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Forward Looking Dynamics in Spatial CGE Modelling

Johannes Bröcker and Artem Korzhenevych

Abstract:

This paper sets up a multiregional dynamic framework by combining the optimal savings model with investment adjustment costs and the spatially disaggregated model with Dixit-Stiglitz structure in the modern sector. Because of increasing product diversity on the dynamic equilibrium path, the model belongs to the category of semi-endogenous growth models. The distinction of goods, factors, firms, and households by location, and the incorporation of trade costs in the model allow to study a variety of issues in regional and transport economics. We describe the model calibration and a taylor-made solution algorithm. The functionality is demonstrated using two illustrative examples.

Keywords: spatial dynamic CGE modeling, multiple optimizing agents, transport costs.

JEL classification: C68, D58, O41, R13

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1 Introduction

In recent years, spatial computable general equilibrium (SCGE) models have become a popular tool of regional impact analysis of policies, in particular in the area of transport economics[1] (Bröcker et al., 2010). The concept of an SCGE model implies the distinction of commodities, factors, firms, and households by location. This alone, however, is not the main distinguishing feature. Decisive for the ability of the SCGE models to bring forward new insights about the effects of policies is the incorporation of the fundamental principles of regional and spatial economics: factor mobility, economies of scale, and the presence of transport costs. These ideas are also central to modern trade and growth theory and to new economic geography. The corresponding theoretical framework is largely based on the work of Paul Krugman (1979, 1980, 1991).

A drawback of most existing SCGE models is that they are still static; dynamic extensions are rare and "recursive" (e.g., Ivanova (2007)), which means to concatenate static equilibria for each period by ad-hoc saving and investment functions. The development of consistent dynamic CGE models incorporating several or many regions has been rather slow due to substantial analytical and computational difficulties involved. In particular, a design fully consistent with the neoclassical basis of SCGE modelling would require to derive saving as well as investment behaviour from intertemporal optimization of households and firms in all locations. Furthermore, an appropriate solution method preserving the dynamic features of the model must be designed.

This paper sets up a dynamic SCGE framework by assuming households in every region to maximise a utility functional over time, taking their respective intertemporal budget constraints, prices and interest rates varying over time and space into account. Similarly, firms maximise present firm values. Adjustment of capital stocks to shocks is smoothed by assuming the existence of adjustment costs for the capital stock. The specification of the production and household sectors as well as of the goods market is close to an earlier static model (Bröcker, 1998) which has been widely applied under the brand name CGEurope in transport policy evaluation (Korzhenevych and Bröcker (2009); Bröcker et al. (2010)). Like in the earlier model, we assume monopolistic competition in Dixit-Stiglitz style in the "modern sector". Because of increasing product diversity on the dynamic equilibrium path the model belongs to the category of semi-endogenous growth models in the sense of (Jones, 2005).

In addition to distinction of goods, factors, firms, and households by location, the spatial dimension in the model comes in through the costs for goods movement depending on geography. The total trade costs for goods to be delivered from one region to another is asusmed to amount to a share in the traded value. The model is thus applicable for studying the spatial effects arising due to both, regional and transport policy measures. The way trade costs are modelled resembles - but is not identical with - the "iceberg" approach (Samuelson, 1954).

[1]An SCGE model was probably first defined by Friesz et al. (1993), who thus denoted a CGE model including an explicit representation of a transportation network.

Two important issues arise when operationalizing a perfect foresight model with multiple optimizing agents (e.g. one per region): the approximation of the infinite horizon and asset ownership.

Two broad approaches to determine a finite-time alternative for the equilibrium conditions at infinity are compatible with the nonlinear solution strategy. The first is stemming from Auerbach and Kotlikoff (1987) and suggests fixing the terminal values of some of the variables at their steady-state values. As this requirement would have to bind after certain time, there is no assurance, however, that the system will thus converge to a point close to the steady state. The majority of models in the literature use this method (e.g., Devarajan and Go (1998) or Diao and Somwaru (2000)).

The second method is based on the use of the *local stable manifold theorem* of dynamic systems theory, presented, for example by Irwin (1980). The theorem says that, in general, what is true of the linearized system (in terms of determinacy and stability of equilibrium) is true of the original nonlinear system in some open neighbourhood of the steady state. It is applied, for example, in Kehoe and Levine (1990) for the case of overlapping generations model. A useful corollary of this theorem is that the stable subspace of the linearized system is the best affine approximation to the stable manifold of the nonlinear system around the steady state. Requiring a dynamic system to reach this stable subspace is therefore an instrument of choice for obtaining a precise approximation of the model dynamics.[2] The application of this theorem for the transformation of the boundary conditions at infinity in the CGE models is computationally more demanding than the first method. However, as we show, the use of modern solvers makes the exercise feasible. We operationalize this method in the infinitely-lived agent setting.

Another major issue in a multiregional model that, however, is not given appropriate attention in the modelling literature is the ownership of capital. A precise approximation of the infinite horizon equilibrium requires the net asset positions of the households to be determined endogenously within the model (Lau et al., 2002). The existing multiregional models do not posess this property, because imposing steady-state restrictions at an arbitrary time point in the future requires ad-hoc assumptions about the value of terminal assets. In contrast, in our model the only necessary constraint is that the total value of capital stock must be always equal to the total value of asset holdings by households.

In the next Section, we present all the steps of the model setup and the accompanying derivations. The resulting mathematical problem requires a tailor-made solution algorithm, which we describe in Section 3. Section 4 studies the predictions of the model using an experimental 3-region setup. Section 5 concludes.

[2] In-between these two methods there is also an approach to impose a balanced growth constraint in the terminal period. It is applied e.g. in Bernstein et al. (1999) and Böhringer and Welsch (2004). Although applicable to a wider class of models than the first method, this approach however lacks the theoretical foundation provided by the local stable manifold theorem.

2 Model formulation

The model we are going to describe is a dynamic version of the earlier static model Bröcker (1998). Therefore, we will concentrate on the dynamic elements of the model and only shortly describe other parts. Agents of the economy are firms and households. The starting point is an open-economy version of the Ramsey optimal savings model, combined with the adjustment costs for investment framework (Abel and Blanchard, 1983). Thus both, households and firms make intertemporal decisions and have perfect foresight. As in the static model, the neoclassical structure is altered by the introduction of monopolistic competition in the "modern sector". The state is not modelled as an own sector.

The intertemporal problem is formulated in continuous time. All variables refer to one region and are functions of time. Real quantities are denoted by caligraphic letters, prices by lower-case Latins, and values (product of real quantity and price) by upper-case Latins. Exogenous parameters mostly do not have a regional index, are constant over time, and are denoted by Greek letters. Exceptions are explicitly mentioned. If not needed for understanding, the regional and time subscripts (r and t) are omitted to avoid notational clutter.

