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JEL classification: E31, E37, E58, C22

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1 Introduction

It is well known that the stance and the strategy of monetary policy in many advanced economies underwent important structural changes over the past decades. Many of these changes can be associated with specific dates in history, such as the early 1980s Volcker disinflation in the US. Other changes are gradual and thus more difficult to identify. The changing nature of monetary policy is likely to be reflected in the time series properties of inflation dynamics. While shifts in the mean and the variance of inflation could be interpreted in light of monetary policy changes, another indicator is the degree of persistence, or inertia, in the inflation process. Persistence refers to the speed at which the inflation rate returns to its mean following a shock. Even if the mean inflation rate falls due to the central bank’s increased focus on price stability, fluctuations in inflation can be short-lived or long-lasting - in part depending on the determination of monetary policy to bring inflation back on an implicit or explicit target rate.¹

Hence, the persistence properties of inflation have received considerable attention in the empirical literature. In a recent survey article, Fuhrer (2011, p. 448) summarizes the abundant literature on the nature and the sources of persistent inflation dynamics in the US. He argues ”that the contribution to inflation from its unit root component has diminished significantly in recent decades. [...] With regard to the specific autocorrelation properties of a stationary inflation rate, the picture is considerably murkier.”

We revisit the changing nature of inflation persistence in the US. We add to the literature on inflation persistence in three ways. First, we use a quantile regression approach which allows us to examine the degree of inflation persistence at different conditional quantiles of inflation. Thus far the literature focuses on persistence evaluated at the conditional mean. This neglects the fact that inflation following shocks drawn from the tails of the shock distribution might exhibit a different pattern of inertia than inflation close to the mean. Second, we draw on techniques recently developed by Oka and Qu (2011) to estimate structural changes in regression quantiles at unknown dates to detect structural changes in persistence for different inflation quantiles. This allows us to examine whether changes in persistence are synchronized across inflation quantiles and whether shifts in persistence at the mean inflation rate are informative about the entire distribution of inflation outcomes. Third, we use the quantile autoregression unit root tests developed in Koenker and Xiao (2004) and Galvao (2009) to test whether inflation follows a unit root process—at different quantiles and globally—in the different regimes identified with the structural break tests.

In contrast to standard estimates of inflation persistence at the conditional mean, our empirical specification allows us to estimate the degree of inflation persistence conditional on different magnitudes and signs of shocks. Such differences in inflation persistence over the conditional inflation distribution could be for example caused by nominal wage rigidities, menu costs or asymmetric monetary policy. Standard estimates at the conditional mean cannot distinguish between these important differences in the effects of shocks to inflation.

¹Other factors such as a reduction in the degree of wage indexation since the early 1980s, see Hoffman, Peersman and Straub (2010), would also lead to a reduction in persistence, although a more gradual decline.
Quantile regressions offer a natural way to empirically assess the importance of these asymmetries. In particular, quantile regressions allow us to analyze whether the timing and the nature of changes of inflation persistence have been synchronous at different parts of the conditional distribution.

Based on a battery of post-war US inflation rates at monthly and quarterly frequency, we derive two key findings:

First, for all monthly inflation rates we provide evidence for a structural break in persistence at all quantiles of the inflation process occurring in the early 1980s. Persistence at the conditional mean as well as persistence at the outer quantiles is significantly lower after the Volcker disinflation. This result is robust with respect to changes in mean inflation and the variance of inflation. We do not find, in contrast, breaks in the persistence of quarterly inflation series, i.e. deflator-based inflation rates.

Second, quantile autoregression based unit root tests show that since the end of the Volcker disinflation the unit root can be rejected at every quantile, both for monthly and quarterly rates. Prior to 1980 the unit root hypothesis could not be rejected over the whole conditional inflation distribution. This sheds light on the recent work on the changing forecastability of inflation. Stock and Watson (2007) model the inflation rate as an integrated moving average process and argue that the difficulty to forecast inflation rates might stem from changing relative roles of a permanent and a transitory component in the inflation process. In our paper, however, we show that the empirical support for the assumption of a unit root component in the inflation process has disappeared not only at the mean inflation rate but at all quantiles.

The remainder of the paper is organized as follows: section 2 briefly surveys the available empirical literature. Our empirical approach is presented in section 3. Section 4 introduces the data set and discusses the main results. A set of robustness checks is documented in section 5. The final section concludes.

2 A brief review of the literature on inflation persistence

In his survey on inflation persistence, Fuhrer (2011, p. 448) summarizes the empirical evidence: "All authors agree that in the US and many other developed countries inflation exhibited considerable persistence from the 1960s to the mid-1980s. After that time, the statistical evidence is mixed. For both the US and other countries studies fall on both sides of the argument about the possibility of declining reduced-form persistence."

Let us briefly survey the key contributions to the picture described by Fuhrer. In one of the earliest studies, Taylor (2000) finds a break in US inflation persistence that coincides with the Volcker disinflation. Cecchetti and Debelle (2006) and Levin and Piger (2006) assess inflation persistence for major industrial economies and find that conditional on a break in the intercept inflation is much less persistent than previously thought. Both papers stress the need to account for shifts in mean inflation. Neglecting shifts in mean inflation could lead to spuriously high estimates of the sum of the autoregressive coefficients in the inflation process.
These contributions do not examine structural changes in inflation persistence at potentially unknown points in time. Cogley, Primiceri and Sargent (2010) use a time-varying vector autoregression to estimate the nonstationary trend component of inflation, which they associate with the Fed’s inflation target. Changes in trend inflation could then lead to changes in persistence of aggregate inflation. They are able to show a reduced persistence in the gap between inflation and the pure random walk component of the inflation rate in the Volcker-Greenspan era.\(^2\) Pivetta and Reis (2007), in contrast, use Bayesian methods and do not find a change in the persistence of GDP deflator inflation in the US. According to their results based on rolling-window and recursive samples inflation persistence is high and unchanged over the past decades. Benati (2008) systematically evaluates the impact of regime shifts in monetary policy on the persistence properties of inflation. His estimates of the sum of the autoregressive coefficients in a univariate process of CPI inflation drop significantly in the post-Volcker period. Persistence of PCE and deflator inflation, however, remains high even after the Volcker-era.

An alternative approach to modeling nonlinearities in the persistence properties of inflation is to specify a smooth transition autoregressive (STAR) model where a nonlinear transition function governs the shift between different regimes. Following this line, Nobay, Paya and Peel (2010) find US inflation to be more mean reverting the further away inflation is from its mean. It is important to note that the STAR approach rests on the assumption of a specific functional form of the transition function. Quantile regressions, in contrast, offer a particularly attractive alternative as they do not require a priori assumptions.

O’Reilly and Whelan (2005) focus on the Euro area and show the difficulty to find empirical support for a reduction in inflation persistence there. Evidence on inflation persistence at the level of disaggregate inflation is provided by Clark (2006). His results reveal that the aggregation process induces persistence into the aggregate inflation series despite disaggregate inflation exhibiting little persistence.

