Ramsey Monetary Policy and International Relative Prices*

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Abstract

We analyze welfare maximizing monetary policy in a dynamic two-country model with price stickiness and imperfect competition. In this context, a typical terms of trade externality affects policy competition between independent monetary authorities. Unlike the existing literature, we remain consistent to a public finance approach by an explicit consideration of all the distortions that are relevant to the Ramsey planner. This strategy entails two main advantages. First, it allows an exact characterization of optimal policy in an economy that evolves around a steady-state which is not necessarily efficient. Second, it allows to describe a full range of alternative dynamic equilibria when price setters in both countries are forward-looking and households' preferences are not restricted. In this context, and in response to productivity shocks, deviations from price stability generally occur under Nash-competition. The size of such inefficiency is directlly related to parameters indexing openness, such as the elasticity of substitution between domestic and foreign goods and to the degree of home bias in consumption.

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1 Introduction

In the classic approach to the study of optimal policy in dynamic economies (Ramsey (1927), Atkinson and Stiglitz (1980), Lucas and Stokey (1983), Chari, Christiano and Kehoe (1991)), and in a typical public finance spirit, a Ramsey planner maximizes household's welfare subject to a resource constraint, to the constraints describing the equilibrium in the private sector economy, and via an explicit consideration of all the distortions that characterize both the long-run and the cyclical behavior of the economy.

Recently there has been a resurgence of interest for a Ramsey-type approach in dynamic general equilibrium models with nominal rigidities. Khan, King and Wolman (2003) analyze optimal monetary policy in a closed economy where the relevant distortions are imperfect competition, staggered price setting and monetary transaction frictions. Schmitt-Grohe and Uribe (2003), and Siu (2003) focus on the joint optimal determination of monetary and fiscal policy. The robust conclusion of these studies - that optimal policy is associated to the prescription of stable prices - is indeed rooted in the principle that the planner tries to eliminate the distortions induced by fluctuations in the aggregate price level, whether stemming from relative price misalignments or from resource costs of resetting prices.

In this paper we aim at taking this approach to the analysis of policy interdependence in open economies. We characterize welfare maximizing monetary policy in a two-country world where financial markets are complete, policymakers act under commitment and compete in a Nash equilibrium. Both economies are characterized by two main distortions: output is inefficiently low (due to the presence of monopolistic competitive goods markets) and firms face quadratic costs of adjusting prices. However, and relative to a cooperative setting enforced by a world Ramsey planner, openness per se adds a further inefficiency typical of the outcome under a Nash equilibrium. This inefficiency stems from the monopoly power that each country can exert on its own terms of trade, and therefore from an externality that the policy competition motive necessarily entails.[1]

Relative to the corresponding closed economy literature, a Ramsey-type approach has received much less attention in the study of optimal monetary and exchange rate arrangements for open economies. Cooley and Quadrini (2003) analyze monetary policy interaction in a two-country model with perfectly competitive goods markets, flexible prices and limited financial markets participation. Their model is essentially static in nature and highlights the presence of a systematic inflation bias induced by international policy competition. Our framework differs from theirs in the fact that prices are sticky (so that *nominal* exchange rate movements exert an effect on international

[1] The idea that terms of trade spillovers generate an externality and therefore room for international (monetary and/or fiscal) policy coordination is already discussed (although within ad-hoc models) in Canzoneri and Henderson (1991), Persson and Tabellini (1995) and dates back in the trade literature at least to Johnson (1954). Chari and Kehoe (1990) discuss the specific role of terms of trade distortions for optimal fiscal policy in a two-country general equilibrium model. More recently, see Corsetti and Pesenti (2001), Benigno and Benigno (2003b), Sutherland (2002).

relative prices), goods markets are imperfectly competitive and agents operate in a fully dynamic environment.

A Ramsey-type approach has also been employed in a certain stream of the so-called New Open Economy Macroeconomics literature (which instead typically features nominal rigidities and imperfect competition). This is the case - for instance - in the work of Benigno and Benigno (2003), Corsetti and Pesenti (2002), and Devereux and Engel (2003). In these papers the analysis of optimal policy is simplified by the assumption that prices (or wages) are *predetermined one-period*. Such an assumption is restrictive, for it typically generates a Lucas-type aggregate supply curve in which the forward-looking nature of inflation is neglected, and along with it the channel through which the anticipation of future policy conduct comes to play a role.[2]

The present paper differs from the aforementioned contributions in that it employs optimizing producers' price setting decisions that are *forward-looking*, thereby rendering the corresponding optimal policy problem inherently *dynamic*. Modelling forward-looking price setting decisions affects the nature of the optimal policy problem in a fundamental way. In practice it entails that a Ramsey planner faces a set of constraints exhibiting future expectations of control variables. The lack of recursivity which occurs under those circumstances affects the nature of the policy program since future actions "limit" the set of feasible actions which are currently available to the planner.[3]

Our analysis can be summarized in terms of two main contributions. First, we show that policy competition in an international setting leads welfare maximizing but independent policymakers to generally deviate from the prescription of price stability. Intuitively, in an open economy, the wedge between the marginal rate of substitution (between consumption and leisure) and the marginal rate of transformation depends not only on the fact that markups are time-varying (due to monopolistic competition coupled with sticky prices), but also on the dynamic behavior of the terms of trade. Hence either country tries to engineer price level movements to try to tilt relative prices in its own favour, and increase real net income (and consumption) for any given level of disutility of labor. On the other hand, when policy is set in a centralized fashion by a world Ramsey planner, the two countries manage to coordinate their actions in such a way to replicate closely the equilibrium dynamics that would prevail under purely flexible prices (thereby mimicking the outcome of a corresponding world closed economy).

Second, and more generally on a methodological ground, our approach allows to study optimal policy in dynamic open economies that evolve around a steady-state which is not necessarily efficient. In that, it differs crucially from a recurrent approach in the recent New-Keynesian litera-

[2] It is by now well understood that this entails a major consequence in that it neglects the sense in which (time consistent) discretionary policies are suboptimal in dynamic environments with forward-looking price (and/or wage) decisions (Woodford, 2003).

[3] See Marcet and Marimon (1999). The main consequence of this is that the set of equilibria that obtains in the case of an optimal policy constrained by predetermined price setting decisions differs from the one obtained in the presence of forward-looking price setting.

ture that forces another (complementary) policy instrument (e.g., fiscal subsidies) to offset second order effects of stochastic uncertainty on variables' mean levels.[4] The same approach resorts to a two-step strategy that involves, at first, taking a log-linear approximation of the competitive equilibrium conditions, and then a quadratic approximation of the correct households' utility function. In particular, resorting to such an approximation method in an open economy requires specific assumptions on preferences, such as log-utility and unitary elasticity of substitution between goods produced in different countries. Yet precisely these assumptions already constrain the form of the optimal policy to coincide, somewhat artificially, with the one that implements the flexible price allocation. Furthermore, if not satisfied, the same conditions do not allow to study each country's policymaker's problem independently, forcing to ignore those equilibria that emerge under policy competition and to restrict the analysis only to the world planner's policy design problem. In fact, relative to the existing literature (e.g., Benigno and Benigno (2003)), a contribution of this paper is precisely in the general characterization of the optimal stabilization policy under Nash competition within a fully dynamic setting and in the presence of uncorrected steady-state distortions.[5]

The remainder of the paper is organized as follows. Section 2 and 3 describe respectively the economic environment and the features of the equilibrium. Section 4 derives the form of the constraints that are relevant to the planner's policy problem. Section 5 analyzes optimal policy under commitment. Section 6 explores the welfare gains from cooperation. Section 7 concludes.

2 The Model

The world economy consists of two countries, that we label Home and Foreign. Each economy is populated by infinitely-lived agents, whose total measure is normalized to unity.

2.1 Domestic Households

Let's denote by $C_t \equiv [(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}}$ a composite consumption index of domestic and imported bundles of goods, where α is the share of imported goods (i.e., an inverse measure of *home bias* in consumption preferences), and $\eta > 0$ is the elasticity of substitution between domestic and foreign goods. Each bundle is composed of imperfectly substitutable varieties (with elasticity

[4] See, for instance, Rotemberg and Woodford (1997), Woodford (2003), Benigno and Benigno (2003), Clarida, Gali and Gertler (2002).

[5] More recently, Benigno and Woodford (2004) show (within a closed economy model) how to preserve a quadratic approximation of the household's welfare objective in the case in which the economy fluctuates around a non-efficient steady-state. This per se requires taking a second order approximation also of (some of) the underlying equilibrium conditions. Benigno and Benigno (2004) and Pappa (2004) apply this approximation method to a two-country optimal policy dynamic model. The key difference of the approach employed in this paper is that, while maintaining general features such uncorrected steady-state distortions and forward-looking price setting decisions, we are able to describe the optimal policy problem in its *exact* form, without resorting to approximation methods.

of substitution $\varepsilon > 1$). Optimal allocation of expenditure within each variety of goods yields:

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \ ; C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t} \tag{1}$$

where $C_{H,t} \equiv \int_0^1 [C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di]^{\frac{\epsilon}{\epsilon-1}}$ and $C_{F,t} \equiv \int_0^1 [C_{F,t}(i)^{\frac{\epsilon-1}{\epsilon}} di]^{\frac{\epsilon}{\epsilon-1}}$.

Optimal allocation of expenditure between domestic and foreign bundles yields:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t; C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{2}$$

where

$$P_t \equiv [(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}} \tag{3}$$

is the CPI index.

We assume, both within and across country, the existence of complete markets for state-contingent claims expressed in domestic units of account.[6] Let $h^t = \{h_0, ...h_t\}$ denote the history of events up to date t, where h_t is the event realization at date t. The date 0 probability of observing history h^t is given by $\rho(h^t)$. The initial state h^0 is given so that $\rho(h^0) = 1$.

Agents maximize the following expected discounted sum of utilities over possible paths of consumption and labor:[7]

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right) \right\} \tag{4}$$

where $E_0 \{\}$ denotes the mathematical expectations operator conditional on h_0 and N_t is labor hours. [8] The function $U(\bullet)$ features regularity conditions and is assumed to be separable in its arguments. To insure their consumption pattern against random shocks at time t households spend

[6] Given that, in our setting, the law of one price holds continually, the unit of denomination of the payoffs of state-contingent assets is not strictly relevant. Alternatively, e.g., in the case in which deviations from the law of one price are due to consumer currency pricing, as in Devereux and Engel (2003), the distinction between nominal and real payoffs would be relevant for the specification of the equilibrium.

[7] Asset market completeness at the national level allows to express the household maximization program in terms of a *representative* consumer. The underlying idea is that the initial holdings of financial wealth adjust as a residual endogenous variable in such a way that, as of time zero, each national household faces, in the presence of aggregate shocks, the same present value budget constraint (and therefore chooses the same consumption stream). See Woodford (2003).

[8] Hence the expression for lifetime utility is equivalent to writing

$$\sum_{t=0}^{\infty} \sum_{h^t} \beta^t U\left(C(h^t), N(h^t)\right) \rho(h^t)$$

where $\rho(h^t) = \rho(h_t|h_0)$.

$\nu_{t+1,t}\ B_{t+1}$ in nominal state contingent securities where $\nu_{t,t+1} \equiv \nu(h^{t+1}|h^t)$ is the period-t price of a claim to one unit of currency in state h^{t+1} divided by the probability of occurrence of that state. Each asset in the portfolio B_{t+1} pays one unit of domestic currency at time $t+1$ and in state h^{t+1}.

By considering the optimal expenditure conditions (1) and (2), the sequence of budget constraints assumes the following form:

$$P_t C_t + \sum_{h^{t+1}} \nu_{t+1,t} B_{t+1} \leq W_t N_t + \tau_t + B_t + \int_0^1 \Gamma_t(i) \tag{5}$$

where τ_t are government net transfers of domestic currency and $\Gamma_t(i)$ are the profits of monopolistic firm i, whose shares are owned by the domestic residents.[9] Households choose processes $\{C_t,\, N_t\}_{t=0}^{\infty}$ and bonds $\{B_{t+1}\}_{t=0}^{\infty}$ taking as given the set of processes $\{P_t,\, W_t,\, v_{t,t+1}\}_{t=0}^{\infty}$ and the initial wealth B_0 so as to maximize (4) subject to (5).

