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by Ingrid Ott and Susanne Soretz

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Keywords: public inputs; agglomeration; integration

JEL classification: O33, Z13

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1 Introduction

Supporting convergence and intensifying European territorial cooperation are among the key objectives of European regional policy for the period of 2007-2013. One of the instruments to reach these goals is the further improvement of the transport infrastructure which is funded from structural and cohesion funds. Considering whether such an instrument is apt to reach the goal of convergence is part of both theoretical and empirical analysis. Aschauer (1989) provided a seminal work in which he derives a strong positive relationship between infrastructure and growth. This could basically speed up convergence. However, more recent contributions in the macroeconomic literature find more modest returns to infrastructure investment (see e. g. Gramlich (1994) for an overview). Within endogenous growth theory, those models strongly influenced by Barro (1990) analyze fiscal policies if a productive governmental input serves as a growth determinant. These models have been continuously refined to allow for different characteristics, especially congestion, of the public input (see e. g. Glomm and Ravikumar (1994a, 1994b) or Turnovsky (2000) for an overview): However, all these considerations focus on the view of a single country and, if they analyze convergence at all, they view it as the process leading to an equilibrium growth path. Consequently it is not possible to explain the distribution of economic activity across space as a mere consequence of interacting regions.

This concern lies at the heart of models known as 'new economic geography' (see Krugman (1995)). These models single out imperfect competition, increasing returns and transportation costs as fundamental resources shaping the economic landscape, but few focus on governmental activity. An exception is the work of Martin and Rogers (1995): They focus on the role of infrastructure as facilitating transactions, i. e. the trade within and between countries. Consequently agglomeration is reinforced as result of governmental activity. Puga (2002) analyzes the impact of regional policy expenditures on mitigating regional disparities and highlights that a undifferentiated consideration of infrastructure neglects that different characteristics of infrastructure also operate differently. Consequently, a thorough analysis of the impact of regional productive governmental policy also requires a sophisticated modelling of the public input.

However, though all these new economic geography models include regional governmental policies, they exclusively consider infrastructure in reference to reduced

transportation costs; by contrast, the Barro type models assume a productive governmental input but neglect regional interaction. The European Union primarily regards infrastructure as production input that enhances the productivity of the other local inputs. Consequently, viewing infrastructure as reducing to transportation costs is too narrow if one wishes to analyze whether the newly intended European regional policy will be successful in reducing regional disparities.

These shortcomings of the existing literature are the starting point for this model: We analyze the impact of regional policy on agglomeration. In doing so, regional policy thereby includes the provision of infrastructure that basically may be interpreted in a broad sense as comprising any facility, good or institution provided by the government that enhances the productivity of the other private inputs. This allows for a consideration of physical infrastructure such as roads, airports, telecommunication networks, but also basic research and training networks of education infrastructure. These different types formally may be represented by integrating a congestion function adopted from Eicher and Turnovsky (2000) which includes relative and absolute congestion as well as capital spillovers. We use this modelling of the governmental input and implement it in a modified version of the regional growth model of Bröcker (2003), who for his part focusses on learning-by-doing and inter-regional knowledge diffusion.

Integration between the two regions is modelled as the extent to which one region may benefit from the other region's public input. With this formulation we rely on Alesina and Spolaore (2003, chapter 6) and are broader than the usual approach of the new economic geography which assumes that integration predominantly reduces transport costs and thereby strengthens agglomeration. Our setting is in line with the goal of the European regional policy mentioned before, namely of enhancing European territorial cooperation. Integration may be also achieved, for example, by increasing the flow of ideas between regions as already argued by Rivera-Batiz and Romer (1991) and others. We assume identical production technologies with constant returns to the private inputs for the two regions. Mobile labor migrates between the regions while capital accumulation is taking place in the region with the higher productivity. The resulting equilibrium is based on equalized productivities of mobile labor and capital, and it determines the equilibrium capital distribution. Depending upon the interaction between agglomeration and dispersion forces, multiple equilibria with different stability characteristics may arise. It is shown that the bifurcation point is a function of congestion, capital

spillovers and integration. The endowment with immobile labor acts as threshold value that determines which equilibrium capital distribution finally results. Agglomerations reflect equilibrium capital distributions with different regional capital stocks. In analogy to Krugman (1991), the region displaying the bigger capital stock may then be interpreted as being the core, while the other region is the periphery.

In the light of this model, convergence in the sense of the European Union may be interpreted as a reduction in concentration. Basically this may be derived by integration or by the type of the governmental input provided, i. e. the choice about the degrees of congestion and spillovers. The following relationships become evident from numerical simulations: Integration reduces concentration since it allows the periphery to access the core's public input and hence to benefit from its productivity. In contrast to this, relative congestion is associated with a negative capital externality and aggravates concentration. As a consequence, the resulting market equilibrium ends up in suboptimally high concentration. The impact of capital spillovers may be ambiguous: Basically agglomeration forces are strengthened by capital spillovers since the productivity advantage of the core gains importance. Nevertheless, strong spillovers may smooth concentration if combined with a high degree of relative congestion. This is the consequence of decreasing marginal returns in the governmental input.

The remainder of the paper is a follows: After presenting the analytical framework the in the next section, balanced steady states are derived in Section 3. Section 4 explores the determinants of agglomeration, while Section 5 carries out numerical simulations. Efficiency arguments are discussed in Section 6. Section 7 concludes, while formal derivations are relegated to the appendix.

2 The analytical framework

2.1 Firms

Firms in both regions $\imath = 0,1$ produce the homogenous good, $Y_\imath$, by the same Cobb-Douglas technology. The inputs used in each region are mobile labor, $M_\imath$, immobile labor, $L_\imath$, and private capital, $K_\imath$. Furthermore, output depends upon regional access to a global public input that is measured by an index, $D_\imath$. The production

function for a representative firm in region $\imath$ is given by

$$Y_\imath = L_\imath^\lambda M_\imath^\mu K_\imath^\alpha D_\imath^\gamma, \quad \lambda \geq 0, \mu \geq 0, \quad 1 \geq \alpha \geq 0, 1 \geq \gamma \geq 0 \tag{1}$$

The global public input, $D_\imath$, includes the regional public inputs, $G_{s\imath}$, that are separately provided by both regions. The firm's access to the other region's public input may be limited as parameterized by $0 < \beta < 1$, and we assume

$$D_1 = G_{s1} + \beta G_{s2} \tag{2a}$$

$$D_2 = G_{s2} + \beta G_{s1} \tag{2b}$$

Correspondingly, the parameter β may be interpreted as a measure for the extent of integration between the two regions: If $\beta = 0$, firms in each region only benefit from the public input provided by their local governments, and consequently the scope of governmental policy is restricted to their own region. In contrast to this, $\beta > 0$ implies that firms in one region also have (at least partial) access to the other region's public input. What we have in mind is the following: If the government of a certain region provides education for the early childhood, with the goal to increase the productivity in its own region, the impact on the other region's productivity probably will not be affected significantly (at least if labor is immobile). Formally, β will be close to zero. The same argument applies to the provision of a university that restricts the access to students stemming from its own region. If, in contrast to this, the government of region 1 provides universities which are open to students from region 2 (and if graduates return to their home region), productivity in both regions will increase as consequence of governmental activity in one single region. Then, β will be positive. Another example could also be given by the provision of a public infrastructure. Consider two countries that both provide a road network as public input. As long as these networks are not connected, the spatial scope of governmental policy is restricted to its own region. Firms in region $\imath$ only benefit from their own region's roads, $\beta = 0$. But if now, e.g., ferries, connecting roads, tunnels or bridges are established, the road network in region 1 may be also used by firms of region 2. Formally, β increases up to $\beta = 1$; this reflects the other polar case in which firms in both regions have access to the entire public inputs provided in both regions. Then the global public input covers both road networks, $D_\imath = G_{s1} + G_{s2}$, and the two regions are perfectly integrated.[1]

[1]Note that both limiting cases, $\beta = 0$ and $\beta = 1$, characterize an extreme and unrealistic world but may be well useful as benchmark cases.

Another example can be given by the validity area of patents that describe another facet of the spatial scope of governmental activity.

The modelling of the governmental input is adopted from Eicher and Turnovsky (2000), and the public input provided by the local government in region $\imath$ may be characterized as follows

$$G_{s\imath} = G_\imath \left(\frac{K_\imath}{\bar{K}_\imath} \right)^{\varepsilon_R} \bar{K}_\imath^{\varepsilon_A}, \quad 0 \leq \varepsilon_R \leq 1, \quad -\alpha \leq \varepsilon_A \leq 1 \tag{3}$$

Thereby $\bar{K}_\imath$ denotes the aggregate stock of private capital in region $\imath$, and $G_\imath$ denotes the aggregate flow of government expenditure. Function (3) incorporates the potential for the public good to be associated with alternative types and degrees of scale effects or congestion as denoted by ε_A and ε_R. In contrast to Eicher and Turnovsky (2000), we do not restrict the sign of ε_A to be negative, but we allow for positive and negative externalities at the aggregate level.[2] Nevertheless, in order to allow for ongoing growth, $-\alpha \leq \varepsilon_A$ has to be satisfied, as will be explained below. Altogether, the public services can be classified into the following categories.