A separate strand of the literature examines the degree of fractional integration of the inflation process. Kumar and Okimoto (2007) find a break in the order of fractional integration of monthly US CPI inflation in 1982. Long run persistence—or long memory—of inflation is much lower after the Volcker disinflation period. Recently, Hassler and Meller (2011) extend this line of research and present a test for multiple structural changes in the degree of fractional integration applied to monthly CPI inflation in the US. Conditional on a shift in mean inflation, which they locate in 1981, they find a break in 1973 only. At this date, that coincides with the collapse of the Bretton Woods system and the first oil crisis, inflation persistence significantly increases. A second break in 1980 proves to be insignificant.\(^3\)

Taken together, the literature indeed supports Fuhrer’s (2011) cautious view. The present paper revisits the changing nature of inflation persistence. A potential explanation behind


\(^3\) Since the long memory property of inflation examined in these studies can be approximated by an autoregressive process of very high order, a change in the degree of fractional integration can be interpreted as a shift in inflation persistence over the very long run. The approach taken in this paper, however, focuses on autoregressive processes of lower order.
these divergent results could be a sizable degree of heterogeneity of inflation persistence at different conditional quantiles of inflation. If persistence differs according to the size or the sign of the shocks driving inflation, the mean inflation rate would not be informative about the true nature of persistence. To address this issue, we model inflation persistence at different quantiles of the inflation process.

Our study is closely related to the recent work of Tsong and Lee (2011). These authors also model inflation in a quantile framework but do not assess time-variation in persistence at individual quantiles. Our key contribution is to apply recently developed tests for structural breaks at unknown time to the question of inflation persistence. We find that the results for a long sample as analyzed in Tsong and Lee (2011) and for subsamples before and after important structural breaks are very different.

3 The quantile approach to inflation persistence

In this section we introduce the measurement of inflation persistence and sketch the test for structural breaks in persistence at different quantiles and the quantile autoregression based unit root test.

3.1 Measuring inflation persistence

Our preferred measure of persistence is the sum of the autoregressive coefficients in a univariate process of inflation. By using this reduced form measure of inflation persistence we do not take a stand on the structural sources of persistence. A change in persistence detected by our measure is consistent with a variety of structural changes in the conduct of monetary policy, the nature of nominal rigidities or the properties of shocks hitting the economy. Let \( \pi_t \) be the inflation measure, \( \alpha \) an intercept term and \( \varepsilon_t \) a serially uncorrelated error term. The AR(\( q \)) process is

\[
\pi_t = \alpha + \sum_{k=1}^{q} \beta_k \pi_{t-k} + \varepsilon_t \tag{1}
\]

The sum of autoregressive coefficients is \( \rho = \sum_{k=1}^{q} \beta_k \). According to Andrews and Chen (1994), \( \rho \) is the preferred scalar measure of persistence in \( \pi_t \), since a monotonic relationship exists between \( \rho \) and the cumulative impulse response function of \( \pi_{t+j} \) to \( \varepsilon_t \). We rewrite expression (1) as

\[
\pi_t = \alpha + \rho \pi_{t-1} + \sum_{k=1}^{q-1} \gamma_k \Delta \pi_{t-k} + \varepsilon_t \tag{2}
\]

where \( \Delta \pi_t = \pi_t - \pi_{t-1} \). If \( \rho = 1 \), the inflation process contains a unit root. If \( |\rho| < 1 \), the process is stationary. In the empirical application below we set the lag length to \( q = 4 \) for quarterly data and \( q = 12 \) for monthly data.

Estimates of \( \rho \) obtained from least squares estimation suffer from a bias as \( \rho \) approaches unity. Therefore, the literature typically resorts to Hansen’s (1999) median unbiased estimator of \( \rho \). This estimator, however, has not yet been developed for quantile autoregressive models.

We follow Tsong and Lee (2011) and, when referring to estimates for conditional quantiles,
report results based on the standard quantile regression estimates by Koenker and Bassett (1978).

### 3.2 Persistence at different quantiles

Quantiles are values that divide a distribution such that a given proportion of observations is located below the quantile. The $\tau$-th quantile is defined as the value $q_{\tau}(\pi_t|\pi_{t-1}, \ldots, \pi_{t-q})$ such that the probability that the conditional inflation rate will be less than $q_{\tau}(\pi_t|\pi_{t-1}, \ldots, \pi_{t-q})$ is $\tau$ and the probability that it will be more than $q_{\tau}(\pi_t|\pi_{t-1}, \ldots, \pi_{t-q})$ is $1-\tau$. The AR$(q)$ process of inflation dynamics at quantile $\tau$ can be written as a quantile autoregression, QAR$(q)$

$$q_{\tau}(\pi_t|\pi_{t-1}, \ldots, \pi_{t-q}) = \alpha(\tau) + \rho(\tau) \pi_{t-1} + \sum_{k=1}^{q-1} \gamma_k(\tau) \Delta \pi_{t-k}.$$  

(3)

Estimating the persistence parameter at different quantiles of the distribution instead of the mean can be done with quantile regressions as introduced by Koenker and Bassett (1978). Following the work of Koenker and Xiao (2004, 2006), this gives us the persistence parameter conditional on a grid of values for $\tau$. Quantile regressions impose no functional form constraints on parameter values over the conditional distribution of the inflation rate.

The interpretation of the quantile regression approach to inflation persistence is straightforward: estimates of $\rho(\tau)$ reveal the extent of inflation persistence at the quantile $\tau$ conditional on past values of inflation $\pi_{t-1}, \ldots, \pi_{t-q}$. Thus, shocks to the inflation process of different size and magnitude are allowed to lead to different patterns of persistence. If inflation is for example very high relative to recent inflation realizations this means that a large positive shock to inflation has occurred and that inflation is located above the mean conditional on past observations somewhere in the upper conditional quantiles. If inflation is lower than in the previous quarters, this means that a negative shock to inflation has occurred and that inflation conditional on past observations is located below the mean somewhere in the lower conditional quantiles. It is important, however, not to confuse the unconditional inflation distribution and the inflation distribution conditional on past inflation data. We cannot interpret persistence at, say, the $\tau = 0.2$ quantile as reflecting persistence at low absolute levels of inflation. Rather, it measures persistence when inflation exhibits a large negative deviation from its conditional mean.

### 3.3 Breaks in persistence at different quantiles

We test for structural breaks in inflation persistence that might show up in any part of the conditional inflation distribution. Qu (2008) and Oka and Qu (2011) have developed tests for structural change with unknown timing in regression quantiles. The tests are subgradient based and have good properties in small samples.

