Valuation effects and long-run real exchange rate dynamics

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This paper uses the Johansen test for cointegration to check the prediction of a portfolio balance model that predictable valuation effects are associated with a saddle-path dynamic relationship between the net foreign asset position and the real exchange rate. The analysis uses newly constructed quarterly series on the net foreign position as a percentage of the nominal gross domestic product, together with data on real effective exchange rate indices for a sample of developed countries which borrow in their own currency. The results indicate that the net foreign asset position and the real exchange rate are not cointegrated for all the countries in the sample. The rejection of saddle-path dynamics suggests that predictable valuation effects are quantitatively small in developed countries. The rejection of cointegration suggests that the net foreign asset position is not a determinant for long-run real exchange rates in developed countries.

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1. Introduction and literature review

Valuation effects are at the heart of recent discussions on external imbalances and long-run dynamics of real exchange rates. Recent data show that the traditional current account does not coincide with the change in the net foreign assets (NFA). The reason is that the change in the NFA also reflects changes in the market value of claims and liabilities underlying a country’s net position while the

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current account only reflects changes in claims and liabilities at face value. Lane and Milesi-Ferretti (2001) show that these valuation changes in the market value of the NFA have become tremendously important for developed countries since the 1980s to the point where over a given period, their fluctuations outweigh the current account balance. However, valuation changes would not matter much for the underlying process of external adjustment if they were purely unexpected and random. It is the predictable component of valuation effects that affects the process of external adjustment and long-run dynamics of the real exchange rate.

Modeling predictable valuation effects is a challenge for standard international finance models that incorporate a parity condition in one form or the other because predictable valuation effects could only be important for the external adjustment process in models with large deviations from standard arbitrage conditions. Recent progress has been made with a revival of the portfolio balance literature associated with Masson (1981), Henderson and Rogoff (1982), and Kouri (1983). Blanchard et al. (2005a) build a model where domestic and foreign assets are assumed to be imperfect substitutes and in the long run predictable valuation effects are associated with a saddle-path dynamic relationship between the real exchange rate and the NFA. I exploit the special properties of this saddle-path dynamic relationship in order to test the importance of predictable valuation effects for the long-run behavior of the real exchange rate. I use a methodology developed by Cheung et al. (2005) who note that both cointegration and saddle-path dynamics depend on the roots of the system's characteristic polynomial. Such similarity suggests the adoption of the Johansen procedure to test for saddle-path dynamics because this test exploits the implications of cointegration for the rank of the coefficient matrix defined by the characteristic polynomial and uses the rank condition to infer system dynamics.

I linearize the equilibrium conditions of the Blanchard et al. (2005a) model around the steady state and put them in a vector error correction (VEC) form. Then, I use the Johansen test for cointegration to check whether the data supports the theoretical prediction of Blanchard et al. (2005a). I test for a saddle-path dynamic relationship between the NFA position as a percentage of the nominal gross domestic product (GDP) and the real effective exchange rate for five developed countries. The choice of countries was motivated by the model which assumes that the home country does not suffer from “original sin” and can borrow internationally in its own currency. According to Hausmann and Panizza (2003) these countries are the United States (US), the United Kingdom (UK), Switzerland, Japan and Germany. The results are remarkably robust for all countries over the sample period. The null hypothesis that the coefficient matrix has a zero rank cannot be rejected at the one percent level, neither by the maximum eigenvalue test, nor by the trace test.

The failure to reject the hypothesis of zero rank could be due to low power of the Johansen test when the substitutability of domestic and foreign assets is too low. When this is the case, the characteristic roots of the system are close to zero and the set of initial conditions that puts the system on a stable saddle path shrinks. Cheung et al. (2005) show that the Johansen test has lower power to reject the null hypothesis of zero rank if the characteristic roots are close to zero and the initial condition is far from the steady state. However, the analysis is performed on a relatively long sample of newly constructed quarterly data which increases the power of the test. In addition, the robustness of the result for different countries and specifications indicates that the result is not due to low power of the statistical test but to the fact that there is in fact no stable long-run link between the NFA and the real exchange rate.

The absence of saddle path dynamics is in line with recent empirical results that predictable valuation effects are too small to affect the long-run dynamic relationship between the NFA and the real exchange rate. The result contributes to an ongoing debate in the literature that concerns the size of predictable valuation effects on the US returns differential. The US returns differential means that there are predictable excess returns on some component of country’s gross assets relative to the same component on its liabilities. Empirical estimates on the importance of predictable valuation effects range from exorbitant to quite small. One set of papers finds large differentials in favor of the US, differentials exceeding three percent per year, with most of the differential being attributed to predictable valuation effects (see Obstfeld and Rogoff (2005), Meissner and Taylor (2006), Lane and Milesi-Ferretti (2007a), Gourinchas and Rey (2007), Forbes (2010), Habib (2010) and Gourinchas et al. (2010)). Another set of papers finds much smaller returns differentials, barely exceeding one percent and attributes most of the differential to a difference in yields on foreign direct investment (FDI) assets and
liabilities (see Lane and Milesi-Ferretti (2009), Curcuru et al. (2008), Curcuru et al. (2010), and Gohrband and Howell (2013)). Most of these papers back international returns out of data on NFA and international flows. Their results depend on assumptions about the difference between current account and NFA measures of external imbalances, whether the difference can be attributed entirely to valuation effects or in part to statistical discrepancies between stock and flow data. The empirical results indicating the absence of saddle-path dynamics suggest that predictable valuation effects are quantitatively small without making an explicit assumption about the difference between current account and NFA measures of external imbalances.

The empirical results also reject the hypothesis of cointegration which implies that for developed countries there is no long-run relationship between the NFA and the real exchange rate. The absence of cointegration is robust to including the domestic real interest rate in the empirical specification. This seems at odds with a number of empirical studies that have documented a stable long-run link between the two variables (see Faruquee (1995), Gagnon (1996), Broner et al. (1997), and Alberola Ila et al. (1999)). However, these studies have either used data on NFA measured at face value rather than market value or sample periods that include the Bretton Woods system collapse. The most recent empirical studies use newly constructed datasets of NFA at market value and similar sample periods and employing panel cointegration tests show the opposite. Lane and Milesi-Ferretti (2004) and Christopoulos et al. (2012) find that for developed countries whose liabilities are mostly denominated in domestic currency, there is no significant relationship between the NFA and the real exchange rate in the long run. Christopoulos et al. (2012) attribute this to the fact that developed countries do not face constraints on capital inflows so that in the long run the real exchange rate is only determined by cross-sector productivity differentials (Balassa-Samuelson effect). The empirical result rejecting cointegration is in line with this theoretical hypothesis.

A testable VEC form of the theoretical model is presented in the next section. Section 3 describes the data and the methodology. The empirical results are presented and interpreted in Section 4. Section 5 concludes.

