Product Replacement Bias in Inflation and Its Consequences for Monetary Policy: Appendix

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Contents

A Solution of the NT model

This appendix provides details on solving the NT model in Weber (2011b). The nonlinear equilibrium conditions of the NT model are the equations (1), (2), (5), (6), (7), (9), firms' technology, the market clearing conditions, the consolidated budget constraint, and a specification of monetary policy. Moreover, the analyst knows the aggregate price level (11) and aggregate output $Y_t = n\,(P_{Nt}/P_t)\,C_{Nt} + (1-n)\,(P_{Tt}/P_t)\,C_{Tt}$.

A.1 Symmetric flexible-price steady state

This appendix derives the symmetric steady state with flexible prices. The pricing equation (9) implies that, with flexible prices, all product prices are equal to the optimal price $P^\star$. This optimal price is equal to a constant markup over marginal costs, which are the same for all firms.

Symmetric price levels: Imposing $P_N = P_T$ on the definitions of the price levels (1) and using the fact that product prices equal $P^\star$ determines the scalar Γ such that it normalizes the mass of products in C_N to unity:

$$\Gamma = \int_{J_t} \gamma^{s_{jt}} dj \ .$$

To solve this integral recall that the mass of new products with age s_{jt} equal to unity is δ at each date t. The N household consumes a fraction γ of these products. Therefore, the mass of new products that N consumes is $\delta\gamma$. The mass of products with age equal to two that N consumes is $\delta(1-\delta)\gamma^2$. This term accounts for the initial mass of the cohort, the death shock, and the fact that N only keeps γ of the products it consumed in the previous period. More generally, N consumes $\delta(1-\delta)^z\gamma^{z+1}$ of the products with age $z+1$ at time t. Summing over all entry cohorts $z = 0, \dots \infty$ delivers $\Gamma = \sum_{z=0}^{\infty} \delta(1-\delta)^z\gamma^{z+1}$ or $\Gamma = \delta\gamma/(1-(1-\delta)\gamma)$. By construction, this value of Γ also normalizes the mass of products in C_T to unity.

Symmetric consumption levels: With $P_N = P_T$, equation (7) is $u_C(C_N, \xi) = u_C(C_T, \xi)$ and implies $C_N = C_T$. The remaining equilibrium conditions yield:

$$u_C(C_T, \xi) = \frac{\theta - 1}{\theta(1-\tau_L)} \frac{h_L(L)}{A} \quad , \quad AL = nC_N + (1-n)C_T \ .$$

Imposing explicit functional forms for u and h yields consumption and labor in steady state.

Constant firm-specific output: In general, while aggregate variables are constant, firm-specific output $\bar{Y}_{jt}$ varies over time in the symmetric steady state:

$$\frac{\bar{Y}_{jt}}{C_T} = \gamma^{s_{jt}} \left(\frac{n}{\Gamma}\right) + (1-\gamma^{s_{jt}}) \left(\frac{1-n}{1-\Gamma}\right) \ .$$

Over the lifetime of firm j, demand shifts from the N household to the T household. If both households demand different amounts then firm-specific steady-state output varies over time. To prevent this, I set the relative population mass n of households equal to Γ which yields $\bar{Y}_{jt} = C_T$. Furthermore, with n equal to Γ, the flexible-price symmetric steady state coincides with the sticky-price symmetric steady state with zero aggregate inflation (SZISS), and this coincidence simplifies the welfare analysis below.

A.2 Linearized equilibrium conditions

This appendix calculates the equilibrium conditions of the NT model accurate to the first order and at the SZISS and yields the linearized NT model of Proposition 1.

Pricing equation: Calculated to the first order and denoting $a_t = \hat{A}_t$, the pricing equation (9) implies that all adjusting firms set the same optimal price:

$$\hat{P}^\star_t - \hat{P}_t = (1-\kappa\beta)(\hat{W}_t - \hat{P}_t - a_t) + \kappa\beta E_t[\hat{P}^\star_{t+1} - \hat{P}_{t+1} + \hat{\pi}_{t+1}] \ .$$

Aggregate price level: Calculated to the first order, the aggregate price level P_t in equation (11) equals $\hat{P}_t = \int_0^1 \hat{P}_{jt}\ dj$. The price level integrates prices over the unit mass that is composed of infinitely many entry cohorts. Each cohort has mass δ in the entry period but diminishes over time by firm exit: $1 = \delta \sum_{z=0}^{\infty}(1-\delta)^z$.

Consider the average price of the cohort that entered $z \geq 0$ periods ago at time $t-z$, and normalize the mass of this cohort to unity. At time t and calculated to the first order, this price is determined by a truncated geometric distribution:

$$\hat{\Lambda}_t(z) = \begin{cases} (1-\alpha)\sum_{k=0}^{z-1} \alpha^k \hat{P}^\star_{t-k} + \alpha^z \hat{P}^\star_{t-z} & \text{if} \quad z \geq 1 \ , \\ \hat{P}^\star_t & \text{if} \quad z = 0 \ . \end{cases} \tag{A.1}$$

Now weight the average price $\hat{\Lambda}_t(z)$ by the mass of the $t-z$ cohort at time t and sum over all cohorts to obtain the aggregate price level:

$$\hat{P}_t = \delta \sum_{z=0}^{\infty} (1-\delta)^z \hat{\Lambda}_t(z) \ . \tag{A.2}$$

Simplifying this expression yields the recursive law of motion: $\hat{P}_t = (1-\kappa)\hat{P}_t^{\star} + \kappa \hat{P}_{Nt}$.

Pricing equation and aggregate price level: Combine the pricing equation and the recursive law of motion of the aggregate price level to obtain:

$$\hat{\pi}_t = [(1-\kappa\beta)(1-\kappa)/\kappa](\hat{W}_t - \hat{P}_t - a_t) + \beta E_t \hat{\pi}_{t+1} \ . \tag{A.3}$$

Marginal costs and output: Calculated to the first order and by using $g_t = -\hat{\xi}_t$, the household optimality conditions (6) and (7) yield:

$$\hat{W}_t - \hat{P}_{Tt} = \nu \hat{L}_t + (\hat{C}_{Tt} - g_t) \quad , \quad \hat{P}_{Nt} - \hat{P}_{Tt} = (\hat{C}_{Tt} - g_t) - (\hat{C}_{Nt} - g_t) \ .$$

Use the definition of the aggregate price level $\hat{P}_t = \Gamma \hat{P}_{Nt} + (1-\Gamma)\hat{P}_{Tt}$ and the one of aggregate output $\hat{Y}_t = \Gamma \hat{C}_{Nt} + (1-\Gamma)\hat{C}_{Tt}$ calculated to the first order, and rearrange the optimality conditions according to:

$$\hat{W}_t - \hat{P}_t \quad = \quad \nu \hat{L}_t + (\hat{Y}_t - g_t)$$

In order to replace aggregate labor by aggregate output, combine labor market clearing, firms' technology, and households' demand functions to the aggregate technology:

$$A_t L_t = C_{Nt} \int_0^1 \gamma^{s_{jt}} \left(\frac{P_{jt}}{P_{Nt}} \right)^{-\theta} dj + C_{Tt} \int_0^1 (1-\gamma^{s_{jt}}) \left(\frac{P_{jt}}{P_{Tt}} \right)^{-\theta} dj \ .$$

Calculate it to the first order and exploit the definition of aggregate output to obtain that $a_t + \hat{L}_t = \hat{Y}_t$. The relative price terms in the aggregate technology are zero to the first order by the equations (1). Combine aggregate technology and the real wage to obtain real marginal costs:

$$\hat{W}_t - \hat{P}_t - a_t \quad = \quad (1+\nu)\hat{Y}_t - g_t - (1+\nu)a_t \ .$$

New Keynesian Phillips curve: Combine real marginal costs with the equation (A.3) to obtain the NKPC:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + [(1-\kappa\beta)(1-\kappa)/\kappa](1+\nu)x_t \ . \tag{A.4}$$

Here, the output gap is $x_t = \hat{Y}_t - \hat{Y}_t^{na}$ and $\hat{Y}_t^{na} = \frac{1}{1+\nu}g_t + a_t$ is the natural level of output in the flexible-price economy absent u_t shocks. The natural level of output is obtained by going through analog steps in the NT model with flexible prices. Equation (A.4) corresponds to the first equation in (12) in the main text.

Aggregate IS relation: Combine household optimality conditions (5) and (7) with the condition $(1+i_t)^{-1} = \beta E_t \Omega_{t,t+1}$ to obtain:

$$(1+i_t)^{-1} = \beta E_t \frac{u_c(C_{Nt+1}, \xi_{t+1})}{u_c(C_{Nt}, \xi_t)} \frac{P_{Nt}}{P_{Nt+1}} \ , \quad (1+i_t)^{-1} = \beta E_t \frac{u_c(C_{Tt+1}, \xi_{t+1})}{u_c(C_{Tt}, \xi_t)} \frac{P_{Tt}}{P_{Tt+1}} \ .$$

Calculate these conditions to the first order and use the definitions of aggregate output and aggregate inflation to obtain:

$$\hat{Y}_t - g_t \quad = \quad E_t(\hat{Y}_{t+1} - g_{t+1}) - (\hat{i}_t - E_t \hat{\pi}_{t+1}) \ .$$

Obtain by analog steps in the flexible-price model that $\hat{Y}_t^{na} - g_t = E_t(\hat{Y}_{t+1}^{na} - g_{t+1}) - \hat{r}_t^{na}$. Subtract the flexible-price equation from the sticky-price equation to obtain:

$$x_t \quad = \quad E_t x_{t+1} - (\hat{i}_t - E_t \hat{\pi}_{t+1} - \hat{r}_t^{na}) \ . \tag{A.5}$$

The natural real rate $\hat{r}_t^{na} = -E_t(1-L^{-1})(a_t - \frac{\nu}{1+\nu}g_t)$ is the flexible-price real interest rate absent u_t shocks. Equation (A.5) corresponds to the second equation in (12) in the main text.

A.3 N inflation and T inflation

This appendix derives the mapping between aggregate inflation and N inflation or T inflation in Proposition 2.

N inflation: Calculated to the first order, P_{Nt} in equations (1) is:

$$\hat{P}_{Nt} = \Gamma^{-1} \int_{J_t} \gamma^{s_{jt}} \hat{P}_{jt} \ dj \ .$$

This price level integrates prices over the unit mass that is composed of infinitely many entry cohorts. Each cohort has mass δ in the entry period but diminishes over time by firm exit. Moreover, only the fraction γ of the total demand for the products in a new cohort comes from N households. Accordingly, express the unit mass of prices in $\hat{P}_{Nt}$ as $1 = \Gamma^{-1}\delta\gamma \sum_{z=0}^{\infty}[(1-\delta)\gamma]^z$.

Along the lines of the derivation for aggregate price level in Appendix A.2, the price level $\hat{P}_{Nt}$ can thus be rearranged as:

$$\hat{P}_{Nt} = \Gamma^{-1}\delta\gamma \sum_{z=0}^{\infty}[(1-\delta)\gamma]^z \hat{\Lambda}_t(z) \ , \tag{A.6}$$

$\hat{\Lambda}_t(z)$ denotes the average price of the $t-z$ cohort at time t that is defined in equation (A.1). Simplify the N price level as $\hat{P}_{Nt} = (1-\kappa\gamma)\hat{P}_t^{\star} + \kappa\gamma\hat{P}_{Nt-1}$ and rewrite it in terms of inflation rates as $\hat{\pi}_{Nt} = (1-\kappa\gamma)\hat{\pi}_t^{\star} + \kappa\gamma\hat{\pi}_{Nt-1}$, with $\hat{\pi}_t^{\star} = \hat{P}_t^{\star} - \hat{P}_{t-1}^{\star}$. Recall the recursive law of motion $\hat{P}_t = (1-\kappa)\hat{P}_t^{\star} + \kappa\hat{P}_{t-1}$ derived in Appendix A.2 and rewrite it in terms of inflation rates as $\hat{\pi}_t = (1-\kappa)\hat{\pi}_t^{\star} + \kappa\hat{\pi}_{t-1}$. Substitute for $\hat{\pi}_t^{\star}$ by the equation $\hat{\pi}_{Nt} = (1-\kappa\gamma)\hat{\pi}_t^{\star} + \kappa\gamma\hat{\pi}_{Nt-1}$. This delivers the mapping between $\hat{\pi}_{Nt}$ and $\hat{\pi}_t$ in Proposition 2.

T inflation: To map $\hat{\pi}_{Tt}$ and $\hat{\pi}_t$, rearrange the equation for $\hat{\pi}_{Nt}$ in Proposition 2 as:

$$\hat{\pi}_{Nt} = \frac{1-\kappa\gamma}{1-\kappa}\left(\frac{1-\kappa L}{1-\kappa\gamma L}\right)\hat{\pi}_t \ .$$

Substitute it for $\hat{\pi}_{Nt}$ in the definition $\hat{\pi}_t = \Gamma\hat{\pi}_{Nt} + (1-\Gamma)\hat{\pi}_{Tt}$ and simplify the result to obtain:

$$(1-\Gamma)\hat{\pi}_{Tt} = \hat{\pi}_t - \Gamma\left(\frac{1-\kappa\gamma}{1-\kappa}\right)\left(\frac{1-\kappa L}{1-\kappa\gamma L}\right)\hat{\pi}_t \ .$$

Multiply through by $(1-\kappa\gamma L)$ and simplify the coefficients to obtain the mapping between $\hat{\pi}_{Tt}$ and $\hat{\pi}_t$ in Proposition 2.

B Product replacement bias

This appendix derives the PRB in Proposition 3 in three steps. First, calculate the measured price levels at the SZISS accurate to the first order. Second, compute measured inflation and expresses it as a lag polynomial of aggregate inflation. Third, expresse PRB in measured inflation as a function of aggregate inflation. Finally, this appendix also derives the PRB in the measured price level in Proposition 5.

B.1 Measured price levels

Calculated to the first order, the measured price levels in equations (10) are equal to:

$$\hat{P}^m_{t,t-1}/Q = \int_{\mathcal{N}_{t-1}} \hat{P}_{jt}\; dj \quad , \quad \hat{P}^m_{t-1,t-1}/Q = \int_{\mathcal{N}_{t-1}} \hat{P}_{jt-1}\; dj \ .$$

Let weights Q_j be the same for all products and equal to Q since products are consumed in equal amounts in the SZISS. The scalar Q normalizes the mass of products in the basket $\mathcal{N}_{t-1}$ to unity and is determined below.

Measured price levels as infinite sums: Consider the measured price level $\hat{P}^m_{t,t-1}$ first. At time $t-1$, the SB samples $(1-\gamma)$ of the δ new products. Account for the discontinued products to obtain that a mass $\delta(1-\delta)(1-\gamma)$ of the products that were new at time $t-1$ is contained in the $\mathcal{N}_{t-1}$ basket. Similarly, a mass $\delta(1-\delta)^2(1-\gamma)(1+\gamma)$ of the products that were new at time $t-2$ is contained in the $\mathcal{N}_{t-1}$ basket. That is, $(1-\gamma)$ products are sampled at time $t-2$, $(1-\gamma)$ of the γ remaining products are sampled in $t-1$, and all subsamples are subject to equal mortality risk. More generally, a mass $\delta(1-\delta)^z(1-\gamma^z)$ of the products that belongs to the $t-z$ cohort is contained in $\mathcal{N}_{t-1}$.

