How the Baby Boomers' Retirement Wave Distorts Model-Based Output Gap Estimates

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Abstract:

Hours per capita measures based on the private sector as usually included in the set of observables for estimating macroeconomic models are affected by low-frequent demographic trends and sectoral shifts that cannot be explained by standard models. Further, model-based output gap estimates are closely linked to the observable hours per capita series. Hence, hours per capita that are not measured in concordance with the model assumptions can distort output gap estimates. This paper shows that sectoral shifts in hours and the changing share of prime age individuals in the working-age population lead indeed to erroneous output gap dynamics. Regarding the aftermath of the global financial crisis model-based output gaps estimated using standard hours per capita series are persistently negative for the US economy. This is not caused by a permanently depressed economy, but by the retirement wave of baby boomers which lowers aggregate hours per capita. After adjusting hours for changes in the age composition to bring them in line with the model assumptions, the estimated output gap gradually closes in the years following the global financial crisis.

Keywords: output gap estimates, DSGE models, hours per capita measurement, demographic trends, Bayesian estimation

JEL classification: C11, C54, E32, J11

Maik H. Wolters
Kiel Institute for the World Economy
Kiellinie 66
24105 Kiel, Germany
maik.wolters@ifw-kiel.de

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1 Introduction

Macroeconomic models used in academic research as well as at policy institutions are often estimated rather than calibrated. A large literature deals with the estimation of Dynamic Stochastic General Equilibrium (DSGE) models (see, e.g., An and Schorfheide, 2007; Fernández-Villaverde, 2010; Herbst and Schorfheide, 2015) and how to use estimated models for policy analysis and forecasting (see, e.g., Adolfson et al., 2007; Alvarez-Lois et al., 2008; Smets et al., 2010; Sbordone et al., 2010; Del Negro and Schorfheide, 2013; Wieland and Wolters, 2013). However, relatively few papers have studied the impact of the choice of observable time series, their measurement and time series characteristics for the estimation outcome. Instead, often a standard set of observables along the lines of Smets and Wouters (2007) is used to estimate DSGE models.

Guerron-Quintana (2010) shows that a careful selection of the observables is important, because alternative combinations of observables can lead to very different estimated key parameters and change the economic implications of estimated models substantially. A number of papers have considered that some data series might be imprecisely measured and have therefore included measurement errors in addition to the models’ structural shocks (see, e.g., Ireland, 2004; Edge et al., 2008). Boivin and Giannoni (2006) question whether economic variables can be properly measured by single indicators at all and introduced techniques to estimate DSGE models based on rich datasets. Gali et al. (2012) and Justiniano et al. (2013) look specifically at the measurement of wages and propose combining two different wage measures with very different time series properties. Justiniano et al. (2013) show that using only one wage measure based on highly volatile compensation leads to an implausibly large standard deviation of estimated wage mark-up shocks and misleading implications for the inflation-output trade-off.

Regarding hours per capita, many researchers have analysed changes in labor supply that imply that hours per capita are non-stationary. This can be caused by low-frequent structural changes like, for example, demographic trends, sectoral shifts between the public and private sector, changes in the tax code and changing preferences. Christiano et al. (2003), Chari et al. (2005), Basu et al. (2006) and Francis and Ramey (2009) show that the correct treatment of such a low frequency component of hours worked is crucial for avoiding erroneous findings regarding propagation mechanisms and the sources of business cycles in structural VARs analysis.

Much less work has been conducted on the implications of using measures of hours per capita that include low-frequent dynamics in the estimation of DSGE models. Regarding the effects of the changing share of prime age workers in the working-age population caused by the baby boomer cohort, Francis and Ramey (2009) show using a simple model that this can affect aggregate hours per capita. They further show that sectoral shifts between the private and public sector lead to mismeasured aggregate hours per capita if one uses hours in the private sector—as is done in the estimation of most DSGE models—rather than total hours of all sectors. Different cohorts and sectoral shifts are not modeled in standard DSGE models. If hours per capita are nevertheless used without correction for demographic trends and sectoral shifts, then the resulting low-frequent dynamics of aggregate hours per capita are erroneously interpreted through the model’s lens as cyclical variations in hours.

To my knowledge Chang et al. (2007) are the only ones that have directly addressed the dis-
crepancy between observed non-stationary hours per capita and the assumptions regarding hours in standard models by adding non-stationary labor supply shocks to their model. However, only in a version without frictions in the adjustment of labor inputs the specification with non-stationary labor supply shocks is supported by the data, while with such frictions the specification with stationary labor supply shocks is preferred even though this implies a mismatch between the model assumptions and the data characteristics.

Despite the possible non-stationarity of standard measures of hours per capita, they are regularly used as an observable in the estimation of DSGE models without adjusting the model accordingly or correcting the data to exclude low-frequent movements that cannot be explained by the model. I show that this can lead to erroneous findings of model-based analysis. In particular, I focus on the implications of the measurement of hours per capita on model-based output gap estimates. I show that model-based output gap estimates are closely linked to observable hours per capita. Sala et al. (2010) explain that this is because hours per capita are the main determinant of the labor wedge which in turn is the main determinant of the output gap in standard DSGE models. Hence, variations in aggregate hours that are caused by demographic trends, are erroneously interpreted by the model as inefficiencies in the labor market and are thus included in the output gap rather than interpreting them as a change in steady state hours.

Not accounting for low-frequent hours dynamics can have large effects on output gap estimates. A number of recent papers document a persistently negative US output gap since the global financial crisis of 2008/2009. Barsky et al. (2014) estimate the output gap in a modified version of the model by Smets and Wouters using hours in the non-farm business sector to remain at -15% after the financial crisis until their sample end in 2013. Del Negro et al. (2015a) show that the DSGE model of the New York Fed implies a persistently negative output gap that only very gradually moves from -5% in 2010 to -3% in 2015 and is projected to remain there until 2017. Per capita hours in non-farm payrolls which are restricted to the private sector are used for the estimation. Using a very similar model and average weekly hours of production and nonsupervisory employees for total private industries multiplied with the employment-population ratio Del Negro et al. (2015b) find an output gap that moves from -10% in 2010 to -8% in 2013. By contrast, output gap estimates with other methods than DSGE models, like the production function approach by the Congressional Budget Office or the state space models proposed by Laubach and Williams (2003), Fleischman and Roberts (2011), and Kiley (2015), imply a gradually closing output gap after the financial crisis as documented in Kiley (2015) and Laubach and Williams (2015).