2. Theoretical model

This section provides a brief description of the Blanchard et al. (2005a) model and puts it into an empirically testable VEC form. Time is discrete. The world consists of two countries, a home and a foreign country. Each country can invest either in an exogenously fixed stock of domestic assets ($X^*$) measured in terms of home goods or in foreign assets ($X^*/E_t$) in terms of foreign goods. $F_t$ represents the net foreign liabilities of the home country measured in units of domestic goods, $W_t = X - F_t$ domestic wealth and $W_t/E_t = X^*/E_t + F_t$ foreign wealth. The stock of foreign assets and foreign wealth, measured in terms of foreign goods, are converted in units of domestic goods by dividing by the real exchange rate $E_t$. The real exchange rate is defined as the price of home goods in terms of foreign goods such that an appreciation corresponds to a rise in $E_t$, $r$ and $r^*$ denote the domestic and foreign interest rates, each measured in terms of local goods and assumed to be constant. Let us also define the relative expected gross rate of return on holding home assets versus foreign assets $R_t$, as a function of the gross rate of return on home assets $(1 + r)$, the gross rate of return on foreign assets $(1 + r^*)$ adjusted for the expected real depreciation in the home currency, i.e. as $R_t = 1 + r - \frac{E_t^{r}}{E_t^{r^*}}$. When domestic and foreign assets are perfect substitutes $R_t$ is always equal to one and the uncovered interest parity holds. When domestic and foreign assets are imperfect substitutes, as is the case in this model, $R_t$ deviates from one.

Home investors allocate a share $\alpha_t$ of their wealth to home assets and, by implication, a share $(1 - \alpha_t)$ to foreign assets. Symmetrically, foreign investors invest a share of their wealth, $\alpha_t^*$ in foreign assets and $(1 - \alpha_t^*)$ in home assets. The share of home assets in domestic portfolios $\alpha_t$ depends on $R_t$ and on $s_t$, a portfolio shifter which includes all factors other than $R_t$ that can affect the relative demand for home assets. An increase in $s_t$ indicates a positive shock to the relative demand for home assets and a rise in $R_t$ also results in an increase in the share of home assets in the home portfolio. The opposite is true for the effects of $s_t$ and $R_t$ on $\alpha_t^*$. An important feature of the model is that home and foreign investors display home bias captured by the assumption that the sum of shares falling on own country assets exceeds one, such that $\alpha_t(R_t,s_t) + \alpha^*(R_t,s_t) > 1$. 
Equilibrium in the market for domestic assets implies:

\[ X = \alpha(R_t, s_t)(X - F_t) + \left(1 - \alpha^*(R_t, s_t)\right)\left(\frac{X^*}{E_t} + F_t\right). \]  

Equation (1)\(^1\) is the first relation, henceforth called the portfolio balance relation (PBR), that describes the joint dynamics of \(F_t\) and \(E_t\). Conceptually, the PBR states that the supply of home assets \(X\) should be equal to the sum of domestic and foreign demands for home assets. Domestic demand for home assets (the first term on the right hand side) is the share \(\alpha(R_t, s_t)\) of financial wealth \(W_t = X - F_t\) that home investors allocate to home assets. Foreign demand for home assets is the share \((1 - \alpha^*(R_t, s_t))\) of financial wealth \(W_t^*/E_t = X^*/E_t + F_t\) that foreign investors allocate to home assets.

The implications of the PBR are clearer if we consider the limiting case where the degree of substitutability is zero, so that the shares \(\alpha\) and \(\alpha^*\) are constant and do not depend on the relative rate of return. In this case, the PBR fully determines the exchange rate as a function of the world distribution of wealth, \((X - F_t)\) and \([X^*/E_t + F_t]\). Thus, news about current or future current account balances has no effect on the current exchange rate. Over time current account deficits or surpluses lead to changes in \(F_t\) and thus to changes in the exchange rate. The slope of the relation between the exchange rate and net liabilities is given by \(\frac{dE_t/F_t}{dF_t} = -\frac{(\alpha^* + \alpha - 1)}{(1 - \alpha - \alpha^*)X/E_t}\). So, in the presence of home bias, an increase in net debt is associated with a lower exchange rate. The reason is that, as wealth is transferred from the home country to the rest of the world, home bias leads to a decrease in the demand for home assets, which in turn requires a decrease in the exchange rate.

Outside this limiting case, the PBR describes the dynamic relationship between \(F_t\) and \(E_t\) for a given expected rate of depreciation. The exchange rate is no longer determined myopically. But the two insights from the limiting case remain: On the one hand, the exchange rate will respond less to news about the current account. On the other hand, it will respond to changes either in the world distribution of wealth or in portfolio preferences.

Home and foreign goods are also imperfect substitutes and the home trade deficit in terms of home goods is \(D(E_t, z_{t+1})\). The trade deficit is assumed to depend positively on the next period’s exchange rate and on a shifter variable \(z_{t+1}\) which stands for all factors other than the exchange rate that can result in an increase in the home trade deficit, i.e. \(z_{t+1}\) indicates a shift of relative demand away from home goods and toward foreign goods. The international goods trade is described by the second key equation of this model, the current account relation (CAR):

\[ F_{t+1} = (1 + r)F_t + (1 - \alpha(R_t, s_t))(X - F_t)(1 + r)\left(1 - \frac{1 + r^*}{1 + r}\frac{E_t}{E_{t+1}}\right) + D(E_{t+1}, z_{t+1}). \]  

(2)

The first and last terms on the right are standard: next period’s net debt is equal to this period’s net debt times the gross rate of return, plus the trade deficit next period. The term in the middle reflects predictable valuation effects. If there is an unexpected real depreciation of the home currency, the value of home holdings of foreign assets increases. The home net debt position improves in two ways, the conventional one, through an improvement in the trade balance, and the second one, through asset revaluation. The strength of the predictable valuation effect depends on the assumption that home gross liabilities are denominated in home currency, which means that their value is unaffected by a home currency depreciation. It also depends on gross rather than net positions, and thus on the share of the home portfolio in foreign assets \((1 - \alpha(R_t, s_t))\) and on home wealth \(W_t = X - F_t\).