2.1 Firms

Two types of goods are distinguished in the model: local and tradable. Local goods can only be sold within the region of production, while tradables are sold everywhere in the world (whereby trade costs arise), including the own region. Identical firms located in the region produce gross output $\mathcal{M}$ by combining capital $\mathcal{K}$, effective amount of labour service $\mathcal{E}$, local goods $\mathcal{L}$, and a CES composite $\mathcal{T}$ of tradable goods coming from all regions, in a Cobb-Douglas (CD) technology,

$$\mathcal{M} = \phi \mathcal{K}^{\chi} \mathcal{E}^{\theta} \mathcal{L}^{\beta} \mathcal{T}^{\gamma}, \tag{2.1}$$

with positive elasticities, where $\chi + \theta + \beta + \gamma = 1$. The level of productivity ϕ_r may be different across regions, and no technological convergence is assumed. The regional populaton is assumed to be immobile and constant at $\bar{\mathcal{E}}_r$. Effective amount of labour input is assumed to grow with an exogenous rate of technological progress, $\tilde{\xi}$, i.e.

$$\mathcal{E}_r(t) = \bar{\mathcal{E}}_r e^{\tilde{\xi} t}. \tag{2.2}$$

Homogenous gross output serves a double purpose: first, it is one-to-one transformed into the local good (without a price mark-up), and secondly, it is used as the only input in the production of a variety of tradable goods under increasing returns to scale. The market for local goods is perfectly competitive, while monopolistic competition with free entry in Dixit-Stiglitz style prevails in the tradables market. Each firm thus produces a different variety of a tradable good. The number of varieties supplied by the region is endogenously determined by the free entry condition, which means that all firms earn zero profit in the equilibrium. By choice of units the mill price of tradable and local goods is the same and is denoted by p. A CES composite price of all tradables available in the region is denoted by g.

Firms do not only produce, they also invest. The evolution of the capital stock employed by firms is given by

$$\dot{\mathcal{K}} = \mathcal{I} - \delta\mathcal{K}, \tag{2.3}$$

where $\mathcal{I}$ is real gross investment, and δ is the depreciation rate. For the sake of simplicity, the investment good is the same as the consumption bundle, a CD composite of non-tradables and tradables, with expenditure shares ϵ and $1-\epsilon$, respectively. However, investment is costly. Following the literature, we assume quadratic adjustment costs,

$$J = h\mathcal{I}\left(1 + \frac{\zeta}{2}\frac{\mathcal{I}}{\mathcal{K}}\right). \tag{2.4}$$

J is nominal investment cost, h is the price of the consumption bundle, and ζ is the adjustment cost parameter. By introducing adjustment costs of investment, we rule out an implausible outcome of the basic open-economy version of the Ramsey model, where the adjustment of capital stock is done through an instantaneous jump to other locations. The larger is ζ, the more sluggish investment is going to respond to changes in capital returns.

The existence of investment costs implies that the stock of capital in a region has a unit market price q that in general differs from the price of the investment good, by which the stock is built up. Taking the stock price at any point in time as given, firms invest until the marginal cost of investment equals its marginal return q, leading to the investment function

$$\frac{\mathcal{I}}{\mathcal{K}} = \frac{q/h - 1}{\zeta}. \tag{2.5}$$

q/h is called "Tobin's Q" in the literature.

For using the capital stock, firms have to pay a rental rate v to their shareholders that is equal to the marginal value product of capital. We assume a perfect completely integrated asset market, such that nominal capital stocks earn the same nominal interest rate ρ everywhere, constant by choice of dynamic numéraire. Hence, a no-arbitrage condition has to hold, namely:

$$\rho q = \dot{q} - \delta q + v, \tag{2.6}$$

with rental rate

$$v = \chi M/\mathcal{K} + h\frac{\zeta}{2}\left(\frac{\mathcal{I}}{\mathcal{K}}\right)^2, \tag{2.7}$$

where M is nominal output, i.e. $M = p\mathcal{M}$. The first additive term on the right-hand side of (2.7) is the marginal value product in production of goods. The other is the marginal investment cost reduction brought about by an extra unit of capital installed.

The evolution of the capital stock is subject to two boundary conditions. First, capital stock is inherited and thus given at t=0,

$$\mathcal{K}(0) = \bar{\mathcal{K}}. \tag{2.8}$$

Second, the transversality condition requires that the market value of a firms's capital stock must converge to zero, in present values,

$$\lim_{t\to\infty} \mathcal{K}(t)q(t)e^{-\rho t} = 0. \tag{2.9}$$

2.2 Consumers

The aggregate consumer in the region maximizes discounted CIES utility over an infinite time horizon,

$$\max_{\mathcal{C}} \int_0^\infty \frac{\mathcal{C}(t)^{1-1/\varphi} - 1}{1 - 1/\varphi} e^{-\rho t} dt$$

subject to the flow budget constraint

$$\dot{A} = \theta M + \rho A - C, \tag{2.10}$$

with real consumption $\mathcal{C}$ and nominal consumption $C = h\mathcal{C}$. Real consumption is a CD composite of loca land tradable goods with respective expenditure shares ϵ and $1 - \epsilon$. θM is thus wage income. $\varphi > 0$ is the intertemporal elasticity of substitution. We choose the time path of the numéraire such that the nominal interest rate ρ equals the rate of time preference. The real interest rate in terms of the consumption composite is thus $r^c = \rho - \hat{h}$, hats denoting growth rates.

Solving household's optimization problem yields the optimality condition for consumption,

$$\hat{\mathcal{C}} = \varphi(r^c - \rho), \tag{2.11}$$

which is known as Keynes-Ramsey rule. Substituting for r^c yields

$$\hat{\mathcal{C}} = -\varphi\hat{h}. \tag{2.12}$$

The dynamics of the consumption is also subject to two boundary conditions. At $t = 0$, households inherit predefined shares of the initial capital stocks,

$$A_r(0) = \sum_s \omega_{rs} q_s(0)\mathcal{K}_s(0), \tag{2.13}$$

where parameter ω_{rs} gives the share of region r in the property of capital stock in region s. Parameter restrictions $\sum_s \omega_{rs} = 1\ \forall r$ guarantee that, at $t = 0$, the asset total in the entire economy equals the total value of capital stocks. Interestingly, one can show (see Appendix A.1) that this condition automatically holds for all times, if it holds for one point in time. This is Walras' law. At any time point, however, the value of assets held by the households in a region is in general not restricted by the value of regional capital stock.

Finally, the transversality condition says that assets must have zero present value in the long run,

$$\lim_{t\to\infty} A(t)e^{-\rho t} = 0. \tag{2.14}$$

2.3 Trade

Consumers and firms buy a CES composite of tradable varieties produced everywhere and sold under condition of Dixit-Stiglitz monopolistic competition. The CES bundle of tradables is used as a production input, and as a component of the composite consumption and investment good. As already mentioned above, we assume investments to be composed of local and tradable goods just in the same way as consumption. The corresponding price index h is given by

$$h = (p)^{\epsilon}(g)^{1-\epsilon} \tag{2.15}$$

The tradables are also used to produce the transport service. Transport cost is added to the sales price p_r leading to the inclusive price $p_r\tau_{rs}$ in destination s for a good coming from origin r. It is assumed that nominal transport cost for a given origin-destination pair is a fixed share of the nominal value of the good, valued at mill price. We call this the "modified iceberg assumption". It differs from the standard iceberg assumption in that we assume the CES composite - not the variety itself - to be used for the trnasport service of an individual variety. This is more plausible than the often criticised (McCallum, 1995) iceberg assumption, though the results differ only slightly.

