Kiel

Working Papers

Kiel Institute
for the World Economy

Applications of Statistical Physics in Finance and Economics

by Thomas Lux

No. 1425 | June 2008

Web: www.ifw-kiel.de

Kiel Institute for the World Economy, Düsternbrooker Weg 120, 24105 Kiel, Germany

Kiel Working Paper No. 1425 | June 2008

Applications of Statistical Physics in Finance and Economics

Thomas Lux

Abstract:
This chapter reviews recent research adopting methods from statistical physics in theoretical or empirical work in economics and finance. The bulk of what has recently become known as 'econophysics' in broader circles draws its motivation from observed scaling laws in financial markets and the abundance of data available from the economy's financial sphere. Sec. 2 of this review presents the robust power laws encountered in financial economics and discusses potential explanations for scaling in finance derived from models of stochastic interactions of traders. Sec. 3 provides an overview over other applications of statistical physics methodology in finance and attempts to evaluate the impact they have had so far on financial economics. With the following section, the review turns to recent work on the emergence of wealth and income heterogeneity and the recent inception of new strands of research on this topic, both within econophysics and the neoclassical economics tradition. Sec. 5 reviews the new stylized facts that have been identified in cross-sectional data of firm characteristics and agent-based approaches to industrial organization and macroeconomic dynamics that have been motivated by these findings. We conclude with an assessment of the major methodological contributions of this new strand of research.

Keywords: stylized facts, power laws, agent-based models, econophysics

JEL classification: C10, C51, G12

Thomas Lux
Kiel Institute for the World Economy
24100 Kiel, Germany
Phone: +49 431-8814 278
E-Mail: thomas.lux@ifw-kiel.de
E-Mail: lux@bwl.uni-kiel.de

Applications of Statistical Physics in Finance and Economics

Thomas Lux*

June 2, 2008

Department of Economics, University of Kiel
and
Kiel Institute for the World Economy

Abstract: This chapter reviews recent research adopting methods from statistical physics in theoretical or empirical work in economics and finance. The bulk of what has recently become known as 'econophysics' in broader circles draws its motivation from observed scaling laws in financial markets and the abundance of data available from the economy's financial sphere. Sec. 2 of this review presents the robust power laws encountered in financial economics and discusses potential explanations for scaling in finance derived from models of stochastic interactions of traders. Sec. 3 provides an overview over other applications of statistical physics methodology in finance and attempts to evaluate the impact they have had so far on financial economics. With the following section, the review turns to recent work on the emergence of wealth and income heterogeneity and the recent inception of new strands of research on this topic, both within econophysics and the neoclassical economics tradition. Sec. 5 reviews the new stylized facts that have been identified in cross-sectional data of firm characteristics and agent-based approaches to industrial organization and macroeconomic dynamics that have been motivated by these findings. We conclude with an assessment of the major methodological contributions of this new strand of research.

prepared for the *Handbook of Research on Complexity*, J. Barkley Rosser, ed.

---

*Contact adress: Thomas Lux, Department of Economics,
University of Kiel, Olshausen Str. 40, 24118 Kiel, Germany,
E-Mail: lux@bwl.uni-kiel.de

Contents

1 Introduction

"The economy" easily comes to one's mind when looking for examples of a *"complex system with a large ensemble of interacting units"*. The layman usually feels that terms like *"out-of-equilibrium dynamics"*, *"critical states"* and *"self-organization"* might have a natural appeal as categories describing interactions in single markets and the economy as a whole. When dealing with the economy's most opalescent part, the financial sphere with its bubbles and crashes, *"life at the edge of chaos"* and *"self-organized criticality"* equally easily enter the headlines of the popular press. However, this proximity of the keywords of complexity theory to our everyday perception of the economy is in contrast to the relatively slow and reluctant adaptation of the ideas and tools of complexity theory in economics. While there has been a steady increase of interest in this topic from various subsets of the community of academic economists, it seems that the physicists' wave of recent research on financial markets and other economic areas has acted as an obstetrician for the wider interest in complexity theory among economists. Physicists entered the scene around 1995 with the ingenious invention of the provocative brand name of *econophysics* for their endeavors in this area. Both the empirical methodology and the principles of theoretical modelling of this group were in stark contrast to the mainstream approach in economics so that broad groups of academic economists were initially quite unappreciative of this new current. While the sheer ignorance of mainstream economics by practically all econophysists already stirred the blood of many mainstream economists, the fact that they seemed to have easy access to the popular science press and as representatives of a 'hard science' were often taken more seriously by the public than traditional economists, contributed to increased blood pressure among its opponents[1]. At the other end of the spectrum, the adaptation of statistical physics methods has been welcomed by economists critical of some aspects of the standard paradigm. Econophysics, in fact, had a close proximity to attempts at allowing for

[1] The author of this review once received a referee report including several pages of refutation of the econophysics approach. Strangely enough, the paper under review was a straight econometric piece and the referee's scolding seemed only to have been motivated by the author's association with some members of the econophysics community in other projects.

heterogeneous interacting agents in economic models. It is in this strongly increasing segment of academic economics where complexity theory and econophysics have made the biggest impact.

In the following I will review the econophysics contribution to various areas of economics/finance and compare it with the prevailing traditional economic approach.

2 Power Laws in Financial Markets: Phenomenology and Explanations

2.1 Financial Power Laws: Fat Tails and Volatility Clustering as Scaling Laws

Scaling laws or power laws (i.e., hyperbolic distributional characteristics of some measurements) are the most sought imprint of complex system behavior in nature and society. Finance luckily offers a number of robust scaling laws which are well accepted among empirical researchers. The most pervasive finding in this area is that of a ubiquitous power-law behavior of large price changes which had been confirmed for practically all types of financial data and markets. In applied research, the quantity one typically investigates is relative price changes or returns: $r_t = \frac{p_t - p_{t-1}}{p_{t-1}}$ (p_t denoting the price at time t). For daily data, the range of variability of r_t is roughly between -0.2 and +0.2 which allows replacement of r_t by log-differences (called continuously compounded returns) $r_t \cong \ln(p_t) - \ln(p_{t-1})$ which for high frequency data would practically obey the same statistical laws. Statistical analysis of daily returns offers overwhelming evidence for a hyperbolic behavior of the tails:

$$Pr(\mid r_t \mid > x) \sim x^{-\alpha} \tag{1}$$

Figure 1 illustrates this finding with a selection of stock indices and foreign exchange rates. As one can see the linearity in a loglog plot imposed by eq. (1) is a good approximation for a large fraction of both the most extreme positive and negative observations. Obviously, the power law of large returns is of outmost importance not only for researchers in complex system theory, but also for anyone investing in financial assets: eq. (1) allows a probabilistic assessment of the chances of catastrophic losses as well as the chances for similarly large gains and, therefore, is extremely useful in such mundane occupations like risk management of portfolios. To be more precise, our knowledge concerning the scaling law eq. (1) can be concretized as follows:

- the overall distribution of returns looks nicely bell-shaped and symmetric. It has, however, more probability mass in the center and the extreme regions (tails) than the benchmark bell-shaped Normal distribution,
- the tails have a hardly disputable hyperbolic shape starting from about the 20 to 10 percent quantiles at both ends,
- the left and right hand tail have a power-law decline with about the same decay factor α (differences are mostly not statistically significant),
- for different assets, estimated scaling parameters hover within a relatively narrow range around $\alpha = 3$. Fig. 1 exhibits this benchmark of a 'cubic law' of large returns together with a sample of empirical data scattered around it.

The literature on this *scaling law* is enormous. It starts with Mandelbrot's (1963) and Fama's (1963) observation of leptokurtosis in cotton futures and their proposal of the Levy distributions as a statistical model for asset returns (implying a power law tail with exponent $\alpha < 2$). For thirty years, the empirical literature in finance has discussed evidence in favor and against this model. A certain clarification has been achieved (in the view of most scientists involved in this literature) by moving from parametric distributions to a semi-parametric analysis of the tail region. Pertinent studies (e.g. Jansen and de Vries, 1991; Lux, 1996) have led to a rejection of the stable distribution model demonstrating that α is typically significantly above 2. While this controversy was going to be settled in the empirical finance literature, the emergent econophysics approach had repeated the thirty-year development in economics within a shorter time interval. Both an early paper by Mantegna (1991) and one of the first widely acknowledged econophysics papers by Mantegna and Stanley (1995) have advocated the Levy distribution, but subsequent work by the same group pointed out 'the universal cubic law' of asset returns (Gopikrishnan et al., 1998).

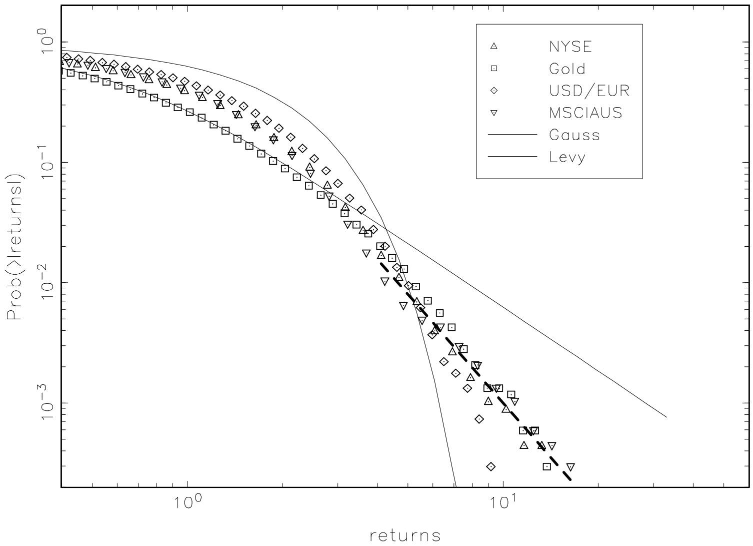

Figure 1: The Scaling Law of Large Returns: log-log plot of the complement of the cumulative distribution of daily returns from a sample of representative financial markets: the NYSE composite index, the MSCI index of the Australian stock market, the price of gold and the USD against EURO exchange rate (pre 1999 the DEM was used instead of the EURO). All series cover the period 1979 to 2004 and were obtained from Datastream. Despite some variations between these series, their tail regions are all close to a scaling law with index $\alpha \approx 3$ (demarcated by the broken line). This universal behavior of financial returns is intermediate between the exponential decline of the Normal distribution and the more pronounced tail fatness of members of the Levy stable family. Our example of the latter family of distributions has $\alpha = 1.7$, a value characteristically obtained when estimating the parameters of these distributions for financial data.

The finding of a power law according to eq. (1) is remarkable as it identifies a truly *universal* property within the social universe. Note also that in contrast to many other power laws claimed in social sciences and economics (cf. Cioffi, 2008, for an overview), the statistical basis of this law compares favorably to those of similar universal constants in the natural sciences: financial markets provide us with huge amounts of data at all frequencies and the power-law scaling has been confirmed over space and time without any apparent exception.

The power law in the vicinity of $\alpha = 3$ to 4 is also remarkable since it implies a kind of universal pre-asymptotic behavior of financial data at certain frequencies. In order to see this, note that according to the central limit law, random variates with $\alpha > 2$ fall into the domain of attraction of the Normal distribution, while random variates with $\alpha < 2$ would have the Levy stable distributions as their attractors. Under aggregation, returns with their leptokurtotic shape at daily horizons should, therefore, converge to a standard Gaussian. Aggregation of returns generates returns over longer time horizons (weakly, monthly) which, in fact, appear to be the closer to the Normal the higher the level of aggregation (time horizon). On the other hand, our benchmark daily returns can be conceived as aggregates of intra-daily returns. Since the tail behavior should be conserved under aggregation, the scaling laws at the daily horizon should also apply to intra-daily returns which is nicely confirmed by available high-frequency data.

The power law for large returns has as its twin a similarly universal feature which also seems to characterize all available data sets without exception: hyperbolic decay of the auto-covariance of any measure of *volatility* of returns. The simplest such measures are *absolute* or *squared* returns which preserve the extent of fluctuations but disregard their direction. Taking absolute returns as an example, this second pervasive power law can be characterized by

$$Cov(|r_t|, |r_{t-\Delta t}|) \sim \Delta t^{-\gamma} \tag{2}$$

The estimated values of γ have received less publicity than those of α, but reported statistics also show remarkable uniformity across time series with $\gamma \cong 0.3$ being a rather typical finding. It is worthwhile pointing out that eq. (2) implies very strong correlation of volatility over time. Hence, expected fluctuations of market prices in the next periods would be the higher the more volatile today's market is. Visually, one observes typical switches between tranquil and more turbulent episodes in the data (*"volatility clustering"*). This dependency can be exploited for prediction of the future development of volatility which would also be important information for risk and portfolio management. Again, the literature on this topic is huge. For quite some time, the *long-term dependency* inherent in power-law decline of eq. (2) had not been properly taken into account. Available models in financial econometrics like the legacy of GARCH models (Engle, 1983, Bollerslev, 1986) have rather modeled the volatility dynamics as a stochastic process with exponential decay of the autocovariance of absolute (or squared) returns. Long-term dependence has been demonstrated first by Ding, Engle and Granger (1993) in the economics literature and, independently, by Liu et al. (1997) and Vandewalle and Ausloos (1997) in contributions in physics journals. The measurement of long-range dependence in the econophysics publications is mostly based on estimation of the Hurst exponent from Mandelbrot's R/S analysis or the refined detrended fluctuation analysis of Peng et al. (1994). The financial engineering literature has taken long-term dependence into account by moving from the original GARCH to FIGARCH and long-memory stochastic volatility models (Breidt et al., 1998) which allow for hyperbolically decaying autocorrelations.

Besides the two above universal features, the literature has also pointed out additional power-law features of financial data. From the wealth of statistical analyses it seems that long-range dependence of *trading volume* is as universal as long-term dependency in volatility (Lobato and Velasco, 2000). Although exponents do not appear to be entirely identical, it is likely that the generating mechanism for both should be related (since trading volume is the ultimate source of price changes and volatility).
Additional power-laws have been found for high-frequency data from the U.S. stock market (Gopikrishnan et al., 2001):

(i) the *unconditional* distribution of volume in this data is found to follow a scaling law with exponent ~ 1.5 and,

(ii) the number of trades per time unit has been claimed to follow a power-law with index ~ 3.4.

2.2 Possible Explanations of Financial Power Laws

Gabaix et al. (2003) have offered a theoretical framework in which the above findings are combined with the additional observation of a Zipf's law for the size distribution of mutual funds (i.e., power-law with index ~ 1) and a square root relationship between transaction volume (V) and price changes (Δp): $\Delta p \sim V^{0.5}$. In this theory the power law of price changes is derived from a simple scaling arithmetic: combining the scaling of portfolios of big investors (mutual funds) with the square root price impact function and the distribution of trading volume in U.S. data, one obtains a cubic power-law for returns. Although their model adds some behavioral considerations for portfolio changes of funds, in the end the unexplained Zipf's law is at the origin of all other power-laws. Both the empirical evidence for some of the new power-laws and the power-law arithmetic itself are subject to a controversial discussion in recent econophysics literature (e.g. Farmer and Lillo, 2004). In particular, it seems questionable whether the power law with exponent 1.5 for volume is universal (it has not been confirmed in other data, cf. Farmer and Lillo, 2004; Eisler and Kertész, 2005) and whether the above linkage of power laws from the size distribution of investors to volume and returns is admissible for processes with long-term dependence à la eq. (2).