The equilibrium dynamics is described by equations (1) and (2). Intuitively, the model implies that a negative shock to the trade balance of the home country (a rise in \(z_t\)) would lead to a depreciation of the home currency. Valuation effects imply that this unexpected depreciation leads immediately to an unexpected fall in \(F_{t+1}\) where the size of the valuation effects is larger if the share of foreign assets in the home portfolio is larger and if the home wealth is larger, \(-\left(1 - \alpha(R_t, s_t)\right)(X - F_t)(\Delta E_t/E_{t+1})\). But this

\(^1\) Note that variables pertaining to the foreign country are denoted with an asterisk.
immediate unexpected depreciation does not fully offset the shock. If it did, there would be excess demand for home assets since the supply of assets $X$ is assumed to be fixed and in domestic terms the value of foreign wealth ($W/Ex = X/Ex + F$) rises (the PBR would not hold). Instead, there is a less than offsetting drop in the home currency and the foreign demand for home assets is kept in check by a further expected depreciation of the home currency towards its long-run steady state value. Foreigners anticipate that exchange rate depreciation will reduce the valuation of their wealth, $R$, will fall and they will allocate a smaller share $(1 - \alpha^*(Rm,St))$ of their wealth to home assets. The home country keeps accumulating more debt along the depreciation path so that the long-run level of its currency will be below that which would have been needed to offset the negative shock at once. The key assumption here is that home and foreign assets are imperfect substitutes. If they were perfect substitutes then the exchange rate would have immediately jumped to offset the negative shock fully. The imperfect substitutability of assets implies slow adjustment of the portfolios together with expected exchange rate changes.

An additional condition must hold in equilibrium, namely that the home trade deficit be equal to minus home saving which is a function of the fixed interest rate and other unspeciﬁed factors. Treating the interest rates as given implies that the government takes measures to adjust saving as the trade deﬁcit changes, for example by reducing the ﬁscal deﬁcit as the trade deﬁcit is reduced in order to maintain output at its natural level.2

Some additional assumptions are necessary to solve the model. The trade deﬁcit is a linear function of the real exchange rate and the home relative goods demand shifter $z_t$, such that $D(E_t, Z_t) = \theta E_t + z_t$ with the parameter $\theta$ characterizing the relationship between the trade deﬁcit and the real exchange rate.3 The home demand for home assets and foreign demand for foreign assets are symmetric linear functions of the relative return $R_t$ and the relative asset demand shifter $s_t$, where $\alpha(Rm,St) = a + bRt + st$ and $\alpha^*(Rm,St) = \alpha^* - bRt - st$, where $a, \alpha^*$ and $b$ are parameters characterizing the properties of the demand relationships. The resulting solutions of the real exchange rate and net foreign debt paths are given by a system of two nonlinear ﬁrst-order simultaneous difference equations.

I put the bivariate system in a VEC form in order to characterize the dynamics graphically and apply the Johansen procedure. I use a ﬁrst order Taylor approximation around the steady state. The solution of the model in a matrix form is written as:

$$\Delta Y_t = \mu + AY_{t-1} + V_t,$$

where $\Delta = (1 - L)$, $L$ is the lag operator, $Y_t = \begin{bmatrix} E_t \\ F_t \end{bmatrix}$, and $\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$ is a 2x1 vector of constants which are functions of the parameters and the steady state values of the model. $V_t = \begin{bmatrix} V_{1t} \\ V_{2t} \end{bmatrix}$ is a 2x1 vector whose elements are zero-mean disturbances that are linear combinations of the trade deﬁcit shocks $z_t$, portfolio demand shocks $s_t$ and prediction errors. $A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}$ is a 2x2 matrix of coeﬃcients which are also functions of the steady state values and the underlying parameters of the model.

The dynamics of the model is determined by the roots of the characteristic equation $|A - \lambda I| = 0$ where the roots are $\lambda_{1,2} = \frac{1}{2}(A_1 + A_4 \pm \sqrt{A_1^2 + 4A_2A_3 - 2A_1A_4 + A_4^2})$. Note that both the PBR and the CAR imply a downward sloping relationship between the real exchange rate and the net foreign debt. When valuation effects are present the PBR is steeper than the CAR and the system exhibits saddle-

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2 See p. 9–10 Blanchard et al. (2005b) for a more detailed discussion.

3 If $\theta > 0$, the Marshall-Lerner condition holds. In the model the trade balance is measured in terms of domestic goods. The model also does not allow for output growth. Thus, calibrating $\theta$ requires adjusting the trade balance for output growth and inﬂation. Then $\theta$ is deﬁned as the derivative of the ratio of the trade balance to GDP with respect to a proportional change in the real exchange rate, $\theta = (dD/Exports)/(dE/E)$. Then, the Marshall Lerner condition implies that $\theta = (\eta_{im} - \eta_{exp} - 1)$ where $\eta_{im}$ and $\eta_{exp}$ are the import and export elasticities with respect to the real exchange rate. Wealth effects are assumed not to aﬀect the trade deﬁcit but if they were introduced in the model, the saddle-path relationship between the NFA and the real exchange rate would still hold in the long run as long as home and foreign assets are imperfect substitutes.
path dynamics. Then, the slope of PBR ($-A_2/A_1$) is greater than the slope of CAR ($-A_4/A_3$) implying that $\det(A) = \lambda_1 \lambda_2 = A_1 A_4 - A_2 A_3 < 0$ and the roots have different algebraic signs such that $\lambda_2 < 0 < \lambda_1$. In this case, $\lambda_1$ is the explosive root and $\lambda_2$ is the stationary root. Intuitively, saddle path dynamics arises when financial markets are assumed to clear faster than goods markets because the adjustment speed of the real exchange rate in response to the net foreign debt under the PBR is faster than under the CAR at the steady state. Fig. 1 gives a typical phase diagram for a saddle-path system. The arrows indicate the system dynamics. The unique trajectory that brings the system to its steady state is the saddle-path line, denoted by the SP line in Fig. 1.

The dynamics of the system depends critically on the degree of substitutability between domestic and foreign assets. The degree of substitutability is governed by the parameter $b$ which determines the responsiveness of asset demands to changes in the expected relative return (i.e. the exchange rate). The smaller $b$ is, the less substitutable domestic and foreign assets are. As $b$ approaches zero, asset demands become more inelastic and the slope of the CAR rises. On Fig. 1, the locus ($\Delta E_t = 0$) rotates clockwise and comes closer to the locus ($\Delta F_t = 0$), so that the saddle-point path also comes closer to the locus ($\Delta E_t = 0$), the PBR. As the degree of substitutability falls the stable region shrinks while the explosive region expands so that the set of initial conditions of $F_t$ and $E_t$ that puts the system on a stable saddle path is smaller. Note that as $b$ approaches 0, the slopes of the CAR and the PBR tend to equality which implies that $\det(A)$ approaches zero while the explosive root falls where $\lambda_1 / A_1 + A_4$ and the stationary root rises where $\lambda_2 / 0$.

As the parameter $b$ increases the degree of substitutability rises. The larger $b$ is, the closer the ($\Delta F_t = 0$) locus is to the locus given by the punctured line on Fig. 1 where $0 = r F_t + D(E_t, z_t)$, and the closer the saddle-point path is to that locus as well. The limiting case of perfect substitutability is degenerate. The rate of adjustment to an unexpected, permanent shift in $z_t$ goes to zero. The economy is then always on the locus $0 = r F_t + D(E_t, z_t)$. For any level of net debt, the exchange rate adjusts so that net debt remains constant, and, in the absence of shocks, the economy stays at that point. There is no unique steady state, and where the economy is depends on history. For simplicity assume that the

Fig. 1. Saddle path dynamics.