The test is run in two stages as recommended by Qu (2008) and Oka and Qu (2011). First, we test for structural stability across a range of quantiles using the $DQ$-test. This is a general test for changes in the entire conditional distribution of inflation. Since we do not have any prior information as to which part of the conditional inflation rate distribution is subject
to a break we test for a relatively large range of \( \tau \in \{0.20, 0.25, \ldots, 0.80\} \). An even larger range would have the disadvantage that the power of the test decreases as opposed to the case where prior information is used to trim the range of quantiles. In a second step we test for structural change in prespecified quantiles using the \( SQ_\tau \)-test. If the \( DQ \)-test rejects the null hypothesis of no structural break, the \( SQ_\tau \)-test can reveal structural breaks in different parts of the conditional inflation rate distribution. In this way we can detect in which parts of the distribution the actual change takes place and obtain a full picture about the stability of persistence across quantiles.

The tests allow for multiple structural breaks with unknown timing. The test procedure runs sequentially: first, for a given number of breaks, the break dates and the AR parameters are estimated jointly by minimizing the quantile check function over all permissible break dates. The range of permissible break dates excludes the first and the final 5% of the observations. We repeat this procedure for one to a maximum of ten possible structural breaks. Second, we use the \( DQ \)- and \( SQ_\tau \)-test to test how many structural breaks have occurred in the sample. This test proceeds stepwise and first tests the existence of one structural break against the null hypothesis of no structural break. Let \( \xi(\tau) = (\alpha(\tau), \rho(\tau), \gamma_1(\tau), \ldots, \gamma_{q-1}(\tau)) \) denote the vector of parameters in equation (3) at quantile \( \tau \) and suppose our inflation series contains \( T \) observations. Then the hypotheses for the \( DQ \)-test are:

\[
H_0^* : \xi_i(\tau) = \xi_0(\tau) \text{ for all } i \text{ and for all } \tau \in \{0.20, 0.25, \ldots, 0.80\}
\]

\[
H_1^* : \xi_i(\tau) = \begin{cases} 
\xi_1(\tau) & \text{for } i = 1, 2, \ldots, t \\
\xi_2(\tau) & \text{for } i = t + 1, \ldots, T.
\end{cases} \text{ for some } \tau \in \{0.20, 0.25, \ldots, 0.80\}
\]

The two hypotheses for the \( SQ_\tau \)-test are given by:

\[
H_0 : \xi_i(\tau) = \xi_0(\tau) \text{ for all } i \text{ for a given } \tau \in \{0.20, 0.25, \ldots, 0.80\}
\]

\[
H_1 : \xi_i(\tau) = \begin{cases} 
\xi_1(\tau) & \text{for } i = 1, 2, \ldots, t \\
\xi_2(\tau) & \text{for } i = t + 1, \ldots, T.
\end{cases} \text{ for a given } \tau \in \{0.20, 0.25, \ldots, 0.80\}
\]

First, we estimate the AR parameters at the different conditional quantiles under the null of no structural break. Afterwards, we estimate the AR parameters at the different conditional quantiles separately for the subsamples based on the previously estimated break dates. If a structural break exists, the estimated parameters under the null hypothesis are not close to the true values for at least one subset of the sample. The estimated residuals will persistently fall below (or above) the true quantile, forcing the subgradient to take a large value.

If the null hypothesis of no structural change is rejected, we test in sequential steps the null hypothesis of 1 break against the alternative hypothesis of 2 breaks using the \( DQ(l+1|l) \)- and the \( SQ_\tau(l+1|l) \)-tests. If we find evidence in favor of 2 breaks we check the null hypothesis of 2 breaks against the alternative hypothesis of 3 breaks and so on. Tables for critical values are provided in Qu (2008).
3.4 Quantile autoregression-based unit root testing

Once different regimes have been identified using the structural break tests, we examine differences in persistence in more detail. In particular we are interested in the question whether in a specific regime the inflation process contains a unit root, i.e. shocks to inflation have permanent effects, or whether inflation is stationary. This information is of crucial importance for central banks to assess which actions are needed to achieve a certain inflation target. While confidence bands are useful to check whether differences in persistence across quantiles are statistically significant, formal tests are more reliable specifically under the null hypothesis of a unit root.

We use the quantile autoregression-based unit root test developed in Koenker and Xiao (2004) and Galvao (2009). With this test we can check whether inflation displays unit root behavior within certain quantiles. The test is run in two stages. First we test for unit root behavior in specific quantiles. Afterwards, we use a Kolgomorov-Smirnov (KS) type test to check global mean reversion. Even if inflation is best described as a unit root process in a range of quantiles, but is mean reverting in the rest of the conditional distribution this may be sufficient to ensure global mean reversion. The test gives more detailed results than the augmented Dickey-Fuller (ADF) test as it allows for heterogeneous effects on inflation. The test is therefore able to show differences in persistence at different quantiles when inflation is hit by shocks of different size and sign.

Besides allowing for asymmetric effects of shocks on inflation, an important advantage of QAR-based unit root tests over standard unit root tests is an increase in power as shown by Koenker and Xiao (2004). To understand this power gain a comparison with Hansen (1995) is helpful. He shows that including more covariates can lead to substantial power gains when compared to univariate unit root tests. Interestingly the limiting distribution of the t-statistic of Koenker and Xiao (2004) and Galvao (2009) resemble the limiting distribution of tests discussed in Hansen (1995). Hence QAR-based unit root tests could be seen as a tool for systematically resorting to the framework of Hansen (1995) without including additional covariates.

Having estimated equation (3) our interest lies in the null hypothesis $H_0 : \rho(\tau) = 1$. We can test this at different values of $\tau$ to analyze the persistence of the inflation impact of positive and negative shocks and shocks of different magnitude.

To test $H_0 : \rho(\tau) = 1$ we use the t-stat for $\hat{\rho}(\tau)$ proposed by Koenker and Xiao (2004) which can be written as

$$t_n(\tau) = \frac{f(F^{-1}(\tau))}{\sqrt{\tau(1-\tau)}} (\pi'_{-1} M_Z \pi_{-1})^{1/2} (\hat{\rho}(\tau) - 1),$$

where $f(F^{-1}(\tau))$ is a consistent estimator of $f(F^{-1}(\tau))$, with $f$ and $F$ representing the probability and cumulative density functions of $\varepsilon_t$ in equation (2), $\pi_{-1}$ is the vector of lagged inflation observations and $M_Z$ is the projection matrix onto the space orthogonal to $Z = (1, \Delta \pi_{t-1}, \Delta \pi_{t-2}, ..., \Delta \pi_{t-q+1})$.

An application of the quantile autoregression-based unit root test to GDP can be found in Hosseinkouchack and Wolters (2013).

