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Escaping the Unemployment Trap - The Case of East Germany -

Christian Merkl and Dennis J. Snower

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Kiel Institute for World Economics

Duesternbrooker Weg 120 24105 Kiel (Germany)

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Escaping the Unemployment Trap - The Case of East Germany -

Abstract

This paper addresses the question of why prolonged regional unemployment differentials tend to persist even after their proximate causes have been reversed (e.g., after wages in the highunemployment regions have fallen relative to those in the low-unemployment regions). We suggest that the longer people are unemployed, the greater is the likelihood of falling into a low-productivity "trap," through the attrition of skills and work habits. We develop and calibrate a model along these lines for East Germany and examine the effectiveness of three employment policies in this context: (i) a weakening of workers' position in wage negotiations due to a drop in the replacement rate or firing costs, leading to a fall in wages, (ii) hiring subsidies, and (iii) training subsidies. We show that the employment effects of these policies depend crucially on whether low-productivity traps are present.

Keywords: labor markets; labor market traps; calibration; East Germany

JEL: E24, J30, J31, J64

^{*}Christian Merkl, Dennis J. Snower

The Kiel Institute for the World Economy (IfW), Düsternbrooker Weg 120, 24105 Kiel, Germany Telephone: +49 431 8814 235, Fax: +49 431 8814 501 E-Mail: christian.merkl@ifw-kiel.de, dennis.snower@ifw-kiel.de

1 Introduction

The persistence of large European regional unemployment differentials - particularly within the large European economies, France, Germany, Italy and Spain - remains a challenge to economists, despite a prodigious literature on the subject (e.g. Decressin and Fatás, 1995, Elhorst, 2005, Faini et al., 1997, Gray, 2004, Sinn and Westermann, 2001, Taylor and Bradley, 1997). The mystery is not how these unemployment differentials arose, for usually regions of relatively high unemployment are generally ones in which labor costs have been relatively high in relation to productivity. Rather, the mystery is why unemployment differentials far outlive their original causes. Specifically, once the unemployment differentials have persisted for a long time, then they do not go away, even after labor costs fall relatively to productivity. Why?

East Germany is a good case example. After German reunification in 1991, East German real wages rose dramatically relative to productivity and unemployment jumped upwards in response. With the social and monetary union in October 1990, East German labor costs jumped from 7% (using the informal exchange rate) to about one half of the West German level (see e.g., Franz and Steiner, 2000, Sinn, 2002). Since then, however, labor costs have fallen steadily in relation to productivity, but the employment rate has remained stubbornly low, hovering near 20 percent for the past decade (see figure 1¹). Traditional labor market analysis has trouble accounting for this experience.

This paper suggests a simple explanation²: Once people remain unemployed for a long time, they tend to fall into a "trap" representing a contraction of their employment opportunities. Snower and Merkl (2006) describe several such traps, but do not model them. Consider a few examples.

Immediately after German reunification, East German wage bargaining was conducted primarily by *West German* unions and employers, and these had strong incentives to push East German wages up, in order to reduce migration of East German workers to West Germany and of West German firms to the East. Given the low short-run elasticity of labor demand, this "bargaining by proxy" was not only in the interests of West German unions, but also West German firms who feared the entry of new firms sparked by the new migration flows. The upward wage pressure was reinforced through generous unemployment benefits and associated welfare entitlements. The resulting East German wage hike led to a sharp fall in East German employment, and this effect was prolonged through the introduction of generous job security provisions and costly hiring regulations, which raised the persistence of employment (i.e. made current employment depend more heavily on past employment). The persistently low employment was mirrored in long-term unemployment.³

This is where possibility of traps arises. The long-term unemployed are prone to attrition of skills and work habits and they are of course unable to get on-the-job training. As their productivity falls, they find more difficult to find jobs, even if labor costs fall relative to the average productivity of the employed workforce.

Naturally, if these "efficiency labor costs," i.e. labor costs deflated by average productivity, fell sufficiently to more than compensate for the drop in the productivity of the long-term unemployed, then their employment opportunities would improve; but the data appear to suggest that these costs did not fall enough.

Furthermore, the massive East German investment subsidies that were granted in the aftermath of reunification - often paid to prevent uncompetitive firms to lay off their employees - resulted in the creation of capital that was relatively unproductive and prone to underutiliza-

¹Sources: Bundesagentur für Arbeit (2006a, b) and Statistische Ämter des Bundes und der Ländern (2006), own calculations.

 $^{^{2}}$ For an alternative explanation see Uhlig (2006).

³The share of long-term unemployed (with a duration of more than one year) has increased from one quarter in 1992 to roughly one half today (Sachverständigenrat, 2004).



Figure 1: East German labor cost normalized by productivity and the employment rate for dependently employed workers.

tion (see, for example, Sinn, 1995). The labor cooperating with this capital became similarly unproductive and underutilized, even if efficiency labor costs subsequently fall.

What these traps have in common is that they are both associated with low productivity⁴: the long-term unemployed are prone to become less productive and this traps them in unemployment. The drop in productivity may arise either because workers lose skills or because they lose access to "good jobs" (i.e. highly productive, well-paying ones).

This paper models such a trap, and examines its implications for labor market activity and employment policy. We build an analytical model of the low-productivity trap and calibrate it for the East German labor market. In this context, we inquire which policies are effective in creating employment.

The trap highlights a major, often ignored, cost of long-term unemployment. A specific rise in efficiency labor costs sends employees into short-term unemployment; but should this state persist and thus turn into long-term unemployment, then an equal and opposite fall in efficiency labor costs may be insufficient to bring these workers back into employment.

Our notion of a labor market "trap" is related to the literature on segmented labor markets, for example, models that divide the labor market into a high-wage "primary sector" and a "secondary sector" that is market clearing.⁵

This paper contributes to this literature by explaining sources of mobility between the two sectors and examining the implications for employment and unemployment dynamics. As noted, our model describes a labor market where workers in the primary sector who become unemployed risk losing their skills or their access to high-productivity jobs (for instance, because they become stigmatized and demotivated through their unemployment spell), and thereby they risk sinking into the "trapped" sector. The longer they are unemployed, the greater this risk becomes. On the other hand, workers who are employed in the trapped sector may gain skills

⁴See Fuchs-Schündeln and Izem (2007) and Ragnitz (2007) for a thorough analysis of the low labor productivity in East Germany. See Burda (2006) for a neo-classical model of economic integration with adjustment costs, which explains the "capital deepening" and the "labor thinning" in the East.

⁵See, for example, Bulow and Summers (1986), McDonald and Solow (1985), Weitzman (1989), Dickens and Lang (1988) for the early foundations of this literature and Kleven and Sorensen (2004) and Lommerud et al. (2004) for more recent contributions. For the empirical literature see, for example, Dickens and Lang (1985), Saint-Paul (1996) for a survey and Ghilarducci and Lee (2005) for a recent contribution.

or access to high-productivity jobs (e.g. by using their jobs to gain information and contact to other employment opportunities), and thereby they may rise into the primary sector. The longer they remain employed, the greater is the likelihood of rising. In short, unemployment is the road to bad jobs and long-term unemployment, whereas employment is the road to good jobs and shorter unemployment spells.

As shown below, these dynamic relations have important implications not only for the persistence of employment and unemployment, but also for the effectiveness of labor market policies. Specifically, we show that

- the existence of low-productivity traps implies that reductions in wages in the trapped sector (induced, say, by cuts in unemployment benefits or firing costs), on their own, are relatively ineffective in raising the corresponding employment rate (both in relation to the primary sector and an economy without low-productivity traps).
- hiring subsidies for the trapped unemployed have a relatively strong positive influence on employment, i.e. for a given subsidy size (both absolute and relative to the wage) they are more cost-effective⁶ than hiring subsidies for primary unemployed. There are two driving forces: The presence of traps reduces the deadweight effects of hiring subsidies and hiring subsidies enable more trapped workers to move to the primary sector via on the job training.
- training subsidies and programs that raise the productivity of workers in the trapped sector, thereby improving their chances of entering the primary sector, may also have a relatively strong employment long-run effect, but this effect takes a long time to manifest itself.

The paper is organized as follows. Section 2 presents our model. In Section 3 this model is calibrated for the East German labor market. Section 4 considers the policy implications. Finally, Section 5 concludes.

2 The Model

Our labor market has a "primary" sector and a "trapped" sector. The average productivity per worker in the trapped sector is assumed to be lower in the trapped (a_T) than in the primary sector (a_P) . Moreover, firms face a random cost ε_t , iid across workers and time, with a constant cumulative distribution $\Gamma(\varepsilon_t)$. This cost may be interpreted as an operating cost or as a negative productivity shock.

Decisions in the labor market are made in the following sequence: First, workers move between sectors. Specifically, each unemployed worker in the primary sector has an exogenously given probability ν of losing productivity and thereby entering the trapped sector (due either to skill attrition or loss of access to good jobs); and each employed worker in the trapped sector has an exogenously given probability ϖ of gaining productivity and thereby ascending to the primary sector.⁷ Second, the wage is determined through bargaining. Third, the value of the random cost ε_t is revealed. Finally, firms make their hiring and firing decisions.

Let the hiring rates of workers in the primary and trapped sectors be η_P and η_T , respectively, and let their firing rates from these sectors be ϕ_P and ϕ_T , respectively. (These hiring and firing

⁶We call a policy more "cost effective" than another policy when it generates more employment, for a given net government expenditure outlay.

⁷Thus the cumulative probability of that an unemployed primary worker falls into the low-productivity trap rises with the duration of unemployment, and the cumulative probability of an employed trapped worker to escape from the trap rises with employment duration.

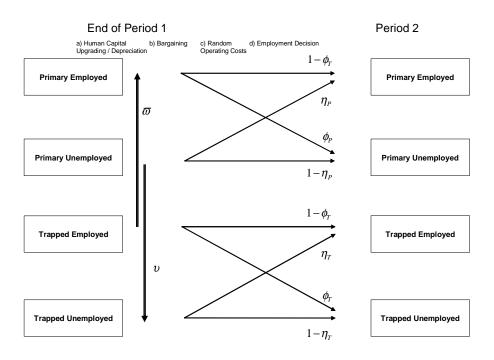


Figure 2: Transition Probabilities

rates will be derived choice-theoretically below.) The transitions between the various economic states are pictured in figure 2. Each employed primary and trapped worker remains employed with probability $(1 - \phi_P)$ and $(1 - \phi_T)$, respectively; she becomes unemployed with probability ϕ_P and ϕ_T , respectively. Each unemployed primary and trapped worker remains unemployed with probability $(1 - \eta_P)$ and $(1 - \eta_T)$, respectively; she becomes employed with probability η_P and η_T , respectively.