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$\lambda_1 \lambda_2 = A_1 A_4 - A_2 A_3 < 0$ implies $\lambda_2 < 0 < \lambda_1$.
domestic and foreign interest rates are equal. As \( b \) approaches infinity, the elements in the first row of the \( A \) matrix tend to zero, so that the \( \det(A) = 0 \) if \( A \) is rank one and \( \lambda_1 = A_4 \) and \( \lambda_2 = 0 \). The sign of \( \lambda_1 \) depends on \( A_4 \) which describes how \( \Delta F_t \) depends on \( F_{t-1} \). We would observe cointegration if \( A_4 < 0 \), so that ceteris paribus (independent of exchange rate movements) a country with a large net foreign debt in period \( t-1 \) would experience a decrease in its net foreign debt in period \( t \). However, a larger debt in period \( t-1 \) should increase debt in period \( t \) as it implies larger interest rate payments. But, if the model is expanded to include wealth effects, then the VEC specification would not change except for the coefficient \( A_4 \) which is more likely to be negative. Wealth effect implies that the trade balance also depends on domestic wealth such that \( D(E_t, X_t, F_t) \) where the derivative of the trade balance with respect to domestic wealth is positive. Larger debt in period \( t - 1 \) then means less wealth and less spending on imports, lower trade balance and lower \( \Delta F_t \). Therefore, if we include a wealth effect in the model it is more likely to observe cointegration because the characteristic root \( \lambda_1 = A_4 \) is more likely to be negative.

Finally the rank of matrix \( A \) is null when all elements of \( A \) are zero. In that case, there is no long-run relationship between the two variables.

### 3. Methodology and data

The Johansen procedure uses the rank of \( A \) to infer system dynamics and distinguish between the saddle-path and stationary systems from a cointegrated system. There are two types of Johansen test, either with trace or with eigenvalue, and the inferences might be a little bit different. Johansen (1991) provides a more detailed account of the methodology of this cointegration test.

In the context of a bivariate difference-stationary system, the maximum eigenvalue statistic of the Johansen procedure tests the null hypothesis that \( H_0: \text{rank}(A) = 0 \) against the alternative hypothesis that \( H_1: \text{rank}(A) = 1 \). Under the null hypothesis the unit root components of the two individual series are driven by two separate \( I(1) \) processes and there is no cointegration. Under the alternative hypothesis the two variables are cointegrated and driven by one common \( I(1) \) process and one stationary process. If the null hypothesis of the first test is rejected, the Johansen procedure considers a second test where the new null hypothesis is \( H_1: \text{rank}(A) = 1 \) and the new alternative hypothesis is \( H_2: \text{rank}(A) = 2 \). Cointegration would be observed if \( H_1 \) is not rejected. However, either a stationary system or a saddle-path system implies the rejection of \( H_1 \).

The trace test is less restrictive than the maximum eigenvalue test. First, it considers the same null hypothesis that \( H_0: \text{rank}(A) = 0 \) but against a different alternative hypothesis that \( H_1: \text{rank}(A) > 0 \). This time the rejection of the null is consistent not only with cointegration but also with saddle path dynamics. If the null hypothesis of this first test is rejected, the procedure considers the null hypothesis that \( H_1: \text{rank}(A) = 1 \) against the alternative hypothesis that \( H_2: \text{rank}(A) > 1 \). The rejection of the null hypothesis is consistent with a full rank for the \( A \) matrix and a system that displays saddle-path dynamics.

Cheung et al. (2005) design a Monte Carlo experiment and demonstrate that the Johansen tests have a reasonable power to detect saddle-path dynamics. The power of the test to detect saddle-path dynamics increases with the size of the sample and is not significantly affected by the choice of test statistics, i.e. whether the maximum eigenvalue or the trace statistic is preferred. Note also, that the power of the test to detect saddle path dynamics depends on the magnitude of the characteristic roots and on the distance between the initial condition of the system from its steady state. This is important as the dynamics of the model presented in the previous section depends on the degree of substitutability between domestic and foreign assets. Recall that as the degree of substitutability falls, the stationary root of the system tends to zero and the explosive root tends to fall. Furthermore, the set of initial conditions that put the system on a stable saddle path also tends to shrink. Cheung et al. (2005) demonstrate that the power of the Johansen test to reject the null hypothesis of a zero rank in the presence of saddle path dynamics is reduced if the roots are close to zero and the initial condition puts the system far from its steady state. Therefore, as domestic and foreign assets become less substitutable, it may be more difficult to detect saddle path dynamics empirically.
I use the Johansen procedure to test the portfolio balance model presented in Section 2. Since the model assumes that the home country gross liabilities are denominated in its own currency, the model’s predictions only apply to countries with the ability to borrow in their own currencies. According to Hausmann and Panizza (2003) only a limited number of developed countries do not suffer from “original sin”, such as the US, the UK, Switzerland, Japan and Germany. The model uses the net foreign liabilities as a measure of the net position of a country while the standard empirical measure is net foreign assets and has the opposite sign. Since the sign of the variable does not change the theoretical prediction of saddle-path dynamics, I choose to use the more standard measure in the analysis. However, samples of officially published data for the NFA position of these countries are short. With limited number of observations the power of the Johansen test to detect saddle-path behavior decreases and as noted previously it is important to ensure higher power, especially if the degree of substitutability between home and foreign assets is lower. That is why, I constructed quarterly data series based on the Chow and Lin (1971) interpolation method. Data on the current accounts of the countries were retrieved from the Federal Reserve Economic Data of the Federal Reserve Bank of Saint Louis and used as indicator series. Annual estimates of the NFA positions were taken from the updated and extended version of the dataset constructed by Lane and Milesi-Ferretti (2007b). Current accounts data were converted into current US dollars using quarterly bilateral exchange rates because the annual NFA series are measured in current US dollars. The correlation between constructed times series and officially published estimates ranges from 0.9 for Japan to 0.98 for Switzerland. In the model the NFA position is measured in terms of the home country goods. That is why, the constructed nominal NFA positions were divided by the countries nominal GDPs in order to convert them into real variables. In addition, the model does not allow for output growth and, therefore, any measure of NFA used in the empirical analysis would not only have to be adjusted for inflation but also for output growth. Data on nominal GDPs were retrieved from the OECD and converted in current US dollars using quarterly bilateral dollar exchange rates.

