Q4								
BE	0.02916	0.02249	**0.02888**	**0.02215**	0.17509	0.15137	0.02925	0.02262
DK	**0.04036**	0.03221	0.04001	0.03197	0.47826	0.40125	0.04067	**0.03129**
DE	0.03280	0.02425	**0.03239**	**0.02401**	0.50231	0.47217	0.03338	0.02497
IE	0.19240	**0.15105**	**0.19006**	0.15114	0.20172	0.17083	0.19222	0.15244
EL	**0.10388**	**0.08324**	0.10519	0.08439	0.11216	0.09168	0.11301	0.08833
FR	0.02762	0.02260	0.02790	0.02266	0.29539	0.27448	**0.02672**	**0.02192**
IT	0.05730	0.04488	0.05667	**0.04409**	0.13318	0.11536	**0.05618**	0.04518
LU	0.07811	0.06394	0.07785	0.06322	0.13168	0.10302	**0.07770**	**0.06180**
NL	**0.04113**	**0.03357**	0.04146	0.03381	0.10610	0.08656	0.04527	0.03697
FI	0.13002	0.09895	0.12761	0.09779	0.39744	0.35258	**0.12240**	**0.09561**
UK	0.03907	0.03238	0.03893	0.03246	0.12580	0.11568	**0.03707**	**0.03078**
Cns.Q4								
BE	0.05641	0.04399	**0.05602**	**0.04342**	0.14855	0.12578	0.05648	0.04366
DK	0.05445	0.03424	0.05455	0.03403	0.15902	0.13965	**0.05350**	**0.03321**
DE	0.03371	0.02801	**0.03347**	0.02797	0.08820	0.07452	0.03400	**0.02787**
IE	**0.05262**	**0.04273**	0.05278	0.04303	0.06989	0.05719	0.05407	0.04462
EL	0.04691	0.03517	0.04734	0.03559	0.04722	0.03523	**0.04685**	**0.03453**
FR	0.05037	0.03737	**0.04986**	**0.03652**	0.09868	0.07829	0.05068	0.03860
IT	0.03174	0.02542	0.03171	0.02546	0.03790	0.03030	**0.03148**	**0.02494**
NL	**0.05659**	**0.04468**	0.05695	0.04501	0.16798	0.13644	0.05834	0.04628
UK	0.03957	0.02974	0.03963	0.02989	0.03908	0.02955	0.03866	**0.02892**
RTr.Q4								
BE	**0.05974**	0.04676	0.05975	0.04667	0.07086	0.05266	0.06184	**0.04645**
DE	**0.05548**	0.04579	0.05558	**0.04559**	0.13057	0.10435	0.05830	0.04828
FR	0.05096	0.03864	**0.05085**	0.03895	0.22687	0.19629	0.05223	**0.03830**
UK	**0.08004**	**0.06712**	0.08014	0.06733	0.07971	0.06709	0.08056	0.06767