The value of tradables supply equals the value of gross output less the total value of demand for local goods (that is the local goods part of intermediate demand, investment demand, and consumption demand):

$$S = (1-\beta)M - \epsilon(C+J) \tag{2.16}$$

The value of intermediate, investment, and consumption demand for tradables (valued inclusive of transport costs) is equal to:

$$D = \gamma M + (1-\epsilon)(C+J) \tag{2.17}$$

The CES form of demand implies a composite price of tradables in the destination region s

$$g_s = \psi \left(\sum_{r=1}^{n} S_r p_r^{-\sigma} \tau_{rs}^{1-\sigma} \right)^{1/(1-\sigma)}, \tag{2.18}$$

with elasticity of substitution $\sigma > 1$ common for all regions. ψ is an arbitrary scaler. The choice does not affect any result, but it offers a degree of freedom to choose the average level of prices. From (2.18) and the above considerations follows the trade equation

$$T_{rs} = \frac{S_r (p_r \tau_{rs})^{-\sigma}}{\sum_i S_i (p_i \tau_{is})^{-\sigma}} D_s. \tag{2.19}$$

2.4 Equilibrium

Due to CD technology and perfect competition on the input markets, firms with sales value M spend θM for labour, βM for non-tradables, and γM for composites of tradables. The remaining part χM goes to the shareholders as remuneration for the service of capital in goods production. The formula for gross output can thus be rewritten as

$$p = \mu w^{\theta}(\chi M/\mathcal{K})^{\chi}(p)^{\beta}(g)^{\gamma}, \tag{2.20}$$

with the wage rate w and inverse productivity parameter $\mu_r = (\phi_r \theta^{\theta} \chi^{\chi} \beta^{\beta} \gamma^{\gamma})^{-1}$. Equilibrium on the labour market requires

$$\theta M = w\mathcal{E}. \tag{2.21}$$

Finally, equilibrium in the tradables market requires the value of supply, S_r in the region to equal the value of demand of all regions for tradables from region r, i.e.

$$S_r = \sum_{s=1}^{n} T_{rs}. \tag{2.22}$$

Equations (2.2)-(2.7), (2.10)-(2.12), and (2.15)-(2.22) give us 16 equations to determine the 16 unknowns $\mathcal{E}$, $\dot{\mathcal{K}}$, J, $\mathcal{I}$, $\dot{q}$, v, $\dot{A}$, $\dot{C}$, h, S, D, g, T, $\mathcal{M}$, w, and p. Four of these equations ((2.3), (2.6), (2.10), and (2.12)) are differential equations to determine $\dot{\mathcal{K}}$, $\dot{q}$ $\dot{A}$, and $\dot{C}$, respectively, the others are algebraic to determine the remaining 12 unknowns. The price level is defined by setting a GDP-weighted average of regional price indices at $t = 0$ equal to 1. According to Walras' law, one of the market clearing conditions in (2.22) can then be dropped for $t = 0$.

We thus have a differential-algebraic equation (DAE) system to find the time paths of all endogenous variables. The differential equations have to be augmented by the boundary conditions for the dynamic variables. These are given by the initial values of capital stock (2.8) and of households' assets (2.13), as well as by the transversality conditions (2.9) and (2.14).

The system has just the right number of equations and boundary conditions to determine the equilibrium. To be sure, this is just a hint at a good chance to be able to solve the model, no existence proof. In particular, it is not obvious whether the transversality conditions make sure that only one, not many trajectories would converge in the way required. We shall return to this point later.

A question we would like to address now is whether this dynamic system possesses a steady state, in which all underlying variables grow at constant rates. If that is the case, the next step would be to solve for this steady state.

2.5 Steady state growth rates

In the multiregional setup, the steady state should be a situation, in which all variables grow at constant rates that are homogeneous across regions. We can show that such a steady state for our model in fact exists by setting up a system of linear relations between the growth rates that have to hold in this equilibrium. Most of them are derived by taking log-derivatives of the model equations and evaluating them at the (hypothetical) steady state. The detailed derivations are contained in Appendix A.2. The resulting system is (the stars denote the steady-state values):

$$\begin{cases} \hat{g}^* = \dfrac{1}{1-\sigma}\hat{S}^* - \dfrac{\sigma}{1-\sigma}\hat{p}^*; \\ \hat{\mathcal{K}}^* = (\eta+1)\hat{p}^* - \eta\hat{g}^* + \alpha\hat{\mathcal{K}}^* + (1-\alpha)\tilde{\xi} - \hat{h}^*; \\ \hat{q}^* = \hat{h}^*; \\ \hat{\mathcal{C}}^* = \hat{\mathcal{K}}^*; \\ \hat{S}^* = \hat{\mathcal{C}}^* + \hat{h}^*; \\ \hat{h}^* = \epsilon\hat{p}^* + (1-\epsilon)\hat{g}^*; \\ \hat{\mathcal{C}}^* = -\phi\hat{h}^*. \end{cases} \tag{2.23}$$

Above, we introduced $\alpha = \frac{\chi}{\chi+\theta}$ and $\eta = \frac{\gamma}{\chi+\theta}$. This linear equation system can be uniquely solved for the growth rates of the seven variables involved. In particular, one gets $\hat{\mathcal{K}} = \hat{\mathcal{C}} = \hat{\mathcal{I}} = \xi$ and $\hat{h} = \hat{q} = -\xi/\varphi$ with

$$\xi = \frac{\theta(\sigma-\epsilon)}{\sigma\theta + \chi\epsilon + \beta - 1} \cdot \tilde{\xi} \tag{2.24}$$

being the rate of real growth of consumption, capital, and investment. Moreover, ξ/φ is the rate of deflation, and the real interest rate is given by $\rho + \xi/\varphi$. All nominal values, such as M, C or A, grow at the rate $(1 - 1/\varphi)\xi$. The nominal wage per worker grows at this rate as well. The inequalities

$$\sigma\theta + \chi\epsilon + \beta - 1 < \sigma\theta + \chi + \beta - 1 = \sigma\theta - \theta - \gamma < \theta(\sigma-\epsilon) > 0$$

prove the denominator in (2.24)) to be less than the numerator. Thus, the fraction is larger than one if the denominator is positive. The latter is guaranteed with any sensible choice of parameters. $\theta\sigma > 1$ is already sufficient. The labour share in the output value is usually in the order of 1/3, and σ is suggested to be considerably larger than 3, so that the condition is satisfied. Note that if this condition fails to hold and the denominator approaches zero, the factor amplifying the rate of Harrod neutral technical growth, how small ever $\tilde{\xi}$ may be, would let growth explode. We can thus interpret this restriction as a kind of "no-black-hole" condition.

Real steady-state growth is faster in this economy than the rate of Harrod neutral technical progress, unlike the standard Solow model, where both are the same.[3] The deviation is due to the fact that in our model there is an aggregate economies of scale effect. If the economy grows, product diversity increases, which makes production and investment more productive and consumers more satisfied. The factor amplifying the rate of Harrod neutral technical progress gets larger, if σ gets smaller or the share of tradables in production or consumption and investment gets larger.

[3]However, note that $\lim_{\sigma\to\infty}\xi = \tilde{\xi}$. That is, if we drop the assumption of imperfect substitution between the tradable goods, we will get the familiar result of the classical Ramsey model.

The existence of the scale effect allows us to link our model to the literature on semi-endogenous growth. As Li (2000) puts it, semi-endogenous growth means that (i) technological change itself is endogenous, but (ii) long-run growth is pinned down by exogenous parameters, the consequence being that straightforward policies do not affect the long-run growth rate. In our case, endogenous technological change has the scale economies in the tradables sector as its source. This is basically the same mechanism as in the famous model by Romer (1990), an important difference being however the absence of an explicit R&D sector in our model. In contrast, the expansion of product diversity is here driven by the economies of variety on the consumer and producer side. The expression in (2.24) is similar to the outcome of the models with "weak scale effects", like Jones (1995), Kortum (1997), and Segerstrom (1998) in that the productivity growth is the growth of effective labour amplified by a factor larger than one.

