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by Christopher Phillip Reicher

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This paper documents the short run and long run behavior of the search and matching model with staggered Nash wage bargaining. It turns out that there is a strong tradeoff inherent in assuming that previously bargained sticky wages apply to new hires. If sticky wages apply to new hires, then the staggered Nash bargaining model can generate realistic volatility in labor input, but it predicts a strong counterfactually negative long run relationship between inflation and unemployment. This finding is robust to including a microeconomically realistic degree of indexation of wages to inflation. The lack of a negative long run relationship between trend inflation and unemployment provides indirect evidence against the proposed mechanism that high inflation systematically makes new hiring more profitable by depressing the real wages of new hires.

Keywords: Sticky wages, staggered Nash bargaining, trend inflation, unemployment, search and matching.

JEL classification: E24, E25, J23, J31.

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# Sticky wages in search and matching models in the short and long run

### Christopher Phillip Reicher <sup>a</sup> Kiel Institute for the World Economy

This version: July 22, 2011

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#### Sticky wages in search and matching models in the short and long run

#### 1. Introduction

This paper shows that there is an important but overlooked tradeoff inherent in introducing sticky wages into search and matching models. A search and matching model with staggered Nash bargaining over nominal wages, when fed a realistic array of shocks and calibrated to a microeconomically plausible process of wage formation, can generate a plausible degree of macroeconomic volatility. This is what has motivated Gertler and Trigari (2009), Hall (2005), Shimer (2005), and many others to include sticky wages in search and matching models. However, such models can generate realistic volatility only at the cost of predicting a strong and counterfactually negative relationship between unemployment and inflation in the long run. In other words, the model can generate a high degree of volatility in employment and vacancies, or it can generate no long run relationship between unemployment and inflation; it cannot do both. This finding has important implications with regard to the debate over the degree to which the wages of new hires are sticky—the long run evidence suggests that sticky wages do not substantially feed through into hiring decisions.

The logic underlying this tradeoff is simple. If wages are sticky and sticky wages apply to new hires, then sticky wages make it possible for the profitability of a new hire to rise after a positive shock to productivity or prices. However, a rise in trend inflation will tend to place continual downward pressure on real wages, permanently increasing the profitability of a new hire and raising the number of vacancies and the amount of labor input demanded. In fact, labor input (as measured by the unemployment rate) and inflation seem to covary negatively in the long run in the United States. Something about the logic which links inflation to low real wages and profitable vacancy creation does not hold in the long run for the United States.

Wages are sticky at the micro level. Kahn (1997) and Barratieri, Basu, and Gottschalk (2010), as well as everyday experience, suggest that nominal wages adjust infrequently

and in a staggered manner. Gertler and Trigari (2009), Gertler, Sala, and Trigari (2008) and Martins, Snell, and Thomas (2009) argue that the wages of new hires are sticky in relation to previously bargained wages. In modeling sticky wages in a search framework, these authors preserve individual rationality at the match level, and their model is are not subject to the Barro (1977) critique. This approach at generating volatility operates through the assumption that new hires are paid the going wage; if the real wage is temporarily "too low"; hiring becomes profitable; firms post more vacancies; and employment rises after a favorable shock. If new hires instead get to bargain over their wages as argued by Haefke, Sonntag, and van Rijns (2009), Pissarides (2009), and Rudanko (2009), then sticky wages should have no effect on real aggregates. In that case, real allocations should mimic the allocations found in a model with flexible wages. The performance of the sticky wage model over the cycle depends crucially on whether sticky wages are allocational with respect to new hires. It turns out that the long run evidence favors the latter view—if the wages for new hires were sticky, then changes to trend labor demand and hiring should mimic changes in trend inflation. In fact, they seem to go in opposite directions in the long run.

The rest of this paper follows the traditional format. Section 2 lays out the sticky wage model and briefly discusses its long run implications. Section 3 lays out the empirical strategy; section 4 discusses the results; and section 5 concludes.

#### 2. The model

I take a standard individualistic search and matching model similar to that of Mortensen and Pissarides (1994) and Hagedorn and Manovskii (2008) but add sticky nominal wages. I follow Gertler and Trigari (2009) in modeling wage determination according to staggered Nash bargaining, where the wages of new hires are drawn from the prevailing wage distribution. Prices are flexible; the only source of nominal propagation is through the effect of sticky wages on the profitability of a new hire. The model has five structural disturbances which are commonly used in business cycle models and for which reasonable data exist: A shock to total factor productivity, a shock to the relative price of

investment goods, a Taylor rule shock, a shock to trend inflation, and a shock to government spending.

#### 2.1 The household sector

Individuals within households, indexed by j, supply labor indivisibly; they either work for a set number of hours per week or do not work at all. They also have the choice between consuming in a given period and saving in nominal bonds to consume in the future. They each seek to maximize the objective function:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ \ln(C_{j,t+i}) - A n_{j,t+i} \right], \tag{1}$$

where  $C_{j,t+i}$  equals the household's real consumption and  $n_{j,t+i}$  is the proportion of workers in the household who work at the end of a given period. For the sake of tractability, households are large and pool consumption efficiently. Households buy vacancies from firms at a price  $p_t^{\nu}$  and own the resulting firms.

