Inequity Aversion, Welfare Measurement And the Gini Index*

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Abstract Integrating *individual* inequity aversion (Fehr and Schmidt, 1999) into a utilitarian *social* welfare function, we derive a simple welfare measure which comprises *both* GDP *and* income inequality as measured by the Gini-index.

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1 Introduction

This paper demonstrates a simple connection between individual inequity aversion a la Fehr and Schmidt (1999) and the Gini index (Gini, 1912). In doing so, we add to the current lively debate regarding the (formal) integration of other-regarding preferences into social welfare measures and the appropriate trade-off between efficiency and equality (e.g. Fleurbaey, 2012; Decerf and van der Lindne, 2016; Treibich, 2018; Daly and Cobb, 1989; Neumayer, 1999; Fleurbaey and Blanchet, 2013).[1]

Here, we assume that the agents of a society have a preference for inequity aversion as proposed by Fehr and Schmidt (1999).[2] By simple aggregation, we then derive a social welfare function which combines average income with the Gini index (Gini, 1912).

2 The Model

Consider a society with n subjects who are inequity averse in the sense of Fehr and Schmidt (1999) and assume that each subject has income x_i, $i = 1, \ldots, n$, with incomes arranged in increasing order, i.e. $x_i > x_m$ for $i > m$. Utility of person i, then, is given by

$$V_i = x_i - \frac{\alpha}{n-1} \sum_{j>i} (x_j - x_i) - \frac{\beta}{n-1} \sum_{k<i} (x_i - x_k), \tag{1}$$

where the parameter α (β) measures the individual i's distaste of disadvantageous (advantageous) inequality.

Using the average utilitarian rule, we obtain the following expression for aggregate social welfare (W):

$$W = \sum_i x_i [\frac{1}{n} + \frac{\alpha + \beta}{n(n-1)} (n + 1 - 2i)]. \tag{2}$$

Thus, once we assume inequity aversion on an individual level, this also shines through in social welfare: each income is weighted by a factor which depends on the income's rank in the distribution, i.e. incomes below the median are overweighted (com-

[1] The measurement of economic welfare has also received substantial attention in the current political debate. An example is the "beyond GDP initiative" of the European Commission, the European Parliament, the Club of Rome, OECD and WWF which has promoted the development of welfare indicators which are as clear and appealing as the traditionally used GDP, but more inclusive of environmental and social aspects of progress (see https://ec.europa.eu/jrc/en/event/beyond-gdp-measuring-progress-true-wealth-and-well-being-nations-7763).

[2] See Rohde (2010) for an axiomatic foundation of the Fehr and Schmidt model.

pared to the equal weighting of $1/n$) and incomes above the median underweighted.[3] Consequently, the obtained social welfare measure exhibits inequality aversion, i.e. even distributions of income in society are preferred to uneven ones.

Moreover, further rearranging the expression for social welfare in equation (2), we can rewrite W as

$$W = \mu\big[1 - (\alpha + \beta)G\frac{n}{n-1}\big] \tag{3}$$

where μ is average income and G is the Gini index (Gini, 1912), i.e.

$$G := \frac{\sum_i 2ix_i}{n\sum_i x_i} - \frac{n+1}{n} \tag{4}$$

Finally, taking limits for $n \to \infty$, we get the following simple expression for a social welfare measure:

$$W = \mu\big[1 - (\alpha + \beta) \cdot G\big]. \tag{5}$$

Thus, integrating inequity aversion into a utilitarian welfare function, we are able to derive a simple welfare measure, which is based on average income – as GDP per capita – but also accounts for income inequality, measured by the Gini index.[4]

Proposition 1 *Considering a society of n individuals (n large) and assuming individual utility to reflect inequality aversion as proposed by Fehr and Schmidt (1999), i.e. individual utility is given by equation (1). Then, simple aggregation leads to a social welfare function that can be approximated as a combination of aggregate income in society, μ, and a measure of inequality in society, namely the Gini index, G:*

$$W \approx \mu\big[1 - (\alpha + \beta) \cdot G\big].$$

Note, as a final remark regarding the eventual expression for social welfare, that a similar measure, namely $W = \mu(1 - G)$, had already been proposed by Sen in 1974. Also, the derived expression turns out to be similar to the one obtained in an

[3]For example, the lowest income is weighted by $\frac{1}{n} + \frac{\alpha+\beta}{n} > \frac{1}{n}$, the highest income by $\frac{1}{n} - \frac{\alpha+\beta}{n} < \frac{1}{n}$ whereas the median income receives precisely the weight $1/n$.

[4]For this specific example, the trade-off between average income and equality is determined by the sum of α and β, i.e. the sum of parameters of aversion to disadvantageous and advantageous inequity. This is arguably an artefact of the simplifications made in the choice of individual utility functions (for $\alpha + \beta = 1$, we obtain the special case of $W = \mu(1 - G)$; cf. Sen, 1974). In general, we would indeed expect the relative weighing to be more complex. The purpose of this paper, however, was not to determine the best possible social welfare function but only to demonstrate that once we acknowledge monetary *and* distributional preferences on an individual level this translates rather simply – by aggregation – into a combination of well known social measures of aggregate wealth and distributional aspects.

axiomatic approach to social welfare proposed by Ben Porath and Gilboa (1994).[5] The present argument adds to this by highlighting the technical connection between a *social* welfare measure integrating the Gini index and a simple *individual* utility function reflecting inequality aversion (Fehr and Schmidt, 1999).[6]

3 Concluding Remark

The previous result – demonstrating the technical connection between individual inequity aversion (Fehr and Schmidt, 1999) and the Gini index (Gini, 1912) – complements the growing discussion about axiomatic approaches to social welfare which account also for other-regarding preferences (e.g. Fleurbaey, 2012; Decerf and van der Linden, 2016; Treibich, 2018). In particular, while axiomatic foundations for inequity aversion (Rohde, 2010) as well as the Gini index (Ben Porath and Gilboa, 1994) have been provided previously, this paper shows how both tools (Fehr and Schmidt preferences on an individual and the Gini index on an aggregate level), in fact, are connected.

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[5] Ben Porath and Gilboa define various axioms for a social preference relation, including inequity aversion regarding the distribution of income in society. As a result, they obtain a social welfare function, similar to the one derived here, combining aggregate income and the Gini index. Different from the present approach, they do not consider individual preferences, though. By contrast, the present approach shows that if individual preference relations display inequity aversion such that they can be represented by Fehr-Schmidt utility, the Gini coefficient arises naturally as welfare measure.

[6] See Footnote 4.

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