Along the lines of the derivation for aggregate price level in Appendix A.2, the measured price level $\hat{P}^m_{t,t-1}$ can thus be rearranged as:

$$\hat{P}^m_{t,t-1}/Q = \sum_{z=1}^{\infty} \delta(1-\delta)^z(1-\gamma^z)\hat{\Lambda}_t(z) \ . \tag{B.1}$$

$\hat{\Lambda}_t(z)$ denotes the average price of the $t-z$ cohort at time t defined in equation (A.1).

Now consider the measured price level $\hat{P}^m_{t-1,t-1}$. This price level refers to the same basket as $\hat{P}^m_{t,t-1}$. However, the average price of a particular entry cohort differs from $\hat{P}^m_{t,t-1}$ because the price distribution of any cohort is less fanned out at time $t-1$ than at time t. Obtain:

$$\hat{P}^m_{t-1,t-1}/Q = \sum_{z=1}^{\infty} \delta(1-\delta)^z(1-\gamma^z)\hat{\Lambda}_{t-1}(z-1) \ . \tag{B.2}$$

Normalize the mass of products in measured price levels as $Q^{-1} = \sum_{z=1}^{\infty} \delta(1-\delta)^z(1-\gamma^z)$ or $Q^{-1} = (1-\gamma)(1-\delta)/(1-(1-\delta)\gamma)$.

Measured price levels in terms of N and T price levels: Rearrange the measured price level in equation (B.1) according to:

$$\hat{P}^m_{t,t-1}/Q = \delta \sum_{z=0}^{\infty}(1-\delta)^z \hat{\Lambda}_t(z) - \delta \sum_{z=0}^{\infty}[(1-\delta)\gamma]^z \hat{\Lambda}_t(z) \ .$$

Substitute for the infinite sums by $\hat{P}_t$ in equation (A.2) and $\hat{P}_{Nt}$ in equation (A.6), respectively. Employ the definition of the aggregate price level and simplify coefficients to obtain:

$$\hat{P}^m_{t,t-1} = \frac{1}{1-\delta}\hat{P}_{Tt} - \frac{\delta}{1-\delta}\hat{P}_{Nt} \ .$$

Rearrange the measured price level in equation (B.2) according to:

$$\hat{P}^m_{t-1,t-1}/Q = \delta(1-\delta) \sum_{z=0}^{\infty}(1-\delta)^z \hat{\Lambda}_{t-1}(z) - \delta\gamma(1-\delta) \sum_{z=0}^{\infty}[(1-\delta)\gamma]^z \hat{\Lambda}_{t-1}(z) \ .$$

Simplify it along similar lines as for $\hat{P}^m_{t,t-1}$ to obtain:

$$\hat{P}^m_{t-1,t-1} = \hat{P}_{Tt-1} \ .$$

B.2 Measured inflation and aggregate inflation

Calculate the definition of measured inflation to the first order as $\hat{\pi}^m_t = \hat{P}^m_{t,t-1} - \hat{P}^m_{t-1,t-1}$ and substitute for measured price levels expressed in terms of the N and T price level to obtain:

$$\hat{\pi}^m_t = \frac{1}{1-\delta}\hat{\pi}_{Tt} - \frac{\delta}{1-\delta}(\hat{P}_{Nt} - \hat{P}_{Tt-1}) \ . \tag{B.3}$$

In the next couple of steps, replace the relative price $\hat{P}_{Nt} - \hat{P}_{Tt-1}$ by the inflation rates $\hat{\pi}_{Nt}$ and $\hat{\pi}_t$. Start by employing the definition of the aggregate price level rearranged according to $P_{Tt-1} = \frac{1}{1-\Gamma}P_{t-1} - \frac{\Gamma}{1-\Gamma}P_{Nt-1}$ and obtain:

$$\hat{P}_{Nt} - \hat{P}_{Tt-1} = \frac{1}{1-\Gamma}(\hat{P}_{Nt} - \hat{P}_{t-1}) - \frac{\Gamma}{1-\Gamma}\pi_{Nt} \ . \tag{B.4}$$

Rewrite the relative price $\hat{P}_{Nt} - \hat{P}_{t-1}$ on the right-hand side in terms of inflation rates by subtracting $\hat{P}_{t-1}$ from the recursive law of motion of $\hat{P}_{Nt}$:

$$\hat{P}_{Nt} - \hat{P}_{t-1} = (1-\kappa\gamma)(\hat{P}^\star_t - \hat{P}_{t-1}) + \kappa\gamma(\hat{P}_{Nt-1} - \hat{P}_{t-1}) \ .$$

The recursive law of motion of $\hat{P}_t$ implies $\hat{P}_t^\star - \hat{P}_{t-1} = (1-\kappa)^{-1}\hat{\pi}_t$. Employ this to obtain:

$$\hat{P}_{Nt} - \hat{P}_{t-1} = \frac{1-\kappa\gamma}{1-\kappa}\hat{\pi}_t + \kappa\gamma(\hat{P}_{Nt-1} - \hat{P}_{t-1}) \ .$$

Rewrite the relative price $P_{Nt-1} - \hat{P}_{t-1}$ on the right-hand side in terms of inflation rates by subtracting the recursive law of motion of P_t from the recursive law of motion of $\hat{P}_{Nt}$:

$$\hat{P}_{Nt} - \hat{P}_t = \frac{\kappa(1-\gamma)}{1-\kappa}\frac{\hat{\pi}_t}{1-\kappa\gamma L} \ . \tag{B.5}$$

Substitute this expression lagged once for $\hat{P}_{Nt-1} - \hat{P}_{t-1}$ into $\hat{P}_{Nt} - \hat{P}_{t-1}$ to obtain:

$$\hat{P}_{Nt} - \hat{P}_{t-1} = \frac{1-\kappa\gamma}{1-\kappa}\hat{\pi}_t + \kappa\gamma\frac{\kappa(1-\gamma)}{1-\kappa}\frac{\hat{\pi}_{t-1}}{1-\kappa\gamma L} \ .$$

Substitute this expression for $\hat{P}_{Nt} - \hat{P}_{t-1}$ into $\hat{P}_{Nt} - \hat{P}_{Tt-1}$ to obtain:

$$\hat{P}_{Nt} - \hat{P}_{Tt-1} = \frac{1}{1-\Gamma}\left(\frac{1-\kappa\gamma}{1-\kappa}\hat{\pi}_t + \kappa\gamma\frac{\kappa(1-\gamma)}{1-\kappa}\frac{\hat{\pi}_{t-1}}{1-\kappa\gamma L}\right) - \frac{\Gamma}{1-\Gamma}\pi_{Nt} \ .$$

Substitute this expression for $\hat{P}_{Nt} - \hat{P}_{Tt-1}$ into measured inflation to obtain:

$$\hat{\pi}_t^m = \frac{1}{1-\delta}\hat{\pi}_{Tt} - \frac{\delta}{1-\delta}\left(\frac{1}{1-\Gamma}\frac{1-\kappa\gamma}{1-\kappa}\hat{\pi}_t + \frac{1}{1-\Gamma}\kappa\gamma\frac{\kappa(1-\gamma)}{1-\kappa}\frac{\hat{\pi}_{t-1}}{1-\kappa\gamma L} - \frac{\Gamma}{1-\Gamma}\pi_{Nt}\right) \ .$$

Employ the definition of aggregate inflation $\hat{\pi}_{Tt} = \frac{1}{1-\Gamma}\hat{\pi}_t - \frac{\Gamma}{1-\Gamma}\hat{\pi}_{Nt}$ to substitute for T inflation. Employ the equation $\pi_{Nt} = \frac{1-\kappa\gamma}{1-\kappa}\frac{1-\kappa L}{1-\kappa\gamma L}\hat{\pi}_t$ derived in Appendix A.3 to substitute for N inflation. This yields that measured inflation is a lag polynomial of aggregate inflation:

$$\hat{\pi}_t^m = \left(\frac{1-\alpha}{1-\kappa}\right)\left(\frac{1-(1-\delta)\kappa\gamma L}{1-\kappa\gamma L}\right)\hat{\pi}_t \ .$$

Rearranging this lag polynomial of aggregate inflation yields $\hat{\pi}_t^m = a(L)\hat{\pi}_t$. The lag polynomial $a(L)$ is the one defined in Proposition 3.

B.3 Product replacement bias in measured inflation

Calculate the definition of the bias $B_t = \pi_t^m/\pi_t$ to the first order and substitute for measured inflation by using $\hat{\pi}_t^m = a(L)\hat{\pi}_t$ to obtain the product replacement bias $\hat{B}_t$ as the lag polynomial of aggregate inflation in Proposition 3.

The properties of $a(L)$ are derived as follows. The polynomial $a(L)$ is invertible if all coefficients are positive and if their sum is finite, $a(1) < \infty$. The definition of $a(L)$ readily implies that all coefficients are positive. To show $a(1) < \infty$, replace L by unity in $a(L)$ and simplify. Obtain:

$$a(1) = \left(\frac{1-\alpha}{1-\alpha(1-\delta)}\right)\left(\frac{1-\alpha(1-\delta)^2\gamma}{1-\alpha(1-\delta)\gamma}\right) < \infty \ .$$

To show that $a(1) = 1$ for $\alpha = 0$, $\delta = 0$ or both, plug these parameter values into $a(1)$ and simplify. To show that the sum of the coefficients fulfills the inequality $a(1) < 1$ for the admissible parameter values, impose this inequality on $a(1)$ and simplify. For the inequality to be fulfilled the condition $\gamma < \frac{1}{1-\delta}$ must hold, and this condition holds for all admissible parameter values.

B.4 Product replacement bias in the measured price level

This appendix derives the PRB in the measured price level in Proposition 5. Employ the definition of the aggregate price level $\hat{P}_t = (1-\Gamma)\hat{P}_{Tt} + \Gamma\hat{P}_{Nt}$ and subtract from it the measured price level $\hat{P}^m_{t,t-1} = \frac{1}{1-\delta}\hat{P}_{Tt} - \frac{\delta}{1-\delta}\hat{P}_{Nt}$ derived in Appendix B.1 to obtain:

$$\hat{P}_t - \hat{P}^m_{t,t-1} = \left(1 - \Gamma - \frac{1}{1-\delta}\right)(\hat{P}_{Tt} - \hat{P}_{Nt}) \ .$$

The definition of $\hat{P}_t$ further implies $\hat{P}_{Tt} - \hat{P}_{Nt} = \frac{1}{1-\Gamma}(\hat{P}_t - \hat{P}_{Nt})$. Combine this with equation (B.5) and simplify the coefficients to obtain:

$$\hat{P}_t - \hat{P}^m_{t,t-1} = \frac{\delta}{1-\delta}\,\frac{\kappa}{1-\kappa}\,\frac{\hat{\pi}_t}{1-\kappa\gamma L} \ .$$

Rearrange the lag polynomial to obtain the lag polynomial $b(L)$ in Proposition 5. The lag polynomial $b(L)$ is invertible because all of its coefficients on aggregate inflation are positive and the sum of the coefficients is finite, $b(1) < \infty$.

C Partition of aggregate and measured inflation

This appendix derives the partition of aggregate inflation and measured inflation in terms of terminated price spells with different durations. Combine equation (B.3), equation (B.4), and the relationship $\pi_{Tt} = \frac{1}{1-\Gamma}\pi_t - \frac{\Gamma}{1-\Gamma}\pi_{Nt}$ to:

$$\hat{\pi}_t^m = \frac{1}{1-\delta}\frac{1}{1-\Gamma}\pi_t - \frac{\Gamma}{1-\Gamma}\pi_{Nt} - \frac{\delta}{1-\delta}\frac{1}{1-\Gamma}(\hat{P}_{Nt} - \hat{P}_{t-1}) \ . \tag{C.1}$$

Replace each component of measured inflation by a sum over terminated price spells with different durations. The first component is aggregate inflation, and the recursive law of motion of $\hat{P}_t$ implies $\hat{\pi}_t = (1-\kappa)[\hat{P}_t^\star - \hat{P}_{t-1}]$ or:

$$\hat{\pi}_t = (1-\kappa)[\hat{P}_t^\star - (1-\kappa)\sum_{s=0}^{\infty}\kappa^s \hat{P}_{t-s-1}^\star] \ .$$

Rearrange this to obtain the partition (17) of aggregate inflation in the main text. The second component in equation (C.1) is N inflation. Proceed along the same lines as for aggregate inflation to obtain $\hat{\pi}_{Nt} = (1-\kappa\gamma)^2 \sum_{i=0}^{\infty}(\kappa\gamma)^i(\hat{P}_t^\star - \hat{P}_{t-1-i}^\star)$.

The third component in equation (C.1) is the price differential $\hat{P}_{Nt} - \hat{P}_{t-1}$. Rearrange it by using the recursive law of motion of $\hat{P}_{Nt} = (1-\kappa\gamma)\hat{P}_t^\star + \kappa\gamma\hat{P}_{Nt-1}$:

$$\hat{P}_{Nt} - \hat{P}_{t-1} = (\hat{P}_t^\star - \hat{P}_{t-1}) - \kappa\gamma(\hat{P}_t^\star - \hat{P}_{Nt-1}) \ .$$

Replace the $\hat{P}_{Nt-1}$ on the right-hand side by the infinite discounted sum of current and past optimal prices that follows from iterating backward the recursive law of motion for $\hat{P}_{Nt}$, and do the same for the $\hat{P}_{t-1}$ that occurs on the right-hand side. This yields:

$$\begin{aligned}\hat{P}_{Nt} - \hat{P}_{t-1} &= \big(\hat{P}_t^\star - (1-\kappa)\sum_{i=0}^{\infty}\kappa^i \hat{P}_{t-1-i}^\star\big) - \kappa\gamma\big(\hat{P}_t^\star - (1-\kappa\gamma)\sum_{i=0}^{\infty}(\kappa\gamma)^i \hat{P}_{t-1-i}^\star\big) \\ &= \sum_{i=0}^{\infty}\left[(1-\kappa)\kappa^i - \kappa\gamma(1-\kappa\gamma)(\kappa\gamma)^i\right](\hat{P}_t^\star - \hat{P}_{t-1-i}^\star) \ .\end{aligned}$$

In order to express measured inflation in terms of terminated price spells with different durations, substitute for the three components $\hat{\pi}_t$ and $\hat{\pi}_{Nt}$ and $\hat{P}_{Nt} - \hat{P}_{t-1}$ the respective expressions of terminated price spells with different durations into equation (C.1). Summarizing coefficients and simplifying them yields the partition (17) of measured inflation in the main text. In terms of the primitive parameters, the function $F_m(i)$ is equal to:

$$F_m(i) = (1-\alpha)^{-1}\left[\left(1 + \frac{\delta\gamma}{1-\gamma}\right)\left(\tfrac{(1-\kappa)^2\kappa^i}{1-\delta} - \tfrac{\delta[(1-\kappa)\kappa^i - (1-\kappa\gamma)(\kappa\gamma)^{i+1}]}{1-\delta}\right) - \left(\frac{\delta\gamma}{1-\gamma}\right)(1-\kappa\gamma)^2(\kappa\gamma)^i\right] \ .$$

It is straightforward to show that $\sum_{i=0}^{\infty} F_m(i) = 1$.