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\footnote{An application of the quantile autoregression-based unit root test to GDP can be found in Hosseinkouchack and Wolters (2013).}
While the test statistic in equation (4) allows us to test the unit root hypothesis at prespecified quantiles, we want to analyze in addition global persistence of inflation by testing jointly over a large range of quantiles $\tau \in T$, with $T = \{0.20, 0.25, ..., 0.80\}$. Koenker and Xiao (2004) and Galvao (2009) extend the testing procedure to a KS type test, which is given by:

$$QKS = \sup \{t_n(\tau)\},$$

(5)

where $t_n(\tau)$ is the t-statistic in equation (4).

The limiting distribution of $t_n(\tau)$ and $QKS$ are nonstandard. As discussed by Galvao (2009), the limiting distribution of $t_n(\tau)$ can be written as

$$t_n(\tau) \xrightarrow{D} \delta \left( \int_0^1 W_2^2 (r) dr \right)^{-1/2} \int_0^1 W_1 (r) dW_1 (r) + \sqrt{1 - \delta^2} N(0,1),$$

where $\xrightarrow{D}$ signifies convergence in distribution, $W_1 (r)$ is a Brownian motion, $W_2 (r) = W_1 (r) - \left[ \int_0^1 (4 - 6s) W_1 (s) ds - r \int_0^1 (6 - 12s) W_1 (s) ds \right]$ and $N(0,1)$ is a standard normal random variable independent from the first component of the limiting distribution. Let $\psi_\tau (x) = \tau - I(x < 0)$ and $\varepsilon_{t\tau}$ be the residuals from the regression equation (3), then the weighting parameter, $\delta$, is

$$\delta = \frac{\sigma_{\varepsilon\psi}}{\sigma_{\varepsilon} \sqrt{\tau (1 - \tau)}},$$

where $\sigma_{\varepsilon}$ is the long-run variance of $\varepsilon_t$ and $\sigma_{\varepsilon\psi}$ is the long-run covariance of $\varepsilon_t$ and $\psi_\tau (\varepsilon_{t\tau})$.

We follow the suggestions of Koenker and Xiao (2004) and Galvao (2009) to estimate the nuisance parameters.

The limiting distribution of $QKS$ can be obtained by a simulation strategy as described in Galvao (2009):

1. For each $\tau_i \in T$ compute $\delta(\tau_i)$.

2. For each $\delta(\tau_i)$ simulate one realization from the Dickey-Fuller and standard normal distributions independently and compute $t(\tau_i) = \delta(\tau_i) \left( \int_0^1 W_2^2 dr \right)^{-1/2} \int_0^1 W_1 dW_1 + \sqrt{1 - \delta(\tau_i)^2} N(0,1)$, and finally, take the maximum of the absolute values over $\tau_i$.

3. Repeating the last step a large number of times (we run 10000 repetitions), one can compute the critical value on the 5% level as the corresponding 0.95 quantile from the empirical distribution of the suprema.

4 Data and results

4.1 The data set

We use measures of post-war US inflation based on three alternative price indices with two different frequencies. The first measure is inflation expressed as the annualized quarter-on-quarter or month-on-month percentage change of the Consumer Price Index (CPI) for all urban consumers. The second measure is the annualized quarter-on-quarter or month-on-month percentage change of the Personal Consumption Expenditure (PCE) chain-type price
index. Our third measure is the quarter-on-quarter percentage change of the GDP deflator. All data series are taken from the FRED database at the Federal Reserve Bank of St. Louis. All data series are seasonally adjusted.

The time series for monthly CPI starts in 1947M2 and goes through 2013M12 (796 observations). The quarterly CPI and the GDP deflator series cover 1947Q2-2013Q4 (267 observations). The monthly and quarterly PCE series start later and cover 1959M2-2013M12 (647 observations) and 1959Q2 to 2013Q4 (219 observations), respectively. We use four lags for quarterly data and 12 lags for monthly data to compute a persistence measure that covers one year.\(^5\)

To contrast selected results with inflation dynamics in the euro area (EA), we also collect different series for EA inflation. We use the Harmonized Index of Consumer Prices (HICP) for the EA and the EA GDP deflator. The EA GDP deflator is available on a quarterly basis, while we can use quarterly and monthly versions of the HICP series. For the HICP series we use two versions. First, we use the HICP series for all 18 current member countries (HICP EA 18). As not all countries were members of the EA over the whole sample, this means that series includes developments outside the EA for the earlier part of the sample. Second, we use the HICP series with a changing composition of member countries (HICP EA CC). This series includes at each point in time the current members of the EA. Thus, this series reflects inflation dynamics within the EA only. So, overall we have five inflation measures for the euro area: quarterly GDP deflator inflation, quarterly and monthly HICP EA 18 and quarterly and monthly HICP EA CC. The samples are 1996Q2-2013Q3 (70 observations) for the GDP deflator series, 1996Q2-2013Q4 (71 observations) for the quarterly HICP EA 18 series, 1996M2-2013M12 (215 observations) for the monthly HICP EA 18 series, 1990Q2-2013Q4 (95 observations) for the quarterly HICP EA CC series, and 1990M2-2013M12 (287 observations) for the monthly HICP EA CC series, respectively. Data sources are Eurostat for the GDP deflator series and HICP EA 18 and the European Central Bank for the HICP EA CC inflation series. All inflation series are seasonally adjusted.

### 4.2 Results

As a first step, we study the behavior of inflation persistence at the conditional median and mean of the series. Figures (1) and (2) present rolling-window estimates of US inflation persistence together with bootstrapped confidence bands for a 10-year window. The bootstrap algorithm is a version of the block-bootstrap algorithm for quantile regression presented in Fitzenberger (1997).\(^6\) The solid line shows estimates at the conditional median, while the dotted line shows estimates at the conditional mean (without confidence band).

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\(^5\)These choices of the lag order are sufficient to eliminate serial correlation from the residuals. Durbin-Watson tests for the total sample and the subsamples do not indicate any serial correlation left in the estimation residuals.

\(^6\)For each bootstrap blocks of the variables are drawn randomly from the whole sample. For each of the 1000 bootstraps the QAR estimates are computed to get an estimate of the distribution of \(\rho(\tau)\). Choosing a block size larger than one allows to capture autocorrelation. However, we found that there is no autocorrelation in the residuals as we allow for a sufficiently high number of lags in the AR(\(q\)) process for inflation. Therefore the confidence bands computed with a standard bootstrap (with block size equal to one) yield extremely similar confidence bands as a block bootstrap so that we only present results for the former.
The results reflect the consensus view portrayed before: US inflation has become less persistent since the early 1980s. This tendency is more pronounced for monthly inflation rates and, in particular, for CPI inflation. Inflation persistence based on the GDP deflator, instead, exhibits fewer signs of instability, which is consistent with Pivetta and Reis’s (2007) finding and Benati’s (2008) result that persistence of CPI inflation has decreased, while it has remained high for PCE and deflator inflation. Prior to the early 1980s, when the correlation between all three measures of inflation was high, persistence behaved similarly. Since then, however, persistence diverged across alternative series. Moreover, some confidence bands still contain the unit root case.