The household's budget constraint is the usual one with a couple of additions.  $B_t$  equals the number of bonds that households buy; households also can consume or invest (or buy vacancies) out of beginning-of-period wealth and gross income  $Q_t$ . The government consumes  $G_t$  units of output which it finances with lump-sum taxes. Bonds earn the gross nominal interest rate  $R_t$ . The budget constraint is standard, with the addition of vacancies into the budget constraint:

$$B_{t+1} + P_t C_t + P_t I_t + P_t G_t + p_t^{\nu} V_t = P_t Q_t + R_{t-1} B_t.$$
 (2)

The household's first-order conditions are also standard. Optimization in bonds generates the usual intertemporal asset pricing relationship:

$$\lambda_{t} = E_{t} \beta R_{t} \frac{P_{t}}{P_{t+1}} \lambda_{t+1}, \tag{3}$$

where the household's marginal utilities of consumption and wealth are equal:

$$\frac{1}{C_t} - \lambda_t = 0. (4)$$

#### 2.2 The wholesale sector

The wholesale sector produces and distributes output originating from a set of worker-firm matches. Workers and firms separate for exogenous reasons, and firms search for workers based on expectations of future profitability. Using standard notation, the number of searchers  $U_t$  equals  $1 - N_t$ , with the population normalized to one. There is a constant probability  $\rho$  that a match will end exogenously, for a constant survival rate of  $\varphi$ . The remaining  $\varphi N_t$  matches must decide to continue or separate, though they never separate in equilibrium.

Capital is measured in consumption units; its real price is  $p_t^k$ , so the real stock of capital in capital units is given by  $k_{it}/p_t^k$ . If the relationship continues, the match produces  $y_{it} = z_t^{1-\alpha} (k_{it}/p_t^k)^{\alpha}$  units of output which are sold competitively to the households. Matches rent capital in a competitive rental market at a rate  $\rho_t^k$  after all aggregate shocks are realized. The surplus of a match at period t equals the real value of the match's production, minus the disutility of work in product terms, plus the expected discounted continuation value of the match (denoted by  $q_{it}$ ), minus the match's capital rental payments:

$$s_{it} = z_t^{1-\alpha} (k_{it} / p_t^k)^{\alpha} - \frac{A}{\lambda_t} + q_{it} - \rho_t^k k_{it}.$$
 (5)

The outside option is compatible with balanced growth; labor market aggregates exhibit no trend while output and productivity tend to grow over time.

The value of  $k_{ii}$  is determined optimally by the match, so that:

$$\frac{k_{it}}{p_t^k} = z_t \left[ \frac{\alpha}{\rho_t^K p_t^k} \right]^{\frac{1}{1-\alpha}}.$$
 (6)

In equilibrium, capital earns a share  $\alpha$  of gross output. As a result, firms and workers bargain over the worker's marginal product:

$$s_{ii} = (1 - \alpha) z_t^{1 - \alpha} (k_{ii} / p_t^k)^{\alpha} - \frac{A}{\lambda_t} + q_{ii}.$$
 (7)

Substituting in the firm's capital demand, one can find the reduced form of the surplus:

$$s_{it} = (1 - \alpha) z_t \left[ \frac{\alpha}{\rho_t^K p_t^K} \right]^{\frac{\alpha}{1 - \alpha}} - \frac{A}{\lambda_t} + q_{it}.$$
 (8)

A high rental rate  $\rho_t^k$  or a high price of capital  $p_t^k$  acts like a negative technology shock and reduces the equilibrium surplus, all else equal.

Entrepreneurial firms owned directly by the households can post vacancies at a marginal cost  $p_t^v$  but face no other barriers to entry. These vacancies get filled at a gross rate  $k_t^f$ . A firm's portion of the surplus at any given date is given by  $s_{it}^f$ , with an economywide average given by  $s_t^f$ . New firms choose at random from the prevailing distribution of wages (and hence the prevailing distribution of  $s_{it}^f$ ).

Free entry in vacancies equates the present return from posting a vacancy with its cost:

$$p_{t}^{v} = (1 - \rho)k_{t}^{f} \beta E_{t} \frac{\lambda_{t+1}}{\lambda_{t}} s_{t+1}^{f}, \qquad (9)$$

The probability of an unemployed worker actually finding a match equals  $k_t^w$ . After some algebra, the continuation value of the surplus is given by:

$$q_{t} = (1 - \rho)\beta E_{t} \frac{\lambda_{t+1}}{\lambda_{t}} (1 - k_{t}^{w}) s_{t+1} + \frac{k_{t}^{w} p_{t}^{v}}{k_{t}^{f}}.$$

$$(10)$$

Firms have a simple vacancy production technology. They can either produce final goods or they can produce  $1/\gamma$  vacancies using their rented labor and capital. Households buy these vacancies competitively. In equilibrium the price of a vacancy, in output units, equals  $\gamma$  times the gross output of a match:

$$p_t^{\nu} = \frac{\gamma Y_t}{\varphi N_t} \,. \tag{11}$$

The average surplus (i.e. market value) earned by firms in the aggregate is given by the present value of profits, expressed as a difference equation:

$$s_{t}^{f} = (1 - \alpha)z_{t}^{1 - \alpha} (k_{it} / p_{t}^{k})^{\alpha} - W_{t} + (1 - \rho)\beta E_{t} \frac{\lambda_{t+1}}{\lambda_{t}} s_{t+1}^{f}.$$
(12)

Equation (12) determines the value of the average firm in present value terms.

#### 2.3 Wage formation and the effect of trend inflation

With probability 1 -  $\nu$ , wages in an existing match are bargained such that the worker receives a share  $\eta$  of the bilateral surplus, and the firm receives the remainder. Otherwise, the nominal wage does not change. Firms and workers separate only when it

is efficient to do so; otherwise they stay together. New firms choose a wage at random from the prevailing wage distribution, so when wages are "too high" relative to their flexible values, new hires will become less profitable, and hiring will fall.

