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# When Inflation Persistence Really Matters: Two examples

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## Abstract

In this paper we present two examples where the presence of inflation persistence could influence the qualitative nature of monetary policy. In the first case the desirability of a monetary policy regime comes under question when extensive inflation persistence exists. In the second case the direction in which interest rates move following a cost push shock changes when inflation persistence becomes important. In both cases, inflation persistence is central to the process influencing policy.

Key Words: Inflation Persistence, Macroeconomic Stabilisation

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# 1 Introduction

The benchmark model of inflation dynamics in New Keynesian models is now the New Keynesian Phillips curve (NKPC). It is widely agreed, however, that there is more persistence than what is shown by this equation. There is a large body of empirical evidence on this (see Gali and Gertler (1999), Benigno and Lopez-Salido (2006), Mehra (2004) among many others) but little agreement on how much persistence there is, as estimates of its extent vary widely. There have also been a number of attempts to microfound this type of persistence (Steinsson (2003), Amato and Laubach (2003)).

If inflation persistence is important empirically, how much will this influence what policy makers do? Here it is useful to distinguish between changes in policy that are qualitative and quantitative. There is no doubt that adding inflation persistence will influence optimal policy in a quantitative way. As Clarida et al. (1999) among others have noted, and we illustrate below, when inflation is more persistent any policy reaction is likely to be more aggressive. However does inflation persistence influence the character of what monetary policy might do?

In terms of the benchmark New Keynesian model, the answer appears to be no. For example, in their survey of optimal monetary policy in New Keynesian models under discretion, Clarida et al. (1999) state ‘Results 1 through 4 that describe optimal monetary policy under discretion within the baseline model also apply in the case with endogenous output and inflation persistence’.<sup>1</sup> In his discussion of inflation persistence, Woodford (2007) concludes ‘policy advice that would be derived from an assumption that inflation dynamics under alternative policies will be correctly predicted by the NKPC under the assumption of rational expectations remains sound advice, despite the limitations of the simple model as an account of recent historical experience with inflation.’<sup>2</sup>

These statements *imply* that the character of monetary-policy systems is unaffected by inflation persistence. In this paper we present two examples to show that such a claim may be false. In the first case, the desirability of a monetary policy regime comes under question when extensive inflation persistence exists. In the second, the direction in which interest rates optimally move following a cost push shock changes when inflation persistence becomes important. In both cases, inflation persistence is central to the process influencing policy.

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<sup>1</sup>Results 1-4 are that there is a short run output inflation trade-off, that policy involves flexible inflation targeting, the Taylor Principle, and offsetting demand shocks but accommodating supply shocks.

<sup>2</sup>The advice Woodford has in mind is that a central bank should commit itself not to allow shocks of any kind to disturb the projected path of the output-gap-adjusted price level, which is the optimal target criterion derived from the standard NKPC.

Both our examples are generalisations of a basic New Keynesian closed economy model, which we set out in the next section. We model inflation persistence using the framework set out in Steinsson (2003). Such persistence obviously makes an inflation shock harder to control.

Our first extension (Section 3) takes this closed economy model and applies it to a (small) member of a monetary union, or equivalently to a small open economy under fixed exchange rates. We show that if the majority of price setters are backward looking, then this economy will exhibit severe cycles following an idiosyncratic shock. Given the number of major economies that are now members of a monetary union, this is a potentially important result.

Our second example (Section 4) moves back to a closed economy, but adds fiscal policy and debt to the model in a non-trivial way. We look at an economy where government spending is valued by consumers as providing public goods, but where it is also used as one of the instruments that a benevolent policy maker can use to stabilise the economy (including debt). Lump sum taxes cannot be varied in response to shocks. Leith and Wren-Lewis (2007) have shown that, under optimal discretion policy, debt will be returned to its pre-shock level, and that, following a positive cost-push shock, interest rates may initially *fall* rather than rise. We argue that this potentially counter-intuitive result depends on the forward looking nature of the NKPC. Following Stehn and Vines (2007), we show that it disappears once we have inflation persistence. In this example, therefore, inflation persistence changes the direction of the response of monetary policy to a shock.

Our two examples are quite different, both in the way that they extend the basic New Keynesian model, and in the way that inflation persistence becomes important. In the open economy case, inflation persistence might call into question the desirability of a fixed exchange rate/monetary union regime. In the case where we add fiscal policy, the direction of response of monetary policy changes in a qualitative way. In a final section we conclude not only that the examples illustrate why inflation persistence is important, but we also note a common feature that these two apparently very different extensions of the basic model share.

## 2 The Steinsson model

In this section we outline a simple closed economy model that includes inflation persistence, which is due to Steinsson (2003). This serves two purposes. First, our two later models are extensions of this set-up, and so we avoid repeating derivations of equations in these extensions. Second, we use this model to illustrate the idea that in this basic model, inflation persistence influences the optimal response to shocks in a quantitative but not a qualitative way.

## 2.1 Consumers

Our economy is inhabited by a large number of individuals, who specialize in the production of a differentiated good (indexed by  $z$ ), and who spend  $h(z)$  of effort in its production. They consume a basket of goods  $C$ . Individuals' maximization problem is

$$\max_{\{C_v^s, h_v^s\}_{v=t}^{\infty}} \mathcal{E}_t \sum_{v=t}^{\infty} \beta^{v-t} [u(C_v) - v(h_v(z))] \quad (1)$$

The price of a differentiated good  $z$  is denoted by  $p(z)$ , and the aggregate price level is  $P$ . An individual chooses optimal consumption and work effort to maximise this criterion (1) subject to the demand system and the intertemporal budget constraint:

$$P_t C_t + \mathcal{E}_t (Q_{t,t+1} \mathcal{A}_{t+1}) \leq \mathcal{A}_t + w_t(z) h_t(z) + \Pi_t(z) \quad (2)$$

where  $P_t C_t = \int_0^1 p(z) c(z) dz$  is nominal consumption,  $\mathcal{A}_t$  are nominal financial assets of a household and  $\Pi_t$  is profit. Here  $w$  is the wage rate, and  $\tau$  a tax rate on labour income.  $Q_{t,t+1}$  is the stochastic discount factor which determines the price in period  $t$  to the individual of being able to carry a state-contingent amount  $\mathcal{A}_{t+1}$  of wealth into period  $t + 1$ . The riskless short term nominal interest rate  $i_t$  has the following representation in terms of the stochastic discount factor:

$$\mathcal{E}_t(Q_{t,t+1}) = \frac{1}{(1 + i_t)}.$$

Each individual consumes the same basket of goods. Goods are aggregated into a Dixit and Stiglitz (1977) consumption index with the elasticity of substitution between any pair of goods given by  $\epsilon_t > 1$  (which is a stochastic elasticity with mean  $\epsilon^3$ ),  $C_t = \left[ \int_0^1 c_t^{\frac{\epsilon_t-1}{\epsilon_t}}(z) dz \right]^{\frac{\epsilon_t}{\epsilon_t-1}}$ .