Figures (3) and (4) show the same measures of persistence for EA inflation. The crucial question is whether the establishment of European Monetary Union (EMU) in 1999 has led to a drop in inflation persistence. Interestingly, however, the fall in persistence observable in the late 1990s is not statistically significant. Nevertheless, for most EA inflation rates the case of \( \rho \) equal to unity is no longer covered by the bootstrapped confidence bands. The creation of EMU coincides with inflation being no longer described by a unit root process.

In order to assess whether these findings are informative for the entire range of inflation quantiles, we now turn to the results from quantile regressions.\(^7\) To test for the existence of structural breaks in persistence at different quantiles, we report results for the DQ-test introduced in the previous section. Table (1) shows the estimated break dates over the conditional inflation distribution \( (\tau \in \{0.20, 0.25, ..., 0.80\}) \).

Three findings stand out. First, we do not find evidence for structural breaks in quarterly US inflation rates after the early 1950s, i.e. neither in the 1970s nor after the Volcker disinflation.\(^8\) Second, a highly significant break is found in both monthly series of US inflation which is clearly associated with the Volker disinflation in the early 1980s. It seems that the aggregation process that transforms monthly price level data to quarterly series is not innocuous with respect to the time series properties of inflation. Third, for the EA there is not a single structural break date.

Because the previous test for a joint break at all quantiles might be too restrictive, figure (5) reports the results of the \( SQ_\tau \)-test for breaks at specific quantiles for the US monthly inflation series for which the DQ-test signaled a structural break.

Monthly CPI inflation exhibits breaks over the entire range of inflation quantiles. While there is also evidence for a break in CPI inflation in the early 1950s, the breaks occurring in the 1980s are more interesting for our purposes. The shift in monetary policy under Paul Volcker had a profound impact on the inflation process leading to a break in persistence at all conditional quantiles. The considerably weaker evidence for a regime shifts in inflation dynamics for other inflation series reflects a general difficulty. Based on time series evidence

\(^7\)The figures discussed before show that quantile regressions at the conditional median, i.e. least absolute deviation estimates, are useful complements to least squares regressions even if one is only interested in characterizing the location of the conditional distribution rather than estimates in the tails of the conditional distribution. The sharp deflationary spike in the US series in 2008Q4 leads to a decrease in estimated inflation persistence at the conditional mean (Hansen’s (1999) unbiased median estimator), but not at the conditional median. The latter estimate is robust against outliers.

a structural break is often difficult to locate despite convincing narrative evidence for the
existence of a policy change. This points to the gradual effect of policy changes on observed
inflation dynamics.\textsuperscript{9}

While we see evidence for structural breaks, we do not know both the size and the sign of these
shifts yet. Figures (1) and (2) showed a decrease in inflation persistence at the conditional
median and mean of the inflation distribution only. To illustrate the nature of these breaks
for the whole range of quantiles, figures (6) to (10) plot the estimated constant $\alpha(\tau)$ and
the persistence parameter $\rho(\tau)$ together with bootstrapped 95\% confidence bands at different
quantiles $\tau \in \{0.20, 0.25, ..., 0.80\}$ for different subsamples. For the quarterly US inflation
rates we show estimates before and after 1981Q3, while the subsamples for the monthly US
inflation rates are based on the break points as suggested by the DQ-test in table (1).

In all figures the level-shifts in persistence across subsamples are apparent. The level-shift
is visible even for those series for which the DQ-test does not signal structural instability.
After 1981 persistence unanimously falls. This fall is more pronounced for CPI inflation
and less clear for PCE and deflator inflation.\textsuperscript{10} Our quantile regressions reveal that the fall
in persistence is indeed a characteristic of all inflation quantiles. Not only mean inflation
became significantly less inertial, but also inflation following particularly large shocks, either
negative or positive ones.

We also find that persistence is typically not equal across the conditional quantiles. For some
cases, i.e. for quarterly CPI inflation, persistence at high quantiles significantly deviates from
persistence evaluated at the mean inflation rate. In this case large shocks to inflation generate
stronger inertia than smaller shocks. This asymmetric nature of inflation persistence across
conditional quantiles cannot be detected by conventional measures of persistence.\textsuperscript{11}

Finally, we are particularly interested in the question whether inflation follows a unit root
process or is characterized as stationary. Tables (2) to (4) show point estimates at different
quantiles and results for the QAR-based unit root test and QKS-test for the whole sample
and different subsamples. The subsamples are again chosen based on the break points as
suggested by the DQ-test for monthly inflation and we split the sample again in 1981Q3 for
quarterly inflation series.

The point estimates show sizable changes in inflation persistence. For all three inflation
measures including the quarterly and the monthly series, inflation persistence is much lower
after 1981 than before. The difference is most pronounced for CPI inflation.\textsuperscript{12} Our quantile
regressions reveal that the fall in persistence is indeed a characteristic of all inflation quantiles.
Not only mean inflation became significantly less inertial, but also inflation following
particularly large shocks, either negative or positive ones. The QAR-based unit root tests
confirm this observation. Prior to 1981 the unit root hypothesis cannot be rejected over al-
most the whole conditional inflation distribution for all three inflation measures. After 1981,

\textsuperscript{9}Wieland and Wolters (2011), for example, show that following the Volcker disinflation forecasters overes-
timated inflation until the 1990s.

\textsuperscript{10}This is consistent with evidence provided by Zhang and Clovis (2009).

\textsuperscript{11}Inflation persistence being higher following large shocks is also consistent with menu cost models as
pioneered by Ball and Mankiw (1994).

\textsuperscript{12}This is consistent with evidence provided by Zhang and Clovis (2009).
however, we can rule out a unit root in the inflation process at all conditional quantiles. Even in the aftermath of large shocks inflation is mean reverting. Tsong and Lee (2011) also find a unit root for inflation in the upper conditional quantiles, but mean reversion in the lower conditional quantiles. Their finding is based on a long sample. Our results suggest that this finding disappears once we allow for structural breaks. The mixed test results for the whole sample shown in tables (2) to (4) disguise the subsample results according to which we can clearly split the sample into a unit root and a mean reverting regime.

5 Robustness

In this section we evaluate the robustness of our findings with respect to two modifications of the empirical specification: changes in the range of quantiles considered and breaks in mean and the variance of inflation, respectively.