Now knowing the steady state rates of growth of all variables, we introduce new variables called *stationary transforms* of the original ones. For example the transformed asset stock $\tilde{A}$ is defined as

$$\tilde{A}(t) = A(t)\exp(-(1 - 1/\varphi)\xi t).$$

As A grows at the steady state rate $(1 - 1/\varphi)\xi$, $\tilde{A}$ is stationary in steady state. We thus replace A in all equations with $\tilde{A}\exp((1-1/\varphi)\xi t)$, and do the same with the other variables. In the non-dynamic equations of the model this just leads to a replacement of all variables by their stationary transforms, because the exponential expressions on both sides cancel. We thus dispense from rewriting all equations with tilde symbols; from now on variables (including those plotted in Section 4) are rather understood as being the stationary transforms. Only the dynamic equations change. As A now denotes the stationary transform of the asset value rather than the asset value itself, equation (2.10) becomes

$$\dot{A} = \theta M - C + \Big(\rho - \xi(1 - 1/\varphi)\Big)A. \tag{2.10'}$$

Similarly, capital grows at rate ξ in steady state. As $\mathcal{K}$ now denotes the stationary transform of capital, (2.3) becomes

$$\dot{\mathcal{K}} = \mathcal{I} - (\delta + \xi)\mathcal{K}. \tag{2.3'}$$

Finally, as the non-transformed share price grows at the steady state rate $-\xi/\varphi$, (2.6) becomes

$$\dot{q} = (\rho + \delta + \xi/\varphi)q - v. \tag{2.6'}$$

The dynamic equations all come in the standard form of an explicit differential equation, with time derivatives ("dotted variables") on the left hand side and levels on the right hand side, with one exception, equation (2.12) having growth rates on both sides. Integrating it yields

$$C = mh^{1-\varphi}. \tag{2.12'}$$

m is an unknown that has to make sure that, given the start values of the households' assets $A(0)$, the transversality condition (2.14) holds. This unknown in other words scales the regional consumption level in such a way that the intertemporal budget constraint is not violated. After this modification there is one differential equation less per region, but an additional vector of static unknowns, one per region, gathered in vector $\mathfrak{m} = \{m_r\}$.

Gathering the variables A, $\mathcal{K}$, q in the vector x and all other (except $\mathfrak{m}$) in the vector y, the system reads

$$\dot{x} = \tilde{f}(x, y), \tag{2.25}$$

$$g(x, y, \mathfrak{m}) = 0. \tag{2.26}$$

$\tilde{f}$ denotes the remaining three differential equations (after stationary transformation) (2.3'), (2.6'), and (2.10') determining $\dot{\mathcal{K}}$, $\dot{q}$, and $\dot{A}$. g is the system of algebraic equations determining y, given x and $\mathfrak{m}$.

3 Solution algorithm

Numerically (implicitly) solving (2.26) for y and inserting into (2.25) yields

$$\dot{x} = f(x, \mathfrak{m}) = \tilde{f}(x, y(x, \mathfrak{m})). \tag{3.1}$$

$x \in \mathbb{R}^{3n}$ is the state vector with components A, $\mathcal{K}$, and q, each of length n. As $\mathfrak{m}$ has also length n, there are $4n$ degrees of freedom. We have boundary conditions at $t = 0$,

$$b(x(0), \mathfrak{m}) = 0, \tag{3.2}$$

fixing initial values for A and $\mathcal{K}$, thus closing $2n$ degrees of freedom. $2n$ degrees of freedom are left to be closed by transversality conditions (2.9) and (2.14).

The mathematical problem is thus a two-point boundary value problem (BVP), with the extra difficulty that one of the boundary points is at infinity. We proceed by first transforming the boundary condition at infinity into one at a finite horizon, and then solving the resulting nonlinear two-point boundary value problem by a collocation method.

The first step is to linearize the original system around the steady state, and then to fix a finite horizon $\bar{t}$ far enough in the future, such that the linearized version of the model can be used as a good approximation to the original system. From that time point on, we will require the system to move along the stable manifold of the linear approximation towards the steady state.

The linearized system is

$$\dot{x} = f_x(x^*, \mathfrak{m})(x - x^*), \tag{3.3}$$

where $x^*(\mathfrak{m})$ is the steady state vector solving

$$f(x, \mathfrak{m}) = 0.$$

x^* thus depends on $\mathfrak{m}$, which we write explicitly when necessary. f_x is the jacobian of f. Due to the stationary transformation the steady state vector is actually stationary. (3.3) has the general solution (assuming full rank of the eigenvector matrix)

$$x(t) = x^*(\mathfrak{m}) + \Big(V \cdot \mathrm{diag}(\exp(t\lambda_i))\Big)\mathfrak{w}.$$

V is the $(3n \times 3n)$-matrix of eigenvectors of f_x, $\lambda_1, \ldots, \lambda_{3n}$ are the corresponding eigenvalues, and $\mathrm{diag}(\exp(t\lambda_i))$ is a diagonal matrix with components $\exp(t\lambda_i)$ on the main diagonal. $\mathfrak{w} \in \mathbb{R}^{3n}$ is a yet undetermined vector of weights. For saddle-path stability, these weights must be zero for all components that have positive real parts of their respective eigenvalues.

It is thus natural to force the system to attain a point $x(\bar{t})$ that is on a solution path of the linearised system converging to the steady state, that lies in other words on the so-called stable manifold of the linearised system. The stable manifold is the linear subspace spanned by the stable columns of V. The stable columns are those that correspond to eigenvalues with non-positive real parts. More formally, partitioning V into its stable und unstable columns V^s and V^u such that $V = [V^s\ V^u]$, we want there to be weights $\mathfrak{v}^s$ such that $x(\bar{t}) - x^*(\mathfrak{m}) = V^s\mathfrak{v}^s$. This is to say that in the solution of the linear equation system

$$x(\bar{t}) - x^*(\mathfrak{m}) = V \begin{pmatrix} \mathfrak{v}^s \\ \mathfrak{v}^u \end{pmatrix}$$

we want to have $\mathfrak{v}^u = 0$. This amounts to the restriction

$$(V^{-1})_u(x(\bar{t}) - x^*(\mathfrak{m})) = 0.$$

$(V^{-1})_u$ must contain rows $n+1$ to $3n$ of the inverse V^{-1}. This delivers $2n$ restrictions which, jointly with the $2n$ initial boundary conditions, close the $4n$ degrees of freedom. If the number of rows in $(V^{-1})_u$ turned out to be more than $2n$, the system would be globally unstable, and if it were less than $2n$, the system would be underidentified, with undetermined properties. We have to rely upon a computational argument (successful simulations in the next Section) to demonstrate that the number of positive eigenvalues is exactly $2n$.

V may be complex. Complex columns come in conjugate pairs. If this happens to be the case, the respective columns of a complex conjugate pair must be replaced with two real columns formed by the real and the imaginary parts of one of these two complex columns. Note that it does not matter which to take because they are the same, except for the sign of the imaginary part which will only affect the sign of the respective weight.

