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Uncertainty in Integrated Assessment Models of Climate Change: Alternative Analytical Approaches

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Abstract Uncertainty plays a key role in the economics of climate change, and research on this topic has led to a substantial body of literature. However, the discussion on the policy implications of uncertainty is still far from being settled, partly because the uncertainty of climate change comes from a variety of sources and takes diverse forms. To reflect the multifaceted nature of climate change uncertainty better, an increasing number of analytical approaches have been used in the studies of integrated assessment models of climate change. The employed approaches could be seen as complements rather than as substitutes, each of which possesses distinctive strength for addressing a particular type of problems. We review these approaches—specifically, the non-recursive stochastic programming, the real option analysis, and the stochastic dynamic programming—their corresponding literatures and their respective policy implications. We also identify the current research gaps associated with the need for further developments of new analytical approaches.

Keywords Uncertainty · Learning · Economics of climate change · Integrated assessment models · Real options · Dynamic programming

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1 Introduction

Although a large body of scientific evidence confirms the existence of the problem, the detailed mechanism and impacts of climate change are still uncertain. Such uncertainty poses a significant problem for policy making about climate change, and examination of this issue has yielded a body of literature in the field of economics of climate change as well. Those studies revolve around the basic question of how uncertainty and the reduction of uncertainty in the future affect optimal climate policy. This question can be broken down into the following subquestions: How do individual types of uncertainty influence the optimal timing and stringency of climate policy? How does future learning influence optimal policy? What is the value of information about different uncertainties?

Theoretical studies have shed some light on these questions (for reviews, see e.g., [11, 27, 56]) but the estimation of quantitative benchmarks usable in the policy debate necessitates the application of detailed models featuring the key mitigation technologies and key uncertainties of climate change. Therefore, an increasing number of studies utilize numerical integrated assessment models (IAMs), the primary tool for the investigation of complex climate-economy interactions.

This article reviews the approaches that have been applied in the literature of those integrated assessment model studies, and summarizes their respective policy implications. It follows the previous review articles by Kann and Weyant [31], Heal and Kriström [26], and Peterson [54]. However, we set a particular focus on the complementarity of varied analytical approaches to analyze the first-best optimal decisions of climate policy under uncertainty, for which a model needs to consider issues intrinsic to decision making under uncertainty, such as learning and the agent's risk aversion. Modeling of this aspect is one of the areas that have seen the most progress since those earlier reviews were published, and differences in methodological approaches play an important role for such analysis.

Indeed, climate change embodies various types and sources of uncertainty, and there is not yet an all-purpose analytical approach to represent all those types and sources. In every link of the cause-effect relationship of climate change from emissions to global warming and impacts, there are parametric uncertainty, which means incomplete knowledge of model parameters, and stochastic uncertainty or stochasticity, which is persistent randomness of the system because of unresolved processes. Another layer of complexity is the fact that knowledge is continuously updated because of scientific progress and measurements. The various uncertainties and their updating can be called the information dynamic complexity of the problem. As a result, estimation of realistic probability distributions for mitigation costs and for avoided benefits under different policies is difficult. This can be called the system dynamic complexity of the problem. The information and system dynamic complexity of the problem suggests that multiple complementary analytical approaches with different tradeoffs between the two complexities are required to grasp the full implications of uncertainty for climate policy.

Near-term climate policy has to be in place before major uncertainties are resolved. A selected policy should balance the near-term abatement costs and the correction costs in the future along with irrecoverable damages and also sunk abatement costs. Quantitative methods should help decision-makers process all available information at the moment and assign a value to each feasible policy by taking into account the projected costs and correction costs in response to learning. Even if understanding of the future is incomplete it should be represented in the model. In fact, without any value assigned to the irreversible nature of climate change, there are little benefits of learning, and thus uncertainties on the climate act as a driver for delaying abatement. Learning is beneficial only if policy-makers preserve flexibility to adjust initially selected policy trajectories. Thus flexibility has a value that should be explicitly presented in cost-benefit analysis.