5.1 The role of the chosen range of quantiles

We repeat the DQ-test and the SQ\(_\tau\)-test for a wider range of quantiles \(\tau \in \{0.05, 0.10, \ldots, 0.95\}\). While this wider range covers also the more extreme tails compared to our baseline case a disadvantage is a decrease in the power of the DQ-test. Furthermore, it might be too ambitious to try to detect structural change in the 5th and 95th percentile using a relatively small number of observations. The test results for the DQ-test are, however, almost unchanged.\(^{13}\) The DQ-test finds the same break in 1981 for monthly US CPI inflation and no break in the quarterly US inflation series nor the EA inflation series. Regarding US monthly PCE inflation, the break in 1980M4 vanishes and instead a break in 1990M10 is found, reflecting that uncertainty regarding structural breaks is considerably higher for US PCE than for US CPI inflation. Running the SQ\(_\tau\)-test yields a number of additional breaks in the extreme tails of the conditional inflation distribution. However, as these do not translate into breaks detected by the DQ-test, these are probably spurious and due to the relatively low number of observations in the outer quantiles.

5.2 The role of changes in mean inflation

Neglecting a break in mean inflation could lead to spuriously high estimates of the sum of the autoregressive coefficients. In their analysis of changes in the degree of fractional integration of US inflation, Hassler and Meller (2011) demean the inflation rate before and after 1981 separately to control for a level shift. To corroborate the robustness of our results, we follow Hassler and Meller (2011) and subtract the mean from the monthly US CPI inflation and US PCE inflation series separately for each subsample as identified by the DQ-test. We then employ again the DQ-test to detect breaks in the entire conditional distribution. The breaks found, see table (5), include the dates found in the baseline case. Note that we find many more break dates over the sample period. Many of them, however, will disappear

\(^{13}\)To save space these findings are available upon request.
below when we also control for changes in inflation volatility. The $SQ_\tau$-test, see figure (11), shows that persistence of monthly CPI inflation still changes at all conditional quantiles. In addition, monthly PCE inflation now also exhibits signs of changes in persistence, at least for persistence evaluated at the mean of the distribution and some neighboring quantiles. These findings support the notion that the baseline results are not obscured by structural breaks in the mean inflation rate.

The estimates of the persistence parameter as a function of the quantile in the different regimes, see figures (13) and (14), are broadly unchanged. The inflation process changed at all conditional quantiles in 1951 and 1981. Furthermore, all other results remain valid. Taken together, accounting for shifts in the mean inflation rate does not affect our main results.

5.3 The role of changes in mean inflation and the variance of inflation

The estimates of inflation persistence might not only be obscured by changes in the mean of the inflation process, but also by shifting volatilities. To account for this, we not only subtract the subsample-specific means but also normalize the series by their subsample-specific variance. Each inflation series then has a mean of zero and a variance of one. The results should thus be immune with respect to shifts in first and second moments. Table (6) shows the results of the DQ-test in this case. The test detects again a break in monthly CPI and PCE inflation in the early 1980s and in addition a break in PCE inflation in 1991 which is, however, only significant on the 10% level. The test does not detect the additional breaks found previously when adjusting the inflation series for changes in mean inflation, but not the volatility of inflation. So, the large number of breaks found in section 5.2 reflect changes in volatility of inflation. In contrast, the structural break in the early 1980s is not caused by changes in mean inflation or the volatility in inflation, but is indicative of structural changes in persistence.

Figure (12) shows the results from the $SQ_\tau$-test and confirms that the breaks in all quantiles of monthly CPI inflation remain unchanged once we control for variance shifts. All other breaks detected before, also those in the 1950s, disappear. As regards monthly PCE inflation we find that in the early 1980s only the shifts at $\tau = 0.50$ and the lower conditional quantiles exhibit a break.

The persistence estimates as a function of the conditional quantile, which are shown in figures (15) and (16), corroborate our baseline findings. Since the Volcker disinflation inflation persistence is lower at each quantile and each quantile-specific persistence estimate is statistically indistinguishable from persistence at $\tau = 0.50$. Thus, we conclude that our baseline results are robust with respect to shifts in first and second moments of inflation dynamics.

6 Concluding remarks

We draw on recently developed methods to identify structural breaks at conditional quantiles to study the changing nature of US inflation persistence. The framework is flexible enough to allow for asymmetries of inflation persistence across inflation quantiles - a characteristic of the data that accords well with several theoretical foundations.
We find strong and robust evidence for a reduction in persistence at all conditional quantiles of US inflation in the early 1980s. Thus, we contributed to the literature on inflation persistence by providing a missing key element: when there are shifts in monetary policy not only persistence at the conditional mean changes. Rather, the results support the notion that the entire inflation process reflects shifts in monetary policy. While Benati (2008) documents that shifts in monetary policy reduce persistence, we show that the new monetary policy regime in the US left its footprint on the entire conditional distribution of inflation.

Our results add to our knowledge about the conduct of monetary policy. The reduction in persistence is consistent with monetary policy successfully stabilizing inflation around the mean. Even shocks drawn from the tails of the distribution have only a short-lived impact on inflation as monetary policy keeps the inflation rate under control. It remains to be seen how the recent shift towards unconventional monetary policy measures since 2008 or the prolonged period of very low inflation rates are reflected in the conditional distribution of inflation. We leave that issue for future research.

References


Table 1: Tests for structural breaks in regression quantiles (DQ-Test)

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Notes: Results for the DQ-Test for breakpoints over the conditional inflation distribution ($\tau \in \{0.20, 0.25, ..., 0.80\}$). *, **, *** refer to the 10%, 5%, and 1% significance level. EA 18 denotes time series for the Euro area containing all 18 countries. EA CC denotes time series for the Euro area with changing country composition.
Table 2: Results for quantile unit root tests on quarterly US inflation rates

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<td>cv=2.89</td>
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</table>

Notes: The table shows estimates of inflation persistence of three different quarterly US inflation series (column 1) and different subsamples (columns 2) at different quantiles (columns 4-10). Statistics reported are point estimates of inflation persistence, \( \hat{\rho}(\tau) \), whether the unit root cannot be rejected (yes) or is rejected (no), the according t-statistic and corresponding critical value at the 5% significance level. The null hypothesis is rejected if the t-statistic is smaller than the critical value. The last row for each sample shows results for the KS-test: The entry unit root shows whether the unit root cannot be rejected (yes) or is rejected (no) over the whole conditional inflation distribution. QKS is the test statistic and cv indicates the critical value on the 5% significance level. The unit root hypothesis is rejected if QKS is larger than the critical value.
Table 3: Results for quantile unit root tests on monthly US CPI inflation rates