For any given state of the world, the following set of efficiency conditions must hold:

$$U_{c,t} \frac{W_t}{P_t} = -U_{n,t} \tag{6}$$

$$\beta \frac{P_t}{P_{t+1}} \frac{U_{c,t+1}}{U_{c,t}} = \nu_{t,t+1} \tag{7}$$

$$\lim_{j \to \infty} \ E_t \left\{ \nu_{t,t+j} \ B_{t+j} \right\} = 0 \tag{8}$$

where $U_{j,t}$ defines the first order derivative of utility with respect to its argument $j = C, N$. Equation (6) equates the CPI-based real wage to the marginal rate of substitution between consumption and leisure. Equation (7) describe a set of optimality conditions for state contingent securities indexed by state h_{t+1}. Optimality requires that the first order conditions (6), (7) and the no-Ponzi game condition (8) are simultaneously satisfied.

Taking conditional expectations of equation (7) allows to define a gross nominal interest rate (or return on the corresponding riskless one-period bond) as

$$\begin{aligned} R_t &= E_t \left\{ \nu_{t+1,t} \right\}^{-1} \\ &= \left[\beta E_t \left\{ \frac{P_t}{P_{t+1}} \frac{U_{c,t+1}}{U_{c,t}} \right\} \right]^{-1} \end{aligned} \tag{9}$$

which is a familiar consumption Euler equation.

[9] Each domestic household owns an equal share of the domestic monopolistic firms. We abstract from trade in shares.

Notice that, following large part of the recent literature, we do not introduce money explicitly, but rather think of it as playing the role of nominal unit of account. This modelling choice has two major consequences. First, it allows us, for the sake of simplicity, to abstract from an additional distortion stemming from the presence of transactions frictions. In *Appendix A* we introduce a more general model in which real balances enter the utility function as a proxy of the transactions benefits of holding money. We show that the optimal policy problem nests our specification in the particular case in which the weight of real money balances in the utility is arbitrarily small. Second, it makes natural to think of nominal interest rates R_t and R_t^* as the underlying instruments of monetary policy.

2.2 Law of One Price and Foreign Demand

We assume throughout that the law of one price holds, implying that $P_{F,t}(i) = \mathcal{E}_t \; P^*_{F,t}(i)$ for all $i \in [0,1]$, where $\mathcal{E}_t$ is the *nominal exchange rate*, i.e., the price of foreign currency in terms of home currency, and $P^*_F(i)$ is the price of foreign good i denominated in foreign currency. It is important to notice that the holding of the law of one price does *not* necessarily imply that PPP holds, unless we make the further restrictive assumption of *absence of home bias,* which entails $\alpha = \alpha^* = \frac{1}{2}$.[10]

Let's denote by B^F foreign households' holdings of the state contingent bond denominated in domestic units of account. The budget constraint of the foreign representative household will read:

$$P_t^* C_t^* + \sum_{h^{t+1}} \nu_{t,t+1} \frac{B_{t+1}^F}{\mathcal{E}_t} \leq W_t^* N_t^* + \tau_t^* + \frac{B_t^F}{\mathcal{E}_t} + \int_0^1 \Gamma_t^*(i) \tag{10}$$

The efficiency condition for bonds' holdings is:

$$\beta \frac{P_t^* \mathcal{E}_t}{P_{t+1}^* \mathcal{E}_{t+1}} \frac{U_{c,t+1}^*}{U_{c,t}^*} = \nu_{t,t+1} \tag{11}$$

Foreign demand for domestic variety i must satisfy:

$$\begin{aligned} C_{H,t}^*(i) &= \left(\frac{P_{H,t}^*(i)}{P_{H,t}^*} \right)^{-\varepsilon} C_{H,t}^* \\ &= \left(\frac{P_{H,t}^*(i)}{P_{H,t}^*} \right)^{-\varepsilon} \alpha^* \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\eta} C_t^* \end{aligned} \tag{12}$$

[10] This differs, e.g., from Benigno and Benigno (2003) and Devereux and Engel (2003) for which PPP holds at all times. One can easily verify that PPP holds in our case only in the absence of home bias by manipulating the CPI expression and substituting conditions $P_{H,t} = \mathcal{E}_t P_{H,t}^*$ and $P_{F,t} = \mathcal{E}_t P_{F,t}^*$, which are implied by the fact that the law of one price holds.

Taking conditional expectations of (11) and defining $R_t^* = \left(E_t \left\{ \nu_{t,t+1} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\} \right)^{-1}$ one can write:

$$R_t^* = \left[\beta \ E_t \left\{ \frac{P_t^*}{P_{t+1}^*} \frac{U_{c,t+1}^*}{U_{c,t}^*} \right\} \right]^{-1} \tag{13}$$

The remaining efficiency conditions characterizing the foreign economy are then exactly symmetric to the ones of the domestic economy described above.

2.3 Real Exchange Rate and the Terms of Trade

The terms of trade is the relative price of imported goods:

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}} \tag{14}$$

while the real exchange rate is defined as $Q_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t}$. The terms of trade can be related to the CPI-PPI ratio as follows

$$\frac{P_t}{P_{H,t}} = [(1-\alpha) + \alpha {S_t}^{1-\eta}]^{\frac{1}{1-\eta}} \equiv g(S_t) \tag{15}$$

with $g^{'}(S_t) > 0$.

The terms of trade and the real exchange rate are linked through the following expression:

$$\begin{aligned} Q_t &= S_t \frac{P_t^*}{P_{F,t}^*} \left(\frac{P_t}{P_{H,t}} \right)^{-1} \\ &= S_t \ \frac{g^*(S_t)}{g(S_t)} \equiv q(S_t) \end{aligned} \tag{16}$$

where

$$\frac{P_t^*}{P_{F,t}^*} = [(1-\alpha^*) + \alpha^* {S_t}^{\eta-1}]^{\frac{1}{1-\eta}} \equiv g^*(S_t) \tag{17}$$

with $q^{'}(S_t) > 0$ and $g^{*'}(S_t) < 0$.

2.4 Budget Constraints and Risk Sharing

By iterating (5) forward and imposing (8) we can write the infinite sequence of period-by-period budget constraints as a single present value constraint (in domestic units of account) as follows:

$$B_0 + \sum_{t=0}^{\infty} \left(\sum_{h^t} \nu_{0,t} \right) [W_t N_t + \Gamma_t] = \sum_{t=0}^{\infty} \left(\sum_{h^t} \nu_{0,t} \right) P_t C_t \tag{18}$$

where the price system $\nu_{0,t}$ is obtained after iteration of equation (7) and can be expressed, for each possible state of the world, as

$$\nu_{0,t} = \beta^t \rho_t \frac{U_{c,t}}{P_t} \frac{P_0}{U_{c,0}} \tag{19}$$

Equation (19) states that the sum of initial financial wealth and expected present discounted net income must match the expected present discounted value of consumption.

We proceed in a similar fashion for the Foreign household. The price system $\nu_{0,t}^F$ (expressed in domestic units of account) obtained from the forward iteration of (11) can be written:

$$\upsilon_{0,t}^F = \left(\beta^t \rho_t \frac{U_{c^*,t}^*}{P_t^*} \frac{P_0^*}{U_{c^*,0}^*} \right) \frac{\mathcal{E}_0}{\mathcal{E}_t} \equiv \nu_{0,t}^* \frac{\mathcal{E}_0}{\mathcal{E}_t} = \upsilon_{0,t} \tag{20}$$

Iterating (10) and imposing a corresponding transversality condition we obtain (in domestic units of account)

$$B_0^F + \sum_{t=0}^{\infty} \left(\sum_{h^t} \upsilon_{0,t} \right) \ \mathcal{E}_t \ [W_t^* N_t^* + \Gamma_t^*] = \sum_{t=0}^{\infty} \left(\sum_{h^t} \nu_{0,t} \right) \ P_t^* C_t^* \mathcal{E}_t \tag{21}$$

Equating (19) with (20) yields the following condition linking the real exchange rate to the ratio of the marginal utility of consumption:

$$\kappa \frac{U_{c^*,t}^*}{U_{c,t}} = \frac{\mathcal{E}_t P_t^*}{P_t} \equiv Q_t \tag{22}$$

where $\kappa \equiv \frac{\mathcal{E}_0 \ P_0^* \ U_{c,0}}{P_0 U_{c^*,0}^*}$. This is a typical condition that emerges in the presence of international asset markets where households engage in risk-sharing via the trading of state contingent securities.[11]

An issue concerns the determination of the risk-sharing parameter κ. Below we show that the value of κ, and in turn the characteristics of the underlying risk-sharing arrangements, are tied to the initial distribution of wealth.

[11] As a consequence of complete markets whether the same trading is undertaken at time zero or sequentially is irrelevant for the specification of the equilibrium.

Consider representative Households in the two countries entering the initial period zero with predetermined (at *time -1*) wealth distribution given by a pair $\{B_0,\ B_0^F\}$. Below we clarify how the *time -1* risk sharing arrangement affects the initial distribution of wealth.

Definition 1. *International risk-sharing requires a time -1 trading of assets such that, as of time zero, both households face the same present value budget constraint.*

The above definition amounts to assuming that, for given equilibrium allocations defining the world competitive equilibrium, $\{B_0, B_0^F\}$ adjusts endogenously (and residually) to insure that such risk sharing arrangement is achieved.[12] Below we show that our assumption on risk-sharing allows to pin down the level of the constant κ in equation (22).

Lemma 1. *If the risk sharing arrangement is consistent with Definition 1, then* $\kappa = 1$.

Proof. Consider the domestic household maximizing (4) subject to (18). Efficiency requires:

$$\beta^t U_{c,t} = \Omega\ v_{0,t} P_t \tag{23}$$

where Ω is the Lagrange multiplier on constraint (18). Notice that this multiplier is *constant* across time and states. Symmetrically, for the Foreign household we have:

$$\beta^t U_{c,t}^* = \Omega^*\ v_{0,t}\ \varepsilon_t P_t^* \tag{24}$$

By combining (22), (23) and (24) one can write the risk-sharing parameter as:

$$\kappa = \frac{\Omega}{\Omega^*} \tag{25}$$

Thus the requirement that, as of time zero, intertemporal budget constraints be equalized across households in different countries implies that the shadow value of nominal wealth be also equalized, which in turn implies, given (25), that $\kappa = 1$.■[13]

2.5 Domestic Producers

Each monopolistic firm i produces a homogenous good according to:

$$Y_t(i) = A\ N_t(i) \tag{26}$$

[12] At this stage a clarification is in order. Although natural, one may consider the above definition of international risk-sharing as restrictive. An alternative specification would be to consider the case where the initial risk-sharing trading of assets is in a competitive fashion. We leave to a companion paper the analysis of optimal policy in which the initial distribution of wealth is determined endogenously.

[13] This is the normalization adopted in Chari et al. (2002). The difference here is that we show that κ must endogenously assume a unitary value under the risk-sharing arrangement described above.

The cost minimizing choice of labor input implies:

$$\frac{W_t}{P_{H,t}(i)} = \frac{MC_t}{P_{H,t}(i)}\ A_t \tag{27}$$

where MC denotes the nominal marginal cost. Notice that since households supply a homogenous type of labor the nominal wage and marginal cost are common across firms.

Changing output prices is subject to some costs. We follow Rotemberg (1982) and model the cost of adjusting prices for each firm i equal to:

$$\psi_t(i) \equiv \frac{\theta}{2}\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)} - 1\right)^2 \tag{28}$$

where the parameter θ measures the degree of price stickiness. The higher θ the more sluggish is the adjustment of nominal prices. If $\theta = 0$ prices are flexible.