D Product replacement bias when new products are small

This appendix contains the derivation of the steady state bias in measured inflation in Proposition 6 that arises when new products are small in terms of their market share. Furthermore, this appendix contains the derivations that belong to Proposition 7 and to Proposition 8, respectively.

D.1 Steady state bias when new products are small

Measured price levels correspond to:

$$P^m_{t,t-1}/q = \int_{\mathcal{N}_{t-1}} g^{\theta s_{jt}} P_{jt} \; dj \quad , \quad P^m_{t-1,t-1}/q = \int_{\mathcal{N}_{t-1}} g^{\theta s_{jt}} P_{jt-1} \; dj \ .$$

In order to integrate over firms' prices, I combine the approach in Weber (2011a) with the derivations in Appendix B.1. Recall that the firms in the FIP model differ regarding the length of their price spell and regarding the level of their productivity. The productivity level depends on the age of a firm. I introduce notation for firms' prices that reflects this two-dimensional heterogeneity. Denote the current price P_{jt} of the firm j as:

$$P_{jt} = P^{\star}_{t-(n+k),t-k} \ , \quad n = 0,1,2,\dots, \ k = 0,1,2,\dots \ .$$

Price P_{jt} equals either the optimal price of the current period or the optimal price of some previous period. The first subscript $t-(n+k)$ indicates the date of market entry. The second subscript $t-k$ indicates the date of the last price change. Index n denotes the time between entry and price change.

Computing $P^m_{t,t-1}$: Consider the average price of the cohort that entered $s \geq 0$ periods ago at time $t-s$, and normalize the mass of this cohort to unity. At time t, this average price is:

$$\Lambda_{1t}(s) = \begin{cases} (1-\alpha)\sum_{k=0}^{s-1} \alpha^k P^{\star}_{t-s,t-k} + \alpha^s P^{\star}_{t-s,t-s} & \text{if} \quad s \geq 1 \ , \\ P^{\star}_{t,t} & \text{if} \quad s = 0 \ . \end{cases} \tag{D.1}$$

The first subscript is the same for all prices because all firms belong to the $t-s$ entry cohort. The second subscript differ across prices because, within that cohort, price durations differ.

Appendix B.1 derives the mass of each $t-s$ entry cohort that is contained in the basket $\mathcal{N}_{t-1}$ of the SB. These derivations apply here, too. Therefore, rewrite the measured price level $P^m_{t,t-1}$ as sum over the cohort-specific average prices $\Lambda_{1t}(s)$ weighted by the appropriate cohort mass:

$$P^m_{t,t-1}/q = \sum_{s=1}^{\infty} \delta(1-\delta)^s(1-\gamma^s)g^{\theta s}\Lambda_{1t}(s) \ . \tag{D.2}$$

Expand this expression according to:

$$P^m_{t,t-1}/q = \delta \sum_{s=0}^{\infty}[(1-\delta)g^{\theta}]^s\Lambda_{1t}(s) - \delta \sum_{s=0}^{\infty}[(1-\delta)\gamma g^{\theta}]^s\Lambda_{1t}(s) \ ,$$

where summations now start at zero. Rewrite the measured price level as follows:

$$P^m_{t,t-1} = qZ_1P_{1t} - qZ_{\gamma}P_{\gamma t} \ . \tag{D.3}$$

Here, Z_{γ} denotes a scalar that normalizes the mass of products in $P_{\gamma t}$ to unity. Define:

$$P_{\gamma t} = \frac{1}{Z_{\gamma}}\delta \sum_{s=0}^{\infty}[(1-\delta)\gamma g^{\theta}]^s\Lambda_{1t}(s) \ . \tag{D.4}$$

Moreover, P_{1t} is the special case of $P_{\gamma t}$ for $\gamma = 1$, and Z_1 is the special case of Z_{γ} for $\gamma = 1$.

Computing $P_{\gamma t}$: First, determine Z_{γ} such that it normalizes the mass of products in $P_{\gamma t}$ to unity. Obtain $Z_{\gamma} = \delta \sum_{s=0}^{\infty}[(1-\delta)\gamma g^{\theta}]^s$ or $Z_{\gamma} = \delta/[1-(1-\delta)\gamma g^{\theta}]$, with $(1-\delta)\gamma g^{\theta} < 1$. Plug Z_{γ} into equation (D.4) and summarize parameters as $\psi_{\gamma} = (1-\delta)\gamma g^{\theta}$ to obtain:

$$P_{\gamma t}[1-\psi_{\gamma}]^{-1} = \sum_{s=0}^{\infty}\psi_{\gamma}^s\Lambda_{1t}(s) \ .$$

Substitute for $\Lambda_{1t}(s)$ by equation (D.1) and simplify the result by using the mapping between optimal prices of firms with different age that follows from the optimal pricing equation of firms, $P^{\star}_{t-k,t-k} = g^n \ P^{\star}_{t-(n+k),t-k}$.[1] For example, with $k = 0$, this mapping says that the optimal price of a new firm $P^{\star}_{t,t}$ is proportional to the optimal price $P^{\star}_{t-n,t}$ of a firm with age

[1] The optimal pricing equation of firms is stated explicitly in the equation (G.2). See also the Appendix G.2. Further details are contained in Weber (2011a).

n. The factor of proportionality g^n exceeds unity and depends on the age difference n across firms. Use this mapping to obtain, after some algebra:

$$P_{\gamma t}[1-\psi_\gamma]^{-1} = \left(\frac{1-\alpha\psi_\gamma/g}{1-\psi_\gamma/g}\right)\sum_{s=0}^{\infty}(\alpha\psi_\gamma)^s P^\star_{t-s,t-s}$$

Express this equation recursively as:

$$P_{\gamma t} =(1-\psi_\gamma)\left(\frac{1-\alpha\psi_\gamma/g}{1-\psi_\gamma/g}\right)P^\star_{t,t} + \alpha\psi_\gamma P_{\gamma t-1} \ .$$

Summarizing $P^m_{t,t-1}$: The parameter q in equation (D.3) normalizes the mass of products in the measured price level $P^m_{t,t-1}$ to unity. Obtain $1/q = \delta\sum_{s=0}^{\infty}[(1-\delta)g^\theta]^s - \delta\sum_{s=0}^{\infty}[(1-\delta)\gamma g^\theta]^s$ or $q\delta = \frac{(1-\psi_1)(1-\psi_\gamma)}{\psi_1-\psi_\gamma}$. Collect the variables that belong to the measured price level in equation (D.3). Define relative price levels as deviations from the aggregate price level according to $p^m_{t,t-1} = P^m_{t,t-1}/P_t$ and $p_{\gamma t} = P_{\gamma t}/P_t$, and define the optimal relative price of a new firm according to $p^\star_t = P^\star_{t,t}/P_t$. Obtain:

$$\begin{aligned}
p^m_{t,t-1} &= \left(\frac{1-\psi_\gamma}{\psi_1-\psi_\gamma}\right)p_{1t} - \left(\frac{1-\psi_1}{\psi_1-\psi_\gamma}\right)p_{\gamma t}\\
p_{\gamma t} &= (1-\psi_\gamma)\left(\frac{1-\alpha\psi_\gamma/g}{1-\psi_\gamma/g}\right)p^\star_t + \alpha\psi_\gamma\pi_t^{-1}p_{\gamma t-1}\\
p_{1t} &= (1-\psi_1)\left(\frac{1-\alpha\psi_1/g}{1-\psi_1/g}\right)p^\star_t + \alpha\psi_1\pi_t^{-1}p_{1t-1} \ .
\end{aligned} \tag{D.5}$$

Computing $P^m_{t-1,t-1}$: The price level $P^m_{t-1,t-1}$ refers to the same basket as $\hat{P}^m_{t,t-1}$ in equation (D.2) while the average price of a particular entry cohort differs from $\hat{P}^m_{t,t-1}$ because the price distribution of any cohort is less fanned out at time $t-1$ than at time t. Obtain:

$$P^m_{t-1,t-1}/q = \sum_{s=1}^{\infty}\delta(1-\delta)^s(1-\gamma^s)g^{\theta s}\Lambda_{1t-1}(s-1) \ .$$

Simplify the right-hand side along the same lines as for $\hat{P}^m_{t,t-1}$, and define the relative measured price level as $p^m_{t-1,t-1} = P^m_{t-1,t-1}/P_{t-1}$. Obtain:

$$p^m_{t-1,t-1} = \left(\frac{1-\psi_\gamma}{\psi_1-\psi_\gamma}\right)\psi_1 p_{1t-1} - \left(\frac{1-\psi_1}{\psi_1-\psi_\gamma}\right)\psi_\gamma p_{\gamma t-1} \ . \tag{D.6}$$

Computing the steady state bias: The bias is defined as measured inflation over aggregate inflation, $B_t = \pi_t^m / \pi_t$. Measured inflation is defined as $\pi_t^m = P^m_{t,t-1}/P^m_{t-1,t-1}$. Rearrange it according to $\pi_t^m = \pi_t p^m_{t,t-1}/p^m_{t-1,t-1}$ and obtain for the bias that $B_t = p^m_{t,t-1}/p^m_{t-1,t-1}$. Denote the relative measured price levels in steady state with an overbar to obtain the steady state bias:

$$B = \bar{p}^m_{t,t-1}/\bar{p}^m_{t-1,t-1} \ .$$

Equations (D.5) and (D.6) deliver the steady state values of the relative measured price levels:

$$\bar{p}^m_{t,t-1} = g^{-1}\frac{(1-\psi_\gamma)(1-\psi_1)}{(1-\psi_\gamma/g)(1-\psi_1/g)}p^\star \ , \quad \bar{p}^m_{t-1,t-1} = \frac{(1-\psi_\gamma)(1-\psi_1)}{(1-\psi_\gamma/g)(1-\psi_1/g)}p^\star \ .$$

Plugging them into the steady state bias B and simplifying it yields the Proposition 6.

D.2 Measured inflation and aggregate inflation

This appendix derives the lag polynomial in Proposition 7 that maps aggregate inflation into measured inflation. Calculate the relative price levels in equations (D.5) and (D.6) to the first order to obtain:

$$\begin{aligned}
(\psi_1-\psi_\gamma)\hat{p}^m_{t,t-1} &= g(1-\psi_\gamma/g)\hat{p}_{1t} - g(1-\psi_1/g)\hat{p}_{\gamma t} \\
(\psi_1-\psi_\gamma)\hat{p}^m_{t-1,t-1} &= \psi_1(1-\psi_\gamma/g)\hat{p}_{1t-1} - \psi_\gamma(1-\psi_1/g)\hat{p}_{\gamma t-1} \\
\hat{p}_{\gamma t} &= (1-\alpha\psi_\gamma/g)\hat{p}^\star_t + (\alpha\psi_\gamma/g)(\hat{p}_{\gamma t-1}-\hat{\pi}_t) \\
\hat{p}_{1t} &= (1-\alpha\psi_1/g)\hat{p}^\star_t + (\alpha\psi_1/g)(\hat{p}_{1t-1}-\hat{\pi}_t)
\end{aligned} \tag{D.7}$$

The price level $\hat{p}_{1t}$ can be simplified by the recursive law of motion of aggregate inflation, which implies $(1-\alpha\psi_1/g)\hat{p}^\star_t = (\alpha\psi_1/g)\hat{\pi}_t$ when calculated to the first order.[2] Substitute this expression into $\hat{p}_{1t}$ to find that $\hat{p}_{1t} = (\alpha\psi_1/g)\hat{p}_{1t-1}$. This difference equation has the nonexplosive solution $\hat{p}_{1t} = 0$ for all t's.

Calculate measured inflation to the first order, $\hat{\pi}^m_t = \hat{p}^m_{t,t-1} - \hat{p}^m_{t-1,t-1} + \hat{\pi}_t$, and substitute for measured price levels to obtain:

$$(\psi_1-\psi_\gamma)(\hat{\pi}^m_t - \hat{\pi}_t) \quad = \quad -g(1-\psi_1/g)\hat{p}_{\gamma t} + \psi_\gamma(1-\psi_1/g)\hat{p}_{\gamma t-1} \ .$$

Express $\hat{p}_{\gamma t}$ in terms of aggregate inflation by using $(1-\alpha\psi_1/g)\hat{p}^\star_t = (\alpha\psi_1/g)\hat{\pi}_t$ and rearrange

[2] See equation (G.3) for the recursive law of motion of aggregate inflation in the FIP model.

it to obtain:

$$\hat{p}_{\gamma t} \quad = \quad \alpha/g\frac{\psi_1-\psi_\gamma}{1-\alpha\psi_1/g}(1-(\alpha\psi_\gamma/g)L)^{-1}\hat{\pi}_t \ . \tag{D.8}$$

Plug this into measured inflation and rewrite it according to:

$$(1-(\alpha\psi_\gamma/g)L)(\hat{\pi}_t^m-\hat{\pi}_t) \quad = \quad -\alpha\frac{1-\psi_1/g}{1-\alpha\psi_1/g}\left(\hat{\pi}_t-\psi_\gamma/g\hat{\pi}_{t-1}\right) \ .$$

Solve for measured inflation:

$$\hat{\pi}_t^m \quad = \quad \frac{1-\alpha}{1-\alpha\psi_1/g}\left(\frac{1-(\alpha\psi_\gamma/g)(\psi_1/g)L}{1-(\alpha\psi_\gamma/g)L}\right)\hat{\pi}_t \ .$$

Rearrange the expression in the large brackets according to:

$$(1-(\alpha\psi_\gamma/g)(\psi_1/g)L)\sum_{i=0}^{\infty}(\alpha\psi_\gamma/g)L)^i = 1+(1-\psi_1/g)\sum_{i=1}^{\infty}(\alpha\psi_\gamma/g)L)^i \ .$$

Finally, substitute the definitions of ψ_1 and ψ_γ to obtain the mapping between measured inflation and aggregate inflation in Proposition 7.

The properties of $a_\rho(L)$ in Proposition 7 are derived as follows. The polynomial $a_\rho(L)$ is invertible if all coefficients are positive and if their sum is finite, $a_\rho(1) < \infty$. The definition of $a_\rho(L)$ readily implies that all coefficients are positive. To show $a_\rho(1) < \infty$, replace L by unity in $a_\rho(L)$ and simplify. Obtain:

$$a_\rho(1) = \left(\frac{1-\alpha}{1-\kappa\rho}\right)\left(\frac{1-(1-\delta)\kappa\gamma\rho^2}{1-\kappa\gamma}\right) < \infty \ .$$

To show that the sum of coefficients fulfills the inequality $a_\rho(1) < 1$, impose this inequality on $a_\rho(1)$ and simplify. For the inequality to be fulfilled the condition $(1-\delta)\gamma g^{\theta-1} < 1$ must hold. Plug into that condition the maximum value of growth $g = (1-\delta)^{-\frac{1}{\theta}}$ to obtain $\gamma < (1-\delta)^{-\frac{1}{\theta}}$. This last condition holds for all admissible parameter values.