We assume no Ponzi schemes and that the net present value of individual's future income is bounded. We assume that the nominal interest rate is positive at all times. These assumptions rule out infinite consumption (either because of infinite future income, or because money would pay a higher return than bonds) and allow us to replace the infinite sequence of flow budget constraints of the individual by a single intertemporal constraint:

$$\mathcal{E}_t \sum_{v=t}^{\infty} Q_{t,v} C_v P_v = \mathcal{A}_t + \mathcal{E}_t \sum_{v=t}^{\infty} Q_{t,v} (w_v(z) h_v(z) + \Pi_v(z)).$$

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<sup>3</sup>We make this parameter stochastic to allow us to generate shocks to the mark-up of firms.

We assume the specific functional form for utility from consumption component,  $u(C_v) = \frac{(C_v)^{1-1/\sigma}}{1-1/\sigma}$ . This household optimisation problem leads to the following consumption Euler equation:

$$C_t = \mathcal{E}_t \left( \left( \frac{1}{\beta} \frac{P_{t+1}}{P_t} Q_{t,t+1} \right)^\sigma C_{t+1} \right). \quad (3)$$

We linearise equations (3) around the steady state (for each variable  $X_t$  with steady state value  $X$ , we use the notation  $\hat{X}_t = \ln(X_t/X)$ ). Equation (3) leads to the following Euler equation (intertemporal IS curve):

$$\hat{C}_t = \mathcal{E}_t \left( \hat{C}_{t+1} \right) - \sigma (\hat{i}_t - \mathcal{E}_t \hat{\pi}_{t+1}) \quad (4)$$

Inflation is  $\pi_t = \frac{P_t}{P_{t-1}} - 1$  and we assume inflation is zero in equilibrium.

## 2.2 Firms and Price Setting

Price setting is based on an extension to Calvo contracting set out in Steinsson (2003). Each period agents recalculate their prices with fixed probability  $1 - \gamma$ . If prices are recalculated, then a proportion of agents  $\omega$  use a backward looking rule of thumb to reset prices, while the remainder calculate the optimum price. If prices are not recalculated (with probability  $\gamma$ ), they rise at the average rate of inflation.

We use an asterisk to denote those firms that do reset their price. Their average price set is a weighted average of forward and backward-looking components:  $P_t^* = (P_t^F)^{1-\omega} (P_t^B)^\omega$ . Backward-looking agents set their prices  $P_t^B$  according to the rule of thumb:

$$P_t^B = P_{t-1}^* \Pi_{t-1} \left( \frac{Y_{t-1}}{Y_{t-1}^n} \right)^\delta \quad (5)$$

where:  $\Pi_t = P_t/P_{t-1}$  and  $Y_t^n$  is the flexible-price equilibrium level of output (defined later). The forward-looking agents are able to solve the first order conditions for profit maximization and obtain an optimal solution  $P_t^F$ , see Rotemberg and Woodford (1997). For the rest of the sector the price will rise at the steady state rate of domestic inflation  $\bar{\Pi}$  with probability  $\gamma$ ,  $P_t = \bar{\Pi} P_{t-1}$ . For the sector as a whole, the price equation can be written as:

$$P_t = [\gamma(\bar{\Pi} P_{t-1})^{1-\epsilon_t} + (1-\gamma)(1-\omega)(P_t^F)^{1-\epsilon_t} + (1-\gamma)\omega(P_t^B)^{1-\epsilon_t}]^{\frac{1}{1-\epsilon_t}}. \quad (6)$$

All optimising producers reset prices in period  $t$  according to the following approximate (log-linear) rule:

$$p_{Ht}^F = (\beta\theta) E_t p_{Ht+1}^F + (1-\beta\theta) (mc_t + p_{Ht}) \quad (7)$$

This is formula (B.2) in Steinsson (2003). To determine marginal costs we assume that the production function for good  $z$  is  $y_t(z) = h_t(z)$  and the cost of supplying a good is given as  $w_s(z)h_s(z)$ . So the (nominal) marginal cost is determined as:

$$S_t(z) = \frac{\partial Cost(z)}{\partial y_t} = w_t(z) = \frac{v_y(y_t(z))}{u_C(C_t)} P_t. \quad (8)$$

The formula for the log-linearised real marginal cost is then

$$mc_t = \frac{1}{\psi} \hat{Y}_t + \frac{1}{\sigma} \hat{C}_t. \quad (9)$$

Steinsson (2003) has shown that (formula (A.3))

$$p_{Ht}^B = (1 - \omega) (p_{Ht-1}^F) + \omega (p_{Ht-1}^B) + \pi_{H,t-1} + \delta y_{t-1} \quad (10)$$

and average inflation is defined as

$$\pi_{Ht} = \frac{(1 - \gamma)}{\gamma} ((1 - \omega) p_{Ht}^F + \omega p_{Ht}^B - p_{Ht}) \quad (11)$$

Manipulations with formulae (7), (10) and (11) (see Steinsson (2003), formulae (A.5), (A.3) and (A.1)) lead to the following inflation equation

$$\hat{\pi}_t = \chi^f \beta \mathcal{E}_t \hat{\pi}_{t+1} + \chi^b \hat{\pi}_{t-1} + \kappa_c \hat{C}_t + \kappa_{y0} \hat{Y}_t + \kappa_{y1} \hat{Y}_{t-1} + \hat{\mu}_t \quad (12)$$

where the shock  $\hat{\mu}_t$  is a mark-up shock . The coefficients of the Phillips curve are:

$$\begin{aligned} \chi^f &= \frac{\gamma}{\gamma + \omega(1 - \gamma + \gamma\beta)}, & \chi^b &= \frac{\omega}{\gamma + \omega(1 - \gamma + \gamma\beta)}, \\ \kappa_c &= \frac{(1 - \gamma\beta)(1 - \gamma)(1 - \omega)\psi}{(\gamma + \omega(1 - \gamma + \gamma\beta))(\psi + \epsilon)\sigma}, & \kappa_{y1} &= \frac{(1 - \gamma)\omega}{\gamma + \omega(1 - \gamma + \gamma\beta)}\delta, \\ \kappa_{y0} &= \frac{(1 - \gamma\beta)(1 - \gamma)(1 - \omega)}{(\gamma + \omega(1 - \gamma + \gamma\beta))(\psi + \epsilon)} - \frac{(1 - \gamma)\gamma\beta\omega}{\gamma + \omega(1 - \gamma + \gamma\beta)}\delta, \\ \delta &= \frac{(1 - \gamma\beta)(\psi + \sigma)}{\gamma\sigma(\psi + \epsilon)}. \end{aligned}$$

where  $\psi = v_y/v_{yy}y$ .

### 2.3 Aggregate Demand and The Economy as a Whole

We now write down the final system of equations for the ‘law of motion’ of the out-of-equilibrium economy. As our economy has no government sector, the national income identity is  $Y_t = C_t$  and its linearised version:

$$\hat{Y}_t = \hat{C}_t. \quad (13)$$



We simplify notation by using lower case letters to denote ‘gap’ variables, where the gap is the difference between actual levels and natural levels i.e.  $x_t = \hat{X}_t - \hat{X}_t^n$ . We omit the expectational superscript, assuming rational expectations,  $\mathcal{E}_t X_{t+1} = X_{t+1}$  for any variable  $X$ .

$$c_t = c_{t+1} - \sigma(i_t - \pi_{t+1}) \quad (14)$$

$$\pi_t = \chi^f \beta \pi_{t+1} + \chi^b \pi_{t-1} + \kappa_{y0} y_t + \kappa_{y1} y_{t-1} + \hat{\mu}_t \quad (15)$$

$$y_t = c_t \quad (16)$$

The model consists of an intertemporal IS curve (14), the Phillips curve (15), and an aggregate demand equation (16).