(i) If $\varepsilon_A = \varepsilon_R = 0$, government services constitute a pure public good in the sense of Samuelson (1954) and $G_{s\imath} = G_\imath$. The public input is available equally to each individual within region $\imath$, independent of the usage of others.[3] Governmentally provided basic research may serve as an example. Its usage by one firm does not affect the possible usages of the others. The same is true for the usage of the public input by firms from other regions.

(ii) Relative congestion arises if $\varepsilon_R > 0$: This reflects situations in which the level of the public input available to the individual is tied to this individual's usage of capital. As already explained, $\varepsilon_R = 0$ corresponds to a nonrival pure public input, while $\varepsilon_R = 1$ reflects a situation of proportional relative congestion. Accordingly, the cases $0 < \varepsilon_R < 1$ correspond to situations of partial relative congestion, in the sense that given the individual stock of capital, government spendings can increase at slower rate than does $\bar{K}_\imath$ and still provide a fixed level of services to the firm. An example for $\varepsilon_R \leq 1$ could be the provision of a road network. In extreme, it

[2] Note that the integration parameter β is also a measure for the extent to which the arising externalities of one region have a bearing on the other region. Above, the actual level of ε_A is of major importance for the resulting equilibria.

[3] Since only few examples of such pure public goods exist, this case should be treated primarily as a benchmark.

is proportionally congested and each of the N_ι individuals within region ι may use $1/N_\iota$ parts of the entire public input, G_ι, for production.[4] Relative congestion reflects the disadvantages of concentration: For a given amount of governmental input (e. g. infrastructure), the individually available amount is smaller, the more individuals make use of it or – put differently – the larger the aggregate capital stock. A single-lane highway is more productive for the individual firm, the less other trucks (aggregate capital) use it.

(iii) Intra-regional spillovers given that $\varepsilon_A > -1$: In any dynamic equilibrium, aggregate capital and governmental expenditures grow at the same constant rate, as will be demonstrated in the context of (19). Hence, with $\varepsilon_A > -1$, positive effects of capital accumulation arise, and individuals benefit from the accumulation of the others. This externality can be interpreted as a net externality or in the sense of Romer (1986); and an example could be the outcomes of research centers that are financed by non-distortionary taxes.[5] The positive effects of the governmental input increase with the absolute size of the economy: Learning by doing is promoted by governmentally provided schools and universities; and the productivity increase induced by schools and universities is enhanced by a high degree of automation displayed by high capital intensity.

For the production technology (1) to allow for endogenous growth in both regions, an additional constraint has to be imposed, namely $\alpha+\gamma(1+\varepsilon_A)=1$. This ensures constant returns to private capital, the accumulable factor.[6]

From (1), (2) and (3), the output of the individual (representative) firm in re-

[4] As Turnovsky (1996, p. 364) argues, the case $\varepsilon_R > 1$ describes a situation where congestion is so great that the public input must grow faster than the economy in order for the level of services provided to the individual firm to remain constant. This case is unlikely at the aggregate level, but may well be plausible for local public goods (see also Edwards (1990)). A local public good could be a harbor that is provided by the regional government. Nevertheless it also may be used by individuals coming from outside the region. However, Turnovsky (1996) argues in the context of a one-country model; hence it is not possible to apply the argumentation carried out there 1:1 to our framework. Here, possible utilization of an input that is provided by the other region is parameterized by $\beta > 0$ and not by $\varepsilon_R > 1$.

[5] Note that the positive spillovers in the model of Romer (1986) do not exactly correspond to the framework of this model since there the spillovers arise independent of governmental activity.

[6] This interdependence between the parameters implies an adjustment of the values of α or γ whenever a change in absolute congestion, ε_A, is analyzed. Besides, together with $0 < \gamma \leq 1$ another constraint, namely $-\alpha \leq \varepsilon_A$ has to be imposed to enable ongoing growth. Otherwise capital productivity would not suffice to promote endogenous growth.

gion 1 is given by

$$Y_1 = L_1^{\lambda} M_1^{\mu} K_1^{\alpha} (G_1 K_1^{\varepsilon_R} \bar{K}_1^{\varepsilon_A - \varepsilon_R} + \beta G_2 K_2^{\varepsilon_R} \bar{K}_2^{\varepsilon_A - \varepsilon_R})^{\gamma} \tag{4}$$

and output of the representative firm in region 2 may be derived accordingly. If $\beta = 0$, the scale elasticity of Y_ι is given by $\lambda + \mu + \alpha + \gamma(1 + \varepsilon_A)$. Hence, for all feasible levels of ε_A, production is characterized by increasing returns to the local inputs and this is reinforced with increasing ε_A. The private (average) capital productivities in both regions evolve according to

$$\frac{Y_1}{K_1} = L_1^{\lambda} M_1^{\mu} \left(1 + \frac{\beta}{g_s}\right)^{\gamma} \left(\frac{G_1}{K_1}\right)^{\gamma} N_1^{\gamma(\varepsilon_A - \varepsilon_R)} \tag{5a}$$

$$\frac{Y_2}{K_2} = L_2^{\lambda} M_2^{\mu} (1 + \beta)^{\gamma} \left(\frac{G_2}{K_2}\right)^{\gamma} N_2^{\gamma(\varepsilon_A - \varepsilon_R)} \tag{5b}$$

Thereby the following variables are utilized

$$g \equiv G_1/G_2, \quad g_s \equiv G_{s1}/G_{s2} = g k^{\varepsilon_R} \bar{k}^{\varepsilon_A - \varepsilon_R} \quad \text{with} \quad k \equiv K_1/K_2, \quad \bar{k} = \bar{K}_1/\bar{K}_2 \tag{6}$$

Average productivities thus depend on the distribution of capital and governmental activity across regions, as incorporated within g_s, the ratio G_ι/K_ι, as well as on the number of firms located in each region and on the type of the public input, as incorporated within the term $N_\iota^{\gamma(\varepsilon_A - \varepsilon_R)}$.

2.2 Households and regional growth

Households are identical across regions and are comprised of either immobile or mobile workers. Immobile workers receive wages denoted by $w(t)$, while mobile workers receive wages denoted by $m(t)$. Mobile workers do not face any relocation cost and choose the location offering the highest value of $m(t)$. Since perfect competition at the factor markets is assumed, wages for mobile labor are highest where the private marginal productivity of mobile labor is highest.

The infinitely lived households possess identical isoelastic preferences, and the representative household maximizes lifetime utility out of consumption, $C(t)$, according to[7]

$$U = \int_0^{\infty} \frac{\sigma}{\sigma - 1} C(t)^{\frac{\sigma - 1}{\sigma}} e^{-\rho t} dt \tag{7}$$

[7] As the households' preferences are homothetic, we prefer to analyze the optimization problem of the collectivity of the households in order to avoid too many indices.

The subjective discount rate is denoted by ρ, and σ is the elasticity of intertemporal substitution. Households save by accumulating a risk free asset. The asset value equals the value of the stock of capital at any point in time and hence the asset value of the two regions at time t equals $V(t) \equiv q_1(t)K_1(t) + q_2(t)K_2(t)$, where $q_\imath$ denotes the stock price of capital installed in region $\imath$.

Mobile and immobile workers earn labor income as well as capital income from investment in both regions. Their total income evolves according to

$$\dot{V}_w(t) + \dot{V}_m(t) = w(t)L(t) + m(t)M(t) + (r(t) - \delta)V(t) - C(t) - T(t) \tag{8}$$

with $r(t)$ denoting the interest rate determined in capital market equilibrium, δ as the constant depreciation rate of private capital and $T(t)$ as constant lump-sum tax that is used to finance the provision of the public input. To fully describe the optimization problem, the transversality condition

$$\lim_{t-\infty} K_\imath(t)\xi_\imath(t) = 0 \tag{9}$$

has to be met, where $\xi_\imath$ denotes the shadow value of capital. Maximizing (7) subject to the accumulation constraint (8) leads to the well known growth rate of consumption as[8]

$$\frac{\dot{C}}{C} = \sigma(r - \delta - \rho) \tag{10}$$

Due to constant average returns of capital (see (5)), the consumption-wealth ratio is constant and hence the growth rates of consumption and capital coincide.

3 Balanced steady states

The equilibrium is based on equalized productivities of mobile labor and private capital. It thus includes the migration of $M_\imath$ and the accumulation of $K_\imath$. In order to keep the analysis simple, we assume that labor mobility neither induces mobility costs nor requires time. In contrast, physical capital is only mobile as long as it is not yet nailed down. Hence, the marginal return of mobile labor is equalized across regions in each time increment, whereas the adjustment process of marginal capital returns takes time.

[8] In what follows time indices will be suppressed.