The cost of price adjustment renders the domestic producer's pricing problem dynamic. Each producer chooses the price $P_{H,t}(i)$ of variety i to maximize its total market value:

$$E_t\left\{\sum_{t=0}^{\infty}\beta^t\lambda_t\ \frac{D_t(i)}{P_{H,t}}\right\} \tag{29}$$

subject to the constraint

$$Y_t(i) \leq \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon}\ (C_{H,t} + C^*_{H,t}) \tag{30}$$

where $\beta^t\lambda_t$ measures the marginal utility value to the representative producer of additional profits (expressed in domestic units of account) and where real profits in units of domestic goods are:

$$\frac{D_t(i)}{P_{H,t}} \equiv \frac{P_{H,t}(i)Y_t(i)}{P_{H,t}} - \frac{W_t}{P_{H,t}}N_t(i) - \frac{\theta}{2}\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)} - 1\right)^2$$

The first order condition of the above problem reads

$$\begin{aligned} 0 \ = \ & \lambda_t\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon}\frac{C_t^W}{P_{H,t}}\left((1-\varepsilon) + \varepsilon\frac{W_t}{A_tP_{H,t}}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-1}\right) \\ & -\lambda_t\theta\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)} - 1\right)\frac{1}{P_{H,t-1}(i)}\ +\ \beta\ E_t\left\{\lambda_{t+1}\theta\left(\frac{P_{H,t+1}(i)}{P_{H,t}(i)} - 1\right)\frac{P_{H,t+1}(i)}{P_{H,t}(i)^2}\right\} \end{aligned} \tag{31}$$

where $C_t^W \equiv C_{H,t} + C_{H,t}^*$.

Let's define $\widetilde{p}_{H,t} \equiv \frac{P_{H,t}(i)}{P_{H,t}}$ as the relative price of domestic variety i and $\pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$ as the gross domestic *producer inflation rate*. It is useful to see that the above condition can be rewritten as

$$\begin{aligned} 0 \;=\; & \lambda_t\, C_t^W\, \widetilde{p}_{H,t}{}^{-\varepsilon} \left((1-\varepsilon) + \varepsilon \frac{W_t}{A_t P_{H,t}} \right) - \\ & \lambda_t \theta \left(\pi_{H,t} \frac{\widetilde{p}_{H,t}}{\widetilde{p}_{H,t-1}} - 1 \right) \frac{\pi_{H,t}}{\widetilde{p}_{H,t-1}} \\ & + \beta\, E_t \left\{ \lambda_{t+1}\, \theta \left(\pi_{H,t+1} \frac{\widetilde{p}_{H,t+1}}{\widetilde{p}_{H.t}} - 1 \right) \pi_{H,t+1} \frac{\widetilde{p}_{H,t+1}}{\widetilde{p}_{H,t}{}^2} \right\} \end{aligned} \tag{32}$$

3 World Competitive Equilibrium in Primal Form

In this section we show how to derive a compact form of the world competitive equilibrium in terms of a minimal set of relations involving only real allocations in the spirit of the primal approach described in Lucas and Stokey (1983). Khan, King and Wolman (2003) adopt a similar structure to analyze optimal monetary policy in a closed economy with market power, price stickiness and monetary frictions, while Schmitt-Grohe and Uribe (2002) to analyze a problem of joint determination of optimal monetary and fiscal policy.

We focus our attention on a *symmetric* equilibrium where all domestic producers charge the same price, adopt the same technology and therefore choose the same demand for labor. This implies that:

$$\widetilde{p}_{H,t} = 1, \text{ for all t} \tag{33}$$

$$N_t(i) = N_t, \text{ for all } i,\ t \tag{34}$$

$$\Gamma_t(i) = \Gamma_t, \text{ for all } i,\ t \tag{35}$$

and corresponding conditions for $\widetilde{p}_{F,t}^*$, $N_t^*(i)$ and $\Gamma_t^*(i)$ in Foreign.

In equilibrium state contingent bonds are in zero net supply:

$$B_t + B_t^F = 0 \tag{36}$$

3.1 Pricing and Market Clearing Conditions

In such an equilibrium equation (32) will simplify to:

$$\lambda_t \pi_{H,t}(\pi_{H,t} - 1) = \beta E_t \{\lambda_{t+1}\pi_{H,t+1}(\pi_{H,t+1} - 1)\} + \frac{\lambda_t \varepsilon A_t N_t}{\theta}\left(mc_t - \frac{\varepsilon - 1}{\varepsilon}\right) \tag{37}$$

where $mc_t \equiv \frac{MC_t}{P_{H,t}}$ is the *real* marginal cost.

Substituting (6), (15) and (27) we can write:

$$\begin{aligned} U_{c,t}\ \pi_{H,t}\ (\pi_{H,t} - 1) \ &= \ \beta E_t \{U_{c,t+1}\pi_{H,t+1}(\pi_{H,t+1} - 1)\} + \\ &\quad \frac{U_{c,t}\varepsilon A_t N_t}{\theta}\left(-\frac{U_{n,t}}{U_{c,t}A_t} g(S_t) - \frac{\varepsilon - 1}{\varepsilon}\right) \end{aligned} \tag{38}$$

where in particular the expression for the real marginal cost is:

$$mc_t = -\frac{U_{n,t}}{U_{c,t}A_t} g(S_t) \tag{39}$$

An analogous condition will hold in Foreign:

$$\begin{aligned} U^*_{c,t}\ \pi^*_{F,t}(\pi^*_{F,t} - 1) \ &= \ \beta E_t \left\{U^*_{c,t+1}\ \pi^*_{F,t+1}(\pi^*_{F,t+1} - 1)\right\} + \\ &\quad \frac{U^*_{c,t}\ \varepsilon\ A^*_t N^*_t}{\theta}\left(-\frac{U^*_{n,t}}{U^*_{c,t}\ A^*_t} g^*(S_t) - \frac{\varepsilon - 1}{\varepsilon}\right) \end{aligned} \tag{40}$$

Market clearing for domestic variety i must satisfy:

$$\begin{aligned} Y_t(i) \ &= \ C_{H,t}(i) + C^*_{H,t}(i) + \psi_t(i) \\ &= \ \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon}\left[\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}(1-\alpha)C_t + \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta}\alpha^* C^*_t\right] + \psi_t(i) \end{aligned} \tag{41}$$

for all $i \in [0, 1]$ and t.

Plugging (41) into the definition of aggregate output $Y_t \equiv \left[\int_0^1 Y(i)^{1-\frac{1}{\varepsilon}}\ di\right]^{\frac{\varepsilon}{\varepsilon-1}}$, recalling that $P_{H,t} = \mathcal{E}_t P^*_{H,t}$ and using (15) and (16) we can express market clearing for domestic goods as:

$$\begin{aligned} A_t N_t \ &= \ \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}(1-\alpha)\ C_t + \left(\frac{P_{H,t}}{\mathcal{E}_t P^*_t}\right)^{-\eta}\alpha^* C^*_t \ + \ \psi_t \\ &= \ (1-\alpha)\ [g(S_t)]^{\eta}\ C_t \ + \ \alpha^*\ [S_t\ g^*(S_t)]^{\eta}\ C^*_t \ + \ \psi_t \end{aligned} \tag{42}$$

The corresponding market clearing condition for Foreign will read:

$$A_t^* N_t^* = (1-\alpha^*)\ \ [g^*(S_t)]^\eta\ C_t^* + \alpha\ \left(\frac{g(S_t)}{S_t}\right)^\eta\ C_t\ +\ \psi_t^* \tag{43}$$

The CPI level can be linked to the domestic price level and the terms of trade as follows: $P_t = P_{H,t}\ g(S_t)$. Let's define gross *CPI inflation* as $\pi_t \equiv \frac{P_t}{P_{t-1}}$. This is related to domestic producer inflation and the change in the the terms of trade as follows:

$$\pi_t = \pi_{H,t}\frac{g(S_t)}{g(S_{t-1})} \tag{44}$$

Similarly for CPI inflation in Foreign:

$$\pi_t^* = \pi_{F,t}^*\frac{g^*(S_t)}{g^*(S_{t-1})} \tag{45}$$

Substituting into (9) and (13) one obtains the following modified expressions for the Euler equations:

$$U_{c,t} = \beta\ R_t\ E_t\left\{\frac{U_{c,t+1}}{\pi_{H,t+1}\frac{g(S_{t+1})}{g(S_t)}}\right\} \tag{46}$$

$$U_{c,t}^* = \beta\ R_t^*\ E_t\left\{\frac{U_{c,t+1}^*}{\pi_{F,t+1}^*\frac{g^*(S_{t+1})}{g^*(S_t)}}\right\} \tag{47}$$

At this stage we can propose the following definition of world competitive equilibrium in primal form:

Definition 2. *For given interest rate policies* $\{R_t,\ R_t^*\}$ *and shock processes* $\{A_t,\ A_t^*\}$ *a world (imperfectly) competitive equilibrium is a set of allocations* $\{C_t,\ C_t^*,\ N_t,\ N_t^*,\ \pi_{H,t}, \pi_{F,t}^*,\ S_t\}_{t=0}^\infty$ *that solves equations (9), (13), (38), (40), (42), (43).*

Few observations on this form of the competitive equilibrium are in order. First, nominal interest rates are residual, for they only show up in equations (9) and (13). This is a typical effect of the cashless environment adopted here.[14] Second, present value budget constraints (18) and (21)

[14] One may notice that this is the parallel in the cashless economy of the real money balances being residual in the competitive equilibrium with transaction frictions when utility is separable in consumption and real balances. See Appendix A.

are not strictly included in the definition of the competitive equilibrium. In fact, once equilibrium allocations and policy rules are computed, one can determine residually the initial distribution of assets $\left\{B_0, B_0^F\right\}$ in such a way that our assessment of risk-sharing as in Definition 1 is satisfied.

The above definition of the equilibrium includes the terms of trade as a part of the competitive allocations. However one can express the terms of trade as a result of the equilibrium consumption allocations. By using (15), (16) and (17) one can write:

$$Q_t = q(S_t) = \left(\frac{\alpha^* + (1-\alpha^*)\ {S_t}^{1-\eta}}{(1-\alpha) + \alpha\ {S_t}^{1-\eta}} \right) \tag{48}$$

Notice that if $\alpha = \alpha^* = \frac{1}{2}$ it follows immediately that $Q_t = 1$ (i.e., PPP holds) regardless of the equilibrium value of S_t. By substituting (22) one can express the terms of trade as a function of the consumption allocations only:

$$S_t = \left(\frac{\alpha^* - (1-\alpha)\left(\kappa \frac{U^*_{c,t}}{U_{c,t}}\right)^{1-\eta}}{\alpha \left(\kappa \frac{U^*_{c,t}}{U_{c,t}}\right)^{1-\eta} - (1-\alpha^*)} \right) = S(C_t, C_t^*) \tag{49}$$

with $\frac{\partial S(\bullet)}{\partial C} > 0$ and $\frac{\partial S(\bullet)}{\partial C^*} < 0$. By substituting (49) into (15) and (17) one obtains:

$$g(S_t) = \left(\frac{\alpha \left(\kappa \frac{U^*_{c,t}}{U_{c,t}}\right)^{1-\eta} - (1-\alpha^*)}{\alpha\alpha^* - (1-\alpha^*)(1-\alpha^*)} \right)^{\frac{1}{\eta-1}} \equiv \widetilde{g}(C_t, C_t^*) \tag{50}$$

$$g^*(S_t) = \left(\frac{\alpha^* \left(\kappa \frac{U^*_{c,t}}{U_{c,t}}\right)^{\eta-1} - (1-\alpha)}{\alpha\alpha^* - (1-\alpha^*)(1-\alpha^*)} \right)^{\frac{1}{\eta-1}} \equiv \widetilde{g}^*(C_t, C_t^*) \tag{51}$$

with $\frac{\partial \widetilde{g}(\bullet)}{\partial C} > 0$, $\frac{\partial \widetilde{g}(\bullet)}{\partial C^*} < 0$, $\frac{\partial \widetilde{g}^*(\bullet)}{\partial C} < 0$, $\frac{\partial \widetilde{g}^*(\bullet)}{\partial C^*} > 0$.

In the following, we formulate a proposition that establishes a mapping between the minimal form summarized by conditions *(42), (43), 38, (40),(9), (13).*expressed above and all remaining real allocations and prices. Establishing such a mapping is a crucial preliminary step for the setup of the optimal policy problem in the section below.