I use a standard medium-scale DSGE model and show that estimating this model with standard hours per capita measures leads indeed to a persistently negative output gap after the financial crisis of 2008/2009. Comparing the estimated output gap and the hours per capita series confirms the finding by Sala et al. (2010) that hours per capita are the main determinant of the output gap.

Next, I replicate and update the analysis in Francis and Ramey (2009), who correct hours per capita for low-frequent movements. First, they propose using total hours rather than hours in the private sector as will be shown in the next sections.

1 I find that the exact level of the output gap following the financial crisis depends on whether and how one includes data on interest rate expectations in the estimation to account for the zero lower bound. The high persistence of the negative output gap since the financial crisis is, however, independent of this specific choice and is caused by persistently low hours per capita in the private sector as will be shown in the next sections.
private sector. Second, they correct hours per capita for the effects caused by the changing share of prime age workers in the working-age population due to the baby boomer cohort. Both adjustments avoid a large decrease in the low-frequent component of hours between 1960 and 1990. The large decrease in private hours from 1950 to about 1970, is corrected by accounting for the increase in government hours during the same time. Low per capita hours caused by the baby boomer cohort of which a large fraction was young and working less hours than prime age workers between 1960 and 1990 is corrected via the demographic adjustment of the hours series. Similarly, the large increase in per capita hours during the 1990s when the baby boomer cohort moved to the prime age worker group is corrected.

The sample analysed by Francis and Ramey (2009) ends in 2007, while I have updated their adjustment of hours per capita until the end of 2015. I show that sectoral and demographic shifts are of particular importance for the period after 2007. While private business hours decreased by more than 10% during the financial crisis and only very gradually increased afterwards, hours in the government sector stayed constant and hours in the non-profit sector even increased by 3%. Total hours decreased by about 7.5% so that focusing on private hours would overstate the decrease in hours during the financial crisis. I show that this leads to too low output gap estimates during the financial crisis.

Even more important than sectoral shifts are demographic changes after 2007. I show that the population share of people aged 65 and over has started to increase substantially since 2006 from 15.9% to 18.8% in 2015. Hence, the beginning of the retirement wave of the baby boomer cohort coincides roughly with the beginning of the financial crisis. People of ages 65 and over work substantially less than prime age workers so that the retirement wave of the baby boomer cohort induces a decrease in aggregate hours and hence depresses model-based output gap estimates after the financial crisis. Once, I apply the demographic correction proposed by Francis and Ramey (2009) aggregate hours increase after the financial crisis and the output gap does not remain persistently negative, but rather closes gradually until 2015.

Hence, the mismatch between the model assumptions and the data characteristics can lead to substantial distortions in estimated output gaps. Insofar such estimates are used in the policy process at central banks, erroneously low output gap estimates can have far reaching implications. I further show, that the demographic effects via hours on estimated output gaps will continue and intensify in the future. The population share of people aged between 55 and 64 has steadily increased from 10% in the mid-1990s to 16% in 2015 so that the retirement wave of the baby boomer cohort will continue and intensify over the next decade. Without demographically adjusting hours per capita output gap estimates of standard DSGE models will be too low over the coming years.

The remainder of this paper proceeds as follows. Section 2 outlines the medium scale DSGE model used for estimating the output gap. Section 3 shows that the model-based output gap estimates are closely linked to hours per capita and explains the reasons for this. In section 4 I replicate and update the correction of low-frequent movements in hours proposed by Francis and Ramey (2009) and show the effects of sectoral shifts and demographic adjustments on the measurement of hours and estimates of the output gap after the financial crisis. Finally, section 5 summarizes the findings and concludes.
2 A Medium-Scale DSGE Model with Financial Frictions

The model used is a standard DSGE model based on Smets and Wouters (2007) which is extended to include financial frictions as in Bernanke et al. (1999). The same model has been used in Del Negro and Schorfheide (2013) and Del Negro et al. (2015). It is a medium-scale DSGE model which is similar to models that are regularly used at central banks.

Long-run growth is described by a neoclassical core model and business cycle fluctuations are generated by a variety of structural shocks combined with a number of nominal and real frictions. Nominal frictions include sticky prices and wages, price and wage indexation and the financial accelerator mechanism and real frictions include habit formation, investment adjustment costs and capital utilization adjustment costs. Other specific features of the model are the non-separability of utility in consumption and leisure, the usage of the aggregator by Kimball (1995) which implies a non-constant elasticity of demand rather than the Dixit-Stiglitz aggregator and fixed costs in production. The model contains eight structural shocks and is fit to eight time series. Among the shocks are a total factor productivity shock, a risk premium shock, an investment-specific technology shock, a wage mark-up shock, a price mark-up shock, a government spending shock, a monetary policy shock, and a spread shock. All shock processes are serially correlated.

2.1 Model Equations

The model is so well known that I only shortly describe the log-linearized equations and refer the reader for more details to the literature cited above. All variables in the following are expressed in log deviations from their non-stochastic steady state.

\[ \tilde{z}_t = \rho \tilde{z}_{t-1} + \sigma z \varepsilon_{z,t}. \]

Non-stationary variables are detrended by

\[ Z_t = e^{\gamma t + \frac{1}{1-\alpha} \tilde{z}_t}, \]

where \( \gamma \) denotes the steady state growth rate. \( z_t \) denotes the growth rate of \( Z_t \) in deviations from \( \gamma \) and follows the process

\[ z_t = \ln\left(\frac{Z_t}{Z_{t-1}}\right) = \frac{1}{1-\alpha} (\rho z - 1) \tilde{z}_{t-1} + \frac{1}{1-\alpha} \sigma z \varepsilon_{z,t}. \]

The consumption Euler equation can be derived from combining the households’ first order conditions for consumption and bond holdings and is given by:

\[ c_t = c_1 (c_{t-1} - z_t) + (1 - c_1) E_t [c_{t+1} + z_{t+1}] + c_2 (L_t - E_t [L_{t+1}]) - c_3 (R_t - E_t [\pi_{t+1} + \epsilon^b_t]). \]  

The parameters are

\[ c_1 = (he^{-\gamma})/(1 + he^{-\gamma}), \]

\[ c_2 = [(\sigma c - 1)(w L/c*)]/[\sigma c (1 + he^{-\gamma})], \]

\[ c_3 = (1 - he^{-\gamma})/[(1 + he^{-\gamma})\sigma c]. \]