## 2.4 Central Bank’s decisions

The Central Bank’s control variable is the short-term interest rate. We assume that the central bank explicitly maximises the aggregate utility function:

$$\max_{\{i_s\}_{s=t}^{\infty}} \frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ u(C_s, \xi_s) - \int_0^1 v(h_s(z), \xi_s) dz \right]. \quad (17)$$

We show in Appendix A that (17) implies the following loss function

$$\min_{\{i_s\}_{s=t}^{\infty}} \frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ a_{\pi} \pi_s^2 + a_y y_s^2 + d_{\Delta\pi} (\Delta\pi_s)^2 + d_y y_{s-1}^2 + d_{y\Delta\pi} y_{s-1} \Delta\pi_s \right] + \mathcal{O}(3) \quad (18)$$

where  $\mathcal{O}(3)$  collects terms of higher than second order and terms independent of policy. This quadratic approximation to social welfare is obtained assuming that there is a production subsidy  $\mu^w$  that eliminates the distortion caused by monopolistic competition, again following Steinsson (2003).

This loss function introduces terms in the change in inflation, and lags in output, compared to the more traditional alternative. As Steinsson (2003) discusses, if the proportion of backward-looking price setters ( $\omega$ ) is high, this puts a large weight on stabilising the change in inflation. The presence of inflation inertia also implies some output smoothing, in that we have a coefficient on past output. We assume that the central bank acts under discretion.

## 2.5 Inflation Persistence has no Qualitative Effect

Figure 1 presents the optimal discretionary response to a non-persistent 1% cost push shock under discretion for two values of the parameter  $\omega$ , which measures the proportion of backward

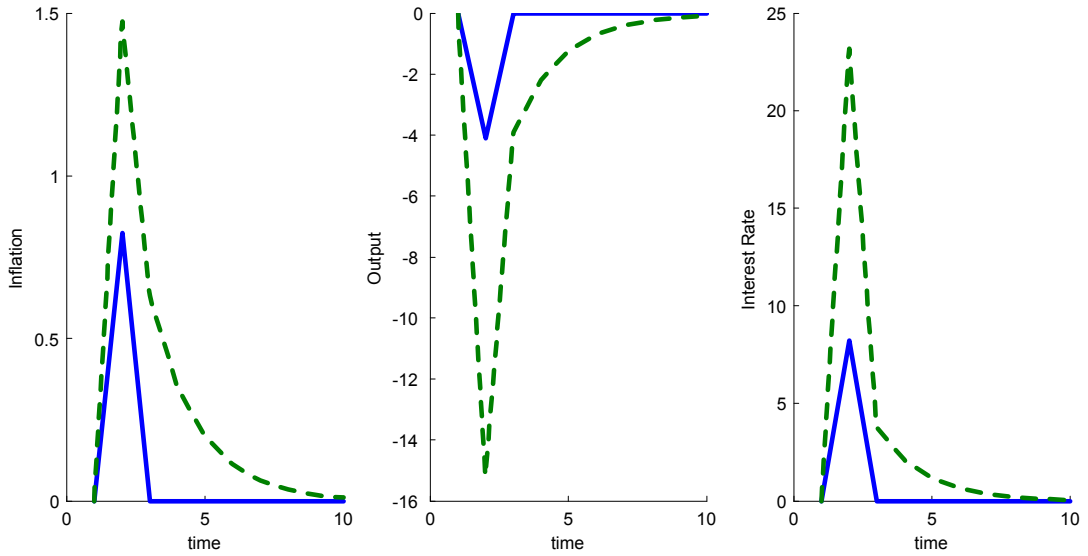


Figure 1: Impulse responses to a unit cost-push shock. Solid line –  $\omega = 0$ , dashed line –  $\omega = 3/4$ .

looking consumers.<sup>4</sup> When  $\omega = 0$ , we have the standard NKPC, and real interest rates rise to dampen the impact of the shock on inflation. The time consistency requirement implies that interest rates only change in the period of the shock. When  $\omega = 0.75$ , so the majority of price setters are backward-looking, we get the same qualitative response, but its size is greater. Monetary policy is much more aggressive, because the impact of the cost-push shock will more long lasting as a result of inflation persistence. Interest rates also rise in the periods following the shock. In both cases the monetary policy maker is prepared to be very aggressive in responding to this inflationary shock, which indicates how much more important inflation is than output in the social welfare function.

These results are standard (see Clarida et al. (1999) for example), and provide evidence for the claim that inflation persistence leads to quantitative but not qualitative differences in optimal policy.

### 3 Extension to a fixed exchange rate regime.

We now extend this model to a small open economy, which either operates a fixed exchange rate regime, or is a member of a monetary union. Following Gali and Monacelli (2005), we assume

<sup>4</sup>Calibration of parameters is the following:  $\gamma = 0.75, \sigma = 0.5, \psi = 1.0, \epsilon = 5.0, \beta = 0.99$ .

that international risk sharing holds.<sup>5</sup> Our derivation of the Euler equation for consumption remains unaffected, apart from the fact that we now need to distinguish between consumer and output prices, with the former defining real interest rates. We also need to allow for the fact that some goods will be imported/exported, both in the national income identity and the derivation of the Phillips curve, and that the fixed nominal exchange rate implies a simple dynamic relationship between inflation differentials and the real exchange rate. It also implies, of course, that nominal interest rates are fixed at the union level.

More specifically, the model in the previous section is modified as follows.

### 3.1 Households

In the presence of imported goods the aggregate consumption bundle is defined as

$$C = \frac{C_H^{1-\alpha} C_F^\alpha}{(1-\alpha)^{(1-\alpha)} \alpha^\alpha} \quad (19)$$

where, if we drop the time subscript, all variables are commensurate.  $C_H$  is a composite of domestically produced goods given by

$$C_H = \left( \int_0^1 C_H(z)^{\frac{\epsilon-1}{\epsilon}} dz \right)^{\frac{\epsilon}{\epsilon-1}} \quad (20)$$

where  $z$  denotes the good's type or variety. The aggregate  $C_F$  is an aggregate across overseas countries  $i$

$$C_F = \left( \int_0^1 C_i^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}} \quad (21)$$

where  $C_i$  is an aggregate similar to (20). The elasticity of substitution between varieties  $\epsilon > 1$  is common across countries. The parameter  $\alpha$  is (inversely) related to the degree of home bias in preferences, and is a natural measure of openness.

Optimisation of expenditure for any individual good implies the demand functions

$$C_H(z) = \left( \frac{P_H(z)}{P_H} \right)^{-\epsilon} C_H, \quad C_i(z) = \left( \frac{P_i(z)}{P_i} \right)^{-\epsilon} C_i \quad (22)$$

where we have price indices given by

$$P_H = \left( \int_0^1 P_H(z)^{1-\epsilon} dz \right)^{\frac{1}{1-\epsilon}}, \quad P_i = \left( \int_0^1 P_i(z)^{1-\epsilon} dz \right)^{\frac{1}{1-\epsilon}} \quad (23)$$

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<sup>5</sup>In contrast, Kirsanova et al. (2006a) consider an economy in which UIP rather than IRS holds. In addition, Kirsanova et al. (2006a) assume that a proportion of consumers are credit constrained, but still hold some debt.

