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Intermediates Trade and Knowledge Flows

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Abstract

This paper introduces a novel mechanism by emphasizing benefits for firms through participation in buyer networks among firms that source the same locally produced inputs. In a first step, we utilize register-based data from Denmark to generate a firm-specific buyer network variable which relies on firms' industrial input structures and imports. Utilizing this proxy we provide evidence of cost savings from network participation, as larger buyer networks reduce firms' input demand. Subsequently, we develop a trade model incorporating vertical linkages and introduce network effects that result in savings in intermediate costs. Our theory posits that the magnitude of these savings may be associated with the effectiveness of knowledge transmission among network participants. Consequently, firms operating in regions with efficient knowledge transmission networks may realize greater savings in intermediate input costs, leading to increased profits from local and export sales. In a last step, we provide empirical evidence supporting our theoretical predictions by demonstrating the positive impact of buyer networks based on relationship-specific products on domestic firm revenues.

Keywords:New Trade Theory, Vertical Linkages, Network Effects

JEL Code: F12, F15, R12

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1 Introduction

In this paper, we introduce a novel mechanism to global and local supply chains, by highlighting the potential for firms to achieve cost savings by participating in buyer networks. Specifically, we demonstrate that firms can realize significant savings in intermediate inputs when operating within a local buyer network structure. We propose that the extent of these savings is closely linked to the efficacy of knowledge transmission among network participants. In regions (or countries) where knowledge dissemination within the network is efficient, firms stand to benefit from larger savings in their expenditures for intermediates. This underscores the importance of considering not only the physical flow of goods but also the flow of knowledge within supply chains. Furthermore, we emphasize the implications of these findings for firms' profitability and market competitiveness. By reducing their expenditures on intermediates, firms can enhance their profit margins and potentially lower their prices, thereby gaining a competitive edge in both local and export markets. Overall, our study sheds light on the complex dynamics of supply chain networks and emphasizes the importance of fostering efficient knowledge exchange within the network, thereby highlighting the role for policy makers and the need for industrial policies aimed at reducing the costs of local production networks.

In a first step, we make use of register based data from Statistics Denmark. We combine information on the input structure of industrial production with imports, to generate a firm-specific buyer network proxy based on the number of firms within a firms region (and industry) that make use of the same locally produced inputs in their production process. We use this buyer-network measure, to investigate how it affects expenses for intermediate inputs. Exploiting within firm-variation and controlling for shocks that are specific to region-industry pairs within a year, we reveal a negative impact of our network proxy on input demand. Our results are robust to different versions of our buyer network proxy, alternative specifications, different regional divisions, and different clustering. Overall, we provide novel evidence which supports the underlying assumption in our theoretical model, namely that firms can gain from being part of a network, as it allows them to reduce their intermediate input demand.

Subsequently, we develop a trade model incorporating vertical linkages that are present only in fixed costs in following the short-run version of Ottaviano and Robert-Nicoud

(2006) new economic geography model. Our theory posits that the magnitude of savings may be associated with the effectiveness of knowledge transmission among network participants. Consequently, firms operating in regions with efficient knowledge transmission networks may realize greater savings in intermediate input costs, leading to increased profits from local and export sales.

Finally, we test specific predictions of our theory. Our novel theory highlights a distinct effect of the network size on firm revenues, depending on the cost of a network. To proxy such costs, we make use of a recent product-level index of relationship stickiness, which is provided by Martin et al. (2023) using firm-to-firm relationship duration data. We argue that one expects the cost of a production network to be lower for relationship specific inputs, as it is easier to share knowledge among those inputs in contrast to relationship unspecific inputs. We therefore compute our buyer network measures separately for relationship specific and unspecific products. By doing so we reveal that networks, based on relationship specific products, are increasing domestic revenues, while networks based on non-relationship specific products, are not affecting firm sales. This is intuitive and in line with our model predictions, as we expect the cost of a production network to be lower for relationship specific inputs.

In the present paper, the network of production is exogenously given as we do not have buyer-supplier data that take into account explicitly the relationship between sellers and buyers. Therefore, our approach falls within the stream of the literature where extensive margin of firm-to-firm linkages are considered exogenous, and, consequently, the network structure is taken as given. This is in line with the work by König et al. (2019), who consider a framework in which firms that are part of a network determine their production level and their R&D effort by treating the network as exogenously given. Specifically, we assume that firms have a given number of firms in their network and each of them determines its demand of manufacturing inputs - demand that is falling in increases in the use of intermediates by other firms in the same network. This allow us to capture the fact that firms may benefit of 'processes of transmission of knowledge' in their local networks that allow them to reduce their expenditures on intermediate inputs.

The type of network we consider in this work is different from that analyzed in other papers on 'production networks', which, nevertheless, may present an exogenous structure. For instance, Dhyne et al. (2023, p.3) point out that "[b]uilding on the seminal work by Hulten (1978), several works aim to derive the importance of networks for understanding aggregate outcomes arising from nonlinearities (e.g. Baqaee and Farhi, 2019) or distortions (e.g. Baqaee and Farhi, 2020). Baqaee and Farhi (2024) study the transmission of trade and technology shocks in a multi-country and multi-industry general equilibrium model without fixed costs. Huneeus et al. (2021) and Dhyne et al. (2022) take the extensive margin of the domestic production network as given when studying worker-level outcomes in response to trade shocks." Dhyne et al. (2023, p.3) highlight that, making use of an alternative structure with endogenous networks, their "quantitative findings imply that a tractable and well-behaved endogenous network formation model can lead to aggregate real income changes that are very similar to the ones obtained from models that take the extensive margin of firm-to-firm linkages as exogenous." Thus, since our main research question is to understand how knowledge transmission among firms in regional/national networks may affect firms' expenditures in intermediates, we rely on a model taking the number of firms in a network as exogenously given. Moreover, as in other models that consider endogenous production networks with fixed costs (i.e. Huneeus (2018), Lim (2018), Bernard et al. (2022), we have a continuum of firms.

The rest of the paper is organized as follows. Section 2 is introducing the different data sources we employ throughout our empirical analysis. In Section 3, we develop and introduce a novel buyer network measure and relate it to firms demand for intermediate inputs. Section 4 develops a new theoretical framework with the aim to analyze the implications of savings in intermediate costs incurred by firms generated by buyer network effects due, for instance, to better transmission of knowledge. Section 5 analyzes the implications of network effects for firms profits and sales. Finally, in Section 6 we test some of the model predictions, while Section 7 concludes.

2 Data

We combine different register data from Statistics Denmark for our empirical analysis across the years 2000 to 2018. General information on firms, such as industry classification and location, and information on employment, revenues, stock of assets or value added is obtained from the register FIRM. Industries are classified according to the 6-digit Danish version of EU's nomenclature (NACE). The first four digits refer to NACE rev. 2, while the last two represent the Danish subdivision. We harmonize industry codes over time and exclude companies from the public sector (i.e., utility services, public administration and defense, education, health services, culture and entertainment services). In our descriptive and empirical analysis, we will distinguish between 69 (470) 2(4)-digit NACE rev. 2 classification industries.

Based on firms municipality code, we assign firms to different regions within Denmark. Specifically, we divide Denmark into five main regions following the NUTS 2 classification, i.e., Capital region of Denmark, Region Zealand, Region of Southern Denmark, Central Denmark Region, and North Denmark Region. As an alternative, we distinguish between 11 provinces at the NUTS 3 level of aggregation: Copenhagen City, Copenhagen Surrounding, North Zealand, East Zealand, West & South Zealand, Bornholm, Funen, South Jutland, East Jutland, West Jutland, and North Jutland.

Information about the input structure of industrial production is obtained from the register VARK. Inputs are goods and services regardless of whether they are imported or purchased domestically and goods include raw materials, components, parts and auxiliaries used in the processing and production. The data are divided by detailed industrial groups (i.e., NACE-groups). Specifically, the inputs have a 6-digit code, where the first four digits are equivalent to the CN-code.[1] For example, inputs within the group "73 – Goods of iron and steel", could be twisted wire, cables, ropes, braided bands, straps, or radiators for central heating. Inputs within the group of "84 – 'Boilers; machines and apparatus and mechanical tools" are, for example, steam turbines, parts for washing machines, or parts for machines/apparatus for processing textiles.

Likely, as Denmark is a small open economy which is importing parts and components from abroad, we combine information coming from the VARK register, with trade data. Specifically, we can combine it with the imports of firms at the same 4-digit level. Information about imports are obtained from the register UHDI, which provides data on firms' imported (and exported) products at the Combined Nomenclature (CN) 8-digit product level. We aggregate firm-level imports up to the 4-digit level, and thus have the same classification in VARK and UHDI. While the survey on Manufacturers' Purchases of Goods and Services does not distinguish between domestic and imported inputs, the

[1] For a detailed list see https://www.dst.dk/da/Indberet/oplysningssider/industriens_kob_varer/industriens-koeb-af-varer-og-tjenester.

combination with the import data allows us to check if expenses on 4-digit products exceed the one from reported imports, or if the input is only imported and thus not produced in Denmark. For our network measures that we introduce below, we exclude those goods and services that are purely imported.[2] Based on the information provided in VARK and the UHDI register, we compute total expenses across all 4-digit purchases and the number of distinct 4-digit products, both at the firm-year level. As can be seen in Table 1, on average firms buy 16.65 different goods and services.

Finally, we make use of the VARS register, the Danish version of the European PRODCOM survey, which provides information on firms' sales at the product level. VARS covers firms in the manufacturing and raw material extraction sectors with 10 or more employees. Even though it just represents a subset of firms available in the FIRM, VARK and UHDI register, it allows us to compute domestic sales of firms. Specifically, subtracting export sales in UHDI from total sales in VARS, we obtain domestic sales of firms, which is used in our empirical analysis in Section 6. Summary statistics of the main variables are provided in Table 1.