2.1 Wage Determination

We assume that the wage is the outcome of a Nash bargain between the median insider and her firm in the respective sector.⁸ The median insider faces no risk of dismissal at the negotiated wage.⁹

There are constant returns to labor.¹⁰ Under bargaining agreement, the insider receives the

⁹This assumption is made merely for analytical convenience; various other assumptions would lead to similar results. The wage could e.g. be the outcome of a bargain between the firm and the marginal worker, or between the firm and a union representing all employees. In this last case, the insiders' objective in the bargain will depend on their retention rate.

¹⁰In what follows, only those variables have time subscripts that, for given parameter values, actually vary through time in our model. j is the index for the sector. It can either be P (primary sector) or T (trapped

⁸The critical reader may object that insider power has been seriously eroded in East Germany due to the fall in union membership since reunification. The first response to this objection is that we should not confuse our insider bargaining with union bargaining, since our Nash bargaining problem could be interpreted as the individual median insider bargaining with her firm. Second, much of the erosion of East German insider power since reunification has resulted from the replacement of bargaining by proxy (in which West German unions and firms had dominant influence on negotiations about East German wages) by self-sufficient bargaining (in which East German workers and firms have taken control of East German wage determination). In our model, we assume that East German wage determination is entirely self-sufficient in this sense. And finally, although union membership has dropped in East Germany, union wage agreements still have very broad coverage. For example, in 2003 firms that were covered by a firm level or sectoral wage agreement employed 54 percent of all workers in East Germany. A large share of the other firms followed existing wage agreements voluntarily, covering 52 percent of the remaining employees (Schnabel, 2005).

wage $w_{T,t}$ and the firm receives the expected profit $(a_T - w_{T,t})$ in each period t. The expected present value of returns to a trapped insider under bargaining agreement $(V_{T,t}^I)$ is

$$V_{T,t}^{I} = w_{T,t} + \delta \left(\begin{array}{c} (1 - \varpi) \left(1 - \phi_{T,t+1} \right) V_{T,t+1}^{I} + (1 - \varpi) \phi_{T,t+1} \left(1 - \varpi \right) V_{T,t+1}^{U} \\ + \varpi \left(1 - \phi_{P,t+1} \right) V_{P,t+1}^{I} + \varpi \phi_{P,t+1} V_{P,t+1}^{U} \end{array} \right)$$
(1)

where δ is the discount factor and $V_{T,t+1}^U(V_{P,t+1}^U)$ is the expected present value of returns of an unemployed trapped (primary) worker and $V_{T,t+1}^I(V_{P,t+1}^I)$ is the expected present value of returns of an employed trapped (primary) worker, respectively. Note that with probability ϖ a trapped worker is upgraded to the primary sector and thus has a higher future present value. The expected present value of returns to the firm under bargaining agreement is

$$\widetilde{\Pi}_{T,t}' = (a_T - w_{T,t}) + \delta \left(\begin{array}{c} (1 - \varpi) \left(1 - \phi_{T,t+1} \right) \widetilde{\Pi}_{T,t+1}^I - (1 - \varpi) \phi_{T,t+1} f_{T,t+1} \\ + \varpi \left(1 - \phi_{P,t+1} \right) \widetilde{\Pi}_{P,t+1}^I - \varpi \phi_{P,t+1} f_{P,t+1} \end{array} \right)$$
(2)

where $\widetilde{\Pi}_{T,t+1}^{I}$ ($\widetilde{\Pi}_{P,t+1}^{I}$) is the future profit in the trapped (primary) sector, weighted with the probability that the worker stays in the respective sector.

Under disagreement, the insider's fallback income is $b_{T,t}$, assumed equal to the unemployment benefit. The firm's fallback profit is $-f_{T,t}$, which is the firing cost per employee (in the trapped sector). In words, during disagreement the insider imposes the maximal cost on the firm (e.g. through strike, work-to-rule, sabotage) short of inducing dismissal. Assuming that disagreement in the current period does not affect future returns, the present values of insider's returns under disagreement is

$$V_{T,t}^{\prime I} = b_{T,t} + \delta \left(\begin{array}{c} (1-\varpi) \left(1-\phi_{T,t+1}\right) V_{T,t+1}^{I} + (1-\varpi) \phi_{T,t+1} \left(1-\varpi\right) V_{T,t+1}^{U} \\ + \varpi \left(1-\phi_{P,t+1}\right) V_{P,t+1}^{I} + \varpi \phi_{P,t+1} V_{P,t+1}^{U} \end{array} \right)$$
(3)

and the present value of the firm's agreement under disagreement is

$$\widetilde{\Pi}_{T,t}^{\prime} = -f_{T,t} + \delta \left(\begin{array}{c} (1 - \varpi) \left(1 - \phi_{T,t+1} \right) \widetilde{\Pi}_{T,t+1}^{I} - (1 - \varpi) \phi_{T,t+1} f_{T,t+1} \\ + \varpi \left(1 - \phi_{P,t+1} \right) \widetilde{\Pi}_{P,t+1}^{I} - \varpi \phi_{P,t+1} f_{P,t+1} \end{array} \right)$$
(4)

Thus the insider's bargaining surplus is

$$V_{T,t}^{I} - V_{T,t}^{\prime I} = w_{T,t} - b_{T,t}$$
(5)

and the firm's bargaining surplus is

$$\widetilde{\Pi}_{T,t} - \widetilde{\Pi}_{T,t}^{I} = a_T - w_{T,t} + f_{T,t}$$
(6)

The negotiated wage maximizes the Nash product (Λ)

$$\Lambda = (w_{T,t} - b_{T,t})^{\gamma} \left(a_T^I - w_{T,t} + f_{T,t} \right)^{1-\gamma},$$
(7)

where γ represents the bargaining strength of the insider relative to the firm. Thus the negotiated wage is

$$w_{T,t} = (1 - \gamma) b_{T,t} + \gamma \left(a_T + f_{T,t} \right).$$
(8)

The bargaining problem is analogous in the primary sector (see Appendix), so that the negotiated primary wage is

$$w_{P,t} = (1 - \gamma) b_{P,t} + \gamma (a_P + f_{P,t}).$$
(9)

sector).

2.2 Employment Decision

Having determined the wage, we now proceed to derive the hiring and firing rates for the primary and trapped sector.

2.2.1 Primary Sector

Given the realized value of the random cost variable ε_t , which is iid across individuals and time and whose mean is normalized to zero, an insider generates the following present value of expected profit:¹¹

$$\Pi_t = -\varepsilon_t + \sum_{t=0}^{\infty} \delta^t \left(1 - \phi_P\right)^t \left(a_P - w_P\right) - \delta\phi_P f_P \sum_{t=0}^{\infty} \delta^t \left(1 - \phi_P\right)^t.$$
(10)

i.e. with probability $(1 - \phi_P)$ the insider is retained and generates profit $(a_P - w_P)$, whereas with probability ϕ_P is fired and generates the firing cost f_P (constant per employee).

The insider is fired when her generated profit is less than the firing cost: $\Pi_t < -f_P$, so that $\varepsilon_t > (a_P - w_P + (1 - \delta) f_P) / (1 - \delta (1 - \phi_P))$. Recalling that $\Gamma(\varepsilon_t)$ is the cumulative density of the random cost ε_t , the firing rate is given by the following implicit function:¹²

$$\phi_P = 1 - \Gamma\left(\frac{a_P - w_P + (1 - \delta) f_P}{1 - \delta (1 - \phi_P)}\right)$$
(11)

The firm faces a hiring cost of h, constant per worker. An entrant is hired when his generated profit exceeds this hiring cost: $\Pi > h_P$. Thus the hiring rate is

$$\eta_P = \Gamma\left(\frac{a_P - w_P - \delta\phi_P f_P}{1 - \delta\left(1 - \phi_P\right)} - h_P\right)$$
(12)

2.2.2 The Trapped Sector

As noted, each worker in the trapped sector is assumed to have an average productivity a_T that is lower than the one of his counterpart in the primary sector. Furthermore, trapped workers have a probability ϖ of moving into the primary sector. Thus, the present value of the profit generated by an entrant in the trapped sector is¹³

$$\Pi_{t} = -\varepsilon_{t} + \frac{a_{T} - w_{T} - \delta(1 - \varpi)\phi_{T}f_{T}}{1 - \delta(1 - \phi_{T})(1 - \varpi)} - \phi_{P}\delta\varpi\frac{f_{P}}{(1 - \delta(1 - \varpi)(1 - \phi_{T}))} + (1 - \phi_{P})\delta\varpi\left(\frac{a_{P} - w_{P} - \delta\phi_{P}f_{P}}{(1 - \delta(1 - \phi_{P}))(1 - \delta(1 - \varpi)(1 - \phi_{T}))}\right)$$
(13)

Along the same lines as before, a worker is fired if her expected profits are smaller than minus the firing costs $(\pi_t < -f_T)$:

$$\phi_T = 1 - \Gamma \left(\begin{array}{c} \frac{a_T - w_T - \delta(1 - \varpi)\phi_T f_T}{1 - \delta(1 - \phi_T)(1 - \varpi)} + f_T - \phi_P \delta \varpi \frac{f_P}{(1 - \delta(1 - \varpi)(1 - \phi_T))} \\ + (1 - \phi_P) \delta \varpi \left(\frac{a_P - w_P - \delta \phi_P f_P}{(1 - \delta(1 - \phi_P))(1 - \delta(1 - \varpi)(1 - \phi_T))} \right) \end{array} \right)$$
(14)

And she is hired if the expected profits are bigger than the hiring costs in the trapped sector $(\pi_t > h_T)$.

¹¹In what follows, only those variables have time subscripts that, for given parameter values, actually vary through time in our model.

¹²We assume that $(\partial \Gamma / \partial \phi) > -1$, so that a rise in (a - w) or f both reduce the firing rate.

¹³See the Appendix for a detailed derivation.

$$\eta_T = \Gamma \left(\begin{array}{c} \frac{a_T - w_T - \delta(1 - \varpi)\phi_T f_T}{1 - \delta(1 - \phi_T)(1 - \varpi)} - h_T - \phi_P \delta \overline{\omega} \frac{f_P}{(1 - \delta(1 - \varpi)(1 - \phi_T))} \\ + (1 - \phi_P) \delta \overline{\omega} \left(\frac{a_P - w_P - \delta \phi_P f_P}{(1 - \delta(1 - \phi_P))(1 - \delta(1 - \varpi)(1 - \phi_T))} \right) \end{array} \right)$$
(15)

2.3 Employment Dynamics

We allow for the possibility that the employed workers in the trapped sector may raise their productivity - through learning-by-doing, improved work motivation, better work habits and so forth - and then move into the primary sector. Specifically, we also allow for the possibility that unemployed workers in the primary sector may lose productivity - through attrition of human capital, reduced work motivation, lost work habits, etc. - and then fall into the trapped sector. In particular, we assume that, in each period, a constant proportion ϖ of the employed workers in the trapped sector ascend to the primary sector, and a constant proportion v of the unemployed primary workers descend into the trapped sector.