\( h \) governs the degree of habit formation, \( \sigma c \) is the inverse of the intertemporal elasticity of substitution and parameters with a * subscript denote steady state values. \( \epsilon^b_t \) denotes an AR(1) shock process on the premium over the central bank controlled interest rate. Consumption is a weighted average of past and expected future consumption due to habit formation. Consumption depends on hours worked, \( L_t \), because of their nonseparability in the utility function. When consumption and hours are complements (\( \sigma c > 1 \)), consumption increases with current hours and decreases with expected hours next period. The real interest rate and the shock term affect aggregate demand by inducing intertemporal substitution in consumption.
The investment Euler equation is given by:

\[ i_t = i_1(i_{t-1} - z_t) + (1 - i_1)E_t[i_{t+1} + z_{t+1}] + i_2q_t + \epsilon_t, \]  

where \( i_1 = 1/(1 + \beta e^{(1-\sigma_c)\gamma}) \) and \( i_2 = 1/((1 + \beta e^{(1-\sigma_c)\gamma})e^{2\gamma} \phi) \). \( \beta \) denotes the discount factor, \( \phi \) the elasticity of the capital adjustment cost function, \( q_t \) Tobin’s Q and \( \epsilon_t \) an investment specific technology shock that follows an AR(1) process. Current investment is a weighted average of past and expected future investment due to the existence of capital adjustment costs. It is positively related to the real value of the existing capital stock. This dependence decreases with the elasticity of the capital adjustment cost function.

The law of motion for physical capital is given by:

\[ k_t = k_1(k_{t-1} - z_t) + (1 - k_1)i_t + k_2\epsilon_t, \]  

where \( k_1 = (1 - i_*/k_*) \) and \( k_2 = i_*/k_*(1 + \beta e^{(1-\sigma_c)\gamma})e^{2\gamma} \phi. \)

The introduction of financial frictions leads to a replacement of the standard arbitrage condition between the return to capital and the riskless rate with the two following conditions:

\[ E_t \left[ \tilde{R}_t^k - R_t \right] = b_t + \zeta_{sp,b} \left( q_t^k + k_t - n_t \right) + \sigma_{w,t} \]  

and

\[ \tilde{R}_t^k - \pi_t = q_1r_t^k + q_2q_t^k - k_{t-1}, \]

where \( q_1 = r_t^k / (r_t^k + (1 - \delta)) \) and \( q_2 = (1 - \delta) / (r_t^k + (1 - \delta)) \). \( \tilde{R}_t^k \) denotes the gross nominal return on capital for entrepreneurs and \( n_t \) denotes equity of entrepreneurs. \( \sigma_{w,t} \) denotes an AR(1) shock process that captures mean-preserving changes in the cross-section dispersion of entrepreneurial equity. Equation (5) shows that the real value of the existing capital stock is a positive function of the rental rate of capital and a negative function of the real interest rate and the external finance premium. Equation (4) determines the spread between the expected return on capital and the riskless interest rate. The net worth of entrepreneurs evolves according to the following law of motion:

\[ n_t = \zeta_{n,\tilde{R}_t^k} \left( \tilde{R}_t^k - \pi_t \right) - \zeta_{n,R} (R_{t-1} - \pi_t) + \zeta_{n,qK} \left( q_{t-1}^k + k_{t-1} \right) + \zeta_{n,n}n_{t-1} - \frac{\zeta_{n,\sigma_{w,t}}}{\zeta_{sp,\sigma_{w,t}}} \sigma_{w,t-1}. \]

Capital used in production depends on the capital utilization rate and the physical capital stock of the previous period as new capital becomes effective with a lag of one quarter:

\[ k_t^e = k_{t-1} + u_t - z_t. \]

\( k_t^e \) denotes effective capital (physical capital adjusted for the capital utilization rate) and \( u_t \) the capital utilization rate.

Household income from renting capital services to firms depends on \( r_t^k \) and changing capital
utilization is costly so that the capital utilization rate depends positively on the rental rate of capital:

\[ u_t = (1 - \psi)/\psi r^k_t, \quad (8) \]

where \( \psi \in [0, 1] \) is a positive function of the elasticity of the capital utilization adjustment cost function.

Real marginal costs are given by:

\[ mc_t = w_t + \alpha L_t - \alpha k_t, \quad (9) \]

where \( \alpha \) is the income share of capital in the production function.

The capital-labor ratio is the same across all firms:

\[ k_t = w_t - r^k_t + L_t. \quad (10) \]

The production process is assumed to be determined by a Cobb-Douglas production function with fixed costs:

\[ y_t = \Phi(\alpha k^2_t + (1 - \alpha)L_t) + (\Phi - 1)/(1 - \alpha)\zeta_t. \quad (11) \]

The resource constraint is given by:

\[ y_t = c_y c_t + i_y i_t + u_y u_t + \epsilon^p_t - 1/(1 - \alpha)\zeta_t, \quad (12) \]

where output \( y_t \) is the sum of consumption, \( c_t \), and investment, \( i_t \), weighted with their steady state ratios to output \( c_y = c_*/y_* \) and \( i_y = i_*/y_* \), the capital-utilization adjustment cost which depends on the capital utilization rate, \( u_t \), and the steady state ratio of this cost to output \( u_y = r^k_*k_*/y_* \), and an exogenous government spending shock \( \epsilon^p_t \). \( \epsilon^p_t \) follows an AR(1) process and is also affected by the technology shock.

Monopolistic competition, Calvo-style price contracts, and indexation of prices that are not free to be chosen optimally combine to yield the following Phillips curve:

\[ \pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t[\pi_{t+1}] + \pi_3 mc_t + \epsilon^p_t, \quad (13) \]

with \( \pi_1 = t_p/(1 + \beta e^{(1-\sigma_c)\gamma} t_p) \), \( \pi_2 = \beta e^{(1-\sigma_c)\gamma} / (1 + \beta e^{(1-\sigma_c)\gamma} t_p) \), \( \pi_3 = 1/(1 + \beta e^{(1-\sigma_c)\gamma} t_p) \), \( (1 - \beta e^{(1-\sigma_c)\gamma} t_p)/(e^\gamma (\Phi - 1) \epsilon_p + 1) \). This Phillips curve contains not only a forward-looking but also a backward-looking inflation term because of price indexation. Firms that cannot adjust prices optimally either index their price to the lagged inflation rate or to the steady-state inflation rate. Note, this indexation assumption ensures also that the long-run Phillips curve is vertical. \( \xi_p \) denotes the Calvo parameter, \( t_p \) governs the degree of backward indexation, \( \epsilon_p \) determines the curvature of the Kimball aggregator. The mark-up shock \( \epsilon^p_t \) follows an ARMA(1,1) process.