D.3 Bias in the measured price level

Calculated to the first order, measured output is equal to $\hat{Y}_t^m = \hat{Y}_t - \hat{p}_{t,t-1}^m$ by using the definition $p_{t,t-1}^m = P_{t,t-1}^m/P_t$. By using equations (D.7) and exploiting that $\hat{p}_{1t} = 0$ as derived in Appendix D.2, the measured price level and the auxiliary price level $\hat{p}_{\gamma t}$ are related according to $(\psi_1 - \psi_\gamma)\hat{p}^m_{t,t-1} = -g(1 - \psi_1/g)\hat{p}_{\gamma t}$. Substitute for $\hat{p}_{\gamma t}$ by equation (D.8) to obtain:

$$-\hat{p}^m_{t,t-1} = \alpha \frac{1 - \psi_1/g}{1 - \alpha\psi_1/g}(1 - (\alpha\psi_\gamma/g)L)^{-1}\hat{\pi}_t \ .$$

Rearrange this expression and use $\hat{p}^m_{t,t-1} = \hat{P}^m_{t,t-1} - \hat{P}_t$ to obtain:

$$\hat{P}_t - \hat{P}^m_{t,t-1} = \frac{\alpha(1 - (1 - \delta)\rho)}{1 - \kappa\rho} \sum_{i=0}^{\infty} (\kappa\gamma\rho)^i \hat{\pi}_{t-i} \ .$$

This equation corresponds to the equation in Proposition 8.

E Welfare-based loss function

Weber (2011b) examines in Section 5 the ad hoc loss function with inflation and the output gap as arguments, in order to examine the consequences of product replacement bias for monetary policy. This appendix and the Appendix F revisit the monetary policy consequences of product replacement bias for the case of the welfare-based loss function. The main finding is that the results obtained in the paper are fairly insensitive with respect to using the ad hoc loss function versus the welfare-based loss function.

This appendix contains the derivation of the welfare-based loss function, which approximates the expected discounted lifetime utility of the N and the T household in the NT model that is described in Section 2 of the paper. The Appendix F revisits the policy analysis in Section 5 of the paper but employs the welfare-based loss function.

This appendix calculates, accurate to the second order and at the symmetric steady state with zero aggregate inflation (SZISS), the joint expected discounted lifetime utility of the N and the T household that corresponds to equation (3) in the paper:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\Gamma u(C_{Nt}, \xi_t) + (1-\Gamma) u(C_{Tt}, \xi_t) - h(L_t) \right] \ .$$

In a first step, calculate the labor argument of utility to the second order. In a second step, calculate the two consumption arguments to the second order. A variable with a hat is generically defined as $\hat{z}_t = \log(z_t/z)$.

E.1 Labor

Calculating the labor argument to the second order yields:

$$h(L_t) = Y u_C \left(\hat{L}_t + \tfrac{1}{2}(1+\nu)\hat{L}_t^2 \right) + tip + o_3 \ .$$

Here, tip denotes terms independent of policy, o_3 indicates terms of order three or higher, and $\nu = \frac{L h_{LL}}{h_L}$. Impose $1 - \tau_L = \theta/(\theta - 1)$ and exploit the households' optimality conditions that imply $A u_C = h_L$ or, by aggregate technology, $Y u_C = L h_L$. Variables without a time subscript denote steady state values.

In order to substitute aggregate labor for N and T consumption, consider the aggregate technology that is derived in Appendix A.2:

$$L_t = \Gamma C_{Nt} \Delta_{Nt} / A_t + (1-\Gamma) C_{Tt} \Delta_{Tt} / A_t \ .$$

Employ the following definition of N price dispersion:

$$\Delta_{Nt} = \frac{1}{\Gamma} \int_0^1 \gamma^{s_{jt}} \left(\frac{P_{jt}}{P_{Nt}} \right)^{-\theta} dj \ . \tag{E.1}$$

Moreover, employ the analog definition for Δ_{Tt}. Calculating aggregate technology to the second order yields:

$$\hat{L}_t = \Gamma \hat{C}_{Nt} + (1-\Gamma)\hat{C}_{Tt} - a_t + \tfrac{1}{2}\Gamma \hat{C}_{Nt}^2 + \tfrac{1}{2}(1-\Gamma)\hat{C}_{Tt}^2 - \tfrac{1}{2}\hat{Y}_t^2 + \Gamma \hat{\Delta}_{Nt} + (1-\Gamma)\hat{\Delta}_{Tt} + tip + o_3 \ .$$

Squaring this equation yields:

$$\hat{L}_t^2 = \hat{Y}_t^2 - 2\hat{Y}_t a_t + tip + o_3 \ .$$

Here, use the definition of aggregate output $\hat{Y}_t = \Gamma \hat{C}_{Nt} + (1-\Gamma)\hat{C}_{Tt} + o_2$ that is accurate to the first order. Employ the equation for $\hat{L}_t$ and the one for $\hat{L}_t^2$ to substitute for aggregate labor in $h(L_t)$. This yields the disutility of labor expressed in terms of N and T consumption, output, and price dispersion:

$$\begin{aligned} h(L_t) = Y u_C \Big(& \Gamma \hat{C}_{Nt} + (1-\Gamma)\hat{C}_{Tt} - a_t + \tfrac{1}{2}\Gamma \hat{C}_{Nt}^2 + \tfrac{1}{2}(1-\Gamma)\hat{C}_{Tt}^2 + \tfrac{1}{2}\nu \hat{Y}_t^2 - (1+\nu)\hat{Y}_t a_t \\ & + \Gamma \hat{\Delta}_{Nt} + (1-\Gamma)\hat{\Delta}_{Tt} \Big) + tip + o_3 \ . \end{aligned}$$

E.2 Consumption

Calculating the N consumption argument of joint utility to the second order yields:

$$u(C_{Nt}, \xi_t) = C_N u_C \left(\hat{C}_{Nt} + \tfrac{1}{2}(1-\sigma^{-1})\hat{C}_{Nt}^2 + \sigma^{-1}\hat{C}_{Nt} g_t \right) + tip + o_3 \ .$$

Define $\sigma^{-1} = -\frac{C_N u_{CC}}{u_C}$ and $g_t = -\frac{\xi u_{C\xi}}{C_N u_{CC}}\hat{\xi}_t$.[3] Along the same lines, obtain for the T consumption argument:

$$u(C_{Tt}, \xi_t) = C_T u_C \left(\hat{C}_{Tt} + \tfrac{1}{2}(1-\sigma^{-1})\hat{C}_{Tt}^2 + \sigma^{-1}\hat{C}_{Tt} g_t \right) + tip + o_3 \ .$$

Combining both arguments yields:

$$\begin{aligned}
&\Gamma u(C_{Nt}, \xi_t) + (1-\Gamma) u(C_{Tt}, \xi_t) \\
&\quad = Y u_C \left(\Gamma \hat{C}_{Nt} + (1-\Gamma)\hat{C}_{Tt} + \tfrac{1}{2}(1-\sigma^{-1})(\Gamma \hat{C}_{Nt}^2 + (1-\Gamma)\hat{C}_{Tt}^2) + \sigma^{-1}\hat{Y}_t g_t \right) + tip + o_3 \ .
\end{aligned}$$

E.3 Combining labor and consumption

Combine the labor argument and the two consumption arguments to the utility function and simplify to obtain:

$$\begin{aligned}
&\Gamma u(C_{Nt}, \xi_t) + (1-\Gamma) u(C_{Tt}, \xi_t) - h(L_t) \\
&= Y u_C \Big(-\tfrac{1}{2}\sigma^{-1}[\Gamma \hat{C}_{Nt}^2 + (1-\Gamma)\hat{C}_{Tt}^2] - \tfrac{1}{2}\nu \hat{Y}_t^2 + \hat{Y}_t[\sigma^{-1} g_t + (1+\nu) a_t] - \Gamma \hat{\Delta}_{Nt} - (1-\Gamma)\hat{\Delta}_{Tt} \Big) \\
&\quad + tip + o_3 \ .
\end{aligned}$$

In this equation, link $[\Gamma \hat{C}_{Nt}^2 + (1-\Gamma)\hat{C}_{Tt}^2]$ and $\hat{Y}_t^2$ by squaring $\hat{Y}_t = \Gamma \hat{C}_{Nt} + (1-\Gamma)\hat{C}_{Tt} + o_2$ and rearranging it according to:

$$[\Gamma \hat{C}_{Nt}^2 + (1-\Gamma)\hat{C}_{Tt}^2] = \hat{Y}_t^2 + \Gamma(1-\Gamma)(\hat{C}_{Nt} - \hat{C}_{Tt})^2 + o_3 \ .$$

Use this link and simplify the utility function to obtain:

$$\begin{aligned}
&\Gamma u(C_{Nt}, \xi_t) + (1-\Gamma) u(C_{Tt}, \xi_t) - h(L_t) \\
&\quad = Y u_C \left(-\tfrac{1}{2}(\sigma^{-1}+\nu) x_t^2 - \tfrac{1}{2}\sigma^{-1}\Gamma(1-\Gamma)(\hat{C}_{Nt} - \hat{C}_{Tt})^2 - \Gamma \hat{\Delta}_{Nt} - (1-\Gamma)\hat{\Delta}_{Tt} \right) + tip + o_3 \ .
\end{aligned}$$

Define the natural level of output as $\hat{Y}_t^{na} = \frac{\sigma^{-1}}{\sigma^{-1}+\nu} g_t + \frac{1+\nu}{\sigma^{-1}+\nu} a_t$, and denote the output gap as $x_t = \hat{Y}_t - \hat{Y}_t^{na}$. Employ the households' optimality conditions and the definition of the

[3] For log utility of consumption, $\sigma = 1$.

aggregate price level to obtain the second order accurate relationship:

$$\sigma^{-1}(\hat{C}_{Nt} - \hat{C}_{Tt})^2 = \frac{\sigma}{(1-\Gamma)^2}(\hat{P}_{Nt} - \hat{P}_t)^2 + o_3 \ .$$

Plug it into the approximate utility function to obtain:

$$\begin{aligned} &\Gamma u(C_{Nt}, \xi_t) + (1-\Gamma)u(C_{Tt}, \xi_t) - h(L_t) \qquad \text{(E.2)} \\ &= Y u_C \Big(-\tfrac{1}{2}(\sigma^{-1} + \nu)x_t^2 - \tfrac{1}{2}\sigma \frac{\Gamma}{1-\Gamma}(\hat{P}_{Nt} - \hat{P}_t)^2 - \Gamma \hat{\Delta}_{Nt} - (1-\Gamma)\hat{\Delta}_{Tt} \Big) + tip + o_3 \ . \end{aligned}$$

E.4 N price dispersion

In order to obtain a recursive expression for the N price dispersion Δ_{Nt}, define $p_{Njt} = P_{jt}/P_{Nt}$ and calculate the definition (E.1) to the second order:

$$\hat{\Delta}_{Nt} + \frac{1}{2}\hat{\Delta}_{Nt}^2 = -\theta \frac{1}{\Gamma}\int_0^1 \gamma^{s_{jt}} \hat{p}_{Njt} \ dj + \tfrac{1}{2}\theta^2 \frac{1}{\Gamma}\int_0^1 \gamma^{s_{jt}} \hat{p}_{Njt}^2 \ dj + o_3 \ .$$

Define weighted first and second moments according to:

$$E_j(\hat{p}_{Njt}) = \frac{1}{\Gamma}\int_0^1 \gamma^{s_{jt}} \hat{p}_{Njt} \ dj \ , \quad V_j(\hat{p}_{Njt}) = \frac{1}{\Gamma}\int_0^1 \gamma^{s_{jt}} \hat{p}_{Njt}^2 \ dj \ .$$

The relative mass of current and past optimal prices in the N price level and the fact that optimal prices are the same for all adjusting firms imply that first and second moments can be expressed as:

$$E_j(\hat{p}_{Njt}) = (1-\kappa\gamma)\sum_{z=0}^{\infty}(\kappa\gamma)^z \hat{p}_{Nt-z}^{\star} \ , \quad V_j(\hat{p}_{Njt}) = (1-\kappa\gamma)\sum_{z=0}^{\infty}(\kappa\gamma)^z (\hat{p}_{Nt-z}^{\star})^2 \ . \qquad \text{(E.3)}$$

Here, the relative optimal price is denoted as $\hat{p}_{Nt-z}^{\star} = \hat{P}_{t-z}^{\star} - \hat{P}_{Nt}$. Plug the first and the second moment into the second order expansion of the N price dispersion and exploit that $\hat{\Delta}_{Nt}^2 = 0 + o_3$ to obtain:

$$\hat{\Delta}_{Nt} = -\theta E_j(\hat{p}_{Njt}) + \tfrac{1}{2}\theta^2 V_j(\hat{p}_{Njt}) + o_3 \ .$$

By the equations (E.3), obtain $E_j(\hat{p}_{Njt}) = \frac{1}{2}(\theta-1)V_j(\hat{p}_{Njt}) + o_3$ and thus $\hat{\Delta}_{Nt} = \frac{1}{2}\theta V_j(\hat{p}_{Njt}) + o_3$. Rearranging $V_j(\hat{p}_{Njt})$ further and exploiting the recursive law of motion for $\hat{P}_{Nt}$ yields the following recursive expression for N price dispersion:

$$\hat{\Delta}_{Nt} = \kappa\gamma\hat{\Delta}_{Nt-1} + \tfrac{1}{2}\theta\frac{\kappa\gamma}{1-\kappa\gamma}\hat{\pi}^2_{Nt} + o_3 \ .$$

E.5 T price dispersion

In order to derive a similar expression for T price dispersion, consider the definition of it, $\Delta_{Tt} = \frac{1}{1-\Gamma}\int_0^1 (1-\gamma^{s_{jt}})p_{Tjt}^{-\theta}\, dj$, with $p_{Tjt} = P_{jt}/P_{Tt}$, and calculate it to the second order:

$$(1-\Gamma)(\hat{\Delta}_{Tt}+\frac{1}{2}\hat{\Delta}^2_{Tt}) = -\theta\int_0^1 \hat{p}_{Tjt}\, dj + \theta\Gamma\frac{1}{\Gamma}\int_0^1 \gamma^{s_{jt}}\hat{p}_{Tjt}\, dj + \tfrac{1}{2}\theta^2\int_0^1 \hat{p}^2_{Tjt}\, dj - \tfrac{1}{2}\theta^2\Gamma\frac{1}{\Gamma}\int_0^1 \gamma^{s_{jt}}\hat{p}^2_{Tjt}\, dj + o_3 \ .$$