It is worthwhile to emphasize that the power-laws in the financial area would, in this theory, be due to other power-laws characterizing the size distribution of investors. The latter would probably have to be explained by a more complete theory of the economy along with the distribution of firm sizes and other macroeconomic characteristics. This somewhat subordinate role of the financial market in Gabaix et al. (2003) is certainly at odds with perceptions of *"criticality"* and *"phase transitions"* displayed in this market. The derivative explanation is also in contrast to what economists call the *"disconnect paradox"*, i.e. the observation that share markets and foreign exchange markets seem to develop a life of their own and at times appear entirely disconnected from their underlying economic fundamentals. Aoki and Yoshikawa (2007, c.10) point out that the power law behavior of financial returns is not shared by macroeconomic data which are rather characterized by exponentially distributed increments. They argue that the wedge between the real economy and financial markets stems from the higher level of activity of the latter. They demonstrate that, when modeling the dynamics of economic quantities as truncated Levy flights, different limiting distributions can emerge depending on the frequency of elementary events. Beyond a certain threshold, the limiting distribution switches from exponential to power law. The conjecture, then, is that this dependency on the number of contributing random events is responsible for the difference between exponential increments of macroeconomic data and power-law tails in financial markets. From an economic perspective the excess of relevant micro events in the financial sphere would be due to the decoupling from the real sector and the autonomous speculative activity in share and foreign exchange markets.

Most models proposed in the behaviorally orientated econophysics literature attempt to model this speculative interaction via simple models that are designed along certain prototypical behavioral types found in financial markets. Behavioral models of speculative markets have been among the first publications inspired by statistical physics. However, contrary to some claims from the physics community, physicists have not been the first and foremost in simulating markets. Earlier examples in the economics literature include Stigler (1964) and Kim and Markowitz (1989). The earliest econophysics example is Takayasu et al. (1992) who in a continuous double auction, let agents' bid and ask prices change according to given rules and studied the statistical properties of the resulting price process. A similar approach has been pursued by Sato and Takayasu (1998) and Bak, Paczuski and Shubik (1997). Independently, Levy, Levy and Solomon (1994, 1995, 2000) have developed a multi-agent model inspired by statistical physics which, at first view, looks more conventional than the previous ones: agents possess a well-defined utility function which they attempt to maximize by choosing an appropriate portfolio of stocks and bonds. They adopt a particular expectation formation scheme (expected future returns are assumed to be identical to the mean value of past returns over a certain time horizon), and impose short-selling and credit constraint as well as idiosyncratic stochastic shocks to individuals' demand for shares. Under these conditions, the market switches between periodic booms and crashes whose frequency depends on agents' time horizons. Although this model and its extensions produce spectacular price paths, their statistical properties are not really in line with the empirical findings outlined above, nor are those of the other early models (cf. Zschischang and Lux, 2001). Somewhat ironically, these early econophysics papers on financial market dynamics have been in fact similarly ignorant of the stylized facts (i.e. the scaling of eqs. 1 and 2) like most of the traditional finance literature.

The second wave of models was more directly influenced by the empirical literature and had the declared aim of providing candidate explanations for the observed scaling laws. Mostly, they performed simulations of 'artificial' financial markets with agents obeying a set of plausible behavioral rules and demonstrated that pseudo-empirical analysis of the generated time series yields results close to empirical findings. To our knowledge, the model by Lux and Marchesi (1999, 2000) has been the first which generated both the (approximate) cubic law of large returns and temporal dependence of volatility (with realistic estimated decay parameters) as emergent properties of their market model. This model had its roots in earlier attempts at introducing heterogeneity into stochastic models of speculative markets. It had drawn some of its inspiration from Kirman's (1993) model of information transmission among ants which had already been used as a model of interpersonal influences in a foreign exchange market in Kirman (1991). While Kirman's model had been based on pair-wise interaction, Lux and Marchesi had a mean-field approach in which an agent's opinion was influenced by the average opinion of all other traders. Using statistical physics methodology, it could be shown that a simple version of this model was capable of generating bubbles with over- and undervaluation of an asset as a reflection of the emergence of a majority opinion among the pool of traders. Similarly, periodic oscillations and crashes could be explained by the breakdown of such majorities and the change of *"market sentiment"* (Lux, 1995). A detailed analysis of second moments can be found in Lux (1997) where the explanatory power of stochastic multi-agent models for time-variation of volatility has been pointed out (Ramsey, 1996, also emphasizes the applicability of statistical physics methods for deriving macroscopic laws for second moments from microscopic behavioral assumptions). The group dynamics of these early interaction models have been enriched in the simulation studies by Lux and Marchesi (1999, 2000) and Chen, Lux and Marchesi (2001) by allowing agents to switch between a chartist and fundamentalist strategy in response to differences in profitability of both strategies. Interpersonal influences enter via chartists' attempts to trace out information from both ongoing price changes as well as the observed 'mood' of other traders. Assuming that the market is continuously hit by news on fundamental factors, one could investigate in how far price changes would reflect incoming information (the traditional view of the efficient market paradigm) or would be disconnected from fundamentals. The answer to this question turned out to have two different aspects: on the one hand, the speculative market on average kept close track of the development of the fundamentals. All new information was incorporated into prices relatively quickly as otherwise fundamentalist traders would have accepted high bets on reversals towards the fundamental value. On the other hand, however, upon closer inspection, the output (price changes) differed quite significantly from the input (fundamental information) in that price changes were always characterized by the scaling laws of eq. (1) and eq. (2) even if the fundamental news were modeled as a white noise process without these features. Hence; the market was never entirely decoupled from the real sphere (the information), but in processing this information it developed the ubiquitous scaling laws as *emergent properties* of the macroscopic market statistics from the distributed activity of its independent subunits.

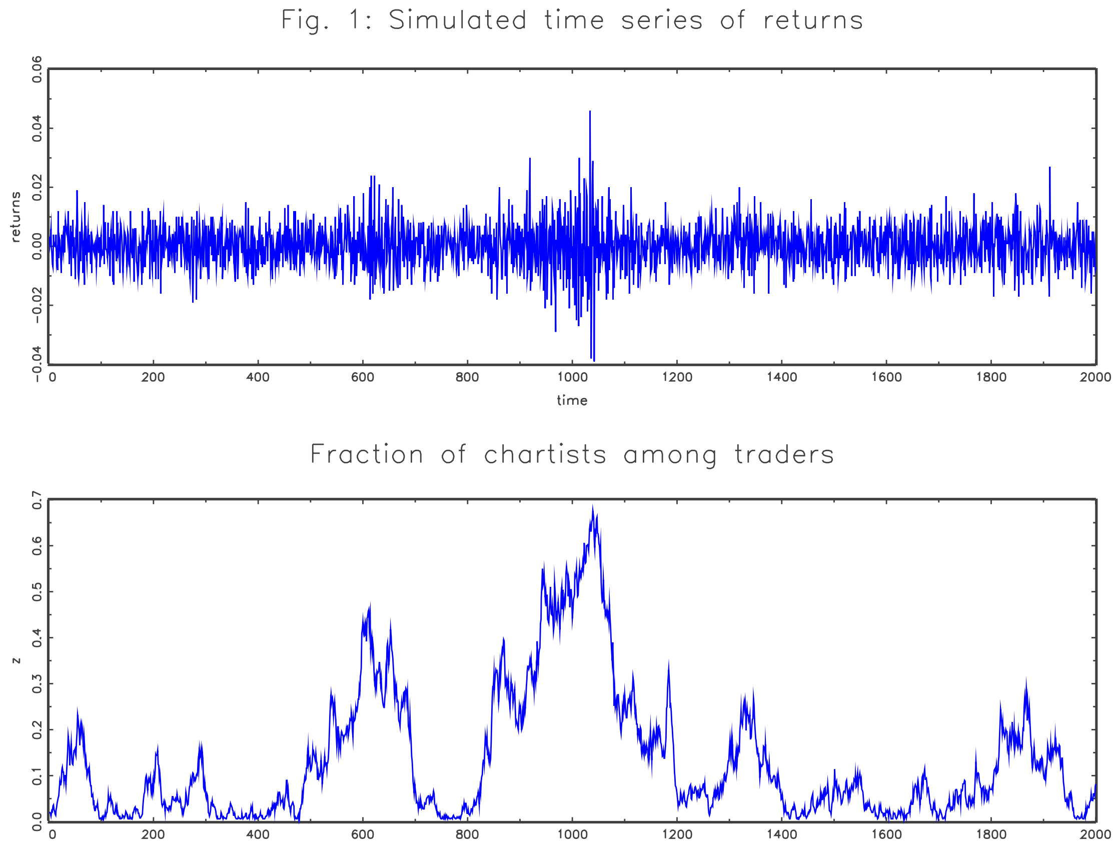

Figure 2: A snapshot of the evolution of prices and the composition of the pool of traders in the microscopic market model proposed by Lux and Marchesi (1999). The upper panel exhibits returns (relative price changes between unit time intervals), the lower panel shows the simultaneous changes of the fraction of chartists within the population. Note that the remaining part of the population follows a fundamentalist strategy. As can be seen a higher fraction of chartists leads to an increase in the volatility of price changes.

This result could also be explained to some extent via an analysis of approximate differential equations for the mean value dynamics of state variables derived from the mean-field approach (Lux, 1998; Lux and Marchesi, 2000). In particular, one finds that, in a steady state, the composition of the population is *indeterminate*. The reason is that, in a stationary environment, the price has to equal its fundamental value and no price changes are expected by agents. In such a situation, neither chartists nor fundamentalists would have an advantage over the other group as neither mispricing of the asset nor any discernible price trend prevails. In the vicinity of such a steady state, movements between groups would, then, only be governed by stochastic factors which would lead to a *random walk in strategy space.* However, in this model (and in many related models), the composition of the population determines stability or instability of the steady state. Quite plausibly (and in line with a large body of literature in behavioral finance), a dominance of chartists with their reinforcement of price changes will be destabilizing. Via bifurcation analysis one can identify a threshold value for the number of chartists at which the system becomes unstable. The random population dynamics will lead to excursions into the unstable region from time to time which leads to an onset of severe fluctuations. The ensuing deviations of prices from the fundamental value, however, will lead to profit differentials in favor of the fundamentalist traders so that their number increases and the market moves back to the stable subset of the strategy space. As can be seen from Fig. 2, the joint dynamics of the population composition and the market price has a close resemblance to empirical records. With the above mechanism of intermittent switching between stability and (temporal) instability the model does not only exhibit interesting *emergent properties*, but it also can be characterized by another key term of complexity theory: *criticality*. Via its stochastic component, the system approaches a critical state where it temporally looses stability and the ensuing out-of-equilibrium dynamics give rise to stabilizing forces. One might note that *"self-organizing criticality"* would not be an appropriate characterization as the model does not have any systematic tendency towards the critical state. The trigger here is a purely stochastic dynamics without any preferred direction.

Lux and Marchesi argue that, irrespective of the details of the model, indeterminateness of the population composition might be a rather general phenomenon in a broad range of related models (because of the absence of profitability of *any* trading strategy in *any* steady state). Together with dependency of stability on the population composition, the intermittent dynamics outlined above should, therefore, prevail in a broad class of microscopic interaction models. Support for this argument is offered by Lux and Schornstein (2005) who investigate a multi-agent model with a very different structure. Adopting the seminal Kareken and Wallace (1983) approach to exchange rate determination, they consider a foreign exchange market embedded into a general equilibrium model with two countries and overlapping generations of agents. In this setting, agents have to decide simultaneously on their consumption and savings together with the composition of their portfolio (domestic vs. foreign assets). Following Arifovic (1996), agents are assumed to be endowed with artificial intelligence (genetic algorithms) which leads to an evolutionary learning dynamics in which agents try to improve their consumption and investment choices over time. This setting also features a crucial indeterminacy of strategies: in a steady state, the exchange rate remains constant so that holdings of domestic and foreign assets would earn the same return (assuming that returns are only due to price changes). Hence, the portfolio composition would be irrelevant as long as exchange rates do not change (in steady state) and any configuration of the GA for the portfolio part would have the same pay-off. However, out-of-equilibrium different portfolio weights might well lead to different performance as an agent might profit or loose from exchange rate movements depending on the fraction of foreign or domestic assets in her portfolio. The resulting dynamics again shares the scaling laws of empirical data. Similarly, Giardina and Bouchaud (2003) allow for more general strategies than Lux and Marchesi (1999) but also found a random walk in strategy space to be at the heart of emergent realistic properties.

A related branch of models with critical behavior has been launched by Cont and Bouchaud (2000). Their set-up also focuses on interaction of agents. However, they adapt the framework of percolation models in which agents are situated on a lattice with periodic boundary conditions. In percolation models, each site of a lattice might initially be occupied (with a certain probability p) or empty (with probability $1-p$). Clusters are groups of occupied neighboring sites (various definitions of neighborhood could be applied). In

Cont and Bouchaud occupied sites are traders and trading decisions (buying or selling a fixed number of assets or remaining inactive) are synchronized within clusters. Whether a cluster buys or sells or does not trade at all, is determined by random draws. Given the trading decisions of all agents, the market price is simply driven by the difference between overall demand and supply. Being modelled closely along the lines of applications of percolation models in statistical physics, it follows immediately that the distribution of returns (price changes) is connected to the scaling laws established for the cluster size distribution. Therefore, if the probability for connection of lattice sites, say q, is close to the so-called percolation threshold q_c (the critical value above which an infinite cluster will appear), the distribution will follow a power law. As detailed in Cont and Bouchaud, the power-law index for returns at the percolation threshold will be 1.5, some way apart from the *"cubic law"*. As shown in subsequent literature, finite-size effects and variations of parameters could generate alternative power laws, but a cubic law would emerge only under particular model designs (Stauffer and Penna, 1998). Autocorrelation of volatility is entirely absent in these models, but could be introduced by sluggish changes of cluster configurations over time (Stauffer et al., 1999). If clusters dissolve or amalgamate after transactions, more realistic features could be obtained (Eguiluz and Zimmerman, 2000). As another interesting addition, Focardi et al. (2002) consider latent connections which only become active in times of crises. Alternative lattice types have been explored by Iori (2002) who considers an Ising type model with interactions between nearest neighbors. This approach appears to generate more robust outcomes and seems not to suffer from the percolation models' essential need to fine tune the parameter values at criticality for obtaining power laws. Another recent alternative is a cellular automaton model of percolation proposed by Bartolozzi and Thomas (2004). In this model each cell is occupied by one trader who might buy, sell or remain inactive at any time step. Traders influence their neighbors, become inactive or are activated spontaneously with certain probabilities. It is shown that time series from this model have realistic properties if the probability for natural influence among traders is sufficiently high. If traders are subjected to strong interpersonal influences, relatively large clusters of homogenous trading activity will emerge and these clusters of agents will lead to clusters of volatility.

As long as no self-organizing principles are offered for the movements of the system towards the percolation threshold, the extreme sensitivity of percolation models with respect to parameter choices is certainly unsatisfactory. Sweeping these systems back and forth through a critical state is an interesting variation (explored by Stauffer and Sornette, 1999) that gets rid of the necessity for fine-tuning of parameters. In the context of a stock market model, Stauffer and Sornette are able to get a robust cubic power law for returns. However, the behavioral underpinnings for such sweeping dynamics remain to be elucidated.

3 Other Applications in Financial Economics

The contributions of physicists to financial economics are voluminous. A great part of it is of a more applied nature and does not necessarily have any close relationship to the methodological view expressed in the manifesto of the Boston group of pioneers in this field:

"Statistical physicists have determined that physical systems which consist of a large number of interacting particles obey laws that are independent of the microscopic details. This progress was mainly due to the development of scaling theory. Since economic systems also consist of a large number of interacting units, it is plausible that scaling theory can be applied to economics" (Stanley et al., 1996).