Table 1: Summary Statistics

Variable	Observations	Mean	Std.Dev.
(log) total revenues	130,992	17.092	1.692
Employment	130,992	73.150	551.918
(log) value added	130,992	15.724	1.710
(log) wage-bill	130,992	15.395	1.747
(log) assets per worker	130,992	11.966	1.535
Indic. Exporting	130,992	0.680	0.466
Number of inputs	130,992	16.651	26.958
(log) expenses on inputs	130,992	14.225	3.209
(log) domestic revenues	18,909	17.417	1.595

Notes: The table reports means and standard deviations on firm-year observations over the period 2000 - 2018 based on data from Statistics Denmark. Exporting is an indicator variable equal to one if a firm is exporting. Number of (expenses on) inputs measures the number of (overall expenses across all) different 4-digit goods and services a firm is purchasing.

[2] We do so, as our theory presented below, is highlighting the role of networks from sourcing *locally* produced intermediates.

3 Facts on Local Input Buyer Networks in Denmark

Our theoretical framework presented below in Section 4 and 5, highlights the implications of vertical linkages on the production side. Specifically, the model introduces positive network effects, that depend among other things on the number of firms within a region that purchase the same locally produced intermediate input. Consequently, there is a size effect of the regional network, such that aggregate purchases of a firm are decreasing in the size of the network. Before we turn towards the theory, we make use of the Danish register based data introduced in Section 2, to motivate this crucial model ingredient.

3.1 A Measure for Local Input Buyer Networks

To obtain a proxy for a firms' local network, we compute in a first step the number of firms that report positive expenses on the same 4-digit inputs from the VARK register within different cells. We distinguish between 4 different cells, by looking at the number of firms that report positive expenses for the same input within (i) 5 regions in Denmark; (ii) 5 regions $\times$ 69 industries; (iii) 11 regions; and (iv) 11 regions $\times$ 69 industries. Importantly, we restrict the potential network effects to those products that are (at least to some extend also) produced in Denmark, and thus not only imported from abroad.[3] This gives us a firm-product specific network proxy, under the idea that there exist vertical linkages among firms purchasing the same domestically produced input, such as knowledge spillovers among the same input users within a region (and industry). As firms report to use on average more than 16 different inputs (see above), we take in a second step the average across the different inputs to obtain a firm-specific local input user network measure. Furthermore, we compute a similar measure, considering Denmark as one region. For this measure, every firm within Denmark that purchases the same input is considered to be part of the network, irrespective of the location.

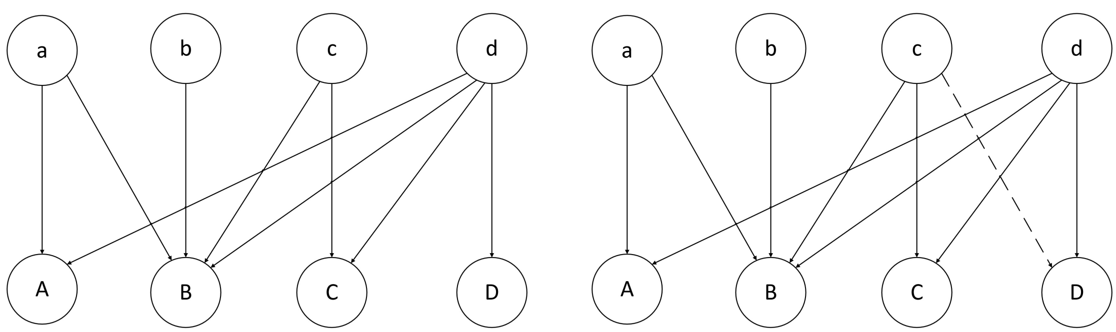
An illustrative example is provided in Figure 1, that provides two different scenarios where different firms (A, B, C, D) can buy inputs (a, b, c, d). In scenario I, inputs a and d are bought by 2 and 4 firms, respectively. Thus, our network proxy for firm A that is using these two inputs would be 3 (=(2+4)/2). In a similar way, we can compute the network

[3] Notably, only a very small share of inputs is solely produced in Denmark, and thus not imported. Restricting the analysis to those only locally produced is thus not feasible.

proxies under scenario I for firm B, C, and D as 2.25 (=(2+1+2+4)/4), 3 (=(2+4)/2), and 4 (=4/1), accordingly. Under scenario II, firm D now also purchases input c. As firm B and C are also using this input, they benefit, as captured by an increase in the network proxy to 2.5 and 3.5, respectively. Thereby, as firm C is using relatively fewer inputs as firm B, it is benefiting more from the fact that firm D starts to buy input c. Put differently, how firms are affected by changes in the input purchases of other firms in a local area, also depends on the overall number of their inputs. Contrary, firm A remains unaffected, as it is not making use of this specific input in the production process, i.e., knowledge spillovers are limited to firms with vertical linkages.

Figure 1: *Example of Input User Network*

(a) Scenario I (b) Scenario II



Summary statistics on our different network proxies are provided in Table 2. When distinguishing between 5 regions, the average number of vertical linkages across the different inputs of a firm is 4.57. Distinguishing between more regions in Denmark, we mechanically get a lower number (2.33), while it is higher if we consider Denmark as one region (18.35). Similar, the proxy numbers are way smaller, if we only consider network effects among firms within the same region *and* industry. In that case, we restrict vertical linkages to those firms that also operate in the same industry.

In Figure 2, we plot the kernel densities of the network proxy based on Denmark as one region (Panel a) and when distinguishing between the 5 different NUTS-2 regions (Panels b-f).[4] Restricting vertical linkages and, thus, network effects to the regional level, reduces the likelihood of high values of our network proxy, especially in less populated

[4] Due to security and confidentiality regulations from Denmark Statistics, we cannot plot the exact distribution (i.e., histogram), but only the approximation for the shapes of the distributions.

Table 2: Summary Statistics - Input Buyer Networks

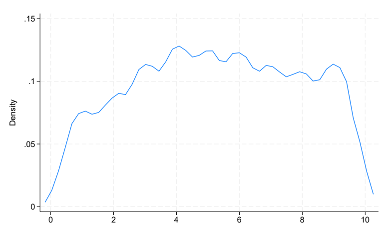
Variable	Observations	Mean	Std.Dev.
Input Buyer Network: 5-reg.	130,992	4.570	4.464
Input Buyer Network: 11-reg.	130,992	2.328	2.456
Input Buyer Network: Denmark	130,992	18.346	18.160
Input Buyer Network: 5-reg. & 69 Ind.	130,992	0.2889	0.833
Input Buyer Network: 11-reg. & 69 Ind.	130,992	0.179	0.547
Input Buyer Network: Denmark & 69 Ind.	130,992	1.015	2.866

Notes: The table reports means and standard deviations on firm-year observations over the period 2000 - 2018 based on data from Statistics Denmark. Input Buyer Network measures for the different cells (5 NUTS-2 regions; 11 NUTS-3 regions; Denmark; 5 NUTS-2 regions and 69 2-digit industries; 11 NUTS-3 regions and 69 2-digit industries; Denmark and 69 2-digit industries) are obtained by first, computing for each input a firm is using the number of firms within a cell that purchase the same input, and secondly, take the average of this number across the inputs a firm is purchasing.

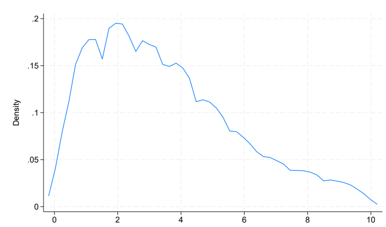
regions, such as Zealand or North Denmark. As we show in a next step, network effects are limited to vertical linkages at the regional level.

Figure 2: *Input Buyer Network Distribution in Denmark*

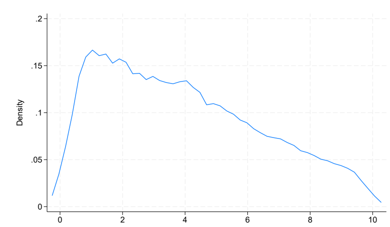
(a) Denmark



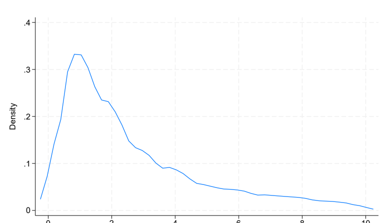
(b) Capital Region



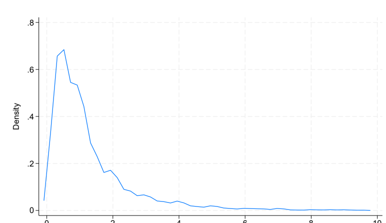
(c) Central Denmark



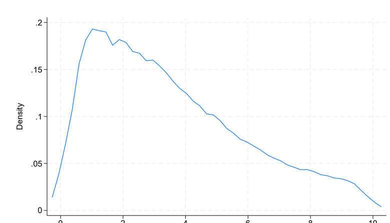
(d) North Denmark



(e) Zealand



(f) Southern Denmark



Notes: The figure plots the kernel densities of the network proxy based on Denmark as one region (Panel a) and when distinguishing between the 5 different NUTS-2 regions (Panels b-f).

3.2 Network Size and Demand for Inputs

We use our proxy for the size of a firms' network, to investigate how it affects the (log of) expenses for inputs. Specifically, we estimate different variants of the following equation

$$\text{input}_{it} = \beta \text{Network}_{it} + \gamma \mathbf{X}_{it} + \mu_i + \mu_{trs} + \epsilon_{it}, \tag{1}$$

where Network are different variants of our input buyer network measures defined and introduced above, and $\mathbf{X}_{it}$ are time varying controls. Specifically, we include the (log of) value added per worker, (log) employment, an indicator for exporting, the (log of) number of distinct 4-digit inputs, the (log) wage bill, and the (log) assets per worker as controls.[5] Furthermore, we include firm fixed effects μ_i and year-region-industry fixed effects μ_{trs}, and ϵ_{it} is an error term with zero conditional mean. Notably, by including year-region-industry fixed effects, we control for shocks that are specific to a region and industry within a year, such as import competition or the emergence of a new input that all other firms in the same region (and industry) use. These fixed effects also control for exit or entry of new firms within a industry and thus measures of market concentration such as the Herfindahl-Hirschman Index. The parameter of interest is β, capturing the impact of an increase in the (log of) our network measure on a firms' input demand. Regressions results for our different network proxies are reported in Table 3 and Table 4.