Proposition 1. **[Part A]**. *For given policy instruments* $\{R_t, R_t^*\}_{t=0}^{\infty}$, *equilibrium allocations* $\{C_t, N_t, mc_t, C_{F,t}, C_{H,t}, C_t^*, N_t^*, mc_t^*, \pi_{F,t}^*, C_{H,t}^*, C_{F,t}^*\}_{t=0}^{\infty}$, *prices* $\left\{v_{t,t+1}, W_t, W_t^*, P_{H,t}, P_{F,t}, P_{H,t}^*, P_{F,t}^*, P_t^*, P_t\right\}$ *and relative prices* $\{\mathcal{E}_t, Q_t, S_t\}_{t=0}^{\infty}$ *satisfying equations (1), (2), (6), (7), (8), (11), (12), (13), (14),* *(15), (16), (17), (18, (21), (22), (27), (37), (42), (43) also satisfy equations (9) (13), (38), (40), (42), (43).* **[Part B]**. *By reverse, using allocations* $\left\{C_t, N_t, \pi_{H,t}, C_t^*, N_t^*, \pi_{F,t}^*\right\}_{t=0}^{\infty}$ *that satisfy equations (9) (13), (38), (40), (42), (43) it is possible to construct all the remaining equilibrium (nominal and real) allocations, prices and policy instruments for Home and Foreign.*

Proof. See Appendix B■

4 Optimal Monetary Policy

Optimal policy is determined by a monetary authority that maximizes the discounted sum of utility of the representative agent under the constraints that characterize the competitive economy. As in the classical literature on optimal taxation (see Chari, Christiano and Kehoe (1994)) or more recently in the monetary policy closed-economy analysis of Khan et al. (2003), the policy problem takes the form of a constrained *allocation problem*, in which the government can be thought of choosing directly a feasible allocation subject to those constraints that ensure the existence of instruments and prices which make the same allocation consistent with optimality. In our cashless economy, the minimal set of constraints that are relevant for the Ramsey allocation problem are the ones described in Definition 2. In practice, they consist of an appropriately characterized price implementability constraint (Phillips curve) and of an appropriately characterized resource constraint.[15].

A distinctive feature of the Ramsey-type analysis undertaken here is that we allow the relevant distortions characterizing the private sector economy to remain explicit. The private sector world economy is characterized by *three distortions*The first two, market power and price stickiness, are common to both the closed and the open economy version of our model. The price stickiness distortion, summarized by the quadratic term in inflation in the resource constraint, is obviously minimized at zero net inflation (i.e., $\pi_{H,t} = \pi_{F,t}^* = 1$ for all t). On the other hand, the presence of market power renders the level of output and employment inefficiently low.

As also emphasized in Corsetti and Pesenti (2001), what in general distinguishes the analysis of an open economy is the presence of an additional inefficiency, that we label *international relative price* distortion. This stems from the possibility for each country, in the presence of rigid nominal prices, of strategically affecting the terms of trade, and in turn of reducing the (expected) disutility of labor effort for any given level of consumption. This externality creates per se room for policy competition. The interesting aspect concerns the extent to which such policy competition motive may lead each policymaker to try to deviate from the prescription of price stability that would typically characterize optimal policy in the closed economy version of our model.

Below we solve the optimal allocation problem under the assumption that *commitment* is

[15] In Appendix A we show how this set of constraints needs to be extended in a more general economy featuring monetary transactions frictions.

feasible.[16] We study two international policy arrangements. In the first, policymakers in the two countries conduct policy in a competitive fashion by choosing allocations independently. Relative to the existing literature, it is of particular interest to characterize a Nash-equilibrium under policy competition and within the context of general consumption preferences, forward-looking price setting and uncorrected steady-state distortions.[17] In the second policy regime, a world social planner forces the two policymakers to coordinate their actions to maximize global welfare.

4.1 Optimal Policy under Nash Competition

We begin with the analysis of policy competition. We have the following definition of a Nash-Ramsey allocation problem:

Definition 2. *Let $\{\lambda_{p,t},\, \lambda_{f,t}\}_{t=0}^{\infty}$ represent sequences of Lagrange multipliers on the constraints (38) and (42) respectively. For any given allocation $\{C_t^*\}_{t=0}^{\infty}$ and stochastic processes $\{A_t,\, A_t^*\}_{t=0}^{\infty}$, plans for the control variables $\{C_t,\, \pi_{H,t},\, N_t\}_{t=0}^{\infty}$ and for the costate variables $\{\lambda_{p,t},\, \lambda_{f,t}\}_{t=0}^{\infty}$ represent a Nash-constrained allocation if they solve the following maximization problem:*

$$Max\ E_0\{\sum_{t=0}^{\infty}\beta^t U(C_t, N_t) \tag{52}$$

$$+\beta^t\lambda_{p,t}\ \left[U_{c,t}\pi_{H,t}(\pi_{H,t}-1)-\beta E_t\left\{U_{c,t+1}\pi_{H,t+1}(\pi_{H,t+1}-1)\right\}+\frac{U_{c,t}\ \varepsilon\ A_t N_t}{\theta}\left(\frac{U_{n,t}\ \widetilde{g}_t}{U_{c,t}A_t}+\frac{\varepsilon-1}{\varepsilon}\right)\right]$$
$$+\beta^t\lambda_{f,t}\ \left[A_t N_t-\widetilde{g}_t^{\ \eta}\left[(1-\alpha)\ C_t\ +\ \alpha^*\left(\frac{U_{c,t}^*}{U_{c,t}}\right)^{\eta}\ C_t^*\right]\ -\frac{\theta}{2}\left(\pi_{H,t}-1\right)^2\right]\}$$

where $\widetilde{g}_t$ is compact notation for $\widetilde{g}(C_t,\, C_t^*)$.

Notice that the distinctive feature of the commitment problem under Nash competition is that the Home policymaker *does not internalize* that the relative price $\widetilde{g}_t$ depends also on the level of consumption in Foreign (see equation (50) above), and that, symmetrically, the relative price $\widetilde{g}_t^*$ is affected by its own consumption choice. It is clear that the externality works via foreign consumption affecting international relative prices and in turn domestic real marginal cost. For any given level of foreign consumption, a fall (rise) in *domestic* consumption leads to an appreciation (depreciation) of the terms of trade and therefore to a fall (rise) in labor effort and real marginal cost (see equation (39)).

[16] This allows to abstract from any incentive of policymakers to resort to surprise deflation as in the work of Corsetti and Pesenti (2000).

[17] Benigno and Benigno (2003), for instance, focus on policy competition in a model with one-period predetermined prices, and limit the analysis only to Nash equilibria conditional on one particular strategy (i.e., price stability).

4.1.1 Non-recursivity and Initial Conditions

As a result of the constraint (38) exhibiting future expectations of control variables, the maximization problem as spelled out in (42) is intrinsically non-recursive.[18] As first emphasized in Kydland and Prescott (1980), and then developed in Marcet and Marimon (1999), a formal way to rewrite the same problem in a recursive stationary form is to enlarge the planner's state space with additional (pseudo) costate variables. Such variables, that we denote χ_t and χ_t^* for Home and Foreign respectively, bear the crucial meaning of tracking, along the dynamics, the value to the planner of committing to the pre-announced policy plan.

A related aspect concerns the specification of the law of motion of these Lagrange multipliers. For in our case the forward-looking Phillips curve constraint features a simple one-period expectation, the same costate variables have to obey the laws of motion:[19]

$$\begin{aligned} \chi_{t+1} &= \lambda_{p,t} \\ \chi^*_{t+1} &= \lambda^*_{p,t} \end{aligned} \tag{53}$$

A further point concerns the definition of the appropriate initial conditions for χ_t and χ_t^*. Marcet and Marimon (1999) show that for the modified (recursive) Lagrangian to generate a global optimum under time zero commitment it must hold:

$$\chi_0 = 0 = \chi_0^* \tag{54}$$

The above condition states that there is no value to the policy planner, in either country and as of time zero, attached to prior commitments.

In *Appendix C* we show how to reformulate the optimal plan in an equivalent recursive stationary form. First order conditions for time $t \geq 1$ for the choice of C_t, N_t and $\pi_{H,t}$ imply respectively:

$$\begin{aligned} 0 \;=\; & U_{c,t} \;+\; U_{cc,t}\, \pi_{H,t}\, (\pi_{H,t}-1)\, (\lambda_{p,t}-\chi_t) \;+\; \lambda_{p,t} \frac{N_t}{\theta} \left(\varepsilon\, U_{n,t}\, \widetilde{g}_{c,t} \;+\; (\varepsilon-1) A_t\, U_{cc,t}\right) \\ & -\lambda_{f,t} \left(\eta\, \widetilde{g}_t^{\,\eta-1}\, \widetilde{g}_{c,t} \left[(1-\alpha)\, C_t + \left(\frac{U^*_{c,t}}{U_{c,t}} \right)^{\eta} \alpha^*\, C_t^* \right] \right) \\ & -\lambda_{f,t}\, \widetilde{g}_t^{\,\eta} \left((1-\alpha) \;-\eta\, U^*_{c,t}\, \alpha^*\, C_t^*\, U_{c,t}^{\eta-1}\, U_{cc,t} \right) \end{aligned} \tag{55}$$

[18] See Kydland and Prescott (1977), Calvo (1978). As such the system does not satisfy per se the principle of optimality, according to which the optimal decision at time t is a time invariant function only of a small set of state variables.

[19] The law of motion of the additional costate variables would take a more general form if the expectations horizon in the forward looking constraint(s) featured a more complicated structure, as, for instance, in the case of constraints in present value form. See Marcet and Marimon (1999).

$$0 = U_{n,t} + \frac{\lambda_{p,t}\ \varepsilon\ \widetilde{g}_t}{\theta}\left(U_{n,t} + N_t U_{nn,t}\right)\ +\ \lambda_{p,t}\left(\frac{\varepsilon - 1}{\theta}\right) U_{c,t}\ A_t\ +\ \lambda_{f,t} A_t \tag{56}$$

$$0 = U_{c,t}\ (2\pi_{H,t} - 1)\ (\lambda_{p,t} - \chi_t) - \theta\ (\pi_{H,t} - 1)\ \lambda_{f,t} \tag{57}$$

where $\widetilde{g}_{c,t} \equiv \frac{\partial \widetilde{g}_t(\bullet)}{\partial C_t} > 0$.

The system of efficiency conditions in Home is completed by the law of motion (53), the initial condition (54) and by the constraints (38) and (42) holding with equality.[20]

Once defined a completely symmetric problem for the policy maker in Foreign, we can state the following definition of a Nash equilibrium:

Definition 4. (Nash equilibrium under commitment). *The set of processes* $\{\lambda_{p,t}, \lambda_{f,t}, \lambda^*_{p,t}, \lambda^*_{f,t}\}_{t=0}^{\infty}$, $\{C_t, \pi_{H,t}, N_t, C^*_t, \pi^*_{F,t}, N^*_t\}_{t=0}^{\infty}$, $\equiv_{t=0}^{\infty}$ *fully describe a Nash equilibrium under commitment if they solve the system of equations (55) - (57), equations (38), (42) holding with equality, along with a symmetric set of conditions jointly holding for Foreign.*

4.1.2 Nash-Optimal Inflation Rate in the Long-Run

To determine the long-run optimal inflation rate associated to the Nash-game described above, one needs to solve the steady-state version of the set of efficiency conditions (55)-(57). To develop an analogy with the Ramsey-Cass-Koopmans model, this amounts to computing the *modified golden rule* steady state. This per se contrasts with the *golden rule* inflation rate, which would correspond to the one that maximizes households' instantaneous utility under the constraint that the steady-state conditions are imposed ex-ante. It is well known that in dynamic economies with discounted utility the two concepts of long-run optimal policy cannot coincide.[21]

The following Lemma establishes the average optimal inflation rate under Nash-competition:

Lemma 2. *The (net) producer inflation rate associated to the steady-state of the Nash-Ramsey policy problem is zero.* To see this consider the steady-state version of equation (57). In that case it holds:

$$\lambda_p = \chi$$

Given that $\theta > 0$ and that, in this second-best environment, $\lambda_F > 0$, we immediately conclude that $\pi_H = 1$■

[20] As a consequence of the underlying time consistency problem the system of first order conditions takes a different form at time $t = 0$. One can easily obtain that system by considering that, at $t = 0$, equation (54) must hold.

[21] See King and Wolman (1999) and Khan, King and Wolman (2003) for a closed-economy analysis on this point.