Since the model includes two countries only, the home country is treated as the country under study, in this case the US, the UK, Switzerland, Japan or Germany while the foreign country is the rest of the world. Then, the best measure for the real exchange rate in the model is the real effective exchange rate (REER) index of these countries. The REER is the weighted average of a country’s currency relative to an index or basket of other major currencies adjusted for the effects of inflation using the consumer price index (CPI). The weights are determined by comparing the relative trade balances, in terms of one country’s currency, with each other country within the index. Quarterly data were retrieved from the OECD database.

The data series span the period from the first quarter of 1973 until the fourth quarter of 2012, except for Japan where due to lack of current account data the sample only starts from the first quarter of 1977. Only data after 1973 are considered because the failure of the Bretton Woods system of fixed exchange rates in the beginning of the 1970s is a regime change and is likely to jeopardize the inferences of the Johansen test about long-run dynamics.

4. Testing a portfolio balance model

In this section, the Johansen procedure is used to infer whether the saddle path and the related valuation channel dynamics are an appropriate description of the joint adjustment paths of the NFAt/GDPt and the REERt. First the data series for individual countries are tested for unit root with the Augmented Dickey Fuller test. The appropriate order of the underlying autoregressive model is selected using the Akaike Information Criteria (AIC).

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5 The analysis Section 2 remains the same because rewriting the system in terms of net foreign assets instead of net foreign liabilities does not change the determinant and the characteristic roots of A. Note that NFA/GDP is the empirical counterpart of $-F_t$ and the coefficient matrix is $egin{bmatrix} A_1 & -A_2 \\ -A_3 & A_4 \end{bmatrix}$.

6 Deflating by the US CPI instead of the nominal GDP does not affect the empirical results presented in Section 4.
Fig. 2. Time series plots.
Fig. 2 suggests that except for Germany there are trends in the series of the NFAt/GDPt. That is why, the unit root test was performed under a specification where the underlying regression includes both a constant and a trend term. The specification is only different for Germany where the constant and trend terms are suppressed as the coefficients in front of them were not significant. The first panel of Table 1 presents the results for NFAt/GDPt. The test statistic is consistently greater than critical values at all significance levels and the null hypothesis of unit root cannot be rejected. The first differenced series D(NFAt/GDPt) was also tested for unit root but results in the second panel of Table 1 clearly reject the presence of unit roots.

Fig. 2 suggests that the time series for the REERt contain drifts but not trends. That is why, the unit root test was performed under a specification where the underlying regression only includes a constant term. The third panel of Table 1 reports the results for REERt. The test statistic is consistently greater than critical values at the 1% significance level and the null hypothesis of unit root cannot be rejected. However, the evidence for the presence of unit roots in REERt varies across countries. The null hypothesis of unit root cannot be rejected at all significance levels for the US and Japan. For the UK it is rejected at the 10% while for Germany and Switzerland it is rejected at the 5% level. The first difference series ΔREERt was also tested for unit root but results in the fourth panel of Table 1 reject the presence of unit roots. Overall, the evidence in favor of unit root both for NFAt/GDPt and REERt is strong enough to justify a Johansen test for their joint dynamics.

Before proceeding to test the dynamics with the Johansen method, for notational convenience the bivariate system in Equation (3) is rewritten in its general form:

**Table 1**

Augmented Dickey–Fuller test results.

<table>
<thead>
<tr>
<th></th>
<th>ADF test statistic</th>
<th>Drift t-statistic</th>
<th>Trend t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>NFAt/GDPt</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>United States</td>
<td>−2.711918</td>
<td>2.273156**</td>
<td>−2.816741*</td>
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<td>United Kingdom</td>
<td>−2.162015</td>
<td>1.553327</td>
<td>−1.891116***</td>
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<tr>
<td>Switzerland</td>
<td>−2.655035</td>
<td>2.368488**</td>
<td>2.752881*</td>
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<tr>
<td>Japan</td>
<td>−1.229113</td>
<td>−1.544999</td>
<td>2.167804**</td>
</tr>
<tr>
<td>Germany</td>
<td>0.149752</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δ (NFAt/GDPt)</td>
<td>−7.707560*</td>
<td>−2.617070*</td>
<td></td>
</tr>
<tr>
<td>United States</td>
<td>−4.698253*</td>
<td>−0.500799</td>
<td></td>
</tr>
<tr>
<td>United Kingdom</td>
<td>−8.362813*</td>
<td>0.855647</td>
<td></td>
</tr>
<tr>
<td>Switzerland</td>
<td>−4.321639*</td>
<td>3.273620*</td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>−6.184301*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>REERt</td>
<td></td>
<td></td>
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<tr>
<td>U.S. Dollar</td>
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<td>2.134151**</td>
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<tr>
<td>U.K. Pound</td>
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<tr>
<td>Swiss Franc</td>
<td>−3.372176**</td>
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<td>Japanese Yen</td>
<td>−2.440802</td>
<td>2.498017**</td>
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<tr>
<td>Germany</td>
<td>−2.903780**</td>
<td>2.882024*</td>
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<tr>
<td>ΔREERt</td>
<td>−9.813655*</td>
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<tr>
<td>U.S. Dollar</td>
<td>−10.32531*</td>
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<tr>
<td>U.K. Pound</td>
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<tr>
<td>Swiss Franc</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>German, Yen</td>
<td>−5.340838*</td>
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Notes to Table 1: The Augmented Dickey–Fuller test for unit root for the levels and first differences of the NFAt/GDPt and REERt of 5 countries (US, UK, Switzerland, Japan, and Germany). The ADF procedure tests the null hypothesis of unit root against the alternative of a stationary root. The ADF test statistic is reported under the first column and its significance is compared to MacKinnon critical values. The lag parameter “Lag =” is selected using the Akaike information criterion. In cases where the underlying regression included a drift and a trend terms, the standard t-test statistics of the regression coefficients in front of the drift and trend terms are reported under columns two and three, respectively. Significance at the 10%, 5% and 1% levels are indicated by ****, ***, ** and *.
\[ \Delta Y_t = \mu_t + AY_{t-1} + \sum_{i=1}^{k-1} A_i \Delta Y_{t-i} + V_t, \]  

(4)

where no parameter restrictions are imposed on matrices A and \( A_i \). The lagged first differences of \( Y_t \)'s are included to control for serial correlation in \( V_t \) and to guarantee that it is a white noise process. Before implementing the test for each separate country, the lag parameter \( k \) needs to be selected; it refers to the order of the underlying vector autoregressive (VAR) model that is fitted to the first differenced series of \( Y_t \). The lag \( k \) parameter is selected based on the AIC.