All columns in Table 3, report a negative impact of our network proxy on a firm's demand for inputs, which is statistically significant different from zero. According to the estimates reported in column (2), a 10% increase in the number of firms that buy the same inputs within a region, reduces demand for inputs by around 0.6%. The effect is also present, if we add as additional control the size of the network, if one would not distinguish between the five regions in Denmark. As the estimates in column (3) reveal, it is the local network that affects demand for inputs, not vertical linkages across all firms in Denmark. As estimates in columns (4) to (6) reveal, the effects are of similar size, if we distinguish between 11 regions in Denmark. In Table 4, we present similar results, where we now restrict vertical linkages only to firms that are active in the same 2-digit

[5] Note, that our sample is via construction of our network measure restricted to firms that import. The reason is that our network measure is obtained from information on expenses for (domestic or foreign) inputs, and all firms that a part of the register VARK, are also importing other inputs from abroad, according to the UHDI register.

Table 3: Network size and firms aggregate demand for inputs

	Dependent variable: (log) expenses on inputs					
	(1)	(2)	(3)	(4)	(5)	(6)
(log) IBN 5-reg.	-0.0671***	-0.0606***	-0.0524**			
	(0.00853)	(0.00843)	(0.0211)			
(log) IBN 11-reg.				-0.0615***	-0.0562***	-0.0290*
				(0.00839)	(0.00850)	(0.0156)
(log) IBN Denmark			-0.00896			-0.0328**
			(0.0212)			(0.0160)
Observations	115526	115526	115526	109265	109265	109265
adj. R^2	0.879	0.885	0.885	0.880	0.887	0.887
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Time-varying controls	Yes	Yes	Yes	Yes	Yes	Yes
Year×region FE	Yes	Yes	Yes	Yes	Yes	Yes
Year×industry×region FE	No	Yes	Yes	No	Yes	Yes

Notes: The sample covers firms in the private sector for the years 2000 to 2018. IBN computes for each input a firm is using the number of firms within a cell that purchase the same input, and secondly, take the average of this number across the inputs a firm is purchasing. Cells are 5 NUTS-2 regions, 11 NUTS-3 regions, or Denmark as a whole. Time-varying controls include (log) value added per worker, (log) employment, an indicator variable for exporting, (log) number of distinct 4-digit inputs, (log) wage bill, and (log) assets per worker. Industry classification follows the 4-digit NACE rev. 2 version. Standard errors in parentheses are clustered at the firm level. ***, **, * denote significance level at 1%, 5% and 10% respectively.

industry. Compared to the estimates in Table 3, the magnitude of our network effects are now larger. This is intuitive, as one would expect stronger knowledge spillovers among firms that are active in the same industry.

In the Appendix, we also show that these results are robust to different specifications and sample restrictions. In columns (1) and (2) of Appendix Table A1, we show that results are robust to clustering standard errors at the region level. In columns (3) and (4) we replace our network measure by first computing for each input a firm is using the number of firms within a cell that purchase the same input, and secondly, take the *median* of this number across the inputs a firm is purchasing. Finally, in column (5) and (6), we restrict the sample to those firms in the manufacturing sector, thereby excluding for example wholesalers, that might import and just re-sale many goods. One concern could be that our network proxy just reflects a negative externality in a situation where the availability of inputs is limited to firms. If other firms within a region (and industry) start to buy the same input, it mechanically reduces a firms demand on inputs, as fewer

Table 4: Network size and firms aggregate demand for inputs

	Dependent variable: (log) expenses on inputs					
	(1)	(2)	(3)	(4)	(5)	(6)
(log) IBN 5-reg. & 69 Ind.	-0.0885***	-0.0734***	-0.115***			
	(0.0199)	(0.0225)	(0.0359)			
(log) IBN 11-reg. & 69 Ind.				-0.132***	-0.127***	-0.144***
				(0.0255)	(0.0298)	(0.0386)
(log) IBN Denmark & 69 Ind.			0.0523			0.0237
			(0.0383)			(0.0410)
Observations	26739	26739	26739	19695	19695	19695
adj. R^2	0.884	0.890	0.890	0.882	0.889	0.889
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Time-varying controls	Yes	Yes	Yes	Yes	Yes	Yes
Year×region FE	Yes	Yes	Yes	Yes	Yes	Yes
Year×industry×region FE	No	Yes	Yes	No	Yes	Yes

Notes: The sample covers firms in the private sector for the years 2000 to 2018. IBN computes for each input a firm is using the number of firms within a cell that purchase the same input, and secondly, take the average of this number across the inputs a firm is purchasing. Cells are 5 NUTS-2 regions, 11 NUTS-3 regions, or Denmark as a whole, times 69- 2-digit NACE rev. 2 industries. Time-varying controls include (log) value added per worker, (log) employment, an indicator variable for exporting, (log) number of distinct 4-digit inputs, (log) wage bill, and (log) assets per worker. Industry classification follows the 4-digit NACE rev. 2 version. Standard errors in parentheses are clustered at the firm level. ***, **, * denote significance level at 1%, 5% and 10% respectively.

might be available. This effect might be even more pronounced, if a firm is only using few inputs, and thus the network proxy is based only on a few number of inputs. To check for differences among firms that use few or many inputs, we split the sample according to the median number of inputs firms are using. Results are presented in Appendix Table A2 for the four different network proxies. However, no clear pattern emerges. Looking at network measures based on a region (and industry), it seems to be present for firms that use few (many) inputs.

Taking stock, results presented here provide strong evidence in favor of our underlying assumption that firms can gain from being part of a network, as it allows them to reduce their intermediate input demand. This assumption is one of the crucial inputs to our theory, presented in the next Section.

4 Theory

We consider a world of two countries/regions denoted by r and v, with $r, v = 1, 2$, and two sectors, manufactures M and agriculture (traditional sector) A. Both sectors employ labour, and, in addition, firms in the manufacturing sector use the output produced by other firms as intermediate inputs.

Workers are internationally immobile and L_W is world's population. Labour endowments in country r and country v are, respectively, given by $L_r = \lambda_r L_W$ and $L_v = (1 - \lambda_r) L_W$ workers, with λ_r denoting the share of workers living in country r. Clearly, the two countries are symmetric in labour endowments only when $\lambda_r = 1/2$.

There are also other dimensions across which the two countries can differ, as, for instance, the given number (mass) of firms producing in each country that corresponds, respectively, to N_r and N_v in country r and country v. Consequently, given that the manufacturing varieties are bought not only by consumers (workers), but also by other firms using them as intermediate inputs, the two countries are potentially asymmetric in their market size.

Goods produced in both sectors are traded; however, while trade in the agriculture sector is frictionless, trade in the manufacturing sector is inhibited by iceberg trade costs and a firm producing in r must ship $\tau_{rv} > 1$ units of its output to sell abroad, in the other country v, one unit of the same good.

To model 'network effects' on the production side, we modify the Ottaviano and Robert-Nicoud (2006) model with vertical linkages (VL) only in fixed costs by introducing positive 'network' effects that reduce the fixed cost of production of a firm involved in a network.[6] Moreover, as the main aim of this paper is to understand the interaction that exists between trade in goods and knowledge flows, we consider the case in which the mass of firms in each country is given and this corresponds to the analysis of the so-called short-run equilibrium in Ottaviano and Robert-Nicoud (2006). In so doing, we adopt a

[6] Specifically, if we assume $\delta_r = \rho_r = 0$ for $r = 1, 2$ in the expressions in the following paragraphs, we get the supply side of the New Economic Geography (NEG) model by Ottaviano and Robert-Nicoud (2006) with vertical likages only in fixed costs (i.e. $\alpha = 0$ and $\mu \neq 0$ in their setup). More precisely, in this paper we add network effects among firms in Ottaviano and Robert-Nicoud (2006) model with vertical linkages which apply only to fixed costs (which is a "footloose entrepreneur model with vertical linkages", FEVL). It is not suitable for us to add vertical linkages also in variable costs since, as the two models are isomorphic, Ottaviano and Robert-Nicoud (2006) recommend to work with $\alpha = 0$, that is with vertical linkages considered only for fixed costs.

macroeconomic approach to analyze the role of 'production' networks in a monopolistic competition model since we focus on how countries' or regions' macroeconomic conditions are determined given their production networks.

4.1 The production side

The technology in the agricultural sector is such that one unit of labour is required to produce one unit of the agricultural/traditional good, and we assume that the agricultural good is the numeraire of the model. This, together with the assumption that the traditional good is freely traded, implies that the wage of workers is equal to one in both countries ($w_r = w_v = 1$).

The manufacturing sector produces a continuum of horizontally differentiated varieties, indexed by s, under increasing returns to scale and Dixit-Stiglitz monopolistic competition. Specifically, each manufacturing firm produces a variety of the differentiated good that is not only consumed, but it is also used as intermediates by other manufacturing firms. Manufacturing firms use as inputs both labour and intermediates produced by other domestic and foreign manufacturing firms. These manufacturing firms can be considered as 'innovative' firms in the 'modern' sector of the economy.[7] In particular, N_W is the total number (mass) of firms producing differentiated varieties in the two countries and it is equal to the number of firms producing in r (N_r) plus the number of firms producing in v (N_v), that is $N_W = N_r + N_v$, with $r, v = 1, 2$.

Each manufacturing variety is produced by a monopolistically competitive firm with increasing returns to scale employing both labour and intermediate goods. Specifically, to produce $x_r(s)$ units of variety s the firm located in r incurs in a fixed input requirement of $\bar{F}$ units of a composite of labour and intermediate (manufacturing) goods independently of the production level, and employs $a_M x_r(s)$ units of workers. Thus, if w_r is the unit wage in r, $w_r a_M x_r(s)$ is the variable cost. The composite input is a Cobb-Douglas composite of manufacturing goods (a share μ) and labour (a share $1 - \mu$). The manufacturing input is itself a constant elasticity of substitution (CES) composite of all the varieties of the manufacturing good available in the world and $\sigma > 1$ is the elasticity of substitution between any two pair of varieties.