It follows that

$$\int_0^1 P_H(z)C_H(z)dz = P_H C_H, \quad \int_0^1 P_i(z)C_i(z)dj = P_i C_i \quad (24)$$

Optimisation across imported goods by country implies

$$C_i = \left( \frac{P_i}{P_F} \right)^{-\eta} C_F \quad (25)$$

where

$$P_F = \left( \int_0^1 P_i^{1-\eta} di \right)^{\frac{1}{1-\eta}} \quad (26)$$

This allows us to write

$$\int_0^1 P_i C_i di = P_F C_F \quad (27)$$

Optimisation between imported and domestically produced goods implies

$$P_H C_H = (1 - \alpha)PC, \quad P_F C_F = \alpha PC \quad (28)$$

where

$$P = P_H^{1-\alpha} P_F^\alpha \quad (29)$$

is the consumer price index (CPI).

Maximisation of household utility (1) yields (3) and its linearised version (4) but where  $P_t$  is a CPI price index and  $\pi_t$  is CPI inflation.

### 3.2 Identities with PPP

The bilateral terms of trade are the price of country  $i$ 's goods relative to home goods prices.

The effective terms of trade are given by

$$S = \frac{P_F}{P_H}. \quad (30)$$

The CPI and domestic price level are related as

$$P = P_H S^\alpha. \quad (31)$$

We also define the nominal bilateral rate  $EX_i$

$$EX_i = \frac{P_{H,i}}{P_H}$$

and the bilateral real exchange rate  $Q_i$  :

$$Q_i = \frac{EX_i P_i}{P}$$

### 3.3 Firms and Price Setting

Marginal costs are now influenced by the degree of the openness of the economy. From formula (8) it follows that the real marginal cost is

$$\frac{S_t(z)}{P_{Ht}} = \frac{v_y(y_t(z))}{u_C(C_t)} \frac{P_t}{P_{Ht}} \quad (32)$$

so the log-linearised marginal cost is given as

$$mc_t = \frac{1}{\psi} \hat{Y}_t + \alpha \hat{S}_t + \frac{1}{\sigma} \hat{C}_t. \quad (33)$$

The forward-looking agents are able to solve the first order conditions for profit maximization and obtain an optimal solution  $P_t^F$ . Openness of the economy does not affect the behaviour of the backward-looking producers. Manipulations with formulae (7), (10) and (11) where the marginal cost is determined by (33) yield:

$$\hat{\pi}_{Ht} = \chi^f \beta \mathcal{E}_t \hat{\pi}_{Ht+1} + \chi^b \hat{\pi}_{Ht-1} + \kappa_c \hat{C}_t + \kappa_{y0} \hat{Y}_t + \kappa_c \hat{S}_t + \kappa_{y1} \hat{Y}_{t-1} + \hat{\mu}_t \quad (34)$$

where the shock  $\hat{\mu}_t$  is a mark-up shock . The coefficients of the Phillips curve are:

$$\begin{aligned} \chi^f &= \frac{\gamma}{\gamma + \omega(1 - \gamma + \gamma\beta)}, & \chi^b &= \frac{\omega}{\gamma + \omega(1 - \gamma + \gamma\beta)}, \\ \kappa_c &= \frac{(1 - \gamma\beta)(1 - \gamma)(1 - \omega)\psi}{(\gamma + \omega(1 - \gamma + \gamma\beta))(\psi + \epsilon)\sigma}, & \kappa_{y1} &= \frac{(1 - \gamma)\omega}{\gamma + \omega(1 - \gamma + \gamma\beta)}\delta, \\ \kappa_{y0} &= \frac{(1 - \gamma\beta)(1 - \gamma)(1 - \omega)}{(\gamma + \omega(1 - \gamma + \gamma\beta))(\psi + \epsilon)} - \frac{(1 - \gamma)\gamma\beta\omega}{\gamma + \omega(1 - \gamma + \gamma\beta)}\delta, \\ \kappa_c &= \frac{(1 - \beta\gamma)(1 - \gamma)(1 - \omega)}{(\gamma + \omega(1 - \gamma + \beta\gamma))(\psi + \epsilon)}\alpha, & \delta &= \frac{(1 - \gamma\beta)(\psi + \sigma)}{\gamma\sigma(\psi + \epsilon)}. \end{aligned}$$

### 3.4 Capital markets

Under the assumption of complete securities markets, a first order condition to the household optimisation problem must hold for the representative household in any country. This assumption implies complete International Risk Sharing:

$$C = C_i Q_i^\sigma$$

and its linearised version:

$$\hat{C}_t = \hat{C}_t^* + \sigma(1 - \alpha)\hat{S}_t. \quad (35)$$

### 3.5 Aggregate Demand

Goods market clearing requires

$$Y(z) = C_H(z) + \int_0^1 C_i(z) dz \quad (36)$$

Symmetrical preferences imply

$$C_{Hi}(z) = \alpha \left( \frac{p_H(z)}{P_H} \right)^{-\epsilon} \left( \frac{P_H}{EX_i P_i} \right)^{-1} C_i \quad (37)$$

which allows us to write

$$Y(z) = \left( \frac{p_H(z)}{P_H} \right)^{-\epsilon} \left[ (1 - \alpha) \frac{PC}{P_H} + \alpha \int_0^1 \frac{EX_i P_i C_i}{P_H} di \right] \quad (38)$$

Defining aggregate output as

$$Y = \left[ \int_0^1 Y(z)^{\frac{\epsilon-1}{\epsilon}} dz \right]^{\frac{\epsilon}{\epsilon-1}} \quad (39)$$

allows us to write

$$Y = (1 - \alpha) \frac{PC}{P_H} + \alpha \int_0^1 \frac{EX_i P_i C_i}{P_H} di = S^\alpha \left[ (1 - \alpha) C + \alpha \int_0^1 Q_i C_i di \right] = CS^\alpha \quad (40)$$

### 3.6 The Complete Model

Our system consists of equations (4) (35), (40), (30), and (34). Log-linearising and assuming a fixed exchange rate regime, we obtain the following:

$$c_t = c_{t+1} + (1 - \alpha) \pi_{Ht+1} + \alpha \pi_{Ht+1}^* \quad (41)$$

$$y_t = (1 - \alpha) c_s + 2\eta\alpha (1 - \alpha) s_t + \alpha c_t^* \quad (42)$$

$$s_t = \pi_{Ht}^* - \pi_{Ht} + s_{t-1} \quad (43)$$

$$\pi_{Ht} = \chi^f \beta \mathcal{E}_t \pi_{Ht+1} + \chi^b \pi_{Ht-1} + \kappa_c c_t + \kappa_s s_t + \kappa_{y0} y_t + \kappa_{y1} y_{t-1} \quad (44)$$

where we denote with an asterisk all external variables, which are exogenous to the model.

### 3.7 Severe Cycles if Inflation is Persistent

Figure 2 shows the response of the economy to a world consumption shock, again for two different values of  $\omega$ , representing different degrees of inflation persistence.<sup>6</sup> (Our choice of shock is unimportant.) We again find that the impact of the shock is greater with persistence.