A monopolistic labor market yields the condition that the wage mark-up \( \mu^w_t \) equals the real wage minus the marginal rate of substitution \( mr s_t \):

\[ \mu^w_t = w_t - mr s_t = w_t - [\sigma L_t + 1/(1 - he^{-\gamma}) (c_l - he^{-\gamma}(c_{t-1} - z_t))] \quad (14) \]
where $\sigma_l$ characterizes the curvature of the disutility of labor.

The wage Phillips-Curve is given by:

$$ w_t = w_1(w_{t-1} - z_t) + (1 - w_1)E_t[w_{t+1} + z_{t+1} + \pi_{t+1}] - w_2\pi_t - w_3\pi_{t-1} - w_4\mu^m_t + \epsilon_t^w, \quad (15) $$

where $w_1 = 1/(1 + \beta e^{(1-\sigma_c)\gamma})$, $w_2 = (1 + \beta e^{(1-\sigma_c)\gamma}e_{tw})/((1 + \beta e^{(1-\sigma_c)\gamma}))$, $w_3 = e_{tw}/(1 + \beta e^{(1-\sigma_c)\gamma})$, and $w_4 = 1/(1 + \beta e^{(1-\sigma_c)\gamma})(1 - \beta e^{(1-\sigma_c)\gamma}\xi_{tw})(1 - \xi_{tw})/((\phi_w - 1)\epsilon_{tw} + 1)$. The parameter definition is analogous to the price Phillips curve.

The monetary policy rule reacts to inflation, the output gap and the change in the output gap and incorporates partial adjustment:

$$ R_t = \rho R_{t-1} + (1 - \rho)(\phi_\pi\pi_t + \phi_x x_t) + \phi_{\Delta x}(x_t - x_{t-1}) + r^m_t. \quad (16) $$
r$^m_t$ is a monetary policy shock that follows an AR(1) process. The output gap $x_t$ is defined as the log difference between output and potential output.

Potential output is described by an allocation without nominal rigidities, i.e. with flexible prices and wages, without financial frictions, and without inefficient price and wage mark-up shocks and financial friction shocks. This allocation is obtained by setting $\xi_p = 0$, $\xi_w = 0$, $\epsilon^p_t = 0$ and $\epsilon^w_t = 0$ and replacing equations (4), (5), and (6) with

$$ q_{f,t} = q_1 E_t \left[ r_{f,t+1}^k \right] + (1 - q_1)E_t[q_{f,t+1}] - r_{f,t} + \epsilon_t^b, \quad (17) $$

where $q_1 = r^k_{sf} / (r^k_s + 1 - \delta)$. The $f$ subscript denotes that this allocation refers to flexible prices and wages and $r_{f,t}$ denotes the real natural interest rate. This allocation is efficient except for the constant inefficiency caused by monopolistic competition.

### 2.2 Estimation

The model is solved using the assumption of rational expectations. In addition to equations (1) to (17) measurement equations that relate the model variables to the data are added and these are given by:

- **output growth** \[ \gamma + 100(y_t - y_{t-1} + z_t) \]  \quad (18)
- **consumption growth** \[ \gamma + 100(c_t - c_{t-1} + z_t) \]  \quad (19)
- **investment growth** \[ \gamma + 100(i_t - i_{t-1} + z_t) \]  \quad (20)
- **real wage growth** \[ \gamma + 100(w_t - w_{t-1} + z_t) \]  \quad (21)
- **hours** \[ L_s + 100L_t \]  \quad (22)
- **inflation** \[ \pi_s + 100\pi_t \]  \quad (23)
- **federal funds rate** \[ R_s + 100R_t \]  \quad (24)
- **spread** \[ SP_s + 100E_t \left[ \tilde{R}_{t+1}^k - R_t \right] \]  \quad (25)
\( \pi_s, R_s, L_s \) and \( SP_s \) denote the steady state level of inflation, the federal funds rate, hours and the spread.

I further include four measurement equations that link model-based interest rate expectations with those from financial market participants to account for the zero lower bound on nominal interest rates and the effects of forward guidance:

\[
\text{federal funds rate expectations } t+k = R_s + 100E_t [R_{t+k}], \quad k = 1, \ldots, 4. \tag{26}
\]

To make estimation feasible with these four additional measurement equations I augment the model with four anticipated monetary policy shocks. The monetary policy shock process is thus given by:

\[
r_t^m = \rho r_{t-1}^m + \epsilon_t^m + \sum_{k=1}^{4} \epsilon_{t,t-k}^m. \tag{27}
\]

\( \epsilon_t^m \) is a standard monetary policy shock, where \( \epsilon_t^m \sim N(0, \sigma_r^2) \), and \( \epsilon_{t,t-k}^m \) are anticipated monetary policy shocks, where \( \epsilon_{t,t-k}^m \sim N(0, \sigma^2_{k,r}) \). They are known to agents at time \( t-k \), but affect the policy rule only at time \( t \).

The model is estimated using Bayesian techniques. I use the same prior distribution as in Del Negro and Schorfheide (2013) and Del Negro et al. (2015). This is essentially also the same prior as used in Smets and Wouters (2007), except for a wider prior distribution for the steady state inflation rate and additional priors for the financial friction parameters.

The sample goes from the first quarter of 1959 to the fourth quarter of 2015. The data series on per capita real output growth, consumption growth, investment growth, wage growth, inflation and the federal funds rate are constructed as in Smets and Wouters (2007). Following Del Negro et al. (2015) I use the difference between the Moody’s Seasoned Baa Corporate Bond Yield and the 10-Year Treasury Note Yield at Constant Maturity to measure the spread. I further use different measures of hours per capita that are described in the following sections and are documented in detail in the appendix. Interest rate expectations are taken from the Blue Chip Financial Forecast Survey for the period from 1992 to 2011 and from the New York Fed’s Survey of Primary Dealers from 2011 onwards. Interest rate expectations prior to 1992 are treated as unobserved.