[7] See the chapter by Audretsch and Feldman (2004) in the Handbook of Regional and Urban Economics.

'Local or domestic' network effects are modelled as follows. We consider the case in which firms in a country/region have a specific number or 'mass' of links with other firms producing within the same country/region and that their demand of manufacturing inputs is falling in the use of intermediates by other firms with which they are connected in the network.

These networks are costly, since, for instance, networks characterized by a larger number of firms require higher coordination costs. Thus, we assume that the cost for each firm participating in a network increases in the 'mass' of the links that the firm has in the network (because, for instance, of higher coordination costs). However, ceteris paribus, increasing the 'mass' of firms with which the firm has a link in the network allows the firm to rise the endogenous savings due to their participation to a larger network. As in the work by König et al. (2019), who consider a framework in which firms decide about their production and R&D effort by treating the network as exogenously given, we also assume that the network is exogenously given. Specifically, we assume that, for a given number or 'mass' of their links with other firms, that is for a given network structure, each firm determines its demand of manufacturing inputs - demand that is falling in increases in the use of intermediates by other firms in the network. Among other things, this capture the fact that firms may benefit of 'processes of transmission of knowledge' in their local networks that allow them to save on their expenditures for intermediates.

Let us consider $\Lambda_r(s)$ as the 'given' set of firms that are in the network of production of firm s in country r . Firm's demand of manufacturing inputs decreases if there is a local increase in the average manufacturing input used locally by all firms $i \in \Lambda_r(s)$ with which the firm interacts in the network, which is given by $\int_{\Lambda_r(s)} X_r(i)di / \int_{\Lambda_r(s)} di$, multiplied by the size effect of the network $S(\Omega(\Lambda_r(s)))$. We assume that $S()$ is an arbitrary non decreasing function taking values between 0 and $\Omega(\Lambda_r(i))$, with $\Omega(\Lambda_r(i))$ denoting the Lebesgue measure of the set of firms in the network. To model this, we assume that the fixed 'input' $\bar{F}$ is a composite input that is Cobb-Douglas in labor $L_r(s)$ and in the composite of manufactured goods Φ, with shares $1-\mu$ and μ respectively, given by

$$\bar{F} = F(L_r(s), \Phi\left(X_r(s), X_{-sr}, \Lambda_r(s)\right)) = \frac{\left[L_r^f(s)\right]^{1-\mu} \left[\Phi\left(X_r(s), X_{-sr}, \Lambda_r(s)\right)\right]^{\mu}}{(1-\mu)^{1-\mu}\mu^{\mu}} \tag{2}$$

with

$$\Phi\left(X_r(s), X_{-sr}, \Lambda_r(s)\right) \equiv X_r(s) + \delta_r S(\Omega(\Lambda_r(s))) \frac{X_{-sr}}{\int_{\Lambda_r(s)} di} \tag{3}$$

where the positive parameter δ_r describes the magnitude of the 'externality' effect that reduces the use of own intermediates by firm s because this firm benefits from the use of intermediates by other domestic firms in its local network given by the term in the numerator of (3)

$$X_{-sr} \equiv \int_{\Lambda_r(s)} X_r(i) di.$$

The value of the parameter δ_r for country r can potentially differ from that of the other country δ_v, implying that the two countries can be characterized by a 'different efficiency in the process of transmission of knowledge' in their local networks. The mass of firms which are in the network of firms s in country r is given by $\int_{\Lambda_r(s)} di = \Omega(\Lambda_r(s))$; and $\int_{\Lambda_r(s)} X_r(i)di$ is the total manufacturing input locally used by all other firms with which firm s interacts in its network. As usual, the composite of all varieties of the manufacturing good available in the location r is defined as follows

$$X_r(s) \equiv \left(\int_{j \in N} d_r^s(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the elasticity of substitution between any two pair of varieties.

Making use of the definition of X_{-sr}, the definition of Φ in expression (3) can be rewritten as follows

$$\Phi\left(X_r(s), X_{-sr}, \Lambda_r(s)\right) = X_r(s) + \delta_r S(\Omega(\Lambda_r(s))) \frac{\int_{\Lambda_r(s)} X_r(i)di}{\int_{\Lambda_r(s)} di}. \tag{4}$$

Expression (2) implies that each firm s uses $\left[\frac{\mu^\mu (1-\mu)^{1-\mu} \bar{F}}{(\Phi(X_r(s), X_{-sr}, \Lambda_r(s)))^\mu}\right]^{\frac{1}{1-\mu}}$ units of workers for the fixed input requirement $\bar{F}$, and $a_M x_r(s)$ units of workers for the production of $x_r(s)$ units of variety s. Therefore, the total labour required by firm s is given by

$$L_r(s) = \left[\frac{\mu^\mu \left(1-\mu\right)^{1-\mu} \bar{F}}{\left(\Phi\left(X_r(s), X_{-sr}, \Lambda_r(s)\right)\right)^\mu}\right]^{\frac{1}{1-\mu}} + a_M x_r(s),$$

with the implied production function representing the technology used to produce $x_r(s)$ given by

$$x_r(s) = \frac{1}{a_M}\left\{L_r(s) - \left[\frac{\mu^\mu (1-\mu)^{1-\mu}\bar{F}}{(\Phi(X_r(s), X_{-sr}, \Lambda_r(s)))^\mu}\right]^{\frac{1}{1-\mu}}\right\}. \tag{5}$$

Hence, each firm s minimizes its production cost given the production technology (5), with $\Phi(X_r(s), X_{-sr}, \Lambda_r(s))$ defined as in (4). As it is shown in the Theoretical Appendix, the ensuing labour $L_r(s)$ and intermediates $X_r(s)$ demands are affected by the presence of network effects.

Before discussing the properties of firms' labour and intermediates demands, let us define the *potential value* of 'real savings in terms of intermediates' for firm s in country r when it is part of a network as follows

$$G_r(s) \equiv S(\Omega(\Lambda_r(s)))\frac{\int_{\Lambda_r(s)} X_r(i)di}{\Omega(\Lambda_r(s))} \tag{6}$$

The expression for $G_r(s)$ depends on how the size effect of the network $S(\Omega(\Lambda_r(s)))$ is specified. In the case of *the additive specification* that we will consider, $S(\Omega(\Lambda_r(i))) = \Omega(\Lambda_r(i)) = \Lambda_r(s)$, and it allows to capture the fact that the potential real savings (of fixed production costs) $G_r(s) = \int_{\Lambda_r(s)} X_r(i)di$ increase for firm s if, ceteris paribus, the mass of its links in its local network is larger.

The 'total production cost' of a firm producing variety s in country r is given by the following expression

$$TC_r(s) = w_r a_M x_r(s) + P_r^\mu w_r^{1-\mu}\bar{F} - \delta_r P_r G_r(s), \tag{7}$$

where P_r is the price index of manufactured variety in country r, which is defined as $P_r \equiv \left(\int\limits_{j\in N} p_r(j)^{1-\sigma}dj\right)^{\frac{1}{1-\sigma}}$. In the Theoretical Appendix it is explained how expression (7) is derived. Here, it is key to notice that the term $-\delta_r P_r G_r(s)$ in (7) describes the impact on total production cost of firm s of network effects in production, and it disappears when $\delta_r = 0$. This term represents the effective 'savings of the fixed production costs' that firm s has from taking part in its network; savings that are larger, the larger the value of δ_r is. We assume that, besides other technological factors, δ_r is larger, the more efficient the transmission of knowledge among firms in r is.

In the Theoretical Appendix it is also shown how to derive:

(i) the intermediate demand for variety h by a firm s producing in country r

$$d_r^s(h) = \frac{p_r(h)^{-\sigma}}{P_r^{1-\sigma}} \left[\mu P_r^{\mu} \left(w_r \right)^{1-\mu} \bar{F} - \delta_r P_r G_r(s) \right]; \tag{8}$$

(ii) the aggregate intermediate input demand by firm s producing in r

$$X_r(s) = \left(\frac{w_r}{P_r} \right)^{1-\mu} \mu \bar{F} - \delta_r G_r(s) \tag{9}$$

where the presence of a positive network externality (when $\delta_r > 0$) implies that intermediate demand by each firm in the network is smaller with respect to the case in which there are no network effects;[8] and

(iii) labour demand by firm s

$$L_r(s) = a_M x_r(s) + (1-\mu) \bar{F} \left(\frac{P_r}{w_r} \right)^{\mu},$$

which does not directly depend on δ_r.

Let us recall that the savings generated by the network effects on the total cost of production $TC(s)$ of firm s, which has a mass of links $\Lambda_r(s)$ with other firms in the same country/region r, are given by the last term $\delta_r P_r G_r(s)$ in expression (7). The costs of the network are given by $\rho_r \left(\Lambda_r(s) \right)^2$ and they are larger, the larger are both the mass of links with other firms in the network and the parameter $\rho_r > 0$ that represents the country specific cost parameter of 'production network'. Potentially, the value of this parameter may differ across countries, with $\rho_r \neq \rho_v$.

Given the empirical findings in Section 3, we assume that there is a size effects of networks with $S(\Omega(\Lambda_r(s))) > 1$, and from now onward we continue our analysis focusing

[8] Clearly, the value of δ_r should be sufficiently small in order to have a positive value of the aggregate intermediate input demand by firm s, $X_r(s)$ in (9). This will also ensure that the intermediate demand for variety h by a firm s producing in country r in (8) will be positive. See the following text for the upper limit of δ_r in the case of the symmetric additive specification.

on the specific case of *the additive specification* with $S(\Omega(\Lambda_r(s))) = \Lambda_r$,[9] assuming that all firms producing within a country are symmetric.[10] Hence, with symmetric firms in r and the additive specification, potential real savings in expression (6) are given by

$$G_r(s) = \Lambda_r X_r, \tag{10}$$

as symmetry implies that $X_r(i) = X_r$ and that $\Lambda_r(s) = \Lambda_r$ for each firm in its local network.[11]

With symmetric firms, multiplying $d_r^s(h)$ in (8) for the number of firms producing in r, N_r, we get the total local intermediate demand for variety h that is

$$d_r^T(h) = d_r^s(h) N_r$$

Then, the intermediate demand for a variety h produced in r by a firm s producing in the other country v is given by

$$d_r^{s*}(h) = \frac{p_r^*(h)^{-\sigma}}{P_v^{1-\sigma}} \left[\mu P_v^{\mu} (w_v)^{1-\mu} \bar{F} - \delta_v P_v G_v(s)\right] \tag{11}$$

where $p_r^*(h)$ is the price set in the foreign market v for the variety h produced in r. Thus, the total foreign (in v) intermediate demand for variety h produced in r is

$$d_r^{T^*}(h) = d_r^{s*}(h) N_v$$

Making use of (9) and (10), yields that with symmetric firms the aggregate interme-

[9] On the other hand, in the case of the *average specification* ($S(\Omega(\Lambda_r(s))) = \Omega(\Lambda_r(s)) = 1$) with symmetric firms, potential real savings for each firm s would be given by $G_r(s) = X_r$. However, we do not consider this specification because it is not consistent with the empirical findings in Section 3.