The relative mass of current and past optimal prices in the N price level and in the aggregate price level and the fact that optimal prices are the same for all adjusting firms imply:

$$\begin{aligned}(1-\Gamma)(\hat{\Delta}_{Tt}+\frac{1}{2}\hat{\Delta}^2_{Tt}) = &-\theta(1-\kappa)\sum_{z=0}^{\infty}\kappa^z\hat{p}^{\star}_{Tt-z} + \theta\Gamma(1-\kappa\gamma)\sum_{z=0}^{\infty}(\kappa\gamma)^z\hat{p}^{\star}_{Tt-z} \\ &+ \tfrac{1}{2}\theta^2(1-\kappa)\sum_{z=0}^{\infty}\kappa^z(\hat{p}^{\star}_{Tt-z})^2 - \tfrac{1}{2}\theta^2\Gamma(1-\kappa\gamma)\sum_{z=0}^{\infty}(\kappa\gamma)^z(\hat{p}^{\star}_{Tt-z})^2 + o_3 \ .\end{aligned}$$

Define $p_{jt} = P_{jt}/P_t$. Moreover, define $E_j(\hat{p}_{jt})$ and $V_j(\hat{p}_{jt})$ analog to $E_j(\hat{p}_{Njt})$ and $V_j(\hat{p}_{Njt})$, respectively, and rewrite the approximate expression for T price dispersion as:

$$\begin{aligned}(1-\Gamma)(\hat{\Delta}_{Tt}+\frac{1}{2}\hat{\Delta}^2_{Tt}) = &-\theta E_j(\hat{p}_{jt}) + \theta\Gamma E_j(\hat{p}_{Njt}) - \theta[\hat{P}_t - \Gamma\hat{P}_{Nt} - (1-\Gamma)\hat{P}_{Tt}] \\ &+ \tfrac{1}{2}\theta^2 V_j(\hat{p}_{jt}) - \tfrac{1}{2}\theta^2\Gamma V_j(\hat{p}_{Njt}) + \tfrac{1}{2}\theta^2(\hat{P}_t - \hat{P}_{Tt})^2 - \tfrac{1}{2}\theta^2\Gamma(\hat{P}_{Nt} - \hat{P}_{Tt})^2 + o_3 \ .\end{aligned}$$

Employ the definition of the aggregate price level $\hat{P}_t = \Gamma\hat{P}_{Nt} + (1-\Gamma)\hat{P}_{Tt} + o_2$ to obtain:

$$\begin{aligned}(1-\Gamma)(\hat{\Delta}_{Tt}+\frac{1}{2}\hat{\Delta}^2_{Tt}) = &-\theta E_j(\hat{p}_{jt}) + \theta\Gamma E_j(\hat{p}_{Njt}) - \theta[\hat{P}_t - \Gamma\hat{P}_{Nt} - (1-\Gamma)\hat{P}_{Tt}] \\ &+ \tfrac{1}{2}\theta^2 V_j(\hat{p}_{jt}) - \tfrac{1}{2}\theta^2\Gamma V_j(\hat{p}_{Njt}) - \tfrac{1}{2}\theta^2\frac{\Gamma}{1-\Gamma}(\hat{P}_{Nt} - \hat{P}_t)^2 + o_3 \ .\end{aligned}$$

To rearrange the term in square brackets, calculate the definition of the aggregate price level $P_t^{1-\theta} = \Gamma P_{Nt}^{1-\theta} + (1-\Gamma)P_{Tt}^{1-\theta}$, which is equation (11) in Weber (2011b), accurate to the second order:

$$\hat{P}_t - \Gamma\hat{P}_{Nt} - (1-\Gamma)\hat{P}_{Tt} = \tfrac{1}{2}(1-\theta)\frac{\Gamma}{1-\Gamma}(\hat{P}_{Nt} - \hat{P}_t)^2 + o_3 \ .$$

Plug it into the approximate expression of T price dispersion to obtain:

$$(1-\Gamma)(\hat{\Delta}_{Tt}+\frac{1}{2}\hat{\Delta}_{Tt}^2) = -\theta E_j(\hat{p}_{jt}) + \theta\Gamma E_j(\hat{p}_{Njt}) \\ + \tfrac{1}{2}\theta^2 V_j(\hat{p}_{jt}) - \tfrac{1}{2}\theta^2\Gamma V_j(\hat{p}_{Njt}) - \tfrac{1}{2}\theta\frac{\Gamma}{1-\Gamma}(\hat{P}_{Nt}-\hat{P}_t)^2 + o_3 \ .$$

This equation implies $\hat{\Delta}_{Tt}^2 = 0 + o_3$. Moreover, employ $E_j(\hat{p}_{Njt}) = \frac{1}{2}(\theta-1)V_j(\hat{p}_{Njt}) + o_3$ from before and, correspondingly, $E_j(\hat{p}_{jt}) = \frac{1}{2}(\theta-1)V_j(\hat{p}_{jt}) + o_3$, to obtain:

$$(1-\Gamma)\hat{\Delta}_{Tt} = \tfrac{1}{2}\theta V_j(\hat{p}_{jt}) - \tfrac{1}{2}\theta\Gamma V_j(\hat{p}_{Njt}) - \tfrac{1}{2}\theta\frac{\Gamma}{1-\Gamma}(\hat{P}_{Nt}-\hat{P}_t)^2 + o_3 \ .$$

Use $\hat{\Delta}_{Nt} = \frac{1}{2}\theta V_j(\hat{p}_{Njt}) + o_3$ to find:

$$(1-\Gamma)\hat{\Delta}_{Tt} + \Gamma\hat{\Delta}_{Nt} = \tfrac{1}{2}\theta V_j(\hat{p}_{jt}) - \tfrac{1}{2}\theta\frac{\Gamma}{1-\Gamma}(\hat{P}_{Nt}-\hat{P}_t)^2 + o_3 \ .$$

Similar to $V_j(\hat{p}_{Njt})$, the cross-sectional variance $V_j(\hat{p}_{jt})$ evolves recursively according to:

$$V_j(\hat{p}_{jt}) = \kappa V_j(\hat{p}_{jt-1}) + \frac{\kappa}{1-\kappa}\hat{\pi}_t^2 + o_3 \ .$$

Plug this into $(1-\Gamma)\hat{\Delta}_{Tt} + \Gamma\hat{\Delta}_{Nt}$. Then, compute the infinite sum that is discounted with β, and impose the initial condition $V_j(\hat{p}_{j,-1}) = o_3$ to obtain:

$$\sum_{t=0}^{\infty}\beta^t\left((1-\Gamma)\hat{\Delta}_{Tt} + \Gamma\hat{\Delta}_{Nt} + \tfrac{1}{2}\theta\frac{\Gamma}{1-\Gamma}(\hat{P}_{Nt}-\hat{P}_t)^2\right) = \tfrac{1}{2}\theta\frac{\kappa}{(1-\kappa)(1-\kappa\beta)}\sum_{t=0}^{\infty}\beta^t\hat{\pi}_t^2 + o_3 \ . \tag{E.4}$$

E.6 Summarizing terms

Define the welfare-based loss function $\mathcal{L}_{WB}$ as:

$$\mathcal{L}_{WB} = -E_0\sum_{t=0}^{\infty}\beta^t\Big(\Gamma u(C_{Nt},\xi_t) + (1-\Gamma)u(C_{Tt},\xi_t) - h(L_t)\Big) \ .$$

Combine equation (E.2) and equation (E.4) and rearrange them to obtain:

$$\mathcal{L}_{WB} = \tfrac{1}{2}\Omega E_0\sum_{t=0}^{\infty}\beta^t\Big(\hat{\pi}_t^2 + \lambda x_t^2 - \lambda_R(\hat{P}_{Nt}-\hat{P}_t)^2\Big) + tip + o_3 \ .$$

Parameters are defined as:

$$\Omega = (\sigma^{-1}+\nu)\frac{\theta}{\phi}Yu_C \ , \quad \lambda = \frac{\phi}{\theta} \ , \quad \lambda_R = \lambda \ \frac{\theta - \sigma}{\sigma^{-1} + \nu} \ \frac{\gamma\delta}{1-\gamma} \ , \quad \phi = \frac{(1-\kappa)(1-\kappa\beta)}{\kappa}(\sigma^{-1}+\nu) \ .$$

In order to express the price differential $\hat{P}_{Nt} - \hat{P}_t$ in the welfare-based loss function in terms of inflation rates, rewrite it using the recursive laws of motion of the N price level and the aggregate price level according to:

$$\hat{P}_{Nt} - \hat{P}_t = (\hat{P}_t^\star - \hat{P}_t) - (\hat{P}_t^\star - \hat{P}_{Nt}) = \frac{\kappa}{1-\kappa}\hat{\pi}_t - \frac{\kappa\gamma}{1-\kappa\gamma}\hat{\pi}_{Nt} \ .$$

Plug this expression for the price differential into the welfare-based loss function to obtain:

$$\mathcal{L}_{WB} = \tfrac{1}{2}\Omega E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{\pi}_t^2 + \lambda x_t^2 - \lambda_R [\tfrac{\kappa}{1-\kappa}\hat{\pi}_t - \tfrac{\kappa\gamma}{1-\kappa\gamma}\hat{\pi}_{Nt}]^2 \right) + tip + o_3 \ .$$

The welfare-based loss function attributes a positive weight to variations in inflation and in the output gap. Both arguments occur in the ad hoc loss function that underlies the policy analysis in Section 5 of Weber (2011b). However, in addition to inflation stabilization and output gap stabilization, the welfare-based loss function also attributes the weight λ_R to the squared quasi-differential of aggregate inflation and N inflation.

E.7 Welfare-based loss function versus ad hoc loss function

The ad hoc loss function corresponds to the special case $\lambda_R = 0$ of the welfare-based loss function $\lambda_R \neq 0$. The weight λ_R is positive as long as $\theta > \sigma$. Accordingly, the squared quasi-differential between inflation rates enters the welfare-based loss function negatively.

There are three reasons for the inflation differential to occur in the welfare-based loss function. First, the variations in the inflation differential reflect the variations in the relative price $\hat{P}_{Nt} - \hat{P}_{Tt}$.[4] If the relative price between N and T consumption differs from zero, then the N household and the T household find it optimal to consume different amounts of their respective consumption composites $\hat{C}_{Nt}$ and $\hat{C}_{Tt}$. From a welfare perspective, however, this is inefficient. Accordingly, the central bank reduces the welfare loss by stabilizing the relative price between N and T consumption, and this corresponds to stabilizing the inflation differential. This stabilization motive is captured by the parameter σ in the weight λ_R. If σ

[4]Recall that the inflation differential is linked to the price differential $\hat{P}_{Nt} - \hat{P}_t$. This price differential can be rearrange according to $\hat{P}_{Nt} - \hat{P}_t = \hat{P}_{Nt} - \Gamma\hat{P}_{Nt} - (1-\Gamma)\hat{P}_{Tt} = (1-\Gamma)(\hat{P}_{Nt} - \hat{P}_{Tt})$.

is large enough, this motive dominates the weight λ_R and implies $\lambda_R < 0$. In this case, the squared inflation differential enters the loss function positively.

Second, price dispersion that arises from differences in the two prices $\hat{P}_{Nt}$ and $\hat{P}_{Tt}$ matters neither for the N household nor for the T household because none of them consumes both composites $\hat{C}_{Nt}$ and $\hat{C}_{Tt}$. However, by stabilizing aggregate inflation $\hat{\pi}_t$, the central bank implicitly penalizes variation in the relative price $\hat{P}_{Nt} - \hat{P}_{Tt}$ because the aggregate price level is a combination of the household-specific price levels. The weight λ_R corrects for this overemphasis on stabilizing aggregate inflation, and therefore the inflation differential enters the loss function negatively as long as $\theta > \sigma$.

The third reason for the inflation differential to occur in the welfare-based loss function is also related to price dispersion. Dispersion among prices of all products $j \in J_t$ affects both inflation rates $\hat{\pi}_{Tt}$ and $\hat{\pi}_{Nt}$ at the same time because each inflation rate comprises all prices even though each inflation rate weights these prices differently. At least to some degree, thus, stabilizing one inflation rate also stabilizes the other inflation rate, and the weight λ_R also corrects for this "double-counting" effect. To sum up, by imposing $\lambda_R = 0$, the ad hoc loss function abstracts from benefits and losses that arise because the N and the T household differ from each other.

F Monetary policy using the welfare-based loss function

This appendix revisits how the results in Section 5 of Weber (2011b) change when the NT policy corresponds to the optimal policy that is derived from the welfare-based loss function $\mathcal{L}_{WB}$ rather than from the ad hoc loss function.

F.1 Optimal commitment

The lagrangian to the policy problem with the objective function $\mathcal{L}_{WB}$ corresponds to:

$$\begin{aligned} L \;=\; & E_0 \sum_{t=0}^{\infty} \beta^t \big[\tfrac{1}{2} \big(\hat{\pi}_t^2 + \lambda x_t^2 - \lambda_R \big(\tfrac{\kappa}{1-\kappa} \hat{\pi}_t - \tfrac{\kappa\gamma}{1-\kappa\gamma} \hat{\pi}_{Nt} \big)^2 \big) \\ & - \psi_{1t} (\hat{\pi}_t - \beta \hat{\pi}_{t+1} - \phi x_t - u_t) \\ & - \psi_{2t} (\hat{\pi}_{Nt} - \kappa\gamma \hat{\pi}_{Nt-1} - \tfrac{1-\kappa\gamma}{1-\kappa} \hat{\pi}_t + \tfrac{1-\kappa\gamma}{1-\kappa} \kappa \hat{\pi}_{t-1}) \big] \; . \end{aligned} \tag{F.1}$$

The first constraint is the NKPC. The second constraint is the mapping between aggregate inflation and N inflation that is established in the Proposition 2 of Weber (2011b). For the case $\gamma = 1$, there is no tradeoff between stabilizing aggregate inflation and stabilizing the inflation differential because, in this case, the N household is the representative household, such that $\hat{\pi}_t = \hat{\pi}_{Nt}$. Accordingly, in this case, the second constraint will not be binding. For the general case $0 < \gamma < 1$, however, the second constraint will be binding because aggregate inflation and N inflation no longer coincide.

Deriving the optimality conditions with respect to $\hat{\pi}_t, \hat{\pi}_{Nt}, x_t$ and rearranging them yields the optimal NT policy under commitment:

$$E_t(1-\kappa\gamma\beta L^{-1})\left[\hat{\pi}_t + \tfrac{\lambda}{\phi}(x_t - x_{t-1})\right] \quad = \quad (1-\gamma)\tfrac{\kappa}{1-\kappa}\lambda_R\left(\tfrac{\kappa}{1-\kappa}\hat{\pi}_t - \tfrac{\kappa\gamma}{1-\kappa\gamma}\hat{\pi}_{Nt}\right) \ . \tag{F.2}$$

The optimal NT policy depends on aggregate inflation and on the aggregate output gap but it also depends on the inflation differential. Evidently, the optimal NT policy that is derived from the ad hoc loss function in Section 5 of Weber (2011b), $\hat{\pi}_t + \frac{\lambda}{\phi}(x_t - x_{t-1}) = 0$, is embedded into the left-hand side of the optimal NT policy that is derived from the welfare-based loss function.