We discuss analytical approaches in association with the issues for which each of them has a methodological advantage. More precisely, our focus lies in the following: (1) non-recursive stochastic programming (NSP) is the most common approach for finding optimal decision under uncertainty in IAMs and particularly useful for investigating the implications of parametric uncertainty. We discuss NSP studies both with and without learning about uncertainty. In many NSP analyses, policy-making processes are represented as a binary decision tree with anticipation of learning and possibility to correct policy at the node of the tree, and this schematic presentation of learning and irreversibility allows qualitative analysis. By and large, NSP in IAMs shows that uncertainty without learning favors stronger mitigation. The climate damage uncertainty dominates the mitigation cost uncertainty. Future learning about uncertainty has only a small effect on the optimal level of mitigation unless a highly nonlinear climate threshold is included in the analysis. Hence, learning serves as an argument for neither deferring nor advancing mitigation action. (2) This last conclusion changes in another approach presented in Section 3, which applies real options analysis (ROA) to IAMs. ROA highlights the value of flexibility in future actions in the face of uncertain climate change and could be regarded as an extension of a binary tree approach. In fact, a binary tree is the most refined way to explain and option value formation. The analysis shows that substantially stricter interim targets become economical if the value of the option to switch to a laxer target later on is taken into account. This result stems from the skewness and long upper tail in the probability distribution of avoided damages. (3) Stochastic dynamic programming (SDP) is the most comprehensive approach to uncertainty and discussed in Section 4. The use of SDP is practically a necessity for investigating implications of stochasticity, as it allows simulating many plausible states of the world and selecting a policy that is less exposed to uncertainties than others. However, given its technical complexity, applications of this approach to IAMs are still relatively few. Two studies show that learning about key uncertainties in the climate problem might take a long time if it takes a form of Bayesian learning from the stochastic climate. SDP is also applied to analysis of a few other problems, such as effects of unpredictable arrival of a breakthrough clean technology on climate policy, effects of climatic fluctuations on the decisions of the risk-averse agent, and tipping-point risks of the climate or economic system under climate change.

Studies that use those three approaches all suggest that an intensity of abatement policy is “an increasing function” of uncertainties on the climate and highlight importance of irreversibility in climate policy analysis. All three methods can offer some valuable qualitative insights to policy makers and provide some guidance in ongoing efforts to perform dynamic stochastic optimization of IAMs.

This review limits its scope to the body of IAM studies on uncertainty and does not cover the entire range of literature about uncertainty and the economics of climate change. Considering a methodological focus of the paper, we do not discuss in detail the theoretical literature on the economics of climate change and uncertainty, which is reviewed by other authors such as Pindyck [56], Baker [11], and Heal and Millner [27]. Also, in this paper, we only review the studies on first-best climate policy, but there is an extensive literature on the choice of policy instruments under uncertainty (see, e.g., [29]). A recent special issue of this journal [25] also contains a number of articles concerning policy instruments or portfolios of emission-reducing technologies.[1]

As the basis for the discussion of different approaches below, we here give a generic formulation of an IAM. We denote the utility function of the representative agent by u. This could also be a multi-regional welfare function and would then depend on the average consumption in the different regions. We denote the pure rate of time preference by δ, time-step length by Δt, the vector of state and decision variables at time t by X_t and I_t, respectively, consumption by c_t, the vector of uncertain parameters by θ, stochastic shocks by η_t and the measurement error by γ_t. The state variables include the production capital and the atmospheric carbon stock, for instance, while investments are formulated as decision variables. We omit a time-varying population in this article, which would simply lead to a time-dependent factor to the utility function. Finally, the vector of messages containing information about the uncertain parameters up to time t is denoted by $m^t=(m_1,\ldots,m_t)$. A generic stochastic first-best IAM can then be written as

$$\begin{aligned} \max_{\{I_t(m^t)\}} \quad & \left\{ E_0 \sum_{t=0}^{\infty} e^{-\delta t \Delta t} u(c_t(X_t, I_t(m^t))) \right\} \\ \text{s.t.} \quad & X_{t+1} = f(X_t, I_t(m^t), \theta) + g(X_t)\eta_t \\ & m_t = X_t + h(X_t)\gamma_t \end{aligned} \tag{1}$$

The expectation (E_0) of utility is taken conditional on the information available at time $t=0$. The first constraint in Eq. (1) specifies the system dynamics, which contains both uncertain parameters θ and a stochastic term η_t. The measurement error in the second constraint has not been considered in IAMs yet and will also be neglected in the following. What makes Eq. (1) difficult to solve is the fact that decisions I_t generally depend on the history of messages that have been received.

[1] As for the effects of uncertainty on policy instruments, the special issue specifically includes two articles that address the issue of allocation of emission allowances. Allowances allocation under uncertainty needs to seek two competing goals of containing costs of climate policy and of controlling the damage of climate change. Golub and Keohane [23] solve the problem of allocation and size of allowances reserve for a given countrywide emission trading scheme for containing price of carbon allowances at the level not higher than a politically acceptable level with a reasonable probability, Meanwhile, Aubin et al. [7] deal with the issue of how to translate an overall climate mitigation objective into an allocation of emission reduction objectives among polluters. The study proposes a method for dynamically allocating pollutant emissions rights among polluters, given that the emissions growth rates of the various polluters cannot be controlled, or even predicted. The problem is solved with mathematical and algorithmic tools of viability theory. With given maximum growth rates of emissions of each polluter in the worst case, the method of these authors provides the allocation rule for emissions rights and the required initial emissions.

2 Implications of Parameter Uncertainty: Non-recursive Stochastic Programming

Numerical modeling of climate change uncertainty is so far mostly based on stochastic programming, which denotes an optimization including random parameters, whether it be uncertain model parameters or stochastic shocks. We denote methods that do not use dynamic programming, which is discussed separately in Section 4, by non-NSP.