The average nominal wage rate lags the rebargained wage rate. Those who do not rebargain index their wages according to the previous period's inflation with an intensity  $\chi$ . A value of  $\chi$  of one would indicate full indexation, and a value of zero would imply that no indexation occurs. Aggregate wages per worker are given by the law of motion:

$$P_{t}W_{t} = \nu P_{t-1}W_{t-1} \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\chi} + (1-\nu)P_{t}W_{t}^{*}.$$
(13)

To solve for the rebargained real wage, one could note that those firms which pay the rebargained wage  $W_t^*$  have an average surplus of  $(1-\eta)s_t$  from Nash bargaining. The only difference between firms is the wage that they pay. The surplus of a firm paying  $w_{it}$  conditional on the state of the economy  $x_t$  is given by the following expression:

$$s^{f}(w_{it}, x_{t}) = (1 - \alpha)y_{t} - w_{it}$$

$$+ \beta \varphi E_{t} \frac{\lambda_{t+1}}{\lambda_{t}} \left[ \upsilon s^{f} \left( w_{it} \left( \frac{P_{t}}{P_{t-1}} \right)^{\chi}, x_{t+1} \right) + (1 - \upsilon) s^{f} \left( W_{t+1}^{*}, x_{t+1} \right) \right]. \tag{14}$$

Subtracting the surplus of a given firm from the surplus of a rebargaining firm gives the following expression:

$$s^{f}(w_{it}, x_{t}) - s^{f}(W_{t}^{*}, x_{t}) = W_{t}^{*} - w_{it}$$

$$+ \beta \varphi v E_{t} \frac{\lambda_{t+1}}{\lambda_{t}} \left[ s^{f} \left( w_{it} \left( \frac{P_{t}}{P_{t-1}} \right)^{\chi}, x_{t+1} \right) - s^{f} \left( W_{t}^{*} \left( \frac{P_{t}}{P_{t-1}} \right)^{\chi}, x_{t+1} \right) \right]. \tag{15}$$

The difference between the average firms' surplus and the rebargained surplus is given by iterating (15) forward and averaging over all firms:

$$s_{t}^{f} - (1 - \eta)s_{t} = W_{t}^{*} - W_{t} + E_{t} \sum_{i=1}^{\infty} (\beta \varphi \upsilon)^{i} \frac{\lambda_{t+i}}{\lambda_{t}} \left( \frac{P_{t+i-1}}{P_{t+i-2}} \right)^{\chi} \frac{P_{t}}{P_{t+i}} \left( W_{t}^{*} - W_{t} \right)$$
(16)

When wages equal their rebargained wage, then the firms' portion of the surplus equals the rebargained portion of the surplus, and the standard Nash bargaining outcome occurs. Equation (16) implicitly pins down the rebargained wage when included in the system.

The interaction of equations (13) and (16) implies that an increase in trend inflation will have large effects on steady state vacancy creation and employment. The wage formation equation (13) implies that in steady state, the real wage relates to the rebargained wage in the following manner:

$$W = \frac{(1-\nu)}{1-\nu\Pi^{\chi-1}}W^*,\tag{17}$$

where  $\Pi$  gives the gross rate of trend inflation. The derivative of W with respect to  $\Pi$  is negative, so real wages lag rebargained wages in response to high inflation so long as wages are sticky ( $\upsilon > 0$ ) and there is not full indexation ( $\chi < 1$ ). Based on (16), firms capture more of the surplus than before. Hiring becomes more profitable, so vacancies and employment will rise. There should be a very strong positive relationship between employment and inflation in the long run, so long as wages are sticky and those sticky wages apply to new hires.

#### 2.4 Equilibrium and aggregation in the labor and product markets

The total number of unemployed in a period equals the starting stock of unemployed plus those who separate at the beginning of the period, with a constant labor force:

$$u_t \equiv U_t + \rho N_t = 1 - \varphi N_t. \tag{18}$$

The matching function follows the Cobb-Douglas form  $m(u_t, v_t) = \varsigma u_t^a v_t^{1-a}$ . The vacancy-filling rate equals:

$$k_t^f = \frac{m(u_t, v_t)}{v_t},\tag{19}$$

and the worker's job-finding rate is given by:

$$k_t^w = \frac{m(u_t, v_t)}{u_t}. (20)$$

The number of matches evolves according to the accounting identity:

$$N_{t+1} = (1 - \rho)N_t + m(u_t, v_t), \tag{21}$$

and the gross output of the matched firms and workers is given by:

$$Y_{t} = (1 - \rho) N_{t} z_{t} \left[ \frac{\alpha}{\rho_{t}^{K} p_{t}^{K}} \right]^{\frac{\alpha}{1 - \alpha}}, \tag{22}$$

based on the demand for capital given by (6).

Capital depreciates at a constant rate  $\delta$ . Capital pricing comes from the household's optimal choice of investment:

$$1 = \beta E_{t} \frac{p_{t+1}^{k} \lambda_{t+1}}{p_{t}^{k} \lambda_{t}} \left[ 1 + \rho_{t+1}^{K} - \delta \right]. \tag{23}$$

and the capital accumulation equation in capital units is given by:

$$\frac{K_{t}}{p_{t}^{k}} = (1 - \delta) \frac{K_{t-1}}{p_{t-1}^{k}} + \frac{I_{t-1}}{p_{t-1}^{k}}.$$
(24)

In the background, one could think of  $p_t^k$  as determined in a three-sector model where one sector produces capital goods; one sector produces vacancies; and one sector produces consumption goods, all under constant returns to scale. In such a situation,  $p_t^k$  equals the inverse of investment-specific productivity.