Labor market equilibrium A migration equilibrium requires that mobile labor is distributed such that the wages of mobile workers, $m(t)$, are equalized across regions. As already mentioned above, perfect mobility of mobile workers is assumed. Hence, the migration equilibrium is given when marginal productivities of $M_\imath$ in both regions coincide, and is realized instantaneously. It is characterized by

$$\frac{\partial Y_1}{\partial M_1} = \frac{\partial Y_2}{\partial M_2} \quad \Longrightarrow \quad \frac{Y_1}{Y_2} = \frac{M_1}{M_2} \tag{11}$$

Perfect labor mobility thus implies that the output ratio equals the ratio of mobile labor. Utilizing (4) and denoting $l \equiv L_1/L_2$, the output ratio of both regions can be written as

$$\frac{Y_1}{Y_2} = \left[l^{\lambda} k^{\alpha} \left(\frac{g_s + \beta}{1 + \beta g_s} \right)^{\gamma} \right]^{\frac{1}{1-\mu}} \tag{12}$$

Lower case letters reflect the distribution of the respective variable across the two regions as given by (6). In the context of (12), only the relative sizes of the aggregate variables, and not their absolute levels gain importance. For given production elasticities and given l, the distribution of mobile labor across the regions only depends on the distribution of private capital, k, as well as on governmental activity. The latter also includes the spatial scope via spillovers, ε_A, the congestion parameter, ε_R, and the extent of inter-regional integration as measured by β.

Capital market equilibrium Individuals in the two regions are able to hold capital either in region 1 or in region 2. Hence, not only mobile labor, $M_\imath$, but also capital is mobile as long as it is not yet nailed down. Consequently, capital is immobile once being installed and may not be relocated to the other region. Therefore net investment in either region is nonnegative and given by

$$I_\imath = \dot{K}_\imath - \delta K_\imath \geq 0 \tag{13}$$

With $q_\imath$ denoting the stock price of capital installed in region $\imath$, the following conditions are complementary and must be fulfilled for sustained investment in region $\imath$

$$I_\imath \geq 0, \quad q_\imath \leq 1, \quad I_\imath (1 - q_\imath) = 0 \tag{14}$$

No-arbitrage applies if capital in both regions yields identical rates of private return

$$(r + \delta) q_\imath = \dot{q}_\imath + \frac{\partial Y_\imath}{\partial K_\imath} \tag{15}$$

Since we abstract from adjustment costs, the marginal costs for installing an additional unit of capital in region ι is unity. Consequently marginal costs and marginal returns of an additional unit of capital are equalized if $q_\iota = 1$, and as long as $q_\iota = 1$, private investors are willing to invest in region ι.[9] Then $\dot{q}_\iota = 0$; and according to (15), the interest rate equals the net marginal product of capital, $r = \partial Y_\iota / \partial K_\iota - \delta$ and investment is positive, $I_\iota > 0$. If instead $q_\iota < 1$, no investment will take place. Then $I_\iota = 0$. Since individuals only invest in the region with the higher capital return, positive investment in both regions is only feasible if marginal capital productivities coincide. Then both capital stocks grow at the same rate and the capital ratio, k, is constant.

Denote the ratio of marginal capital productivities with

$$R \equiv \frac{\partial Y_1}{\partial K_1} \Big/ \frac{\partial Y_2}{\partial K_2} \tag{16}$$

A balanced steady state is characterized by a stationary capital distribution, i. e. by $R = 1$. Then ongoing positive investment in both regions arises and capital stocks in both regions grow according to (10), with r being derived from (4). In case of initial productivity disparities (i. e. $R \neq 1$), the prevailing capital ratio is not stationary; but over time transitions to a steady state with k increasing (if $R > 1$) or decreasing (if $R < 1$) will take place. Hence an equilibrium is only attained after a certain transition period, but k converges to a stable equilibrium in finite time. Since we assumed that capital is immobile once it has been nailed down, a transition with increasing k implies that during the transition period there is only investment in region 1 and no investment in region 2. The capital stock in region 2 then declines with the depreciation rate, δ.

Assume that initially capital in region 1 is more productive. Then the transition may be described by the following differential equations

$$\dot{K}_1 = Y_1 + Y_2 - \delta K_1 - C - (G_1 + G_2) \tag{17a}$$

$$\dot{K}_2 = -\delta K_2 \tag{17b}$$

$$\frac{\dot{C}}{C} = \sigma \left(\frac{\partial Y_1}{\partial K_1} - \delta - \rho \right) \tag{17c}$$

$$\dot{q}_2 = \left(\frac{\partial Y_1}{\partial K_1} - \delta \right) q_2 - \frac{\partial Y_2}{\partial K_2} \tag{17d}$$

[9] If $q_\iota > 1$, investment would be infinite; hence to analyze balanced steady states and the corresponding transitions, it is sufficient to deal with $q_\iota = 1$.

which hold as long as $q_2 < 1$. (17a) is the goods market equilibrium condition; (17b) is due to exclusive investment in region 1; (17c) describes the Keynes-Ramsey rule; and (17d) is the equilibrium condition of the asset market. Note that in (17a) it is assumed that the provision of $G_1 + G_2$ is realized out of global income $Y_1 + Y_2$.[10]

4 Determinants of agglomeration: core and periphery

4.1 Equilibrium and government expenditure

To study the model's dynamics, we focus on stable steady states, and on transitions to stable steady states. Formally one has to analyze how productivities of private capital in both regions depend on the regional distribution of capital as well as on governmental activity, provided that mobile labor is distributed such that the corresponding wages, $m(t)$, are equalized across regions any time. To do so, the ratio R may be derived from the specified production function (4), together with (11) and (12). Note that since we focus on a growing economy, we assume that the governments in both regions set the aggregate expenditure levels, G_ι, as a constant fraction, θ_ι, of aggregate capital, $\bar{K}_\iota$, namely[11]

$$G_\iota = \theta_\iota \bar{K}_\iota, \quad 0 < \theta_\iota < 1 \tag{18}$$

An expansion in government expenditure is then parameterized by an increase in the capital share, θ_ι. Additionally we have to take into account that in equilibrium $\bar{K}_\iota = N_\iota K_\iota$ applies. Then

$$\tilde{g}_s = \theta k^{1+\varepsilon_A} n^{1+\varepsilon_A-\varepsilon_R} \tag{19}$$

defines the equilibrium ratio of governmental activity, and $\theta \equiv \theta_1/\theta_2$. In equilibrium the ratio of marginal capital productivities turns out to equal

$$R = \left[l^\lambda k^{\mu+\alpha-1} \left(\frac{\tilde{g}_s + \beta}{1+\beta\tilde{g}_s} \right)^{\mu+\gamma-1} \right]^{\frac{1}{1-\mu}} \cdot \left(\frac{\alpha(\tilde{g}_s+\beta)+\gamma\varepsilon_R\tilde{g}_s}{\alpha(1+\beta\tilde{g}_s)+\gamma\varepsilon_R} \right) \tag{20}$$

[10] The regional decision about the governmental input is described in Section 6.

[11] The derived results are equivalent to assuming $G_\iota = \theta Y_\iota$ but the formulation in (18) keeps the formal analysis much simpler.

Taking logarithms, after some simple manipulations, yields

$$R \gtrless 1 \quad \Longleftrightarrow \quad i(k) \gtrless -\lambda \ln l \tag{21}$$

with

$$i(k) \equiv (\mu+\alpha-1)\ln k + (\mu+\gamma-1)\ln\left(\frac{\tilde{g}_s+\beta}{1+\beta\tilde{g}_s}\right) + (1-\mu)\ln\left(\frac{\alpha(\tilde{g}_s+\beta)+\gamma\varepsilon_R\tilde{g}_s}{\alpha(1+\beta\tilde{g}_s)+\gamma\varepsilon_R}\right) \tag{22}$$

Referring to the equilibrium concept, balanced steady states are attained at those capital ratios, k^*, that solve $i(k^*) = -\lambda \ln l$. Then $R = 1$ and the marginal capital productivities are equalized across the regions. Since both regions then grow at constant rates, the capital ratio stays constant.[12] The initial endowment with immobile labor, $-\lambda \ln l$, reflects a threshold value that does not only affect the equilibrium capital ratio, but may also have a major impact on the number of the finally resulting equilibria. The threshold value is independent of the capital ratio, k, and decreases in l and λ. In case of symmetric distribution of immobile labor, $l = 1$, the term vanishes and $R \gtrless 1$ if $i(k) \gtrless 0$. The intuition for this is that, all things being equal, an increase in l increases the relative productivity in region 1. Hence, the relative capital productivity stemming from the other inputs included in $i(k)$ has to be lower in equilibrium in order to balance capital productivity in both regions ($R = 1$).