To summarize, we now have the two-point boundary value problem

$$\left\{ \begin{array}{rcl} \dot{x} & = & f(x, \mathfrak{m}) \\ b(x(0), \mathfrak{m}) & = & 0 \\ (V^{-1})_u(x(\bar{t}) - x^*(\mathfrak{m})) & = & 0. \end{array} \right.$$

For the nonlinear solution we use a collocation method to solve two-point boundary value problems, that are allowed to depend on unknown parameters ($\mathfrak{m}$ in our case). We use the MATLAB code `bvp6c` (Hale and Moore, 2008), a refinement of MATLAB's original code `b4p4c` (Kierzenka and Shampine., 2001). This collocation method finds a functional (piecewise cubic polynomial) approximation of the time path of our system, which exactly satisfies the differential equations at some chosen points in $[0, \bar{t}]$ (so-called mesh points). The solver automatically chooses the mesh points, and extends or reduces its number, depending on the size of the residual. This method provides a uniform prescribed accuracy throughout the computational interval. Moreover, this method allows to solve for the endogenous time-invariant parameter vector $\mathfrak{m}$, and to handle discontinous changes in the values of exogenous parameters (as in the case of a time lag between policy announcement and realization).

4 Applications

In the following we describe the calibration of a test three-region model and the results of two simulation exercises. The first exersice involves a transport cost shock between two regions. The second represents a shock on capital stocks.

4.1 Calibration

Table 1 lists the assumed values of elasticities and share parameters. Cobb-Douglas parameters are calculated from the aggregate GTAP data. The default value of the parameter of the adjustment cost function ζ is chosen taking account of the implications for the value of the TOBIN's Q and the convergence speed. The econometric estimates of the TOBIN's Q (e.g. Blanchard et al. (1993)) usually do not exceed 1.5, while the plausible speed of convergence for the capital stock should not be higher than 0.05 per year. With chosen parameterization, both criteria are fulfilled for our test model. Furthermore, Barro and Sala-i Martin (1995, p.122-125) point at the possibility of TOBIN's Q reaching implausibly high transitional values if chosen adjustment cost parameter is too large. We will verify whether this happens in the next Sections.

Parameter	χ	θ	γ	β	ϵ	δ	σ	φ	ζ
Value	0.19	0.24	0.29	0.28	0.60	0.05	12.0	0.80	6.0

Table 1: Assumed parameter values for the test model

Iceberg costs τ_{rs} are in our test model set at 1.2 (20% trade cost mark-up) for the interregional flows, and at 1.05 for intraregional flows. The scaling factor ψ is set at unity for all regions. The rate of growth of the efficient labour stock $\tilde{\xi}$ is calibrated according to (2.24), where the real growth rate of consumption ξ is set at 2% per year. The rate of time preference, ρ, is calibrated from (2.11), assuming the real interest rate of 5% per year.

It remains to specify region-specific prodcutivity parameters and initial values for assets and capital. This is done by inserting data characterizing the base year (benchmark) into the model equations.

The majority of dynamic models are designed to initially start in the steady state. For the implementation of an alternative approach - to start outside of the steady state (which may be regarded as more plausible for real-world applications) - additional data requirements arise. In particular, the values of households' assets and the capital stocks must be known. This information is usually not avaialble. Although our method is not bound to starting from the steady state, we will thus describe the most frequently used calibration strategy.

All variables in the equations to follow have to be thought of as evaluated at the initial steady state. We omit the notation with explicit time indices and stars, however, for the clarity of exposition.

First, we can use the model equations to express the value of assets in a convenient form. Specifically, equations (2.16)-(2.17) yield

$$D - S = C + J - (\theta + \chi)M. \tag{4.1}$$

Next, the budget constraint (2.10), evaluated in the steady state, suggests

$$\theta M - C + \Big(\rho - \xi(1 - 1/\varphi)\Big)A. \tag{4.2}$$

At the time point $t = 0$, $(D - S)$ is the benchmark regional trade deficit. For any needed base year it can be computed from the trade data, which is regularly available if the regions are countries.

Combining (4.1) and (4.2), we get

$$\bar{A} = \frac{1}{\rho - \xi(1 - 1/\varphi)}(D - S - J + \chi M). \tag{4.3}$$

We assume that all regions initially hold shares in a perfectly deversified global portfolio of assets, that is, $\omega_{rs} = \bar{A}_r / \sum_i \bar{A}_i \ \ \forall r, s$.

The productivity parameter (time-invariant) is determined from the requirement that, at $t = 0$, the base-year regional GDP value is reproduced, which is equal to the primary factors' income. No technological convergence is assumed. Based on (2.20)-(2.21), we have

$$M = \frac{GDP}{\theta + \chi} = \tilde{\mu}(\mathcal{K})^{\alpha}(p)^{\eta+1}(g)^{-\eta}, \tag{4.4}$$

where $\tilde{\mu}_r = \bar{\mathcal{E}}_r^{1-\alpha}(\phi_r \beta^{\beta}\gamma^{\gamma})^{\frac{1}{\theta+\chi}}$, $\alpha = \frac{\chi}{\chi+\theta}$, $\eta = \frac{\gamma}{\chi+\theta}$.

Inserting (4.3)-(4.4) into our DAE system evaluated at the initial steady state, we then can solve for the initial values of all variables, as well as for regional productivity parameter ϕ. The solution vector can be used to feed the BVP solver with initial values.

4.2 Effects of a transport cost shock

We demonstrate the functioning of the model using a test setup with three symmetric regions. We assume all regions to have initially equal values of $GDP = 1$, and zero trade deficits. As a consequence, each region initially holds 1/3 of global assets. The matrix of trade cost mark-ups is assumed to have the following symmetric form:

$$\tau = \begin{pmatrix} 1.05 & 1.2 & 1.2 \\ 1.2 & 1.05 & 1.2 \\ 1.2 & 1.2 & 1.05 \end{pmatrix}.$$

The first experiment that we perform is the simulation of an infrastructure improvement, leading to a 50% trade cost reduction between regions 1 and 2 in both directions (the corresponding values of τ_{rs} thus reducing to 1.10). We only look at the phenomena arising after the project completion, thus ignoring the effects during the construction phase. The infrastructure improvement is announced and realized at time $t = 0$.

Due to the shock, the spatial symmetry is destroyed. We can use the diagrams of the time paths of stationary transformed variables to demonstrate the subsequent adjustment process predicted by our model. The (equal) responses of the variables in the two directly affected regions are plotted using the solid lines, while the response of the indirectly affected third region is plotted using the dashed lines. Moreover, we display the post-shock steady state positions of the respective variables using the solid and dashed straight lines. The pre-shock steady state values (common to all regions) are displayed using the dotted lines.

-Figure 1 about here-

In Figure 1, the results for consumption and capital stock are displayed. The adjustment process of consumption and capital stock is characterized by smooth convergence towards the steady state. As should be expected, the level of consumption jumps at $t = 0$ (when the shock occurs), after which the convergence process begins. In contrast, the time path of capital stock starts at the predefined level $\bar{\mathcal{K}}$, which in our test setup is common to all regions. Under the chosen parameterization, it takes about 20 years for consumption and capital to cover half of the distance to the new steady state. Main parameters that control the speed of convergence are the elasticity of intertemporal substitution φ and the parameter of adjustment cost function ζ. The results of the sensitivity analysis that we performed with respect to the values of φ and ζ (not reported here) are straightforward. The speed of convergence is quite sensitive to large changes in these parameters, which suggests that a search for the most appropriate estimates for the study area under investigation (for the case of real policy analysis) is desirable.

The simulated reduction of trade costs allows all regions to increase consumption in the new steady state. However, immediately after the shock, the consumption in the directly and indirectly affected regions jumps in the opposite directions. In the case of directly affected regions 1 and 2, the biggest effect comes through the reduced prices of tradable goods. For region 3, the initial negative effect on consumption is due to the expectation of lower income flows in the future (as more demand shifts towards the other two regions and more output is produced there). In course of time, the expanding variety of products available from regions 1 and 2 as well as higher returns from foreign assets lead to some gains for region 3, and materialize in the higher long-run equilibrium level of consumption there.