However, rather than investigating the underlying forces responsible for the universal scaling laws of financial markets, a relatively large part of the econophysics literature mainly adopts physics tools of analysis to more practical issues in finance. This line of research is the academic counterpart to the work of *"quants"* in the financial industry who mostly have a physics background and are occupied in large numbers for developing quantitative tools for forecasting, trading and risk management. The material published in the journal *Quantitative Finance* (launched in 2000) provides ample examples for this type of applied work. Similarly, some of the monographs and textbooks from the econophysics camp have a strong focus on applied quantitative work in financial engineering. A good example is the well-known monograph by Bouchaud and Potters (2000) whose list of contents covers an introduction to probability and the statistics of financial prices, portfolio optimization and pricing of futures and options. While the volume provides a very useful and skilled introduction to these subjects, it has only cursory references to a view of the market as a complex system of interacting subunits. Much of this literature, in fact, fits well into the mainstream of applied and empirical research in finance although one often finds a scolding of the carefully maintained straw man image of traditional finance. In particular, ignoring decades of work in dozens of finance journals, it is often claimed that "economists believe that the probability distribution of stock returns is a Gaussian", a claim that can easily be refuted by a random consultation of any of the learned journals of this field. In fact, while the (erroneous) juxtaposition of scaling (physics!) via Normality (economics!) might be interpreted as an exaggeration for marketing purposes, some of the early econophysics papers even gave the impression that what they attempted was a first quantitative analysis of financial time series ever. If this was, then, performed on a level of rigor way below established standards in economics (a revealing example is the analysis of supposed day-of-the-week effects in high-frequency returns in Zhang, 1999)[2] it clearly undermined the standing of econophysicists in the economics community.

However, among the (sometimes reinventive and sometimes original) contributions of physicists to empirical finance, portfolio theory and derivative pricing, a few strands of research stand out which certainly deserve a more detailed treatment. These include the intricate study of the microstructure of order book systems, new approaches to determining correlations among assets, the proposal of a new type of model for volatility dynamics (so-called multifractal models), and the much promoted attempts at forecasting financial downturns.

[2]The reader might compare this paper with the more or less simultaneous paper by Sullivan, White and Golomb (2001) which is quite representative of the state-of-the-art in this area in empirical finance.

3.1 The Dynamics of Order Books

The impact of the institutional details of market organization is the subject of market microstructure theory (O'Hara, 1995). According to whether traders from the demand and supply side are getting into direct contact with each other or whether trades are carried out by middlemen (called market makers or specialists) one distinguishes between order-driven and quote-driven markets. The latter system is characteristic of the traditional organization of the U.S. equity markets in which trading has been organized by designated market makers whose task it was and still is to ensure continuous market liquidity. This system is called quote-driven since the decisive information for traders is the quoted *bid* and *ask prices* at which the market makers would accept incoming orders. In most European markets, these active market makers did not exist and trading was rather organized in a continuous *double auction* in which all orders of individual traders are stored in the *order book*. In this system, traders could either post limit orders which would have to be carried out when a pre-specified limiting price is reached over time and are stored in the book until execution, or market orders which are carried out immediately. The order book, thus, covers a whole range of limit-orders on both the demand and supply sides with a pertinent set of desired transaction volumes and pertinent prices. This information can be viewed as a complete characterization of the demand and supply schedules with the current transaction price and volume being determined at the intersection of both curves. Most exchanges provide detailed data records with all available information on the development of the book, i.e. time-stamped limit and market orders with their volumes, limit bid and ask prices and cancellation or execution times. The recent literature contains a wealth of studies of order book dynamics, both empirical analyses of the abundant data sets of various exchanges as well as numerical and theoretical studies of agents' behavior in such an institutional framework.

Empirical research has come up with some insights on the distributional properties of key statistics of the book. The distribution of incoming new limit orders had been found to obey a power-law in the distance from the current best price in various studies. There is, however, disagreement on the coefficient of this scaling relationship: while Farmer and Zovko (2002) report numbers around 1.5 for a sample of fifty stocks traded at the London

Stock Exchange, Bouchaud et al. (2002) rather find a common exponent of ~ 0.6 for three frequently traded stocks of Paris Bourse. The average volume in the queue of the order book was found to follow a Gamma distribution with roughly identical parameters for both the bid and ask side. This hump-shaped distribution with a maximum at a certain distance from the current price can be explained by the fact that past fluctuations might have thinned out limit orders in the immediate vicinity of the mid-price while those somewhat further away had a higher survival probability.

A recurrent topic in empirical studies of both quote-driven and order-driven systems had been the shape of the price impact function: as has been reported above, Gopikrishan et al. found a square-root dependency on volume in NYSE data: $\Delta p \sim V^{0.5}$. Conditioning on *volume imbalance* (Ω), i.e. the difference between demand and supply, Plerou et al. (2003) found an interesting bifurcation: while the conditional density of Ω on its first absolute moment (Σ) was uni-modal for small Σ it developed into a bi-modal distribution for larger volume imbalances. They interpret this finding as indication of two different phases of the market dynamics: an *equilibrium phase* with minor fluctuations around the current price and an out-of-equilibrium phase in which a predominance of demand or supply leads to a change of the mid price in one or the other direction. However, Matia and Yamazaki (2005) show that this feature appears quite naturally in simulation experiments if the distribution of volume follows a power law simply because large positive and negative realisations of Ω give rise to a bimodal distribution. This feature can, therefore, be explained almost mechanically and need not be due to the alleged presence of critical phenomena. Matia and Yamazaki also criticize the sloppy use of the concept of a phase transition in Plerou et al.'s *Nature* paper: While phase transitions in physics have an independent variable as the control parameter, here it is a moment of the order parameter itself.

The empirical work on order book dynamics is often accompanied by theoretical work or simulation studies trying to explain the observed regularities. Some of the earliest models in the econophysics literature already contained simple order book structures with limit orders being posted according to some simple stochastic rule. In Bak *et al.* (1997), bid and ask prices of individual agents change randomly over time with equal probabilities for upward and downward movements. If bid and ask prices cross each other, a trade between two agents will be induced. The two agents' bids and asks are subsequently cancelled. The agents are then reinjected into the market with randomly chosen new limit orders between the current market price and the maximum ask or minimum bid, respectively. Like in many early models this stochastic design of the market amounts to a process that has been studied before (it is isomorphic to a reaction-diffusion process in chemistry). For this process, the time variation of the price can be shown to follow a scaling law with Hurst exponent $1/4$. Since this is clearly an unrealistic behavior, the authors expand their model by allowing for volatility feedback (the observed price change influencing the size of adjustment of bid and ask prices). In this case $H = 0.65$ is estimated for simulated time series which would rather speak for long-term dependence in the price dynamics. Although it is plausible that the volatility feedback could lead to this outcome, this is also different from the well-known martingale behavior of financial markets with $H = 0.5$ (Bak *et al.* quote some earlier econometric results which indeed found $H \approx 0.6$, but these are viewed as being due to biases of their estimation procedure nowadays).

A number of other papers have followed this avenue of analyzing simple stochastic interaction processes without too many behavioral assumptions: Tang and Tian (1999) provided analytical foundations for the numerical findings of Bak *et al.*. Maslov (1999) seemed to have been the first to attempt to explain fat tails and volatility clustering from a very simple stochastic market model. His model is built around two essential parameters: in every instant a new trader appears in the market who with probability q places a limit order and with the complementary probability $1 - q$ trades at the current market price. New limit orders are chosen from a uniform distribution with a support on $[0, \Delta_m]$ above or below the price of the last transaction. Slanina (2001) provides theoretical support for a power-law decline of price changes in this model. The mechanics of volatility clustering in this set-up might be explained as follows: if prices change only very little, more and more new limit orders in the vicinity of the current mid price built up which leads to persistence of low levels of volatility. Similarly, if price movements have been more pronounced in the past, the stochastic mechanism for new limit orders generates a more dispersed distribution of entries in the book which also leads to persistence of a high-volatility regime. Very similar models have been proposed by Smith *et al.* (2002) and Daniels *et al.* (2003) whose main message is that a concave price impact can emerge from such simple models without any behavioral assumptions.

What is the insight from this body of empirical research and theoretical models? First, the analysis of the huge data sets available from most stock markets might allow to identify additional stylized facts. So far, however, evidence for robust features, applying to more than one market, appears sparse. It rather seems that some microstructural features do vary between markets such as, e.g., the distribution of limit orders within the book. Second, the hope of simple interaction models is to explain stylized facts via the organization of the trading process. In a sense, this line of research is similar to earlier work in economics on markets with *zero-intelligence traders* (Gode and Sunder, 1993) and, in fact, physicists have often adopted this label for their pertinent research. However, the zero-intelligence literature in economics had a clear interest in the allocative efficiency of markets in the presence of agents without any understanding of the market mechanism. Such a criterion is absent in the above models: while one gets certain distributions of market statistics under certain assumptions on arrival probabilities of traders and the distribution of their limit orders, it is not clear how to compare different market designs. A clear benchmark both for the evaluation of the explanatory power of competing models as well as for normative conclusions to be drawn from their outcomes are entirely absent. As concerns explanatory power, most models feature some stylized facts. However, what would be a minimum set of statistical properties and how robust they would have to be with respect to slight variations of the distributional assumptions has not been specified in this literature. Any normative evaluation, for example with respect to excessive volatility caused by certain market mechanisms, is impossible simply because prices are not constrained at all by factors outside the pricing process. Recent papers by Chiarella and Iori (2002) have made some progress in this perspective by considering different trading strategies (chartist, fundamentalist) in an order-book setting. They note that incorporation of these behavioral components is necessary in their model for generating realistic time series.

3.2 Analysis of Correlation Matrices

The study of cross-correlations between assets has attracted a lot of interest among physicists. This body of research has a strong resemblance to portfolio theory in classical finance. Consider the portfolio choice problem of an investor in an economy with an arbitrary number N of risky assets. One way to formulate this problem is to minimize the variance of the portfolio for a given required expected return $\bar{r}$. Solving this quadratic programming problem for all $\bar{r}$ leads to the well-known efficient frontier which depicts the trade-off the investor faces between the expected portfolio return and its riskyness (i.e., the variance). A central but problematic ingredient in this exercise (the so-called Markowitz problem, Markowitz, 1952) is the $N \times N$ covariance matrix. Besides its sheer size (when including all assets of a developed economy or, as one should do in principle, all assets available around the world's financial markets), stability and accuracy of historical estimates of cross-asset correlations to be used in the Markowitz problem are problematic in applied work. Furthermore, the formulation of the problem assumes either quadratic utility functions (so that investors only care about the first and second moments) or Normally distributed returns (so that the first and second moments are sufficient statistics for the entire shape of the distribution). Of course, both variants are easily criticized: returns are decisively non-Normal at least at daily frequency and developments like value-at-risk are a clear indication of more complex objective functions than mean-variance optimization.

The econophysics literature has contributed to the literature at various ends: first, some papers took stock of theoretical results from random matrix theory. Random matrix theory allows to establish bounds for the eigenvalues of a correlation matrix under the assumption that the matrix has random entries. As has been shown only a few eigenvalues 'survive' above the noise bands (Laloux *et al.*, 1999). In a comprehensive study of the U.S. stock market, Plerou *et al.* (2000) found that the deviating non-random eigenvalues were stable in time and that the largest eigenvalue corresponded to a common influence on all stocks (in line with the market portfolio of the *Capital Asset Pricing Model*). Various studies have proposed methods for identification of the non-random elements of the correlation matrix (Laloux *et al.*, 1999; Noh, 2000; Gallucio *et al.*, 1998). It can easily be imagined that efficient frontiers from the original correlation matrix might differ strongly from those generated from a correlation matrix that has been cleaned by eliminating the eigenvalues within the noise band. Quite plausible, the incorporation of arguably unreliable correlations may lead to an illusory high efficient frontier. According to the underlying argument, standard covariance matrix estimates might then vastly overstate the chances of diversification so that better performance could be expected from using cleaned-up matrices. An interesting recent contribution uses random matrix theory for complexity reduction in large multivariate GARCH models (Rosenow, 2008). As demonstrated in this paper, determination of the small number of significant components allows to easily estimate multivariate models with hundreds of stocks and to forecast portfolio volatility on the base of these estimates.

A closely related branch of empirical studies attempts to extract information on hierarchical components in the correlation structure of an ensemble of stocks. This line of research was pioneered by Mantegna (1999) who used an algorithm known as *minimum spanning tree* to visualize the correlation structures between stocks in a connected tree. Alternative methods for cluster identification have been proposed by Kullmann *et al.* (2002) and Onnela *et al.* (2003). A visualization is provided in Fig. 3 adopted from Onnela *et al.* From an economics point of view these approaches are germaine to so-called *factor models* that incorporate common risk factors (e.g. sector-specific or country-specific ones) into asset pricing models (Chen, Roll and Ross, 1986). The clustering algorithms could, in principle, provide valuable inputs for the implementation of such factor models. Unfortunately, the major weakness of available research in this area is that it has confined itself to illustrating the application of a particular methodology. However, it had hardly ever tried a rigorous comparison of refined methods of portfolio optimization or asset pricing based on random matrix theory or clustering algorithms (a remarkable exception is the mentioned contribution by Rosenow, 2008). There seems to be some cultural difference between the camps of economists/statisticians and physicists that makes the former insist on rigorous statistical tests while the later inexplicably shy away from such evaluations of their proposed theories and methods.

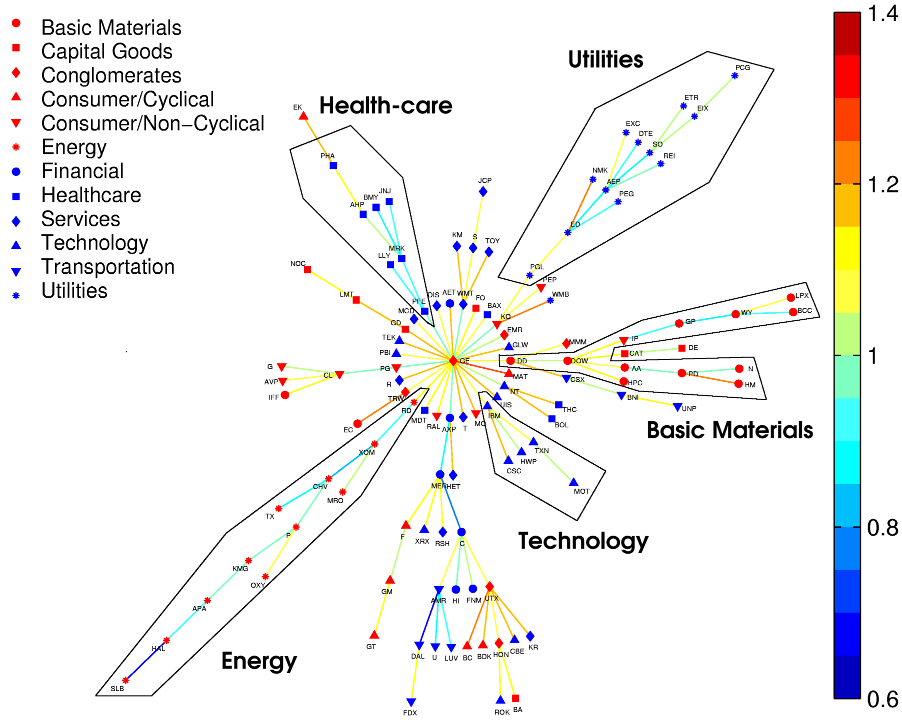

Figure 3: The hierarchical structure of the major U.S. stocks as indicated by a cluster identification algorithm. This taxonomy of a sample of 116 stocks has been obtained by constructing a so called minimum spanning tree for the mean correlation coefficients of stock returns over a certain time window (1996 to 1999 in the present case). By courtesy of J.-P. Onnela. Reprinted with permission from J.-P. Onnela et al., *Physical Review* E 68, 2003, 056110, ©2003 by the American Physical Society.