Under the assumption $\alpha = \alpha^*$, the solution to the steady-state of the Nash game is symmetric and features $\pi_H = \pi_F^* = 1$, $C = C^*$, $g(S) = g^*(S) = 1$, $N = N^*$. As a corollary of Lemma 2 and of the fact that in the symmetric steady state $S = g(S) = g^*(S) = 1$, we also have that the *CPI (net) rate of inflation* is zero, i.e., $\pi = \pi^* = 1$.

Hence the steady state of the solution to the Nash-optimal policy indicates that, if unconstrained, *both* policymakers would choose to set the economy along a path that would lead to a long-run net inflation rate of zero. The intuition for this result is simple. One can view the modified golden rule as the long-run state of the economy when the discount rate β converges to 1. In this case the steady-state version of the Phillips curve relation (38) is vertical, and the policymaker of either country cannot exert any effect on markups by setting inflation rates different from zero. Hence a second-best policy requires to choose the rate of inflation that minimizes the price adjustment cost distortion. It is intuitive to see that this constitutes the only long-run Nash equilibrium.

4.1.3 Optimal Stabilization Policy around the Long-Run Steady-State

We are now in the position to analyze the dynamic features of the optimal commitment policy under Nash competition. In this section we interpret optimal policy in the sense of *optimal stabilization* in response to shocks. The motivation driving our analysis is to explore the extent to which, in response to shocks, Nash competition among policymakers generates deviations from price stability, which we have shown above to be the *average* optimal policy outcome.

To this end, we proceed in the following way. First, we compute (for both countries) the stationary allocations that characterize the deterministic steady state of the first order conditions (55)-(57) (and symmetric ones for Foreign). We then compute a log-linear approximation of the respective policy functions in the neighborhood of the same steady state.

The spirit of this exercise deserves some further comments. In concentrating on (log-linear) dynamics in the neighborhood of the steady state associated to the efficiency conditions, we deviate from the initial condition (54) in the fact that we set the initial value of the lagged Lagrange multipliers equal to their deterministic steady state values, i.e.,:

$$\chi_0 = \overline{\chi}_0 \ ; \ \ \chi_0^* = \overline{\chi}_0^* \tag{58}$$

where $\overline{\chi}_0$ and $\overline{\chi}_0^*$ are endogenously determined from the solution of the system given by the steady-state version of (55)-(57), along with symmetric conditions for Foreign.[22].

It is important to understand that this strategy corresponds to focussing on a particular dimension. Namely, optimal stabilization policy in response to bounded shocks that hit in the

[22]See Khan et al. (2003) for an application of this strategy to an optimal monetary policy problem in a closed economy.

neighborhood of the long-run steady state. This amounts to implicitly assuming that the economy has been evolving and policy been conducted around such a steady already for a long period of time.[23] [24]

4.1.4 Parameterization

In conducting our simulations we employ the following form of the period utility: $U(C_t, N_t) = \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\gamma} N_t^{1+\gamma}$. The time unit is meant to be quarters. The discount factor β is equal to 0.99. The degree of risk aversion σ is 1 (which implies log-utility and in turn consistence with a balanced growth path), the inverse elasticity of labor supply γ is equal to 3, which is a common values in the real business cycle literature. As a benchmark value (see below for a discussion) we set $\eta = 2$. As in Bergin and Tchakarov (2003), and consistent with estimates by Ireland (2001), we set the price stickiness parameter θ equal to 50. The elasticity ε between varieties produced by the monopolistic sector is 6. The (inverse) degree of home bias α, identified by the share of foreign imported goods in the domestic consumption basket, is set to a default value of 0.4. This implies the existence of a *mild home bias*, which is assumed to be symmetric across countries ($\alpha = \alpha^*$). Finally (log) productivity in both countries is assumed to follow an autoregressive process:

$$\log(A_t) = \varphi \ \log(A_{t-1}) \ + \ \varepsilon_t^a$$

$$\log(A_t^*) = \varphi^* \ \log(A_{t-1}^*) \ + \ \varepsilon_t^{a^*}$$

where $\varphi = \varphi^* = 0.9$ and $\left\{\varepsilon_t^a, \varepsilon_t^{a^*}\right\}$ are i.i.d. shocks with standard deviation $\sigma^a = \sigma^{a^*} = 0.01$. We choose to keep the source of stochastic volatility limited to productivity shocks in order to maintain as a benchmark the closed economy optimal response to those shocks, which is known to correspond to (producer) price stability.[25]

4.1.5 Response to Asymmetric Productivity Shocks

Figure 1 displays impulse responses of selected Home and Foreign variables to a one percent *rise in Home productivity* in the case of *Nash-commitment*. Since productivity is higher in Home, the

[23] From a methodological point view, it is also of independent interest to notice that we reverse the logic of the so-called linear-quadratic approach to the optimal policy problem. In fact, we employ standard log-linear approximation methods to describe the policy function only when *the exact optimal Ramsey allocation has been already characterized.*

[24] Technically speaking, one should add the following further clarification. Assuming initial conditions as (58) renders the approximation valid only if $\overline{\chi}_0$ and $\overline{\chi}_0^*$ lie in the neighborhood of a stochastic steady state which includes the values $\chi_0 = \chi_0^* = 0$. This is most likely the case in the presence of bounded shocks of the size employed in this paper.

[25] See Khan et al. (2003). However, and more generally, Adao et al (2003) and Khan et al. (2003) show that in the presence of (real) demand shocks, such as exogenous variations in government purchases, strict price stability ceases to correspond to the optimal policy even in the prototype sticky price model with imperfect competition.

adjustment to the equilibrium requires an increase in the demand of domestic goods relative to foreign goods. This is achieved by means of a terms of trade depreciation, captured by a rise in the CPI to PPI ratio $g(S)$.

Hence we see that price (markup) stability is *not* the Nash equilibrium of the underlying policy game in response to an asymmetric shock. Consumption rises both in Home and Foreign as a result of higher productivity and risk-sharing. However the rise in Home consumption exceeds the one in Foreign in order to achieve the necessary real exchange rate depreciation. The only equilibrium is one in which the same real (terms of trade) depreciation is achieved via an increase in prices in both countries. In fact, and due to risk sharing, both countries face the incentive to increase prices to tilt the terms of trade in their own favor, thereby achieving a relatively higher real income for any given level of labor effort.

The intuition for why the policymaker will not find it optimal to replicate the flexible price allocation is simple, and is best understood in the case of logarithmic utility. In that case, in fact, and in response to a rise in productivity, real income Y_t will exceed (fall short of) consumption depending on whether $\eta >$ or < 1. To see this, notice that by combining (22) with (41) one obtains, in the case $\eta = 1$, $P_{H,t}Y_t = P_tC_t$, or, in different terms , $\frac{Y_t}{g(S_t)} = C_t$ for all t. This is the knife-edge case of continuously balanced trade analyzed in Corsetti and Pesenti (2000).

More generally, if $\eta > 1$ (< 1), *at the margin and relative to the flexible price allocation*, the planner can make the consumer better off by contracting (expanding) employment and output below their natural levels (i.e., appreciate the terms of trade relative to its natural level) in response to a rise (fall) in real income above (below) consumption (i.e., in response to a trade surplus (deficit)). However, in a Nash-game where policy objectives collide, neither country succeeds in manipulating the terms of trade beyond the adjustment that is necessary in response to the asymmetric productivity shock. The resulting effect is an inefficient level of consumption. In general the marginal welfare gain of appreciating the terms of trade (relative to the equilibrium response under purely flexible price) rises with the elasticity of substitution η.

It is also interesting to notice that policy competition generates a dynamic behavior of the price level that resembles the one in response to a *cost-push shock*. The novel aspect of this result is that the same dynamics are obtained in response to a productivity shock, which is not aimed per se (like in many recent New Keynesian studies of optimal monetary policy) to induce the artificial effect of exogenously drifting the economy away from the efficient allocation. The fact that productivity shocks are a source of price variability under the optimal policy is here an endogenous outcome of the competition on international relative prices.[26]

[26] For an analysis of the optimal policy setting in response to this type of shocks see Woodford (2003) and Clarida et al. (1999). For open economy models with one-period predetermined prices see Sutherland (2001).

4.1.6 Sensitivity Analysis

While the result above suggests that markup variability is an equilibrium outcome of policy competition (unless, with log utility, $\eta == 1$), deviations from a price stability policy appears quantitatively small. *Figure 2* illustrates how the incentive to generate price movements vary with the critical elasticity parameter, namely η.

The figure displays impulse responses (under Nash-commitment) of the same selected variables to a productivity shock for alternative values of $\eta = [1, 2, 5]$. The first case corresponds to the benchmark one of Cobb-Douglas preferences typically employed in the linear-quadratic approach to the study of optimal policy for open economies. The literature lacks a consensus on the value of this parameter. Lai and Trefler (1999) suggest an empirical value as high as 5. Collard and Dellas (2002) derive an estimated value of 2.5. In their quantitative (theoretical) study, Backus, Kehoe and Kydland (1995) explore a range of η between 0 and 5. Chari et al. (2002) set $\eta = 1.5$, while Bergin and Tchakarov (2003) set $\eta = 5$. Anderson and Van Wincoop (2003) argue for much larger values, in the order of 8 to 10. Overall, there seems to exist both empirical and theoretical support for the hypothesis that the value of η lies well above unity.

Thus the figure highlights the coincidence of the Nash-optimal response with a close-to-price stability strategy only in the particular case of $\eta = 1$. In this knife-edge case, the income effect of the required terms of trade depreciation (given the relatively higher productivity in Home) balances the incentive to switch expenditure towards Home goods.[27] In general, the higher the elasticity of substitution, the larger the deviations from price stability.

Figure 3 conducts a sensitivity analysis to illustrate how Nash-optimal inflation volatility varies with both parameters that index openness in our economy, namely the (inverse of) home bias α and the elasticity η. In conducting this experiment we keep our assumption of symmetric degree of home bias, i.e.,. $\alpha = \alpha^*$. In this case we allow η to assume values ranging from the benchmark case of 1 to the extreme value of 10 suggested by the recent survey of Anderson and Van Wincoop (2003). Hence we see that Nash-optimal producer inflation volatility is increasing in both the degree of trade openness (inverse of home bias) and the elasticity of substitution. One may notice, though, that with values of η beyond 4 and reaching the extreme values suggested by the recent literature inflation volatility tends to level off. Thus we see that the lower the degree of home bias (i.e., the higher α), the larger the equilibrium deviations from price stability. To gather an intuition, recall that the terms of trade affects the real marginal cost (thereby activating the externality in our setting) via the relative price $g(S_t)$. Log-linearizing we have $\widehat{g}_t = (1 - \alpha)\widehat{S}_t$, which is decreasing in α. Hence the larger α the larger the variation in $\widehat{S}_t$ necessary to achieve

[27] Another way of seeing this is to notice that in the case $\eta = 1$ it holds that $A_t N_t = g(S_t)\ C_t$ at all times. Substituting this into equation (39) one can see that the domestic real marginal cost ceases to depend on any foreign variable, thereby shutting down any policy competition motive on the terms of trade.

any required variation in $\widehat{g}_t$, hence the larger the expenditure switching effect that generates the externality in our setting

4.2 Optimal Policy under Cooperation

Under cooperation, a social planner explicitly recognizes the channel of interdependence that works through the relative prices $g(S_t)$ and $g^*(S_t)$. Below we define the world Ramsey planner problem, under the assumption that the same planner aims at maximizing the *average* level of utility of the two countries. We also assume that both countries receive equal weight in the planner's objective function.