[10] Firm producing within a country/region can be considered symmetric since they face the same cost functions, with the same cost parameters (included δ_r and ρ_r) and factor prices ($w_r = 1$ and P_r), and they produce a differentiated variety that enters symmetrically both the utility function (as it will be shown below) and the production function.

[11] Given (9) and (10), the value of δ_r in the case of the symmetric additive specification must be such that $X_r = \frac{1}{X_r}\left(\left(\frac{w_r}{P_r}\right)^{1-\mu} \mu\bar{F} - \delta_r \Lambda_r\right) X_r > 0$, which requires that $\delta_r < \left(\frac{w_r}{P_r}\right)^{1-\mu} \frac{\mu\bar{F}}{\Lambda_r}$.

diate input demand by firm s producing in r corresponds to

$$X_r = \left(\frac{w_r}{P_r}\right)^{1-\mu} \frac{\mu \bar{F}}{(1+\delta_r \Lambda_r)}, \tag{12}$$

which is decreasing in the size of the network in r given by Λ_r, and in the efficiency of the transmission of knowledge among firms in r captured by δ_r. This is exactly what our estimates in Section 3 have shown, by relating the different network proxies to a firms expenses on inputs. As It follows that the aggregate expenditures of each firm in intermediates is

$$P_r X_r = P_r^{\mu} w_r^{1-\mu} \frac{\mu \bar{F}}{(1+\delta_r \Lambda_r)},$$

clearly decreasing in Λ_r and in δ_r.

Finally, the total number of firms in the economy is given by

$$N_W = N_r + N_v$$

4.2 Consumers

The representative consumer in each country/region has a two-tier utility function where the upper tier is Cobb-Douglas and the lower tier is CES with

$$U(C_A, C_M) = \frac{C_A^{1-\gamma} C_M^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}, \quad C_M = \left(\int_{j\in N} c(j)^{\frac{\sigma-1}{\sigma}} dj\right)^{\frac{\sigma}{\sigma-1}} \quad \text{with} \quad 0<\gamma<1<\sigma \tag{13}$$

As usual, the indirect utility function for the preferences in expression (13) is

$$V = \frac{I}{P^{\gamma}} \tag{14}$$

where P is the price index defined as $P \equiv \left(\int_{j\in N} p(j)^{1-\sigma} dj\right)^{\frac{1}{1-\sigma}}$ and I is the household's income.

By Roy's identity, the demand for variety j of the representative consumer can be computed from (14) and it is given by

$$c(j) = \frac{p(j)^{-\sigma}}{P^{1-\sigma}} \gamma I.$$

Then, the total demand of variety j by consumers in country r is given by

$$c_r(j) = \frac{p_r(j)^{-\sigma}}{P_r^{1-\sigma}} \gamma I_r, \tag{15}$$

where I_r is the total income of households in r, which is given by the sum of the labour wage and profits of all N_r firms producing in the same country that are owned by local workers/consumers, namely

$$I_r = w_r L_r + N_r \Pi_r \tag{16}$$

where Π_r are pure average profits realized by firms producing in r.

Following Ottaviano and Robert-Nicoud (2006) and chapter 8 of the book by Baldwin et al. (2011), and many others, among which Nocco (2012) and Navas and Nocco (2021), we assume that $\gamma = \mu$, that is we assume that consumers and firms devote the same shares of expenditures to manufactures.

4.3 Sales and Profits

The total quantity sold to consumers and firms in country r by a domestic firm j is given by

$$x_{rr}(j) = c_r(j) + d_r^s(j) N_r \tag{17}$$

Given that $\tau_{rv} > 1$ units of goods have to be shipped from the production country r to have 1 unit that arrives at destination in the foreign country v, the production for the foreign market of a firm located in r is τ_{rv} times the demand, that is

$$x_{rv}(j) = \tau_{rv} \left[c_v(j) + d_r^{s*}(j) N_v \right] \tag{18}$$

So the total equilibrium production of a firm located in r is

$$x_r(j) = x_{rr}(j) + x_{rv}(j) = c_r(j) + d_r^s(j) N_r + \tau_{rv} \left[c_v(j) + d_r^{s*}(j) N_v \right]$$

Operating profits of a firm j producing in r derived from local sales and from foreign sales are, respectively, given by

$$\pi_{rr}(j) = [p_r(j) - a_M w_r]\, x_{rr}(j) \tag{19}$$

and

$$\pi_{rv}(j) = \left[\frac{p_r^*(j)}{\tau_{rv}} - a_M w_r\right] x_{rv}(j) \tag{20}$$

Finally, total operating profits of a firm j producing in r are given by

$$\pi_r(j) = \pi_{rr}(j) + \pi_{rv}(j), \tag{21}$$

while its pure profits are

$$\Pi_r(j) = \pi_r(j) - \rho_r\left(\Lambda_r(j)\right)^2 - \left[\bar{F} P_r^{\mu} w_r^{1-\mu} - \delta_r P_r G_r(j)\right] \tag{22}$$

obtained by subtracting to its total operating profits, the cost of the network and the fixed production costs net of the savings $\delta_r P_r G_r(j)$ due to the participation to the network.

In addition, it can be readily verified, making use of (17), (8), (15), (16) and (22), that, with N_r symmetric firms in country r, revenues from local sales of a firm producing in r are

$$p_r\left(c_r + d_r^s N_r\right) = \frac{{p_r}^{1-\sigma}}{P_r^{1-\sigma}}\left\{\mu\left[w_r L_r + N_r\left(\pi_r - \rho_r \Lambda_r^2 + \delta_r P_r G_r\right)\right] - \delta_r P_r G_r N_r\right\}, \tag{23}$$

while, making also use of (11), revenues from export sales in v of a firm producing in r can be expressed as follows[12]

$$p_r^*\left(c_v + d_r^{s*} N_v\right) = \frac{{p_r^*}^{1-\sigma}}{P_v^{1-\sigma}}\left\{\mu\left[w_v L_v + N_v\left(\pi_v - \rho_v \Lambda_v^2 + \delta_v P_v G_v\right)\right] - \delta_v P_v G_v N_v\right\} \tag{24}$$

Notice that the expressions for revenues (23) and (24) capture two contrasting effects due to the presence of savings generated by 'network' effects. Indeed, from one hand, these savings tend to reduce revenues as shown by the presence of the last terms in the curly brackets in the two expressions above. On the other hand, they tend to increase pure

[12] The indexes related to a firm j have been removed because all firms within a country are symmetric.

profits in the two terms in the round brackets of the same expressions, and, consequently, consumers' expenditures. Which of these two contrasting effects prevail depends on the overall general equilibrium effects of the model, and, ultimately, on the values of all the parameters, including ρ_r and ρ_v. Specifically, the smaller the values of ρ, the larger the strength of the impact of savings due to network effects on pure profits, and, thus, on consumers' expenditures, making it more likely to observe an increase in firms revenues. These type of prescriptions of the model could be eventually tested in case of the availability of data.

Under Dixit-Stiglitz monopolistic competition, transportation costs are fully passed onto consumers and mill pricing is optimal. Choosing the units for manufacturing varieties in such a way that $a_M = \frac{\sigma-1}{\sigma}$ and making use of the assumption on the numeraire (that implies that $w_r = w_v = 1$),[13] Domestic and export prices for a firm producing in r are respectively given by

$$p_r(j) = \frac{\sigma}{\sigma-1} a_M w_r = p_r \quad \text{and} \quad p_r^*(j) = \frac{\sigma}{\sigma-1} \tau_{rv} a_M w_r = p_r^* = \tau_{rv} p_r \tag{25}$$

Hence, the price set by each firm in a country does not depend on its position in the network, as the latter only affects its fixed costs and has no effect on its marginal production costs.

Making use of (20)-(25), we get that total operating profits of firm j producing in r are equal to its total revenues divided by σ, that is

$$\pi_r(j) = \frac{p_r(j) x_{rr}(j) + p_r^*(j) \frac{1}{\tau_{rv}} x_{rv}(j)}{\sigma} \tag{26}$$

As stated above, choosing the units for manufacturing varieties in such a way that $a_M = \frac{\sigma-1}{\sigma}$ and making use of the assumption on the numeraire (that implies that $w_r = w_v = 1$), domestic and export prices for a firm producing in r are respectively given by

$$p_r = 1 \quad \text{and} \quad p_r^* = \tau_{rv} \tag{27}$$

[13] In these choices, we follow Ottaviano and Robert-Nicoud (2006).

Hence, the price index in country r is given by

$$P_r = (N_r + N_v \phi_{rv})^{\frac{1}{1-\sigma}} = \frac{1}{(N_r + N_v \phi_{rv})^{\frac{1}{\sigma-1}}} \tag{28}$$

where $\phi_{rv} = \tau_{rv}^{1-\sigma} \in [0, 1]$ is a direct measure of the *freeness of trade*, with its value equal to zero when trade costs are prohibitively high ($\tau_{rv} \to \infty$), and equal to 1 when markets are perfectly integrated ($\tau_{rv} = 1$).[14] An analogous expression holds for the price index in the other country P_v, that is

$$P_v = (N_v + N_r \phi_{vr})^{\frac{1}{1-\sigma}} \tag{29}$$

Let us recall that, given our assumptions, all firms within a country are symmetric. This is due to the fact that all firms in a country have the same cost functions, with the same cost parameters (including δ_r and ρ_r) and factor prices ($w_r = 1$ and P_r), and each of them produces a differentiated variety that enters symmetrically the utility function and the production function. Hence, from now onward, we will drop the index that denotes the firm and keep only the index that denotes the country in which it is producing.