Due to the nonlinearity in the structural parameter vector \( \theta \) the calculation of the likelihood is not straightforward. The Kalman filter is applied to the state space representation to set up the likelihood function. Combining the likelihood with the priors yields the log posterior kernel

\[
\ln L(\theta | y_{t=1}^{obs}, \ldots, y_{t}^{obs}) + \ln p(\theta),
\]

where \( y_{t}^{obs} \) denotes the vector of observable variables, that is maximized over \( \theta \) using numerical methods to compute the posterior mode. The posterior distribution of the parameters is a complicated nonlinear function of the structural parameters. The Metropolis-Hastings algorithm offers an efficient method to derive the posterior distribution via simulation. Details are provided for example in Schorfheide (2000). I compute 500000 draws from the Metropolis-Hastings algorithm and use the first 25000 of these to calibrate the scale such that an acceptance ratio of 0.3 is achieved. Another 25000 draws are disregarded as a burn in sample. Priors and posterior estimates are documented in the appendix. Output gap estimates shown in the different figures in this paper show the posterior mean of the output gap.
3 Hours per Capita Measures and Output Gap Estimates

In the following, I document that the dynamics of the model-based output gap estimates depend crucially on the dynamics of observable hours per capita and show that the importance of the labor wedge in standard DSGE models is the reasons for this close link.

3.1 The Close Link Between the Output Gap and Hours per Capita

Figure 1 shows in the upper panel two output gap series based on the estimated model. The output gap series shown as a solid line is based on a version of the model where hours per capita are measured using average weekly hours in the nonfarm business sector multiplied with the ratio of civilian employment of persons 16 years of age and older and the civilian noninstitutional population. This is probably the most widely used measure of hours per capita in estimated DSGE models (see, e.g., Smets and Wouters, 2007; Christiano et al., 2011, among many others). The second output gap series is shown as a dashed line and is based on an updated version of the hours per capita series by Francis and Ramey (2009). Both hours per capita measures are shown in percent deviations from the mean in the lower graph.

The hours series by Francis and Ramey (2009) is adjusted for sectoral and demographic shifts that standard DSGE models like the one used in this paper do not account for. The series accounts for sectoral shifts in hours worked between the private and public sector by simply using total hours instead of hours in the private sector. Further, original hours per capita \( H_t \) are adjusted for the cumulated chain-weighted changes in hours that are caused by demographic trends to yield a corrected series \( H_{t}^{\text{demo.adj.}} \) via the following formula:

\[
H_{t}^{\text{demo.adj.}} = H_t - \sum_{\tau=t_0}^{t} \sum_{i=1}^{8} \left( h_{i,\tau} + \frac{h_{i,\tau-1}}{2} \right) \left( \theta_{i,\tau} - \theta_{i,\tau-1} \right),
\]

(28)

where \( h_{i,t} \) denote hours per capita by age-group \( i \) in period \( t \), and \( \theta_{i,t} \) denotes the share of age-group \( i \) of the noninstitutional population ages 16 and over. This approach has been originally suggested by Shimer (1998) to correct the unemployment rate for demographic trends caused by the baby boomer cohort. I use the same eight age groups as in Francis and Ramey (2009): 16-17, 18-21, 22-24, 25-34, 35-44, 45-54, 55-64, and 65 and older. Also all other aspects of the computation follow exactly Francis and Ramey (2009) and I also use Census data from the integrated public use microdata series (IPUMs) so that the series shown in figure 1 replicates their adjustment of hours per capita and updates their series until the end of 2015. Details about the data sources (in particular regarding the update beyond the sample from Francis and Ramey) and computational details can be found in the appendix.

From comparing the upper and the lower graph of figure 1 it is clear that the dynamics of the estimated output gap series are closely related to the dynamics of the hours per capita measures. The correlation coefficients between the estimated output gap series and the hours per capita measures are 0.93 for the nonfarm business sector hours measure and 0.89 for the series based on Francis and Ramey (2009), respectively. Further, the differences between the two hours per capita measures are
reflected by the output gap estimates.² For example, the nonfarm business sector hours series shows a persistent negative deviation from the mean following the global financial crisis of 2008/2009, while the hours series based on Francis and Ramey (2009) gradually returns to its mean towards the end of the sample. Very similar dynamics can be seen for the respective output gap measures.

²Overall, the differences between the two hours series and the implied differences between the two output gap series are relatively modest. If one would instead compare per capita hours in the private sector (rather than the measure based on average hours in the non-farm business sector multiplied with the employment-population ratio) to demographically adjusted per capita hours in all sectors as is done in Francis and Ramey (2009), the differences would be much larger. For the purpose of this paper it is, however, more useful to focus on the measure of hours that is most widely used as an observable for estimating DSGE models.
3.2 The Output Gap, the Labor Wedge and Hours per Capita

In order to understand why there is such a strong link between the dynamics of hours and the output gap, I closely follow Sala et al. (2010) who first analyse how the output gap and the labor wedge are connected and in a second step how the labor wedge is linked to hours per capita.

The output gap measures general inefficiencies in the sticky price, sticky wage and financial friction allocation, whereas the labor wedge measures specifically inefficiencies in the allocation of labor. The labour wedge is defined as the deviation of household’s marginal rate of substitution between consumption and leisure from the firms’ marginal product of labor. Labor would be efficiently allocated if the marginal rate of substitution would equal the marginal product of labor. Any deviations that are measured by the labor wedge are therefore inefficiencies in the allocation of labor (see e.g. Chari et al., 2007).

The marginal rate of substitution is given by:

\[ mrs_t = \sigma L_t - \xi_t, \tag{29} \]

where \( \xi_t = -\frac{1}{1-he^{-\gamma}(c_{t-1} - z_t)} \) denotes the marginal utility of consumption. The marginal product of labor is given by:

\[ mpl_t = \alpha (k^s_t - L_t). \tag{30} \]

Hence, the labor wedge is:

\[
\text{wedge}_t = mrs_t - mpl_t \tag{31}
\]

\[ = (\sigma + \alpha) L_t - \xi_t - \alpha k^s_t \tag{32} \]

\[ = (\sigma + \alpha) (L_t - L_{f,t}) - (\xi_t - \xi_{f,t}) - \alpha (k^s_t - k^s_{f,t}), \tag{33} \]

where the last line uses the fact that the labor wedge is zero in the allocation with flexible prices and wages and without financial frictions and inefficient mark-up shocks.