Let's define the *world Ramsey period utility objective* as $\left\{\frac{\mathcal{V}(C_t,N_t)\,+\,\mathcal{V}^*(C_t^*,N_t^*)}{2}\right\}$,where $\mathcal{V}(C_t,N_t) \equiv E_0\left\{\sum_{t=0}^{\infty}\beta^t\, U(C_t,N_t)\right\}$ and $\mathcal{V}^*(C_t^*,N_t^*) \equiv E_0\left\{\sum_{t=0}^{\infty}\beta^t\, U(C_t^*,\ N_t^*)\right\}$. Then the Ramsey maximization problem can be defined as follows:

Definition 5. *Let* $\left\{\lambda_{p,t},\ \lambda_{p,t}^*,\ \lambda_{f,t},\ \lambda_{f,t}^*\ \right\}_{t=0}^{\infty}$ *represent sequences of Lagrange multipliers on the constraints (38), (42), (40) and (43) respectively. For any given stochastic processes* $\{A_t,\ A_t^*\}_{t=0}^{\infty}$*, plans for the control variables* $\{C_t,\ \pi_{H,t},\ N_t,\ C_t^*,\ \pi_{F,t}^*,\ N_t^*\}$*, and for the costate variables* $\{\lambda_{p,t},\ \lambda_{f,t},\ \lambda_{p,t}^*,\ \lambda_{f,t}^*\}_{t=0}^{\infty}$ *represent a first-best constrained allocation if they solve the following maximization problem:*

$$Max\ E_0\left\{\frac{\mathcal{V}(C_t,N_t)\ +\ \mathcal{V}^*(C_t^*,N_t^*)}{2}\right\} \tag{59}$$

$$\begin{aligned}
&+\beta^t\ \lambda_{p,t}\left[U_{c,t}\pi_{H,t}(\pi_{H,t}-1)-\beta E_t\left\{U_{c,t+1}\pi_{H,t+1}(\pi_{H,t+1}-1)\right\}+\frac{U_{c,t}\ \varepsilon\ A_tN_t}{\theta}\left(\frac{U_{n,t}\ \widetilde{g}_t}{U_{c,t}A_t}+\frac{\varepsilon-1}{\varepsilon}\right)\right]\\
&+\beta^t\ \lambda_{f,t}\left[A_tN_t-\widetilde{g}_t^{\ \eta}\left[(1-\alpha)\ C_t\ +\ \alpha^*\left(\frac{U_{c,t}^*}{U_{c,t}}\right)^{\eta}\ C_t^*\right]\ -\frac{\theta}{2}\left(\pi_{H,t}-1\right)^2\right]\\
&+\beta^t\ \lambda_{p,t}^*\left[U_{c^*,t}^*\ \pi_{F,t}^*\ (\pi_{F,t}^*-1)-\beta E_t\left\{U_{c^*,t+1}^*\pi_{F,t+1}^*(\pi_{F,t+1}^*-1)\right\}\ +\ \frac{U_{c^*,t}^*\ \varepsilon\ A_t^*N_t^*}{\theta}\left(\frac{U_{n^*,t}^*\ \widetilde{g}_t^*}{U_{c^*,t}^*A_t^*}+\frac{\varepsilon-1}{\varepsilon}\right)\right]\\
&+\beta^t\ \lambda_{f,t}^*\left[A_t^*N*_t-\widetilde{g}_t^{*\ \eta}\left[(1-\alpha^*)\ C_t^*\ +\ \alpha\left(\frac{U_{c,t}^*}{U_{c,t}}\right)^{-\eta}\ C_t\right]\ -\frac{\theta}{2}\left(\pi_{F,t}^*-1\right)^2\right]
\end{aligned}$$

We defer to *Appendix D* the description of the first order conditions corresponding to this plan. The discussion on the non-recursivity structure of the problem follows exactly the logic applied above to the re-definition of the Nash-commitment policy setup. In practice, this will entail specifying an equivalent recursive stationary program in the new world planner's state space defined by $\{A_t, A_t^*,\ \chi_t,\ \chi_t^*\}$.

4.2.1 Ramsey Steady-State

A deterministic Ramsey steady state is a set of allocations $\{C, C^*,\ N,\ N^*, \pi_H, \pi_F^*\}$ that solves the steady-state version of the efficiency conditions associated to the program under Definition 4. In *Appendix E* we characterize such system of equations. Once again, this steady state has a symmetric solution in which $\pi_H = \pi_F^* = 1$. Hence, and exactly like in the steady-state version of the efficiency conditions of the Nash problem analyzed above, the unconstrained long run optimal inflation policy is associated with price stability in both countries. It follows that both policy regimes, Nash and Cooperation, share the same deterministic steady state.

4.2.2 Optimal Response to Shocks around the Ramsey Steady-State: Nash vs. Cooperation

Figure 4 displays impulse responses to a normalized one percent increase in home productivity and compares selected variables under *Nash* versus *Cooperation*. Under policy cooperation, the planner coordinates the responses of both policy makers to achieve the required terms of trade depreciation only by means of a nominal exchange rate depreciation. In other words, it is optimal for the Ramsey planner to have both countries targeting very closely the flexible price allocation. This results in a dampened dynamic of the terms of trade under cooperation. The crucial aspect is that this is now compatible with a smooth path of the price level (the response of the price level, measured in percent deviation from steady state, barely deviates from zero) and with a smoother response of employment, for any given variation in consumption. The reason why *perfect* price stability is not the equilibrium outcome under cooperation is due to the underlying asymmetric nature of the shock. In fact, while it is globally optimal for the world planner to minimize the underlying relative price distortion, it remains efficient, at the margin, to have the country experiencing the positive rise in productivity to slightly deviate from price stability and appreciate the terms of trade (relative to the strictly flexible price policy).

5 Conclusions

We have laid out a typical public finance framework for the analysis of welfare maximizing monetary policy within an economy characterized by three distortions: market power, rigidity in the adjustment of producer prices and international terms of trade externality. The main novelty of our approach is methodological. In fact, and relative to the existing literature, it allows to characterize optimal policy in an open economy where all the relevant distortions remain explicit and uncorrected. Hence the presence of forward looking price setting decisions as well as of a general form of household's preferences over the consumption of domestic and foreign goods do not represent a constraint for the *exact* characterization of the optimal commitment policy in an international setting.

Despite the generality of the approach, our modelling framework remains restrictive in three main dimensions. First, in assuming that the law of one price for traded goods holds continually. Second, in allowing households to obtain full risk sharing via international financial markets. Third, in not allowing households to invest in physical capital. Amending on all these features should aim at generating less trivial dynamics of the current account than the ones generated here via the only movements in the trade balance. Such dynamics may be of first order importance, for instance, for the welfare evaluation of alternative (exchange rate) policies along two dimensions. First, they would more critically affect the transition from the time of policy implementation to the long-run steady state of the new policy. Second, they would impinge on the transition from one policy regime to another. For instance from Nash-competition to cooperation, or from the optimal commitment policy to a fixed exchange rate arrangement. We are currently investigating all these issues in our ongoing work.

A General Model with Monetary Friction

In this section we briefly lay out a model in which money entering the utility function is justified by the presence of transactions frictions. We show that if real balances enter the utility function separably it is possible to nest the Nash-Ramsey allocation problem as a particular case. Agents maximize the following expected discounted sum of utilities over possible paths of consumption, money balances and labor:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(C_t, N_t) + \chi H\left(\frac{M_t}{P_t}\right) \right\}$$

where $\frac{M_t}{P_t} \equiv m_t$ is demand for real money balances, and where $H(\bullet)$ is such that $H_m > 0$ for all $m < \overline{m}$ (with the latter being a satiation level of real balances), and $H_{mm} < 0$.

The sequence of budget constraints assumes the following form:

$$P_t C_t + \sum_{h^{t+1}} \nu_{t+1,t} B_{t+1} + M_t \leq W_t N_t + \tau_t + B_t + M_{t-1} + \int_0^1 \Gamma_t(i)$$

The set of efficiency conditions is given by (6), (7), (8) with the addition of

$$\frac{\chi H_{m,t}}{P_t} = \frac{U_{c,t}}{P_t} + \beta E_t \left\{ \frac{U_{c,t+1}}{P_{t+1}} \right\} \tag{60}$$

By combining (60) and (9) one can typically derive a money demand equation:

$$\frac{\chi \; H_{m,t}}{U_{c,t}} = 1 - \frac{1}{R_t} \tag{61}$$

The government budget constraint reads:

$$\frac{M_t^s - M_{t-1}^s}{P_t} = \tau_t \tag{62}$$

where M_t^s denotes money supply. The instruments of policy are defined as the rates of growth of money supply $\{\mu_t, \mu_t^*\}$ which need to satisfy (respectively in the two countries):

$$\begin{aligned} m_t &= \mu_t \frac{m_{t-1}}{\pi_t} \\ &= \mu_t \frac{m_{t-1} \; g(S_{t-1})}{g(S_t)} \end{aligned} \tag{63}$$

$$
\begin{aligned}
m_t^* &= \mu_t^* \frac{m_{t-1}^*}{\pi_t^*} \\
&= \mu_t \frac{m_{t-1}^* \, g^*(S_{t-1})}{g^*(S_t)}
\end{aligned} \tag{64}
$$

The Nash-Ramsey problem differs from the one in the cashless economy in the fact that the set of feasible allocations must include both the nominal interest rates $\{R_t, R_t^*\}$ and the choice of real money balances $\{m_t, m_t^*\}$. This requires extending the number of constraints to include equations (9) and (61), with the equivalent expressions for Foreign. In practice the *Nash-Ramsey allocation problem* in the model with monetary frictions can be defined as follows (in the representative case of Home):

Choose $\{\lambda_{e,t}, \lambda_{m,t}, \lambda_{p,t}, \lambda_{f,t}\}_{t=0}^{\infty}$ and $\{C_t, \pi_{H,t}, N_t, M_t, R_t\}_{t=0}^{\infty}$

$$
Max \; E_0\{\sum_{t=0}^{\infty} \beta^t \left\{ U(C_t, N_t) + \chi H\left(\frac{M_t}{P_t}\right)\right\} \tag{65}
$$

$$
\begin{aligned}
&+\beta^t \lambda_{e,t} \left[\frac{U_{c,t}}{R_t} - \beta \; E_t \left\{ \frac{U_{c,t+1}}{\pi_{H,t+1} \frac{\widetilde{g}_{t+1}}{\widetilde{g}_t}} \right\}\right] \\
&+\beta^t \lambda_{m,t} \left[\frac{\chi \; H_{m,t}}{U_{c,t}} - \left(1 - \frac{1}{R_t}\right)\right] \\
&+\beta^t \lambda_{p,t} \; \left[U_{c,t}\pi_{H,t}(\pi_{H,t}-1) - \beta E_t \left\{U_{c,t+1}\pi_{H,t+1}(\pi_{H,t+1}-1)\right\} + \frac{U_{c,t} \; \varepsilon \; A_t N_t}{\theta}\left(\frac{U_{n,t} \; \widetilde{g}_t}{U_{c,t} A_t} + \frac{\varepsilon - 1}{\varepsilon}\right)\right] \\
&+\beta^t \lambda_{f,t} \; \left[A_t N_t - \widetilde{g}_t^{\,\eta} \left[(1-\alpha) \; C_t \; + \; \alpha^* \left(\frac{U_{c,t}^*}{U_{c,t}}\right)^{\eta} \; C_t^* \right] \; - \frac{\theta}{2} \left(\pi_{H,t}-1\right)^2 \right] \}
\end{aligned}
$$

where $\lambda_{e,t}$, $\lambda_{m,t}$ are the multipliers associated to the constraints (9) and (61).