3.3 Forecasting Volatility: The Multifractal Model

There is, however, one area of application of statistical physics methods, in which researchers have rather successfully connected themselves to the mainstream of research in empirical finance: the small literature on multi-fractal models of asset returns. The introduction of so-called *multifractal models* (MF) as a new class of stochastic processes for asset returns was mainly motivated by the findings of their *multi-scaling* properties. Multi-scaling (often also denoted as multifractality itself) refers to processes or data which are characterized by different scaling laws for different moments. Generalizing eq. (2), these defining features can be captured by dependency of the temporal scaling parameter on the pertinent moment, i.e.

$$E\left[|r_t^q r_{t-\Delta t}^q|\right] \sim \Delta t^{-\gamma(q)}. \quad (3)$$

The phenomenology of eq. (3) has been described in quite a number of early econophysics papers. A group of authors at the *London School of Economics*, Vassilicos, Demos and Tata (1993) deserves credit for the first empirical paper demonstrating multi-scaling properties of financial data. Other early work of a similar spirit includes Ausloos and Vandewalle (1998) and Ghasghaie et al. (1996). The latter contribution estimates a particular model of turbulent processes from the physics literature and has stirred a discussion about similarities and differences between the dynamics of turbulent fluids and asset price changes (cf. Vassilicos, 1995; Mantegna and Stanley, 1996).

Note that eq. (3) implies that different powers of absolute returns (which all could be interpreted as measures of volatility) have different degrees of long-term dependency. In the economics literature, Ding, Engle and Granger (1993) had already pointed out that different powers have different dependence structures (measured by their ensemble of autocorrelations) and that the highest degree of autocorrelation is obtained for powers around 1. However, standard econometric models do not capture this feature of the data. Baseline models like GARCH and so-called stochastic volatility models rather have exponentially declining autocorrelations. While these models have been modified so as to allow for hyperbolic decline of the ACF

according to eq.(2)[3], no models have existed in the econometrics toolbox prior to the proposal of the MF model that generically could give rise to multi-scaling à la eq. (3). However, since data from turbulent flows also exhibit multi-scaling, the literature on turbulence in statistical physics had already developed models with these characteristics. These are known as multifractal cascade models and are generated via operations on probability measures. To model the break-off of smaller eddies from bigger ones one starts with a uniform probability measure over the unit interval $[0, 1]$. In the first step, this interval is split up into two subintervals of equal length (smaller eddies) which receive fractions p_1 and $p_2 = 1 - p_1$ of their 'mother intervals'. In principle, this procedure is repeated ad *infinitum* for the resulting subintervals cf. Fig. 4. What it generates is a heterogeneous structure in which the final outcome after n steps of emergence of ever smaller eddies can take any of the values $p_1^m p_2^{n-m}, 0 \leq m < n$. This process is highly autocorrelated since neighboring values have on average several joint components. In the limit of $n \to \infty$, 'strict' multifractality according to eq. (3) can be shown to hold.
The literature on turbulent flows has investigated quite a number of variants of the above algorithm. The above multifractal measure is called a Binomial cascade. However, instead of taking the same probabilities, one could also have drawn random numbers for the multipliers. An important example of the later class is the Lognormal model in which the two probabilities of the new eddies are drawn from a Lognormal distribution. Note that in this case, the overall mass of the measure is not exactly preserved (as in the Binomial), but is maintained only in expectation (upon appropriate choice of the parameters of the Lognormal distribution). While the mean is, thus, constant in expectation over different steps, other moments might converge or diverge. Other extensions imply transition from the case of two subintervals to a higher number (Trinomial, Quadrunomial cascades) or using irregularly spaced subintervals.

How to apply these constructs as models of financial data? While the multifractal measure generated in Fig. 4 does not exhibit too much similarity with price charts, we know that by its very construction it shares the

[3]The most prominent example is Fractionally Integrated GARCH, cf. Baillie *et al.* (1996).

multi-scaling properties of absolute moments of returns. Since multi-scaling applies to the *extent of fluctuations* (volatility), one would, therefore, interpret the non-observable process governing volatility as the analogue of the multifractal measure. The realizations over small subintervals would, then, correspond to local volatility. A broadly equivalent approach is to use the multifractal measure as a transformation of chronological time. Assuming that the volatility process is homogeneous in transformed time, then, means that via the multifractal transformation, time is compressed and retarded so that the extent of fluctuations within chronological time steps of the same length becomes heterogeneous. This idea was formulated by Mandelbrot, Calvet and Fisher (1997) in three seminal Cowles Foundation working papers which for the first time went beyond a mere phenomenological demonstration of multi-scaling in financial data. They assumed that log price changes follow a compound stochastic process in which the distribution function of a multifractal measure Θ serves as the directing process (transforming chronological time into business time) and the subordinate process is fractional Brownian motion B_H,

$$r(t) = B_H[\Theta(t)]. \tag{4}$$

In contrast to GARCH and stochastic volatility models, this model is scale-free so that one and the same specification can be applied to data of various sampling frequencies. Mandelbrot, Calvet and Fisher (1997) show how the stochastic properties of the compound process reflect those of the directing multifractal measure. They also introduce a so-called scaling estimator for the parameters of the process and apply it to both daily and intra-daily data of the U.S. \$-DEM foreign exchange market. A more systematic analysis of the underlying estimation procedure and additional empirical applications can be found in Calvet and Fisher (2002).

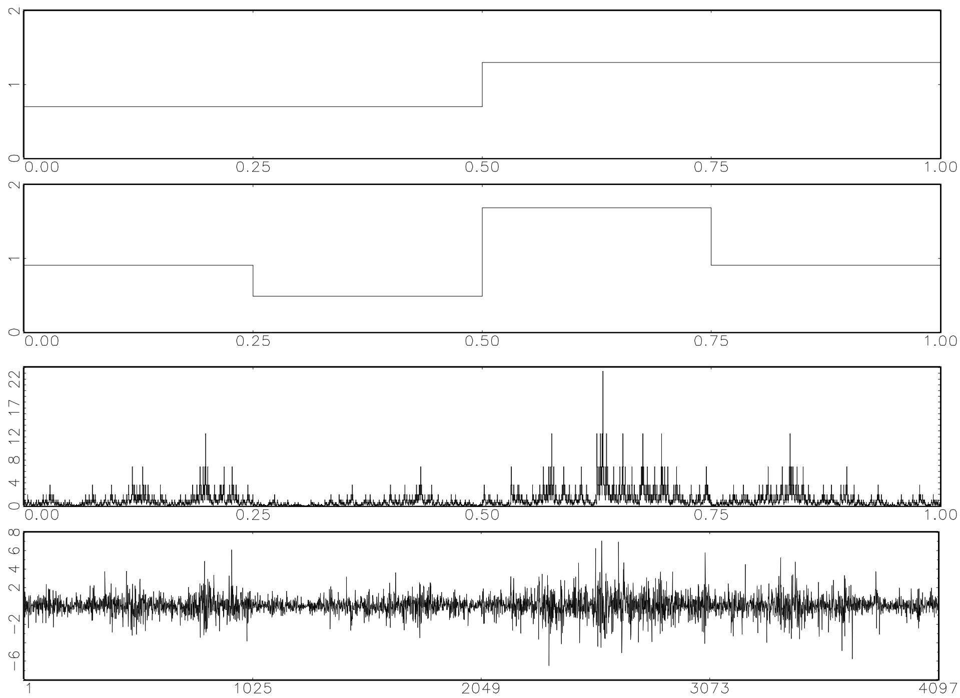

Figure 4: The construction of a multifractal cascade and its use as a time transformation. The first panel illustrates the segmentation of the unit interval into two segments of equal length receiving fractions $p_1 = 0.65$ and $p_2 = 0.35$ of the overall mass. The second panel shows the second stage of the Binomial cascade, while the third panel shows the result after 12 iterations of this process with a total of $2^{12} = 4096$ segments. Using these segments as time transformations in the sense of eq. 4 with $H = 0.5$ generates returns with heterogenous volatility (lower panel).

Unfortunately, the process as specified in eq. (4) has serious drawbacks that limit its attractiveness in applied work: due to its combinatorial origin, it is bounded to a prespecified interval (which in economic applications might be a certain length of time) and it suffers from non-stationarity. Application of many standard tools of statistical inference would, therefore, be questionable and the combinatorial rather than causal nature limits its applicability as a tool for forecasting future volatility.

These restrictions do not apply to a time series model by Calvet and Fisher (2001) which preserves the spirit of a hierarchically structured volatility process but has a much more 'harmless' format. Their *Markov-Switching Multifractal* process (MSM) can be interpreted as a special case of both Markov-switching and stochastic volatility models. Returns over a unit time interval are modeled as:

$$r_t = \sigma_t \cdot u_t \tag{5}$$

with innovations u_t drawn from a standard Normal distribution $N(0,1)$ and instantaneous volatility σ_t being determined by the product of k volatility components or multipliers,$M_t^{(1)}, M_t^{(2)}, \ldots, M_t^{(k)}$ and a constant scale factor σ:

$$\sigma_t^2 = \sigma^2 \prod_{i=0}^{k} M_t^{(i)}. \tag{6}$$

Each volatility component is renewed at time t with probability γ_i depending on its rank within the hierarchy of multipliers. Calvet and Fisher propose the following flexible form for these transition probabilities:

$$\gamma_i = 1 - (1 - \gamma_1)^{(b^k - 1)} \tag{7}$$

with parameters $\gamma_1 \in [0,1]$ and $b \in (1, \infty)$. This specification is derived by Calvet and Fisher (2001) as a discrete approximation to a continuous-time multifractal process with Poisson arrival probabilities and geometric progression of frequencies. They show that when the grid step size of the discretized version goes to zero, the above discrete model converges to the continuous-time process.

Estimation of the parameters of the model involves γ_1 and b as well as those parameters characterizing the distribution of multipliers. If a discrete distribution is chosen for the multipliers (e.g., a Binomial distribution with states

p_1 and $1-p_1$ like in our combinatorial example above), the discretized multifractal process is a well-behaved Markov-switching process with 2^k states. This framework allows estimation of its parameters via maximum likelihood. ML estimation of this process comes along with identification of the conditional probabilities of the current states of the volatility components which can be exploited for computation of one-step and multi-step forecasts of the volatility process using Bayes's rule. The Markov-switching multifractal model, thus, easily lends itself to practical applications. Calvet and Fisher (2004) demonstrate that the model allows for improvements over various GARCH-type volatility processes in a competition for the best forecasts of exchange rate volatility in various currency markets. A certain drawback of the ML approach is that it becomes computationally unfeasible for a number of volatility components beyond 10. Its applicability is also limited to MSM specifications with a finite state space so that it can not be applied to processes where multipliers are drawn from a continuous distribution (e.g., the Lognormal). Recent additions to extant literature introduce alternative estimation techniques that allow to deal with these cases: Calvet, Fisher and Thompson (2006) consider a simulated maximum likelihood approach based on a particle filter algorithm which allows to estimate both models with continuous state space and a new bi-variate MSM (which would be computationally too demanding for exact ML). Lux (2008) proposes a Generalized Method of Moments technique based on a particular selection of analytical moments together with best linear forecasts along the lines of the Levinson-Durbin algorithm. Both papers also demonstrate the dominance of the multifractal model over standard specifications in some typical financial applications. Financial applications of a different formalization of multifractal process can be found in Bacry et al. (2008).

The relatively small literature that has emerged on multifractal processes over the last decade could be seen as one of the most significant contributions of physics-inspired tools to economics and finance. In contrast to some other empirical tools, researchers in this area have subscribed to the level of rigor of empirical work in economics and have attempted to show in how far their proposed innovations provide an advantage over standard tools in crucial applications. Somewhat ironically, this literature is both better known and has had more of an impact in economics than in the econophysics community itself.

Available literature on MF models is altogether empirical in orientation and is not very informative on the origin of multifractality [4]. However, the empirical success of the multifractal model suggests that their basic structural set-up, a multiplicative hierarchical combination of volatility components, might be closer to the real thing than earlier additive models of volatility. Some speculation on the source of this multi-layer structure can be found, for example, in Dacorogna et al. (2001) who argue that different types of market participants with different time horizons are at the heart of the data-generating mechanism. To substantiate such claims would offer a formidable challenge to agent-based models. While there are few papers that demonstrate multi-scaling of artificial data from particular models (e.g. Castiglione and Stauffer, 2001, for a particular version of the Cont/Bouchaud model), it seems clear that most behavioral models available so far do not really have the multi-frequency structure of the stochastic MF models.

3.4 Problematic Prophecies: Predicting Crashes and Recoveries

While the success of MF volatility models has only received scant attention beyond the confines of financial econometrics, attempts at forecasting the time of stock market crashes from precursors became a notorious and highly problematic brand of econophysics activity. This strand of activity started with a number of papers offering ex-post formalizations of the dynamics prior to some major market crashes, e.g. the crash of October 1997 (Vandewalle and Ausloos, 1998; Sornette et al., 1996; Sornette and Johansen, 1999b, 1997). Adapting a formalization similar to that of precursor activity of earthquakes in geophysics, it was postulated that stock prices follow a log-periodic pattern prior to crashes which could be modelled by a dynamic equation of the type:

$$p_t = A + B\left(\frac{t_c - t}{t_c}\right)^{-m}\left[1 + C\cos(\omega \ln\frac{t_c - t}{t_c}) + \Phi\right] \qquad (8)$$

[4]However, Calvet, Fisher and Thomson (2006) relate low-frequency volatility components to certain macroeconomic risk factors.

for $t < t_c$. This equation generates fluctuations of accelerating frequency around an increasing trend component of the asset price. Such a development culminates in the singularity at the *critical time* t_c. Since the log-periodic oscillation breaks down at t_c this is interpreted as the estimate of the occurrence of the subsequent crash. Note that A, B, C, m, ω, t_c, and Φ are all free parameters in estimating the model without imposing a known crash time t_c. A voluminous literature has applied this model (and slightly different versions) to various financial data discovering - as it had been claimed - dozens of historical cases to which the log-periodic apparatus could be applied. This business had subsequently been extended to 'anti-bubbles', mirror-imaged log-periodic downward movements which should give rise to a recovery at criticality. Evidence for this scenario has first been claimed for the Nikkei in 1999 (Johansen and Sornette, 1999a) and had also been extended to various other markets shortly thereafter (Johansen and Sornette, 2001b). Somewhat unfortunately, the details of the estimation algorithm for the many parameters of the highly nonlinear log-periodic equation have not been spelled out exactly in all this literature and an attempt at replicating some of the published estimates reported that the available information was not sufficient to arrive at anything close to the authors' original results (Feigenbaum, 2001a, b). Eventually, the work in this area culminated via an accelerating number of publications and log-periodically increasing publicity in its own critical rupture point: Sornette and Zhou (2002) published a prediction that the U.S. stock market would follow a downward log-periodic pattern for the years to come culminating in a sharp fall in the first half of 2004. Similar predictions were subsequently issued for other important markets (Zhou and Sornette, 2003). However, not much of these predictions did materialize. As can be seen in Fig. 5 for the case of the German DAX, the predicted log periodic evolution was quite different from the actual market development. While the in-sample fit (up to early 2003) seems quite good, the predicted and actual changes appear virtually uncorrelated.