The output gap \( x_t \) can be written as:

\[ x_t = y_t - y_{f,t} = \Phi \left[ \alpha (k^s_t - k^s_{f,t}) + (1 - \alpha) (L_t - L_{f,t}) \right]. \tag{34} \]

One can combine equations (33) and (34) to show the connection between the output gap and the labor wedge:

\[ x_t = \Phi \frac{1 - \alpha}{\alpha + \sigma_t} \left[ \text{wedge}_t + (\xi_t - \xi_{f,t}) + \frac{\alpha(1 + \sigma_t)}{1 - \alpha} (k^s_t - k^s_{f,t}) \right]. \tag{35} \]

Figure 2 shows the output gap and its components according to equation (35). It is visible that the largest part of the inefficiencies that are measured by the output gap are related to the labor wedge. The correlation between the output gap and the labor wedge is 0.96. The figure is based on the version of the model in which observable hours per capita are measured using average hours
in the nonfarm business sector (solid line, figure 1), but the close connection between the output gap and the labor wedge also holds when using alternative observable hours per capita measures. In simpler models without physical capital, government spending, fixed costs in production, and consumption habits, the output gap and the labor wedge are even exactly proportional (see Sala et al., 2010).

The graph also shows that the dynamics of the labor wedge are closely linked to the NBER defined recessions. The gap for the marginal utility of consumption is negatively correlated with the output gap and much smaller than the labor wedge. Finally, the capital services gap is positively correlated with the output gap, but very small.

Having shown that most inefficiencies captured by the output gap are caused by inefficiencies in labor as measured by the labor wedge, I turn now to the relation between the labor wedge and hours per capita. Figure 3 plots the different terms of equation (32). One can see that the labor wedge is mainly explained by the dynamics of hours per capita. Marginal utility of consumption also plays some role, though a much smaller than hours per capita, while capital services are unimportant for the labor wedge. The correlation between the labor wedge and hours per capita is 0.92. Sala et al. (2010) show that in a simpler model without capital, government spending, fixed costs in production, and consumption habits, the labor wedge is even exactly proportional to hours per capita.

One can further show that the labor wedge is dominated by the dynamics in the marginal rate of substitution, while the dynamics of the marginal product of labor are much smaller. As the real wage is acyclical it follows that the wage mark-up is the main cause for the inefficient allocation...
of labor, while the price mark-up plays a minor role.\(^3\) Similar results have been found for example by Gali et al. (2007) and Sala et al. (2010). Thus, the inefficient component is mainly caused by inefficient wage mark-up shocks and wage rigidities. These are needed to reconcile the volatile and strongly procyclical movements of hours and the more stable and acyclical real wages.\(^4\)

Overall, the analysis shows that most dynamics of hours per capita are interpreted by standard DSGE models as being inefficient and therefore hours per capita are the main determinant of the labor wedge. As most inefficiencies in the model are due to the inefficient allocation of labor, the labor wedge is the main determinant of the output gap. Hence, it is very important to measure hours per capita precisely. Dynamics that are caused by an imprecise measurement of hours per capita will be interpreted by the model as inefficiencies in the labor market and will hence be reflected in the labor wedge and will distort the estimated output gap.

### 4 Low Frequent Trends in Hours per Capita

I will now show that standard measures of hours per capita distort estimated output gap estimates and that this can be fixed by adjusting hours per capita for low frequent sectoral and demographic shifts that the model cannot explain.

---

\(^3\)The labor wedge is related to the wage and price mark-up (\(\mu^w_t\) and \(\mu^p_t\)) as follows: \(\text{wedge}_t = (mrs_t - w_t) + (w_t - mpt_t) = -(\mu^w_t + \mu^p_t)\).

\(^4\)Many economists argue that the large role of wage mark-up shocks in explaining recessions is unsatisfactory (see, e.g., Shimer, 2009). DSGE models in which wage mark-up shocks play an important role (the wage mark-up shock explains in the model by Smets and Wouters (2007) 20 percent of the variance in output and over 50 percent of the variance in inflation at a 10-quarter horizon) are nevertheless frequently used in applied work. Therefore, the goal of this paper is studying how one can avoid distortions in estimated output gaps in these models rather than contributing to solving the general well-known problems with some assumptions and features of these models.
Figure 4 shows three different hours per capita measures. For direct comparability of the different hours series I focus on aggregate hours divided by the population rather than the previously used measure based on average weekly hours (see section 3, figure 1). This has also the advantage, that the shown hours measures are exactly the same as used in Francis and Ramey (2009). So, the following results are directly comparable to their analysis and in addition the figure shows the implications of an update of their proposed hours adjustment beyond the global financial crisis of 2008/2009.

The upper graph focuses on the role of sectoral shifts for the measurement of hours by comparing total hours per capita (solid line) with hours per capita in the private business sector (dashed line). I can see large differences between private and total hours in the 1960s, which diminish afterwards because of a decline of private hours as a share of total hours. This sectoral shift is analysed in detail in Francis and Ramey (2009). However, also between 2007 and 2015, which is after the
end of the sample analysed by Francis and Ramey (2009), large differences between private and total hours are visible so that private hours are an inaccurate measure of total hours per capita.

The lower graph focuses on the role of demographic shifts by comparing total hours per capita (solid line, same series as in the upper graph) with total hours demographically adjusted using the formula described in equation (28). It is visible that demographic shifts contributed to the large decrease in total hours in the 1970s and the increase in hours in the 1990s. The graph also shows that demographic shifts have contributed to the persistent decline of hours after the global financial crisis of 2008/2009 and that without demographic shifts hours per capita would have moved more quickly back towards their long-run mean. Overall, the dynamics of the demographically adjusted total hours series are muted compared to the unadjusted series which means that demographic trends lead to dynamics of hours that could falsely be interpreted as cyclical movements.

### 4.1 Sectoral Shifts in Hours per Capita

In the following I analyse the sources of sectoral and demographic shifts in more detail. Francis and Ramey (2009) have already shown that the difference between private and total hours over time is mainly caused by the decrease in hours in the private sector as a share of total hours from the 1950s to the 1970s and an increase in government hours and also in the non-profit sector as a share of total hours during the same time. I focus therefore on the most recent period from 2005 to 2015 that has not been analysed, yet.