Because of market clearing, output equals total absorption:

$$Y_{t} = C_{t} + I_{t} + G_{t} + \gamma v_{t} \left( \frac{Y_{t}}{\varphi N_{t}} \right). \tag{25}$$

The discussion which follows uses the national accounts definition of GDP, which in this case equals everything on the right hand side of (25) except for vacancy costs.

#### 2.5 Driving processes and equilibrium

Labor-specific productivity and real capital prices follow loglinear random walks:

$$\ln(z_t) = \ln(z_{t-1}) + \varepsilon_t^z, \tag{26}$$

and

$$\ln(p_t^k) = \ln(p_{t-1}^k) + \varepsilon_t^{pk}. \tag{27}$$

Government spending follows an AR(1) process and error-corrects toward the prevailing level of output, thus ensuring balanced growth:

$$\ln(G_t) = \rho^G \ln(G_{t-1}) + (1 - \rho^G) \ln(Y_t) + \varepsilon_t^G.$$
(28)

Trend inflation is exogenous to the system and also follows a loglinear random walk:

$$\pi_t^* = \pi_{t-1}^* + \varepsilon_t^{\pi^*}. \tag{29}$$

Deviations of inflation and interest rates from their trend are governed by a Taylor rule with persistence:

$$r_{t} - \pi_{t}^{*} = (1 - \rho_{r})\phi_{\pi}(\pi_{t} - \pi_{t}^{*}) + (1 - \rho_{r})\phi_{y}(\ln(Y_{t}) - \ln(Y_{t-1}))$$

$$+ \rho_{r}(r_{t-1} - \pi_{t-1}^{*}) + \varepsilon_{t}^{r}.$$
(30)

Here I follow Bordo, Erceg, Levin, and Michaels (2007) in taking output in growth rates instead of levels, since output is nonstationary and trend inflation may affect output. All nominal variables in (30) are detrended relative to trend inflation.

The household conditions (2) through (4), the labor market conditions (5) through (17), the aggregation conditions (18) through (25), the shock processes (26) through (30), and the appropriate transversality conditions constitute a rational expectations equilibrium for this economy. Based on a linearized version of this system, is possible to obtain feedback coefficients using the gensys.m program of Sims (2002). In this particular situation, the equilibrium exists and is unique in the neighborhood around the steady state, for the parameter values chosen.

#### 3. Calibration and data

#### 3.1 Calibration and estimation strategy

Most of the parameter values follow the calibrated or estimated values used by Hagedorn and Manovskii (2008), Walsh (2002, 2005), Gertler, Sala, and Trigari (2008), or Gertler

and Trigari (2009). Inflation and growth are zero in steady state—this is not literally true but most models are linearized around such a steady state, and I wish to facilitate comparison with those models. The real interest rate *R* equals 4.72 percent per year based on the average real interest rate in the data. Investment (including residential structures but excluding consumer durables) is 16.0% of GDP based on NIPA data, and depreciation is 2.0% per quarter. Government spending and net exports are 19.2% of GDP.

In the sticky version of the model, wages have an average duration of one year for a value of  $\upsilon$  of 0.75. Gertler, Sala, and Trigari (2008) estimate a value slightly larger than this. From the micro data, Barratieri, Basu, and Gottschalk (2010) find that the most common duration of a nominal wage is one year, with an average quarterly hazard rate of adjustment somewhat below 0.2. A value of  $\upsilon$  of 0.75 therefore seems somewhat conservative. I vary the indexation parameter  $\chi$ ; in the baseline sticky wage model it equals zero. In the baseline model with indexation (the "light indexation") model, the indexation parameter equals 0.2. Sheifer (1979) reports that about than ten percent of workers in the United States during the high-inflation 1970s obtained explicit cost of living adjustments, and those workers did not typically receive a full allowance for inflation. To allow for implicit indexation in the nonunionized labor force and to err on the side of conservatism, I double Sheifer's number to 0.2, which should be regarded as an upper bound on the degree of quarterly wage indexation actually present in the economy.

Following Walsh and others, the total job separation rate  $\rho$  equals 0.10 per quarter, which is near the separation rate found in the JOLTS data. Vacancy posting costs altogether equal 0.20 percent of output, which gives a value of the outside option  $A/\lambda$  equal to 95% of the prevailing wage W. An outside option of 95% of the going wage ends up allowing

<sup>&</sup>lt;sup>1</sup> Holland (1995) and Ragan and Bratsberg (2000) discusses the evolution of indexation over time using data on collective bargaining agreements; about 60% of unionized workers were covered by cost of living adjustments during the late 1970s. Sheifer (1979) reports that unionized workers comprised the vast majority of workers covered by cost of living adjustments and that about ten percent of workers were covered economywide.

the benchmark sticky wage model to match the volatility of labor input extremely well. Hairault (2002) and Andolfatto (1996) use a value of 1.0 percent, value while Hagedorn and Manovskii (2008) use a very small share for vacancy posting costs. A lower outside option has the well-known effect of dampening the fluctuations produced by the model at all frequencies; a low outside option would naturally help the model to match the relative lack of a long run trend in unemployment, but it would also prevent the model from generating a plausible amount of business cycle volatility with or without sticky wages.