The set of first order conditions reads as follows:

$$
\begin{aligned}
0 \;=\; & \text{eq. (55)} + \\
& +\lambda_{e,t} \frac{U_{cc,t}}{R_t} - \lambda_{e,t-1} \; \left(\frac{U_{cc,t}}{\pi_{H,t}} \frac{\widetilde{g}_{t-1}}{\widetilde{g}_t} - \frac{U_{c,t}}{\pi_{H,t}} \widetilde{g}_{t-1} \left(\widetilde{g}_t\right)^{-2} \widetilde{g}_{c,t} \right) \; - \; \lambda_{e,t} \left(\frac{U_{c,t}}{\pi_{H,t}} \frac{\widetilde{g}_t^{-2} \; \widetilde{g}_{c,t}}{\widetilde{g}_{t+1}} \right) \\
& +\lambda_{m,t} \frac{U_{cc,t}}{R_t} - \lambda_{m,t-1} \; \left(\frac{U_{cc,t}}{\pi_{H,t}} \frac{\widetilde{g}_{t-1}}{\widetilde{g}_t} - \frac{U_{c,t}}{\pi_{H,t}} \widetilde{g}_{t-1} \widetilde{g}_t^{-2} \widetilde{g}_{c,t} \right) \; - \; \lambda_{m,t} \left(\frac{U_{c,t}}{\pi_{H,t}} \frac{\widetilde{g}_t^{-2} \; \widetilde{g}_{c,t}}{\widetilde{g}_{t+1}} \right)
\end{aligned} \tag{66}
$$

$$
\text{eq. (56)} \tag{67}
$$

$$0 = \text{eq. (57)} \; - \lambda_{e,t-1} U_{c,t} \frac{\widetilde{g}_{t-1}}{\widetilde{g}_t} (\pi_{H,t}^{-2}) - \lambda_{m,t-1} U_{c,t} \frac{\widetilde{g}_{t-1}}{\widetilde{g}_t} (\pi_{H,t}^{-2}) \tag{68}$$

$$0 = \lambda_{e,t} U_{c,t} (R_t^n)^{-2} + \lambda_{m,t} \; (R_t^n)^{-2} \left(\chi H_{m,t} + U_{c,t} \right) \tag{69}$$

$$0 = \chi H_{m,t} + \chi \; \lambda_{m,t} H_{mm,t} \; (R_t)^{-1} \tag{70}$$

Lemma 3. *If the weight χ on real money balances in the utility function approaches zero, then the first order conditions of the Nash-Ramsey problem in the model with transaction frictions coincide with those of the Nash-Ramsey problem in the cashless model.*

To see this one needs to notice that, for $\chi \to 0$, (69) implies $\lambda_{e,t} = -\lambda_{m,t}$. Substituting this into (66), (68) and (70) one obtains that such conditions coincide with (55)-(57)■

B Proof of Proposition 1

The proof of *part A* follows from the equation manipulations presented in Section 3. As for *part B*, given $\{C_t, C_t\}_{t=0}^{\infty}$ from the Ramsey problem, obtain $g(S_t)$ and $g^*(S_t)$ respectively from (50) and (51). Then derive S_t from (49). Then obtain $C_{H,t}$, $C_{F,t}$, $C_{H,t}^*$, $C_{F,t}^*$ by using (2). Given $P_{H,-1}$ and $P_{F,-1}^*$ obtain $P_{H,t}$ and $P_{F,t}^*$ from allocations $\left\{\pi_{H,t},\ \pi_{F,t}^*\right\}_{t=0}^{\infty}$. Using the fact that the law of one price holds, and given $P_{H,t}$, $P_{F,t}^*$ and S_t, pin down the nominal exchange rate by using the definition of the terms of trade: $\mathcal{E}_t = \frac{S_t P_{H,t}}{P_{F,t}^*}$. Next, obtain Q_t from (22). To obtain P_t and P_t^* use (3) and the symmetric expression for Foreign. Given $\pi_{H,t}$, $\pi_{F,t}^*$, $g(S_t)$ and $g^*(S_t)$, use (44) and (45) to obtain π_t and π_t^*. To derive mc_t and mc_t^* use (39) and the symmetric expression in Foreign. Next, use (27) and the symmetric expression in Foreign to obtain W_t and W_t. Finally use (7) to obtain $v_{t,t+1}$, (9) and (13) to obtain R_t and R_t^*.

C The Stationary Policy Problem

Below we derive the stationary form of the policy problem under Nash commitment. We illustrate the argument only for the Home policymaker's problem, since the problem in Foreign is exactly symmetric. Let's consider the optimal plan as formulated in equation (65) in the text. By applying the law of iterated expectations and by grouping expectations and multipliers that share the same date one obtains:

$$Max \;\; E_0\{U(C_0, N_0)$$

$$
\begin{aligned}
&+\lambda_{p,0}\left[U_{c,0}\ \pi_{H,0}\ (\pi_{H,0}-1)+\frac{U_{c,0}\ \varepsilon\ A_0N_0}{\theta}\left(\frac{U_{n,0}\widetilde{g}_0}{U_{c,0}A_0}+\frac{\varepsilon-1}{\varepsilon}\right)\right] \\
&+\lambda_{f,0}\left[A_0N_0-(1-\alpha)C_0\widetilde{g}_0^{\eta}-\left(\frac{U^*_{c^*,0}}{U_{c,0}}\right)^{\eta}\widetilde{g}_0^{\eta}\alpha^*C_0^*-\frac{\theta}{2}\left(\pi_{H,0}-1\right)^2\right] \\
&+\beta\{U(C_1,N_1)+(\lambda_{p,1}-\beta\lambda_{p,0})(U_{c,1}\pi_{H,1}(\pi_{H,1}-1))+\lambda_{p,1}\left(\frac{U_{c,1}\ \varepsilon\ A_1N_1}{\theta}\left(\frac{U_{n,1}\ \widetilde{g}_1}{U_{c,1}A_1}+\frac{\varepsilon-1}{\varepsilon}\right)\right) \\
&+\lambda_{f,1}\left[A_1N_1-(1-\alpha)C_1\ \widetilde{g}_1^{\eta}-\left(\frac{U^*_{c^*,1}}{U_{c,1}}\right)^{\eta}\ \widetilde{g}_1^{\eta}\alpha^*C_1^*-\frac{\theta}{2}\left(\pi_{H,1}-1\right)^2\right]+...\}\}
\end{aligned}
$$

Notice that this problem is *not time-invariant* due to the fact that the constraints at time zero lack the term $-\beta\lambda_{p,-1}(U_{c,0}\pi_{H,0}(\pi_{H,0}-1))$. For this reason we amplify the state space to introduce a new (pseudo) costate variable χ_t and define a new policy functional $\mathcal{W}(C_t,N_t,\chi_t)\equiv U(C_t,N_t)-\chi_t(U_{c,t}\pi_{H,t}(\pi_{H,t}-1))$. We then write the optimal policy plan in the following form:

Choose $\{\lambda_{p,t},\lambda_{f,t}\}_{t=0}^{\infty}$ and $\{C_t,\ \pi_{H,t},\ N_t\}_{t=0}^{\infty}$ to

$$
Max\ E_0\{\sum_{t=0}^{\infty}\beta^t\{\mathcal{W}(C_t,N_t,\chi_t) \tag{71}
$$

$$
\begin{aligned}
&+\lambda_{p,t}\left[U_{c,t}\ \pi_{H,t}\ (\pi_{H,t}-1)+\frac{U_{c,t}\ \varepsilon\ A_tN_t}{\theta}\left(\frac{U_{n,t}\ \widetilde{g}_t}{U_{c,t}A_t}+\frac{\varepsilon-1}{\varepsilon}\right)\right] \\
&+\lambda_{f,t}\left[A_tN_t-(1-\alpha)\ C_t\widetilde{g}_t^{\eta}-\left(\frac{U^*_{c^*,t}}{U_{c,t}}\right)^{\eta}\ \widetilde{g}_t^{\eta}\ \alpha^*C_t^*-\frac{\theta}{2}\left(\pi_{H,t}-1\right)^2\right]\}\}
\end{aligned}
$$

with law of motion for the new costate

$$
\chi_{t+1}=\lambda_{p,t}
$$

and initial condition

$$
\chi_0=0
$$

Following Marcet and Marimon (1999), one can show that this new maximization program is now saddle point stationary in the amplified state space $\{A_t,\ \chi_t\}$. First order conditions of this problem exactly replicate conditions (55)-(57) in the text. An exactly symmetric argument is applied to the design of the policy problem in Foreign, which will involve specifying an amplified state space $\{A_t^*,\ \chi_t^*\}$, with law of motion $\chi_{t+1}^*=\lambda_{p,t}^*$ and initial condition $\chi_0^*=0$.

D First Order Conditions of the Cooperation-Commitment Problem

First order conditions for the choice of C_t, N_t, $\pi_{H,t}$ for the Ramsey problem under Cooperation at time $t \geq 1$ read:

$$
\begin{aligned}
0 \quad = \quad & -\frac{U_{c,t}}{2} \;+\; \text{eq.}(55) & (72)\\
& +\lambda^*_{p,t}\ \widetilde{g}^*_{c,t}\ \frac{\varepsilon N^*_t U^*_{n,t}}{\theta} \;+\; \lambda^*_{f,t}\left\{-(1-\alpha^*)C^*_t\ \widetilde{g}^*_{c,t}\ -\alpha\ \widetilde{g}^*_{c,t}\left(\frac{U^*_{c,t}}{U_{c,t}}\right)^{-\eta}\ C_t\right\}\\
& +\lambda^*_{f,t}\left\{-\ \widetilde{g}^*_t\ \left(\alpha\ \left(U^*_{c,t}\right)^{-\eta}\eta U^{\eta-1}_{c,t}U_{cc,t}C_t+\alpha\left(\frac{U^*_{c,t}}{U_{c,t}}\right)^{-\eta}\right)\right\} & (73)
\end{aligned}
$$

$$0 = -\frac{U_{n,t}}{2} \;+\; \text{eq.}(56) \tag{74}$$

$$0 = \;\text{eq.}(57) \tag{75}$$

This system is completed by symmetric conditions on C^*_t, N^*_t, $\pi^*_{F,t}$.

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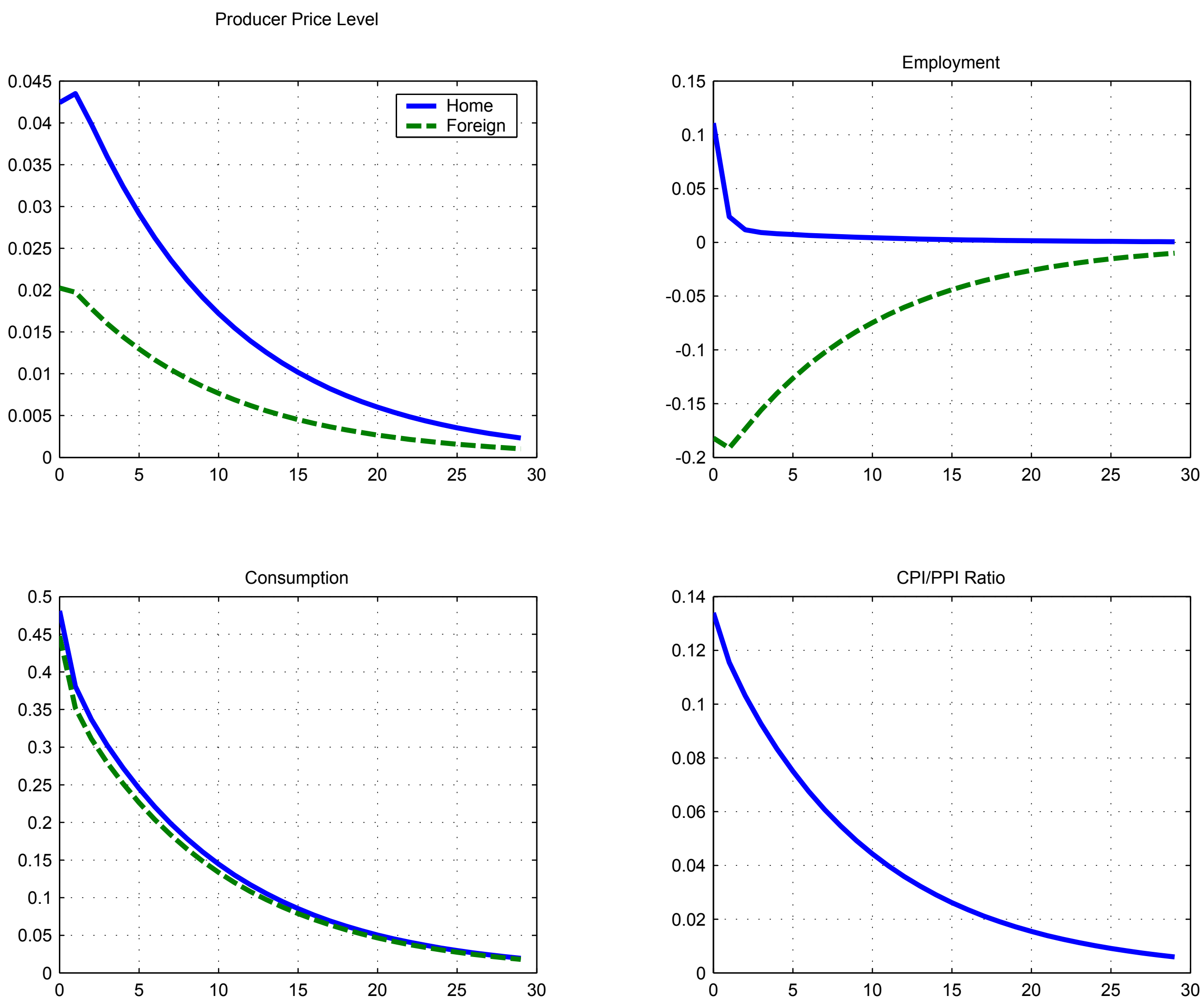