5 More results

We consider the case in which entry and exit of firms are restricted and, as in a short-run equilibrium, the total number of firms N_W and their number in each country, N_r and N_v, is given.[15] In the short run, consumers maximize utility, firms maximize profits, and all markets clear for given numbers of firms, N_r and N_v. Let us focus on this type of equilibrium and define E_r as follows

$$E_r \equiv I_r + N_r \bar{F} P_r^{\mu} w_r^{1-\mu}$$

and observe that it represents the expenditures in country r of consumers (I_r) and of firms before network effects are considered. Indeed, $N_r \bar{F} P_r^{\mu} w_r^{1-\mu}$ represents total fixed cost of firms in r that should be paid were network effects absent. Making use of (16) and (22),

[14] The general expression for the price index in region r is $P_r = \left[N_r p_r^{1-\sigma} + N_v (p_v^*)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$

[15] Following Fujita et al. (2001) we distinguish between 'short-run' and 'long-run' equilibria, and focus our analysis on the short-run equilibrium.

when network effects are present, with $\delta_r > 0$ and and $\rho_r > 0$, we rewrite E_r as follows

$$E_r = L_r + N_r \left(\pi_r - \rho_r \Lambda_r^2 + \delta_r P_r G_r \right) \tag{30}$$

Specifically, when $\rho_r = \delta_r = 0$ there are no network effects and we fall back in a short-run equilibrium in the framework by Ottaviano and Robert-Nicoud. Clearly, the presence of network effects is captured by $\delta_r \neq 0$ and $\rho_r \neq 0$.

Making use of (8), (11), (15)-(18), (22), (26), (27) and (30), operating profits of a firm producing in r can be expressed as follows

$$\pi_r = \frac{1}{\sigma} \left[\frac{1}{P_r^{1-\sigma}} \left(\mu E_r - \delta_r P_r G_r N_r \right) + \phi_{rv} \frac{1}{P_v^{1-\sigma}} \left(\mu E_v - \delta_v P_v G_v N_v \right) \right] \tag{31}$$

with $r \neq v$ and $r, v = 1, 2$.

Recall that labour endowments are, respectively, given by $L_r = \lambda_r L_W$ workers in country r and $L_v = (1 - \lambda_r) L_W$ workers in country v, with L_W denoting world's population and λ_r the share of workers living in country r. Workers are internationally immobile, and the two countries are symmetric not only when $\lambda_r = 1/2$, but all the parameters are symmetric and the two countries host the same number of firms ($N_r = N_v$).

Substituting the value for operating profits given by (31) in (30) for the two countries 1 and 2, we get a system of two equations in two unknowns (E_1 and E_2) that can be solved (as described in the section computation of the Theoretical Appendix) to find

$$E_1 = \frac{\sigma \left\{ \sigma \lambda_1 - \mu \frac{[\lambda_1 N_2 + N_1 \phi_{12} (1 - \lambda_1)]}{P_2^{1-\sigma}} \right\} L_W + N_1 \left(d_{11} \delta_1 P_1 G_1 - c_{11} \rho_1 \Lambda_1^2 - d_{12} \delta_2 P_2 G_2 - c_{12} \rho_2 \Lambda_2^2 \right)}{\sigma^2 - \sigma \mu \left(\frac{N_1}{P_1^{1-\sigma}} + \frac{N_2}{P_2^{1-\sigma}} \right) + \mu^2 \frac{N_1}{P_1^{1-\sigma}} \frac{N_2}{P_2^{1-\sigma}} \left(1 - \phi_{12} \phi_{21} \right)} \tag{32}$$

with:

$$\begin{aligned}
d_{11} &= \sigma^2 - \sigma\left(\frac{N_1}{P_1^{1-\sigma}} + \mu\frac{N_2}{P_2^{1-\sigma}}\right) + \mu\frac{N_1}{P_1^{1-\sigma}}\frac{N_2}{P_2^{1-\sigma}}\left(1-\phi_{12}\phi_{21}\right) > 0 \\
c_{11} &= \sigma\left(\sigma - \mu\frac{N_2}{P_2^{1-\sigma}}\right) > 0 \\
d_{12} &= \left(1-\mu\right)\sigma\phi_{12}\frac{N_2}{P_2^{1-\sigma}} > 0 \\
c_{12} &= \sigma N_2\frac{1}{P_2^{1-\sigma}}\phi_{12}\mu > 0
\end{aligned}$$

where $P_r^{1-\sigma} = N_r + N_v\phi$ and an analogous expression holds for E_2.[16] Clearly, the last expression in the last brackets in the numerator of E_1 disappears when there are no network effects (i.e. $\delta_r = 0$ and $\rho_r = 0$).[17] Notice that these results have been obtained for asymmetric trade costs $\tau_{12} \neq \tau_{21}$ with corresponding asymmetric level of the freeness of trade $\phi_{12} \neq \phi_{21}$.

Let us consider the numerator of E_1 and notice that the term $\delta_1 P_1 G_1$ and $\rho_1\Lambda_1^2$, respectively, represent the benefit and the cost of the network for each firm producing in country 1, while $\delta_2 P_2 G_2$ and $\rho_2\Lambda_2^2$, respectively, represent the benefit and the cost of the network for each firm producing in country 2. Thus, the presence of savings from network effects, and the associated costs, may affect operating profits in (31) in opposite ways, and their overall impact can be eventually empirically assessed. Finally, notice that the values of π_r in the expressions for sales (23) and (24) depends on E_r and E_v that are obtained as described above.

6 Testing the model

According to Eq. (23) network size has contrasting effects on firms domestic revenues. How local revenues respond to changes in the size of the network depends crucially on the parameter ρ. The smaller the values of ρ, the larger the strength of the impact of

[16] Notice that $0 < \frac{N_r}{P_r^{1-\sigma}} = \frac{N_r}{N_r+N_v\phi} < 1$. Moreover, as the maximum admissible value for μ is 1 (or $\sigma - 1$ for the no black hole condition) we can use this information to study the sign of d_{11}. We know that $\frac{\partial d_{11}}{\partial \mu} < 0$. Thus, given that $d_{11} > 0$ when $\mu = 1$, it will be positive for any other value of $\mu < 1$.

[17] In the specific case of $\delta_r = 0$ and $\rho_r = 0$, we get $E_1 + E_2 = \sigma\frac{L_W}{\sigma-\mu}$, that is the value obtained by Ottaviano and Robert-Nicoud (2006).

savings due to network effects on pure profits, and, thus, on consumers' expenditures, making it more likely to observe an increase in firms revenues. Thereby, ρ represents the country specific cost parameter of the production network. To proxy the parameter, we make use of a recent study by Martin et al. (2023), which constructs a product-level index of relationship stickiness using firm-to-firm relationship duration data. They furthermore show, that their measure of product stickiness is positively correlated with alternative measures of product specificity used in the literature, most notably, Rauch (1999) and Nunn (2007). We use the measure for product specificity and match it to our Danish data. This allows us to distinguish between relationship specific or unspecific products, and thus, compute our network proxies separately for the two types of products. The idea is that the cost of the production network, determined by the parameter ρ, depends on the relationship specificity of the involved products. To classify products into relationship specific or not, we use the median value in the data set from Martin et al. (2023), once matched to our products, which is 3.0095 (compared to 2.97 in Martin et al. (2023)).

In a first step, we rerun regression akin to the one presented in Section 3, to check if network measures for relationship specific or unspecific products have different effects on the demand for inputs. According to Eq. (12), the parameter ρ does not affect the relation between network size and demand for inputs. Thus, we expect both to negatively affect the demand for inputs. As we show in Tables 5 and 6, the data confirms this pattern. Irrespective of the type of product - relationship specific, or not-relationship specific - and, thus, the type of network, we find a negative impact on the demand for inputs. The estimated coefficients for the relationship specific or not-relationship specific networks do not differ in terms of sign or magnitude.

However, according to Eq. (23), we expect differential affects of our relationship specific network proxies on domestic revenues. As shown in Tables 7 and 8, the data supports this view. Specifically, these tables reveal that networks, based on relationship specific products, are increasing domestic revenues, while networks based on non-relationship specific products, are not affecting local sales. This is in line with our model predictions. As discussed above, we expect a larger impact of savings due to network effects on firms revenues for smaller values of ρ. If products are relationship specific, the cost of a production network should be lower, as it is easier (or more important) to share knowledge among relationship specific products, compared to non-relationship specific products (or

Table 5: Relationship specific network size and firms aggregate demand for inputs

	Dependent variable: (log) expenses on inputs					
	(1)	(2)	(3)	(4)	(5)	(6)
(log) NRS-IBN 5-reg.	-0.0273***		-0.0288***			
	(0.00832)		(0.00843)			
(log) RS-IBN 5-reg.		-0.0156**	-0.0173**			
		(0.00778)	(0.00787)			
(log) NRS-IBN 11-reg.				-0.0230**		-0.0244**
				(0.00999)		(0.0101)
(log) RS-IBN 11-reg.					-0.0261***	-0.0270***
					(0.00918)	(0.00924)
Observations	69601	69601	69601	54907	54907	54907
adj. R^2	0.893	0.893	0.893	0.895	0.895	0.896
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Time-varying controls	Yes	Yes	Yes	Yes	Yes	Yes
Year×industry×region FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The sample covers firms in the private sector for the years 2000 to 2018. IBN computes for each input a firm is using the number of firms within a cell that purchase the same input, and secondly, take the average of this number across the inputs a firm is purchasing. Cells are 5 NUTS-2 regions or 11 NUTS-3 regions. Furthermore, inputs are split into relationship and non-relationship specific products according to the measure provided in Martin et al. (2023). Time-varying controls include (log) value added per worker, (log) employment, an indicator variable for exporting, (log) number of distinct 4-digit inputs, (log) wage bill, and (log) assets per worker. Industry classification follows the 4-digit NACE rev. 2 version. Standard errors in parentheses are clustered at the firm level. ***, **, * denote significance level at 1%, 5% and 10% respectively.

one might even expect no savings or knowledge sharing among non-relationship specific products).[18]

7 Conclusion

Our study introduces a novel mechanism that sheds light on the potential for firms to achieve cost savings through participation in networks within global and local supply chains. We have demonstrated that firms operating within local buyer networks can realize significant savings in intermediate inputs, with the magnitude of these savings closely linked to the efficacy of knowledge transmission among network participants. Regions

[18] Due to the small number of firms available in the VARS, UHDI and VARK register, for which we can obtain domestic revenues, the number of observations in Tables 7 and 8 is substantial lower, which also explains the lack of significance in our most restrictive estimates in columns (3) and (6).