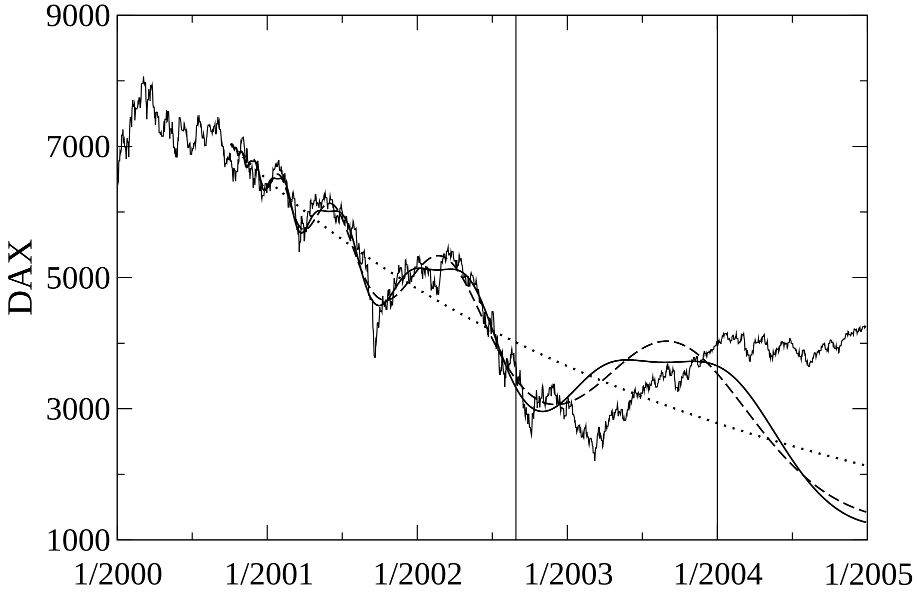

Figure 5: Log-periodic predictions: the figure compares the predictions by Zhou and Sornette (2003) and the subsequent development of the index. By courtesy of J. Voit. Reprinted with permission from Voit, J., *Statistical Mechanics of Financial Markets*, 3rd. ed., Springer 2005. ©2005 Springer Verlag.

The bubble of log-periodicity would certainly constitute an interesting episode for a deeper analysis of sociological mechanisms in scientific research. Within a few years, publications in learned journals (all authored by a very small group of scientists) on this topic reached almost three-digit numbers and the prospect of predicting markets created enormous excitement both in the popular press as well as among scientists. At the same time, almost no one had apparently ever successfully replicated these results. While physicists have often been sympathetic to this approach (due to its foundation in related work in geophysics) economists coming across the log-periodic hypothesis have always been conscious of the amount of 'eye-balling' statistics involved. After all, inspecting a time window with a crash event, one is very likely to see an upward movement prior to the crash (otherwise the event would not qualify itself as a crash). Furthermore, it is well-known that the human eye has a tendency of 'detecting' spurious periodicity even in entirely random data, so that inspection of data prior to a crash might easily give rise to the impression of apparent oscillations with an upward trend. Because of this restriction to an endogenously selected subperiod of the data, a true test of log-periodicity would be highly demanding. On the other hand, one might have the feeling that the idea of a built-up of intensifying pressure and exuberance which can only be sustained for some time and eventually leads to a crash has some appeal. Unfortunately, the literature has not produced behavioral models in which such log-periodic patterns occurred.

The decline in the interest in log-periodic models was due to the poor performance of their predictions. There had, in fact, been strong emphasis within the econophysics community on producing *point predictions* of future events that would confirm the superiority of the underlying models.[5] While this

[5] In relation to Zhou and Sornette's prediction of a steep decline of the U.S. stock market in 2003/04 Stauffer (2002), among others, had emphasized the importance of such "non-trivial statements ... published ... before the event is over" as a kind of litmus test for the significance of econophysics research. While Zhou's and Sornette's predictions are quite remarkably rejected by the data, it is worthwhile to note that quite a variety of actual developments could have been claimed as supporting evidence. The upshot of the log periodic prediction was, in fact, summarized more in the style of an investors newsletter than a rigorous scientific statement: "...in the next two years, we predict an overall contribution of the bearish phase, punctuated by local rallies..." and so on, cf. Sornette and Zhou (2002, p. 468). Luckily, the lack of success

aim is perhaps understandable from the importance of such predictions in the natural sciences, it might be misleading when dealing with economic data. The reason is that it neglects both the stochastic nature of economic systems (due to exogenous noise as well as the endogenous stochasticity of the large number of interacting subunits) as well as their self-referential nature. Rather than testing theories in a dichotomic way in the sense of a clear yes/no evaluation, the approach in economics is to evaluate forecasts statistically via their average success over a larger *out-of-sample* test period or a number of different cases. Unfortunately, the log-periodic theory like many other tools introduced in the econophysics literature has hardly ever been rigorously scrutinized in this manner.[6]

4 The Distribution of Wealth and Income

Although the predominant subjects of the econophysics literature have been various strands of research on financial markets, some other currents exist. Maybe the area with the highest number of publications next to finance is work on microscopic models of the emergence of unequal distributions of wealth and pertinent empirical work.

The frequency distribution of wealth among the members of a society has been the subject of intense empirical research since the days of Vilfredo Pareto (1897) who first reported power-law behavior with an index of about 1.5 for income and wealth of households in various countries. Empirical work initiated by physicists has confirmed these time-honored findings (Levy and Solomon, 1997; Fujiwara et al., 2003; Castaldi and Milakovic, 2005). While Pareto as well as most subsequent researchers have emphasized the power law character of the largest incomes and fortunes, the recent literature has also highlighted the fact that a crossover occurs from exponential behavior for the bulk of observations and Pareto behavior for the outmost tail. A

of stock market predictions also casts doubts on the validity of subsequent far-fetched doomsday scenarios derived from a log-periodic study of non-financial socio-economic data (Johansen and Sornette, 2001a).

[6] Chang and Feigenbaum (2006) make an effort on rigorous statistical tests of the log-periodic model. Their results are hardly supportive.

careful study of U.S. income data locates the cross-over at about the 97 to 99 percent quantiles (Silva and Yakovenko, 2005) as illustrated in Fig. 6 taken from this source. It seems interesting to note that this scenario is similar to the behavior of financial returns which also exhibit an asymptotic power-law behavior in the tails and a relatively well-behaved bell shape in the center of the distribution. The difference between the laws governing the majority of the small and medium-sized incomes and fortunes and the larger ones might also point to different generating mechanisms underlying these two segments of the distribution.

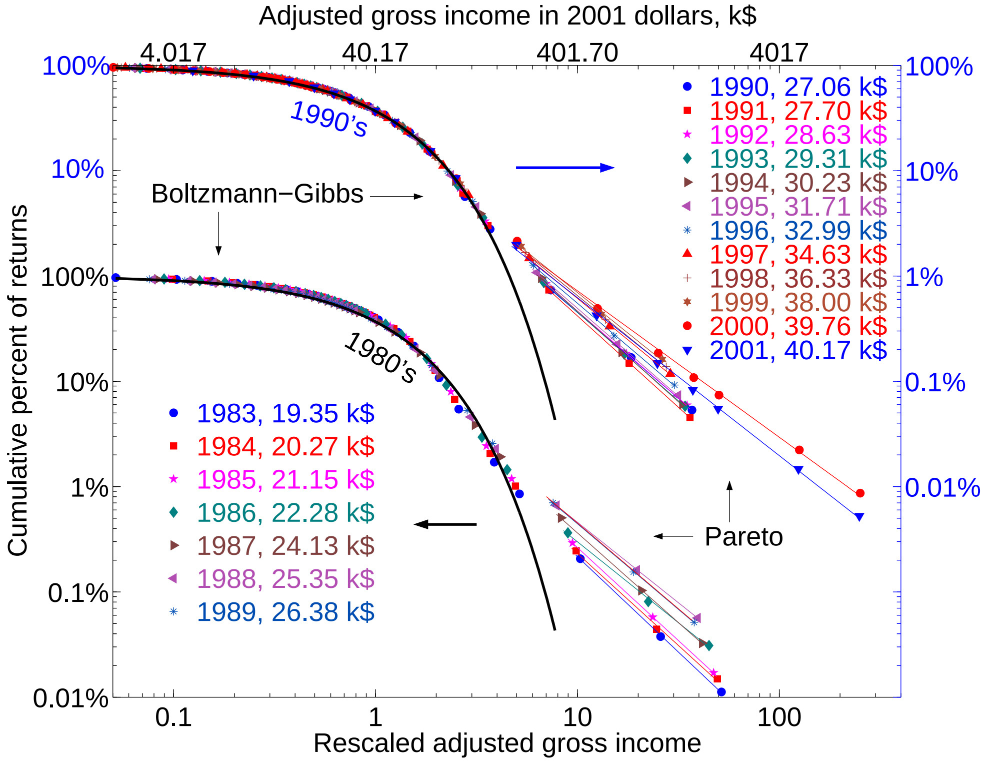

Figure 6: The distribution of gross income in the U.S. compiled from tax revenues over the period 1983-2001. The decomposition shows a pronounced crossover from the bulk of observations to a Pareto tail for the highest incomes. The numbers on the left-hand side give the average income per year. By courtesy of V. Yakovenko. Reproduced with permission from Yakovenko, V. and C. Silva, Two-class structure of income distribution in the U.S.A.: Exponential bulk and power-law tail, in: Chatterjee, A., S. Yarlagadda and B. Chakraborti, eds., *Econophysics of Wealth Distribution*. Springer 2005, ©2005 by Springer Verlag.

In economics, the emergence of inequality had been a hot topic up to the fifties and sixties. Several authors have proposed Markov processes that under certain conditions would lead to emergence of a Pareto distribution. The best known contribution to this literature is certainly Champernowne (1953): his model assumes that an individual's income develops according to a Markov chain with transition probabilities between a set of income classes (defined over certain intervals). As a basic assumption, transitions were only allowed to either lower income classes or the next higher class, and the mean change for all agents was assumed to be a reduction of income (which is interpreted as a stability condition). Champernowne showed that the equilibrium distribution of this stochastic process is the Pareto distribution in its original form. Variations on this topic can be found in Whittle and Wold (1957), Mandelbrot (1961) and Steindl (1965), among others. Over the sixties and seventies, the literature on this topic gradually died out due to the rise of the representative agent approach as the leading principle of economic modeling. From the point of view of this emergent new paradigm, the behavioral foundations of these earlier stochastic processes seemed too nebulous to warrant further research in this direction. Unfortunately, a representative agent framework - quite obviously - does not offer any viable alternative for investigation of distributions among agents. As a consequence, the subject of inequality in income and wealth has received only scant attention in the whole body of economics literature for some decades and lectures in the 'Theory of Income Distribution and Wealth' eventually disappeared from the curricula of economics programs.[7]

The econophysics community recovered this topic in 2000 when three very similar models of 'wealth condensation' (Bouchaud and Mézard, 2000) and the 'statistical mechanics of money' (Dragulescu and Yakovenko, 2000; Chakraborti and Chakrabarti, 2000) appeared. While these attempts at microscopic simulations of wealth formation among interacting agents received an enthusiastic welcome in the popular science press (Buchanan, 2002; Hayes, 2002), they were actually not the first to explore this seemingly unknown territory. The credit for a much earlier analysis of essentially

[7]Another explanation of the decline of interest in distributional issues is that this was not a politically opportune topic in Western countries during the cold war era with its juxtaposition of communist and market-oriented systems.

the same type of structures has to be given to sociologist John Angle. In a series of papers starting in 1986 (Angle, 1986, 1993, 1996, among many others), he explored a multi-agent setting which draws inspiration from two quite distinct sources: particle physics and human anthropology. Particle physics motivates the modelling of agents' interactions as collisions from which one of both opponents emerges with an increase of his wealth at the expense of the other. Human anthropology provides a set of stylized facts that Angle attempts to explain with this 'inequality process'. In particular, he quotes evidence from archeological excavations that inequality among the members of a society first emerges with the introduction of agriculture and the prevalence of food abundance. Once human societies proceed beyond the hunter and gatherer level and production of some 'surplus' becomes possible, the inequality of a 'ranked society' or 'chiefdom' appears. Since this ranked society persists through all levels of economic development, a very general and simple mechanism is required to explain its emergence. The 'inequality process' proposes a mechanism for this stratification of wealth from the following ingredients (Angle, 1986): within a finite population, agents are randomly matched in pairs. A random toss, then, decides which of both agents comes out as the winner of this encounter. In the baseline model, both agents have the same probabilities 0.5 of winning but other specifications have also been analyzed in subsequent papers. If the winner is assumed to take away a fixed portion of the other agent's wealth, say ω, the simplest version of the process leads to a stochastic evolution of wealth of two individuals i and j bumping into each other according to:

$$\begin{aligned} w_{i,t} &= w_{i,t-1} + D_t \omega w_{j,t-1} - (1 - D_t) \omega w_{i,t-1}, \\ w_{j,t} &= w_{j,t-1} + (1 - D_t) \omega w_{i,t-1} - D_t \omega w_{j,t-1}. \end{aligned} \tag{9}$$

Time t is measured in encounters and $D_t \in \{0, 1\}$ is a binary stochastic index which takes the value 1 (0) if $i(j)$ is drawn as the 'winner'. Angle (1986) shows via microscopic simulations that this process leads to a limiting distribution that can be reasonably well fitted by a Gamma distribution. Later papers provide a theoretical analysis of the process, various extensions as well as empirical applications (see Angle, 2006, for a summary).

The econophysics papers of 2000 proposed models that are almost undistinguishable from Angles's. Dragulescu and Yakovenko begin their investigation with a model in which a constant 'money' amount is changing hands rather than a fraction of one agent's wealth. They show that this process leads to a Boltzmann-Gibbs distribution $P(w) \sim e^{-w/T}$ (with T 'effective temperature' or average wealth). Note that this variant of the inequality process is equivalent to a simple textbook model for the exchange of energy of atoms. One generalization of their model allows for a random amount of money changing hands, while another considers the exchange of a fraction of wealth of the losing agent, i.e. Angle's inequality process depicted in eqs. (9). Chakraborti and Chakrabarti (2000) have a slightly different set-up allowing agents to swap a random fraction ε of their total wealth, $w_{i,t} + w_{j,t}$. A more general variant of the wealth exchange process can be found in Bouchaud and Mézard (2000) whose evolution of wealth covers simultaneous interactions between all members of the population. Cast into a continuous-time setting, agent i's wealth, then, develops according to:

$$\frac{dw_{i,t}}{dt} = \eta_i(t) w_{i,t} + \sum_{j(\neq i)} J_{ij} w_{j,t} - \sum_{j(\neq i)} J_{ji} w_{i,t} \tag{10}$$

with: η_i a stochastic term and the matrix J_{ij} capturing all factors of redistribution due to interactions within the population. Solving the resulting Fokker-Planck equation, for the case of identical exchange parameters $J_{ij} = J/N$, the authors show that the equilibrium distribution of this model obeys a power law with the exponent depending on the parameters of the model (J and the distributional parameters of η_i). In the rich literature following Dragulescu and Yakovenko and Chakraborti and Chakrabarti one of the main goals was to replace the baseline models with their exponential tail behavior by refined ones with power law tails. Power laws have been found when introducing 'savings' in the sense of a fraction of wealth that is exempted from the exchange process (Chatterjee, Chakraborti and Manna, 2003) or asymmetry in interactions (Sinha, 2005).