Figure 1. Impulse Responses to a Domestic Productivity Shock under Nash-Commitment

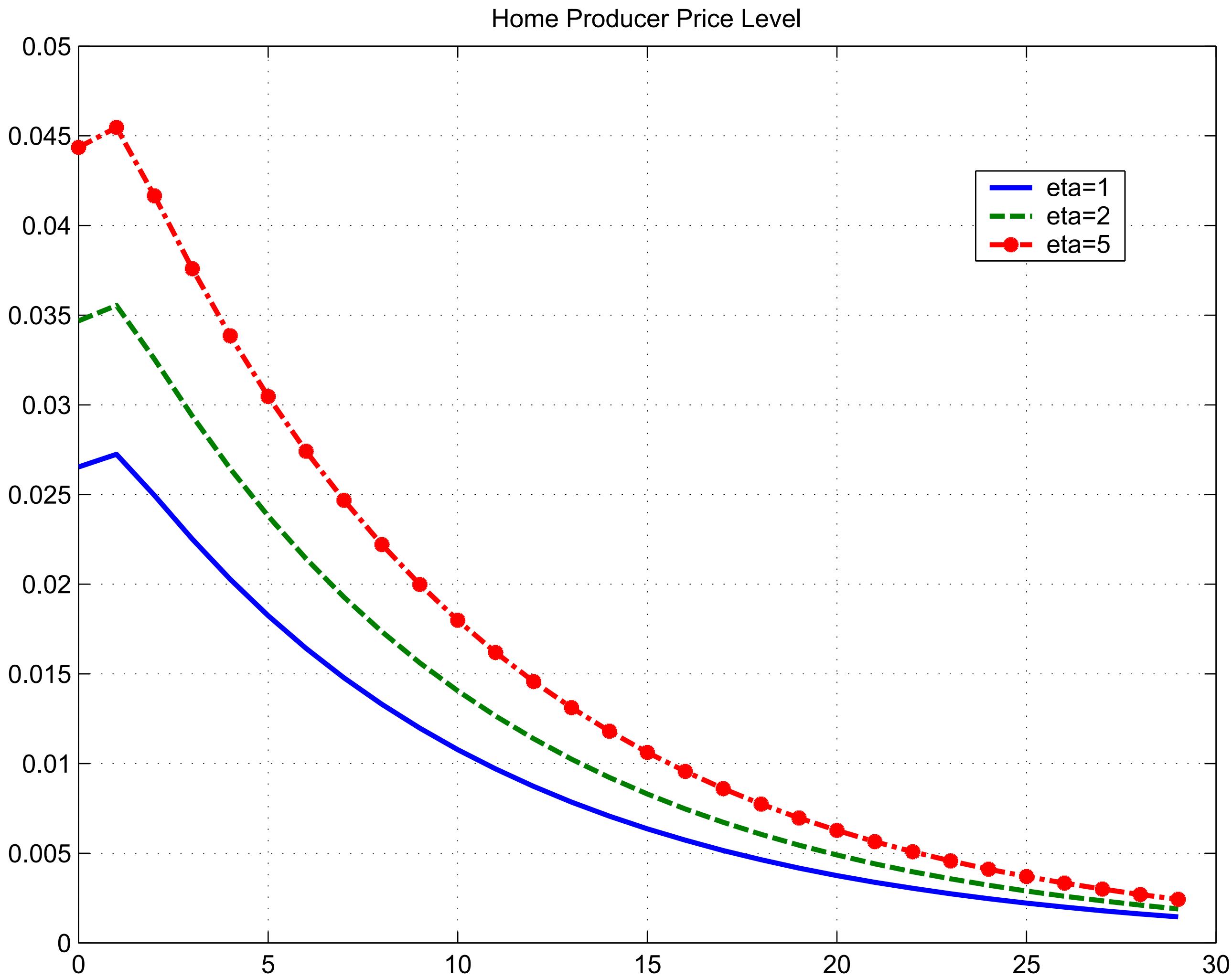

Figure 2. Effect of Varying the Elasticity of Substitution on the Price Level Response to a Productivity Shock

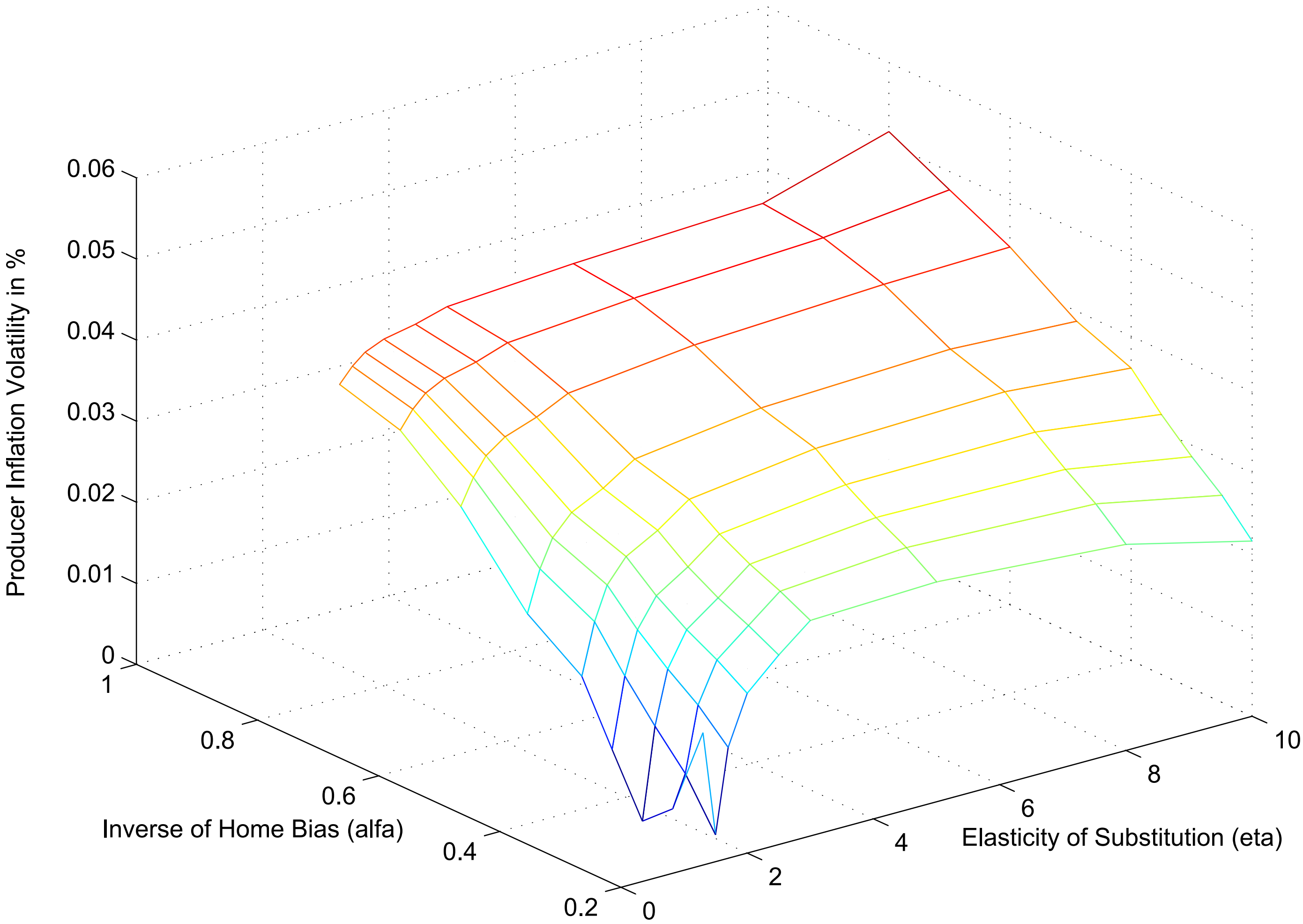

Figure 3. Effect on Nash-Optimal Inflation Volatility of Varying Home Bias and Elasticity of Substitution

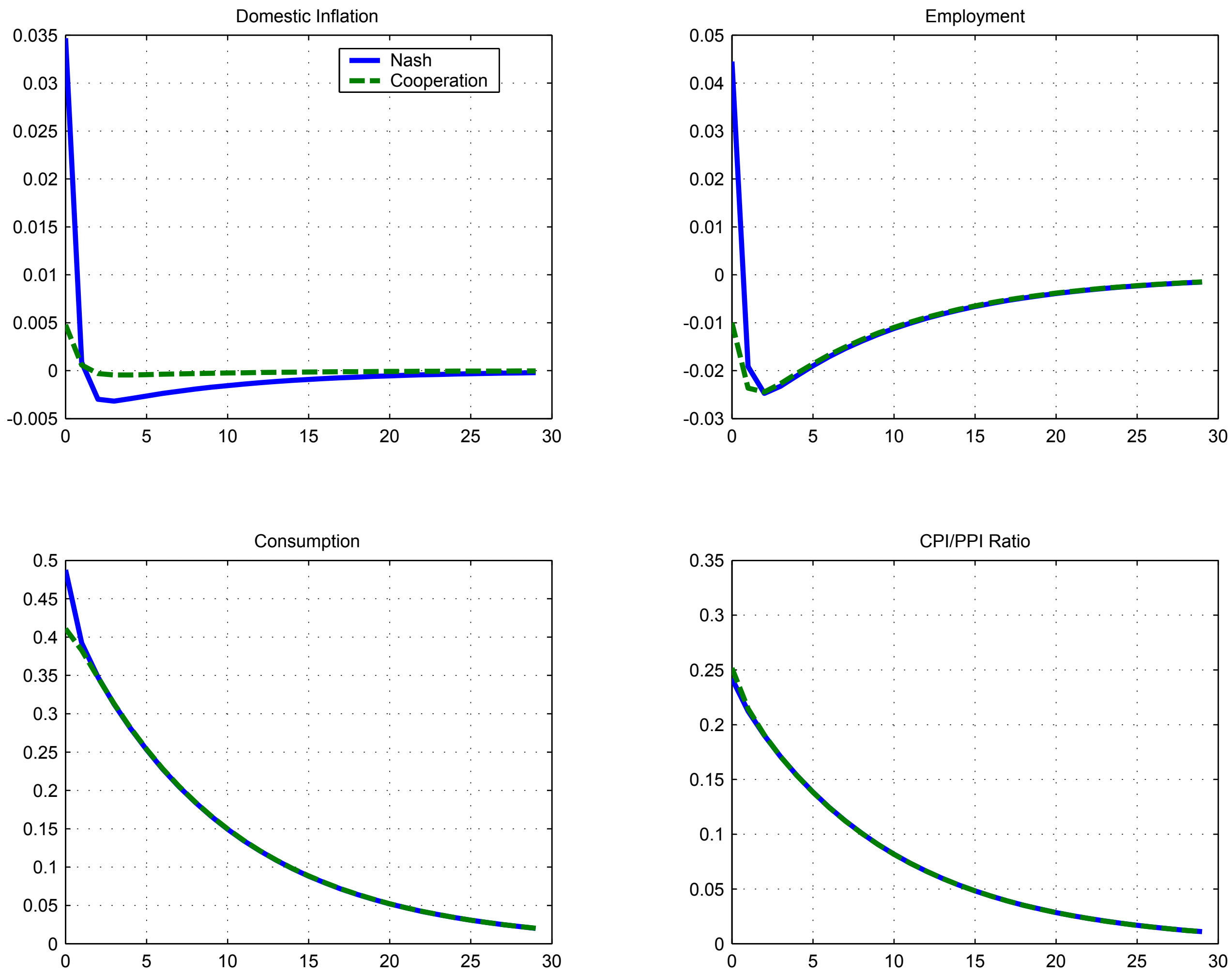

Figure 4. Impulse Responses to a Domestic Productivity Shock: Nash vs Cooperation