During the global financial crisis hours in the private sector decreased by about 10%. However, using only hours for the private sector would lead to an overestimation of the decline in per capita hours. Total hours decreased by about 7.5%. The difference is due to government hours, which roughly remained constant, and hours in the non-profit sector, which even increased by 3% during the global financial crisis. For comparability with the graphs in Francis and Ramey (2009), figure 5 shows hours in the private, government, and non-profit sector as a share of total hours. It is visible that while the share of private hours decreased substantially during the global financial crisis, the share of government and non-profit hours increased at the same time.

![Figure 5: Hours Worked by Sector (Percentage of Total)](image)

During the recovery, focusing on private hours would yield an overoptimistic picture of the return of hours worked towards their long-run mean. Total hours increased more slowly than hours in the private business sector, which is visible in figure 5 through the increase in the share of private
hours after 2010. Hence, sectoral shifts lead to an overestimation of fluctuations in hours worked during and after the global financial crisis if one focusses on the private business sector only instead of using total hours.

### 4.2 Demographic Trends

Figure 6 shows the age composition of the working-age population over time. As demonstrated by Francis and Ramey (2009) there are large changes over time caused by the baby boomer cohort. This cohort led to an increase in the fraction of individuals between ages 16-21 between about 1955 and 1985 and a decrease in the fraction of prime age individuals (ages 22-64) around the same time. As young workers work substantially less hours than prime age workers this decreased aggregate per capita hours. Afterwards, the baby boomer cohort increased the fraction of prime age workers in the working-age population which contributed to the large increase in per capita hours in the 1990s. These demographic effects on aggregate per capita hours were visible in the lower graph of figure 4.

For current and future aggregate per capita hours, the share of older workers who have already retired or will retire over the next decade is more interesting. The share of individuals aged 65 and over has increased over time and an acceleration of this upward trend is visible since 2006 (graph on the upper right of figure 6). Since then the share has increased from 15.9% to 18.8% reflecting the beginning of the retirement wave of the baby boomer cohort. The share of prime age workers has declined from 73.0% in 2006 to 71.1% in 2015. Hence, the retirement wave of baby boomers is a major factor in explaining the persistent decline in aggregate per capita hours after the global financial crisis. The lower panel of figure 4 showed that adjusting total hours for this demographic
trend accelerates the return of hours per capita to their long-run mean after the global financial crisis.\(^5\)

The baby boomer cohort is typically defined as those born between 1946 and 1964. So, in 2015 baby boomers are between 51 and 69 years old. Hence, a large fraction of the baby boomer cohort has not retired, yet. The lower right graph of figure 6 shows that the share of individuals aged between 55 and 64 has increased over recent years even more than that of individuals aged 65 and older. The population share of individuals aged between 55 and 64 has steadily increased from 10% in the mid-1990s to 16% in 2015 so that the retirement wave of the baby boomer cohort will continue and intensify over the next decade and the share of prime age workers will decrease further with possibly large effects on aggregate hours per capita.

### 4.3 Sectoral Shifts, Demographic Trends and Output Gap Estimates

Finally, I analyse to which extent the above documented sectoral shifts and demographic trends affect hours per capita since 2005 and in turn model-based output gap estimates. Figure 7 shows in the graph on the right four different hours per capita measures: hours in the private sector (solid line), total hours (dashed line), total hours with demographical adjustment (dashed-dotted line) and average hours in the nonfarm business sector multiplied with the employment-population ratio as used, for example, in Smets and Wouters (2007) (dotted line). The graph on the left shows the respective model-based output gap estimates for using the different hours per capita measures as observable.

![Figure 7: Output Gap Estimates and Hours (2005-2015)](image)

First, it is visible that sectoral shifts have a large effect on aggregate hours per capita during and after the financial crisis. Between 2008 and 2010 hours in the private sector decreased much

\(^5\)The changes in average hours worked by the different age groups cannot compensate for the change in the population structure. Hours worked by individuals aged 65 and over have increased only slightly from 4.5 hours per week in 2006 to 5.5 hours per week in 2014, while those of prime age workers even decreased from 29.9 in 2006 to 28.6 in 2014.
more than total hours and remained also lower afterwards. In turn, the estimated output gap based on hours in the private sector falls up to -12% during the financial crisis and is still highly negative at about -6% in 2015, while the output gap based on total hours only decreased to -7%. However, also the output gap based on total hours remains persistently negative and is at about -3% in 2015.

Second, the retirement wave of the baby boomer cohort has a large effect on hours. Both, unadjusted and demographically adjusted total hours decreased up to about 8% below their long-run mean in 2010. However, unadjusted hours remained highly negative and are still 4% below their long-run mean in 2015, while demographically adjusted hours increased faster and are only 2% below their long-run mean in 2015. The demographic effects on the estimated output gaps are even larger. The output gap based on total unadjusted hours per capita reached its trough at -7% in 2010. According to this measure output is still 3% below potential in 2015. The output gap based on demographically adjusted hours decreased up to -6% in 2010 and has gradually shrunk since. According to this measure, slack in the US economy has completely disappeared in 2015.

Hence, not accounting for demographic trends leads to the false impression of a permanently depressed economy since the global financial crisis, while actually the output gap has already closed. The output gap based on the demographically adjusted total hours series is also much more in line with output gap estimates based on simpler state space models and the output gap estimates by the Congressional Budget Office as documented in Kiley (2015) and Laubach and Williams (2015), than the permanently negative output gap estimates that have been found in the DSGE literature (see, e.g., Barsky et al., 2014).

The differences between the output gap estimates based on hours in the private business sector and those based on total demographically adjusted hours are very large. However, usually not hours in the private business sector are used to estimate DSGE models, but the most common hours measure is based on average weekly hours in the non-farm business sector multiplied with the employment-population rate (dotted line). Unfortunately, also for this measure I can see that hours are lower than total hours in all sectors and demographically adjusted total hours in all sectors. Hence, output gap estimates based on average hours in the non-farm business sector multiplied with the employment-population rate have been too low after the financial crisis because they do not account for the dynamics of hours in the public sector and the beginning of the retirement wave of the baby boomer cohort. This output gap measure implies that output is 5% below potential output in 2015, while the output gap based on hours in all sectors adjusted for demographic trends has already closed.