Thus, we obtain the following employment equation for the primary sector:¹⁴

$$N_{P,t} = (1 - \phi_P) N_{P,t-1} + (1 - \phi_P) \varpi N_{T,t-1} + \eta_P (1 - \upsilon) U_{P,t-1}$$
(16)

The number of employed in the primary sector $(N_{P,t})$ consists of workers who are retained from the previous period¹⁵ plus the newly hired workers $(\eta_P (1 - v) U_{P,t-1})$.

For the trapped sector the employment dynamics equation is:

$$N_{T,t} = (1 - \phi_T) (1 - \varpi) N_{T,t-1} + \eta_T (U_{T,t-1} + v U_{P,t-1})$$
(17)

The number of employed workers in the trapped sector is equal to those who are retained and have not received a human capital upgrade $((1 - \phi_T)(1 - \varpi)N_{T,t-1})$ plus the newly hired workers $(\eta_T (U_{T,t-1} + vU_{P,t-1}))$.¹⁶

After some re-formulations (see Appendix), we obtain an employment dynamics equation (expressed in employment rates) for the primary sector

$$n_{P,t} = \frac{1}{g_{t,P}} \left[(1 - \phi_P) n_{P,t-1} + (\eta_P (1 - \upsilon)) (1 - n_{P,t-1}) \right] + (1 - \phi_P) \varpi \frac{L_{T,t-1}}{L_{P,t}} n_{T,t-1}$$
(18)

and for the trapped sector

$$n_{T,t} = \frac{1}{g_{t,T}} \left[(1 - \phi_T) \left(1 - \varpi \right) n_{T,t-1} + \eta_T \left(1 - n_{T,t-1} \right) \right] + \eta_T \upsilon \left(1 - n_{P,t-1} \right) \frac{L_{P,t-1}}{L_{T,t}}$$
(19)

where L_P and L_T are the labor forces of the primary and secondary sector. $g_{t,P} = L_{P,t}/L_{P,t-1}$ and $g_{t,T} = L_{T,t}/L_{T,t-1}$ are the labor force growth in the primary and trapped sector.

The labor force in each sector is equal to the previous period's labor force plus the net movement from the other sector:

$$L_{P,t} = L_{P,t-1} - \upsilon u_{P,t-1} L_{P,t-1} + \varpi n_{T,t-1} L_{T,t-1}$$
(20)

and

¹⁴Note that capital letters (N, U) refer to levels, while small letters (n, u) are (un-)employment rates.

 $^{^{15}(1-\}phi_P) N_{P,t-1}$ are the primary employees carried forward from the previous period and $(1-\phi_P) \varpi N_{T,t-1}$ are the previously trapped workers who received a human capital upgrade.

¹⁶Note that the pool of potential recruits is enlarged by those who moved from the primary to the trapped sector $(vU_{t-1,P})$.

$$L_{T,t} = L_{T,t-1} + \upsilon u_{P,t-1} L_{P,t-1} - \varpi n_{T,t-1} L_{T,t-1}.$$
(21)

Setting the sectoral growth rate to zero and omitting time subscripts, we obtain the following steady state value for the employment in the primary sector

$$n_P = \frac{\eta_P \left(1 - \upsilon\right) + \left(1 - \phi_P\right) \varpi \frac{\eta_T \frac{L_T}{L_P} + \eta_T \upsilon}{\left(1 - \left[\left(1 - \phi_T\right)\left(1 - \varpi\right)\right] + \eta_T\right)}}{\phi_P + \left(\eta_P \left(1 - \upsilon\right)\right) + \left(1 - \phi_P\right) \varpi \frac{\eta_T \upsilon}{\left(1 - \left[\left(1 - \phi_T\right)\left(1 - \varpi\right)\right] + \eta_T\right)}}$$
(22)

and in the trapped sector

$$n_{T,t} = \frac{\eta_T + \eta_T \upsilon \left(1 - n_P\right) \frac{L_P}{L_T}}{\left(1 - \left[\left(1 - \phi_T\right) \left(1 - \varpi\right)\right] + \eta_T\right)}$$
(23)

Logically, if we set $v = \varpi = 0$, we have two entirely separated sectors in this economy and the above formula delivers the well-known formula:

$$n_P = \frac{\eta_P}{\phi_P + \eta_P} \text{ and } n_T = \frac{\eta_T}{\phi_T + \eta_T}$$
(24)

3 Calibration of the Model

In 2004, 17.2 percent of the East German full time employed workers were below the low wage income threshold, which is defined a two thirds of the East German median income, i.e. they earned below 7.36 \in per hour (Rhein and Stamm, 2006). We consider these workers as a good proxy for the trapped sector. From Hunt (2004) we know that about 60 to 80 percent of unemployed in East Germany do not "survive" their first year of unemployment, i.e. they leave unemployment within one year, which we interpret as hiring. During the second year of unemployment the non-survival rate drops to much smaller numbers, roughly ranging in the magnitude of 20 to 50 percent (very much dependent on gender and observation period), with even smaller non-survival rates thereafter. It can be assumed that trapped workers represent a large share of the long-term unemployed since they have lower hiring rates and higher firing rates than primary workers. However, they do not do so exclusively, since primary workers in our model can stay unemployed for several periods without becoming employed and trapped (although the probability is decreasing over time). For simplicity, we set the steady state (indicated by the subscript $_{0}$) hiring rate for trapped workers ($\eta_{T,0}$) to 30 percent and the one for primary workers to 80 percent (η_{P0}) , roughly corresponding to Hunt's (2004) non-survival rates for long-term and short-term unemployed. In accordance with a transition table for the European Union (one year transition probability from "low pay" to "no pay", see European Commission, 2004), we set the steady state firing rate for trapped workers equal to $\phi_{T,0} = 0.18$. To obtain an aggregate employment rate of 80 $percent^{17}$, we set the steady state firing rate in the primary sector (ϕ_{P0}) to 12 percent.

Furthermore, we have to choose an exogenous probability of an employed trapped worker to move to the trapped sector (ϖ). According to Rhein et al. (2005) the probability for German low wage income earners to move beyond the low income threshold after 5 years is 32.5 percent.¹⁸ The European Commission (2004) calculates a probability of 50 percent for a low-pay worker to move to a higher pay within seven years.¹⁹ In line with these two pieces of evidence, we set $\varpi = 0.08$. By setting the labor share of primary workers to 76 percent,

¹⁷This corresponds to the employment rate of dependently employed in East Germany (see Bundesagentur, 2006a, b).

¹⁸Corresponding to an average yearly probability of 7.6 percent.

¹⁹Corresponding to an average yearly probability of 9.4 percent.

about 17 percent of all employed workers belong to the trapped sector; thus corresponding to the numbers by Rhein and Stamm (2006). To obtain a stable initial equilibrium, we set the probability of a primary worker to move to the trapped sector (v) to 11.2 percent.²⁰ In our initial equilibrium the unemployment rate in the primary sector is 12 percent, whereas it amounts to 35 percent in the trapped sector.

We set the replacement rates in the primary and trapped sector to 65 and 80 percent, respectively.²¹ Aggregate real productivity (*a*, gross value added per worker) in 2005 was about \in 38,000 and real wages (*w*, measured as real labor costs) were about \in 22,000 in East Germany.²² (All estimates are divided by the German GDP deflator, base year 1991.²³). We set the productivity for trapped workers to 50 percent of the economy's average, while setting the one of primary workers to 110 percent of the average productivity.

Furthermore, we assume that in the long-run the productivity and all real costs (the wage, the hiring and firing costs and the operating cost ε) grow at the same rate of two percent ($\alpha = 1.02$). All future values are discounted (δ) at rate 3%.²⁴

In the literature firing costs (f_t) and hiring costs (h_t) which amount to 60 percent and 10 percent of labor costs, respectively, are proposed (Chen and Funke, 2003). It is well known that the employment duration is one of the most important determinants of firing costs²⁵. Thus, we set them to 40 percent for trapped workers, whose employment duration is shorter due to higher firing rates, and to 70 percent for primary workers. We assume that all workers have the same bargaining bargaining power is set equally for both sectors ($\mu = 0.195$) in order to match the aggregate labor costs in East Germany.

We simulate our model in a linearized form, choosing first derivatives of the cumulative function that replicate the employment path from 1991 to 2004 as closely as possible in the homogeneous model. (For the derivation of the linearized equations see Appendix.)

4 Policy Exercises

We now consider the effects of various labor policies in the context of our calibrated model of the East German labor market. We first examine the employment effects of policies targeted at the trapped sector, and then investigate untargeted policies. In both cases, we explore the influence of (i) a reduction of the ratio of the firing costs to the wage ("firing cost ratio") together with a fall in the replacement ratio²⁶, (ii) hiring subsidies, (iii) training subsidies that raise the probability of moving from the trapped to the primary sector. For the training subsidies the policy can of course only be targeted at trapped employees.

²⁰This is necessary to guarantee that the condition $vU_{NT} = \varpi N_T$ holds, i.e. in the old steady state the number of people moving from the trapped to the non-trapped sector equals those moving into the other direction.

 $^{^{21}}$ The net replacement ratios (unweighted average across six family types) of workers with 67, 100, and 150 percent of average productivity are 78.25, 68.25, and 64.67 percent, respectively (OECD, 2006).

²²Source: Statistische Ämter des Bundes und der Länder (2006).

 $^{^{23}}$ This is done to make numbers comparable to Snower and Merkl (2006).

²⁴This is the average real interest rate over last 15 years, calculated as the yearly money market interest rate minus the inflation rate (using the GDP deflator). Source: International Financial Statistics, International Monetary Fund.

 $^{^{25}}$ See e.g. Grund (2006).

²⁶In Snower and Merkl (2006) we have done several ex-post policy exercises with a model that did not contain traps. Especially during the last years of the observation period (1991-2004), our prediction was more optimistic than the real outcome, suggesting the existence of labor market traps. The first policy exercise is the same as in Snower and Merkl (2006), but the innovation of this paper over Snower and Merkl (2006) is that it models the effects of labor market traps. It turns out that they have far-reaching implications for the effectiveness of employment policies, as shown below.