The unemployment share a of the matching function equals 0.4. Walsh cites Blanchard and Diamond (1989, 1991) who use postwar CPS data to derive an estimate of 0.4. The steady-state unemployment rate u (after separations) equals 0.06 which is just above the average postwar CPS unemployment rate; this value allows for some slight underreporting of unemployment on average. The worker-finding rate  $k^f$  equals 0.7 and the job-finding rate  $k^w$  equals 0.6, both from Walsh's calibration. These imply that there are 0.051 vacancies v in the steady state, which is a bit more than in the data. Vacancies here are considered a flow while they are considered a stock in the data; this higher value includes all of the vacancies which appear and disappear between JOLTS surveys. Workers have a bargaining power  $\eta$  of 0.5. The Taylor rule coefficients on inflation, output, and past interest rate deviations equal 1.5, 0.5, and 0.85. Government spending and net exports have a persistence of 0.99.

To match the model and the data, I assume that labor input and the vacancy-employment ratio are measured with error relative to the model, bringing the number of shocks in the augmented model to seven. The shocks are all mutually independent and iid across time; their variances are estimated by taking advantage of the fact that the errors in the model and data are of the same rank.<sup>2</sup> Estimates of the model-consistent data are formed from the observed data, and then a series of forecast errors from the second quarter onward is

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<sup>&</sup>lt;sup>2</sup> The transition matrix of the model has a rank of 9 versus the dataset, which has a rank of 7. The extra two eigenvalues reflect the fact that capital and employment are state variables which have initial conditions associated with them. In practice, the effect of the initial conditions vanishes after the first two quarters, and using the Kalman filter to extract smoothed shocks becomes highly problematic. The parameter estimates obtained using this simple method of moments strategy are extremely close to their maximum likelihood values, and they are identical for trend inflation.

constructed. The shock variances are given by the observed variance of the realized structural shocks, taken from the second quarter of 1948 onward.

#### 3.2 The data

The first and second series used in the estimation procedure are PCE inflation minus its long run trend (the 1-10 year forward expected inflation rate) and the trend itself. Reicher and Utlaut (2011) calculate a CPI trend inflation series from 1953 onward using a combination of survey data and data on long-term interest rates. To extend the series to 1948, I first calculate a projected long-term forward interest rate using the implied forward interest rate constructed from data on interest rates on 3-5 year and 20-year treasury securities, available from the NBER macrohistory database. Then I multiply that series by 0.688, based on the cointegrating relationship between CPI inflation and interest rates estimated by Reicher and Utlaut. I then difference-splice the implied series to the Reicher-Utlaut series. To obtain trend PCE inflation, I shift the entire composite series downward by 0.11% per quarter, to reflect that PCE inflation is on average 0.11% less than CPI inflation throughout the sample.

The top panel of Figure 1 shows the composite PCE trend inflation series along with actual PCE inflation. These series track each other extremely well over the medium to long run. Trend inflation has ranged from under 2% per year at the beginning and end of the sample to over 6% at the end of 1979. Trend inflation and actual inflation move together very closely, and Reicher and Utlaut (2011) show that it is impossible to reject the hypothesis that trend inflation follows a random walk, given information on past inflation, past trend inflation, commodity prices, output, and interest rates.

The third series is log productivity, which is calculated using real GDP in the numerator and total hours worked in the denominator. Total economywide hours worked come from the updated data of Francis and Ramey (2009), which are based on BLS data. The fourth series is the log price of gross domestic investment relative to personal consumption expenditures, from the NIPA. Cummins and Violante (2002) point out that

a proper measure of quality-adjusted investment goods prices would show a faster downward trend during the postwar period than the NIPA data.<sup>3</sup> However, their quality-adjusted series has the same cyclical behavior as the NIPA series and is not available for the entire postwar period, so I use a detrended quarterly NIPA series instead. The fifth series is the share of government spending plus net exports in output.

The sixth series is detrended labor input. I calculate detrended labor input by taking the log of hours per person aged 16 and over from the Francis and Ramey (2009) data. I project changes in log hours per person using changes in the unemployment rate and take the resulting projection (remeaned to zero) as my labor input deviation series. Detrended labor input is basically an amplified unemployment gap, so statements about "labor input" in this paper are actually statements about the behavior of the (un)employment rate.

The seventh series is vacancies divided by CES employment. I take quarterly vacancy data from the JOLTS and splice it to the adjusted help wanted index of Barnichon (2010) and the Conference Board, using the average log difference between the two series during the overlap period. Then I take the Metropolitan Life Help Wanted Index from the NBER.<sup>4</sup> First I adjust the series by regressing monthly changes in log help wanted on the number of Sundays in the month. Then I seasonally adjust the series. Then I adjust the trend of the series since that series shows a downward trend over time due to the way it was constructed. In particular, I assume that the difference between the Conference Board HWI and the Met Life HWI follows a loglinear trend based on their divergence from January 1951 through August 1960. I extrapolate that trend from 1951 to 1948 and then splice all three series together to form a composite vacancy series.

The two productivity series are detrended linearly, by assuming that they were at trend in the first quarter of 1948 and the final quarter of 2011. Trend inflation, which is a driftless

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<sup>&</sup>lt;sup>3</sup> I wish to thank Gianluca Violante for providing the Cummins and Violante (2002) data.