Another consideration before the Johansen test is applied is the specification of the deterministic term \( \mu_t \) in equation (4). The model presented in section 2 predicts that \( \mu \) is a constant which is a function of the underlying parameters and the steady state values of the variables. Under this specification \( \mu \) is left unrestricted in equation (4), thereby including both deterministic trends in the component series and constants in the cointegrating vectors. This specification is appealing because it not only fits the theoretical predictions of the portfolio balance model but also accounts for the empirical properties of the component series, namely the presence of a drift in \( \text{REER}_t \) and the presence of a trend in \( \text{NFA}_t/\text{GDP}_t \). However, trends are excluded from the cointegrating vector but in reality the long-run relationship between \( \text{NFA}_t/\text{GDP}_t \) and \( \text{REER}_t \) might also have a trend. Thus, in implementing the Johansen procedure to test for saddle path dynamics in the bivariate system described by equation (4), two specifications are considered, one where \( \mu \) is a constant and one where \( \mu_t \) contains a trend. Only the specification where \( \mu \) is constant is reported for Germany because the time series \( \text{NFA}_t/\text{GDP}_t \) for Germany displays neither a drift, nor a trend. Both the Johansen maximum eigenvalue and trace statistics are reported.

The results of the Johansen tests are reported in Table 2. The results are remarkably robust for all countries and specifications. The null hypothesis that the coefficient matrix A has a zero rank cannot be rejected at the 1% level, neither by the maximum eigenvalue test, nor by the trace test. For some countries there is weak evidence of cointegration between the net foreign asset position and the real exchange rate. For the US under the specification where \( \mu_t \) contains a trend the hypothesis of a zero rank of the coefficient matrix can be rejected at the 5% level by the trace test, and at the 10% level by the maximum eigenvalue test. For Japan under the specification where \( \mu_t \) contains a trend and for

### Table 2

<table>
<thead>
<tr>
<th></th>
<th>Max. Eigenvalue statistic</th>
<th>Trace statistic</th>
</tr>
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<tbody>
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<td>Rank(A) = 0</td>
<td>Rank(A) = 1</td>
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<tr>
<td>United States lag = 5</td>
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<tr>
<td>Constant</td>
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<td>Restricted trend</td>
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<tr>
<td>Constant</td>
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<td>Restricted trend</td>
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<td>Restricted trend</td>
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<td>7.535529</td>
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<td>Japan lag = 4</td>
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<td></td>
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<tr>
<td>Constant</td>
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<td>Restricted trend</td>
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<td>Constant</td>
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<td>1.360403</td>
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</table>

Notes to Table 2: The Johansen tests for cointegration between \( \text{NFA}_t/\text{GDP}_t \) and \( \text{REER}_t \) are presented for five countries (US, UK, Switzerland, Japan and Germany). Both the maximum eigenvalue statistic “Max. Eigenvalue Statistic” and the trace statistics “Trace Statistic” are reported. The null hypotheses are given underneath the statistic labels. The alternatives for the maximum eigenvalue statistic are \( \text{Rank}(A) = 1 \) and \( \text{Rank}(A) = 2 \) and those for the trace statistic are \( \text{Rank}(A) > 0 \) and \( \text{Rank}(A) > 1 \). The lag parameter “Lag =” is selected using the Akaike information criterion. For all countries except Germany, two specifications for the deterministic term are considered, including a “constant” and a “restricted trend”. For Germany, only the “constant” specification is reported. Significance at the 10% and 5% levels are indicated by *** and **.
Switzerland under the specification where \( \mu \) is a constant, the hypothesis of a zero rank of \( A \) can be rejected at the 10% level by the maximum eigenvalue statistic. Overall, however, the results seem to suggest that there is no significant long-run relationship between the NFA position and the real exchange rate for the countries under study.

As noted in Section 3 the failure of the Johansen test to reject the hypothesis of a zero rank might be attributed to low power of the test which could arise from the fact the characteristic roots are close to zero and the initial condition is far from the steady state. Recall that as domestic and foreign assets become less substitutable, the characteristic roots tend to approach zero and the set of initial conditions that puts the system on a stable saddle path shrinks. Thus, as the degree of substitutability between domestic and foreign assets falls, the power of the Johansen test to reject the null hypothesis of zero rank might be lower. However, the sample of observations is relatively long which would contribute to the power of the test detect saddle path dynamics. In addition, the fact that the test failed to reject the null hypothesis of a zero rank for all the countries suggests a different interpretation, namely that the failure to reject zero rank is not due to low power but to the fact that the data actually indicates the absence of a long-run relationship between NFA and the real exchange rate for developed countries.

The most recent empirical literature on predictable valuation effects implies that they are very small and are not likely to affect the long-run empirical relationship between the NFA and the real exchange rate. The model described in Section 2 indicates that the strength of the predictable valuation effects is associated with the fact that domestic and foreign assets are imperfect substitutes which leads to saddle path dynamics between the NFA and the real exchange rate. Therefore, when the test indicates the absence of saddle path dynamics, it also suggests the absence of sizable predictable valuation effects in developed countries. There is a literature that estimates the size of predictable valuation effects on US returns differentials, meaning that there are predictable excess returns on some component of country’s gross assets relative to the same component on its liabilities. Estimates range from exorbitant to quite small. The most recent estimates of Curcuru et al. (2013), however, suggest that the return differential arising from predictable valuation effects is near zero. For FDI, the literature estimates persistently higher return on US assets abroad than on foreign assets held in the US, but this mostly stems from differential in yields rather than capital gains. So, this return differential is not due to valuation effects arising from capital gains.

Another reason for the absence of saddle-path dynamics is that the estimated return differential due to predictable valuation effects is entirely due to a composition effect, which is not captured by the empirical specification implied by the model. A composition effect gives rise to excess returns because domestic investors favor foreign assets with higher average return such as equity while foreign investors favor domestic assets with lower average return such as bonds. However, the model assumes that there is only one type of financial asset that is traded across countries. In any case, estimates of this effect for the US also tend to be quite small, about half a percent. As a result, it seems unlikely that the results are driven by a misspecification implied by the model.