NSP is the simplest approach to actual optimization under uncertainty and especially useful for the investigation of parametric uncertainty. As a similar and even simpler modeling method, there is an approach called uncertainty propagation, which performs optimization of policies for a large number of possible parameter combinations individually and estimates their probability-weighted sum (an early study that belongs in this category is [69]). Just as sensitivity (or scenario) analysis, uncertainty propagation is unable to take into account factors intrinsic to decision making under uncertainty, such as learning and the agent's risk aversion. Still, uncertainty propagation (and also sensitivity analysis) can show relative importance of various uncertainties on climate change and thus is still widely used for this purpose. In this paper, however, we do not discuss this approach as our focus is the modeling frameworks in which uncertainty actually influences the incentives of the decision maker. See Kann and Weyant [31] and Peterson [54] for reviews of studies based on uncertainty propagation. Recent studies that adopt those approaches and are not listed in those references include Nordhaus [51], McInerney and Keller [46], Newbold and Daigneault [49], Ackerman et al. [1], Webster et al. [67], and Anthoff and Tol [3].

2.1 Effect of Uncertainty on Optimal Policy

A few studies conduct NSP without taking learning about uncertainty into account. The main question of these studies is how uncertainty affects optimal policy in terms of timing and stringency. Exclusion of learning from the modeling substantially reduces the information dynamic complexity and thus allows including multiple uncertainties and detailed system dynamics.

NSP can be formulated as follows. First, a sample is drawn from the joint probability distribution of all uncertain parameters θ and shocks η_t. The sample points $(\theta_s, \eta_{t,s})$ can be called states of the world. We denote the probability of state s by p_s. Without learning and a with a finite time horizon T, problem (1) then reads as

$$\begin{aligned} \max_{\{I_t\}} \quad & E_0 \left\{ \sum_{t=0}^{T} e^{-\delta t \Delta t} u(c_t(X_t, I_t)) \right\} \\ \text{s.t.} \quad & X_{t+1} = f(X_t, I_t, \theta_s) + g(X_t)\eta_{t,s} \end{aligned} \tag{2}$$

Pizer [57] shows a way to apply NSP by approximating the optimal consumption paths under climate change policy by analytical functions of the state variables. He finds the optimal policy path under uncertainty by evaluating the intertemporal welfare from the optimal consumption paths with differing policies and inclusive of uncertainty. Taking account of various forms of uncertainty, he finds that uncertainty justifies roughly 30 % stricter emissions reductions.

Problem (2) can be further simplified by not actually performing a continuous optimization but only finding the most desirable policy in a given set of policies I^p. This is called policy evaluation. However, whether the resulting policy approximates the optimal choice well is not clear, particularly in models with a high-dimensional decision space. Gjerde et al. [22], for example, use this simplification to show that a potential climate catastrophe justifies substantially stronger mitigation action. They do not separate the effect of uncertainty about this catastrophe, but it can be conjectured that it strengthens the argument, because mitigation costs are not uncertain in their study.

NSP can also be used to estimate the value of the immediate and complete resolution of uncertainty. This is done by comparing expected utility resulting from problem (2) with the expectation of utility over separate deterministic optimizations in each state of the world. Peck and Teisberg [53] report for the CETA model that climate sensitivity and climate damages are the most useful uncertainties to learn about with values of about US$150 and 100 billion, respectively. Gjerde et al. [22] report an even higher value of learning about potential climate catastrophes of almost US$600 billion.

2.2 Effect of Learning on Policy Stringency

By taking learning into account, NSP can address the questions of how future learning changes optimal near-term climate policy and of how valuable future information about different uncertainties is.

Given the increasing computing power, NSP with learning has become more widely applicable to IAMs in recent years. It is still limited to a single or at most a few learning steps, though, and information arrives continuously in reality. However, information pooling and climate policy formation are slow processes. The IPCC publishes its reports every 7 years, it took five years to negotiate the Kyoto Protocol, 15 years to build consensus on the 2 °C threshold as a long-term environmental target, and it may still take several years to get the major developing countries to commit to absolute emission targets. In this light, it might not be unrealistic to assume that an initial near-term climate policy up to 2030 or 2050, for instance, is revised only once or a couple of times.

For one learning step and only parametric uncertainty, we can rewrite Eq. (1) as

$$\begin{aligned} \max_{\{I_t^i\}} \quad & \sum_i q_i \sum_j p_j^i \sum_{t=0}^{T} e^{-\delta t \Delta t} u\left(c_t\left(X_t(\theta_j), I_t^i\right)\right), \\ s.t. \quad & X_{t+1}(\theta_j) = f\left(X_t, I_t^i, \theta_j\right), \\ & \forall t < t_l : I_t^l = I_t^1, \end{aligned} \tag{3}$$

where q_i and p_j^i are the probability of message i and the probability of state of the world j after receipt of message i, respectively. The latter is characterized by the vector of parameter values θ_j.