<sup>&</sup>lt;sup>4</sup> Terleckyi (1961) describes this series in detail. The Met Life HWI is the precursor to the Conference Board's HWI but there are differences in the construction of both series; in particular the Met Life HWI shows a downward trend over time relative to the Conference Board series.

random walk, is assumed to have an initial deviation in 1948 of zero. Log cyclical labor input, log vacancies per worker, the short run inflation rate, and the share of government and net exports in GDP are demeaned. Log cyclical output is taken as log productivity plus log cyclical labor input. Figure 1 shows all of these series together. These series follow a familiar pattern. Productivity grew more quickly before the 1970s than after, while investment prices fell quickly beginning in the 1980s. The share of government and net exports is countercyclical, since the levels of government spending and net exports move slowly relative to private consumption and investment. Labor input and vacancies per employee are highly correlated, with labor input showing particular weakness during the 1970s and 1980s relative to other decades.

#### 4. Results

#### 4.1 Sticky wages and volatility

Table 1 shows the volatility of the observed data along with the simulated data. The leftmost column of numbers shows the volatility of the data. All numbers are HP filtered with a smoothing parameter of 100,000. Cyclical labor input has a volatility of 1.85% in the data, and the vacancy-employment ratio is just under ten times as volatile. Cyclical output has a volatility of 2.47%. The second column shows the simulated moments under flexible wages. Under flexible wages, shocks to trend inflation and to the Taylor rule do not contribute to real fluctuations, and the effects of the other shocks are not large. As one might expect, the flexible wage model does not display much volatility. Labor input has a volatility of 0.64%, and the vacancy ratio has a volatility of 12.6%. Output does somewhat better; its volatility in the flexible wage model is 1.87%. This is not surprising, since a productivity-driven model will display large output fluctuations even when labor input does not fluctuate by much.

Each column in Table 1 shows what happens after increasing the stickiness of wages relative to the previous column. The rightmost column of Table 1 shows what happens under the baseline sticky wage model without indexation. In the baseline sticky wage model, labor input has a volatility of 1.82% and vacancies have a volatility of 40.4%;

output also becomes more volatile. The sticky wage model almost perfectly captures the volatility of employment, though it slightly overstates the volatility of vacancies and output. The models in between (with varying degrees of indexation) perform in between the two extremes with respect to volatility, though they perform more like the baseline sticky wage model than like the flexible wage model. The baseline sticky wage model generates a macroeconomically realistic degree of volatility in response to shocks; this fact explains the increasing popularity of including sticky wages in search and matching models of the business cycle.

#### 4.2 Sticky wages and the effects of inflation in the long run

The top two panels of Figure 2 show what happens after feeding the estimated shocks to trend inflation into the model, beginning in the second quarter of 1948. Labor input comoves strongly at low frequencies with inflation, since high inflation reduces the real wage and increases the profitability of new hires. The model fares particularly poorly in the 1970s. Since the 1970s saw high trend inflation, trend labor input should have risen to more than six percent above its 1948 trend. In fact, labor input appears to have been particularly weak during the 1970s; the unemployment rate was stubbornly high during that decade. If one were to take the sticky wage model literally, the 1970s should have seen the tightest labor markets in the postwar era, since inflation was so high. The 1950s, 1990s, and 2000s should have seen relatively loose labor markets since trend inflation was relatively low; hiring during these low inflation periods should have been especially unprofitable.

Table 2 quantifies this relationship a bit more. Table 2 presents the results of regressing 10-year changes in the Hodrick-Prescott trend for the endogenous variables on 10-year changes in the Hodrick-Prescott trend for trend inflation. Since the data are not assumed to be stationary, it is necessary to come up with a measure of comovement in trends, and looking at decade-over-decade changes in measured trends seems reasonable. In the data (first column), there is a slight negative relationship between labor input and inflation in the long run; for every increase in quarterly trend inflation, labor input falls by about 3.8

percent. Interestingly, the vacancy ratio appears to have been fairly high during the 1970s, and the vacancy ratio might comove positively with inflation to a slight degree.

The other columns of Table 2 show the regression coefficients from the simulated data, along with standard errors. The flexible wage model predicts no relationship between trend inflation and real aggregates; any nonzero estimates are due to simulation noise. Based on the standard errors predicted by the model, it is impossible to reject the hypothesis that the flexible wage model generates the observed behavior in vacancies, and the flexible wage model is not incompatible with the observed long run behavior of labor input either. The data do not clearly indicate that trend inflation stimulates labor demand over the long run.

The baseline sticky wage model (rightmost column) predicts that trend employment should rise by 5.1%, with a standard error of 4.7%, for every one point rise in quarterly trend inflation. This is at odds with the behavior of labor input over the long run. The sticky wage model overshoots the changes in the vacancy ratio, but it is not possible to reject that the trends in the vacancy ratio are compatible with any of the four models. Based on the results from Figure 2 and Table 2, it appears that the sticky wage model generates a long run comovement between employment and inflation which is dramatically at odds with that seen in the data. The sticky wage model predicts that trend inflation should be highly stimulative, but such a stimulative effect of trend inflation does not clearly show up in the data.

There is a strong tradeoff between generating volatility in a search and matching model with sticky wages and matching the lack of a long run positive relationship between labor input and inflation. This is equivalent to saying that trend inflation appears to be positively correlated with unemployment in the data but should be negatively correlated with unemployment in the sticky wage model. While the baseline sticky wage model predicts that inflation is highly stimulative over the long run, it does not appear to be highly stimulative in the data. A flexible wage model, by contrast, matches the long run behavior of the data, but it has trouble generating a realistic amount of cyclical volatility.