To test how restrictive it is to impose $\lambda_R = 0$ in Weber (2011b), compare the optimal NT policy (F.2) to the T policy $\hat{\pi}_{Tt} + \frac{\lambda}{\phi}(x_{Tt} - x_{Tt-1}) = 0$ that is derived in Section 5.1 of this paper. The first difference between these two policies is that the T policy does not refer to aggregate variables. Weber (2011b) analyzes the consequences of this first difference in Section 5.1. The second difference is that the T policy ignores the consequences of household heterogeneity in the welfare-based loss function by imposing $\lambda_R = 0$. Here, it is tested how robust the results in the paper are with respect to this second difference.

Analog to the paper, compute the relative welfare loss $\mathcal{L}_R^{WB} = (\underline{\mathcal{L}}_{WB} - \mathcal{L}_{WB})/\mathcal{L}_{WB}$ for different rates δ of product turnover. $\underline{\mathcal{L}}_{WB}$ is the loss associated with the T policy in the NT model, and $\mathcal{L}_{WB}$ is the loss associated with the the optimal NT policy (F.2) in the NT model. Zero product turnover continues to serve as useful reference point because the relative loss is equal to zero in this case. There are two reasons for this. First, as in the paper, there is no difference between measured and aggregate variables without product turnover. Second, the weight λ_R is proportional to δ and therefore it is equal to zero without product turnover. For both reasons, the T policy and the optimal NT policy (F.2) coincide without product turnover and losses $\underline{\mathcal{L}}_{WB}$ and $\mathcal{L}_{WB}$ are the same.

Table 1 contains the relative loss $\mathcal{L}_R^{WB}$ computed from the welfare-based loss function and the relative loss $\mathcal{L}_R$ computed from the ad hoc loss function at the benchmark calibration of δ

equal to 0.069395. The variable $\mathcal{L}_R$ is the welfare metric in the paper. The table shows relative losses for both the Case I and the Case II. Each case corresponds to a specific assumption of how the central bank translates T variables into measured variables.[5] For optimal policy under commitment, the numerical differences between the relative losses in the table are fairly small. This finding suggests that both the welfare-based loss function and the ad hoc loss function yield very similar results.

Table 1: Relative losses for welfare-based and ad hoc loss function.

	Case	Commitment	Discretion	Interest rule
$100\times\mathcal{L}_R^{WB}$	I	0.09	-18.92	-49.71
$100\times\mathcal{L}_R^{WB}$	II	0.24	-4.65	-32.80
$100\times\mathcal{L}_R$	I	0.07	-19.12	-49.79
$100\times\mathcal{L}_R$	II	0.28	-4.68	-32.87

To make sure that this similarity of results does not only arise at the benchmark calibration, Panel (a) of Figure 1 plots the relative loss $\mathcal{L}_R^{WB}$ computed from the welfare-based loss function for the range $\delta \in [0, 0.15]$. The panel should be compared to the Panel (a) of the Figure 2 in Weber (2011b), which plots $\mathcal{L}_R$. The comparison reveals that the relative losses $\mathcal{L}_R^{WB}$ and $\mathcal{L}_R$ remain fairly similar for the entire range of values considered for δ. It therefore seems reasonable to conclude that the results in Section 5.1 of Weber (2011b) are robust as to whether they are derived from the ad hoc loss function or from the welfare-based loss function.

F.2 Optimal discretion

To obtain the optimal NT policy under discretionary monetary policy, derive the optimality conditions with respect to $\hat{\pi}_t, \hat{\pi}_{Nt}$ and x_t from the lagrangian (F.1) accounting for the fact that the discretionary central bank cannot influence private sector expectations. Rearranging the optimality conditions yields:

$$E_t(1-\kappa\gamma\beta L^{-1})\left[\hat{\pi}_t + \tfrac{\lambda}{\phi}x_t\right] \quad = \quad (1-\gamma)\tfrac{\kappa}{1-\kappa}\lambda_R\left(\tfrac{\kappa}{1-\kappa}\hat{\pi}_t - \tfrac{\kappa\gamma}{1-\kappa\gamma}\hat{\pi}_{Nt}\right) \ . \tag{F.3}$$

As for the optimal policy under commitment, the optimal NT policy under discretion depends on aggregate inflation, the aggregate output gap, and the inflation differential. Again, the optimal NT policy that is derived from the ad hoc loss function in Section 5.2 of Weber (2011b), $\hat{\pi}_t + \frac{\lambda}{\phi}x_t = 0$, is embedded into the optimal NT policy (F.3).

[5] See Section 5 in Weber (2011b) for more details.

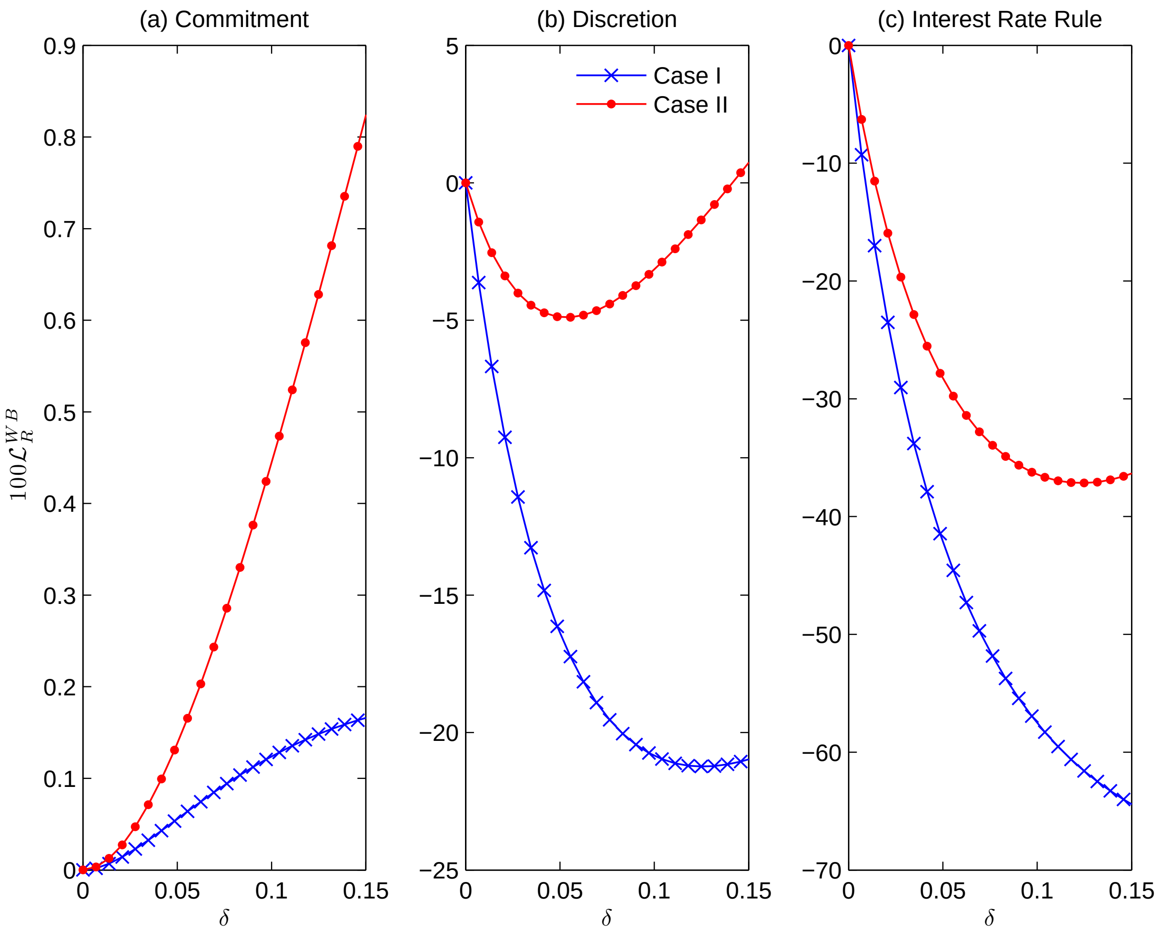

Figure 1: Relative loss $\mathcal{L}_R^{WB}$ computed from the welfare-based loss function as a function of product replacement bias that is captured by δ. Panel (a) shows the relative loss for optimal policy under commitment, Panel (b) shows it for optimal policy under discretion, and Panel (c) shows it for the interest rate rule.

For discretionary monetary policy, Table 1 contains the relative loss $\mathcal{L}_R^{WB}$ computed from the welfare-based loss function and the relative loss $\mathcal{L}_R$ computed from the ad hoc loss function at the benchmark calibration of δ. The table shows that relative losses are negative for discretionary policy, as in Section 5.2 of the paper. Numerically, relative losses are fairly similar irrespectively of whether they are computed from the welfare-based loss function or from the ad hoc loss function. For Case I, the loss $\underline{\mathcal{L}}_{WB}$ is about nineteen percent smaller than the loss $\mathcal{L}_{WB}$ for both loss function. For Case II, the corresponding figure is about 4.7 percent. Moreover, Panel (b) of Figure 1 plots the relative loss that is computed from the welfare-based loss function for the range $\delta \in [0, 0.15]$. The panel should be compared to the Panel (b) of the Figure 2 in Weber (2011b), which plots $\mathcal{L}_R$ for discretionary monetary policy. Again, the comparison reveals that the relative losses $\mathcal{L}_R^{WB}$ and $\mathcal{L}_R$ remain fairly similar for the entire range of values considered for δ.

F.3 Interest rate rule

The functional form of the interest rate rule in Section 5.3 of Weber (2011b) is not affected by the functional form of the loss function because this rule is not derived from first principles. Accordingly, the equilibrium dynamics of aggregate inflation and the aggregate output gap are the same for both the welfare-based loss function and the ad hoc loss function. Nevertheless, when the same equilibrium dynamics are evaluated in terms of welfare by the welfare-based loss function rather than by the ad hoc loss function, this affects the relative loss $\mathcal{L}_R^{WB}$.

Table 1 contains the relative losses $\mathcal{L}_R^{WB}$ and $\mathcal{L}_R$ for the interest rate rule and computed at the benchmark calibration with δ equal to 0.069395. The numerical differences between the relative losses are fairly small. This finding mirrors those obtained for optimal policy under commitment and discretion. Furthermore, Panel (c) in the Figure 1 shows the relative loss $\mathcal{L}_R^{WB}$ that emerges under the interest rate rule for the range $\delta \in [0, 0.15]$. The panel should be compared to the Panel (c) of the Figure 2 in Weber (2011b), which plots $\mathcal{L}_R$ for the interest rate rule. The message from this comparison resembles the one obtained from Table 1, namely, relative losses are fairly similar.

Overall, the analysis in this appendix suggests that the monetary policy consequences of the product replacement bias in Section 5 of Weber (2011b) are not driven by assuming the ad hoc loss function. Rather, fairly similar results emerge when the welfare-based loss function is employed for the analysis.

G The firm-specific productivity (FIP) model

This appendix provides technical details on the derivation of the extended model with firm-specific productivity that occurs in Section 7 of Weber (2011b).

G.1 Intermediate good firms

The technology is

$$Y_{jt} = g^{s_{jt}} \left(A_t L_{jt}\right)^{1/\chi} K_{jt-1}^{1-1/\chi} .$$

The output Y_{jt} of product j is produced with labor L_{jt} and capital K_{jt-1}, with $\chi > 1$. If $g > 1$, then the term $g^{s_{jt}}$ captures productivity growth at the firm level. The productivity grows with the age s_{jt} of firm j. This mechanism of a firm-specific level of productivity is from Weber (2011a). Stochastic aggregate productivity A_t is labor-augmenting and identical across firms.

G.1.1 Input mix

Firms rent capital and labor in perfectly competitive factor markets. Let W_t and $P_t r_t^k$ denote the nominal wage rate and the nominal rental rate on capital services, respectively. Firm j minimizes total costs over L_{jt}, K_{jt-1} and subject to technology:

$$W_t L_{jt} + P_t r_t^K K_{jt-1} \quad s.t. \quad Y_{jt} = g^{s_{jt}} \left(A_t L_{jt}\right)^{1/\chi} K_{jt-1}^{1-1/\chi} \ .$$

Optimality requires that the firm uses the expensive input less:

$$\frac{K_{jt-1}}{L_{jt}} = \left(\frac{1-1/\chi}{1/\chi}\right) \frac{W_t}{P_t r_t^K} \ .$$

Plug the optimal input mix into the technology to obtain the factor demand functions:

$$K_{jt-1} = \frac{Y_{jt}}{g^{s_{jt}} A_t^{1/\chi}} \left(\frac{W_t}{P_t r_t^K}\right)^{1/\chi} \left(\frac{1-1/\chi}{1/\chi}\right)^{1/\chi}, \ L_{jt} = \frac{Y_{jt}}{g^{s_{jt}} A_t^{1/\chi}} \left(\frac{W_t}{P_t r_t^K}\right)^{1/\chi-1} \left(\frac{1-1/\chi}{1/\chi}\right)^{1/\chi-1} .$$

Plug the factor demand functions into total costs and obtain nominal total costs:

$$TC_{jt} = \Xi \frac{Y_{jt}}{g^{s_{jt}} A_t^{1/\chi}} W_t^{1/\chi} \left(P_t r_t^K\right)^{1-1/\chi} \ .$$

Define the constant $\Xi = (1-1/\chi)^{-(1-1/\chi)} (1/\chi)^{-(1/\chi)}$. Total costs are firm-specific because output and productivity is firm-specific. Nominal marginal costs are:

$$MC_{jt} = \Xi \frac{W_t^{1/\chi} \left(P_t r_t^K\right)^{1-1/\chi}}{g^{s_{jt}} A_t^{1/\chi}} \ .$$

Also, $TC_{jt} = MC_{jt} Y_{jt}$.