The last constraint in (3) ensures that decisions can only be tailored to the individual messages after receiving them at time t_l. This “trick” is sometimes called “discrete stochastic programming” and was first proposed by Cocks [15]. It allows solving recourse problems such as (3) by efficient optimization solvers in modeling systems such as AMPL and GAMS. This in turn allows using IAMs with comparably high system dynamic complexity and often without changing the modeling system. However, the number of decision variables and constraints increases exponentially for more than one learning step quickly rendering the problem unsolvable. For several learning steps, solution methods based on a recursive formulation are superior (see Section 4).

The formulation in Eq. (3) is particularly suited to consider parameter uncertainty but could in principle also be used to incorporate stochasticity. For example, Fisher and Narain [18] model stochasticity of damages in a two-period setting and investigate the effects of learning and also of sunk abatement capital. They find that the effect of sunk capital is stronger than the effect of uncertainty of future damages and also that of learning. As a stochastic process introduces a random shock for each time-step, however, a sufficient sample would render the sums in Eq. (3) unmanageable. Therefore, recursive methods are the preferred choice if stochasticity is considered (see Section 4).

Considering only one learning point as in (3) simplifies not only the solution but also the interpretation of results. It makes the distinction between the four cases shown in Fig. 1 particularly intuitive: (a) the deterministic case, in which the uncertain parameters are fixed at their expected value. This is the blue line in Fig. 1. (b) The case without learning, in which the parameters are uncertain and uncertainty is not resolved. This is the green line in Fig. 1. (c) The case of non-anticipated learning, in which the uncertainty is at least partly resolved but this is not anticipated. Decisions before learning coincide with decisions without learning. These are the orange lines. (d) The case of anticipated learning, in which learning is additionally anticipated, potentially leading to different optimal pre-learning decisions. These are the red lines in Fig. 1. The key

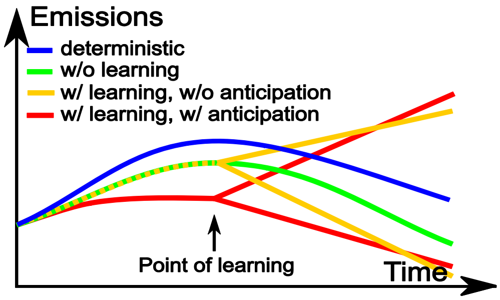

Fig. 1 Scheme of optimal emissions in different scenarios. The cases with learning are depicted for only two messages

results are differences in optimal policies in these scenarios and the associated welfare differences.

More specifically, we can distinguish two effects. Firstly, static uncertainty has an effect on optimal emissions and associated welfare as compared with the deterministic scenario. This is the difference between the blue and the green line in Fig. 1. This effect stems from the nonlinearity of the objective function in the uncertain parameters and is also investigated in uncertainty propagation. It is generally found to be small in studies using NSP [44, 52, 64, 65]. Uncertainty propagation has shown that uncertainty can have a substantial effect on optimal emissions. This indicates that the smallness of the effect of uncertainty in NSP studies is likely to be at least partly because of a crude representation of uncertainty.

Secondly, learning has an effect on optimal emissions as compared with the no-learning case. This is the difference between the green and the red lines. The associated welfare increase is called the expected value of information (EVOI). The EVOI is generally found to be significant. Particularly learning about climate damages and climate sensitivity are found to be very valuable compared with current research budgets. This was shown in different IAMs by Nordhaus and Popp [50] and Lorenz et al. [44] among others.

The effect of learning can be decomposed into two parts. Firstly, optimal policy after learning will depend on what is learned. This is the difference between the orange lines and the green line. The associated welfare difference can be called an option premium, which is further discussed in Section 3.

Secondly, anticipation of future learning changes optimal near-term climate policy before learning. This is the difference between the orange and the red lines. The associated welfare increase can be called the expected value of anticipation. Anticipation of learning is valuable if decisions are irreversible and anticipation generates flexibility. There are two main irreversibilities involved in the climate problem that counteract each other. Investments in mitigation, which are at least partly sunk, and emissions stay in the atmosphere for decades to centuries. As a result, most studies performing cost-benefit analysis find that anticipation of learning has only a small effect on optimal emissions [44, 52, 63–65].

However, a substantial effect of anticipation on optimal near-term policy was shown in the presence of an irreversible climate threshold with uncertain corresponding damages [17, 33, 44]. A stricter policy can then be justified because it keeps the option open to avoid the threshold if it is learned to be severe.

A strong effect of anticipation is also found in studies performing cost-effectiveness analysis in general that find lower optimal emissions with uncertainty and learning (parts of [13, 65]). However, Schmidt et al. [60] argue that these latter results should be taken with caution because they stem from a disputable interpretation of climate targets under uncertainty as strict targets that have to be met with certainty.