#### 4.3 Indexation, volatility, and the effects of inflation

The fourth column of Table 1 shows what happens under light indexation, with the indexation parameter  $\chi$  set to 0.2. The model generates a similar level of volatility as the sticky wage model. A look at the bottom two panels of Figure 2 shows that the model still predicts, counterfactually, that labor input and trend inflation comove positively at low frequencies. A look at Table 2 confirms this. The model with light indexation comes close to mimicking the sticky wage model in its long run behavior; in fact, in most respects it acts like a slightly dampened version of the baseline sticky wage model. Allowing for a realistic degree of indexation does not resolve the tradeoff between generating volatility and matching the long run behavior of the data.

The third column of Table 1 shows what happens under heavy indexation, with the indexation parameter  $\chi$  set to 0.8, which is a much greater degree of indexation than that reported by Sheifer (1979). Gertler, Sala, and Trigari (2008) estimate a very high value of  $\chi$  above 0.8 from the macro data. With such a high indexation parameter, labor input has a simulated volatility of 1.60%, compared with 1.82% for the baseline sticky wage model. The model with heavy indexation actually does reasonably well with respect to volatility. It also does somewhat better at matching trend employment and vacancies. Figure 3 and Table 2 compare the long run properties of the model in response to trend inflation under the two indexation regimes. The model with heavy indexation predicts very little upward trend in labor input during the 1970s compared with the model with light indexation. If one were to estimate the degree of wage indexation from the macro data, the macro data would indicate a high degree of indexation based on the behavior of trend unemployment. In this case, the macro and micro data would say different things.

In short, allowing for indexation does not resolve the tradeoff if one wishes for the model to match the micro facts. Allowing for a microeconomically unrealistic high degree of indexation such as 0.8 does make it possible for the model to generate volatility without generating a strong counterfactual relationship between inflation and (un)employment in the long run. However, a microeconomically realistic degree of indexation, such as a

value of  $\chi$  of 0.2, does not resolve the tradeoff between matching the volatility of employment and matching its long run behavior. If wage indexation were far more prevalent than it actually is at the micro level, then indexation could provide one resolution to the tradeoff.

#### 5. Conclusion

When augmented with sticky wages which affect new hires, the search and matching model can either generate a reasonable degree of volatility in employment, or else it can behave reasonably in the long run in response to changes in trend inflation. It cannot do both. When calibrated with microeconomically realistic degrees of wage stickiness and indexation of wages to past inflation, the model predicts that a rise in trend inflation should be highly stimulative in the long run. The lack of a positive long run relationship between inflation and labor input (or the lack of a negative long run relationship between inflation and unemployment) provides evidence against the mechanism by which sticky wages should stimulate hiring in response to a rise in trend inflation. Sticky wage search and matching models can generate realistic volatility or they can generate realistic long run behavior; they cannot do both.

One could view this paper as providing indirect evidence that the wages of new hires are not allocational. If wages are sticky but not allocational for new hires, then including sticky wages into a search and matching model may help the model to match the behavior of wages, but doing so does not help the model to generate realistic business cycle volatility. In that case, business cycle models which feature a simple search and matching mechanism may still suffer from a systematic lack of volatility which comes from either a lack of interesting shocks or the lack of a substantial propagation mechanism.

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Table 1: Volatility of detrended macroeconomic variables

			Heavy	Light	
	Data	Flexible	indexation	indexation	Sticky
Wage stickiness $v$	-	0	0.75	0.75	0.75
Wage indexation $\chi$	-	0	0.8	0.2	0
Short run inflation	0.45%	0.54%	0.56%	0.56%	0.56%
Trend inflation	0.11%	0.11%	0.11%	0.11%	0.11%
Productivity	1.44%	1.78%	1.89%	1.87%	1.86%
Real investment price	2.15%	1.38%	1.38%	1.36%	1.36%
Govt + NX Share	1.27%	1.11%	1.17%	1.21%	1.20%
Labor input	1.85%	0.64%	1.60%	1.74%	1.82%
Vacancies per employee	17.91%	12.59%	37.85%	38.82%	40.34%
Output	2.47%	1.87%	2.83%	2.83%	2.85%

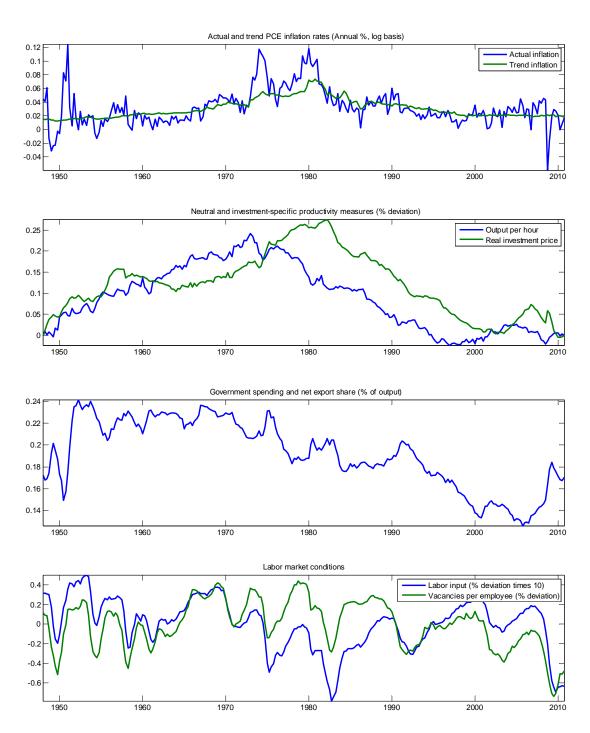
All data are HP filtered with a smoothing parameter of 100,000.