Recent empirical evidence also supports the fact that the failure to reject the absence of a long-run relationship between the NFA position and the real exchange rate is not due to low power of the test to detect saddle path dynamics but to the fact that there is actually no stable long-run link between the two variables for developed countries. A number of papers have documented that real exchange rates should be cointegrated with the NFA position (see Faruquee (1995), Gagnon (1996), Broner et al. (1997), and Alberola Ila et al. (1999)). However, the most recent empirical studies, including Lane and Milesi-Ferretti (2004) and Christopoulos et al. (2012), using panel cointegration tests find that the long-run relationship between real exchange rates and NFA positions tends to be insignificant for developed countries. Christopoulos, Gente and León-Ledesma (2012) suggest that this difference reflects whether countries are or are not credit-constrained in international markets. They argue that countries face a constraint on capital inflows but for developed countries this constraint is not binding which makes the long-run real exchange rate only depend on the productivity spread between sectors (Balassa-Samuelson effect). Since the hypothesis of zero rank of the coefficient matrix cannot be rejected for all countries in the sample, the results are in line with this hypothesis.
To check the robustness of these results several tests were done. First, the model assumes that the real interest rate in a country is constant which implies that the trade balance (which always equals negative saving) does not vary in response to the interest rate but this is not the case in reality. The real interest rate varies over time and it might affect the behavior of NFA_t/GDP_t and the RER_t. Thus, when I analyze the joint dynamics of NFA_t/GDP_t and the RER_t I might also be capturing dynamics introduced by the interest rate, which might affect the roots of the system. Therefore, I included the real interest rate in the empirical specification in order to check whether the absence of cointegration is a result from not conditioning on real interest rates. Data were retrieved from the OECD and the International Financial Statistics for two types of interest rates, 3-month interest rates and money market rates, both adjusted with CPI inflation. Tables with the results are available in the appendix. The dynamic properties displayed by the bivariate systems for various countries remain unchanged after the Johansen test was performed on the expanded system. Specifically, even after the real interest rate was included in the specification, the variables continue not to be cointegrated for Germany, the UK and Japan. Similarly to the bivariate case for Switzerland the evidence is more mixed but the hypothesis of no cointegration cannot be rejected at the 1% level except for certain specifications with the 3 month rate. Similarly to the bivariate system for the US there is no cointegration when \( \mu_t \) is constant but the hypothesis of no cointegration can be rejected at the 1% level in favor of one cointegrating relationship when \( \mu_t \) contains a trend. However, when I estimate a VECM with one cointegrating vector, and test the hypothesis that the real interest rate is weakly exogenous, the likelihood ratio test cannot reject this restriction. This implies that excluding the interest rate from the specification does not impact the parameters of the cointegrating relationship between NFA and REER if it exists. In addition, the joint hypothesis that the real interest rate is weakly exogenous and can be excluded from the cointegrating relationship is also a restriction that cannot be rejected by the likelihood ratio test. This confirms that not including the interest rate in the specification does not affect the results from the Johansen test for the US.

Second, the analysis on the bivariate system was repeated with a sample period which ends in the fourth quarter of 2006 and excludes the Great Recession period. Then, the NFA was deflated using the US CPI\(^7\) instead of the nominal GDP and finally the real exchange rate was measured with a real effective exchange rate deflated with unit labor costs rather than the CPI.\(^8\) The results remain unchanged. The two variables under consideration continue not to be cointegrated.

5. Conclusions

Portfolio balance models suggest that predictable valuation effects can play a prominent role in the external adjustment process of developed countries whose international liabilities are denominated in domestic currency. The strength of predictable valuation effects depends on the assumptions that domestic and foreign assets are imperfect substitutes and that financial markets clear faster than goods markets. Then, the model predicts that the joint behavior of the NFA and real exchange rates in the long run displays saddle-path dynamics. This paper puts the portfolio balance model of Blanchard et al. (2005a) in VEC form and brings it to the data. The Johansen procedure is used to infer long-run dynamics. The empirical result is that NFA and the real exchange rate for a sample of five developed countries (US, UK, Switzerland, Japan and Germany) are not cointegrated. This result has two important implications. First, it suggests that predictable valuation effects are quantitatively small and do not affect the long-run relationship between the NFA and the real exchange rate in developed countries. This is in line with recent empirical findings that predictable valuation effects are negligible for developed countries, even the US. Second, the result suggests that the NFA does not determine the long-run value of the real exchange rate in developed countries. This is in line with recent findings that sectoral productivity differentials are the sole determinant of long-run real exchange rates in countries which do not face borrowing constraints on international financial markets.

\(^7\) Note that the constructed NFA series is measured in current US dollars.

\(^8\) These results are available upon request.
Appendix

Figure A1. Time series plots for 3 month real interest rates.
Figure A2. Time series plots for real money market rates.
Table A1
Augmented Dickey Fuller test results

<table>
<thead>
<tr>
<th>Country</th>
<th>Lag</th>
<th>ADF test statistic</th>
<th>Drift t statistic</th>
<th>Trend t statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>10</td>
<td>-1.891143***</td>
<td></td>
<td></td>
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<td></td>
<td>9</td>
<td>-5.101703*</td>
<td></td>
<td></td>
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<tr>
<td>United Kingdom</td>
<td>11</td>
<td>-1.778703***</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>10</td>
<td>-5.761551*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Switzerland</td>
<td>7</td>
<td>-1.787357***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>-8.474365*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>7</td>
<td>-3.669796*</td>
<td></td>
<td></td>
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<td></td>
<td>6</td>
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<td></td>
<td></td>
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<tr>
<td>Germany</td>
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<td>-3.071235*</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>6</td>
<td>-2.313850**</td>
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Notes to Table A1: The Augmented Dickey–Fuller test for unit root for the levels and first differences of the 3 month real interest rates $r_t$ and of money market interest rates $r^m_t$ for 5 countries (US, UK, Switzerland, Japan, and Germany). The ADF procedure tests the null hypothesis of unit root against the alternative of a stationary root. The ADF test statistic is reported in a separate row for each country and its significance is compared to MacKinnon critical values. The lag parameter "Lag =" is selected using the Akaike information criterion. In cases where the underlying regression included a drift and a trend terms, the standard t-test statistics of the regression coefficients in front of the drift and trend terms are reported in separate rows. Significance at the 10%, 5% and 1% levels are indicated by ****, ***, and *. The samples on 3 month rates for Japan and Switzerland on which the test is performed start from 1977q1 and 1974q1, respectively. The sample for the Swiss money market rate starts on 1975q4.

Table A2
Johansen cointegration test results for 3 month rates

<table>
<thead>
<tr>
<th>Country</th>
<th>Max. Eigenvalue statistic</th>
<th>Trace statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rank(A) = 0</td>
<td>Rank(A) = 1</td>
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<tr>
<td>United States</td>
<td>Lag = 3</td>
<td>18.54424</td>
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<tr>
<td></td>
<td>Constant</td>
<td>30.85807**</td>
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<tr>
<td>United Kingdom</td>
<td>Lag = 4</td>
<td>13.64031</td>
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<tr>
<td></td>
<td>Constant</td>
<td>17.92345</td>
</tr>
<tr>
<td>Switzerland</td>
<td>Lag = 6</td>
<td>28.50371*</td>
</tr>
<tr>
<td>Germany</td>
<td>Lag = 4</td>
<td>15.39836</td>
</tr>
<tr>
<td></td>
<td>Constant</td>
<td>20.07041</td>
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</table>

Notes to Table A2: The Johansen tests for cointegration between NFA/GDP, REER, and $r$, are presented for four countries (US, UK, Switzerland, Japan and Germany). Both the maximum eigenvalue statistic “Max. Eigenvalue Statistic” and the trace statistics “Trace Statistic” are reported. The null hypotheses are given underneath the statistic labels. The alternatives for the maximum eigenvalue statistic are Rank(A) = 1, Rank(A) = 2, and Rank(A) = 3 and those for the trace statistic are Rank(A) > 0, Rank(A) > 1, Rank(A) > 2. The lag parameter “Lag =" is selected using the Akaike information criterion. For all countries two specifications for the deterministic term are considered, including a “constant” and a “restricted trend”. Significance at the 10%, 5% and 1% levels are indicated by *****, ***, and *. Japan is excluded because of the stationarity properties of the Japanese 3 month real interest rate.
Estimates of US VECM and likelihood ratio test on restrictions on interest rates

The following matrices show the estimates of the adjustment coefficients and cointegrating relation coefficients for the US where the estimated VECM includes a restricted trend and the lag of the underlying VAR model is 3. The first set of results is on 3-month rates and the second on money market rates. The first subset (Unrestricted estimates) contains results on $\alpha$, a vector of estimated adjustment coefficients and $\beta$, a vector of estimated coefficients in the cointegrating relation.