In regions 1 and 2, the expansion in consumer demand does not come at the cost of initially lower investment. The increased demand for the products of these regions leads to higher production activity and to additional capital accumulation, which is partly financed by region 3 through an increase in foreign asset holdings. This increase of asset position in region 3 is accompanied by a deficit in trade account. Own rate of investment in region 3, in contrast, goes down, and also its output shrinks. In fact, the decrease in relative attractiveness of region 3 for investment is big enough to make the new steady-state level of capital stock fall below its pre-shock level.

-Figure 2 about here-

The effects on prices are displayed in Figure 2. As expected, the prices of tradable goods in the directly affected regions go down the most. The decline is however not too large in relative terms, because trade costs are only a small fraction of trade value. In the indirectly affected region, the largest price change concerns the local goods, the demand for which from the side of local consumers and producers declines as cheaper products from the other two regions become available.

The last panel in Figure 2 displays the smooth adjustment of Tobin's Q. At the steady state, Tobin's Q is constant and uniform across regions (see equations (2.5) and (2.3')). The initial divergence in the regional values of the Tobin's Q illustrates the change in the relative attractiveness of the regions for investment, with the region untouched by the infrastructure improvement being a clear loser. The graph also suggests that the attained levels of Tobin's Q are plausible.

An issue of interest is the measurement of welfare impact. We use two approaches. First, we compare the before- and after-shock steady states, using the indicators of real GDP and real consumption (equal to the consumption index $\mathcal{C}$). Second, we would like to calculate a measure of dynamic impact of the policy on the consumers. For this, we calculate the relative equivalent variation in consumption (denoted R) for each region, which is defined by

$$\int_{t=0}^{\infty} \frac{\left(\mathcal{C}_t^0(1+\frac{R}{100})\right)^{1-1/\varphi}-1}{1-1/\varphi}\exp^{-\rho t}dt \quad = \quad \int_{t=0}^{\infty} \frac{(\mathcal{C}_t^1)^{1-1/\varphi}-1}{1-1/\varphi}\exp^{-\rho t}dt \qquad (4.5)$$

Thus, this measure gives the percentage change in the time path of pre-shock consumption $\{\mathcal{C}_t^0\}$, such that the discounted utility flows before and after the shock are equalized.

The results are in Table 2. The difference between the "static" and "dynamic" consumption-based welfare measures has a direct relation to the convergence process. The larger the adjustment of consumption during the initial jump, and the faster the convergence, the more close will the two measures be to each other. In our case, the convergence speed is quite moderate, and this leads to substantially different results from the two methods. The new steady state consumption levels for both regions lie above the old steady state. However,

Indicators	Real GDP change, % (comparative steady state)	Real consumption change, % (comparative steady state)	Equivalent variation in consumption, %
Regions 1 & 2	3.96	4.43	3.45
Region 3	-0.54	0.49	-0.40

Table 2: Effects of the transport cost shock: GDP and welfare

the initial jump in the directly and indirectly affected regions goes in different directions. The consumption in the indirectly affected region initially falls short of the old steady-state level. Because the future consumption is weighted less in the lifetime utility, the equivalent variation measure thus turns out to be slightly negative.

The difference between the real GDP and real consumption effects in the comparative steady state calculations is due to different rates of investment, and the changes in asset ownership. The transport cost reduction makes the third region relatively less attractive for investment, and it ends up having lower capital stock than in the pre-shock situation, which also reduces the output and thus the payments to the primary factors. In case of the households, the reduction of wage income is in course of time compensated by the additional income from the assets purchased in the other two regions.

The welfare results are somewhat different if we assume that initially households own all capital stock of the domestic region, and not a share in the global portfolio. In this case the households in the region without infrastructure improvement suffer the whole extent of falling domestic capital prices immediately after the shock, which lowers their income. In turn, the households in the directly affected regions enjoy the full extent of their property appreciation. The welfare impact is then slightly more positive in Regions 1 & 2, $R_1 = R_2 = 3.63\%$, and slightly more negative in Region 3, $R_3 = -0.75\%$.

4.3 Effects of a shock on the capital stock

As a second illustration of model performance, we simulate a 20% drop in the initial capital stock in region 1. This may be regarded as not the most realistic case, but it is quite a standard exercise to demonstrate the dynamic features of the model. The shock is rather big, so the use of a solution method that is more sophisticated than simple linearization approach is well justified. The response of key variables is depicted in Figures 3 and 4. The adjustment process is characterized by smooth convergence of all variables to new respective steady states. Note that the solid lines now refer to region 1 and dashed lines to regions 2 and 3.

-Figure 3 about here-

-Figure 4 about here-

The shock results in capital stock becoming a more scarce resource in region 1. With less capital available, the marginal productivity of labour falls, and so does wage income. Local consumption drops and stays permanently below the old steady state level. On the other hand, the marginal return on investment in region 1, as represented by the TOBIN's Q, jumps up and stays high for a long time. This gives the indirectly affected regions 2 and 3 an incentive to invest in capital of region 1. In the short term, this stategy reduces capital levels in the indirectly affected regions, but in the long term it allows them to enjoy higher steady-state levels of consumption. In the new steady state, the size of capital stock in all regions is the same as before the shock. The asset distribution is however shifted in favour of indirectly affected regions. As an effect, these regions import more than they export. The relative equalivalent variation in consumption is negative for all regions and equals -3.50% for region 1 and -1.41% for regions 2 and 3.

The welfare impact of the assumption about capital ownership is even more pronounced in the case where the shock directly affects the existing stock of capital. If we consider the case of local ownership again, the impact on region 1 reaches -5.30%, while the indirectly affected capital owners in regions 2 and 3 suffer a limited welfare loss of -0.5%.

5 Conclusions

In this paper, we introduce and operationalise a dynamic spatial CGE framework. To our knowledge, this is the first consistent dynamic SCGE model incorporating forward-looking behaviour of firms and houselholds. The instantenous equations with Dixit-Stiglitz structure in the modern sector stem from the earlier static model of ours.

An interesting theoretical result here is that the expanding variety of products produced in the modern sector gives rise to semi-endogenous growth even without an explicit R&D sector in our model. The dynamic adjustment is characterized by smooth convergence towards the steady state. The terminal condition is based on the stable local manifold theorem, and thus has a solid theoretical foundation.

The distinction of goods, factors, firms, and households by location, and the incorporation of trade costs in the model allow to study a variety of issues in regional and transport economics. The inclusion of consistent capital market and the flexibility in terms of choice of asset ownership scheme in particular suggest the use of the model to study regional investment subsidies. Furthermore, the model can be connected to a transport network model, as in Bröcker et al. (2010), to study the dynamic effects of transport infrastructure projects.

Some caveats must also be mentioned. It is known that under perfect competition without externalities saddle path stability holds (Farmer, 1999), which is a kind of local uniqueness condition, saying that any trajectory fulfilling the constraint has no other one in its neighbourhood doing so as well, if the neighbourhood is chosen small enough. This result can fail under increasing returns, but local properties around the steady state found in our numerical experiments never show any such pathology. We highly trust in saddle path stability of the solution of our model.