<table>
<thead>
<tr>
<th>Series</th>
<th>Sample</th>
<th>$\tau$</th>
<th>0.20</th>
<th>0.30</th>
<th>0.40</th>
<th>0.50</th>
<th>0.60</th>
<th>0.70</th>
<th>0.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>US CPI (m)</td>
<td>1947M2-2011M8</td>
<td>$\hat{\rho}(\tau)$</td>
<td>0.70</td>
<td>0.79</td>
<td>0.82</td>
<td>0.87</td>
<td>0.90</td>
<td>0.91</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Unit root</td>
<td>no</td>
<td>no</td>
<td>no</td>
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<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td></td>
<td></td>
<td>t-stat</td>
<td>-5.66</td>
<td>-5.30</td>
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<td>-2.59</td>
<td>-1.96</td>
<td>-2.03</td>
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<tr>
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<td>KS-test</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1947M2-1952M9</td>
<td>$\hat{\rho}(\tau)$</td>
<td>0.03</td>
<td>0.42</td>
<td>0.40</td>
<td>0.56</td>
<td>0.60</td>
<td>0.43</td>
<td>0.52</td>
</tr>
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<td></td>
<td></td>
<td>Unit root</td>
<td>yes</td>
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<td>yes</td>
<td>yes</td>
<td>no</td>
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<tr>
<td></td>
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<td>t-stat</td>
<td>-4.19</td>
<td>-2.34</td>
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<td>-1.97</td>
<td>-1.51</td>
<td>-2.41</td>
<td>-2.34</td>
</tr>
<tr>
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<td></td>
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<td>-2.63</td>
<td>-2.60</td>
<td>-2.53</td>
<td>-2.67</td>
<td>-2.58</td>
<td>-2.23</td>
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<tr>
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</tr>
<tr>
<td></td>
<td>1952M10-1981M9</td>
<td>$\hat{\rho}(\tau)$</td>
<td>0.92</td>
<td>0.91</td>
<td>0.96</td>
<td>1.00</td>
<td>1.00</td>
<td>0.98</td>
<td>0.97</td>
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<td>Unit root</td>
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<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
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<tr>
<td></td>
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<td>t-stat</td>
<td>-1.39</td>
<td>-1.81</td>
<td>-0.96</td>
<td>-0.10</td>
<td>-0.01</td>
<td>-0.36</td>
<td>-0.50</td>
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<td>critical value</td>
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<td>-2.55</td>
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<td>-2.73</td>
<td>-2.72</td>
<td>-2.60</td>
<td>-2.42</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1981M10-2000M11</td>
<td>$\hat{\rho}(\tau)$</td>
<td>0.29</td>
<td>0.36</td>
<td>0.58</td>
<td>0.56</td>
<td>0.57</td>
<td>0.56</td>
<td>0.46</td>
</tr>
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<td>Unit root</td>
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<td>-2.57</td>
<td>-2.53</td>
<td>-2.65</td>
<td>-2.56</td>
<td>-2.52</td>
<td>-2.67</td>
</tr>
<tr>
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<td>KS-test</td>
<td>unit root: no QKS=5.81 cv=2.88</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2000M12-2013M12</td>
<td>$\hat{\rho}(\tau)$</td>
<td>-0.31</td>
<td>0.03</td>
<td>0.08</td>
<td>0.33</td>
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<td>0.34</td>
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<td>Unit root</td>
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<td>no</td>
<td>no</td>
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<td>no</td>
<td>no</td>
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</tr>
<tr>
<td></td>
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<td>t-stat</td>
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<td>-3.93</td>
<td>-3.04</td>
<td>-2.98</td>
<td>-2.34</td>
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<tr>
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<td>KS-test</td>
<td>unit root: no QKS=4.96 cv=2.89</td>
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</tr>
</tbody>
</table>

Notes: The table shows estimates of inflation persistence for monthly US CPI inflation (column 1) and different subsamples (columns 2) at different quantiles (columns 4-10). Statistics reported are point estimates of inflation persistence, $\hat{\rho}(\tau)$, whether the unit root cannot be rejected (yes) or is rejected (no), the according t-statistic and corresponding critical value at the 5% significance level. The null hypothesis is rejected if the t-statistic is smaller than the critical value. The last row for each sample shows results for the KS-test: The entry unit root shows whether the unit root cannot be rejected (yes) or is rejected (no) over the whole conditional inflation distribution. QKS is the test statistic and cv indicates the critical value on the 5% significance level. The unit root hypothesis is rejected if QKS is larger than the critical value.
Table 4: Results for quantile unit root tests on monthly US PCE inflation rates

<table>
<thead>
<tr>
<th>Series</th>
<th>Sample</th>
<th>$\tau$</th>
<th>0.20</th>
<th>0.30</th>
<th>0.40</th>
<th>0.50</th>
<th>0.60</th>
<th>0.70</th>
<th>0.80</th>
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</thead>
<tbody>
<tr>
<td>US PCE (m)</td>
<td>1959M2-2013M12</td>
<td>$\hat{\rho}(\tau)$</td>
<td>0.81</td>
<td>0.85</td>
<td>0.89</td>
<td>0.94</td>
<td>0.94</td>
<td>0.95</td>
<td>1.01</td>
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<td>yes</td>
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<td>yes</td>
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<tr>
<td></td>
<td></td>
<td>t-stat</td>
<td>-4.28</td>
<td>-4.09</td>
<td>-2.89</td>
<td>-1.54</td>
<td>-1.58</td>
<td>-1.40</td>
<td>0.32</td>
</tr>
<tr>
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<td>KS-test</td>
<td>unit root: no</td>
<td>QKS=4.57</td>
<td>cv=2.91</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1959M2-1980M4</td>
<td>$\hat{\rho}(\tau)$</td>
<td>0.86</td>
<td>0.93</td>
<td>0.94</td>
<td>0.99</td>
<td>1.00</td>
<td>1.04</td>
<td>1.11</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>Unit root</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
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<td>t-stat</td>
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<td>-1.55</td>
<td>-1.32</td>
<td>-0.15</td>
<td>-0.08</td>
<td>0.89</td>
<td>1.85</td>
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<td>-2.48</td>
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<td>-2.60</td>
<td>-2.72</td>
<td>-2.59</td>
<td>-2.59</td>
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<td>cv=2.90</td>
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<tr>
<td>1980M5-2013M12</td>
<td>$\hat{\rho}(\tau)$</td>
<td>0.67</td>
<td>0.64</td>
<td>0.59</td>
<td>0.59</td>
<td>0.68</td>
<td>0.72</td>
<td>0.73</td>
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<td>-3.46</td>
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<td>critical value</td>
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<td>-2.43</td>
<td>-2.43</td>
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<td>KS-test</td>
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<td>QKS=5.94</td>
<td>cv=2.92</td>
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</table>

Notes: The table shows estimates of inflation persistence for monthly US PCE inflation (column 1) and different subsamples (columns 2) at different quantiles (columns 4-10). Statistics reported are point estimates of inflation persistence, $\hat{\rho}(\tau)$, whether the unit root cannot be rejected (yes) or is rejected (no), the according t-statistic and corresponding critical value at the 5% significance level. The null hypothesis is rejected if the t-statistic is smaller than the critical value. The last row for each sample shows results for the KS-test: The entry unit root shows whether the unit root cannot be rejected (yes) or is rejected (no) over the whole conditional inflation distribution. QKS is the test statistic and cv indicates the critical value on the 5% significance level. The unit root hypothesis is rejected if QKS is larger than the critical value.