The studies discussed so far in this subsection concern uncertainties in the effects of climate change, but similar modeling frameworks could also be applied to a related question of uncertainty on the potentials of carbon dioxide capture and storage (CCS), which could substantially affect the overall costs of long-term climate change mitigation. Studies that address the latter question utilize stylized scenarios that represent uncertain properties of CCS and of climate change and consider that the decision maker learns at a certain time point which scenario is a true one. Gerlagh and van der Zwaan [21] examine the optimal climate policy and use of CCS under uncertainties in the leakage rates of carbon dioxide from geological reservoirs also in the damage of climate change, They consider that revelation of uncertainties, which are represented by four potential cases, takes place in 2050 and estimate long-term implications of carbon dioxide leakage up to the year 3000. Meanwhile, Keppo and van der Zwaan [35] investigate a similar question to that of Gerlagh and van der Zwaan but treat the availability of CCS storage sites, rather than the leakage rates, as uncertain. By employing a bottom-up model and assuming six potential cases among which a true case becomes known in 2040, they mainly discuss potential deployment of CCS in the first half of the twenty-first century

Although their analysis is not narrowly a study of NSP, Babonneau et al. [9] could also be placed in this strand of literature. They combine NSP and simple Monte-Carlo simulations to assess the impacts of multiple uncertainties in climate change and energy technologies on climate policy. While they model technology uncertainties in a basic Monte-Carlo fashion, they analyze the uncertainty of the climate sensitivity (represented as four potential cases) with NSP by assuming a learning process that reveals the true level of the climate sensitivity in the year 2030.

While NSP is a powerful tool for investigating some important questions of climate policy and uncertainty, it also has limitations. As learning is exogenous in NSP, it is not suitable for studying learning about technologies. Uncertainty about the floor costs and learning rates of technologies is mostly reduced by applying them. However, NSP does not estimate what can be expected to be learned. It uses the learning rates as an input.

3 Value of Flexibility: Real Options Analysis

ROA uses methods from financial option pricing, in particular contingent claims analysis, to value the managerial flexibility inherent in real investment decisions.[2] It sometimes also uses SDP, which we discuss separately in Section 4. ROA also provides a language and intuition for talking about this flexibility.

ROA is widely applied to analyses of the energy market, including those involving climate or renewables policy [20, 37, 40, 58, 59, 61]. In those studies, the authors analyze the portfolio choice of energy technologies by applying ROA. Climate change is sometimes explicitly incorporated in their analysis in the form of stabilization targets of atmospheric carbon dioxide concentrations—in other words, an overall climate policy is given. Meanwhile, the ROA concept is also useful for investigating the targets of the first-best climate policy, which are the focus of this paper. To investigate the first-best policy (climate targets), one can think of it as a continuum of real options on different levels of the stabilization target. Accumulation of an additional amount of emissions in the atmosphere kills some options (for example, any target below 400 ppm might become infeasible) and also makes other targets (say below 500 ppm) less realistic. In other words, by continuing emissions, the economy continuously loses options on the stabilization target and concurrently experiences the damage that is equal to the economic value of lost options. An example of this type of analysis is Anda et al. [2], on which the description below is based.

The simple example depicted in Fig. 2 demonstrates the option value concept. There is uncertainty about climate sensitivity (CS), which can only take one of three possible values. The true value is learned in 2050. The mitigation costs of an interim target up to 2050 are \$35 trillion. Benefits in the form of avoided climate damages accrue only after 2050 and depend on the true value of climate sensitivity. Adoption of the long-term target whatever the value of CS has a negative NPV of -\$35+¼ \$123+½ \$24+¼ (−\$45)=−\$3.5 trillion. This assumes risk-neutrality for simplicity and would have to be risk-adjusted under risk aversion. If we add the option to abandon the target for a looser one with zero NPV if the post-learning NPV of the target is negative, we get an expanded NPV of −\$35+¼ \$123+½ \$24+¼ (−\$0)=\$7.75 trillion. Thus, the option premium is \$11.25 trillion. Given this premium, it is economical to adopt the target as an interim target but not as a one-shot, long-term target.

[2] There is a closely related theoretical literature on the quasi-option value in environmental economics [4, 28]; see [5] for the relation to ROA).

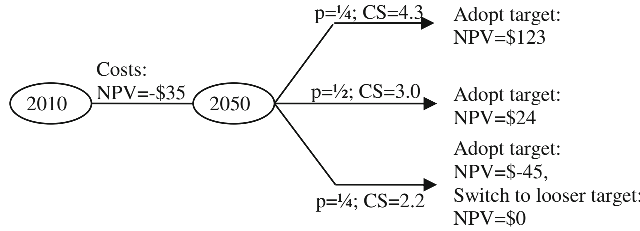

Fig. 2 Simple demonstration of the option value of an interim target. CS=4.3 with probability ¼, CS=3.0 with probability ½, and CS=2.2 with probability ¼

The interim target can be seen as a European option on the long-term target. It is the option but not the obligation to adopt the long-term target at a given future time. The time of expiration is 2050. We say the option is executed if the target is abandoned (put option). The strike price is then the NPV of the best alternative, which is assumed to be zero for simplicity.