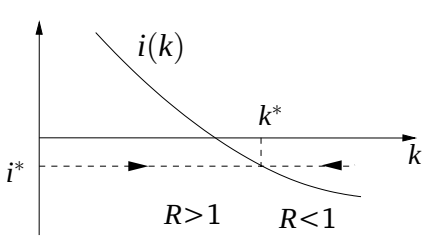
Depending on the characteristics of $i(k)$ it is possible to attain either one unique equilibrium or multiple equilibria, the latter showing different stability characteristics.[13] Stable equilibria arise whenever capital ratios outside the equilibrium strive towards the equilibrium. If, in contrast, the capital ratio continuously departs from the equilibrium, the underlying equilibrium is unstable.

Within Figures 1(a) and 1(b) the equilibrium capital ratios are denoted by k^* and k^{**} respectively and the stability implications are indicated by the arrows at the horizontal axis. The threshold value is denoted by i^*. The intuition for multiple equilibria will be discussed below.

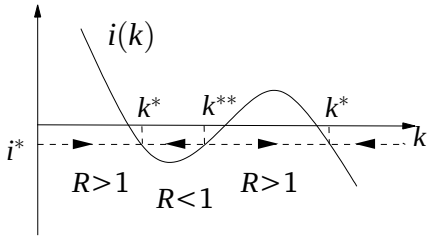
Formally, the underlying equilibrium is unstable whenever function $i(k)$ is positively sloped in the equilibrium capital (see k^{**} in Figure 1(b)). If then, starting from the steady state capital ratio, the relative capital productivity in region 1 increases ($R > 1$), the resulting capital productivity advantage in region 1 attracts

[12] Note that it is anyhow possible that, given identical growth rates, both regions diverge with respect to their absolute levels of output, governmental input and private capital.

[13] These features about the run of the curve $i(k)$ are derived in Appendix A.



(a) $i'(k) < 0$: stability



(b) $i'(k) \gtrless 0$: multiple equilibria

Figure 1: Stability and multiple equilibria

investment and induces further increases of k. Hence the capital distribution departs continuously from the initial steady state and the system diverges from the unstable equilibrium. The argumentation holds analogously if, starting from an initially unstable equilibrium, k^{**}, the capital ratio is reduced and then declines continuously. If on the contrary the function $i(k)$ is negatively sloped for equilibrium capital ratios (see k^* in Figures 1(a) and 1(b)), an increase in k reduces the ratio of capital productivities ($R < 1$), thus giving rise to a productivity advantage in region 2. Then k declines and converges again to its original steady state value.

It is possible to show that within our framework either one stable equilibrium or three equilibria result – the latter exhibiting stability characteristics as indicated within Figure 1(b) and argued above.[14] A more unequal distribution of immobile labor induces a shift of the threshold value away from the k-axis. Hence if the regions sufficiently differ with respect to their endowment of immobile factors, multiple equilibria will not occur even if the run of $i(k)$ would basically allow for multiple equilibria. Instead there is one stable equilibrium and the equilibrium capital ratio reflects the distribution of immobile labor, with k^* increasing in l. The simple reason is that capital and labor are complementary production factors; hence a large amount of immobile labor causes a productivity advantage for physical capital.

4.2 Agglomeration and concentration

To analyze the regional distribution of economic activity, we now focus on stable equilibria with $k^* \neq 1$; hence we consider the case of multiple equilibria. In this context, stable equilibria will be called agglomerations, with a concentration of mobile factors. We consider transitions in which concentration is either increased

[14] See Appendix A for a proof.

or reduced. Following Krugman (1991), the region which holds the higher capital stock then represents the core of the entire economy, whereas the other region is the periphery. The analysis will be carried out for equally distributed immobile labor, $l = 1$; hence the threshold value is given by $-\lambda \ln l = 0$. The argumentation focusses on those determinants that affect the run of function $i(k)$ and the underlying economic effects will be discussed. Two aspects gain especial importance: the sign of $i'(k)$, which determines whether agglomeration forces ($i'(k) > 0$) or dispersion forces ($i'(k) < 0$) prevail; and the multiplier that decides on the extent of the arising forces.

Starting point is $i(k)$ in (22). We then analyze the impact of capital productivity differentials on the development of the capital ratio, k. If this ratio increases, economic activity becomes more concentrated over time. The formal analysis yields the first derivative of (22), as follows

$$\begin{aligned}
\frac{d\,i(k)}{d\,k} = & \underbrace{\frac{\partial i(k)}{\partial k}}_{\text{direct effect}} + \underbrace{\frac{\partial i(k)}{\partial \tilde{g}_s}\frac{\partial \tilde{g}_s}{\partial k}}_{\text{indirect effect}} \\
= & \quad (\mu+\alpha-1)\frac{1}{k} \quad + && \text{(23a)} \\
& \quad (\mu+\gamma-1) \quad (1+\varepsilon_A)\frac{\tilde{g}_s}{k} \quad \frac{(1-\beta)(1+\beta)}{(\tilde{g}_s+\beta)(1+\beta\tilde{g}_s)} \quad + && \text{(23b)} \\
& \quad (1-\mu) \quad (1+\varepsilon_A)\frac{\tilde{g}_s}{k} \quad \frac{(\alpha(1-\beta)+\gamma\varepsilon_R)(\alpha(1+\beta)+\gamma\varepsilon_R)}{[\alpha(\tilde{g}_s+\beta)+\gamma\varepsilon_R\tilde{g}_s][\alpha(1+\beta\tilde{g}_s)+\gamma\varepsilon_R]} && \text{(23c)}
\end{aligned}$$

Eq. (23a) displays the direct effect of an increase in the capital ratio on the relative capital productivity. Due to constant returns of the private inputs, $\mu+\alpha < 1$; hence there are decreasing local returns to mobile labor, M_l, and private capital, K_l, as long as the productivity impact of capital within D_l is neglected. Since $\alpha < 1$, a rise in capital endowment goes along with a decreasing marginal product of capital. If, analogously we focus on the ratio of capital stocks, an unequal distribution of physical capital (large k) ceteris paribus leads to lower capital return in the core, $R < 1$. Hence, investment is more attractive in the periphery, and this results in a decrease of k. The direct effect (23a) contributes to the convergence of the system to equally distributed physical capital, $k = 1$, and tends to cut off nascent concentration.

In addition to this direct effect, there is an indirect effect of an increase in relative capital, k, on $i(k)$, which is included within the terms (23b) and (23c). They

capture the impact of governmental activity, as incorporated within $\tilde{g}_s$, and also consider the impact of integration, β. Starting from an initial equilibrium capital ratio, any increase in k will raise the relative supply of the public inputs, $\tilde{g}_s$ (see (19)). This leads to counterworking effects on the relative productivity of capital. Negative effects result from the decreasing marginal productivity of the public input. Positive effects are due to the complementarity of physical capital and the public input in the production function, $Y_{iK_iG_{si}} > 0$.

For all feasible parameter constellations and provided that $\beta < 1$, the term (23b) is positive (negative) if $\mu+\gamma > 1$ ($\mu+\gamma < 1$). If γ is sufficiently low, this term reinforces dispersion due to decreasing marginal productivity of governmental expenditures. More unequally distributed capital (higher k) unambiguously increases the ratio between individually available public inputs, $\tilde{g}_s$. Due to decreasing marginal productivity of the public input, the ratio of capital productivity tends to decrease. Contrariwise, capital and the public input are complementary production factors. If γ is sufficiently high, this argument prevails and the term (23b) strengthens the agglomeration forces.

The third term (23c) is positive since $\mu < 1$ and $\varepsilon_A > -1$, which both reflect sensible parameter constellations. Hence according to (23c) concentration unequivocally increases. As a consequence, the relative productivity of physical capital continues to rise thus inducing further increases and fostering concentration. The strength of this effect is reinforced if ε_R is increased.

The total effect combines the partial effects. To sum up the implications of (23) one finds forces that foster and those that relax the concentration of economic activity. The entire effect is crucially influenced by the extent of regional integration, as parameterized by the term β: The second as well as the third term ((23b) and (23c)) decrease with rising β; the second term vanishes if $\beta = 1$. Hence the arising forces are the stronger, the less pronounced the regional interdependencies are. The reason therefore is that in more isolated regions (low β), the own region's public input gains relatively more importance for the firm's behavior. If, instead, there is a close relationship between the regions (high β), the relative impact of one's region governmental policy is weaker, but also the amount of governmental input provided by the other region affects the firm's decisions.

4.3 Multiple equilibria and bifurcations

In the following we derive in more detail the formal conditions that are required for the origination of multiple equilibria. One central argument will concern the derivation of the bifurcation point that separates conditions in which one unique equilibrium arises from those that go along with multiple equilibria. Due to the multitude of influencing factors, it is basically possible to describe the bifurcation point as a function of several variables.[15] Since the paper focusses on the impact of regional governmental policies, we derive the bifurcation point as a function that covers all policy parameters, namely $\varepsilon_A, \varepsilon_R$ and β.

Starting points of the considerations are eqs. (22) and (23), and we assume that immobile labor is equally distributed, $l = 1$. From Figure 1(b), we now that $i(k)$ is negatively sloped in the limits $k = 0$ and $k \to \infty$. Moreover, the function $i(k)$ has an unambiguous root at $k = 1$.[16] Therefore, the incidence of multiple equilibria depends on the slope of $i(k)$ in the root at $k = 1$: If the slope is negative, one unique and stable equilibrium arises, whereas multiple equilibria exist if the slope of $i(k^* = 1)$ is positive.