Table 5: Tests for structural breaks in regression quantiles with demeaned data (DQ-test)

<table>
<thead>
<tr>
<th>Series</th>
<th>1st break date</th>
<th>2nd break date</th>
<th>3rd break date</th>
<th>4th Break date</th>
<th>5th break date</th>
</tr>
</thead>
<tbody>
<tr>
<td>US PCE (m)</td>
<td>1981M2*</td>
<td>1990M10**</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: The table shows results for the DQ-Test for breakpoints over the conditional inflation distribution ($\tau \in \{0.20, 0.25, \ldots, 0.80\}$) after controlling for a mean shift. *, **, *** refer to the 10%, 5%, and 1% significance level.

Table 6: Tests for structural breaks in regression quantiles with standardized data (DQ-test)

<table>
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<tr>
<th>Series</th>
<th>1st break date</th>
<th>2nd break date</th>
<th>3rd break date</th>
<th>4th Break date</th>
<th>5th break date</th>
</tr>
</thead>
<tbody>
<tr>
<td>US CPI (m)</td>
<td>1982M4***</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>US PCE (m)</td>
<td>1981M2**</td>
<td>1991M1*</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: The table shows results for the DQ-Test for breakpoints over the conditional inflation distribution ($\tau \in \{0.20, 0.25, \ldots, 0.80\}$) after controlling for shifts in the mean and the variance. *, **, *** refer to the 10%, 5%, and 1% significance level.
Figure 1: US inflation persistence (quarterly). Notes: The graphs show 10-year rolling window estimates of $\rho$ (see equation (2)) at the conditional median (solid line) together with 95% bootstrapped confidence bands (gray areas). The dotted line shows point estimates computed with Hansen’s (1999) median unbiased estimator.

Figure 2: US inflation persistence (monthly). Notes: see figure 1.
Figure 3: Euro area inflation persistence (quarterly). Notes: EA 18 CPI denotes CPI inflation for the Euro area consisting of 18 countries, EA CC CPI denotes CPI inflation for the Euro area with changing composition of countries. See also notes to figure 1.

Figure 4: Euro area inflation persistence (monthly). Notes: EA 18 CPI denotes CPI inflation for the Euro area consisting of 18 countries, EA CC CPI denotes CPI inflation for the Euro area with changing composition of countries. See also notes to figure 1.
Figure 5: Estimated break points at different quantiles (monthly). Notes: The graph shows break-points estimated with the SQ$_\tau$-test at all quantiles $\tau \in \{0.20, 0.25, ..., 0.80\}$ (vertical axis) for monthly US CPI and PCE inflation on the 5% significance level.
Figure 6: Estimated parameters at different quantiles for quarterly US CPI inflation. Notes: The graphs show estimates of the constant $\alpha$ and the persistence parameter $\rho$ at different quantiles $\tau \in \{0.20, 0.25, ..., 0.80\}$ for different subsamples. The gray areas indicate 95% bootstrapped confidence bands. The horizontal line shows Hansen’s (1999) median unbiased estimator together with 95% grid-t bootstrap confidence bands.

Figure 7: Estimated parameters at different quantiles for quarterly US PCE inflation. Notes: see figure 6.
Figure 8: Estimated parameters at different quantiles for quarterly US GDP deflator inflation. Notes: see figure 6.

Figure 9: Estimated parameters at different quantiles for monthly US CPI inflation. Notes: The subsamples are chosen based on the estimated break points of the DQ-test for monthly US CPI inflation. For further notes see figure 6.
Figure 10: Estimated parameters at different quantiles for monthly US PCE inflation. Notes: The subsamples are chosen based on the estimated break points of the DQ-test for monthly US PCE inflation. For further notes see figure 6.

Figure 11: Estimated break points at different quantiles (monthly, demeaned). Notes: The graph shows breakpoints estimated with the $SQ_{\tau}$-test at all quantiles $\tau \in \{0.20, 0.25, ..., 0.80\}$ (vertical axis) after controlling for a mean shift for US CPI and PCE inflation on the 5% significance level.

Figure 12: Estimated break points at different quantiles (monthly, standardized). Notes: The graph shows breakpoints estimated with the $SQ_{\tau}$-test at all quantiles $\tau \in \{0.20, 0.25, ..., 0.80\}$ (vertical axis) after controlling for mean and volatility shifts for US CPI and PCE inflation on the 5% significance level.
Figure 13: Inflation persistence (monthly US CPI) after controlling for a mean shift. Notes: The graphs show estimates of the constant \( \alpha \) and the persistence parameter \( \rho \) at different quantiles \( \tau \in \{0.20, 0.25, ..., 0.80\} \) for different subsamples. The gray areas indicate 95% bootstrapped confidence bands. The subsamples are chosen based on the estimated break points of the DQ-test for US CPI inflation. US CPI inflation has been demeaned separately for the subsamples. The horizontal line shows Hansen’s (1999) median unbiased estimator together with 95% grid-t bootstrap confidence bands.

Figure 14: Inflation persistence (monthly US PCE) after controlling for a mean shift. Notes: The graphs show estimates of the constant \( \alpha \) and the persistence parameter \( \rho \) at different quantiles \( \tau \in \{0.20, 0.25, ..., 0.80\} \) for different subsamples. The gray areas indicate 95% bootstrapped confidence bands. The subsamples are chosen based on the estimated break points of the DQ-test for US PCE inflation. US PCE inflation has been demeaned separately for the subsamples. The horizontal line shows Hansen’s (1999) median unbiased estimator together with 95% grid-t bootstrap confidence bands.
Figure 15: Inflation persistence (monthly US CPI) after controlling for shifts in mean and volatility. Notes: The graphs show estimates of the constant $\alpha$ and the persistence parameter $\rho$ at different quantiles $\tau \in \{0.20, 0.25, ..., 0.80\}$ for different subsamples. The gray areas indicate 95% bootstrapped confidence bands. The subsamples are chosen based on the estimated break points of the DQ-test for US CPI inflation. US CPI inflation has been standardized to have mean 0 and standard deviation 1 in each subsample. The horizontal line shows Hansen’s (1999) median unbiased estimator together with 95% grid-t bootstrap confidence bands.

Figure 16: Inflation persistence (monthly US PCE) after controlling for shifts in mean and volatility. Notes: The graphs show estimates of the constant $\alpha$ and the persistence parameter $\rho$ at different quantiles $\tau \in \{0.20, 0.25, ..., 0.80\}$ for different subsamples. The gray areas indicate 95% bootstrapped confidence bands. The subsamples are chosen based on the estimated break points of the DQ-test for US PCE inflation. US PCE inflation has been standardized to have mean 0 and standard deviation 1 in each subsample. The horizontal line shows Hansen’s (1999) median unbiased estimator together with 95% grid-t bootstrap confidence bands.