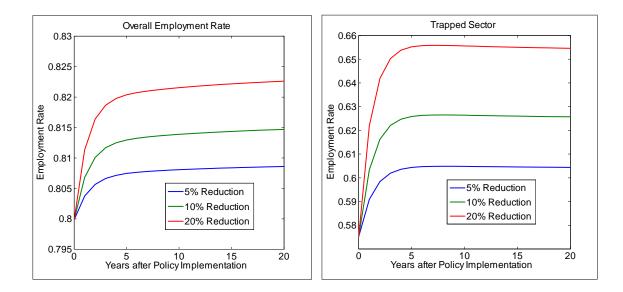


Figure 3: Effects of a Firing Cost Ratio (FCR) and Replacement Ratio (RR) Reduction in the Trapped Sector

4.1 Policies Targeted at the Trapped Sector

4.1.1 Lower Replacement Rate and Firing Costs

Figure 3 shows the effects of a 5, 10 and 20 percent reduction of both the firing cost ratio (the ratio of firing costs to the wage) and the replacement ratio (the ratio of unemployment benefits to the wage) in the trapped sector, which both take place in period 0:

Steady state effects: A lower replacement ratio and a lower firing cost ratio in the trapped sector affect the wage bargaining process. They change the fall-back position of both bargaining parties. As a consequence, insiders bid for lower wages. This improves firms' incentives to hire and retain more of the less productive workers and thus to increase their long-run employment rate in the trapped sector. A 20 percent reduction of the replacement ratio and firing cost ratio²⁷ makes wages fall to about two thirds of their initial steady state value. But this considerable reduction lifts the trapped sector's employment rate only from 58 percent to 65 percent. The reason can be found in the microfounded hiring and firing equations. Since trapped workers face a higher steady state firing rate, the expected future profits of an employed worker in the trapped sector is smaller than in the primary sector. For given operating costs this leads to smaller hiring and hiring sensitivities with respect to wage changes.

There are two reasons why the effects on the overall employment rate are quite moderate: (i) The trapped sector contains only a small share of all workers (24 percent). (ii) Only some of the newly hired workers obtain a human capital upgrade which leads to a higher employment rate, while most of the newly hired trapped workers face a high risk of being fired (compared to primary sector workers). In the long-run a 20 percent reduction of the replacement ratio and firing cost ratio in the trapped sector only reduces the share of trapped workers from 24 to 22 percent.

As a consequence, a 20 percent reduction of the replacement ratio and firing cost ratio (inducing a wage reduction to two thirds of the initial value) in the trapped sector increases the overall long-run employment rate only by 2 percentage points. This very insensitive reaction may explain why the recent reduction of the wages in East Germany (compared to the

²⁷Note that in the trapped sector wages react more sensitively to cuts in the replacement rate and firings costs than in the primary sector.

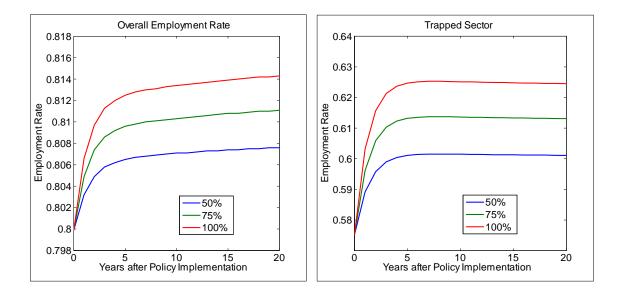


Figure 4: Effects of a Hiring Subsidy in the Trapped Sector

productivity) did not have much of an effect on the employment rate (see figure 1).²⁸

Adjustment dynamics: The increased hiring rate and reduced firing rates do not only lift the employment rate in the trapped sector. With more employed people and an exogenously given probability to move from the trapped to the primary sector, the sectoral upward movement increases. It takes a long time until this development shows its full effects: For a 20 percent reduction of the replacement ratio and the firing cost ratio, 90 percent of the convergence to the new steady state are realized only after 10 years.

If the replacement ratio of the most unemployment-prone group is reduced (the trapped unemployed), the described policy comes at the price of increased income inequality (between high income and low income earners). While this policy may help some trapped workers who would not have found a job otherwise and who get a chance to move to the primary sector, it hurts the insiders in the trapped sector who obtain a lower wage and the trapped workers who remain unemployed and receive lower unemployment benefits (due to lower unemployment benefits).²⁹

4.1.2 Hiring Subsidies

Figure 4 shows the employment effects of a hiring subsidy which is targeted at the trapped sector with different magnitudes (50, 75 and 100 percent of the respective wage).

Steady state effects: A hiring subsidy for trapped workers increases the firms' incentive to hire more workers with lower productivity. Other than in a homogenous economy, hiring subsidies deliver a double dividend. Besides the immediate hiring effects, there is a longer lasting "transition effect," caused by the inter-sectoral movement. The increased employment rate strengthens the upward mobility to the primary sector. A hiring subsidy of 100 percent would for example reduce the share of trapped workers (of the overall workforce) from 24 to 22.5 percent.

Adjustment dynamics: The after effects of the increased movement to the primary sector take some time to work themselves out: for a 100 percent hiring subsidies, 90 percent of the

 $^{^{28}}$ Note that the reduction of the employment rate at the beginning and middle of the ninenties can easily be explained by the initial wage shock. However, it is more difficult to explain the development during the last ten years.

²⁹See Brown, Merkl and Snower (2006) for a more detailed analysis of the inequality effects of different policies.

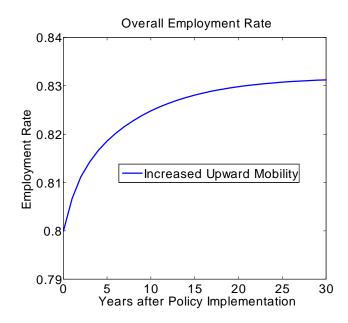


Figure 5: Effects of Training Subsidies

distance to the new steady state is reached after 12 years.

If hiring subsidies are targeted at trapped workers only (as done in the simulation), they are much more cost-effective³⁰ than an untargeted strategy: (i) the deadweight is much lower since the initial steady state hiring rates in the trapped sector are below those in the primary sector, (ii) the replacement ratio of trapped workers is above those of primary workers and thus the savings (in terms of the respective wage) generated by the job creation are much bigger, (iii) the aforementioned "transition effect" strengthens the overall outcome.

Hiring subsidies need to be financed. According to our simulation, long-run net expenditures caused by a 100 percent hiring subsidy³¹ for all trapped workers are about the same as the long-run net savings generated by a 7 percent reduction of the firing cost ratio and replacement ratio.³²

Hiring subsidies increase employment, without worsening the living standard of the poorest workers, namely, the unemployed trapped workers (since they continue to receive the same benefits as before). As a consequence, it may be easier from a political economy point of view to implement hiring subsidies than reducing the replacement ratio, which makes the unemployed workers worse off.

4.1.3 Training Measures

Training subsidies or other measures that improve job-related training (e.g. on the job training, qualification courses, training measures, etc.), could improve trapped workers' productivity and consequently their access to primary good. In our model, better training measures can be captured in terms of an increase in the exogenously given probability of moving from the trapped to the primary sector (ϖ). Figure 5 shows what happens if the probability of moving from the trapped to the primary sector increases from 8 to 16 percent. The latter number roughly corresponds to a rate found in many other European Union countries, such as Belgium, Denmark, France, Italy the Netherlands or Spain (European Commission, 2004).

Steady state effects: The training measures above raise the economy's overall steady state

³⁰Defined as employment effect for a given additional government expenditure.

³¹Of the labor costs in the trapped sector.

³²This calculation is based on an average tax rate of 20 percent and the aforementioned net replacement rates.

employment rate by moving more people to the primary sector which is associated with higher employment rates. Naturally, the steady state employment rate of the trapped sector does not increase, as only the inter-sectoral mobility is affected but not the sectoral hiring and firing rates. Thus, better training measures change the share of workers in the respective sectors. The aforementioned policy would increase the share of primary workers from 74 to 86.5 percent.

Adjustment dynamics: It takes a very long time until such a policy shows its full effects. In our model 90 percent of the distance to the new steady state would be reached 17 years after the implementation of the policy.

Furthermore, in reality it will be a challenge to design training measures in a way that they can effectively improve workers' upward mobility (for empirical work for East Germany see, for example, Lechner, Miquel and Wunsch, 2005, and Lechner and Wunsch, 2007).

4.2 Untargeted Policies

4.2.1 Reduction of Unemployment Benefits and Firing Cost Ratio

If the unemployment benefits and firing cost ratio are reduced for all workers (not just for those in the trapped sector), the employment effects will be modified as follows:

(i) The primary sector's hiring rate increases and the firing rate decreases, as firms' obtain an incentive to hire/retain more of the less productive workers.

(ii) While a higher employment rate in the primary sector is reached quickly, there are longlasting aftereffects through the intersectoral movement of labor. A lower unemployment rate in the primary sector means that fewer people drop into the trapped sector and thus the trapped sector shrinks compared to the primary sector. While a 20 percent cut in unemployment benefits and firing cost ratio for in the trapped sector only would increase the primary sector's share labor share from 76 to 78 percent, extending the policy to the entire economy would increase the primary sector's labor share from 78 to 88 percent.

(iii) If the firing rate in the primary sector goes down, there is a positive spillover effect on the hiring and firing rates in the trapped sector (see equations (14) and (15)). Since trapped workers have a constant probability of getting a human capital upgrade in the future, higher retention rates in the primary sector increase these workers' profitability, giving an incentive to firms to retain/hire more of the less productive workers.

4.2.2 Hiring Subsidies

In this section we compare untargeted hiring subsidies (provided to all workers) to those targeted at the trapped sector (as described in the previous section). Providing a 100 percent hiring subsidy³³ to all workers (instead of trapped workers only) would roughly double the employment effects which are shown in the previous section. However, such an exercise would come at a substantial cost to the government. Specifically, the net costs³⁴ of such an untargeted strategy would be about 9 times higher than those for a 100 percent hiring subsidy targeted at trapped unemployed. The main reason is the very substantial deadweight effect because the hiring rates in the primary sector are much bigger than in the trapped sector.

³³Measured in terms of the respective wage.

 $^{^{34}}$ Defined as the costs for the hiring subsidy minus the increased revenue from higher employment (via higher tax revenues with an assumed tax rate of 20 percent and lower costs for unemployment benefits) in the new steady state.

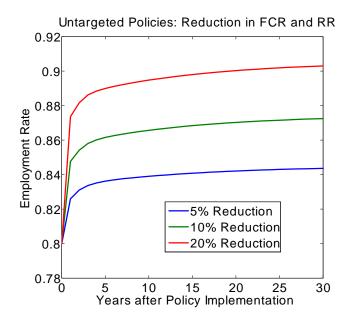


Figure 6: Effects of an Untargeted Reduction of the FCR and the RR

4.3 Summary of Calibration Results

4.3.1 Kick-Starting East Germany

Our calibration exercise shows that even very significant wage reductions in the trapped sector (induced by reductions in the respective replacement ratio and the firing cost ratio) would not be sufficient to bring East Germany to employment levels comparable to West Germany.³⁵ If the replacement ratio and firing cost ratio are reduced in the primary sector as well, this does not only make primary workers more profitable for firms, but also improves the average profitability of the trapped workers (each of them receives a human capital upgrade with a certain probability). Consequently, the employment rate in the trapped sector will rise. Furthermore, the lower unemployment rate in the primary sector will reduce the workers who move to the trapped sector, thus increasing the economy's ratio of primary to trapped workers. Our calibration shows that these spillover effects are very important. Reductions of the replacement ratio and firing cost ratio for all workers can improve the employment rate in the trapped sector and in the economy as a whole much more than a policy that is focused on trapped workers.