What is the contribution of this literature? As pointed out by Lux (2005) and Anglin (2005), economists (even those subscribing to the usefulness of agent-based models) might feel bewildered by the sheer simplicity of this approach. Taken at face value, it would certainly be hard to accept the processes surveyed above as models of the emergence of inequality in market economies. A first objection would be that the processes, in fact, do model what has been called 'theft and fraud' economies (Hayes, 2002). The principles of voluntary exchange to the mutual benefit of both parties are entirely at odds with the model's main building blocks. Human agents with a minimum degree of risk aversion would certainly prefer not to participate at all in this economy. The models also dispense with all facets of *collaborative activity* (i.e. production) to create wealth and merely focus on redistribution of a constant, given amount of wealth (although there is a point to Angle's implicit view that the universality of inequality for all advanced societies may allow to abstract from wealth creation). What is more and what perhaps is at the base of economists' dissatisfaction with this approach is that wealth is a *derived* concept rather than a primitive quantity. Tracking the development of the distribution of wealth, then, requires to look at the more basic concepts of quantities of goods traded and the change of evaluation of these goods via changes of market prices.

Luckily, a few related papers have been looking at slightly more complicated models in which 'wealth' is not simply used as a primitive concept, but agents' wealth is derived from the valuation of their possessions. Silver et al. (2002) consider an economy with two goods and an ensemble of agents endowed with Cobb-Douglas preferences:

$$U_{i,t} = x_{i,t}^{f_{i,j}} y_{i,t}^{1-f_{i,t}} \tag{11}$$

Eq. (11) formalizes the utility function of agent i at time t whose message is that the agent derives pleasure from consuming (possessing) goods x and y and their overall contribution to the agent's well-being depends on the parameter $f_{i,t}$. This parameter is drawn independently, for each agent in each period, from a distribution function with support in the interval $[0, 1]$. This stochasticity leads to changing preferences for both goods which induce agents to exchange goods in an aggregate market. Summing up demand and supply over all agents one can easily determine the relative price between x and y in equilibrium at time t as well as the quantities exchanged between agents. Changes of quantities and prices over time lead to emergence of inequality. Starting from a situation of equal possessions of all agents, wealth stratification is simply due to the favorable or unfavorable development of agents' preferences vis-à-vis the majority of their trading partners (for example, if one agent develops a strong preference for one good in one particular period, he is likely to pay a relatively high price in terms of the other good and might undergo a loss in aggregate wealth if his preferences shift back to more 'normal' levels in the next period).

Note that exchange is entirely voluntary in this economy and allows all agents to achieve their maximum utility possible in any period with given resources and preferences. Silver et al. show both via simulations and theoretical arguments that this process converges to a limiting distribution which is close to the Gamma distribution. A somewhat similar result is already reported in passing in Dragulescu and Yakovenko (2000, p. 725) who besides their simple wealth exchange models reviewed above had also studied a more involved economic model with a production sector.

It, therefore, seems that a certain tendency prevails both in simple physics-sociological models and in more complicated economic models of wealth formation to arrive at an exponential distribution for large fortunes. The analogy to the Boltzmann-Gibbs theory for the distribution of kinetic energy might be at the heart of this (almost) universal outcome of various simple models. All the relevant papers consider *conservative* systems (in the sense of a given amount of 'wealth' or otherwise given resources) governed by random reallocations. The limiting distribution in such a setting, then, reflects the maximization of entropy through the random exchange mechanisms. The important insight from this literature is that the bulk of the distribution can, in fact, be explained simply by the influence of random forces. While the primitive models à la eqs. (9) and (10) are the purest possible formalization of this randomization, the economically more refined version of Silver et al. demonstrates that their results survive in a setting with more detailed structure of trading motives and exchange mediated via markets.

This leaves the remaining Pareto tail to be explained by other mechanisms. Although some power-laws have been found in extended models, these seem to depend on the parameters of the model and do not necessarily yield the apparently universal empirical exponents. In the view of the above arguments, it might also seem questionable whether one could find an explanation of Pareto laws in conservative systems. Economists would rather expect capital accumulation and factors like inheritance to play a role in the formation of big fortunes. Extending standard representative agent models of the business cycle to a multi-agent setting, a few attempts have been made recently to explore the evolution of wealth among agents. A good example of this literature is Krusell and Smith (1998) who study a standard growth model with intertemporally optimizing agents. Agents have to decide about consumption and wealth accumulation and are made heterogeneous via shocks in their labor market participation (i.e. they stochastically move in and out of unemployment) and via shocks to their time preferences (i.e. preferences for consumption vis-à-vis savings). The major contributions of this paper are: the development of a methodology to derive rational (i.e. consistent) expectations in a multi-agent setting and the calibration of their model with respect to selected quantiles of the U.S. Lorenz curve.

Alternative models with a somewhat different structure are to be found in Hugget (1996) and Castañeda et al. (2003). All these models, however, restrict themselves to matching selected moments of the wealth dispersion in the U.S. It is, therefore, not clear so far, whether their structures are consistent with a power law tail or not. While the unduely neglected topic of the emergence of inequality in modern societies has been approached from various sides, none of these new developments has come out with an explanation for the Pareto tails so far. It seems, therefore, to be a worthwhile undertaking to bridge the gap between the extremely simple wealth exchange processes proposed in the econophysics literature and the much more involved emergent new literature on wealth formation in economics. An appropriate middle way might provide useful insights into the potential sources of power-law tails.

5 Macroeconomics and Industrial Organization

Much of the work done by physicists on non-financial data is of an exploratory data-analytical nature. Most of it focuses on the detection of power laws that might have gone unrecognized by economists. Besides high-frequency financial data, another source of relatively large data sets is cross-sectional records of firms' characteristics such as sales, number of employees etc. One such data set, the Standard and Poor's COMPUSTAT sample of U.S. firms has been analyzed by the Boston group around G.

Stanley in a sequence of empirical papers. Their findings include:

(i) the size distribution of U.S. firms follows a Log-normal distribution (Stanley et al. 1995),

(ii) a linear relationship prevails between the log of the standard deviation σ of growth rates of firms and the log of firm size, s (measured by sales or number of employees, cf. Stanley et al., 1996). The relationship is, thus,

$$\ln \sigma \approx \alpha - \beta \ln s \tag{12}$$

with estimates of β around 0.15. This finding has been shown by Canning et al. (1998) to extend to the volatility of GDP growth rates conditioned on current GDP. Due to this surprising coincidence, the relationship has been hypothesized to be a universal feature of complex organizations,

(iii) the conditional density of annual growth rates of firms $p(r_t \mid s_{t-1})$ with s the log of an appropriate size variable (sales, employees) and r its growth rate, $r_t = s_t - s_{t-1}$, has an exponential form

$$p(r_t \mid s_{t-1}) = \frac{1}{\sqrt{2}\sigma(s_{t-1})} \exp\left(-\frac{\sqrt{2} \mid r_t - \overline{r}(s_{t-1}) \mid}{\sigma(s_{t-1})}\right) \tag{13}$$

cf. Stanley et al. (1996), Amaral et al. (1997).

Log-normality of the firm size distribution (finding (i)), is, of course, well-known as Gibrat's law of proportional effect (Gibrat 1931): if firms' growth process is driven by a simple stochastic process with independent, Normally distributed growth rates, the Log-normal distribution governs the dispersion of firm sizes within the economy. The Log-normal hypothesis has earlier been supported by Quandt (1966). However, other studies suggest that the Log-normal tails decrease too fast and that there is excess mass in the extreme part of the distribution that would rather speak in favor of a Pareto law. Pareto coefficients between 1.0 and 1.5 have already been estimated for the size distribution of firms in various countries by Steindl (1965). Okuayama et al. (1999) also report Pareto coefficients around 1 (between 0.7 and 1.4) for various data sets. Probably the most comprehensive data set has been used by Axtell (2001) who reports a Pareto exponent close to 1 (hovering between 0.995 and 1.059 depending on the estimation method) for the total ensemble of firms operating in 1997 as recorded by the U.S. Census Bureau[8].

Finding (ii) has spawned work in economics trying to elucidate the sources of this power law. Sutton (2002) shows that one arrives at a slope coefficient between -0.21 and -0.25 under the assumption that the growth rates of constituent businesses within a firm are uncorrelated. The difference between these numbers and the slightly flatter empirical relationship would, then, have to be attributed to joint firm-specific effects on all business components.

From a broader perspective, a number of researchers have shown the emergence of several empirically relevant statistical laws in artificial economies with a complex interaction structure of their inhabitants. Axtell (1999) building upon the "sugarcube economy" of Epstein and Axtell (1996) allows agents to self-organize into productive teams. Cooptation of additional workers to existing teams is advantageous because of increasing returns, but also provides the danger of suffering from free riding of some group members who might reduce the level of effort invested in team production. This later effect limits the growth potential of firms since agents have less and less incentives to supply effort in growing teams because of the decreasing sensitivity of overall output to individual contributions. In an agent-based model in which workers have to decide adaptively on the formation and break-off of teams, the evolving economy exhibits a number of realistic features: log growth rates of firms (in terms of the number of employees) follow a Laplace distribution (finding (iii)), and the size distribution of firms is skewed to the right. Estimation of the Pareto index yields 1.28 for employees and 0.88 for the distribution of output.

Delli Gatti et al. (2003) arrive at a very similar replication of empirical stylized facts for firm sizes and growth rates. However, their starting point is a framework in which the basic entities are the firms themselves and the heterogeneity of the ensemble of firms with respect to market and financial conditions is emphasized. Focusing on the development of firms' balance sheets, the financial conditions of the banking sector and allowing

[8]The Zipf's law for the size distribution of firms is reminiscent of well-known Pareto law for the distribution of city sizes, cf. Nitsch (2005) for a review of the evidence and Gabaix (1999) for a potential explanation.

for bankruptcies, their model generates business cycle fluctuations driven by the financial sphere of the economy. Simulations and statistical analyses of the synthetic data reveal a reasonable match not only of some of the stylized facts above, but also conformity with other aspects of macroeconomic fluctuations. A third independent approach which not only reproduces IO facts but also a Pareto wealth distribution is the model by Wright (2005). Wright considers a computational model with both workers and firm owners. His framework covers stochastic processes for consumption, hiring and firing decisions of firms and the distribution of agents on classes. Despite the relatively simple behavioral rules for all these components, the resulting macroeconomy seems remarkably close to the empirical data in its statistical features.

While so far the number of papers on agent-based artificial economies is extremely limited, the fact that computational models with very different building blocks have been shown to reproduce stylized facts seems encouraging. Clearly, these promising results still leave a long agenda of investigations in the robustness and generating mechanisms of the macroeconomic power laws.

The agent-based approach to macroeconomic modeling has also been pursued by Aoki in a long chain of publications most of which are summarized in his recent books (Aoki 1996, 2002, Aoki and Yoshikawa, 2007). His approach had initially been rather technically orientated advocating the use of tools from statistical physics like mean-field approximations, Master equations and clustering processes. In some of his work, he had nicely illustrated the potential usefulness of these techniques by revisiting well-known economic models. An example is Diamond's (1982) model of a search economy with multiple equilibria (cf. Aoki, 2002, c.9). With a mean-field approach, the assumption of an infinite population can be dispensed with and one arrives at new results on cyclicity and equilibrium selection in this benchmark model of the Neokeynesian macroeconomics literature.

Recent work by Aoki and Yoshikawa makes an even stronger point for replacing the representative agent paradigm in macroeconomics by an agent-based approach. Most interestingly, the proposed new models have a strong Keynesian flavor revisiting such concepts like the liquidity trap, the role of uncertainty in macroeconomics and the possibility of a slow-down of economic growth due to demand saturation. With their focus on analytical tractability, the models proposed by Aoki and Yoshikawa are more stylized than the computational approaches reviewed above. They are not analyzed from a power-law perspective, but rather from the perspective of other well-known macroeconomic laws like Okun' s (a decrease of unemployment by one percent comes along with an additional increase of GDP by 2.5 percent). Nevertheless, their approach is very similar to that of Axtell, Delli Gatti et al. and Wright in that well-known statistical relationships on the macro level are explained as *emergent* results of a multi-sectoral industrial dynamics.

One particular interesting innovation in Aoki and Yoshikawa's recent work is the application of *ultrametric structures* as an ingredient in a labor market model. Ultrametric structures are tree-like, hierarchical structures pretty similar to the hierarchical structure of the multifractal volatility model exhibited in Fig. 4. Such structures are applied here to measure the distance in terms of specialization of different occupations. A worker at one end node of the tree model has a very similar specialization to that of his neighbor if their branch stems from the same mother nodes at higher hierarchic levels, but they might as well have a large "ultrametric distance" if they originate from different mother nodes (cf. Fig. 7). This hierarchy provides an avenue for explaining the differences in adaptability of certain workers to new jobs offered, the time needed for retraining and the likelihood to find employment in a different occupation. With high ultrametric differences, restructuring of the labor force in the presence of structural change might be a sluggish process. The model, therefore, provides an avenue towards modeling of the much discussed *hysteresis* phenomenon in labor markets: the long-lasting influence of transitory shocks to employment that is held responsible for high levels of unemployment in European countries[9].

[9] While the notion of hysteresis stems from physics and engineering, it has become the standard technical term for this phenomenon in economics already some twenty years ago, cf. Cross (1988).

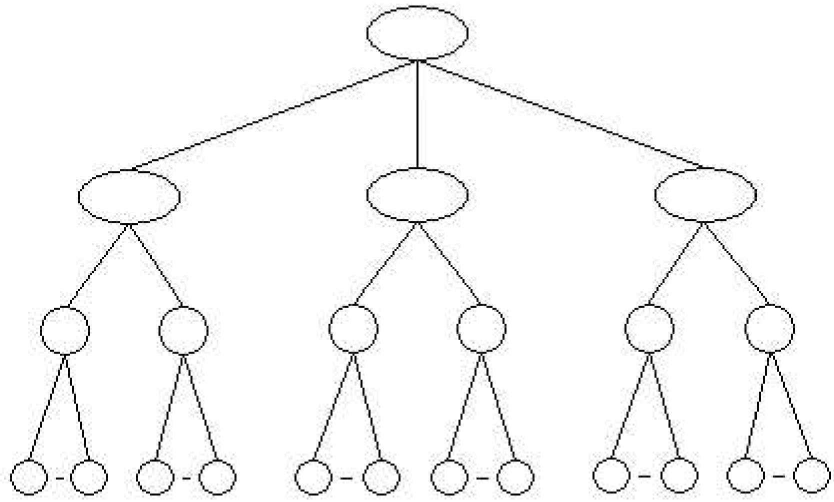

Figure 7: Example of hierarchical structure in an ultrametric space: Such a structure could be contemplated as a formalization of the proximity of professional specializations. In a macroeconomic setting, the ultrametric structure of industries would, then, determine the costs of relocating resources from one sector to another. Note the formal similarity of hierarchical trees to multifractal cascades (depicted in Fig. 5). The cluster formation algorithm underlying Fig. 3 is also based upon an ultrametric concept of distance between companies.