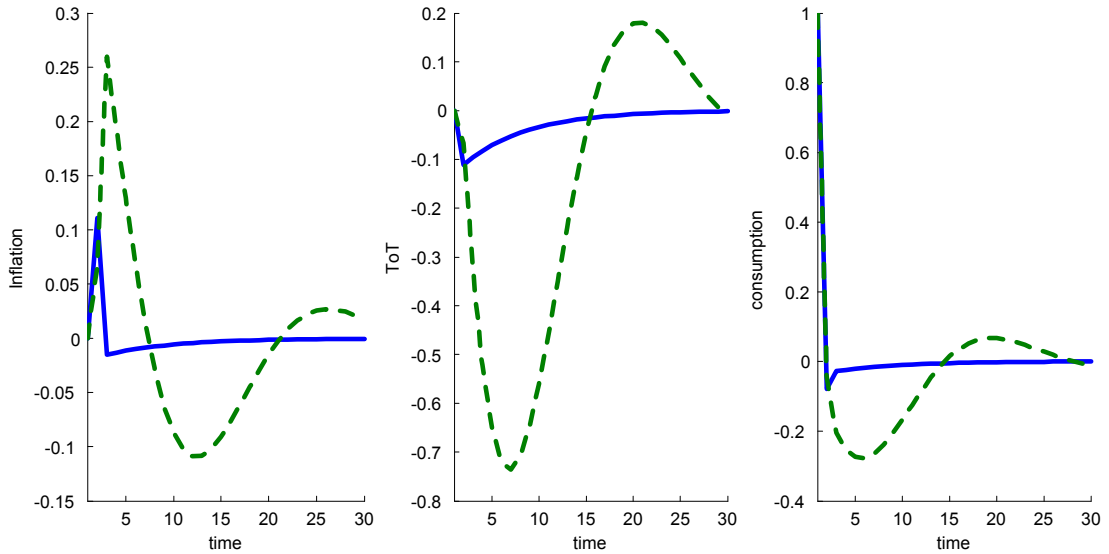


Figure 2: Impulse responses to a unit world output shock. Solid line –  $\omega = 0$ , dashed line –  $\omega = 3/4$ .

However we now have an additional feature when we add persistence, which is that the economy cycles.

Cyclicity that is this severe is clearly undesirable: there is no need to explicitly calculate welfare to see this. It is an intrinsic feature of the fixed exchange rate regime. There are two reasons for this behaviour.

Firstly, the terms of trade can behave in an oscillatory way. With a fixed exchange rate, the terms of trade must return to their initial level, and therefore so must the price level. Along this path, output depends negatively on the price level through the effects of the real exchange rate, and so output falls initially as a result of the initial inflation. This low level of output will cause the price level to return towards its equilibrium; but persistence means that it will fall at an increasing rate. (Persistence makes the Phillips curve ‘accelerationist’.) Thus the price level will tend to overshoot. This behaviour can be oscillatory: we have identified features tending to cause the rate of change of *inflation* to depend negatively on the *price level*; this is the form of equation system which causes the oscillatory behaviour of a pendulum.

Second, under fixed exchange rates, nominal interest rates are fixed. When we have inflation

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<sup>6</sup>Calibration of parameters is the following:  $\gamma = 0.75, \sigma = 0.5, \psi = 1.0, \epsilon = 5.0, \beta = 0.99, \alpha = 0.3$ .

persistence, a positive shock to inflation will raise expected future inflation, which will lead real interest rates (defined in terms of expected inflation) to *fall*. This is a violation of the Taylor principle. If consumption was not determined intertemporally, but instead depended on current income and current interest rates, then the destabilising impact of this fall in real interest rates would be immediate (Kirsanova et al. (2006b)). The danger of this effect, with fixed exchange rates, has become known as ‘Walters’ critique’. But here consumption is forward looking, and so the behavior of consumption contributes to cyclicity, rather than causing instability. Consumption does initially rise following the shock, because real interest rates initially fall. But real interest rates begin to rise after a couple of years, when inflation becomes negative, and the expectation of this soon dominates, causing a fall in consumption. But that will cause inflation to fall; with persistence this will add to the reasons, described above, that cause the price level to overshoot. The inevitable violation of the Taylor principle, together with terms of trade effects, can set up a prolonged cycle.

This problem would not arise under floating exchange rates. In those circumstances there would be no need to return the price level to its starting position (and there would therefore be no aspects of simple harmonic motion). Also, monetary policy could follow the Taylor principle with a floating exchange rate, and ensure that inflation was met by higher real interest rates. This, in turn, would cause consumption to fall initially, and not to rise as it does with a fixed exchange rate. Equally, the problem would not occur without inflation persistence. Without persistence the inflation rate jumps below zero and the price level gradually converges back to its starting position. Also, inflation now has (almost) no implications for future inflation, and therefore there is no impact on real interest rates.

The model is still stable as long as inflation persistence is not complete (i.e. as long as  $\omega < 1$ ), because of the impact of competitiveness on output. When inflation is positive, competitiveness declines, and this reduces the demand for domestically produced goods, which produces a negative output gap which reduces inflation through the Phillips curve. However this stabilising mechanism works as the price level changes, and so cannot directly offset the destabilising movement in real interest rates noted above. As a result, we get cyclical movements in the economy. In the limit, with  $\omega = 1$ , we would have pure cycles, with no return to equilibrium.

We would not want to overstate the importance of this result. Cycles of the size shown in Figure 2 do require a large proportion of price setters to be backward looking: the same chart for  $\omega = 0.5$ , for example, shows much less cyclicity. (The value of  $\omega$  for which the roots of the system become complex is around 0.43.) In addition, it would be possible to mitigate these



cycles through countercyclical fiscal policy (see Kirsanova et al. (2007) for example). However, it remains the case that the existence of inflation persistence clearly makes the choice of a fixed exchange rate regime problematic in a way that is absent without persistence, and in this sense inflation persistence makes a qualitative difference to the choice of policy regime.

## 4 Adding fiscal policy

The closed economy model in Section 2 abstracts from the government budget constraint and the existence of government debt. While this simplification may not matter when lump sum taxes exist and consumers are infinitely lived, the assumption of lump sum taxes is hardly realistic. In this section, following Stehn and Vines (2007), we extend the model of section 2 to include government spending, income taxes and government debt. Although we allow for the fact that income taxes are distortionary, we hold this tax rate  $\tau$  constant, and instead assume that government spending is a policy instrument.

We therefore need to add a government budget constraint to the model. We also need to allow for government spending in the utility function, the national income identity and the Phillips curve. The consumption Euler equation is unaffected, but our derived measure of social welfare will change. Specifically, the model is derived as follows.