While an untargeted strategy is more effective for the reduction of the replacement ratio and firing cost ratio, the opposite is true for hiring subsidies. If they are targeted at the trapped sector, they turn out to be more cost effective than untargeted hiring subsidies, for the following reasons. In the presence of traps, hiring subsidies yield a double dividend of increased hiring and transition to the primary sector. Furthermore, the associated deadweight in the trapped sector is much smaller than in the primary sector. As shown in our calibration, the net budgetary outlay for an targeted subsidy is one ninth as high as the one for an untargeted hiring subsidy, while it delivers one half of the overall the employment effects.

Training measures improve the prospects of trapped workers and thus lift the economy's employment rate in the long-run. But it takes a long time until they show their full effects.

As shown above, a moderate cut in the replacement ratio and a reduction of the firing cost ratio can be combined with a substantial hiring subsidy in a self-financing policy package. Together with improved training measures these labor market policies would help the East to become somewhat more independent of the "caring hand that cripples" (Snower and Merkl,

³⁵This result differs very much from Snower and Merkl (2006) who show in a labor market model without traps that very moderate reforms at the beginning of the nineties would have had substantial positive effects.

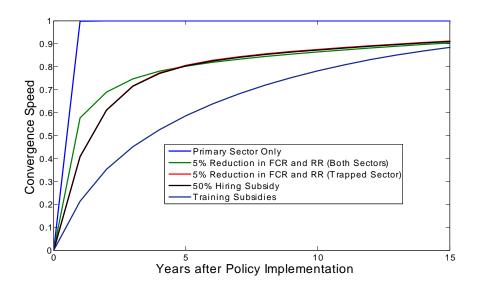


Figure 7: Convergence Speed of Different Policies

2006).

4.3.2 General Lessons for Regional Unemployment Problems

The behavior of the dual labor market, with a primary and a trapped sector differs in two substantial respects from a homogenous labor market:

(i) As shown above, even very substantial reductions in the replacement ratio and the firing cost ratio are not sufficient to reduce the unemployment ratio in the trapped sector to rates which can usually be observed in continental European countries, say around 10 percent.

(ii) The effects of different labor market policies are much more persistent under a dual labor market than under a homogenous labor market. We illustrate this phenomenon in figure 7. It takes at least a decade for policies like the reduction of the replacement ratio and firing cost ratio or hiring subsidies to show 90 percent of their after effects. Training subsidies need even more time to show 90 percent of their full after effects. For a comparison: In an economy which only consists of the primary sector, almost the whole effects of labor market reforms would already be visible after one year ("Primary Sector Only").

5 Concluding Thoughts

The paper explains a puzzling aspect of regional employment and unemployment differentials, namely, that they are very persistent despite changes in wages relative to productivity. Therefore, we develop a dual labor market model with a primary and trapped sector. We show numerically that the trapped sector of the economy, which faces an enormous unemployment rate, reacts very sluggishly to reductions of the wage. We propose additional measures to leave the trap, namely hiring subsidies and better training schemes.

East Germany is simply an extreme example of this phenomenon, which also exists in Spain and Italy and elsewhere. This phenomenon makes the inequality across regions especially persistent and policy makers have been at a loss about how to treat this problem. Our paper provides new insights on which policies are useful and effective under these circumstances and on potential trade-offs which policy makers face.

6 References

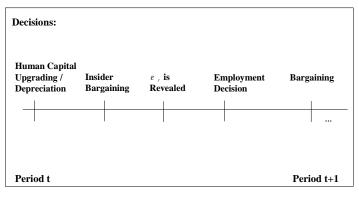
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7 Appendix

7.1 Sequencing of Decisions



Time Axis

Figure 8: Sequencing of Decisions

7.2 Wage Bargaining

7.2.1 Bargaining in the Primary Sector

The expected present value of returns to a primary insider under bargaining agreement $(V_{P,t}^{I})$ is

$$V_{P,t}^{I} = w_{P,t} + \delta \left(\left(1 - \phi_{P,t+1} \right) V_{P,t+1}^{I} + \left(1 - \phi_{P,t+1} \right) V_{P,t+1}^{U} \right)$$
(25)

where δ is the discount factor and $V_{P,t+1}^U$ is the expected present value of returns of an unemployed primary worker and $V_{P,t+1}^I$ is the expected present value of returns of an employed primary worker. The expected present value of returns to the firm under bargaining agreement is

$$\widetilde{\Pi}_{P,t}' = (a_P - w_{P,t}) + \delta \left(\left(1 - \phi_{P,t+1} \right) \widetilde{\Pi}_{P,t+1}^I - \phi_{P,t+1} f_{P,t+1} \right)$$
(26)

where $\widetilde{\Pi}_{P,t+1}^{I}$ is the future profit in the primary.

Under disagreement, the insider's fallback income is $b_{P,t}$, assumed equal to the unemployment benefit. The firm's fallback profit is $-f_{P,t}$, which is the firing cost per employee (in the trapped sector). Assuming that disagreement in the current period does not affect future returns, the present values of insider's returns under disagreement is

$$V_{P,t}^{\prime I} = b_{P,t} + \delta \left(\left(1 - \phi_{P,t+1} \right) V_{P,t+1}^{I} + \left(1 - \phi_{P,t+1} \right) V_{P,t+1}^{U} \right)$$
(27)

and the present value of the firm's agreement under disagreement is

$$\widetilde{\Pi}_{P,t}^{\prime} = -f_{P,t} + \delta\left(\left(1 - \phi_{P,t+1}\right)\widetilde{\Pi}_{P,t+1}^{I} - \phi_{P,t+1}f_{P,t+1}\right)$$

$$(28)$$

Thus the insider's bargaining surplus is

$$V_{P,t}^{I} - V_{P,t}^{\prime I} = w_{P,t} - b_{P,t}$$
⁽²⁹⁾

and the firm's bargaining surplus is

$$\widetilde{\Pi}_{P,t} - \widetilde{\Pi}_{P,t}^{I} = a_P - w_{P,t} + f_{P,t}$$
(30)

The negotiated wage maximizes the Nash product (Λ)

$$\Lambda = (w_{P,t} - b_{P,t})^{\gamma} \left(a_P^I - w_{P,t} + f_{P,t} \right)^{1-\gamma}.$$
(31)

Thus:

$$w_{P,t} = (1 - \gamma) b_{P,t} + \gamma \left(a_P + f_{P,t} \right).$$
(32)

7.2.2 Further Assumptions

We assume that the firing costs are proportional to the wage $f_{i,t} = \rho_{i,t} w_{i,t}$ (where *i* is the index for primary (*P*) and trapped (*T*) workers) with the "firing cost ratio" $\rho_{i,t}$ in the respective sectors and that the unemployment benefit in our model is given by $b_{i,t} = \beta_{i,t} w_{i,t}$ with the net replacement ratio $\beta_{i,t}$ in the respective sectors. Thus, the negotiated wage is

$$w_{i,t} = \frac{\gamma}{\left(1 - \beta_{i,t}(1 - \gamma) - \rho\gamma\right)} a_{i,t} .$$
(33)

7.3 Model Derivation

7.3.1 Profit in the Trapped Sector

In the trapped sector, workers have an average productivity a_T and there is an exogenously given probability ϖ for employed workers to move to the primary sector of the economy. Firms take the regime switch into account (upgrade of trapped to primary workers), which increases the profitability.

State	Probability
Human Capital Upgrading	$\overline{\omega}$
No Upgrading + Firing	$(1-\varpi)\phi_T$
No Upgrading + Retention	$(1-\varpi)\left(1-\phi_T\right)$

The profit function below $(\Pi_{t,regime1})$ corresponds to the first regime (average profits weighted with the probability that workers stay trapped):

$$\Pi_{t,regime1} = -\varepsilon_t + \sum_{t=0}^{\infty} \delta^t \left(1 - \phi_T\right)^t \left(1 - \varpi\right)^t \left(a_T - w_T\right) - (1 - \varpi) \,\delta\phi_T f_T \sum_{t=0}^{\infty} \delta^t \left(1 - \varpi\right)^t \left(1 - \phi_T\right)^t$$
(34)

$$\Pi_{t,regime1} = \frac{a_T - w_T - \delta (1 - \varpi) \phi_T f_T}{1 - \delta (1 - \phi_T) (1 - \varpi)}$$
(35)

In each subsequent period a worker moves with probability ϖ from the trapped to the primary sector. The profit function below ($\Pi_{t,regime2}$) corresponds to the second regime:

$$\Pi_{t,regime2} = \delta \varpi \sum_{t=0}^{\infty} \delta^t \left(1 - \varpi\right)^t \left(1 - \phi_T\right)^t \left(1 - \phi_P f_P + \left(1 - \phi_P\right) \left(\sum_{t=0}^{\infty} \delta^t \left(1 - \phi_P\right)^t \left(a_P - w_P\right) - \delta \phi_P f_P \sum_{t=0}^{\infty} \delta^t \left(1 - \phi_P\right)^t\right)\right) \right]$$
(36)

The second line of the formula describes the present value of a worker if she is upgraded to the primary sector. An upgraded primary worker has the probability ϕ_P of not being fired immediately and a probability $(1 - \phi_P)$ of being retained. If the latter is the case, she has the same expected profit stream as a primary workers: $\sum_{t=0}^{\infty} \delta^t (1 - \phi_P)^t (a_P - w_P) - \delta \phi_P f \sum_{t=0}^{\infty} \delta^t (1 - \phi_P)^t$. Since every period a certain fraction of workers moves the primary sector, we have to write a double sum. A fraction ϖ among those who have not been fired $(1 - \varpi)^t (1 - \phi_T)^t$ moves to the primary sector.

$$\Pi_{t,regime2} = \delta \varpi \sum_{t=0}^{\infty} \delta^t \left(1 - \varpi\right)^t \left(1 - \phi_T\right)^t \left[-\phi_P f_P + (1 - \phi_P) \left(\frac{a_P - w_P - \delta \phi_P f_P}{1 - \delta \left(1 - \phi_P\right)}\right)\right]$$
(37)

$$\Pi_{t,regime2} = -\phi_P \delta \varpi \frac{f_P}{\left(1 - \delta \left(1 - \varpi\right) \left(1 - \phi_T\right)\right)} + \left(1 - \phi_P\right) \delta \varpi \left(\frac{a_P - w_P - \delta \phi_P f_P}{\left(1 - \delta \left(1 - \phi_P\right)\right) \left(1 - \delta \left(1 - \varpi\right) \left(1 - \phi_T\right)\right)}\right)$$
(38)