The value of an option depends on the volatility and price of the underlying asset. For financial options and in standard ROA these characteristics can be observed in markets, or at least the characteristics of a highly correlated twin security. There are, of course, no markets for long-term climate targets, though, but one can derive the characteristics from an IAM.

This is done by a Monte-Carlo simulation of the policy under consideration denoted by I^p. The NPV of the target right after learning the true state of the world θ_s at t_l is obtained by discounting net benefits over business-as-usual (I_{BAU}) at the ex post risk-free rate $r_{s,t}$,

$$\mathrm{NPV}^{ep}(t_l) = \sum_{t=t_l}^{T} e^{-r_{s,t}t}\left(c_t(\theta_s, I^p) - c_t\left(\theta_s, I^{\mathrm{BAU}}\right)\right) \tag{4}$$

$c(\theta_s, I^P)$ denotes the net value of policy I^P at time t and $c(\theta_s, I^{BAU})$ stands for the net value of the business as usual emissions scenario, that is only the damage as there is no abatement cost.

This is the random price of the underlying asset right after learning. The NPV of the target right before learning is obtained by discounting certainty equivalent benefits at the ex ante risk-free rate r_t

$$\mathrm{NPV}^{ea}(t_l) = \sum_{t=t_l}^{T} e^{-r_t t}\left(\overline{c}_t(I^p) - \overline{c}_t\left(I^{\mathrm{BAU}}\right)\right) \tag{5}$$

This is the spot price of the target right before learning. The NPV of the costs of the interim target is given by

$$\mathrm{NPV}^{\cos ts}(t=0) = -\sum_{t=0}^{t_l} e^{-r_t t}\left(\overline{c}_t(I^p) - \overline{c}_t\left(I^{\mathrm{BAU}}\right)\right) \tag{6}$$

$\overline{c}$ denotes the expected value of policy cost.

If the probability distribution of benefits (Eq. 4) can be approximated by a convenient distribution function, one can use analytical option pricing formulas. Most option pricing formulas, including Black–Scholes's, imply non-negative prices. The price of a long-term target, however, can be negative. To deal with those negative values, Anda et al. separated the benefit component of climate policy NPV (i.e., the avoided damage) from the cost component (the abatement cost). Now for each policy target both categories are positive, and more advanced option pricing formula can be applied.

Alternatively, Bachelier's model that allows a negative value for underling asset could be applied. However, in this model, the prices are normally distributed. Restriction on the distribution limits the ability to study tailed risks of climate change and only illustrates the role of the first and the second moments of NPV distribution. If we denote the strike price by K, and the volatility of the asset, i.e. the standard deviation of NPV^{ep}, by σ, Bachelier's formula for the value of the option right before learning reads

$$V^B = (\mathrm{NPV}^{ea} - K)\Phi\left(\frac{\mathrm{NPV}^{ea} - K}{\mathrm{NPV}^{ea}\sigma}\right) + \mathrm{NPV}^{ea}\varphi\left(\frac{\mathrm{NPV}^{ea} - K}{\mathrm{NPV}^{ea}\sigma}\right), \tag{7}$$

where Φ and ϕ are the cumulative distribution function and the probability density function of the standard normal distribution, respectively, and where we have omitted the time argument of the NPV. As we consider an instantaneous resolution of uncertainty, neither the time to expiration nor the interest rate occur in the pricing formula. The interim target should be adopted if

$$\mathrm{NPV}^{\cos ts} + V^B > 0. \tag{8}$$

Considering a long upper tail in the probability distribution of climate damages, a normal distribution for the benefits of climate policy is not realistic and Bachelier's model underestimates the option value of an interim target. Anda et al. [2] show that more sophisticated pricing models taking higher moments of the distribution into account lead to substantially higher option values. ROA has two major advantages over SDP. First, it does not demand a stochastic optimization but only a Monte Carlo simulation. Second, it allows consideration of continuous distribution functions with tails in analytical option pricing formulas. In addition, it provides a clear intuition and quantification of flexibility as an option value.

However, in the method summarized above, the target and the decisions to reach were only evaluated and not derived in an optimization. Thus to this estimation itself does not tell whether the decisions are efficient and the target optimal. However, if the option value could be expressed as a function of the expected value, then options value could be added to the damage function and to the abatement cost function then we can solve an optimization problem as a deterministic problem. See Anda et al. [2] for conditions under which the method can be extended to an optimization.

The ROA of the first-best climate policy targets by Anda et al. [2] reveals that an upper tail in the avoided damage distribution leads to a large option value and thus justifies an aggressive interim target even without risk aversion.