G.1.2 Pricing

Firm j resets its price P_{jt} infrequently according to:

$$\max_{P_{jt}} E_t \sum_{i=0}^{\infty} (\kappa\beta)^i \ \tilde{\Omega}_{t,t+i} \left[P_{jt} Y_{jt+i} - MC_{jt+i} Y_{jt+i}\right] \quad s.t. \quad Y_{jt+i} = \left(\frac{P_{jt}}{P_{t+i}}\right)^{-\theta} Y_{t+i} \ .$$

Factor $\beta^i \tilde{\Omega}_{t,t+i} = \frac{u_c(C_{t+1},\xi^c_{t+1})}{u_c(C_t,\xi^c_t)} \frac{P_t}{P_{t+1}}$ discounts nominal payoffs and $\kappa = \alpha(1-\delta)$ is the probability to produce tomorrow at old prices accounting for exogenous firm death. Optimality requires:

$$0 = E_t \sum_{i=0}^{\infty} (\kappa\beta)^i \frac{u_c(C_{t+i},\xi^c_{t+i})}{u_c(C_t,\xi^c_t)} \left(\frac{Y_{t+i}}{Y_t}\right) \left(\frac{P_{t+i}}{P_t}\right)^{\theta} \left[g^{s_{jt}} \frac{P^\star_{jt}}{P_t} \left(\frac{P_{t+i}}{P_t}\right)^{-1} - \frac{\theta}{\theta-1} \frac{mc^\star_{t+i}}{g^i} \right] .$$

Define the real wage rate as $w_t = W_t/P_t$. Define real marginal costs of a new firm as:

$$mc^\star_t = \Xi (w_t/A_t)^{1/\chi} \left(r^K_t\right)^{1-1/\chi} \ . \tag{G.1}$$

Rearrange the pricing equation according to:

$$g^{s_{jt}} \frac{P^\star_{jt}}{P_t} = \frac{\theta}{\theta-1} \frac{E_t \sum_{i=0}^{\infty} (\kappa\beta/g)^i \frac{u_c(C_{t+i},\xi^c_{t+i})}{u_c(C_t,\xi^c_t)} \left(\frac{Y_{t+i}}{Y_t}\right) \left(\frac{P_{t+i}}{P_t}\right)^{\theta} mc^\star_{t+i}}{E_t \sum_{i=0}^{\infty} (\kappa\beta)^i \frac{u_c(C_{t+i},\xi^c_{t+i})}{u_c(C_t,\xi^c_t)} \left(\frac{Y_{t+i}}{Y_t}\right) \left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}} \ . \tag{G.2}$$

Denote the optimal relative price of a new firm with $s_{jt} = 0$ as $p^\star_t = g^0 P^\star_{jt}/P_t$. Obtain in aggregate steady state with $\pi = g$ that $\frac{p^\star}{mc^\star} = \frac{\theta}{\theta-1}$. Linearize the pricing equation for a new firm at the steady state with $\pi = g$ to obtain:

$$\hat{p}^\star_t = (1 - \kappa\beta\pi^{\theta-1}) \widehat{mc}^\star_t + \kappa\beta\pi^{\theta-1} E_t[\hat{\pi}_{t+1} + \hat{p}^\star_{t+1}] \ . \tag{G.3}$$

G.2 Price level

The aggregate price level P_t is the cost-minimal price of the final good Y_t and is defined in equation (G.10) below as:

$$P_t^{1-\theta} = \int_0^1 P_{jt}^{1-\theta} \, dj \ .$$

Aggregating individual prices to the price level is done exactly as in Weber (2011a). There, aggregation exploits a relationship among the optimal prices of firms with different age that follows from the pricing equation (G.2). For any two firms j and j', the pricing equation implies that optimal time t prices are proportional across firms with different age:

$$P^\star_{jt} = g^{(s_{j't} - s_{jt})} \, P^\star_{j't} \ . \tag{G.4}$$

Let j denote the young firm, $s_{j't} > s_{jt}$, such that $s_{j't} - s_{jt}$ is a positive integer. The equation says that the price of the young firm exceeds the price of the old firm if $g > 1$.

More generally, acknowledge that the current price P_{jt} of a firm j equals either the optimal price of the current period or the optimal price of some previous period. Express this price as:

$$P_{jt} = P^{\star}_{t-(n+k),t-k} \ , \quad n = 0, 1, 2, \ldots, \ k = 0, 1, 2, \ldots \ .$$

The first subscript $t - (n + k)$ indicates the date of market entry of the firm. The second subscript $t - k$ indicates the date of the last price change. Index n denotes the time between entry and price change. With this new notation, rearrange equation (G.4) as follows. Let firm j be of age k and have a price spell of k periods. Let firm j' be of age $n + k$ and have a price spell of k periods. Rearrange equation (G.4) as $P^{\star}_{t-k,t-k} = g^n \ P^{\star}_{t-(n+k),t-k}$.

Employ this relationship among optimal prices of firms with different age and follow the steps outlined in Weber (2011a) to obtain a recursive representation of the price level:

$$P_t^{1-\theta} = n_g(1 - \kappa g^{(\theta-1)})(P^{\star}_{t,t})^{1-\theta} + \kappa P_{t-1}^{1-\theta} \ .$$

The scalar n_g is defined as $n_g = \delta/[1 - (1-\delta)g^{(\theta-1)}]$, with $(1-\delta)g^{(\theta-1)} < 1$.[6] Rearranging the price level recursion in terms of aggregate inflation yields:

$$1 = n_g(1 - \kappa g^{(\theta-1)})(p^{\star}_t)^{1-\theta} + \kappa \pi_t^{-(1-\theta)} \tag{G.5}$$

Aggregate inflation is defined as $\pi_t = P_t/P_{t-1}$ and the optimal relative price of a new firm is equal to $p^{\star}_t = P^{\star}_{t,t}/P_t$. It follows from the price level recursion that, in the steady state with $\pi = g$, the optimal relative price of a new firm is equal to:

$$(p^{\star})^{1-\theta} = \frac{1}{n_g} \, \frac{1 - \kappa\pi^{(\theta-1)}}{1 - \kappa g^{(\theta-1)}} = \frac{1}{n_g} \ .$$

Calculate the price level recursion rearranged in terms of inflation at this steady state to the

[6]Scalar n_g replaces n_γ in Weber (2011a) to avoid confusing notation.

first order to obtain:

$$\hat{p}_t^\star = \frac{\kappa g^{(\theta-1)}}{1-\kappa g^{(\theta-1)}}\hat{\pi}_t \ . \tag{G.6}$$

G.3 New Keynesian Phillips Curve

Combine the pricing equation (G.3) and the price level recursion (G.6) rearranged in terms of inflation to obtain the New Keynesian Phillips curve (NKPC):

$$\hat{\pi}_t = \frac{(1-\kappa g^{(\theta-1)})(1-\kappa\beta g^{\theta-1})}{\kappa g^{(\theta-1)}}\widehat{mc}_t^\star + \beta E_t[\hat{\pi}_{t+1}] \ .$$

Calculate the real marginal costs (G.1) of a new firm to the first order to obtain with $a_t = \hat{A}_t$:

$$\widehat{mc}_t^\star = (1/\chi)(\hat{w}_t - a_t) + (1-1/\chi)\hat{r}_t^K \ .$$

G.4 Aggregate technology of intermediate good firms

Aggregate the optimal input mix across all intermediate firms:

$$\int_0^1 K_{jt-1} dj = \left(\frac{1-1/\chi}{1/\chi}\right)\frac{W_t}{P_t r_t^K}\int_0^1 L_{jt}\ dj$$

Denote aggregate capital as $K_{t-1} = \int_0^1 K_{jt-1}dj$ and aggregate labor as $L_t = \int_0^1 L_{jt}\ dj$.[7] Obtain:

$$r_t^K K_{t-1} = \left(\frac{1-1/\chi}{1/\chi}\right) w_t L_t \ . \tag{G.7}$$

Thus, the labor capital ratio is the same for all firms. Calculate it to the first order to obtain:

$$\hat{r}_t^K + \hat{K}_{t-1} = \hat{w}_t + \hat{L}_t \ .$$

Now rearrange the firm-specific technology according to:

$$Y_{jt}/g^{s_{jt}} = \left(A_t\frac{L_{jt}}{K_{jt-1}}\right)^{1/\chi} K_{jt-1} \ .$$

[7] See also the Appendix G.11 on market clearing.

The the labor capital ratio is the same for all firms such that $\frac{L_{jt}}{K_{jt-1}} = \frac{L_t}{K_{t-1}}$. Aggregate over all intermediate firms and replace Y_{jt} by product demand defined in equation (G.10):

$$Y_t \int_0^1 \left(\frac{P_{jt}}{P_t}\right)^{-\theta} g^{-s_{jt}} \, dj = \left(A_t \frac{L_t}{K_{t-1}}\right)^{1/\chi} \int_0^1 K_{jt-1} \, dj \ .$$

Define the shifter of aggregate productivity according to:

$$\Delta_t = \int_0^1 \left(\frac{P_{jt}}{P_t}\right)^{-\theta} g^{-s_{jt}} \, dj \ .$$

Obtain the aggregate technology:

$$Y_t = \Delta_t^{-1} \left(A_t L_t\right)^{1/\chi} K_{t-1}^{1-1/\chi} \ . \tag{G.8}$$

Calculating the aggregate technology at the steady state to the first order yields:

$$\hat{Y}_t = a_t + (1/\chi)\hat{L}_t + (1-1/\chi)\hat{K}_{t-1} \ .$$

This expression exploits the fact that, to the first order, the endogenous shifter of aggregate productivity Δ_t is constant and equal to zero. I show this next.

G.5 Productivity shifter

Transform the productivity shifter Δ_t into a recursive expression following similar steps as for the aggregate price level.[8] This yields:

$$\Delta_t = n_g(1-\kappa g^{(\theta-1)})(p_t^\star)^{-\theta} + (\kappa/g)\pi_t^\theta \Delta_{t-1} \ .$$

Obtain in the steady state with $\pi = g$ that $\Delta = n_g(p^\star)^{-\theta}$. When calculated to the first order, the productivity shifter evolves according to:

$$\hat{\Delta}_t = \kappa g^{\theta-1}\hat{\Delta}_{t-1} - \theta\left\{(1-\kappa g^{(\theta-1)})\hat{p}_t^\star - \kappa g^{\theta-1}\hat{\pi}_t\right\} \ . \tag{G.9}$$

Equation (G.6) implies that the term in curly brackets is zero to the first order. This yields that $\hat{\Delta}_t = \kappa g^{\theta-1}\hat{\Delta}_{t-1}$, and imposing that $\hat{\Delta}_{-1}$ is sufficiently small, it implies that the pro-

[8]The exact details are contained in Weber (2011a).

ductivity shifter Δ_t is zero to the first order.

G.6 Final good firm

The final good firm buys intermediate goods on the market, produces aggregate output Y_t, and sells aggregate output to consumer, investors, and the government in a perfectly competitive product market. The final good firm solves:

$$\min_{Y_{jt}} \int_0^1 P_{jt} Y_{jt} dj \quad s.t. \quad Y_t = \left(\int_0^1 Y_{jt}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$$

The final good firm's cost-minimal price level and its product demand function are:

$$P_t = \left(\int_0^1 P_{jt}^{1-\theta} di \right)^{\frac{1}{1-\theta}} \quad , \quad Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\theta} Y_t \ . \tag{G.10}$$

Zero profits require that $P_t Y_t = \int_0^1 P_{jt} Y_{jt} dj$.

G.7 Household

Household $i \in [0,1]$ maximizes discounted expected lifetime utility:

$$\max_{\{Z_{it}, B_t, X_t, K_t, W_{it}, C_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t, \xi_t^c) - h(L_{it}) \right] \ , \quad 0 < \beta < 1 \ . \tag{G.11}$$

The flow budget constraint and capital accumulation are:

$$\begin{aligned} E_t[\Omega_{t,t+1} Z_{it+1}] + B_t + P_t(C_t + X_t) = {} & Z_{it} + (1 + i_{t-1}) B_{t-1} \\ & + P_t r_t^K K_{t-1} + (1 - \tau_L) W_{it} L_{it} + \mathcal{T}_t + D_t \end{aligned} \tag{G.12}$$

$$K_t = (1 - \delta_K) K_{t-1} + \epsilon_t^X \left[1 - S(X_t / X_{t-1}) \right] X_t \ . \tag{G.13}$$

Variable Z_{it} denotes household holdings of one-period state-contingent nominal assets. Household preferences over intermediate products j are $C_t = (\int_0^1 C_{jt}^{\frac{\theta-1}{\theta}} dj)^{\frac{\theta}{\theta-1}}$, with $\theta > 1$. Household demand for product j is $C_{jt}/C_t = (P_{jt}/P_t)^{-\theta}$ and the cost-minimal price of C_t corresponds to P_t defined in equation (G.10). Terminal conditions (not shown) require household solvency. I employ the following function forms:

$$u(C, \xi^c) = \frac{C^{1-1/\sigma} (\xi^c)^{-1/\sigma} - 1}{1 - 1/\sigma} \quad , \ h(L) = \eta \frac{L^{1+\nu}}{1+\nu}$$

The shock ξ^c equals unity in steady state.

G.8 Government

Government consumption q_t is exogenous and is the same bundle of individual products j as C_t. The government budget constraint is $(1+i_{t-1})B_{t-1} + \mathcal{T}_t + P_t q_t \leq B_t + \tau_L \int_0^1 W_{it} L_{it}\, di$. The government issues a riskless one-period bond B_t and collects labor taxes τ_L. Government expenses consist of debt service $(1+i_{t-1})B_{t-1}$, consumption expenditure $P_t q_t$, and transfers $\mathcal{T}_t$ to households. Government income consists of new debt B_t and income from taxing labor. Terminal conditions (not shown) require government solvency. The government controls the nominal interest rate i_t.

G.9 Intertemporal optimality conditions of the household

Denote with λ_t^K the multiplier on the capital accumulation constraint (G.13), and define Tobin's Q as $\Theta_t = \lambda_t^K/(P_t \lambda_t)$. Deriving the optimality conditions to the household problem and rearranging them yields:

$$
\begin{aligned}
\Omega_{t,t+1} &= \beta \frac{u_c(C_{t+1}, \xi_{t+1}^c)}{u_c(C_t, \xi_t^c)} \frac{P_t}{P_{t+1}} \\
(1+i_t)^{-1} &= \beta E_t \frac{u_c(C_{t+1}, \xi_{t+1}^c)}{u_c(C_t, \xi_t^c)} \frac{P_t}{P_{t+1}} \\
\Theta_t &= \beta E_t \frac{u_c(C_{t+1}, \xi_{t+1}^c)}{u_c(C_t, \xi_t^c)} [r_{t+1}^K + \Theta_{t+1}(1-\delta_K)] \\
1 &= \Theta_t \epsilon_t^X \left[1 - S(X_t/X_{t-1}) - S'(X_t/X_{t-1}) \frac{X_t}{X_{t-1}}\right] \\
&\quad + \beta E_t \frac{u_c(C_{t+1}, \xi_{t+1}^c)}{u_c(C_t, \xi_t^c)} \Theta_{t+1} \epsilon_{t+1}^X S'(X_{t+1}/X_t) \left(\frac{X_{t+1}}{X_t}\right)^2 \\
K_t &= (1-\delta_K)K_{t-1} + \epsilon_t^X \left[1 - S(X_t/X_{t-1})\right] X_t \ .
\end{aligned} \tag{G.14}
$$

In steady state, obtain:

$$\begin{aligned}
1+i &= \pi/\beta \\
1 &= \beta[r^K + (1-\delta_K)] \\
\Theta &= 1 \\
X &= [1-(1-\delta_k)]K \ .
\end{aligned}$$

Calculate the optimality conditions accurate to the first order and define $g_t = -\hat{\xi}_t^c$ to obtain:

$$\begin{aligned}
\hat{C}_t - g_t &= E_t(\hat{C}_{t+1} - g_{t+1}) - \sigma(\hat{i}_t - E_t\hat{\pi}_{t+1}) \\
\hat{\Theta}_t &= -\sigma^{-1}E_t(\hat{C}_{t+1} - g_{t+1}) + \sigma^{-1}(\hat{C}_t - g_t) + \frac{r^K}{r^K + (1-\delta_K)}E_t\hat{r}^K_{t+1} + \frac{1-\delta_K}{r^K + (1-\delta_K)}E_t\hat{\Theta}_{t+1} \\
(1+\beta)\hat{X}_t &= \hat{X}_{t-1} + \beta E_t\hat{X}_{t+1} + \frac{1}{S''}\hat{\Theta}_t + \frac{1}{S''}\hat{\epsilon}_t^X \\
\hat{K}_t &= (1-\delta_K)\hat{K}_{t-1} + [1-(1-\delta_K)]\hat{\epsilon}_t^X + [1-(1-\delta_K)]\hat{X}_t \ .
\end{aligned}$$

G.10 Labor firm and wage setting of the household

Household i is a monopolistic supplier of its labor service L_{it}. It sells this service to a representative and competitive labor firm.