Table 2: Regressions of 10-year changes in HP trends on the 10-year change in the HP trend of trend inflation

			Heavy	Light	
	Data	Flexible	indexation	indexation	Sticky
Wage stickiness $\upsilon$		0	0.75	0.75	0.75
Wage indexation $\chi$		0	0.8	0.2	0
Short run inflation	0.4	0.0	0.0	0.0	0.0
(Standard error)		0.2	0.2	0.2	0.2
Trend inflation	1.0	1.0	1.0	1.0	1.0
(Standard error)		0.0	0.0	0.0	0.0
Productivity	14.4	-0.3	-0.2	-0.4	-0.5
(Standard error)		11.8	13.5	13.5	14.4
Real investment price	19.5	0.2	-0.3	-0.2	-0.1
(Standard error)		7.9	8.1	7.6	7.9
Govt + NX Share	1.8	0.0	0.0	-0.3	-0.8
(Standard error)		5.1	5.2	5.0	5.5
Labor input	-3.8	0.0	1.0	3.9	5.1
(Standard error)		2.8	3.6	4.0	4.7
Vacancies per employee	35.5	0.0	13.9	56.6	72.2
(Standard error)		39.9	52.7	58.0	66.5
Output	10.6	-0.3	0.8	3.5	4.6
(Standard error)		11.3	13.2	13.4	14.3

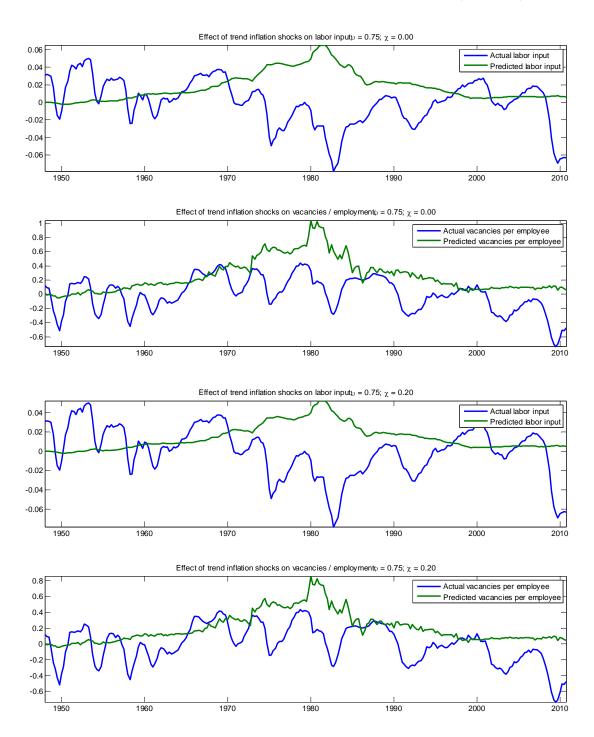
All data are HP filtered with a smoothing parameter of 100,000. Mean regression coefficients and standard errors are calculated using 100,000 draws from the simulated data. Any apparent effect of trend inflation on real aggregates in the flexible model reflects the fact that the estimates are the outcome of a simulation.

Figure 1: Detrended data used in the analysis



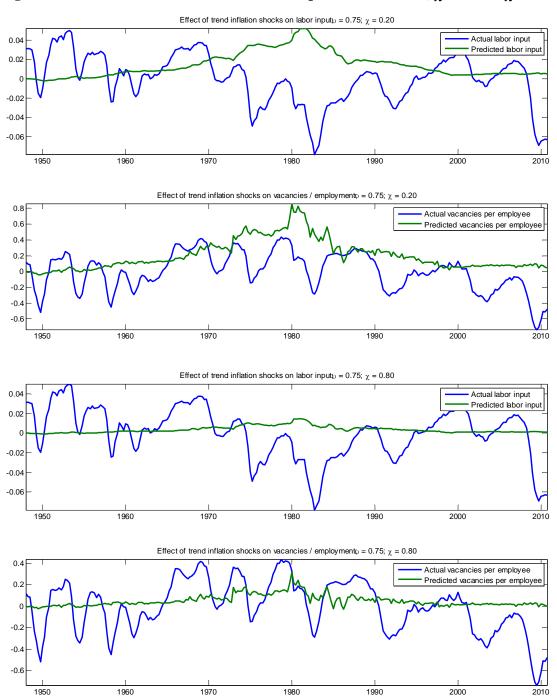
Source for trend inflation: Reicher and Utlaut (2011), extended using interest rate data as described in text. Source for labor input: Francis and Ramey (2009). Source for vacancies: JOLTS and help wanted indices as described in text. Other data come from the NIPA and BLS. All measures are expressed in geometric annual rates.

Figure 2: Effect of trend inflation on labor input and vacancies,  $\chi = 0$  vs.  $\chi = 0.2$ 



This figure shows the effect of shocks to trend inflation on labor input and vacancies per worker under sticky wages. The top two panels show the effect without indexation ( $\chi = 0$ ), and the bottom two panels show the effect with light indexation ( $\chi = 0.2$ ).

Figure 3: Effect of nominal shocks on labor input and vacancies,  $\chi = 0.2$  vs.  $\chi = 0.8$ 



This figure shows the effect of shocks to trend inflation on labor input and vacancies per worker under sticky wages. The top two panels show the effect with light indexation ( $\chi = 0.2$ ), and the bottom two panels show the effect with heavy indexation ( $\chi = 0.8$ ).