Thus, the overall expected profit $(\Pi_t = \Pi_{t,regime1} + \Pi_{t,regime2})$ is:

$$\Pi_{t} = -\varepsilon_{t} + \frac{a_{T} - w_{T} - \delta(1 - \varpi)\phi_{T}f_{T}}{1 - \delta(1 - \phi_{T})(1 - \varpi)} - \phi_{P}\delta\varpi \frac{f_{P}}{(1 - \delta(1 - \varpi)(1 - \phi_{T}))} + (1 - \phi_{P})\delta\varpi \left(\frac{a_{P} - w_{P} - \delta\phi_{P}f_{P}}{(1 - \delta(1 - \phi_{P}))(1 - \delta(1 - \varpi)(1 - \phi_{T}))}\right)$$
(39)

7.3.2 Employment Dynamics

Primary Sector: The (primary) employment in period t is equal to the people who are retained, both from the pool of employed $(N_{P,t-1})$ and from the human capital upgrades $(\varpi N_{T,t-1})$. The two groups have the same retention probability $1 - \phi_P$. A proportion η_P of the unemployed primary workers is hired. The pool of primary unemployed workers is reduced by a share v (workers who move to the trapped sector).

$$N_{P,t} = (1 - \phi_P) N_{P,t-1} + (1 - \phi_P) \varpi N_{T,t-1} + \eta_P U_{P,t-1} - \eta_P v U_{P,t-1}$$
(40)

$$N_{P,t} = (1 - \phi_P) N_{P,t-1} + (1 - \phi_P) \varpi N_{T,t-1} + (\eta_P (1 - \upsilon)) U_{P,t-1}$$
(41)

Next, we introduce $g_{t,P}$ which is the growth rate of the primary workforce from period t-1 to t ($g_{t,P} = L_{P,t}/L_{P,t-1}$).

Dividing by $L_{P,t}$, we obtain:

$$n_{P,t} = \frac{1}{g_{t,P}} (1 - \phi_P) n_{P,t-1} + (1 - \phi_P) \varpi \frac{N_{T,t-1}}{L_{P,t}} + \frac{1}{g_{t,P}} (\eta_P (1 - v)) (1 - n_{P,t-1})$$
(42)

$$n_{P,t} = \frac{1}{g_{t,P}} \left[(1 - \phi_P) n_{P,t-1} + (\eta_P (1 - \upsilon)) (1 - n_{P,t-1}) \right] + (1 - \phi_P) \varpi \frac{L_{T,t-1}}{L_{P,t}} n_{T,t-1}.$$
(43)

The labor force in the primary sector is equal to the previous period's labor force plus the net movement from the trapped sector:

$$L_{P,t} = L_{P,t-1} - \upsilon u_{P,t-1} L_{P,t-1} + \varpi n_{T,t-1} L_{T,t-1}.$$
(44)

In the steady state, the growth rate of the labor force is equal to 0 ($g_{t,P} = L_{P,t}/L_{P,t-1} = 1$) and all time indices can be dropped. Thus, the following equation holds:

$$n_P \left(\phi_P + (\eta_P (1 - \upsilon)) \right) = (1 - \upsilon) \eta_P + \varpi (1 - \phi_P) n_T \frac{L_T}{L_P}.$$
(45)

And the following constraint (human capital upgrades must equal downgrades) has to hold in the steady state:

$$vU_P = \varpi N_T \tag{46}$$

Trapped Sector: The employed in the trapped sector equal the retained workers from the previous period (who did not receive a human capital upgrade: $(1 - \varpi) N_{T,t-1}$) plus the hired trapped unemployed (their number has been enlarged by the human capital depreciation: $\eta_T U_{T,t-1} + \eta_T v U_{P,t-1}$):

$$N_{T,t} = (1 - \phi_T) (1 - \varpi) N_{T,t-1} + \eta_T U_{T,t-1} + \eta_T v U_{P,t-1}$$
(47)

Dividing by $L_{T,t}$:

$$n_{T,t} = \frac{1}{g_{t,T}} \left[(1 - \phi_T) \left(1 - \varpi \right) n_{T,t-1} + \eta_T \left(1 - n_{T,t-1} \right) \right] + \eta_T v \frac{U_{P,t-1}}{L_{T,t}}$$
(48)

$$n_{T,t} = \frac{1}{g_{t,T}} \left[(1 - \phi_T) (1 - \varpi) n_{T,t-1} + \eta_T (1 - n_{T,t-1}) \right] + \eta_T \upsilon (1 - n_{P,t-1}) \frac{L_{P,t-1}}{L_{T,t}}$$
(49)

The labor force in the trapped sector is equal to the previous period's labor force plus the net movement from the primary sector:

$$L_{T,t} = L_{T,t-1} + \upsilon u_{P,t-1} L_{P,t-1} - \varpi n_{T,t-1} L_{T,t-1}.$$
(50)

In the steady state the following relationship holds:

$$n_T = (1 - \phi_T) (1 - \varpi) n_T + \eta_T (1 - n_T) + \eta_T \upsilon (1 - n_P) \frac{L_P}{L_T}$$
(51)

$$n_{T,t} = \frac{\eta_T + \eta_T \upsilon \left(1 - n_P\right) \frac{L_P}{L_T}}{\left(1 - \left(1 - \phi_T\right) \left(1 - \varpi\right) + \eta_T\right)}.$$
(52)

Inserting (52) into (45), we obtain the following steady state relationship:

$$n_{P} \left(\phi_{P} + (\eta_{P} (1 - \upsilon)) \right)$$

$$= \eta_{P} (1 - \upsilon) + (1 - \phi_{P}) \varpi \frac{\eta_{T} + \eta_{T} \upsilon (1 - n_{P}) \frac{L_{P}}{L_{T}}}{(1 - (1 - \phi_{T}) (1 - \varpi) + \eta_{T})} \frac{L_{T}}{L_{P}}$$
(53)

$$n_P = \frac{\eta_P \left(1 - \upsilon\right) + \left(1 - \phi_P\right) \varpi \frac{\eta_T \frac{L_T}{L_P} + \eta_T \upsilon}{\left(1 - (1 - \phi_T)(1 - \varpi) + \eta_T\right)}}{\phi_P + \left(\eta_P \left(1 - \upsilon\right)\right) + \left(1 - \phi_P\right) \varpi \frac{\eta_T \upsilon}{\left(1 - (1 - \phi_T)(1 - \varpi) + \eta_T\right)}}.$$
(54)

If $v = \varpi = 0$, we have two entirely separated sectors in this economy and we obtain the following steady state relationship:

$$n_P = \frac{\eta_P}{\phi_P + \eta_P}.\tag{55}$$

7.4 Derivations for the Calibration

7.4.1 Non-Trapped Sector

The detailed derivations of the steady state firing and hiring rates under different policy exercises is analogous to Snower and Merkl $(2006)^{36}$, providing the following linearized equations:

$$\phi_{P,new} = \phi_{P,0} - A_P \left[(a_{P,new} - w_{P,new}) - (a_{P,0} - w_{P,0}) \right]$$

$$-C_P \left(f_{P,new} - f_{P,0} \right)$$
(56)

and

$$\eta_{P,new} = \eta_{P,0} + G_P \left[(a_{P,new} - w_{P,new}) - (a_{P,0} - w_{P,0}) \right] \\ -I_P \left(\begin{array}{c} f_{P,new} \\ -f_{P,0} \end{array} \right) - K_P \left(\begin{array}{c} h_{P,new} \\ -h_{P,0} \end{array} \right) - L_P \left(\begin{array}{c} \phi_{P,new} \\ -\phi_{P,0} \end{array} \right),$$
(57)

where all coefficients A_P to L_P have a positive sign.

7.4.2 Trapped Sector

Firing Rate: A worker is fired if $\pi_t < -f_T$.

$$\phi_T = 1 - \Gamma \left(\begin{array}{c} \frac{a_T - w_T - \delta(1 - \varpi)\phi_T f_T}{1 - \delta(1 - \phi_T)(1 - \varpi)} + f_T - \phi_P \delta \varpi \frac{f_P}{(1 - \delta(1 - \varpi)(1 - \phi_T))} \\ + (1 - \phi_P) \delta \varpi \left(\frac{a_P - w_P - \delta \phi_P f_P}{(1 - \delta(1 - \phi_P))(1 - \delta(1 - \varpi)(1 - \phi_T))} \right) \end{array} \right)$$
(58)

For the calibration we deflate all variables to their 1991 real value (using German GDP deflator³⁷) and take into account a 2% ($\alpha = 1.02$) growth rate of all variables (a, w, f) and the

 $^{^{36}\}mathrm{See}$ page 39 of the detailed version.

³⁷Source: International Financial Statistics, International Monetary Fund.

operating costs to make the calibration more realistic and comparable to Snower and Merkl (2006).

$$\phi_T = 1 - \Gamma \left(\frac{1}{\alpha^{15}} \begin{pmatrix} \frac{a_T - w_T - \delta\alpha(1 - \varpi)\phi_T f_T}{1 - \delta\alpha(1 - \phi_T)(1 - \varpi)} + f_T - \phi_P \delta\alpha \varpi \frac{f_P}{(1 - \delta\alpha(1 - \varpi)(1 - \phi_T))} \\ + (1 - \phi_P) \delta\alpha \varpi \left(\frac{a_P - w_P - \delta\alpha\phi_P f_P}{(1 - \delta\alpha(1 - \phi_P))(1 - \delta\alpha(1 - \varpi)(1 - \phi_T))} \right) \end{pmatrix} \right)$$
(59)