G.10.1 Labor firm

The labor firm transforms labor services L_{it} into composite labor L_t by the technology:

$$L_t = \left(\int_0^1 L_{it}^{\frac{\theta_w - 1}{\theta_w}} di\right)^{\frac{\theta_w}{\theta_w - 1}} \quad , \ \theta_w > 1 \ .$$

Minimizing costs $\int_0^1 W_{it}L_{it}\ di$ over all L_{it} and subject to the aggregation technology yields the demand function for the labor service of household i, $L_{it} = (W_{it}/W_t)^{-\theta_w} L_t$. Aggregation yields that the average wage W_t is equal to the following wage index:

$$W_t = \left(\int_0^1 W_{it}^{1-\theta_w} di\right)^{\frac{1}{1-\theta_w}} \ .$$

The zero-profit condition implies that $W_tL_t = \int_0^1 W_{it}L_{it}di$.

G.10.2 Wage setting of the household

The wage setting mechanism is similar to Smets and Wouters (2003). The household i sets the nominal wage $W_{it}^{\star}$ for its specialized labor service L_{it} in order to maximize its lifetime utility:

$$\max_{W_{it}^{\star}} E_t \sum_{s=0}^{\infty} (\alpha_w \beta)^s \left[\cdots - h(L_{it+s})\right] \tag{G.15}$$

The household accounts for the fact that it can adjust its wage only with probability α_w in future periods. The dots indicate the consumption component of lifetime utility that does not influence wage setting. Utility maximization with respect to the nominal wage $W_{it}^{\star}$ is subject to the budget constraint (G.12), to the labor firm's demand for household i's labor service, and to wage indexation to aggregate steady state inflation, respectively:

$$\begin{aligned}
\ldots &= \ldots (1-\tau_L) W_{it+s} L_{it+s} \\
L_{it+s} &= \left(W_{it+s}/W_{t+s}\right)^{-\theta_w} L_{t+s} \\
W_{it+s} &= \pi^s W_{it}^{\star} \ .
\end{aligned}$$

The dots indicate the components of the budget constraint that do not influence wage setting. Deriving the optimality condition and rearranging it yields the wage setting equation:

$$(w_t^{\star})^{1+\nu\theta_w} = \frac{\theta_w \eta}{(\theta_w - 1)(1-\tau_L)} \frac{E_t \sum_{s=0}^{\infty} (\alpha_w \beta)^s L_{t+s} w_{t+s}^{\theta_w(\nu+1)} \left(P_{t+s}/(P_t \pi^s)\right)^{\theta_w(\nu+1)} L_{t+s}^{\nu}}{E_t \sum_{s=0}^{\infty} (\alpha_w \beta)^s L_{t+s} w_{t+s}^{\theta_w} \left(P_{t+s}/(P_t \pi^s)\right)^{\theta_w - 1} \left(C_{t+s} \xi_{t+s}^c\right)^{-1/\sigma}} \ . \tag{G.16}$$

The optimal real wage is defined as $w_t^{\star} = W_t^{\star}/P_t$. The right-hand side is independent of the household index i. Therefore, all reoptimizing households choose the same optimal wage. In steady state, obtain:

$$(w^{\star})^{1+\nu\theta_w} = \frac{\theta_w \eta}{(\theta_w - 1)(1-\tau_L)} w^{\nu\theta_w} L^{\nu} C^{1/\sigma} \ .$$

In G.10.3, I show that the wage index implies $w = w^\star$ in steady state. Accordingly, obtain for the wage setting equation in steady state:

$$w = \frac{\theta_w \eta}{(\theta_w - 1)(1-\tau_L)} L^\nu C^{1/\sigma} \ .$$

G.10.3 Wage index and wage setting equation

Exploit the fact that all reoptimizing households choose the same optimal wage rate in order to rearrange the wage index $W_t^{1-\theta_w} = \int_0^1 W_{it}^{1-\theta_w} di$ according to:

$$w_t^{1-\theta_w} = (1-\alpha_w)\left(w_t^\star\right)^{1-\theta_w} + \alpha_w \pi^{1-\theta_w} \pi_t^{\theta_w - 1} w_{t-1}^{1-\theta_w} \ .$$

This equation yields that $w = w^\star$ in the steady state. Calculating the wage index to the first order yields:

$$\hat{w}_t = (1-\alpha_w)\hat{w}_t^\star + \alpha_w(\hat{w}_{t-1} - \hat{\pi}_t) \ . \tag{G.17}$$

Combine this approximate wage index with the wage setting equation (G.16) calculated to the first order. This yields the equation that describes the evolution of the average real wage:

$$\begin{aligned} \hat{w}_t &= \frac{1}{1+\beta}(\hat{w}_{t-1} - \hat{\pi}_t) - \frac{(1-\alpha_w)(1-\alpha_w\beta)}{\alpha_w(1+\beta)(1+\nu\theta_w)}\hat{\mu}_t^w + \frac{\beta}{1+\beta}E_t[\hat{w}_{t+1} + \hat{\pi}_{t+1}] \\ \hat{\mu}_t^w &= \hat{w}_t - [\nu \hat{L}_t + \sigma^{-1}(\hat{C}_t - g_t)] \ . \end{aligned}$$

G.11 Market clearing conditions and resource constraints

Aggregating the households' budget constraints across households and imposing stock market clearing at $\int_0^1 Z_{it} di = 0$ yields:

$$B_t + P_t(C_t + X_t) = (1 + i_{t-1})B_{t-1} + P_t r_t^K K_{t-1} + (1-\tau_L)\int_0^1 W_{it} L_{it} di + \mathcal{T}_t + D_t \ .$$

Consolidate this budget constraint with the government budget constraint and employ the zero-profit condition of the labor firm to obtain:

$$P_t(C_t + X_t + q_t) = P_t r_t^K K_{t-1} + W_t L_t + D_t$$

Aggregating profits in the intermediate goods sector yields:

$$D_t = \int_0^1 D_{jt}\ dj = P_t Y_t - W_t \int_0^1 L_{jt}\ dj - P_t r_t^K \int_0^1 K_{jt-1}\ dj\ .$$

Capital market clearing requires that $K_{t-1} = \int_0^1 K_{jt-1}\, dj$. Regarding labor services, household i supplies service L_{it} to the labor firm. The labor firm aggregates all imperfectly substitutable labor services L_{it} to L_t. Aggregated labor L_t is sold to intermediate good firms, and market clearing on the labor market between the labor firm and the intermediate good firms requires that $L_t = \int_0^1 L_{jt}\ dj$. Plug capital market clearing and labor market clearing and aggregate profits of intermediate firms into the consolidated budget constraint to obtain the aggregate resource constraint:

$$Y_t = C_t + X_t + q_t\ .$$

Calculating it to the first order yields, with $s_c = c/y, s_x = x/y, s_q = q/y$:

$$\hat{Y}_t = s_c \hat{C}_t + s_x \hat{X}_t + s_q \hat{q}_t\ . \tag{G.18}$$

H Solution of the firm-specific productivity model

This appendix provides details on solving the firm-specific productivity (FIP) model, derived in Appendix G. The nonlinear FIP model comprises the equations (G.1), (G.2), (G.5), (G.7), (G.8), (G.9), (G.14), (G.16), (G.17), and (G.18) plus a specification of monetary policy.

H.1 Steady state

From these equations, I compute the great ratios s_c and s_x in the steady state with $\pi = g$. Variables without a time subscript denote steady state values. The following variables were already determined before:

$$\begin{aligned} p^\star &= n_g^{\frac{1}{\theta-1}} \\ \Delta &= n_g (p^\star)^{-\theta} \\ mc^\star &= \tfrac{\theta-1}{\theta} p^\star \\ r^k &= 1/\beta - (1-\delta_k) \\ i &= \pi/\beta - 1 \\ \Theta &= 1 \; . \end{aligned}$$

Now solve for the great ratios s_x and s_c as follows. Marginal costs of a new firm imply:

$$(w/r^K)^{1/\chi} = mc^\star (r^K)^{-1} \Xi^{-1} A^{1/\chi} \; .$$

All variables on the right-hand side are known. The optimal input mix implies:

$$(K/L)^{1/\chi} = \left(\frac{1-1/\chi}{1/\chi}\right)^{1/\chi} (w/r^K)^{1/\chi} \; .$$

Aggregate technology implies:

$$Y/K = \Delta^{-1} A^{1/\chi} (K/L)^{-1/\chi} \; .$$

The investment equation implies determines the investment share of output:

$$\begin{aligned} s_x &= X/Y \\ &= [1-(1-\delta_K)]K/Y \; . \end{aligned}$$

The share s_q of government spending over output is calibrated. Thus, aggregate accounting yields:

$$s_c = 1 - s_x - s_q \; .$$

H.2 Linearized model

There are 12 endogenous variables, $Y, C, X, K, L, i, \pi, \Theta, r^K, w, mc^\star, \mu^w$, and there are 12 equations in the linearized model. I add ad hoc a cost push shock u_t to the NKPC, a wage markup shock u_{wt} to the wage equation, and a monetary policy shock μ_t to the interest rate rule. Thus, there are seven exogenous shocks: $u, u^w, a, g, q, \epsilon^X, \mu$. These are the same seven shocks that are also employed in Smets and Wouters (2007). Exogenous shocks follow AR(1) processes. For the calibration of parameters, see Section 7.4 in Weber (2011b). The 12 linearized equations are:

$$\hat{C}_t - g_t = E_t(\hat{C}_{t+1} - g_{t+1}) - \sigma(\hat{i}_t - E_t\hat{\pi}_{t+1})$$

$$\hat{\Theta}_t = -\sigma^{-1}E_t(\hat{C}_{t+1} - g_{t+1}) + \sigma^{-1}(\hat{C}_t - g_t) + \frac{r^K}{r^K + (1-\delta_K)}E_t\hat{r}^K_{t+1} + \frac{1-\delta_K}{r^K + (1-\delta_K)}E_t\hat{\Theta}_{t+1}$$

$$(1+\beta)\hat{X}_t = \hat{X}_{t-1} + \beta E_t\hat{X}_{t+1} + \frac{1}{S''}\hat{\Theta}_t + \frac{1}{S''}\hat{\epsilon}^X_t$$

$$\hat{K}_t = (1-\delta_K)\hat{K}_{t-1} + [1-(1-\delta_K)]\hat{\epsilon}^X_t + [1-(1-\delta_K)]\hat{X}_t$$

$$(1+\beta)\hat{w}_t = \hat{w}_{t-1} - \hat{\pi}_t - \frac{(1-\alpha_w)(1-\alpha_w\beta)}{\alpha_w(1+\nu\theta_w)}\hat{\mu}^w_t + \beta E_t[\hat{w}_{t+1} + \hat{\pi}_{t+1}] + u^w_t$$

$$\hat{\mu}^w_t = \hat{w}_t - [\nu\hat{L}_t + \sigma^{-1}(\hat{C}_t - g_t)]$$

$$\hat{\pi}_t = \frac{(1-\kappa g^{(\theta-1)})(1-\kappa\beta g^{\theta-1})}{\kappa g^{(\theta-1)}}\widehat{mc}^\star_t + \beta E_t[\hat{\pi}_{t+1}] + u_t$$

$$\widehat{mc}^\star_t = (1/\chi)(\hat{w}_t - a_t) + (1-1/\chi)\hat{r}^K_t$$

$$\hat{r}^K_t + \hat{K}_{t-1} = \hat{w}_t + \hat{L}_t$$

$$\hat{Y}_t = s_c\hat{C}_t + s_x\hat{X}_t + s_q\hat{q}_t$$

$$\hat{Y}_t = a_t + (1/\chi)\hat{L}_t + (1-1/\chi)\hat{K}_{t-1}$$

$$\hat{i}_t = \phi_{i1}\hat{i}_{t-1} + \phi_\pi\hat{\pi}_t + \phi_Y\hat{Y}_t + \phi^m_\pi\hat{\pi}^m_t + \phi^m_Y\hat{Y}^m_t + \mu_t$$

H.3 Measurement equations

Proposition 7 in Weber (2011b) establishes that $\hat{\pi}_t^m = a_\rho(L)\hat{\pi}_t$ for the FIP model. The lag polynomial $a_\rho(L)$ is described in terms of primitive parameters in Proposition 7. Moreover, Proposition 8 in Weber (2011b) establishes that $\hat{P}_t - \hat{P}_{t,t-1}^m = b_\rho(L)\hat{\pi}_t$. The lag polynomial $b_\rho(L)$ is described in terms of primitive parameters in Proposition 8. Measured output is defined as $\hat{Y}_t^m = \hat{Y}_t + \hat{P}_t - \hat{P}_{t,t-1}^m$. Define the auxiliary variable $PD_t = \hat{P}_t - \hat{P}_{t,t-1}^m$ to bring the lag polynomials into a recursive form:

$$\hat{\pi}_t^m = \kappa\gamma\rho\hat{\pi}_{t-1}^m + \left(\frac{1-\alpha}{1-\kappa\rho}\right)\left(\hat{\pi}_t - (1-\delta)\kappa\gamma\rho^2\hat{\pi}_{t-1}\right)$$

$$\hat{Y}_t^m = \hat{Y}_t + PD_t \tag{H.1}$$

$$PD_t = \frac{\alpha(1-(1-\delta)\rho)}{1-\kappa\rho}\hat{\pi}_t + \kappa\gamma\rho PD_{t-1} \ .$$

When mapping the model to the data, I also account for the fact that, in order to obtain the real wage, the nominal wage rate in the data is denominated by the measured price level. Accordingly, define the measured real wage as $w_t^m = W_t/P_{t,t-1}^m$ and rearrange this definition as $w_t^m = w_t P_t/P_{t,t-1}^m$. Calculated to the first order, obtain $\hat{w}_t^m = \hat{w}_t + b_\rho(L)\hat{\pi}_t$. Analog to measured output, rearrange the measured real wage according to:

$$\hat{w}_t^m \ = \ \hat{w}_t + PD_t \ . \tag{H.2}$$

Measurement implies that the four measured variables $\hat{\pi}_t^m, \hat{Y}_t^m, PD_t, \hat{w}_t^m$ and the four measurement equations (H.1) and (H.2) must be added to the linearized model that is summarized in Appendix H.2.

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