### 4.1 Consumers

Individuals consume a basket of goods  $C$ , and derive utility from per capita government consumption  $G$ . Their maximization problem is

$$\max_{\{C_v^s, h_v^s\}_{v=t}^{\infty}} \mathcal{E}_t \sum_{v=t}^{\infty} \beta^{v-t} [u(C_v) + f(G_v) - v(h_v(z))]$$
 (45)

Maximising this utility subject to budget constraint (2) we derive Euler equation (4). Additionally, aggregate (nominal) asset accumulation is given by

$$\mathcal{A}_{t+1} = (1 + i_t)(\mathcal{A}_t + (1 - \tau)P_t Y_t - P_t C_t)$$
 (46)

We denote  $A_t = \mathcal{A}_t/P_{t-1}$  and linearise it as

$$\hat{A}_{t+1} = \hat{i}_t + (1 + i) \left( \hat{A}_t - \hat{\pi}_t + \frac{(1 - \tau)}{A} \hat{Y}_t - \frac{\rho}{A} \hat{C}_t \right),$$

where  $\rho = C/Y$  is the steady state share of private consumption in  $Y$  and  $A$  is the steady state level of real assets as a share of  $Y$ .

## 4.2 Price Setting and Aggregate Demand

Price setting remains unaffected by the introduction of the government sector in the economy and leads to equation (34). Government expenditures constitute part of demand

$$Y_t = C_t + G_t \quad (47)$$

and in steady state  $G = (1 - \rho)Y$ . The linearised aggregate demand equation is then:

$$\hat{Y}_t = (1 - \rho)\hat{G}_t + \rho\hat{C}_t. \quad (48)$$

## 4.3 Fiscal Authorities

The government buys goods ( $G$ ), taxes income (with tax rate  $\tau$ ), and issues nominal debt  $\mathcal{B}$ . The evolution of the nominal debt stock can be written as:

$$\mathcal{B}_{t+1} = (1 + i_t)(\mathcal{B}_t + P_t G_t - \tau P_t Y_t)$$

We assume that the tax rate on income is fixed. This equation can be linearised as (assuming  $B_t = \mathcal{B}_t/P_{t-1}$ ):

$$\hat{B}_{t+1} = \hat{i}_t + (1 + i) \left( \hat{B}_t - \hat{\pi}_t + \frac{1 - \rho}{B} \hat{G}_t - \frac{\tau}{B} \hat{Y}_t \right) \quad (49)$$

## 4.4 The Economy as a Whole

We now write down the final system of equations for the ‘law of motion’ of the out-of-equilibrium economy:

$$c_t = c_{t+1} - \sigma(i_t - \pi_{t+1}) \quad (50)$$

$$\pi_t = \chi^f \beta \pi_{t+1} + \chi^b \pi_{t-1} + \kappa_c c_t + \kappa_{y0} y_t + \kappa_{y1} y_{t-1} + \hat{\mu}_t \quad (51)$$

$$y_t = (1 - \rho)g_t + \rho c_t \quad (52)$$

$$\hat{A}_{t+1} = \hat{i}_t + (1 + i) \left( \hat{A}_t - \pi_t + \frac{(1 - \tau)}{A} y_t - \frac{\theta}{A} c_t \right) \quad (53)$$

$$\hat{B}_t = \hat{A}_t \quad (54)$$

The model consists of an intertemporal IS curve (50), the Phillips curve (51), an aggregate demand equation (52), and an equation explaining the evolution of assets (53). We could use the debt accumulation equation (49) instead of (53) as they are equivalent (equation (54)).

## 4.5 Policy Decisions

The Monetary and Fiscal authorities set their instruments jointly, under cooperative discretionary policy. The Central Bank's control variable is the short-term interest rate  $i_t$  and the fiscal authorities control government spending,  $G_t$ . We assume that the authorities explicitly maximises the aggregate utility function:

$$\max_{\{i_s\}_{s=t}^{\infty}} \frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ u(C_s) + f(G_s) - \int_0^1 v(h_s(z)) dz \right]. \quad (55)$$

We show in Appendix A that (55) implies the following loss function

$$\begin{aligned} \min_{\{i_s\}_{s=t}^{\infty}} \frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} & [a_{\pi} \pi_s^2 + a_c c_s^2 + a_g g_s^2 + a_y y_s^2 \\ & + d_{\Delta\pi} (\Delta\pi_s)^2 + d_y y_{s-1}^2 + d_{y\Delta\pi} y_{s-1} \Delta\pi_s] + \mathcal{O}(3) \end{aligned} \quad (56)$$

where  $\mathcal{O}(3)$  collects terms of higher than second order and terms independent of policy. As in Section 2, this quadratic approximation to social welfare is obtained assuming that there is a production subsidy  $\mu^w$  that eliminates the distortion caused by monopolistic competition and income taxes.

## 4.6 Passive Policies

In this model, the benevolent policy maker has both interest rates and government spending as instruments available to stabilise the economy. If the policy maker is also able to pursue a time inconsistent policy, then Kirsanova and Wren-Lewis (2006) show that optimal policy in this case will involve debt following a random walk, a result obtained for the case where taxes rather than spending is the fiscal instrument by Schmitt-Grohe and Uribe (2004) and Benigno and Woodford (2004). (This, in turn, replicates for New Keynesian models a result from the tax smoothing literature.) If debt follows a random walk, it largely accommodates the impact of any fiscal shock, and so the response of monetary policy to a cost-push shock will be largely unaffected by the need to stabilise debt.

However, if a commitment mechanism for policy does not exist, and policy is forced to be time consistent, then debt no longer follows a random walk. Leith and Wren-Lewis (2007) show this is the case where both government spending and taxes are instruments. The reason for this follows from a close examination of the time inconsistent policy. Although that policy largely accommodates any shock to the government's budget constraint, it does not do so completely: there is some attempt to reduce the long run level of debt. The NKPC implies that it is optimal

to reduce debt in the initial period following the shock: thereafter fiscal instruments move to the new steady state values required to service the new level of debt. This attempt to reduce debt in the initial period is time inconsistent, yet it will be present so long as the long run level of debt differs from the initial steady state (assuming the latter is also an efficient steady state). As a result, under discretion policy must return debt to its pre-shock level.

If debt must return to its initial level, how is this to be achieved: by changing government spending (which is the fiscal instrument in our model) or by changing interest rates? Figure 3 plots optimal policy in our model, once again looking at the no-inflation persistence case, and the case where half price setters are backward looking.<sup>7</sup>

The inflationary shock, for given interest rates and spending, will reduce the stock of government debt. In the case where  $\omega = 0$ , government spending also falls in the initial period, in an attempt to reduce the impact of the cost push shock. Interest rates on the other hand also fall initially. In subsequent periods, the direction of movement of each instrument is reversed, with both government spending and interest rates rising. What can explain this counter-intuitive result, that at least initially fiscal policy tackles the cost-push shock, and monetary policy moves in a perverse direction?

The key to understanding this is in the profile of interest rates over time, coupled with the forward looking nature of parts of the model. Initial cuts in interest rates are subsequently reversed. As a result, the full impact of monetary policy on initial inflation is still negative, as we can see from the path of consumption: higher future interest rates lower current and future consumption, and lower current and future consumption lowers current inflation.

But why not raise current interest rates as well as future interest rates? The reason is that this would fail to bring government debt back to its initial level, or require sub-optimal movements in government spending to do so. Because the government budget constraint is inherently backward looking, falls in current interest rates can offset the impact of future increases on debt, and yet still leave the impact on inflation negative. Policy is therefore exploiting the forward looking nature of consumption and inflation dynamics, relative to the backward dynamics in the budget constraint.<sup>8</sup>

Now consider the case where we have inflation persistence, which is discussed in Stehn and Vines (2007). Interest rates now rise initially, and stay higher for a number of periods. We no

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<sup>7</sup>Calibration of parameters is the following:  $\gamma = 0.75, \sigma = 0.5, \psi = 1.0, \epsilon = 5.0, \beta = 0.99, B = 0.3, \rho = 0.75$ .