Next, we take a first order Taylor approximation for the firing rate (where the subscript "0" refers to old steady state values and the subscript "new" refers to new steady state values). Therefore, we need the first derivatives at the old steady state with respect to the following variables:

$$\frac{\partial \phi_{T,0}}{\partial \left(a_T - w_T\right)} = -\frac{1}{\alpha^{15}} \Gamma'_{f,0} \left[\frac{1}{1 - \alpha \delta \left(1 - \phi_T\right) \left(1 - \varpi\right)} \right]_0 \tag{60}$$

$$\frac{\partial \phi_{T,0}}{\partial \left(a_P - w_P\right)} = -\frac{1}{\alpha^{15}} \Gamma'_{f,0} \left[\frac{\left(1 - \phi_P\right) \delta \alpha \varpi}{\left(1 - \delta \alpha \left(1 - \phi_P\right)\right) \left(1 - \delta \alpha \left(1 - \varpi\right) \left(1 - \phi_T\right)\right)} \right]$$
(61)

$$\frac{\partial \phi_{T,0}}{\partial f_T} = -\frac{1}{\alpha^{15}} \Gamma'_{f,0} \left[\frac{-\delta \alpha \phi_T \left(1 - \varpi\right)}{1 - \alpha \delta \left(1 - \phi_T\right) \left(1 - \varpi\right)} + 1 \right]_0 \tag{62}$$

$$\frac{\partial \phi_{T,0}}{\partial f_P} = -\frac{1}{\alpha^{15}} \Gamma'_{f,0} \left[\begin{array}{c} -\frac{\delta \alpha \varpi \phi_P}{(1-\delta \alpha (1-\varpi)(1-\phi_T))} \\ -\frac{\delta^2 \alpha^2 \varpi \phi_P (1-\phi_P)}{(1-\delta \alpha (1-\phi_P))(1-\delta \alpha (1-\varpi)(1-\phi_T))} \end{array} \right]_0$$
(63)

$$\frac{\partial \phi_{T,0}}{\partial \phi_T} = -\frac{1}{\alpha^{15}} \Gamma'_{f,0} \begin{bmatrix} -\frac{\delta \alpha f_T (1-\varpi) (1-\delta \alpha (1-\phi_T) (1-\varpi)) - \delta \alpha (1-\varphi_T) (1-\varpi) - \delta \alpha (1-\varphi_T) \phi_T f_T]}{(1-\delta \alpha (1-\varphi_T) (1-\varphi_T))^2} \\ - \left(\frac{-\delta^2 \alpha^2 \phi_F \omega_F (1-\varpi)}{((1-\delta \alpha (1-\varphi_T) (1-\varphi_T))^2} \right) \\ + \left(\frac{-(1-\phi_P) \delta \alpha \varpi (\alpha_P - w_P - \delta \alpha \phi_P f_P)}{(1-\delta \alpha (1-\varphi_P)) \delta \alpha (1-\varpi)} \right) \\ + \left(\frac{-(1-\phi_P) \delta \alpha \varpi (\alpha_P - w_P - \delta \alpha \phi_P f_P)}{(1-\delta \alpha (1-\phi_P)) (1-\delta \alpha (1-\varphi_T) (1-\phi_T))^2} \right) \end{bmatrix}$$
(64)
$$\frac{\partial \phi_{T,0}}{\partial \phi_P} = -\frac{1}{\alpha^{15}} \Gamma'_{f,0} \begin{bmatrix} \left[(1-\delta \alpha (1-\phi_P)) (1-\delta \alpha (1-\varpi) (1-\phi_T)) - \delta \alpha (1-\varphi_P) (1-\phi_T) (1-\phi_T) - \delta \alpha (1-\varphi_T) - \delta \alpha (1-\varphi_T) (1-\varphi_T) - \delta \alpha (1-\varphi_T) - \delta \alpha (1-\varphi_T) - \delta \alpha (1-\varphi_T) - \delta \alpha (1-\varphi_T) (1-\varphi_T) - \delta \alpha (1-\varphi_T) - \delta \alpha (1-\varphi_T) (1-\varphi_T) - \delta \alpha (1-\varphi_T) - \delta \alpha (1-\varphi_T) - \delta \alpha (1-\varphi_T) (1-\varphi_T) - \delta \alpha (1-\varphi_T) - \delta \alpha (1-\varphi_T) (1-\varphi_T) - \delta \alpha (1-\varphi_T) (1-\varphi_T) - \delta \alpha (1$$

Thus, we obtain the following expression:

$$\phi_{T,new} = \phi_{T,0} + \frac{\partial \phi_{T,0}}{\partial (a_T - w_T)} \begin{bmatrix} (a_{T,new} - w_{T,new}) \\ - (a_{T,0} - w_{T,0}) \end{bmatrix}$$
(67a)

$$+\frac{\partial\phi_{T,0}}{\partial(a_P - w_P)} \begin{bmatrix} (a_{P,new} - w_{P,new}) \\ -(a_{P,0} - w_{P,0}) \end{bmatrix}$$
(67b)

$$+\frac{\partial\phi_{T,0}}{\partial f_T}\left(f_{T,new} - f_{T,0}\right) \tag{67c}$$

$$+\frac{\partial\phi_{T,0}}{\partial f_P}\left(f_{P,new} - f_{P,0}\right) \tag{67d}$$

$$+\frac{\partial\phi_{T,0}}{\partial\phi_T}\left(\phi_{T,new} - \phi_{T,0}\right) \tag{67e}$$

$$+\frac{\partial\phi_{T,0}}{\partial\phi_P}\left(\phi_{P,new} - \phi_{P,0}\right) \tag{67f}$$

$$+\frac{\partial\phi_{T,0}}{\partial\varpi}\left(\varpi_{new}-\varpi_{0}\right)\tag{67g}$$

By defining

$$V = \frac{1}{\alpha^{15}} \Gamma'_{f,0} \begin{bmatrix} \left(\frac{-\delta \alpha f_T \left(1 - \varpi\right) \left(1 - \delta \alpha \left(1 - \phi_T\right) \left(1 - \varpi\right)\right) - \\ \delta \alpha \left(1 - \varpi\right) \left[a_T - w_T - \delta \alpha \left(1 - \varpi\right) \phi_T f_T\right] \\ (1 - \delta \alpha (1 - \phi_T) (1 - \varpi))^2 \end{bmatrix} \\ - \left(\frac{-\delta^2 \alpha^2 \phi_P \varpi f_P (1 - \varpi)}{\left[(1 - \delta \alpha (1 - \phi_P)) \delta \alpha \varpi \left(a_P - w_P - \delta \alpha \phi_P f_P\right) \\ + \left(\frac{-(1 - \phi_P) \delta \alpha \varpi \left(a_P - w_P - \delta \alpha \phi_P f_P\right)}{\left[(1 - \delta \alpha (1 - \phi_P)) (1 - \delta \alpha (1 - \varpi)) - \phi_T (1 - \phi_T))\right]^2} \right) \end{bmatrix}_0, \quad (68)$$

we obtain:

$$\phi_{T,new} = \phi_{T,0} \tag{69a}$$

$$+\frac{\partial\phi_{T,0}}{\partial(a_T - w_T)} \left(\frac{1}{1+V}\right) \left[\begin{pmatrix} a_{T,new} \\ -w_{T,new} \end{pmatrix} - \begin{pmatrix} a_{T,0} \\ -w_{T,0} \end{pmatrix} \right]$$
(69b)
$$\frac{\partial\phi_{T,0}}{\partial\phi_{T,0}} \left(\begin{array}{c} 1 \\ -w_{T,new} \end{pmatrix} - \begin{pmatrix} a_{P,0} \\ -w_{T,0} \end{pmatrix} \right]$$
(69c)

$$+\frac{\partial \varphi_{T,0}}{\partial (a_P - w_P)} \left(\frac{1}{1+V}\right) \left[\begin{pmatrix} a_{P,new} \\ -w_{P,new} \end{pmatrix} - \begin{pmatrix} a_{P,0} \\ -w_{P,0} \end{pmatrix} \right]$$
(69c)
$$+\frac{\partial \varphi_{T,0}}{\partial \phi_{T,0}} \left(\frac{1}{1+V}\right) \left[\begin{pmatrix} a_{P,new} \\ -w_{P,new} \end{pmatrix} - \begin{pmatrix} a_{P,0} \\ -w_{P,0} \end{pmatrix} \right]$$
(69c)

$$+\frac{\partial f_{T,0}}{\partial f_T} \left(\frac{1}{1+V}\right) \left(f_{T,new} - f_{T,0}\right) \tag{69d}$$

$$+\frac{\partial\phi_{T,0}}{\partial f_P}\left(\frac{1}{1+V}\right)\left(f_{P,new} - f_{P,0}\right) \tag{69e}$$

$$+\frac{\partial\phi_{T,0}}{\partial\phi_P}\left(\frac{1}{1+V}\right)\left(\phi_{P,new}-\phi_{P,0}\right) \tag{69f}$$

$$+\frac{\partial\phi_{T,0}}{\partial\varpi}\left(\frac{1}{1+V}\right)\left(\varpi_{new}-\varpi_{0}\right)$$
(69g)

Or by substituting the coefficients:

$$\phi_{T,new} = \phi_{T,0} - A_T \left[(a_{T,new} - w_{T,new}) - (a_{T,0} - w_{T,0}) \right]$$

$$-B_T \left[(a_{P,new} - w_{P,new}) - (a_{P,0} - w_{P,0}) \right] - C_T \left(f_{T,new} - f_{T,0} \right)$$

$$+D_T \left(f_{P,new} - f_{P,0} \right) + E_T \left(\phi_{T,new} - \phi_{T,0} \right) + F_T \left(\varpi_{new} - \varpi_0 \right)$$
(70)

where A_T to F_T are all positive constants.

Thus, higher productivity and lower wages lead to a reduction of the firing rate. Higher firing costs in the trapped sector reduce firing (not taking their indirect effect via the wage formation into account which outweighs the direct effect), whereas higher firing costs in the primary sector increase firing in the trapped sector. There is a positive spillover effect from the firing rate in the primary sector to the trapped sector, i.e. if the firing rate in the primary sector is reduced, the same is true for the firing rate in the trapped sector. Furthermore, a higher intersectoral mobility reduces firing in the trapped sector (as the average profitability of trapped workers increases).

Hiring Rate: A worker is hired if $\pi_t > h_T$. Thus:

$$\eta_T = \Gamma \left(\begin{array}{c} \frac{a_T - w_T - \delta(1 - \varpi)\phi_T f_T}{1 - \delta(1 - \phi_T)(1 - \varpi)} + h_T + \\ \frac{\delta \varpi \phi_P f_P}{(1 - \delta \varpi)} + (1 - \phi_P) \, \delta \varpi \left(\frac{a_P - w_P + \delta \phi_P f_P}{(1 - \delta(1 - \phi_P))((1 - \delta \varpi))} \right) \end{array} \right)$$
(71)

Analogous to the firing rate the hiring rate is re-written as:

$$\eta_T = \Gamma \left(\frac{1}{\alpha^{15}} \left(\begin{array}{c} \frac{a_T - w_T - \delta\alpha(1 - \varpi)\phi_T f_T}{1 - \delta\alpha(1 - \phi_T)(1 - \varpi)} + h_T + \\ \frac{\delta\alpha \varpi \phi_P f_P}{(1 - \delta\alpha \varpi)} + (1 - \phi_P) \,\delta\alpha \varpi \left(\frac{a_P - w_P + \delta\alpha \phi_P f_P}{(1 - \delta\alpha(1 - \phi_P))((1 - \delta\alpha \varpi))} \right) \end{array} \right) \right)$$
(72)