Table 6: Relationship specific network size and firms aggregate demand for inputs

	Dependent variable: (log) expenses on inputs					
	(1)	(2)	(3)	(4)	(5)	(6)
(log) NRS-IBN 5-reg. & 69 Ind.	-0.130***		-0.115***			
	(0.0324)		(0.0311)			
(log) RS-IBN 5-reg. & 69 Ind.		-0.129***	-0.112***			
		(0.0376)	(0.0362)			
(log) NRS-IBN 11-reg. & 69 Ind.				-0.205***		-0.178***
				(0.0460)		(0.0426)
(log) RS-IBN 11-reg. & 69 Ind.					-0.215***	-0.186***
					(0.0533)	(0.0501)
Observations	11982	11982	11982	8510	8510	8510
adj. R^2	0.888	0.888	0.889	0.885	0.885	0.886
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Time-varying controls	Yes	Yes	Yes	Yes	Yes	Yes
Year×industry×region FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The sample covers firms in the private sector for the years 2000 to 2018. IBN computes for each input a firm is using the number of firms within a cell that purchase the same input, and secondly, take the average of this number across the inputs a firm is purchasing. Cells are 5 NUTS-2 regions or 11 NUTS-3 regions, times 69 2-digit NACE rev. 2 industries. Furthermore, inputs are split into relationship and non-relationship specific products according to the measure provided in Martin et al. (2023). Time-varying controls include (log) value added per worker, (log) employment, an indicator variable for exporting, (log) number of distinct 4-digit inputs, (log) wage bill, and (log) assets per worker. Industry classification follows the 4-digit NACE rev. 2 version. Standard errors in parentheses are clustered at the firm level. ***, **, * denote significance level at 1%, 5% and 10% respectively.

or countries with efficient knowledge dissemination networks are likely to experience larger savings in intermediate input costs. These findings have profound implications for firms' profitability and market competitiveness. By reducing expenditures on intermediates, firms can enhance their profit margins and gain a competitive edge in both local and export markets. Moreover, efficient knowledge transmission within networks can drive innovation and product quality improvements, further strengthening firms' market positioning.

Moving forward, policymakers should consider the role of industrial policies aimed at reducing the costs of local production networks. Fostering efficient knowledge exchange within networks can enhance firms' competitiveness and contribute to economic growth and stability. Additionally, efforts to improve the infrastructure supporting knowledge transmission, such as investments in digital connectivity and education, can further amplify the benefits of participation in supply chain networks. In summary, our study un-

Table 7: Relationship specific network size and domestic sales

	Dependent variable: (log) domestic revenues					
	(1)	(2)	(3)	(4)	(5)	(6)
(log) NRS-IBN 5-reg.	0.0255	0.0268	0.00429			
	(0.0170)	(0.0170)	(0.0154)			
(log) RS-IBN 5-reg.	0.0632***	0.0632***	0.0171			
	(0.0232)	(0.0231)	(0.0205)			
(log) NRS-IBN 11-reg.				0.0125	0.0149	-0.00395
				(0.0228)	(0.0227)	(0.0212)
(log) RS-IBN 11-reg.				0.0714***	0.0703***	0.0221
				(0.0251)	(0.0251)	(0.0220)
Observations	12381	12381	12381	10244	10244	10244
adj. R^2	0.832	0.833	0.847	0.829	0.831	0.844
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Time-varying controls	No	Some	Yes	No	Some	Yes
Year×industry×region FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The sample covers firms in the manufacturing sector for the years 2000 to 2018. IBN computes for each input a firm is using the number of firms within a cell that purchase the same input, and secondly, take the average of this number across the inputs a firm is purchasing. Cells are 5 NUTS-2 regions or 11 NUTS-3 regions. Furthermore, inputs are split into relationship and non-relationship specific products according to the measure provided in Martin et al. (2023). Time-varying controls include (log) value added per worker in columns (2) and (5), plus (log) employment, an indicator variable for exporting, (log) number of distinct 4-digit inputs, (log) wage bill, and (log) assets per worker in columns (3) and (6). Industry classification follows the 2-digit NACE rev. 2 version. Standard errors in parentheses are clustered at the firm level. ***, **, * denote significance level at 1%, 5% and 10% respectively.

derscores the importance of recognizing the role of physical and knowledge flows within supply chains and highlights the potential for targeted policy interventions to enhance firms' competitiveness and drive economic prosperity.

Table 8: Relationship specific network size and domestic sales

	Dependent variable: (log) domestic revenues					
	(1)	(2)	(3)	(4)	(5)	(6)
(log) NRS-IBN 5-reg. & 69 Ind.	-0.0265	-0.0219	-0.00347			
	(0.0230)	(0.0227)	(0.0215)			
(log) RS-IBN 5-reg. & 69 Ind.	0.0677**	0.0681**	0.0111			
	(0.0311)	(0.0309)	(0.0265)			
(log) NRS-IBN 11-reg. & 69 Ind.				-0.0337	-0.0278	-0.00869
				(0.0313)	(0.0310)	(0.0305)
(log) RS-IBN 11-reg. & 69 Ind.				0.0700*	0.0674*	0.00373
				(0.0358)	(0.0354)	(0.0308)
Observations	9502	9502	9502	7616	7616	7616
adj. R^2	0.826	0.828	0.845	0.819	0.821	0.836
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Time-varying controls	No	Some	Yes	No	Some	Yes
Year×industry×region FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The sample covers firms in the manufacturing sector for the years 2000 to 2018. IBN computes for each input a firm is using the number of firms within a cell that purchase the same input, and secondly, take the average of this number across the inputs a firm is purchasing. Cells are 5 NUTS-2 regions or 11 NUTS-3 regions, times 69 2-digit NACE rev. 2 industries. Furthermore, inputs are split into relationship and non-relationship specific products according to the measure provided in Martin et al. (2023). Time-varying controls include (log) value added per worker in columns (2) and (5), plus (log) employment, an indicator variable for exporting, (log) number of distinct 4-digit inputs, (log) wage bill, and (log) assets per worker in columns (3) and (6). Industry classification follows the 2-digit NACE rev. 2 version. Standard errors in parentheses are clustered at the firm level. ***, **, * denote significance level at 1%, 5% and 10% respectively.

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A Appendix

A.1 Empirical Appendix

Table A1: Network size and firms aggregate demand for inputs

	Dependent variable: (log) expenses on inputs					
	(1)	(2)	(3)	(4)	(5)	(6)
(log) IBN 5-reg.	-0.0606***				-0.103***	
	(0.00590)				(0.0246)	
(log) IBN 11-reg.		-0.0562***				-0.0874***
		(0.00374)				(0.0269)
(log) IBN 5-reg. (med)			-0.0978***			
			(0.0125)			
(log) IBN 11-reg. (med)				-0.0964***		
				(0.0163)		
Observations	115526	109265	89920	64628	23189	20590
adj. R^2	0.885	0.887	0.884	0.886	0.873	0.872
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Time-varying controls	Yes	Yes	Yes	Yes	Yes	Yes
Year×industry×region FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The sample covers firms in the private sector for the years 2000 to 2018 in columns (1) to (4) or only the manufacturing sector in columns (5) and (6). IBN computes for each input a firm is using the number of firms within a cell that purchase the same input, and secondly, take the average (median) of this number across the inputs a firm is purchasing in columns (1), (2), (5) and (6) ((3) and (4)). Cells are 5 NUTS-2 regions or 11 NUTS-3 regions. Time-varying controls include (log) value added per worker, (log) employment, an indicator variable for exporting, (log) number of distinct 4-digit inputs, (log) wage bill, and (log) assets per worker. Industry classification follows the 4-digit NACE rev. 2 version. Standard errors in parentheses are clustered at the firm level in columns (3) to (6) or regional level in columns (1) and (2). ***, **, * denote significance level at 1%, 5% and 10% respectively.

Table A2: Network size and firms aggregate demand for inputs

	Dependent variable: (log) expenses on inputs							
Number of Inputs:	few	many	few	many	few	many	few	many
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(log) IBN 5-reg	-0.0500***	-0.00330						
	(0.0122)	(0.0101)						
(log) IBN 11-reg			-0.0300**	-0.00445				
			(0.0133)	(0.00950)				
(log) IBN 5-reg & 69 ind.					-0.00741	-0.0413*		
					(0.0479)	(0.0225)		
(log) IBN 11-reg & 69 ind.							-0.0364	-0.0764**
							(0.0662)	(0.0302)
Observations	52903	52669	47997	48532	6667	16351	4219	11727
adj. R^2	0.825	0.899	0.829	0.904	0.797	0.889	0.790	0.892
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time-varying controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year×industry×region FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The sample covers firms in the private sector for the years 2000 to 2018, and the sample are split according to the median number of inputs firms are using. IBN computes for each input a firm is using the number of firms within a cell that purchase the same input, and secondly, take the average of this number across the inputs a firm is purchasing. Cells are 5 NUTS-2 regions, 11 NUTS-3 regions, and regions times 69- 2-digit NACE rev. 2 industries. Time-varying controls include (log) value added per worker, (log) employment, an indicator variable for exporting, (log) number of distinct 4-digit inputs, (log) wage bill, and (log) assets per worker. Industry classification follows the 4-digit NACE rev. 2 version. Standard errors in parentheses are clustered at the firm level. ***, **, * denote significance level at 1%, 5% and 10% respectively.