<sup>8</sup>Why does government spending not try and play the same trick: of rising initially but falling thereafter? Unlike interest rates,  $G$  works directly on demand, so it cannot exploit the forward looking nature of consumption. Future falls in  $G$  will therefore be less effective at reducing current inflation.

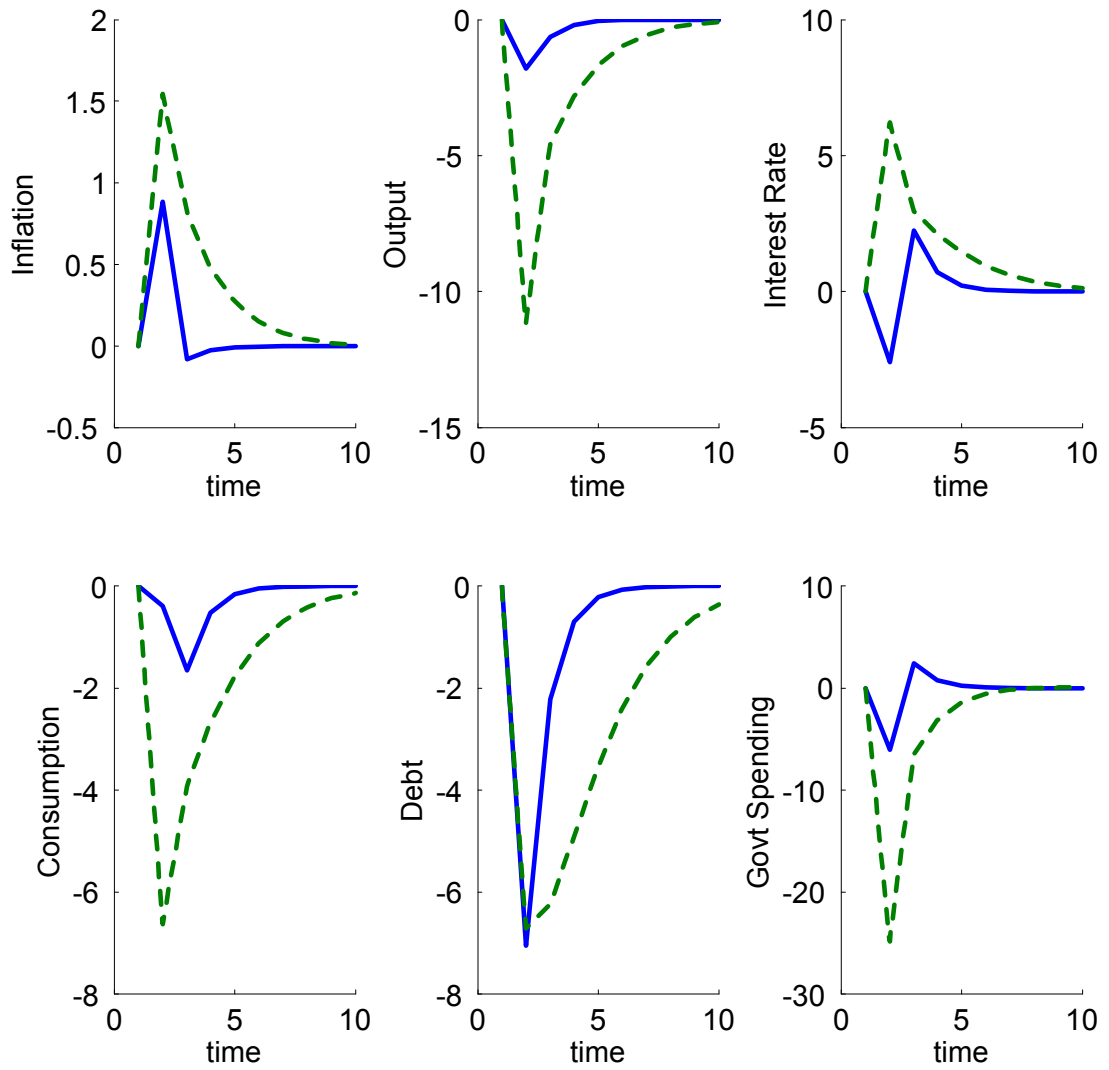


Figure 3: Impulse responses to a unit cost-push shock. Solid line  $-\omega = 0$ , dashed line  $-\omega = 3/4$ .

longer have this reversal of movement in the monetary policy instrument. The reasons follows directly from the previous discussion. When inflation is backward looking, the contrast between the dynamics of inflation and the budget constraint is no longer so marked. The device of achieving deflation while not destabilising debt by shifting forward higher interest rates is no longer effective.

The direction in which interest rates move in this model will obviously be sensitive to other parameters, and particularly assumptions about the initial level of government debt. As Leith and Wren-Lewis (2007) and Stehn and Vines (2007) show, it is the existence of an initial stock of debt which gives interest rates leverage over debt, and which means that monetary policy needs to take account of its impact on debt. Nevertheless, it remains the case that inflation persistence is crucial in understanding why the initial response of monetary policy changes sign so noticeably here, and why it therefore induces a qualitative change in a policy response.

## 5 Conclusions

In the basic New Keynesian model, inflation persistence changes the way policy reacts to shocks in a quantitative way, but the policy framework and the direct of response remain unchanged. In this paper we have extended the model in two different directions, and in each case we have shown that inflation persistence matters in a more fundamental, qualitative sense.

In our first example, we adapt the model to the case of a small open economy under fixed exchange rates, or alternatively as a (small) member of a monetary union. We showed how inflation persistence can lead to a severe cyclical response to shocks in this economy, with obvious negative consequences for welfare. This cyclical response stems from the impact of inflation persistence on real interest rates, and was not negated by competitiveness effects on output. This cyclical response could influence policy choice, in that it might make a fixed exchange rate regime undesirable if inflation persistence was thought to be large.

Our second example moved back to a closed economy, but now added government debt and fiscal policy. In the absence of inflation inertia, we showed that a benevolent policy maker, acting under discretion with both government spending and interest rates as instruments, would cut interest rates following a cost-push shock. This apparently perverse result followed directly from the lack of inflation persistence in the NKPC, and we showed that once inflation persistence was introduced into the model, interest rates rose following the cost-push shock. Introducing inflation persistence in this case changed the sign of the initial response of monetary policy to a shock.

These two examples appear very different. However, there is a rather deep connection between them. Both add a degree of integral control to the basic New Keynesian model. In both examples we have a variable that needs to return to its original level. In the first example, the membership of a fixed exchange rate regime requires that the price level returns to its initial starting position, and this can cause cyclical behavior if there is a high degree of inflation persistence. By contrast, in the analysis in Section 2, in which inflation is controlled, there is no such need to return the price level to its initial level. In the second example, discretionary monetary policy necessarily stabilizes debt at its initial level, and the manner in which this is best done depends on the degree of inflation persistence. By contrast, in the analysis in Section 2, there was no need to control debt, since there was no analysis of fiscal policy.