It is straightforward to show that the slope of function $i(k)$ is unambiguously negative for sufficiently low extents of intra-regional spillovers, $\varepsilon_A \to -1$. This case applies if the public input is characterized by congestion and the available amount of the public input decreases with the size of the economy as given by $G_{sl} = \theta_l N_l^{-\varepsilon_R}$. Hence, dispersion strictly dominates for any capital distribution, k, as given by

$$\left.\frac{di(k)}{dk}\right|_{\varepsilon_A \to -1} = (\mu + \alpha - 1)\frac{1}{k} < 0 \tag{24}$$

With an increase in intra-regional spillovers, ε_A, concentration forces arise due to productivity advantages and scale effects.[17] A rise in ε_A implies an increase in the individually available amount of the public input, hence we have a positive effect of the aggregate capital stock on private capital returns. Moreover, scale effects come into play as the absolute size of aggregate capital affects the individually available amount of public input. A region with a relatively high aggregate capital stock, $\bar{K}_l$, offers a higher amount of the public input, $G_l = \theta_l \bar{K}_l$. This results in more individually available public input and therefore in enhanced productivity.

[15] See (28) below.

[16] See the mathematical Appendix A for a proof.

[17] The slope $i'(1)$ increases in ε_A as derived in Appendix A.

Comparing two regions which differ in their capital endowment, this fosters the concentration forces.

For increasing intra-regional spillovers, ε_A, the agglomeration forces may dominate in the neighborhood of $k = 1$, as will be shown in the following. Provided that symmetry is given ($\theta = n = 1$), the slope of $i(k)$ in $k = 1$ is given by

$$\left.\frac{di(k)}{dk}\right|_{k=1} = \underbrace{\mu+\alpha-1}_{force1} - \underbrace{(\mu-1)(1+\varepsilon_A)\frac{2\beta\gamma\varepsilon_R}{(1+\beta)(\alpha(1+\beta)+\gamma\varepsilon_R)} + (1+\varepsilon_A)\gamma\frac{1-\beta}{1+\beta}}_{force2} \quad (25)$$

Agglomerations arise if agglomeration forces dominate around $k = 1$ and hence if neither agglomeration forces nor dispersion forces unequivocally prevail for all capital ratios. In general, multiple equilibria arise if

$$0 > \mu+\alpha-1 > (\mu-1)(1+\varepsilon_A)\frac{2\beta\gamma\varepsilon_R}{(1+\beta)(\alpha(1+\beta)+\gamma\varepsilon_R)} - (1+\varepsilon_A)\gamma\frac{1-\beta}{1+\beta} \quad (26)$$

and consequently

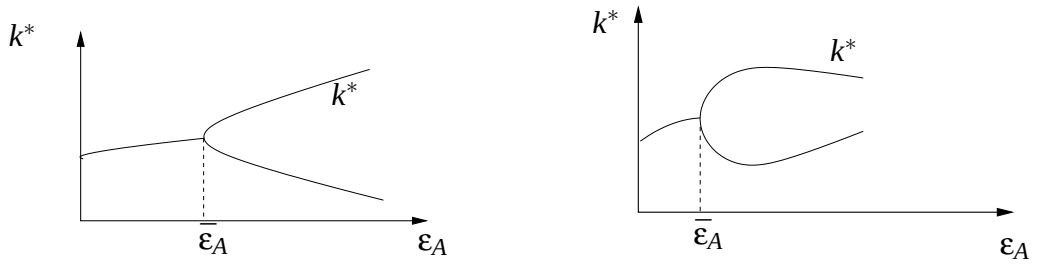
$$\left.\frac{di(k)}{dk}\right|_{k=1} \gtrless 0 \quad \Longleftrightarrow \quad \varepsilon_A \gtrless \bar{\varepsilon}_A(\beta,\varepsilon_R) \quad (27)$$

$$\text{where} \quad \bar{\varepsilon}_A = \frac{\alpha(1+\beta)(2\beta(1-\alpha)-\mu(1+\beta)) - \varepsilon_R(1-\alpha)(\mu(1-\beta)+2\alpha\beta)}{-\alpha(1+\beta)(2\beta(1-\alpha)-\mu(1+\beta)) + \varepsilon_R(1-\alpha)2\beta(1-\mu)} \quad (28)$$

and $\bar{\varepsilon}_A$ denotes the bifurcation point.[18] This threshold value separates the cases in which one unique and stable equilibrium (provided that $\varepsilon_A < \bar{\varepsilon}_A$) or multiple equilibria (in case of $\varepsilon_A > \bar{\varepsilon}_A$) arise. Its level is crucially affected by the (exogenously given) parameters ε_R and β, which both are determined by governmental decisions. Arguing from an analytical point of view, an increase in ε_A rotates the graph of $i(k)$ with center at $k = 1$. Beginning with a sufficiently low level ε_A, dispersion forces dominate for all capital ratios, k, and $i(k)$ is shaped as illustrated within Figure 1(a). If now ε_A increases until it exceeds the value of the bifurcation point as given by (28), the dynamic behavior switches toward a scenario with

[18] To derive equation (28), note that $\alpha+\gamma(1+\varepsilon_A) = 1$, hence $\gamma = (1-\alpha)/(1+\varepsilon_A)$; and then solve equation (26) for ε_A. Note that to ensure the knife-edge condition of endogenous growth, the productivity of government expenditures, γ, has to be reduced whenever an increase in spillovers, ε_A, is considered. In order to prevent the preponderance of this to some extent artificial argument, we restrict to parameter settings which result in a positive denominator of $\bar{\varepsilon}_A$. In contrast to the presentation within (28), one could basically also denote the bifurcation point as $\bar{\beta}(\varepsilon_A,\varepsilon)$ or $\bar{\varepsilon}_R(\beta,\varepsilon_A)$. Qualitatively the results would not change. We calculate different bifurcation points according to (28) in the context of the numerical presentations within Section 5.

agglomeration. The intuition for this is that increasing intra-regional spillovers ($\varepsilon_A \uparrow$) increase local returns, thus strengthening agglomeration forces.[19] Then, the agglomeration forces dominate around $k = 1$, and finally the derivative $di(k)/dk$ becomes positive; multiple equilibria arise. Nevertheless, if capital is distributed more unequally across regions, the dispersion forces eventually dominate and ensure that two stable equilibria exist. Hence, agglomeration arises if (and only if) the derivative of $i(k)$, evaluated at $k = 1$, is positive.



(a) low relative congestion (b) high relative congestion

Figure 2: Bifurcation diagram.

Figure 2 provides a graphical illustration of two bifurcation diagrams. For sufficiently small ε_A the dispersion forces generally dominate and a unique equilibrium ratio k^* results. As soon as ε_A exceeds the threshold value $\bar{\varepsilon}_A$ in (28), the dynamics crucially change and multiple equilibria arise.

Basically, the sign of $\bar{\varepsilon}_A$ within (28) can either be positive or negative, depending predominantly on the integration parameter and on the degree of relative congestion. An increase in territorial cooperation (increase in β) leads to an increase in the critical level of capital spillovers as can be seen from

$$\frac{\partial \bar{\varepsilon}_A}{\partial \beta} = \frac{(1-\alpha)\varepsilon_R(\alpha(2\beta(1-\alpha)-\mu(1+\beta))^2+2\varepsilon_R(1-\alpha)(1-\mu)\mu)}{(-\alpha(1+\beta)(2\beta(1-\alpha)-\mu(1+\beta))+\varepsilon_R(1-\alpha)2\beta(1-\mu))^2} > 0 \qquad (29)$$

Intra-regional spillovers, ε_A, have to be stronger to induce agglomeration if there is more integration. Due to the increased cooperation between the regions, the periphery can benefit from the spillovers arising in the core; hence the agglomeration forces are weakened.

The impact of relative congestion is given by

$$\frac{\partial \bar{\varepsilon}_A}{\partial \varepsilon_R} = -\frac{(1-\alpha)(2\beta(1-\alpha)-\mu(1+\beta))^2}{(-\alpha(1+\beta)(2\beta(1-\alpha)-\mu(1+\beta))+\varepsilon_R(1-\alpha)2\beta(1-\mu))^2} < 0 \qquad (30)$$

[19] This argument will be discussed in the context of Figures 3.

and displays the overestimation of physical capital return due to the congestion externality. Individuals do not take their impact on aggregate capital into account. When they decide about capital accumulation, they take aggregate capital as given and independent from their own decision. Therefore, equilibrium capital accumulation is suboptimally high and reinforces the agglomeration forces. Hence, the level of intra-regional spillovers which is necessary to induce agglomeration decreases.