Furthermore, the assumption of perfect foresight can be viewed as completely unrealistic. The introduction of stochastic rational expectations in the model is regarded as a future task. In terms of deterministic framework, some modellers prefer to impose deviations from optimizing behaviour in their CGE models (e.g. McKibbin and Wilcoxen (1999)). However, for analysis of policies, the latter approach makes the important aspect of welfare measument impossible.

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A Appendix

A.1 Constraint on total asset value

In equilibrium it must be guaranteed that the asset total in the entire economy equals the total value of capital stocks. Misusing our notation for a moment, let non-subscripted expressions denote sums over regions, like $M := \sum_r M_r$ or $q\mathcal{K} := \sum_r q_r \mathcal{K}_r$. What we have so far then implies

$$\begin{aligned} \dot{A} &= \theta M + \rho A - C, && \text{(A.1)} \\ C + J &= (\theta + \chi) M, && \text{(A.2)} \\ v\mathcal{K} - \chi M + J &= q(\dot{\mathcal{K}} + \delta \mathcal{K}), && \text{(A.3)} \\ \dot{q}\mathcal{K} &= (\rho + \delta) q\mathcal{K} - v\mathcal{K}. && \text{(A.4)} \end{aligned}$$

(A.1) is just (2.10). (A.2) is from summing (2.16) and (2.17) up. (A.3) is obtained as follows: multiply (2.7) by $\mathcal{K}$, solve for $v\mathcal{K} - \chi M$, and add J from (2.4) to get for the left hand side $h\mathcal{I}(\zeta\mathcal{I}/\mathcal{K}+1)$, which by (2.5) equals $q\mathcal{I}$, which is $q(\dot{\mathcal{K}}+\delta\mathcal{K})$, thus the right hand side. This holds for each region and therefore also after summing over regions. Finally, (A.4) is just the non-arbitrage condition (2.6) multiplied by $\mathcal{K}$ and summed up. Now take (A.1)−(A.2)+(A.3)−(A.4) to obtain

$$-(\dot{q}\mathcal{K} + q\dot{\mathcal{K}}) + \dot{A} = \rho(A - q\mathcal{K}).$$

Hence, if the right hand side vanishes, so does the left hand side, which is to say that, if $A = q\mathcal{K}$ at one point in time, it is so forever.

A.2 Steady state growth rates

We start by taking equation (2.3), dividing it through by $\mathcal{K}$, and substituting for $\mathcal{I}/\mathcal{K}$ from (2.5). After rearranging this yields $\zeta(\hat{\mathcal{K}} + \delta) = q/h$. Taking now log-derivatives, and evaluating the resulting expressions at the steady state (where all growth rates are constant by assumption) leads to

$$\hat{q}^* = \hat{h}^*. \tag{A.5}$$

Thus, Tobin's Q (q/h) is constant in the steady state. Note that the investement-to-capital ratio $\mathcal{I}/\mathcal{K}$ is then also constant in the steady state.

Inserting (2.7) into (2.6), dividing through by q, taking log-derivatives and using previous results leads to

$$\hat{\mathcal{K}}^* + \hat{q}^* = \hat{\mathcal{M}}^* + \hat{p}^* = \hat{M}^*, \tag{A.6}$$

Next, taking log-derivatives in (2.20) and making use of (2.21) and (2.2) leads to

$$(1 - \beta)\hat{p}^* - \gamma\hat{g}^* + \chi\hat{\mathcal{K}}^* = (\theta + \chi)\hat{M}^* - \theta\tilde{\xi},$$

Dividing by $(\theta + \chi)$ and using (A.6) yields:

$$(1 + \eta)\hat{p}^* - \eta\hat{g}^* + \alpha\hat{\mathcal{K}}^* + (1 - \alpha)\tilde{\xi} = \hat{\mathcal{K}}^* + \hat{q}^*. \tag{A.7}$$

We continue by rearranging (2.4) to get

$$\frac{J}{\mathcal{I}h} = \left(1 + \frac{\zeta}{2}\frac{\mathcal{I}}{\mathcal{K}}\right)$$

As $\mathcal{I}/\mathcal{K}$ is constant in the steady state, so is also the ratio $J/\mathcal{I}h$. Close inspection also suggests that the ratio $J/\mathcal{K}h$ is constant in the steady state, meaning

$$\hat{J}^* = \hat{\mathcal{I}}^* + \hat{h}^* = \hat{\mathcal{K}}^* + \hat{h}^*. \tag{A.8}$$

Further, by construction of investment and consumption composites,

$$\hat{\mathcal{C}}^* = \hat{\mathcal{I}}^*. \tag{A.9}$$

Now, taking time derivatives of equation (2.16) and switching to notation in terms of growth rates we get

$$S\hat{S} = (1-\beta)M\hat{M} - \epsilon C(\hat{\mathcal{C}} + \hat{h}) - \epsilon J\hat{J}$$

Evaluating this expression at the steady state and using (A.5), (A.8), (A.9), and (2.16), we obtain the following relationships:

$$\hat{S}^* = \hat{M}^* = \hat{\mathcal{K}}^* + \hat{q}^* = \hat{\mathcal{C}}^* + \hat{h}^* \tag{A.10}$$

By defition of the steady state, the same rates of growth should prevail in all regions. The definitions of composite prices h and g in (2.15) and (2.18) thus imply the following expressions for their steady-state growth rates:

$$\hat{g}^* = \frac{1}{1-\sigma}\hat{S}^* - \frac{\sigma}{1-\sigma}\hat{p}^* \tag{A.11}$$

and

$$\hat{h}^* = \epsilon\hat{p}^* + (1-\epsilon)\hat{g}^* \tag{A.12}$$

One last equation is given by the Ramsey rule in (2.12):

$$\hat{\mathcal{C}}^* = -\phi\hat{h}^*. \tag{A.13}$$

Summarizing (A.5)-(A.13), we get the system (2.23) with solution $\hat{\mathcal{K}}^* = \hat{\mathcal{C}}^* = \hat{\mathcal{I}}^* = \xi$, $\hat{h}^* = \hat{q}^* = -\xi/\varphi$, and $\hat{M}^* = \hat{C}^* = \hat{A}^* = (1-1/\varphi)\xi$, where

$$\xi = \frac{\theta(\sigma-\epsilon)}{\sigma\theta + \chi\epsilon + \beta - 1} \cdot \tilde{\xi}. \tag{A.14}$$