6 Concluding Remarks

While there had been some crossover of ideas from the econophysics literature into economics and finance, much of the current research still lives in a kind of parallel society that is largely unheard of in the native population of economics departments. Where it had become known, exaggerated claims of the superiority of econophysics and the uselessness of traditional economic thought (McCauley, 2006) together with a sometimes amateurish use of terminology and concepts from economics have inhibited fruitful communication. Economists also often found the empirical analyses in the log-log style to represent substandard methodology compared to the refined methodology developed in econometrics. However, the development of, for example, the literature on multifractal models in econometrics demonstrates that new concepts from statistical physics can be successfully adapted for economic applications and integrated into the econometrician's toolbox. It is particularly remarkable that in this area progress was exactly due to the more rigorous development of methods of statistical inference and forecasting instead of simple copying of the formalism inherited from the turbulence literature. It is worth emphasizing that applications of these models in the economics literature go far beyond contemporaneous econophysics publications that still confine themselves to only demonstrating some scaling laws of empirical data. It could be possible that the methodological developments in this area feed back to the original subject and we will see applications of Markov-switching multifractal processes on turbulent flows in the near future. Other methods brought to the attention of economists via the econophysics literature might undergo a similar transformation.

While the dissemination of various methodological concepts that have been unfamiliar to economists so far, is certainly an important aspect of 'econophysics', there might be an even more seminal influence in its natural emphasis on economies and markets as dispersed systems of interacting units. After the disappointing insights in the (near) impossibility of deriving stable macroeconomic (or macroscopic) behavioral correspondences as the aggregate of individual decisions, much of mainstream economic theory has simply side-stepped this issue by evoking representative agents as the (one or two) single actors in macroscopic models. However, "... there is no plausible formal justification for the assumption that the aggregate of individuals, even maximizers, acts itself like an individual maximizer." (Kirman, 1992), "... macro activity is essentially the result of the *interactions between agents* and as such is not usefully represented by a single 'optimizer' that by definition eliminates all trade between agents and thereby ignores the interactions between them." (Ramsey, 1996). Behavioral work in econophysics typically starts out from the interactions of the elementary units of a system whose macroscopic regulations are emergent properties of the overall dynamics of the system. In empirical research, instead of postulating ad hoc the existence of meaningful macro variables, one can let the data themselves speak and reveal its stable properties. It might well be the case that some scaling laws are more robust characterizations of economic data than the behavior of a simple average of some measurement. The prevalence of Pareto laws in income, wealth, firm size and financial returns supports the view that the scaling view of statistical physics could be fruitful in economics, too. In any case, these emergent properties seem to be much more stable (even quantitatively so) than many of the well-known hypothesized relationships between macro variables in economic theory (take money demand as a striking example, cf. Knell and Stix, 2006, for a recent survey). Since economics deals with statistical ensembles of microscopic configurations, whose exact realization cannot be determined, what can be said about the system as a whole must be based on the statistical laws governing the entire ensemble. These ensemble averages are objects of study in their own right and will - except for trivial cases - not correspond to the behavioral laws of individual members of the ensemble. A satisfactory theory will, therefore, typically require the analysis of both time-varying population averages and their dispersion (second moment). In many cases, even the investigation of the co-evolution of means and (co-)variances of sensible macroscopic measurements might be too rough an approximation and one might want to extend the analysis to higher moments like skewness and kurtosis.[10] Since statistical physics has developed a formal apparatus for dealing with collective phenomena in non-human systems, it provides a rich source of inspiration for the analysis of collective behavior in markets and other areas of social interaction.

Acknowledgement

I am indebted to Simone Alfarano, Jack Angle, Mishael Milakovic, Dietrich Stauffer and Friedrich Wagner for many intense discussions on the relationships between physics and economics. I also wish to thank Maren Brechtefeld, Claas Prelle, Dietrich Stauffer and the editor, J. Barkley Rosser, for their careful reading of a previous version of this chapter and many useful and important comments. Financial support by the European Commission under STREP contract no. 516446 is greatfully acknowledged.

[10]Note that the exponent of a scaling law also gives the highest existing moment of the underlying time series.

References

[1] Amaral, L.A.N., S.V. Buldyrev, S. Havlin, P. Maass, M.A. Salinger, H.E. Stanley, and M.H.R. Stanley (1997), "Scaling behavior in economics: The problem of quantifying company growth", *Physica A*, 244, 1–24.

[2] Anglin, P. M. (2005) "Econophysics of wealth distribution: a comment", in A. Chatterjee et al., eds, *Econophysics of Wealth Distribution: Econophys-Kolkata*, Springer: Milan, 229–238.

[3] Angle, J.(1986),"The surplus theory of social stratification and the size distribution of personal wealth", *Social Forces* 65(2), 293–326.

[4] Angle, J. (1993), "Deriving the size distribution of personal wealth from 'The rich get richer, the poor get poorer'", *Journal of Mathematical Sociology* 18, 27–46.

[5] Angle, J. (1996), "How the gamma law of income distribution appears invariant under aggregation", *Journal of Mathematical Sociology* 31, 325–358.

[6] Angle, J. (2006), "The inequality process as a wealth maximizing process", *Physica A* 367, 388–414.

[7] Aoki, M. (1996), *New Approaches to Macroeconomic Modelling: Evolutionary Stochastic Dynamics, Multiple Equilibria, and Externalities as Field Effects*, University Press: Cambridge.

[8] Aoki, M. (2002) *Modeling Aggregate Behavior and Fluctuations in Economics*, University Press: Cambridge.

[9] Aoki, M., and H. Yoshikawa (2007), *A Stochastic Approach to Macroeconomics and Financial Markets*, University Press: Cambridge.

[10] Arifovic, J. (1996), "The behavior of the exchange rate in the genetic algorithm and experimental economies ", *Journal of Political Economy* 104, 510–541.

[11] Ausloos, M., and N. Vandewalle (1998), "Multi-affine analysis of typical currency exchange rates", *European Physical Journal B* 4, 257–261.

[12] Axtell, R. L. (1999), "The Emergence of Firms in a Population of Agents: Local Increasing Returns, Unstable Nash Equilibria and Power Law Size Distribution, Brookings Institution, Center for Social and Economic Dynamics *Working Paper*

[13] Axtell, R. L. (2001), "Zipf distribution of U.S. firm sizes", *Science* 293, 1818–1820.

[14] Baillie, R. T., T. Bollerslev, and H. Mikkelsen (1996), "Fractionally integrated generalized autoregressive conditional heteroskedasticity", *Journal of Econometrics* 74, 3–30.

[15] Bacry, E., A. Kozhemyak, and J.-F. Muzy (2008), "Continuous cascade models for asset returns", *Journal of Economic Dynamics and Control* 32, 156–199.

[16] Bak, P., M. Paczuski, and M. Shubik (1997), "Price variations in a stock market with many agents", *Physica A* 246, 430–453.

[17] Bartolozzi, M., and A.W. Thomas (2004), "Stochastic cellular automata model for stock market dynamics", *Physical Review E* E69, 046112.

[18] Bouchaud, J.-P., and M. Mézard (2000), "Wealth condensation in a simple model of economy", *Physica A* 282, 536–545.

[19] Bouchaud, J.-P., and M. Potters (2000), *Theory of Financial Risks: From Data Analysis to Risk Management*, University Press: Cambridge.

[20] Bouchaud, J.-P., M. Potters , and M. Mézard (2002),"Statistical properties of stock order bocks: empirical results and models", *Quantitative Finance* 2, 251.

[21] Bollerslev, T. (1986) "A generalized autoregressive conditional heteroskedasticity", *Journal of Econometrics* 31, 307–327.

[22] Breidt, F., N. Crato, and P. de Lima (1998), "On the detection and estimation of long memory in stochastic volatility", *Journal of Econometrics* 83, 325–348.

[23] Breymann, W., S. Ghashghaie, and P. Talkner (2000), "A stochastic

cascade model for FX dynamics", *International Journal of Theoretical and Applied Finance* 3, 357–360.

[24] Buchanan, M. (2002), "Wealth happens", *Harvard Business Review*, April, 49–54.

[25] Buldyrev, S. V., A. L. Goldberger, S. Havlin, C. K. Peng, M. Simons, and H. E. Stanley (1996), "Mosaic organisation of DNA nucleotides", *Physics Review E* 49, 1685–1689.

[26] Calvet, L.E., B. Mandelbrot, and A. Fisher (1997), "Large deviations and the distribution of price changes", *Mimeo: Cowles Foundation for Research in Economics.*

[27] Calvet, L.E., and A. Fisher (2001), "Forecasting multifractal volatility", *Journal of Econometrics* 105, 27–58.

[28] Calvet, L.E., and A. Fisher (2002), "Multifractality in asset returns: theory and evidence", *The Review of Economics and Statistics* 94, 381–406.

[29] Calvet, L.E., and A. Fisher (2004), "How to forecast long- run volatility: regime switching and the estimation of multifractal processes", *Journal of Financial Econometrics* 2, 49–83.

[30] Calvet, L.E., A. Fisher, and S.B. Thompson (2006), "Volatility comovement: a multifrequency approach ", *Journal of Econometrics* 131, 179–215

[31] Canning, D., L. A. N. Amaral, Y. Lee, M. Meyer, and H. E. Stanley (1998), "A power law for scaling the volatility of GDP growth rates with country size", *Economics Letters* 60, 335–341.

[32] Castaldi, C., and M. Milakovic (2007), "Turnover activity in wealth portfolios", *Journal of Economic Behavior and Organization* 63(3), 537–552.

[33] Castañeda, A., J. Díaz-Giménez, and J.-V. Ríos-Rull (2003) "Accounting for the US earnings and wealth inequality" *The Journal of Political Economy* 111(4), 818–857.

[34] Castiglione, F., and D. Stauffer(2001), "Multi-scaling in the Cont-Bouchaud microscopic stockmarket model", *Physica A* 300, 531–538.

[35] Chakraborti, A., B. Chakrabarti (2000), "Statistical mechanics of money: How saving propensities affects its distribution", *European Physical Journal B*17, 167–170.

[36] Champernowne, D. G. (1953)," A model of income distribution", *The Economic Journal* 63, 318–351.

[37] Chang, G. and J. Feigenbaum (2006), "A Bayesian analysis of log-periodic precursors to financial crashes", *Quantitative Finance* 6, 15–36.

[38] Chatterjee, A., B. Chakrabarti, and S. S. Manna (2003), "Money in gas-like markets: Gibbs and Pareto laws", *Physica Scripta* 106, 36–38.

[39] Chen, S.-H., T. Lux, and M. Marchesi (2001), "Testing for non-linear structure in an artificial market", *Journal of Economic Behavior and Organization* 46, 327–342.

[40] Chen, N., R. Roll, and S. A. Ross (1986), "Economic forces and the stock market", *Journal of Business* 59, 3, 383–403.

[41] Chiarella, C., and G. Iori (2002), "A simulation analysis of the microstructure of double auction markets" *Quantitative Finance* 2, 246–353.

[42] Cioffi, C. (2008), ed., *Power Laws in the Social Sciences: Discovering Complexity and Non-Equilibrium Dynamics in Social Universe*, in preparation.

[43] Cont, R. , and J.-P. Bouchaud (2000), "Herd behavior and aggregate fluctuations in financial markets, *Macroeconomic Dynamics* 4, 170–196.

[44] Cross, R., ed. (1988), *Hysteresis and the Natural Rate Hypothesis*, Blackwell: Oxford.

[45] Dacorogna, M., R. Gencay, U. Muller, R. Olsen and O. Pictet (2001), "An Introduction to High-frequency Finance", Academic Press: San Diego.

[46] Daniels, M.G., J. D. Farmer, L. Gillemot, G. Iori, and E. Smith (2003),

"Quantitative model of price diffusion and market friction based on trading as a mechanistic random process", *Physical Review Letters* 90(10), 108102(4).

[47] Delli Gatti, D., M. Gallegati, G. Giuleoni, and A. Palestrini (2003), "Financial fragility, patterns of firms' entry and exit and aggregate dynamics", *Journal of Economic Behavior and Organization* 51, 79–97.

[48] Demos, A., and C. Vassilicos (1994), "The multi-fractal structure of high frequency foreign exchange rate fluctuations", *LSE Financial Markets Group Discussion Paper Series* 195.

[49] Diamond, D. W. (1982), "Aggregate demand management in search equlibrium", *Journal of Political Economy* 90, 881–894.

[50] Ding, Z., R. Engle, and C. Granger (1993), "A long memory property of stock market returns and a new model", *Journal of Empirical Finance* 1, 83–106.

[51] Dragulescu, A. A., and V. M. Yakovenko (2000), "Statistical mechanics of money, income, and wealth", *European Physical Journal B* 17, 723–729.

[52] Eisler, Z., and J. Kertész (2005) "Size matters: some stylized facts of the stock market revisited", *European Physical Journal B*, 51, 1, 145–154.

[53] Egenter, E., T. Lux, and D. Stauffer (1999), "Finite-size effects in Monte Carlo simulations of two stock market models", *Physica A* 268, 250–256.

[54] Eguiluz V. M., and M. G. Zimmermann (2000), "Transmission of information and herd behaviour: an application to financial markets", *Physical Review Letters* 85, 5659–5662.

[55] Epstein, J. M., and R. L. Axtell (1996), *Growing Artificial Societies: Social Science from the Bottom Up*, Washington, DC: MIT Press.

[56] Engle, R. (1983), "Autoregressive conditional heteroskedasticity with estimates of the variance of U.K. inflation", *Econometrica* 50, 987–1008.

[57] Fama, E. (1963), "Mandelbrot and the Stable Paretian Hypothesis", *Journal of Business* 4, 420–429.

[58] Farmer, J. D., and F. Lillo (2004), "On the origin of power law tails in price fluctuations", *Quantitative Finance* 3, C7–C11.

[59] Farmer,J. D. , and I. I. Zovko (2002), "The power of patience: A behavioral regularity in limit order placement", *Quantitative Finance* 2, 387–392.

[60] Feigenbaum, J. A. (2001a), "A statistical analysis of log-periodic precursors to financial crashes", *Quantitative Finance*, 1, 346–360.

[61] Feigenbaum, J. A. (2001b), "More on a statistical analysis of log-periodic precursors to financial crashes", *Quantitative Finance*, 1, 527–532.

[62] Fisher, A., L.E. Calvet, and B. Mandelbrot (1997), "Multifractality of Deutschemark/US Dollar exchange rates", *Mimeo: Cowles Foundation for Research in Economics.*

[63] Focardi, S., S. Cincotti, and M. Marchesi (2002), "Self-organization and market crashes", *Journal of Economic Behavior & Organization* 49, 241–267.

[64] Fujiwara, T., et al. (2003), "Growth and fluctuations of personal income" , *Physica A* 321, 598–604.

[65] Gabaix, X. (1999), "Zipf's law for cities: an explanation", *The Quarterly Journal of Economics* 114(3), 739–767.

[66] Gabaix, X., P. Gopikrishnan, V. Plerou, and H. E. Stanley (2003), "A theory of power-law distributions in financial market fluctuations", *Nature* 423, 267–270.

[67] Galluccio, S. , J.-P. Bouchaud, M.Potters (1998), "Rational decisions, random matrices and spin glasses", *Physica A* 259, 449–456.

[68] Ghasghaie, S., W. Breymann, D. Peinke, P. Talkner, anf Y. Dodge, "Turbulent cascades in foreign exchange markets", *Nature*, 381, 767–770.

[69] Gibrat, R. (1931), *Le inégalités économiques*, Librairie du Recueil: Paris.

[70] Giardina, I., J.-P. Bouchaud (2003), "Bubbles, crashes and intermittency in agent based market models", *European Physical Journal B* 31, 421–437.

[71] Gode, D. K., and S. Sunder (1993), "Allocative efficiency of markets with zero-intelligence traders: market as a partial substitute for individual rationality", *Journal of Political Economy* 101, 119–137.