Nevertheless, integration and congestion do not only influence the bifurcation point, $\bar{\varepsilon}_A$, but additionally impact on the resulting concentration. Increases in ε_A may lead either to a higher or to lower concentration within the equilibrium agglomerations, depending on the degree of relative congestion, ε_R, and on integration, β. Numerical simulations within the next section will help to enlighten these complex interdependencies.

5 Numerical simulations

As argued before, agglomeration only occurs if regional spillovers are sufficiently high, or to argue more precisely, if $\varepsilon_A > \bar{\varepsilon}_A(\beta, \varepsilon_R)$ as represented by the bifurcation point within (28). Nevertheless, higher values of ε_A do not automatically result in more concentration. The following calculations and simulations illustrate the sensitivity of the model with respect to those parameters that represent the externalities, ε_A and ε_R, as well as integration, β. We show their impact on the number of equilibria in the context of Table 1 and analyze their impact on concentration within Figures 3 and 4.

Table 1: Bifurcation points $\bar{\varepsilon}_A(\beta, \varepsilon_R)$

	$\varepsilon_R = 0.2$	$\varepsilon_R = 0.23$	$\varepsilon_R = 0.35$
$\beta = 0.2$	-0.7	-0.722	-0.761
$\beta = 0.25$	-0.104	-0.280	-0.502
$\beta = 0.3$	1.780	0.557	-0.224

Tables 1 and 2 show values of the bifurcation points by solving (25) for ε_A and assuming $\alpha = 0.25$ and $\mu = 0.2$. They illustrate how the levels of the bifurcation

points are affected by integration and relative congestion. The gray values in Table 1 refer to the bifurcation points indicated in Figure 3, while the gray values in Table 2 correspond to Figure 4.

The tables could be interpreted as follows: Increasing integration unequivocally raises the value of the bifurcation point and thus supports the hypothesis that integration mitigates agglomeration forces. The contrary applies with respect to ε_R: There the level of the bifurcation point is reduced with increased congestion, and agglomeration becomes more likely.

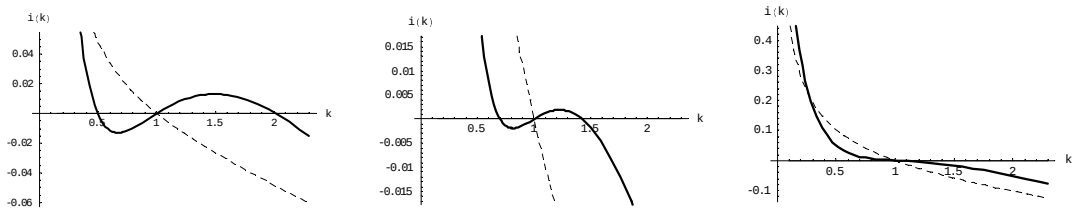
Table 2: Bifurcation points $\bar{\varepsilon}_A(\beta, \varepsilon_R)$

	$\varepsilon_R = 0.4$	$\varepsilon_R = 0.5$	$\varepsilon_R = 0.6$
$\beta = 0.5$	0.895	0.286	0.056
$\beta = 0.6$	2.107	0.699	0.305
$\beta = 0.7$	5.219	1.252	0.580

Within the graphical simulations in Figures 3 and 4, we analyze how ε_A, ε_R and β impact on concentration as measured by the equilibrium level of k^*. As far as possible, we assume symmetry, $\theta = n = l = 1$. Hence the threshold value $i^*(k) = 0$ is represented by the horizontal axis. We consider constant returns to scale in the private inputs ($\alpha + \lambda + \mu = 1$) and make sure that the condition of endogenous growth is fulfilled ($\alpha + \gamma(1 + \varepsilon_A) = 1$). Under these conditions (at least) one equilibrium with equal distribution of capital, i. e. $k^* = 1$, results and no agglomeration takes place within it. If, instead, multiple equilibria arise, the region displaying the higher capital stock represents the core, whereas the other region may be interpreted as being the periphery. The equilibria are symmetric in the sense that one could easily change the region's indices and would have the same implications as before, but now from the point of view of the other region. Higher equilibrium values of k^* are interpreted as reflecting more concentration.

Figures 3(a)–3(c) plot the equilibrium capital distributions for alternative degrees of integration and assume intermediate relative congestion, $\varepsilon_R = 0.5$. The levels of the bifurcation points, $\bar{\varepsilon}_A$, are indicated next to the respective degrees of integration. Solid lines represent high regional spillovers ($\varepsilon_A = 0.9$), while the dashed lines correspond to low levels ($\varepsilon_A = -0.2$).[20] In case of $\varepsilon_A = -0.2 < \bar{\varepsilon}_A$, the pre-

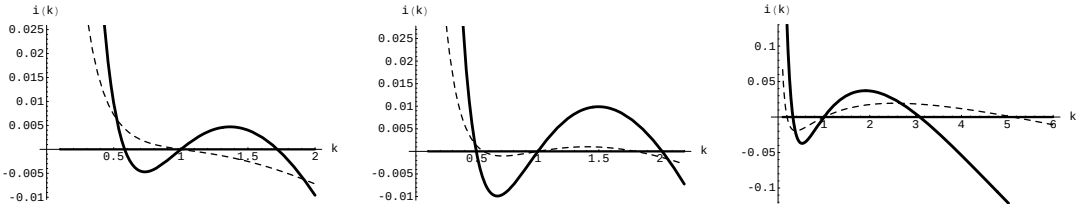
[20] Since the simulations assume $\alpha = 0.25$, we choose this lower benchmark for ε_A to fulfil the



(a) $\beta = 0.5 \quad (\Rightarrow \bar{\varepsilon}_A = 0.286)$ (b) $\beta = 0.6 \quad (\Rightarrow \bar{\varepsilon}_A = 0.699)$ (c) $\beta = 0.7 \quad (\Rightarrow \bar{\varepsilon}_A = 1.251)$

Figure 3: The impact of integration if $\varepsilon_R = 0.5$
parameters: $\alpha = 0.25$, $\mu = 0.2$, $\lambda = 0.55$, $\theta = 1$, $n = 1$, $l = 1 \Rightarrow i^*$: horizontal axis
solid line: $\varepsilon_A = 0.9$, dashed line: $\varepsilon_A = -0.2$

vailing agglomeration forces are too low, capital is equally distributed across the regions, and $k^* = 1$. If, instead, $\varepsilon_A = 0.9$, agglomeration is basically possible (see Figures 3(a) and 3(b)). But more integration reduces concentration (lower k^*) since then the smaller region may also benefit from the spillovers of the bigger region. Consequently, capital accumulation does not move to the core. Figure 3(c) displays a situation in which dispersion forces dominate in either case and $k^* = 1$. As argued before, increasing integration reduces the agglomeration forces.



(a) $\varepsilon_R = 0.2 (\Rightarrow \bar{\varepsilon}_A = -0.104)$ (b) $\varepsilon_R = 0.23 \ (\Rightarrow \bar{\varepsilon}_A = -0.28)$ (c) $\varepsilon_R = 0.35 \ (\Rightarrow \bar{\varepsilon}_A = -0.502)$

Figure 4: The impact of relative congestion if $\beta = 0.25$
parameters: $\alpha = 0.25$, $\mu = 0.2$, $\lambda = 0.35$, $\theta = 1$, $n = 1$, $l = 1 \Rightarrow i^*$: horizontal axis
solid line: $\varepsilon_A = 0.9$, dashed line: $\varepsilon_A = -0.2$

Figures 4(a) – 4(c) emphasize the model's sensitivity and focus on alternative levels of relative congestion for $\beta = 0.25$. Again the levels of the bifurcation points are included in parenthesis below each figure. Solid and dashed lines reflect ε_A in analogy to Figure 3, and equal distribution only arises if $\varepsilon_A < \bar{\varepsilon}_A$. The dashed function in Figure 4(a) is one example. All other combinations of β and ε_R lead to agglomeration, and the following structure may be observed: Increasing relative

condition $-\alpha < \varepsilon_A$.

congestion fosters agglomeration in either case. But note that concentration is even more pronounced for low levels of ε_A. With this, the simulations also confirm the run of the bifurcation diagram within Figure 2(b). The intuition for this result is as follows: On the one hand, we have intra-regional spillovers which foster concentration due to ε_A. But on the other hand, there are decreasing returns not only in private capital but also in the governmental input as discussed in the context of (23b). With an increase in spillovers, ε_A, the ratio of individually available governmental inputs, $\tilde{g}_s$, increases; hence decreasing returns gain importance and reduce concentration. However, as the simulations illustrate, the total effect always implies agglomeration, not only for low but also for high values of relative congestion. Since there is a negative capital externality which goes along with congestion, individuals overestimate private capital return. Hence, agglomeration may even become more concentrated due to an increase in congestion. Nevertheless, concentration is suboptimal as will be shown in the following section.

6 Efficiency

In order to judge the different agglomeration scenarios, it is necessary to compare them with the social optimal situation: Which is the optimal degree of concentration? And is equilibrium concentration suboptimally high or low?