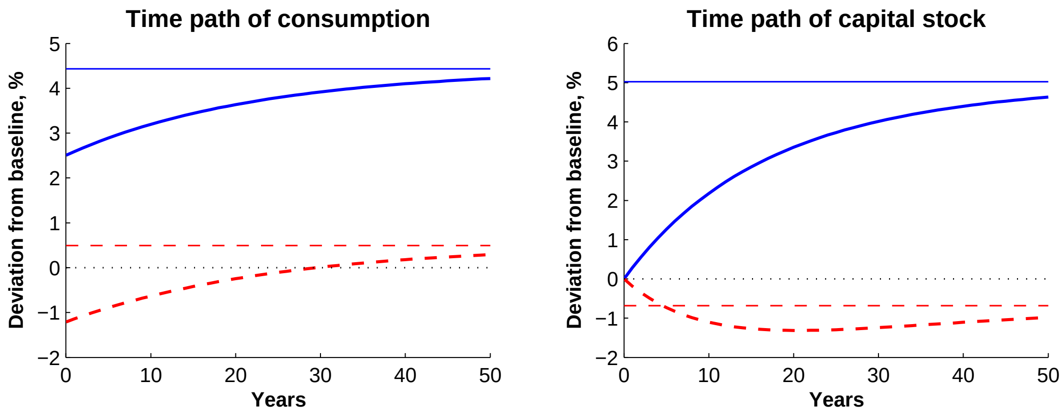

Figure 1: Effects of the transport cost shock: consumption and capital stock

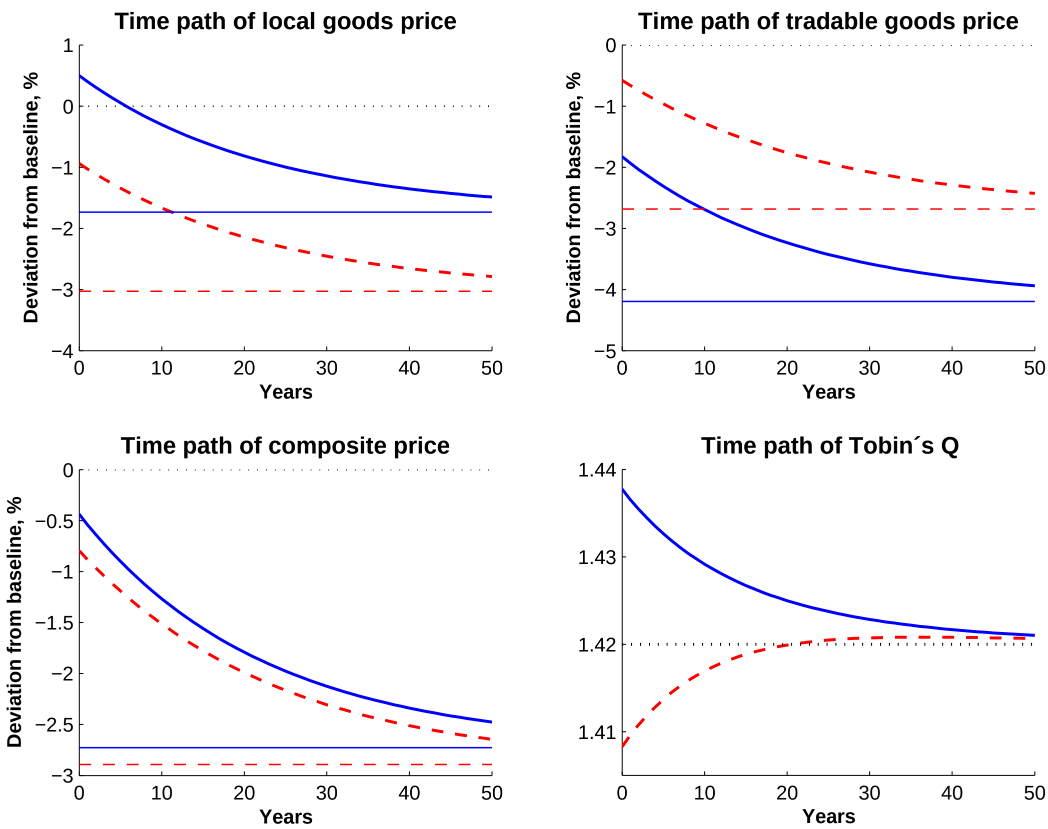

Figure 2: Effects of the transport cost shock: prices

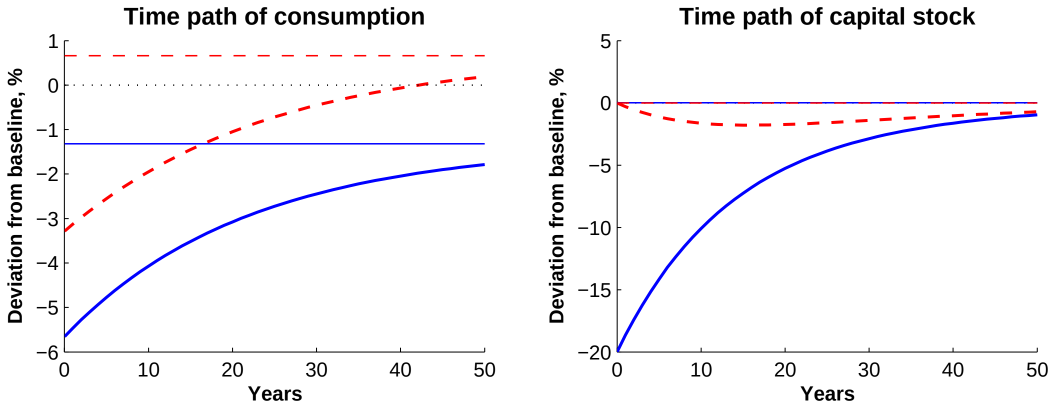

Figure 3: Effects of the capital stock shock: consumption and capital

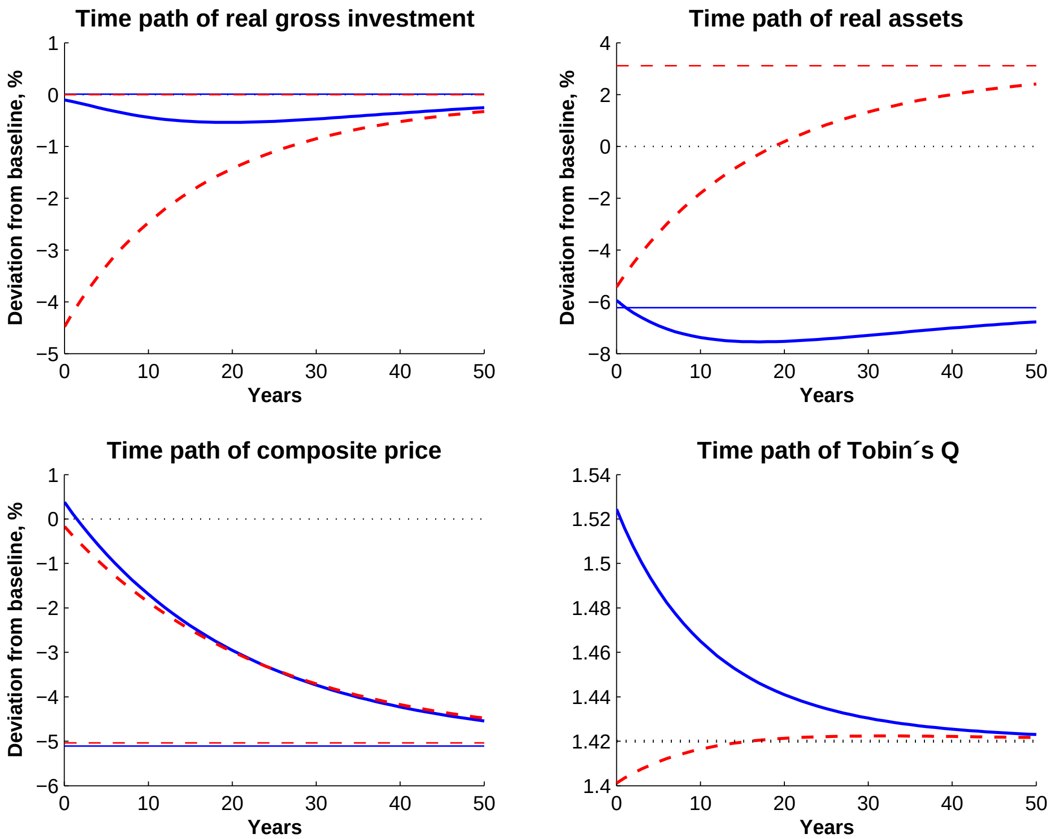

Figure 4: Effects of the capital stock shock: selected variables