[72] Gopikrishnan, P., M. Meyer, L. A. N. Amaral, and H. E. Stanley (1998), "Inverse cubic law for the probability distribution of stock price variations", *European Journal of Physics B* 3, 139–140.

[73] Gopikrishnan P., V.Plerou, X.Gabaix, L.A.N. Amaral, and H.E. Stanley (2001),"Price fluctuations, market activity and trading volume" *Quantitative Finance*, 1, 262–270.

[74] Gopikrishnan, A., V. Plerou, X. Gabaix, L. A. N. Amaral, H. E. Stanley (2002), "Price fluctuations and market activity", in H. Takayasu, editor, *Empirical Science of Financial Fluctuations: The Advent of Econophysics*, Springer: Tokyo, 12–17.

[75] Hayes, B. (2002), "Follow the money", *American Scientist* 90, 400–405.

[76] Huggett, M. (1996), "Wealth distribution in life-cycle economies" *Journal of Monetary Economics* 38(3), 469–494.

[77] Iori, G. (2002), "A micro-simulation of traders' activity in the stock market: the role of heterogeneity of agents' interactions and trade-friction", *Journal of Economic Behavior and Organisation* 49, 269-285.

[78] Jansen, D., and C. de Vries (1991), "On the frequency of large stock returns: Putting booms and busts into perspective", *The Review of Economics and Statistics* 73, 18–24.

[79] Johansen, A., and D. Sornette (1999a), "Financial "anti-bubbles": Log-periodicity in gold and Nikkei collapses", *International Journal of Modern Physics C* 4, 563–575.

[80] Johansen, A., and D. Sornette (1999b), "Predicting financial crashes using discrete scale invariance", *Journal of Risk* 1, 5–32.

[81] Johansen, A. and D. Sornette (2001a), "Finite-time singularity in the dynamics of the world population and economic indices", *Physica A* 294, 405–502

[82] Johansen, A., and D. Sornette (2001b), "Bubbles and anti-bubbles in Latin-American, Asian and Western stock markets: An empirical study", *International Journal of Theoretical and Applied Finance* 4, 853–920.

[83] Kareken, J., and N. Wallace (1981), "On the indeterminacy of equilibrium exchange rates", *The Quarterly Journal of Economics* 96, 207–222.

[84] Kim, G. and H. Markowitz (1989), "Investment rules, margins, and market volatility", *Journal of Portfolio Management* 16, 45–52.

[85] Kirman, A. (1991), "Epidemics of opinion and speculative bubbles in financial markets", in: M. Taylor, editor, *Money and Financial Markets*, Macmillan: London.

[86] Kirman, A. (1992), "Whom or what does the representative agent represent?" *Journal of Economic Perspectives* 6, 117-136.

[87] Kirman, A. (1993), "Ants, rationality, and recruitment", *The Quarterly Journal of Economics* 108(1), 137–156.

[88] Knell, M. and H. Stix, "Three decades of money demand studies: similarities and differences", *Applied Economics* 38, 805-818.

[89] Krusell, P., and A. Smith Jr. (1998), "Income and wealth heterogeneity in the macroeconomy", *Journal of Political Economy*, 106, 867–896.

[90] Kullmann, L., J. Kertész, and K. Kaski (2002), "Time-dependent cross-correlations between different stock returns: A directed network of influence", *Physical Review E*,66,026125.

[91] Laloux, L., C., Pierre, M. Potters, and J.-P. Bouchaud (2000), "Random matrix theory and financial correlations", *International Journal of Theoretical and Applied Finance* 3,391–397.

[92] Levy, H., M. Levy, and S. Solomon (1994), "A microscopic model of the stock market: cycles, booms, and crashes", *Economics Letters* 45, 103–111.

[93] Levy, H., M. Levy, and S. Solomon (1995), "Simulations of the stock market: The effects of microscopic diversity", *Journal de Physique I* 5, 1087–1107.

[94] Levy, M., and S. Solomon (1997), "New evidence for the power-law distribution of wealth", *Physica A*, Vol. 242, 90-94.

[95] Levy, H., M. Levy, and S. Solomon (2000), "Microscopic Simulation of Financial Markets: From Investor Behavior to Market Phenomena", Academic Press: San Diego.

[96] Liu, Y., P. Cizeau, M. Meyer, C.-K. Peng, and G. Stanley (1997)"Correlations in economic time series" *Physica A*, A245, 437–440.

[97] Liu, Y.,P. Gopikrishnan, P. Cizeau, M. Meyer, C.-K. Peng, and H. E. Stanley (1999), "The statistical properties of the volatility of price fluctuations", *Physical Review E* 60, 1390–1400.

[98] Lobato, I. N., and C. Valesco (2000), "Long memory in stock market trading volume", *Journal of Business and Economic Statistics* 18, 410–427.

[99] Lux, T. (1995), "Herd behavior, bubbles and crashes", *The Economic Journal* 105, 881–896.

[100] Lux, T. (1996), "The stable Paretian hypothesis and the frequency of large returns: an examination of major German stocks", *Applied Financial Economics* 6, 463–475.

[101] Lux, T. (1997), "Time variation of second moments from a noise trader/infection model", *Journal of Economic Dynamics and Control* 22, 1–38.

[102] Lux, T. (1998), "The socio-economic dynamics of speculative markets: interacting agents, chaos, and the fat tails of return distributions", *Journal of Economic Behavior and Organization* 33, 143–165.

[103] Lux, T., and M. Marchesi (1999), "Scaling and criticality in a stochastic multi-agent model of a financial market", *Nature* 397, 498–500.

[104] Lux, T., and M. Marchesi (2000), "Volatility clustering in financial markets: a microsimulation of interacting agents", *International Journal of Theoretical and Applied Finance* 3, 675–702.

[105] Lux, T. (2005), "Financial power laws: Empirical evidence, models, and mechanisms", in C. Cioffi, ed., *Power Laws in Social Sciences: Discovering Complexity and Non-Equilibrium in the Social Universe.* In preparation.

[106] Lux, T. (2005), "Emergent statistical wealth distributions in simple monetary exchange models : a critical review", in A. Chatterjee, editor, *Econophysics of Wealth Distributions*, Springer: Milan, 51–60.

[107] Lux, T. (2008), "The Markov-switching multi-fractal model of asset returns: GMM estimation and linear forecasting of volatility, *Journal of Business and Economics Statistics* 26, 194–210.

[108] Lux T., and S. Schornstein (2005), "Genetic algorithms as an explanation of stylized facts of foreign exchange markets", *Journal of Mathematical Economics* 41, 169–196.

[109] McCauley, J. (2006), "Response to 'Worrying trends in econophysics', *Physica A* 371(2), 601-609.

[110] Mandelbrot, B. (1961), "Stable Paretian random functions and the multiplicative variation of income" *Econometrica* 29(4), 517–543.

[111] Mandelbrot, B. (1963), "The variation of certain speculative prices", *Journal of Business* 26, 394–419.

[112] Mandelbrot, B. A. Fisher, and L.E. Calvet (1997), "A Multifractal Model of Asset Returns", *Mimeo: Cowles Foundation for Research in Economics.*

[113] Mantegna, R. H. (1991), "Levy walks and enhanced diffusion in Milan stock exchange", *Physica A* 179, 232–242.

[114] Mantegna, R. (1999), "Hierachical structure in financial markets", *European Physical Journal B*, 11, 193–197.

[115] Mantegna, R. H., and H. E. Stanley (1996), "Turbulence and financial markets", *Nature* 383, 587–588.

[116] Mantegna, R. H., and H. E. Stanley (1995), "Scaling behaviour in the dynamics of an economic index", *Nature* 376, 46–49.

[117] Markowitz, H. (1952) "The utility of wealth", *The Journal of Political Economy* 60(2), 151–158.

[118] Maslov, S. (2000), "Simple model of a limit order-driven market", *Physica A* 278, 571–578.

[119] Matia, K. , and K. Yamazaki (2005), "Statistical properties of demand fluctuations in the financial markets", *Quantitative Finance* 5, 513–517.

[120] Nitsch, V. (2005), "Zipf zipped" *Journal of Urban Economics* 57(1), 86–100.

[121] Noh, J. D. (2000), "Model for correlations in stock markets", *Physical Review E* 61, 5981–5982.

[122] O'Hara, M. (1995), *Market Microstructure Theory*, Blackwell Business: Cambridge.

[123] Okuayama, R., M. Takayasu, and H. Takayasu (1999), "Zipf's law in income distribution of companies ", *Physica A* 269, 125–131.

[124] Onnela, J.-P., A. Chakraborti, K. Kaski, J. Kertesz, A. Kanto (2003), Dynamics of market correlations: Taxonomy and portfolio analysis", *Physical Review E* 68 ,056110.

[125] Pareto, V. (1897), *Cours d'économie politique*, F.Rouge: Lausanne.

[126] Peng, C. K., S. V. Buldyrev, S. Havlin, M. Simons, H. E. Stanley, and A. L. Goldberger (1994), "Mosaic organization of DNA nucleotídes", *Physical Review E*, 49, 1685–1689.

[127] Plerou, V., P. Gopikrishnan, B. Rosenow, L. A. N. Amaral, and H. E. Stanley (2000), "A random matrix theory approach to financial cross-correlations", *Physica A* 287, 374–382.

[128] Plerou, V., P. Gopikrishnan, and H. E. Stanley (2003), "Two-Phase behavior of financial markets," *Nature* 421, 130.

[129] Quandt, R. E. (1966), "On the size distribution of firms", *American Economic Review* 61(3), 416–432.

[130] Ramsey, J. (1996), "On the existence of macro variables and macro relationships", *Journal of Economic Behavior and Organization* 30, 275–299.

[131] Rosenow, B. (2008), "Determining the optimal dimensionality of multivariate volatility models with tools from random matrix theory", *Journal of Economic Dynamics and Control* 32, 279–302.

[132] Sato, A.-H., and H. Takayasu (1998), "Dynamical Models of stock market exchanges: from microscopic determinism to macroscopic randomness", *Physica A* 250, 231–252.

[133] Silva, A. C, and V. M. Yakovenko (2005), "Temporal evolution of the "thermal" and "superthermal" income classes in the USA during 1983-2001", *Europhysics Letters* 69, 2, 304–310.

[134] Silver,J., E. Slud, and K. Takamoto (2002), "Statistical equilibrium wealth distributions in an exchange economy with stochastic preferences", *Journal of Economic Theory* 106(2), 417–435.

[135] Sinha, S. (2005), "The rich are different! Pareto law from asymmetric interactions in asset exchange models", in A. Chatterjee et al., editor, *Econophysics of Wealth Distributions*, Springer: Milan, 177–183.

[136] Slanina, F. (2001), "Mean-field approximation for a limit oder driven market model", *Physical Review E* 64, 5, 56136.

[137] Smith, E., J. D. Farmer, L. Gillemot, and S. Krishnamurthy (2002), "Statistical theory of the continuous double auction", *Quantitative Finance* 3(6) 481–514.

[138] Sornette, D., A. Johansen, and J.-P. Bouchaud, (1996) "Stock market crashes, precursors and replicas", *Journal de Physique I* 6(1), 167–175.

[139] Sornette, D., and W.-X.Zhou (2002), "The US 2000-2002 market descent: How much longer and deeper?", *Quantitative Finance* 2, 468–481.

[140] Sornette, D., and A. Johansen (1997), "Large financial crashes", *Physica A* 245, 411–422.

[141] Stanley, M.H.R., S.V. Buldyrev, S. Havlin, R.N., R.N. Mantegna, M.A. Salinger, and H.E. Stanley (1995), "Zipf plots and the size distribution of firms" *Economic Letters* 49(4) 453–457.

[142] Stanley, M. H. R. , L. A. N. Amaral, S. V. Buldyrev, S. Havlin, H.Leschhorn, P. Maass, M. A. Salinger, and H. E. Stanley (1996), "Can statistical physics contribute to the science of economics?", *Fractals* 4, 415–425.

[143] Stanley, M. H. R. , L. A. N. Amaral, S. V. Buldyrev, S. Havlin, H.Leschhorn, P. Maass, M. A. Salinger, and H. E. Stanley (1996), "Scaling Behavior in the growth of companies", *Nature* 379, 804–806.

[144] Stauffer, D., and T. Penna (1998), "Crossover in the Cont-Bouchaud percolation model for market fluctuations", *Physica A*, 256, 284–290.

[145] Stauffer, D., P. de Oliveira, and A. Bernardes (1999), "Monte-Carlo simulation of volatility clustering in a market model with herding", *International Journal of Theoretical and Applied Finance* 2, 83–94.

[146] Stauffer, D., and D. Sornette (1999), "Self-organised percolation model for stockmarket fluctuations", *Physica A* 271, 496–506.

[147] Stauffer, D. (2002), "How to get rich with Sornette and Zhou", *Quantitive Finance* 2(6), 408.

[148] Steindl, J. (1965), *Random Processes and the Growth of Firms: A Study of the Pareto Law*, Griffin: London.

[149] Stigler, G. (1996), "Public regulation of the securities markets", *Journal of Business* 37, 117–142.

[150] Sullivan, R., H. White, and B. Golomb (2001), "Dangers of data mining: The case of calendar effects in stock returns", *Journal of Econometrics* 105, 249–286.

[151] Sutton, J. (2002), "The variance of firm growth rates: The scaling puzzle", *Physica A* 312, 577–590.

[152] Takayasu, H., H. Miura, T. Hirabayashi, K. Hamada (1992), "Statistical properties of deterministic threshold elements - the case of market price", *Physica A*, 184, 127–134.

[153] Tang, L.H., and G.-S. Tian (1999), "Reaction-diffusion-branching models of stock price fluctuations", *Physica A* 264, 543–550.

[154] Vandewalle, N., and M. Ausloos (1997), "Coherent and random sequences in financial fluctuations", *Physica A* 246, 454–459.

[155] Vandewalle, N., and M. Ausloos (1998), "How the financial crash of October 1997 could have been predicted", *The European Physical Journal* 4, 139–141.

[156] Vassilicos, J. C., A. Demos, and F. Tata (1993), "No evidence of chaos but some evidence of multifractals in the foreign exchange and the stock market", in A. J. Crilly, R. A. Earnshaw and H. Jones, editors, *Applications of Fractals and Chaos*, Springer: Berlin.

[157] Vassilicos, J. C. (1995), "Turbulence and intermittency", *Nature* 374, 408–409.

[158] Voit, J. (2005), *Statistical Mechanics of Financial Markets*, 3rd. ed., Springer: Berlin.

[159] Whittle, P., and H. O. A. Wold (1957), "A model explaining the Pareto distribution of wealth", *Econometrica* 25, 591–595.

[160] Wright, I. (2005), "The social architecture of capitalism", *Physica A* 346, 589–620.

[161] Yakovenko, V., and C. Silva (2005), "Two-class structure of income distribution in the U.S.A.: Exponential bulk and power-law tail", in: Chatterjee, A., S. Yarlagadda, and B. Chakraborti, eds., *Econphysics of Wealth Distribution*, Springer: Berlin.

[162] Zschischang, E. and T. Lux (2001), "Some new results on the Levy, Levy and Solomon microscopic stock market model", *Physica A* 29, 563–573.

[163] Zhang, Y.-C. (1999), "Towards a theory of marginally efficient markets", *Physica A* 269, 30–44.

[164] Zhou, W.-X., and D. Sornette (2003), " Evidence of a worldwide stock market log-periodic anti-bubble since mid-2000" *Physica A* 330, 543–583.