The efficient solution internalizes capital externalities and optimizes government expenditure rates. On the one hand, individuals neglect their influence on aggregate capital; hence they overestimate the individually available amount of the congested governmental input. There is a negative externality of capital accumulation. On the other hand, regional governments usually neglect the productivity impact of governmental activity on the other region. There is a positive externality of governmental activity. We start with the consideration of the congestion externality. In order to evaluate the socially optimal degree of concentration, we have to take into account that private investment increases aggregate capital and hence reduces the individually available amount of the public input. If firms enlarge their truck fleet (private investment), the motorways become more crowded, and there is less infrastructure applicable for each firm. Since all firms in region $\imath$ are identical, aggregate capital is given by $\bar{K}_\imath = N_\imath K_\imath$; hence the congestion function (3) amounts to

$$G_{s\imath} = \theta_\imath N_\imath^{1+\varepsilon_A-\varepsilon_R} K_\imath^{1+\varepsilon_A} \tag{31}$$

The optimal capital distribution, k, is found by maximizing the income of the representative individual $Y = Y_1 + Y_2$ with respect to k. The representative individual's capital stock is given by $K = K_1 + K_2$; hence physical capital in region 1 amounts to $K_1 = kK_2$, and capital in region 2 is given by $K - kK_2$

$$\begin{aligned}\frac{\partial F}{\partial k} =& (F_{K_1} - F_{K_2})K_2 \\ =& \frac{Y_1}{k(g_s+\beta)}\left(\alpha(g_s+\beta)+\gamma(1+\varepsilon_A)(g_s-\beta k)\right) \\ & - \frac{Y_2}{1+\beta g_s}\left(\alpha(1+\beta g_s)\gamma(1+\varepsilon_A)\left(1-\frac{\beta g_s}{k}\right)\right)\end{aligned} \tag{32}$$

This leads to socially optimal capital accumulation determined by

$$\frac{Y_1}{Y_2 k}\,\frac{1+\beta g_s}{g_s+\beta}\,\frac{\alpha(g_s+\beta)+\gamma(1+\varepsilon_A)(1-\beta k)}{\alpha(1+\beta g_s)+\gamma(1+\varepsilon_A)\left(1-\frac{\beta g_s}{k}\right)} - 1 \gtrless 0 \tag{33}$$

$$\iff \quad i(k) + \Delta(k) \gtrless -\lambda \ln l \tag{34}$$

with $i(k)$, as given in equation (22), and $\Delta(k)$ defined as

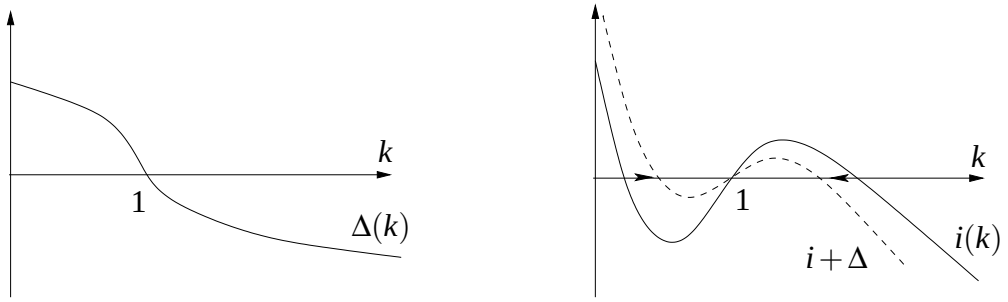
$$\Delta(k) = (1-\mu)\left(\ln\left(\frac{\alpha(1+\beta g_s)+\gamma\varepsilon_R}{\alpha(g_s+\beta)+\gamma\varepsilon_R g_s}\right) + \ln\left(\frac{\alpha(g_s+\beta)+\gamma(1+\varepsilon_A)(1-\beta k)}{\alpha(1+\beta g_s)+\gamma(1+\varepsilon_A)\left(1-\frac{\beta g_s}{k}\right)}\right)\right) \tag{35}$$

$\Delta(k)$ reflects the capital externality and adjusts the ratio of private capital returns to the socially relevant relation. Δ decreases in k and goes through zero for symmetric capital distribution.[21] Furthermore, Δ is bounded from above with $\bar{\Delta} = \ln(\alpha + \gamma\varepsilon_R)$ and from below with $-\bar{\Delta}$. Therefore, the dynamics of optimal concentration are delivered according to Figure 5.

The fact that private investment increases aggregate capital and therefore reduces the availability of the public input alters the ratio between the capital returns in the two regions. Figure 5(b) shows that agglomeration is socially optimal. Nevertheless, concentration is suboptimally high. Since individuals overestimate private capital returns, they react too sensitively with respect to a regional difference in capital returns. As a consequence, the degree of concentration is suboptimally high in market equilibrium.

The remaining point refers to optimal government expenditures: Public inputs such as harbors are supra-regionally productive. If region 1 increases the provision

[21] The calculus is relegated to Appendix B.



(a) Congestion externality. (b) Optimal agglomeration.

Figure 5: Equilibrium versus optimal dynamics.

of public inputs, the productivity in both regions rises. Within the optimal choice of governmental expenditures, G_ι, this impact has to be properly considered. The optimal ratio θ of regional public inputs is found by maximizing the representative individual's income $Y = Y_1 + Y_2$ with respect to θ and taking into account that $G_1 = \theta k n G_2$ and $G_2 = G - \theta k n G_2$ apply. The resulting condition for optimal governmental activity is

$$\begin{aligned} \frac{\partial F}{\partial \theta} =& F_{G_1}\frac{\partial G_1}{\partial \theta} + F_{G_2}\frac{\partial G_2}{\partial \theta} \stackrel{!}{=} 0 \\ \iff & \frac{Y_1}{Y_2}\frac{1+\beta g_s(\theta)}{g_s(\theta)+\beta} = \frac{1-\beta k^{\varepsilon_A} n^{\varepsilon_A-\varepsilon_R}}{k^{\varepsilon_A} n^{\varepsilon_A-\varepsilon_R}-\beta} \end{aligned} \tag{36}$$

Using equation (12) to replace Y_1/Y_2 yields

$$\left(\frac{g_s(\theta^*)+\beta}{1+\beta g_s(\theta^*)}\right)^{\frac{\gamma+\mu-1}{1-\mu}} = \frac{1-\beta k^{\varepsilon_A} n^{\varepsilon_A-\varepsilon_R}}{k^{\varepsilon_A} n^{\varepsilon_A-\varepsilon_R}-\beta}(l^\lambda k^\alpha)^{\frac{1}{\mu-1}} \tag{37}$$

Within the equilibrium analysis given in the last section, the ratio of governmental activity, θ, was assumed to be arbitrarily set. Nevertheless, a regional government would decide about the amount of governmental activity, G_ι, by equating marginal costs and benefits. As the homogenous good may be transformed 1:1 into governmental expenditures, marginal costs of an increase in G_ι are 1. Marginal benefits result from increased productivity. It is self-evident to assume that regional governments are only concerned about the productivity in their own region. They disregard the inter-regional impact of public inputs. Usually, a regional government will only provide a harbor if the productivity gain in its own region is sufficiently high to warrant the harbor. The regional government will not take into account that, due to the harbor, other regions will experience increased productivity.

Hence, both regions equate the marginal benefits and marginal costs of govern-

mental activity according to

$$Y_{1G_1} \stackrel{!}{=} 1 \quad \text{and} \quad Y_{2G_2} \stackrel{!}{=} 1 \quad \Longrightarrow \quad Y_{1G_1} \stackrel{!}{=} Y_{2G_2}$$
$$\Longleftrightarrow \quad \frac{Y_1}{Y_2}\frac{1+\beta g_s(\theta)}{g_s(\theta)+\beta} = \frac{1}{k^{\varepsilon_A} n^{\varepsilon_A-\varepsilon_R}} \tag{38}$$

Replacing again Y_1/Y_2 with (12) leads to

$$\left(\frac{g_s(\tilde{\theta})+\beta}{1+\beta g_s(\tilde{\theta})}\right)^{\frac{\gamma+\mu-1}{1-\mu}} = \frac{1}{k^{\varepsilon_A} n^{\varepsilon_A-\varepsilon_R}} (l^{\lambda} k^{\alpha})^{\frac{1}{\mu-1}} \tag{39}$$

Comparing the optimal ratio of governmental activity, θ^*, and the corresponding equilibrium value, $\tilde{\theta}$, in the symmetric case yields $\theta^* = \tilde{\theta}$. The relative impact of the positive diffusion externality is of the same magnitude in each region. Hence, the ratio between governmental expenditures is unaffected. Nevertheless, the level of governmental expenditures is suboptimally low.[22] Applying this result to Figure 5 demonstrates that selfish governmental behavior has no impact on the degree of agglomeration compared to optimal governmental activity. Nevertheless, other assumptions about regional governmental behavior could be analyzed, but this will be done in another article since issues of political economy are not our main concern here.