4 Implications of Stochasticity: Stochastic Dynamic Programming

The approaches we have discussed up to now are not suitable for taking stochasticity and the repeated and endogenous updating of probability distributions into account. SDP is preferred for the examination of such high information dynamic complexity. The SDP approach models stochasticity by utilizing a recursive formulation of the problem, which is that immediate actions are taken based only on the current situation (rather than the entire history), in other words, the decision problem of the same structure recurs each period. While most current debates on uncertainty in climate change deal with parametric uncertainty, many aggregate processes in the climate system and the economy are also stochastic. Modeling of stochasticity can answer some interesting research questions, such as to what extent stochasticity of the climate system hinders the resolution of parametric uncertainty and how it changes the optimal decisions.

In SDP modeling, the value function is defined as the maximum utility that can be obtained given the current state of the system including the probability distributions on the uncertain parameters. This reads as

$$\begin{aligned} J(X_0) = \max_{\{I_t(m^t)\}} \quad & E_0 \sum_{t=0}^{\infty} e^{-\delta t \Delta t} u(c_t(X_t, I_t(m^t))), \\ \text{s.t.} \quad & X_{t+1} = f(X_t, I_t(m^t), \theta) + g(X_t)\eta_t, \end{aligned} \tag{9}$$

where it is presumed that the time horizon is infinite and the value function does not depend on time explicitly. Using the value function and the principle of optimality,[3] we can rewrite problem (9) recursively as in the following Bellman equation

$$\begin{aligned} J(X_t) = \max_{I} \quad & \{u(c_t(X_t, I) + e^{-\delta \Delta t} E_t J(X_{t+1})\}, \\ \text{s.t.} \quad & X_{t+1} = f(X_t, I_t, \theta) + g(X_t)\eta_t, \end{aligned} \tag{10}$$

where for simplicity and clarity we omitted the system and information dynamics.

For simple models, the Bellman equation can be solved or exploited analytically, whose examples are Dixit and Pindyck [16] and Pindyck [55]. Karp and Zhang [32], who use a linear-quadratic multi-period IAM, could also be placed in this category. They find that anticipation of learning about climate damages decreases optimal abatement by about 10–20 %.

[3] The mathematical conditions for which the principle of optimality holds can be found, e.g., in Stokey and Lucas [62].

In more complex models, the value function has to be analyzed numerically. This requires an approximation method, as Eq. (10) is only a functional equation and does not give the functional form of J. A straightforward computation of backward induction that reflects all nodes of the decision tree is normally not feasible for such problems as the number of nodes becomes extremely large for multiple state, control and random variables and a number of time steps.

As one approach of approximation, Webster et al. [66] utilizes an approximate dynamic programming framework, which is to find the value function through iterations of random sampling on the levels of the policy variable and shocks. By using a seven-stage model, they study the effects of stochasticity on the optimal climate policy and conclude that conventional two-stage models are prone to underestimate the effect of uncertainty.

Meanwhile, most other existing studies of SDP in the climate change context employ another approach, with which the value function is approximated in a composite of functions of given forms, such as Chebyshev polynomials, with a set of coefficients to be specified through a fitting process. To find a solution, one starts with a guess of the value function, applies it to the right-hand side of Eq. (10) for selected combinations of state variables, calculates the error, and obtains a new guess for the value function until the algorithm converges. Thereby, the value function is parameterized. See Judd [30] for details of the methodology.

As for investigating the effect of learning, SDP has the advantage that it can take into account endogenous and repeated updating of uncertainty. So far, however, SDP is only applied to a specific aspect of learning on climate change, which is the Bayesian learning of the climate sensitivity from observed temperature fluctuations. Kelly and Kolstad [34] explicitly model the stochasticity of the temperature process and the Bayesian updating on climate sensitivity in DICE. They find that learning the true value of climate sensitivity takes at least 90 years. They also show a trade-off between emissions control and the speed of learning. Meanwhile, Leach [39] extends the analysis to two uncertain parameters in the temperature process and shows that this can delay learning by hundreds or even thousands of years.

SDP is a suitable method also for investigating how stochasticiy changes the optimal policy decisions. Given the methodological complexity, however, the number of SDP studies that analyze stochasticity is still limited. Bahn et al. [8] discuss an IAM where stochasticity is represented as two jump processes regarding the revelation of climate sensitivity and technological breakthrough. Their model analysis compares two types of precautionary actions, one about climate change mitigation and the other about R&D investment in clean technologies. Meanwhile, Lontzek and Narita [43] examine the optimal climate policy in direct response to continuous fluctuations of the climate system (in some parallel to Kelly and Kolstad and Leach, where the climate sensitivity parameter is estimated from climatic fluctuations) by using a continuous-time SDP model. They show that stochasticity has only a small and ambiguous effect on optimal emissions reductions as compared with the deterministic case (without shocks), while the sign of the effect is partly determined by the level of risk aversion.