A.2 Theoretical Appendix

For a given mass of links of the firm $\Lambda_r(s)$ with other firms in their local network, each firm producing variety s solves the following expenditure minimization problem given the technology

$$\min_{L_r(s),\, d_r^s(j)} \quad w_r L_r(s) + \int_{j\in N} p_r(j) d_r^s(j) dj$$

$$\text{s.t.} \qquad x_r(s) = \frac{1}{a_M}\left\{L_r(s) - \left[\frac{\mu^\mu (1-\mu)^{1-\mu}\bar{F}}{(\Phi(X_r(s), X_{-sr}, \Lambda_r(s)))^\mu}\right]^{\frac{1}{1-\mu}}\right\}$$

$$\text{with } \Phi\left(X_r(s), X_{-sr}, \Lambda_r(s)\right) \equiv \left(\int_{j\in N} d_r^s(j)^{\frac{\sigma-1}{\sigma}} dj\right)^{\frac{\sigma}{\sigma-1}} + \delta_r S(\Omega(\Lambda_r(s)))\frac{\int_{\Lambda_r(s)} X_r(i) di}{\Omega(\Lambda_r(s))}$$

The Langrangian is: $\ell = w_r L_r(s) + \int_{j\in N} p_r(j) d_r^s(j) dj + \lambda\{x_r(s)$

$$-\frac{1}{a_M}\left[L_r(s) - \left[\mu^\mu (1-\mu)^{1-\mu}\bar{F}\right]^{\frac{1}{1-\mu}}\left[\left(\int_{j\in N} d_r^s(j)^{\frac{\sigma-1}{\sigma}} dj\right)^{\frac{\sigma}{\sigma-1}} + \delta_r S(\Omega(\Lambda_r(s)))\frac{\int_{\Lambda_r(s)} X_r(i) di}{\Omega(\Lambda_r(s))}\right]^{-\frac{\mu}{1-\mu}}\right]\}$$

where λ is the Lagrange multiplier.

The first-order condition (FOC) with respect to $L_r(s)$, $\frac{\partial \ell}{\partial L_r(s)} = 0$, implies that

$$w_r = \lambda \frac{1}{a_M} \tag{33}$$

The FOC with respect to variety j,$\frac{\partial \ell}{\partial d_r^s(j)} = 0$, implies that

$$p_r(j) = \frac{\mu}{1-\mu}\lambda \frac{\left[\mu^\mu (1-\mu)^{1-\mu}\bar{F}\right]^{\frac{1}{1-\mu}}}{a_M}\left[\left(\int_{j\in N} d_r^s(j)^{\frac{\sigma-1}{\sigma}} dj\right)^{\frac{\sigma}{\sigma-1}} + \delta_r S(\Omega(\Lambda_r(s)))\frac{\int_{\Lambda_r(s)} X_r(i) di}{\Omega(\Lambda_r(s))}\right]^{-\frac{\mu}{1-\mu}-1} *$$
$$\left(\int_{j\in N} d_r^s(j)^{\frac{\sigma-1}{\sigma}} dj\right)^{\frac{1}{\sigma-1}} d_r^s(j)^{-\frac{1}{\sigma}} \tag{34}$$

The FOC with respect to variety h, $\frac{\partial \ell}{\partial d_r^s(h)} = 0$, implies that

$$p_r(h) = \frac{\mu}{1-\mu}\lambda \frac{\left[\mu^{\mu}(1-\mu)^{1-\mu}\bar{F}\right]^{\frac{1}{1-\mu}}}{a_M}\left[\left(\int_{j\in N} d_r^s(j)^{\frac{\sigma-1}{\sigma}}dj\right)^{\frac{\sigma}{\sigma-1}} + \delta_r S(\Omega(\Lambda_r(s)))\frac{\int_{\Lambda_r(s)} X_r(i)di}{\Omega(\Lambda_r(s))}\right]^{-\frac{\mu}{1-\mu}-1} * \left(\int_{j\in N} d_r^s(j)^{\frac{\sigma-1}{\sigma}}dj\right)^{\frac{1}{\sigma-1}} d_r^s(h)^{-\frac{1}{\sigma}} \tag{35}$$

The FOC with respect to λ, $\frac{\partial \ell}{\partial \lambda} = 0$, implies that

$$x_r(s) = \frac{1}{a_M}\left\{L_r(s) - \left[\mu^{\mu}(1-\mu)^{1-\mu}\bar{F}\right]^{\frac{1}{1-\mu}}\left(\Phi\left(X_r(s), X_{-sr}, \Lambda_r(s)\right)\right)^{-\frac{\mu}{1-\mu}}\right\} \tag{36}$$

From the ratio of (34) and (35), we get that

$$d_r^s(j) = \left(\frac{p_r(h)}{p_r(j)}\right)^{\sigma} d_r^s(h) \tag{37}$$

Moreover, making use of (33), (34), (37) and of the definition of the price index $P_r \equiv \left(\int_{j\in N} p_r(j)^{1-\sigma}dj\right)^{\frac{1}{1-\sigma}}$, we get the intermediate input demand by firm s

$$X_r(s) = \left(\frac{w_r}{P_r}\right)^{1-\mu}\mu\bar{F} - \delta_r S(\Omega(\Lambda_r(s)))\frac{\int_{\Lambda_r(s)} X_r(i)di}{\Omega(\Lambda_r(s))} \tag{38}$$

The expression above for intermediate input demand by firm s shows that when there is a positive externality ($\delta_r > 0$) due to 'network' effects, intermediate demand by each firm s is reduced.

Substituting $X_r(s)$ into (36), yields labour demand of firm s

$$L_r(s) = a_M x_r(s) + (1-\mu)\bar{F}\left(\frac{P_r}{w_r}\right)^{\mu} \tag{39}$$

Thus, labour demand of firm s is indirectly affected by the presence of externalities since these affect the price index (when the number of firms may change) and $x_r(s)$ that has to be equal to the demand for firm s producing in r.

Substituting (37) into intermediate demand (38), yields the demand for each variety h by firm s producing in r

$$d_r^s(h) = \frac{p_r(h)^{-\sigma}}{P_r^{1-\sigma}} \left[\mu P_r^{\mu} (w_r)^{1-\mu} \bar{F} - P_r \delta_r S(\Omega(\Lambda_r(s))) \frac{\int_{\Lambda_r(s)} X_r(i) di}{\Omega(\Lambda_r(s))} \right] \tag{40}$$

Then, making use of (39) and (40), the production cost function $TC_r(s) = w_r L_r(s) + \int\limits_{j \in N} p_r(j) d_r^s(j) dj$ of the firm producing variety s at location r is

$$TC = w_r x_r(s) a_M + \bar{F} P_r^{\mu} w_r^{1-\mu} - P_r \delta_r S(\Omega(\Lambda_r(s))) \frac{\int_{\Lambda_r(s)} X_r(i) di}{\Omega(\Lambda_r(s))}$$

Computations for Section 5 .

Denoted the two country $r, v = 1, 2$, the values of E_1 and E_2 can be found for a given number of firms, number of links of each firm Λ_1 and Λ_2, and savings G_1 and G_2. Specifically, the expressions for operating profits in the two countries π_1 and π_2 from (31), that are respectively given by

$$\pi_1 = \frac{1}{\sigma} \left[\frac{1}{P_1^{1-\sigma}} (\mu E_1 - \delta_1 P_1 G_1 N_1) + \phi_{12} \frac{1}{P_2^{1-\sigma}} (\mu E_2 - \delta_2 P_2 G_2 N_2) \right]$$

and

$$\pi_2 = \frac{1}{\sigma} \left[\frac{1}{P_2^{1-\sigma}} (\mu E_2 - \delta_2 P_2 G_2 N_2) + \phi_{21} \frac{1}{P_1^{1-\sigma}} (\mu E_1 - \delta_1 P_1 G_1 N_1) \right]$$

are substituted into

$$E_1 \equiv L_1 + N_1 \left(\pi_1 - \rho_1 \Lambda_1^2 + \delta_1 P_1 G_1 \right)$$

and

$$E_2 \equiv L_2 + N_2 \left(\pi_2 - \rho_2 \Lambda_2^2 + \delta_2 P_2 G_2 \right)$$

This give a system of two equations in two unknowns (E_1 and E_2), that is

$$\begin{cases} E_1 = \lambda_1 L_W + N_1 \left\{ \frac{1}{\sigma} \left[\frac{1}{P_1^{1-\sigma}} \left(\mu E_1 - \delta_1 P_1 G_1 N_1 \right) + \phi_{12} \frac{1}{P_2^{1-\sigma}} \left(\mu E_2 - \delta_2 P_2 G_2 N_2 \right) \right] - \rho_1 \Lambda_1^2 + \delta_1 P_1 G_1 \right\} \\ E_2 = \left(1 - \lambda_1\right) L_W + N_2 \left\{ \frac{1}{\sigma} \left[\frac{1}{P_2^{1-\sigma}} \left(\mu E_2 - \delta_2 P_2 G_2 N_2 \right) + \phi_{21} \frac{1}{P_1^{1-\sigma}} \left(\mu E_1 - \delta_1 P_1 G_1 N_1 \right) \right] - \rho_2 \Lambda_2^2 + \delta_2 P_2 G_2 \right\} \end{cases}$$

The system can be solved to find E_1 and E_2 for given masses of firms producing in each country, price indexes, population size and network structures with relative savings. The solution for E_1 is given in expression (32) in the main text, while an analogous expression holds for E_2.

ABOUT RETHINK-GSC

The project ‚Rethinking Global Supply Chains: Measurement, Impact and Policy' (RETHINK-GSC) captures the impact of knowledge flows and service inputs in Global Supply Chains (GSCs). Researchers from 11 institutes are applying their broad expertise in a multidisciplinary approach, developing new methodologies and using innovative techniques to analyse, measure and quantify the increasing importance of intangibles in global supply chains and to provide new insights into current and expected changes in global production processes.

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