7 Conclusions

The basic objective of this paper is to analyze the impact of regional policy on the spatial distribution of economic activity. We ask whether integration will increase concentration as usually shown in new economic geography models which interpret integration as a reduction in transport costs. And we ask whether the European regional policy to foster territorial cooperation will reach the goal to support convergence. Within the context of the model presented, regional policy includes the extent of inter-regional cooperation, as well as the type of the governmental input provided. This input affects output not only directly but also indirectly as it enhances the productivity of the other inputs. Since the governmental input is characterized by absolute and by relative congestion, the model may be adopted to a variety of interpretations; two examples are physical infrastructure or research networks. It is shown that either one unique or multiple equilibria arise, with the

[22] This is easily seen since the direct marginal returns, Y_{iG_i}, are lower than the social returns, F_{G_i}.

latter showing different stability characteristics. Whether or not this leads to convergence in the sense of the European Union's regional policy goals depends upon a variety of economic conditions.

The model is very sensitive to the assumed parameter constellations, but nevertheless some basic results are derived: Integration unequivocally reduces concentration since it allows the smaller regions access to the other regions' public input and hence to benefit from its productivity impact. This result stands in strong contrast to those analysis that model infrastructure as facilitating trade. Relative congestion is associated with a negative capital externality and aggravates concentration. As a consequence, the resulting market equilibrium ends up with suboptimally high concentration. This argument reflects the typical discussion within the growth literature about the impact of relative congestion. The effect of intra-regional capital spillovers is more complex. Agglomeration only arises if spillovers are strong enough to overweigh decreasing returns to private capital. Nevertheless, if a high level of capital spillovers applies in a situation of high relative congestion, the impact may be reversed and decrease the resulting concentration.

The model's policy implications could then be summarized as follows: More integration reduces regional disparities, while relative congestion operates in the opposite direction. These congestion externalities could be internalized by a fiscal policy that corrects for the distortions. With this, it is clear that much work is still left to be done. Another open-ended question consists in the implementation of governmental policy that merges the agglomeration effects of a regional governmental policy that provides a productive input that also facilitates inter-regional exchange.

Mathematical appendix

A Shape of $i(k)$

This first part of the appendix is concerned with the derivation of the shape of $i(k)$. The thread is as follows: The limit of i for $k = 0$ is shown to be infinity, with an unambiguously negative slope. The limit of i for $k \to \infty$ is $-\infty$, and the slope eventually approaches zero. Hence, i displays at least one root. One root is

shown to be at $k = 1$. Hence, if the slope of i is positive for $k = 1$, we have two agglomerations, one for $k < 1$ and one for $k > 1$; see Figure 1.

If k tends to zero, $g_s = \theta k^{1+\varepsilon_A} n^{1+\varepsilon_A-\varepsilon_R}$ tends to zero, too. Hence, the limit of i for $k = 0$ is given by

$$\lim_{k\to 0} i(k) = \underbrace{(\mu+\alpha-1)\lim_{k\to 0}\ln(k)}_{\to\infty} + \underbrace{(\mu+\gamma-1)\ln(\beta)}_{\gtrless 0,\quad \text{bounded}} + \underbrace{(1-\mu)\ln\left(\frac{\alpha\beta}{\alpha+\gamma\varepsilon_R}\right)}_{<0,\quad \text{bounded}} = \infty \tag{40}$$

The slope of i at $k = 0$ can be denoted as

$$\lim_{k\to 0} i'(k) = \lim_{k\to 0}\frac{1}{k}\left(\underbrace{\mu+\alpha-1}_{<0} + \underbrace{(\mu+\gamma-1)(1+\varepsilon_A)g_s\frac{(1-\beta)(1+\beta)}{\beta}}_{\gtrless 0,\quad \to 0}\right.$$
$$\left. + \underbrace{(1-\mu)(1+\varepsilon_A)g_s\frac{(\alpha(1-\beta)+\gamma\varepsilon_R)(\alpha(1+\beta)+\gamma\varepsilon_R)}{\alpha\beta(\alpha+\gamma\varepsilon_R)}}_{>0,\quad \to 0}\right) = -\infty \tag{41}$$

For k going to infinity, $g_s \to \infty$, and therefore

$$\lim_{k\to\infty} i(k) = \underbrace{(\mu+\alpha-1)\lim_{k\to\infty}\ln(k)}_{<0,\quad \to 0} + \underbrace{(\mu+\gamma-1)\ln\left(\frac{1}{\beta}\right)}_{\gtrless 0,\quad \text{bounded}} + \underbrace{(1-\mu)\ln\left(\frac{\alpha+\gamma\varepsilon_R}{\alpha\beta}\right)}_{>0,\quad \text{bounded}} = -\infty \tag{42}$$

and

$$\lim_{k\to\infty} i'(k) = \lim_{k\to\infty}\frac{1}{k}\left(\underbrace{\mu+\alpha-1}_{<0} + \underbrace{(\mu+\gamma-1)(1+\varepsilon_A)\frac{(1-\beta)(1+\beta)}{(1+\beta/g_s)(1+\beta g_s)}}_{\gtrless 0,\quad \to 0}\right.$$
$$\left. + \underbrace{(1-\mu)(1+\varepsilon_A)\frac{(\alpha(1-\beta)+\gamma\varepsilon_R)(\alpha(1+\beta)+\gamma\varepsilon_R)}{(\alpha(1+\beta/g_s)+\gamma\varepsilon_R)(\alpha(1+\beta g_s)+\gamma\varepsilon_R)}}_{>0,\quad \to 0}\right) = 0 \tag{43}$$

For a symmetric society, that is $k = n = \theta = 1$, and hence $g_s = 1$, the function unambiguously has a root

$$i(1) = 0 \tag{44}$$

Nevertheless, the slope in this root is indeterminate

$$i'(1) \gtrless 0 \quad \Longleftrightarrow \quad \mu+\alpha-1 \gtrless (\mu-1)(1+\varepsilon_A)\frac{2\beta\gamma\varepsilon_R}{(1+\beta)(\alpha(1+\beta)+\gamma\varepsilon_R)} - \gamma(1+\varepsilon_A)\frac{1-\beta}{1+\beta} \tag{45}$$

and increases in ε_A

$$\begin{aligned}\frac{\partial i'(1)}{\partial \varepsilon_A} =& (\mu+\gamma-1)\frac{1-\beta}{1+\beta} + (1-\mu)\frac{\alpha(1-\beta)+\gamma\varepsilon_R}{\alpha(1+\beta)+\gamma\varepsilon_R} \\ =& \frac{1-\alpha}{1+\varepsilon_A} + (1-\mu)\frac{2\beta\gamma\varepsilon_R}{(\alpha(1+\beta)+\gamma\varepsilon_R)(1-\beta)} > 0\end{aligned} \tag{46}$$

B Shape of $\Delta(k)$

In the following, we will analyze the slope of the function $\Delta(k)$ as given in (35), which determines the discrepancy between equilibrium agglomeration and socially optimal agglomeration. For notational convenience, we define $\Delta(k) \equiv (1-\mu)(\Delta_1(g_s(k)) + \Delta_2(g_s(k),k))$. Hence, the slope of Δ is given by

$$\frac{d\Delta}{dk} = (1-\mu)\left(\left(\frac{\partial \Delta_1}{\partial g_s} + \frac{\partial \Delta_2}{\partial g_s}\right)\frac{\partial g_s}{\partial k} + \frac{\partial \Delta_2}{\partial k}\right) \tag{47}$$

with

$$\frac{\partial \Delta_1}{\partial g_s} = -\frac{(\alpha(1-\beta)+\gamma\varepsilon_R)(\alpha(1+\beta)+\gamma\varepsilon_R)}{(\alpha(g_s+\beta)+\gamma\varepsilon_R g_s)(\alpha(1+\beta g_s)+\gamma\varepsilon_R)} < 0 \tag{48}$$

$$\frac{\partial \Delta_2}{\partial g_s} = \frac{\left(1-\beta^2\left(1-\alpha\gamma(1+\varepsilon_A)\frac{(1+k)^2}{k}\right)\right)}{(\alpha(g_s+\beta)+\gamma(1+\varepsilon_A)(1-\beta k))\left(\alpha(1+\beta g_s)+\gamma(1+\varepsilon_A)\left(1-\frac{\beta g_s}{k}\right)\right)} < 0 \tag{49}$$

$$\frac{\partial g_s}{\partial k} = (1+\varepsilon_A)\frac{g_s}{k} > 0 \tag{50}$$

$$\frac{\partial \Delta_2}{\partial k} = -\frac{\gamma(1+\varepsilon_A)\beta}{\alpha(g_s+\beta)+\gamma(1+\varepsilon_A)(1-\beta k)} - \frac{\gamma(1+\varepsilon_A)\beta g_s}{k^2(\alpha(1+\beta g_s)+\gamma(1+\varepsilon_A)\left(1-\frac{\beta g_s}{k}\right))} < 0 \tag{51}$$

It follows immediately that the slope of Δ is unambiguously negative.

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