To obtain the first order Taylor approximation, we need to calculate the first partial derivatives:

$$\frac{\partial \eta_{T,0}}{\partial \left(a_T - w_T\right)} = \frac{1}{\alpha^{15}} \Gamma_{h,0}' \left[\frac{1}{1 - \alpha \delta \left(1 - \phi_T\right) \left(1 - \varpi\right)} \right]_0$$
(73)

$$\frac{\partial \eta_{T,0}}{\partial \left(a_P - w_P\right)} = \frac{1}{\alpha^{15}} \Gamma_{h,0}' \left[\frac{\left(1 - \phi_P\right) \delta \alpha \varpi}{\left(1 - \delta \alpha \left(1 - \phi_P\right)\right) \left(1 - \delta \alpha \left(1 - \varpi\right) \left(1 - \phi_T\right)\right)} \right]$$
(74)

$$\frac{\partial \eta_{T,0}}{\partial f_T} = \frac{1}{\alpha^{15}} \Gamma'_{h,0} \left[\frac{-\delta \alpha \phi_T \left(1 - \varpi \right)}{1 - \alpha \delta \left(1 - \phi_T \right) \left(1 - \varpi \right)} \right]_0 \tag{75}$$

$$\frac{\partial \eta_{T,0}}{\partial f_P} = \frac{1}{\alpha^{15}} \Gamma'_{h,0} \left[\begin{array}{c} -\frac{\delta \alpha \varpi \phi_P}{(1-\delta \alpha (1-\varpi)(1-\phi_T))} \\ -\frac{\delta^2 \alpha^2 \varpi \phi_P (1-\phi_P)}{(1-\delta \alpha (1-\phi_P))(1-\delta \alpha (1-\varpi)(1-\phi_T))} \end{array} \right]_0$$
(76)

$$\frac{\partial \eta_{T,0}}{\partial h} = \frac{1}{\alpha^{15}} \Gamma'_{h,0} \tag{77}$$

$$\frac{\partial \eta_{T,0}}{\partial \phi_T} = \frac{1}{\alpha^{15}} \Gamma'_{h,0} \begin{bmatrix} \begin{pmatrix} -\delta \alpha f_T \left(1 - \varpi\right) \left(1 - \delta \alpha \left(1 - \phi_T\right) \left(1 - \varpi\right)\right) - \\ \delta \alpha \left(1 - \varpi\right) \left[a_T - w_T - \delta \alpha \left(1 - \varpi\right) \phi_T f_T\right] \\ (1 - \delta \alpha (1 - \phi_T)(1 - \varpi))^2 \end{pmatrix} \\ - \left(\frac{-\delta^2 \alpha^2 \phi_P \varpi f_P (1 - \varpi)}{\left[(1 - \delta \alpha (1 - \omega)(1 - \phi_T))\right]^2} \right) \\ + \left(\frac{-\left(1 - \phi_P\right) \delta \alpha \varpi \left(a_P - w_P - \delta \alpha \phi_P f_P\right)}{\left[(1 - \delta \alpha (1 - \phi_P)) \delta \alpha \left(1 - \varpi\right)\right]} \right) \end{bmatrix}$$
(78)

$$\frac{\partial \eta_{T,0}}{\partial \phi_P} = \frac{1}{\alpha^{15}} \Gamma'_{h,0} \begin{bmatrix}
\frac{\left[(1 - \delta\alpha \left(1 - \phi_P \right) \right) (1 - \delta\alpha \left(1 - \varpi \right) \left(1 - \phi_T \right) \right) \right]}{\left[-\delta^2 \alpha^2 \varpi f_P \left(1 - 2\phi_P \right) \right]} \\
- \left[\frac{(1 - \phi_P) \delta\alpha \varpi \left(a_P - w_P \right) - \delta^2 \alpha^2 \varpi f_P \left(\phi_P - \phi_P^2 \right) \right]}{\left[(1 - \delta\alpha \left(1 - \phi_P \right) \right) \left(\delta\alpha \left(1 - \varpi \right) \right) \right]} \\
- \left[\frac{(1 - \delta\alpha \left(1 - \phi_P \right) \right) \left(\delta\alpha \left(1 - \varpi \right) \right) - \delta\alpha \varpi f_P - \delta\alpha \varpi f_P - \delta\alpha (1 - \varpi) \left(1 - \sigma_T \right) \right] - \delta\alpha \varpi f_P - \delta\alpha (1 - \varpi) \left(1 - \sigma_T \right) \right]}{\left[(1 - \delta\alpha \left(1 - \sigma_T \right) \left(1 - \sigma_T \right) \right]^2 - \frac{\delta\alpha \varpi f_P - \delta\alpha \left(1 - \sigma_T \right) \left(1 - \sigma_T \right) \right]}{\left[(1 - \delta\alpha \left(1 - \sigma_T \right) \left(1 - \sigma_T \right) \right] \right]} \\
\frac{\partial \eta_{T,0}}{\partial \varpi} = \frac{1}{\alpha^{15}} \Gamma'_{h,0} \begin{bmatrix}
\frac{\delta \alpha \phi_T f_T \left[1 - \delta\alpha \left(1 - \phi_T \right) \left(1 - \sigma_T \right) \right] - \delta\alpha \left(1 - \phi_T \right) \left(1 - \sigma_T \right) \right]}{\left[(1 - \delta\alpha \left(1 - \sigma_T \right) \left(1 - \phi_T \right) \right) \right]} \\
\frac{\partial \eta_{T,0}}{\partial \varpi} = \frac{1}{\alpha^{15}} \Gamma'_{h,0} \begin{bmatrix}
\frac{\delta \alpha \phi_T f_T \left[1 - \delta\alpha \left(1 - \sigma_T \right) \left(1 - \sigma_T \right) \right]}{\left[(1 - \delta\alpha \left(1 - \sigma_T \right) \left(1 - \phi_T \right) \right]} \\
\frac{\left[\left(1 - \delta\alpha \left(1 - \phi_P \right) \right) \left(1 - \delta\alpha \left(1 - \sigma_T \right) \right) \right]}{\left[\left(1 - \delta\alpha \left(1 - \phi_P \right) \right) \left(1 - \delta\alpha \left(1 - \sigma_T \right) \right) \right]} \\
\frac{\delta \alpha \left(1 - \phi_P \right) \left(\delta\alpha \left(1 - \phi_T \right) \right)}{\left[\left(1 - \delta\alpha \left(1 - \phi_P \right) \right) \left(1 - \delta\alpha \left(1 - \phi_T \right) \right) \right]} \\
\frac{\delta \alpha \left(1 - \phi_P \right) \left(\delta\alpha \left(1 - \phi_T \right) \right)}{\left[\left(1 - \delta\alpha \left(1 - \phi_P \right) \left(1 - \delta\alpha \left(1 - \phi_T \right) \right) \right]} \\
\frac{\delta \alpha \left(1 - \phi_P \right) \left(\delta\alpha \left(1 - \phi_T \right) \right)}{\left[\left(1 - \delta\alpha \left(1 - \phi_P \right) \left(1 - \delta\alpha \left(1 - \phi_T \right) \right) \right]} \\
\frac{\delta \alpha \left(1 - \phi_P \right) \left(\delta\alpha \left(1 - \phi_T \right) \right)}{\left[\left(1 - \delta\alpha \left(1 - \phi_P \right) \left(1 - \delta\alpha \left(1 - \phi_T \right) \right) \right]} \\
\frac{\delta \alpha \left(1 - \phi_P \right) \left(\delta\alpha \left(1 - \phi_T \right) \right)}{\left[\left(1 - \delta\alpha \left(1 - \phi_T \right) \right) \left(1 - \delta\alpha \left(1 - \phi_T \right) \right) \right]} \\
\frac{\delta \alpha \left(1 - \phi_P \right) \left(\delta\alpha \left(1 - \phi_T \right) \right)}{\left[\left(1 - \delta\alpha \left(1 - \phi_T \right) \left(1 - \delta\alpha \left(1 - \phi_T \right) \right) \right]} \\
\frac{\delta \alpha \left(1 - \phi_P \right) \left(\delta\alpha \left(1 - \phi_T \right) \right)}{\left[\left(1 - \delta\alpha \left(1 - \phi_T \right) \right) \left(1 - \delta\alpha \left(1 - \phi_T \right) \right) \left(1 - \phi_T \right)} \right]} \\$$

Thus, the first order Taylor approximation is

$$\eta_{T,new} = \eta_{T,0} + \frac{\partial \eta_{T,0}}{\partial (a_T - w_T)} \left[(a_{T,new} - w_{T,new}) - (a_{T,0} - w_{T,0}) \right]$$
(81a)

$$+\frac{\partial \eta_{T,0}}{\partial (a_P - w_P)} \left[(a_{P,new} - w_{P,new}) - (a_{P,0} - w_{P,0}) \right]$$
(81b)

$$+\frac{\partial \eta_{T,0}}{\partial f_T} \left(f_{T,new} - f_{T,0} \right) + \frac{\partial \eta_{T,0}}{\partial f_P} \left(f_{P,new} - f_{P,0} \right)$$
(81c)

$$+\frac{\partial\eta_{T,0}}{\partial\phi_T}\left(\phi_{T,new} - \phi_{T,0}\right) + \frac{\partial\eta_{T,0}}{\partial\phi_P}\left(\phi_{P,new} - \phi_{P,0}\right)$$
(81d)

$$+\frac{\partial\eta_{T,0}}{\partial\varpi}\left(\varpi_{new}-\varpi_0\right)\tag{81e}$$

Or by substituting the coefficients

$$\eta_{T,new} = \eta_{T,0} + G_T \left[\begin{pmatrix} a_{T,new} - w_{T,new} \end{pmatrix} - \begin{pmatrix} a_{T,0} - w_{T,0} \end{pmatrix} \right]$$

$$+ H_T \left[\begin{pmatrix} a_{P,new} \\ -w_{P,new} \\ - \begin{pmatrix} a_{P,0} \\ -w_{P,0} \end{pmatrix} \right] - I_T \begin{pmatrix} f_{T,new} \\ -f_{T,0} \end{pmatrix} - J_T \begin{pmatrix} f_{P,new} \\ -f_{P,0} \end{pmatrix}$$

$$- K_T \begin{pmatrix} h_{T,new} \\ -h_{T,0} \end{pmatrix} - L_T \begin{pmatrix} \phi_{T,new} \\ -\phi_{T,0} \end{pmatrix} - M_T \begin{pmatrix} \phi_{P,new} \\ -\phi_{P,0} \end{pmatrix} + N_T \begin{pmatrix} \varpi_{new} \\ -\varpi_0 \end{pmatrix} .$$
(82)

where G_T to N_T are all positive coefficients. The rationale for the signs of the coefficients is the same as for the linearized firing rate in the trapped sector.³⁸

³⁸For the linearized model a value for the first derivative of the cumulative function has to be chosen (Γ'). Snower and Merkl (2006) set the same values for the firing and hiring rate, while we choose ($\Gamma'_f = 6 * 10^{-7}$) and ($\Gamma'_h = 6 * 10^{-6}$), where $_f$ and $_h$ stand for the firing and hiring rate respectively. In the homogenous model, this provides us with a similar labor demand elasticity and thus a similar employment path, but is more in line with the empirical evidence on hiring and firing elasticities (for a summary, see Orszag and Snower, 1999).