The second subset of results (Restricted estimates 1) includes estimates under the null hypothesis that the real interest rate is weakly exogenous ($\beta(2,1) = 0$) with the corresponding Likelihood Ratio (LR) test statistic and p-value in brackets based on which we cannot reject this null hypothesis.

The third subset of results (Restricted estimates 2) includes estimates under the joint null hypothesis that the real interest rate is weakly exogenous ($\beta(2,1) = 0$) and can be excluded from the cointegration relation altogether ($\beta(2,1) = 0$) with the corresponding Likelihood Ratio (LR) test statistic and p-value in brackets based on which we cannot reject this null hypothesis.

Unrestricted estimates for 3-month rates

$$\hat{\alpha} = \begin{bmatrix} -0.1147393 \\ 13.62231 \\ 0.334395 \end{bmatrix}, \quad \hat{\beta} = \begin{bmatrix} -0.0018165 \\ 0.0026932 \\ 0.0020776 \\ 0.096459 \end{bmatrix}$$

Restricted estimates 1 for 3-month rates

$$\hat{\alpha} = \begin{bmatrix} -0.1079887 \\ 0 \\ 0.3801859 \end{bmatrix}, \quad \hat{\beta} = \begin{bmatrix} 0.9269094 \\ 0.0026932 \\ 0.0019368 \\ -0.0938627 \end{bmatrix}$$

Table A3
Johansen cointegration test results for money market rates

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<tr>
<th>Country</th>
<th>Lag</th>
<th>Constant</th>
<th>$\text{Trace statistic}$</th>
<th>$\text{Max. Eigenvalue statistic}$</th>
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<td></td>
<td>Rank(A) = 0</td>
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<td>Restricted Trend</td>
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</tbody>
</table>

Notes to Table A3: The Johansen tests for cointegration between NFA/GDP, REER, and $r^m$ are presented for five countries (US, UK, Switzerland, Japan and Germany). Both the maximum eigenvalue statistic “Max. Eigenvalue Statistic” and the trace statistics “Trace Statistic” are reported. The null hypotheses are given underneath the statistic labels. The alternatives for the maximum eigenvalue statistic are Rank(A) = 1, Rank(A) = 2, and Rank(A) = 3 and those for the trace statistic are Rank(A) > 0, Rank(A) > 1, Rank(A) > 2. The lag parameter “Lag” is selected using the Akaike information criterion. For all countries two specifications for the deterministic term are considered, including a “constant” and a “restricted trend”. Significance at the 10%, 5% and 1% levels are indicated by “***”, “**” and “*”.

Estimates of US VECM and likelihood ratio test on restrictions on interest rates

The following matrices show the estimates of the adjustment coefficients and cointegrating relation coefficients for the US where the estimated VECM includes a restricted trend and the lag of the underlying VAR model is 3. The first set of results is on 3-month rates and the second on money market rates. The first subset (Unrestricted estimates) contains results on $\alpha$, a vector of estimated adjustment coefficients and $\beta$, a vector of estimated coefficients in the cointegrating relation.

The second subset of results (Restricted estimates 1) includes estimates under the null hypothesis that the real interest rate is weakly exogenous ($\beta(2,1) = 0$) with the corresponding Likelihood Ratio (LR) test statistic and p-value in brackets based on which we cannot reject this null hypothesis.

The third subset of results (Restricted estimates 2) includes estimates under the joint null hypothesis that the real interest rate is weakly exogenous ($\beta(2,1) = 0$) and can be excluded from the cointegration relation altogether ($\beta(2,1) = 0$) with the corresponding Likelihood Ratio (LR) test statistic and p-value in brackets based on which we cannot reject this null hypothesis.

Unrestricted estimates for 3-month rates

$$\hat{\alpha} = \begin{bmatrix} -0.1147393 \\ 13.62231 \\ 0.334395 \end{bmatrix}, \quad \hat{\beta} = \begin{bmatrix} -0.0018165 \\ 0.0026932 \\ 0.0020776 \\ -0.096459 \end{bmatrix}$$

Restricted estimates 1 for 3-month rates

$$\hat{\alpha} = \begin{bmatrix} -0.1079887 \\ 0 \\ 0.3801859 \end{bmatrix}, \quad \hat{\beta} = \begin{bmatrix} 0.9269094 \\ 0.0026932 \\ 0.0019368 \\ -0.0938627 \end{bmatrix}$$
Restricted estimates 2 for 3-month rates

\[
\hat{\alpha} = -0.1082454, \quad \beta = 0.3795724
\]

\[
\hat{NFA}/GDP = 0.9283066, \quad r^m = 0
\]

\[
\hat{REER} = -0.0088459, \quad LR = 1.199[0.549] \quad \text{constant}
\]

Unrestricted estimates for money market rates

\[
\hat{\alpha} = -0.1146408, \quad \beta = 0.3390834
\]

\[
\hat{NFA}/GDP = 0.9376348, \quad r^m = 1
\]

\[
\hat{REER} = 0.0022107, \quad LR = 1.066[0.302] \quad \text{constant}
\]

Restricted estimates 1 for money market rates

\[
\hat{\alpha} = -0.1091384, \quad \beta = 0.3780797
\]

\[
\hat{NFA}/GDP = 0.9376348, \quad r^m = 0
\]

\[
\hat{REER} = -0.0088459, \quad LR = 1.066[0.302] \quad \text{constant}
\]

Restricted estimates 2 for money market rates

\[
\hat{\alpha} = -0.1086747, \quad \beta = 0.3808643
\]

\[
\hat{NFA}/GDP = 0.9312044, \quad r^m = 0
\]

\[
\hat{REER} = -0.0094385, \quad LR = 1.069[0.586] \quad \text{constant}
\]

References


Lane, Philip R., Milesi-Ferretti, Gian Maria, December 2001. The external wealth of nations: measures of foreign assets and liabilities for industrial and developing countries. J. Int. Econ. 55, 263–294.


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