5 Conclusion

I have demonstrated the importance of precisely measuring aggregate hours per capita used as an observable in the estimation of macroeconomic models. A mismatch between data measurement and model assumptions can distort model-based analysis. In particular, low-frequent movements in hours per capita that are not accounted for by the model are falsely interpreted through the model’s lens as inefficiencies in the allocation of labor and are in turn erroneously included in the output gap. I show that this is a particular serious problem in standard DSGE models as in these models
the dynamics of observed hours are the main determinant of output gap dynamics.

Sectoral shifts in hours between the private and public sector and the large share of young workers who work much less hours than prime age workers decreased hours per capita in the private sector in the 1970s and 1980s. Standard models do not include sectoral shifts in hours and different age cohorts, so that low aggregate per capita hours lead to too low output gap estimates during that time when using unadjusted private hours as an observable. Similarly, the large increase in the share of prime age workers among the working-age population during the 1990s caused by the baby boomer cohort led to a large increase in aggregate hours per capita and too large model-based output gap estimates.

Private hours decreased more than total hours during the global financial crisis of 2008/2009 so that using private hours as an observable leads to an overly pessimistic view regarding the output gap during that time. I further show, that the financial crisis roughly coincides with the beginning of the retirement wave of the baby boomer cohort. This decreases aggregate hours per capita because the population share of individuals of ages 65 and over increased and these work much less hours than prime age workers. If one does not correct for this demographic trend, estimates of the output gap based on standard DSGE models are permanently negative since the global financial crisis.

I correct hours per capita instead for this important demographic trend using the adjustment proposed by Francis and Ramey (2009) to bring observable hours in line with the model assumptions. The corrected hours series increases much quicker after the financial crisis towards its long-run mean and implies a gradually closing rather than a permanent negative output gap after the global financial crisis. I show, that the retirement of the baby boomer cohort will continue and intensify over the next decade. Hence, to compute non-distorted model-based output gap estimates in the future, it will be crucial to adjust hours per capita for demographic trends or to model different demographic cohorts. Otherwise, DSGE model-based output gap estimates will be erroneously low for the next decade or so.

References


Appendix A: Data Sources

Average Weekly Hours in the Nonfarm Business Sector

- Source: US. Bureau of Labor Statistics, Series ID: PRS85006023. This hours measure is multiplied with the employment-population ratio to measure hours per capita.

Hours per Capita in the Private Business Sector

- Population: Civilian Noninstitutional Population (see description above).

Total Hours per Capita all Sectors

- Population: Noninstitutional Population (sum of civilian noninstitutional population and armed forces)
  - Civilian Noninstitutional Population (see description above).
  - Armed Forces: Data until end of 2011 is taken from data constructed by Cociuba et al. (2012); Data from 2012 onwards is taken from the Defense Manpower Data Center: https://www.dmdc.osd.mil/appj/dwp/dwp_reports.jsp (Active Duty Military Personnel by Service by Rank/Grade).

Total Hours per Capita all Sectors, demographically adjusted

- Until the fourth quarter of 2007 the series from Francis and Ramey (2009) is used. It is available on Valerie A. Ramey’s website: http://econweb.ucsd.edu/~vramey/research/Francis-Ramey_JMCB_Data_09.xls. I have replicated the series and got almost identical numbers.
- Data for Total Hours per Capita all Sectors is described above.
- Data for the demographical adjustment (from 2008 onwards):
  - Population shares of different age groups: US Census Bureau, Annual Data is interpolated to quarterly:
  - Average hours of different age groups: I use Census data from the integrated public use microdata series (IPUMs) based on the yearly American Community Survey from 2007-2014 (Ruggles et al., 2015).
\* Calculating average hours worked per week: For each individual I multiply the number of hours per week (UHRSWORK) with the number of weeks worked and divide the result by 52. Afterwards, I take the mean for all individuals of each age group.

\* The exact number of weeks worked (WKSWORK1) is only available until 2007. Afterwards, only intervals of the number of weeks worked are available in IPUMS (WKSWORK2). For 2007 WKSWORK1 and WKSWORK2 are available. I compute for 2007 for each age group the mean of WKSWORK1 for each interval WKSWORK2. I then use this number as a proxy of the number of weeks worked for each interval WKSWORK2 for the years after 2007.

\* For 2015 I approximate average hours worked by the different age groups with the values from 2014.

\* Annual data is interpolated to quarterly.
## Appendix B: Estimated Parameters

### Table 1: Estimated Structural Parameters

<table>
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<th>Mean (St. Dev.)</th>
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<th>Hours Tot.</th>
<th>H. Demo. Adj.</th>
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Notes: The table shows priors and posterior estimates for different observable hours measures. Hours BS: hours in the private business sector, Hours Tot.: hours in all sectors, H. Demo. Adj.: hours in all sectors demographically adjusted, Avg. H. NFBS: average weekly hours in the nonfarm business sector multiplied with employment-population ratio. The discount factor \(\beta\) is indirectly given through the steady state real interest rate: \(\beta = \left(1/r_\ast\right)^{1/100}\). The following parameters are fixed: \(\delta = 0.025\), \(g_* = 0.18\), \(\phi_w = 1.5\), \(\epsilon_w = 10\), \(\epsilon_p = 10\). The steady-state default probability of entrepreneurs is \(\bar{F}_\ast = 0.03\) and their survival rate is \(\gamma_\ast = 0.99\).
### Table 2: Estimated Shock Process Parameters

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</table>

Notes: The table shows priors and posterior estimates for different observable hours measures. Hours BS: hours in the private business sector, Hours Tot.: hours in all sectors, H. Demo. Adj.: hours in all sectors demographically adjusted, Avg. H. NFBS: average weekly hours in the nonfarm business sector multiplied with employment-population ratio. The different $\sigma$-parameters denote the standard deviation of the structural shocks and the $\rho$-parameters the autocorrelation parameters. $z$: technology, $b$: risk-premium, $g$: government spending, $i$: marginal efficiency of investment, $r$: monetary policy, $p$: price mark-up, $w$: wage mark-up, $\sigma_w$: spread. $\eta_p$ and $\eta_w$ denote the additional MA-parameters in the price and wage mark-up ARMA shock processes. $\eta_{g,z}$ denotes the reaction of government spending to the technology shock. $\sigma_{k,r}$, $k = 1, ..., 4$, denote the standard deviations of anticipated monetary policy shocks.