As the most recent development in this strand of research, Lemoine and Traeger [41] and Cai et al. [14] investigate effects of tipping-point climate change risks, i.e., potential permanent shifts of the climatic or economic system that would set in with an increasing likelihood linked to the degree of climate change. They both extend the deterministic DICE model to allow for stochastic time paths of multiple state variables. Their results show that the inclusion of tipping-point risks in the model estimation generally raises the level of the optimal climate policy substantially.

5 Summary and Conclusions

We have reviewed probabilistic approaches to uncertainty in integrated assessment models and their respective implications for climate policy.

Non-NSP is the simplest way to take uncertainty and learning into account in IAMs. Uncertainty generally, and not surprisingly, justifies stronger emissions reductions. Estimates of the extent, however, vary from very little up to 30 % depending on how many uncertain parameters and sample points are considered. Future learning is generally found not to be a significant factor to promote more or less mitigation unless potential climate thresholds are taken into account. However, learning can have an impact on the efficient mitigation portfolio, and the optimal level of R&D in particular.

We have then discussed a way to apply ROA to IAMs. It is characterized by the use of financial option pricing methods to value the option of adjusting policy to future learning. It allows a more comprehensive consideration of uncertainties than discrete stochastic programming, and the representation of tails in particular. It shows that future learning can then be an argument for substantially stronger short-term emissions reductions. Up to now, it has only been used to evaluate given policies. Its application with a direct optimization might be a promising extension.

Whenever stochasticity is taken into account, possibly in conjunction with the endogenous resolution of uncertainty, dynamic programming is the preferred, or rather mandatory, choice. We have briefly discussed methods of SDP. It has been used to show that learning about climate uncertainty may take a very long time up to thousands of years. SDP is the most complete approach among the three approaches we have discussed and can in principle supersede the others. However, in practice, it could be rather seen as a complement to the other two approaches as it involves complex computation and its results tend to be less clear and intuitive than those of the other two.

The most important general policy implication from the literature is that despite a wide variety of analytical approaches addressing different types of climate change uncertainty, none of those studies supports the argument that no action against climate change should be taken until uncertainty is resolved. On the contrary, uncertainty despite its resolution in the future is often found to favor a stricter policy.

Research on this subject is not complete yet. An important issue that is becoming recognized and has yet to be explored further in the context of IAMs is the optimal climate policy under deep uncertainty. Scientific consensus rarely exists about the probability distributions of climate responses to anthropogenic interference and climate change impacts, but standard approaches based on expected utility theory do not capture such divergence of scenarios [36]. The critical meaning of incomplete information on potential extreme outcomes of climate change is also well illustrated by Weitzman [68], who shows that expected utility optimization may not yield finite solutions when complete information does not exist about the tails of probability distributions for climate change related parameters. Conceptual frameworks on finding favorable climate policy under deep uncertainty have been described by some authors [38, 42, 45], and there are also some recent studies that attempt to find more numerical implications [6, 19, 24, 41, 47, 48].[4] However, with the exception of Lemoine and Traeger [41], these studies are based on simple modeling methods that do not reflect varied degrees of flexibility in policy decisions over time or stochasticity of the climate system. The approaches to model deep uncertainty generally use a range of possible probability distributions for each parameter, which are weighted according to either some decision criteria (such as a focus on the worst cases) or some preference parameter (such as the ambiguity aversion). In principle, such formulations of problems do not prohibit applications of ROA and SDP, and those two approaches could in fact shed more light on problems of deep uncertainty associated with irreversibility of mitigation decisions or with stochasticity of the climate system.

There are also a number of other future research needs concerning first-best climate policy under uncertainty. (1) Learning about some uncertainties is endogenous. Risks of geoengineering options will be fully known only after they are applied. The maximum efficiency of various renewable energy technologies will be learned only if the technologies are applied on a large scale (see also [12]). Modeling endogenous learning demands SDP, in which the inclusion of sufficient climatological or technological detail poses a great challenge. In addition, endogenous technical change generates nonconvexities in the optimization problem, which demand global optimization solvers. (2) What are the implication of uncertainty and learning for first-best climate policy in developing countries? Significant short-term policy of emission control might steer developing countries into low-carbon economic growth and prevent a lock-in to carbon-intensive production capital. The associated benefits could be estimated by discrete stochastic programming or real options analysis. (3) The question of how alternative preferences, such as habit formation, direct utility from an environmental good, distinction between risk aversion, and intertemporal elasticity of substitution and others change optimal policy under uncertainty has not yet received sufficient attention but should be explored. (4) The analysis of the persistent stochasticity both of the climate system and the economy is still in an initial stage, and investigations of its implications for climate policy in more complex IAMs are needed. (5) Finally, there is a strong need for reliable probability estimates for the key parameters of IAMs, especially the climate change damage parameters.

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[4] Although their analysis focuses on energy security (unpredictable energy supply) rather than on the first-best climate policy, Babonneau et al. [10] also attempt robust optimization under uncertainty in the context of climate change, They conduct a type of chance-constrained programming that consider uncertain constraints that are satisfied most of the time but not always.

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