Adding inflation persistence makes inflation control more difficult. In each of our two examples, adding integral control also makes the system more difficult to control. We have shown that the interaction of these two difficulties, can, in comparison with the basic model, make the dynamic properties of policy regime to become qualitatively different or make a qualitative difference to the way in which optimal policy responds to shocks.

## A Social Welfare

We use social welfare in Sections 2 and 4. We show how the social welfare can be derived for the model in Section 4, and then the social welfare function for the model in Section 2 can be obtained by eliminating the government sector. We, of course, will obtain the same welfare metric as derived in Steinsson (2003).<sup>9</sup>

One-period social welfare function  $\mathcal{W}_s$  in (45) can be linearised around the steady state (see Woodford (2003))

$$\begin{aligned} \mathcal{W}_s = & \theta u_C [\hat{C}_s + \frac{1}{2}(1 - \frac{1}{\sigma})\hat{C}_s^2] + (1 - \theta) f_G [\hat{G}_s + \frac{1}{2}(1 - \frac{1}{\sigma})\hat{G}_s^2] \\ & - v_y [\hat{Y}_s + \frac{1}{2}(1 + \frac{1}{\psi})\hat{Y}_s^2 + \frac{1}{2}(\frac{1}{\psi} + \frac{1}{\epsilon})var_z \hat{y}_s(z)] \end{aligned} \quad (57)$$

where we assumed  $\sigma = -u_C(C, 1)/u_{CC}(C, 1)C = -f_G(G, 1)/f_{GG}(G, 1)G$ .

A second-order approximation of aggregate demand (47) can be written as

$$\hat{C} = \frac{1}{\theta}(\hat{Y} - (1 - \theta)\hat{G} - \theta\frac{1}{2}\hat{C}^2 - \frac{1}{2}(1 - \theta)\hat{G}^2 + \frac{1}{2}\hat{Y}^2)$$

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<sup>9</sup>We use the Erratum available at [www.people.fas.harvard.edu/~steinss/papers/STjme03erratum.pdf](http://www.people.fas.harvard.edu/~steinss/papers/STjme03erratum.pdf)

so we can substitute consumption in (57) and obtain

$$\begin{aligned} \mathcal{W}_s = & \theta u_C \left[ \left(1 - \frac{v_y}{u_C}\right) \hat{Y}_s - (1 - \theta) \left(1 - \frac{f_G}{u_C}\right) \hat{G}_s - \frac{\theta}{2\sigma} \hat{C}^2 - \frac{(1 - \theta)}{2} \left(1 + \frac{f_G}{u_C} \frac{(1 - \sigma)}{\sigma}\right) \hat{G}_s^2 \right. \\ & \left. - \frac{1}{2} \left(\frac{v_y}{u_C} \frac{1 + \psi}{\psi} - 1\right) \hat{Y}_s^2 - \frac{1}{2} \frac{v_y}{u_C} \frac{\psi + \epsilon}{\psi \epsilon} \text{var}_z \hat{y}_s(z) \right] \end{aligned}$$

To transform this equation into a more convenient form that does not include linear terms, we proceed as follows (see Beetsma and Jensen (2004)). The aggregate demand relationship (47) is an identity along the dynamic path of the economy, which can be differentiated with respect to government expenditures in order to yield the following condition:

$$\frac{\partial Y_t}{\partial G_t} = \frac{\partial C_t}{\partial G_t} + 1 \quad (58)$$

The first order condition  $(1 - \tau)(\epsilon_t - 1)/\epsilon_t = v_y(Y_t)/u_C(C_t)$  also holds along the dynamic path of the economy. Its differentiation yields:

$$\frac{(1 - \tau)(\epsilon_t - 1)}{\epsilon_t} u_{CC}(C_t) \frac{\partial C_t}{\partial G_t} = v_{yy}(Y_t) \frac{\partial Y_t}{\partial G_t} \quad (59)$$

Both conditions (58) and (59) hold in the steady state and can be solved for  $\frac{\partial C_t}{\partial G_t}$  and  $\frac{\partial Y_t}{\partial G_t}$ :

$$\frac{\partial C_t}{\partial G_t} = -\frac{\theta\sigma}{(\psi + \theta\sigma)}, \quad \frac{\partial Y_t}{\partial G_t} = \frac{\psi}{(\psi + \theta\sigma)}$$

We assume that the steady state level of government expenditures is chosen to maximise utility function (subject to aggregate demand constraint and aggregate supply conditions), so that in the steady state<sup>10</sup>:

$$\frac{\partial}{\partial G} [u(C_s) + f(G) - v(Y_s)] = [u_C(C) \frac{\partial C}{\partial G} + f_G(G) - v_y(Y) \frac{\partial Y}{\partial G}] = 0 \quad (60)$$

Finally, from (58), (59) and (60) it follows that in equilibrium:

$$\frac{f_G}{u_C} = \frac{v_y}{u_C} \frac{\partial Y}{\partial G} - \frac{\partial C}{\partial G} = \frac{\psi(1 - \tau)(\epsilon - 1)/\epsilon + \theta\sigma}{(\psi + \theta\sigma)} \quad (61)$$

Now if the government removes monopolistic distortions *and* distortions from labour income taxation in the steady state using a subsidy, then  $f_G/u_C = 1$  and so the welfare function does not contain linear terms. This yields us the final formula for social welfare

$$\mathcal{W}_s = -\theta u_C \left[ \frac{\theta}{2\sigma} c_s^2 + \frac{(1 - \theta)}{2\sigma} g_s^2 + \frac{1}{2\psi} y_s^2 + \frac{1}{2} \left(\frac{1}{\psi} + \frac{1}{\epsilon}\right) \text{var}_z \hat{y}_s(z) \right]$$

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<sup>10</sup>Derivatives of constraints are equal to zero so we did not include them in the final expression.



Steinsson (2003) have shown that

$$\text{var}_z \hat{y}_s(z) = \frac{\epsilon^2}{(1-\gamma\beta)(1-\omega)} \left( \frac{\gamma(1-\omega)}{1-\gamma} \pi_t^2 + \frac{\omega}{(1-\gamma)} (\Delta\pi_t)^2 + (1-\gamma)\omega\delta^2 y_{t-1}^2 - 2\omega\delta y_{t-1} \Delta\pi_t \right)$$

so we get the final formula for the social welfare function (were we normalise so there is a unit coefficient on inflation variability):

$$\begin{aligned} \mathcal{W}_s = & -\frac{1}{2} \frac{(\epsilon + \psi) \epsilon \theta u_C \gamma}{\psi(1-\gamma\beta)(1-\gamma)} \left( \frac{\psi(1-\gamma\beta)(1-\gamma)}{\epsilon(\epsilon + \psi)\gamma} \left( \frac{\theta}{\sigma} c_s^2 + \frac{(1-\theta)}{\sigma} g_s^2 + \frac{1}{\psi} y_s^2 \right) \right. \\ & \left. + \pi_t^2 + \frac{\omega}{\gamma(1-\omega)} (\Delta\pi_t - \delta(1-\gamma)y_{t-1})^2 \right) \end{aligned}$$

which is formula (56) in the main text.

If there is no government sector in the economy, i.e.  $\theta = 1$ , then the formula above collapses to the one derived in Steinsson (2003) and which is given by formula (18) in the main text.

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