Optimal Monetary Policy in a Small Open Economy with Home Bias

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Abstract

We analyze optimal monetary policy in a small open economy characterized by home bias in consumption. Peculiar to our framework is the application of a Ramsey-type analysis to a model of the recent open-economy New Keynesian literature. We show that home bias in consumption is a sufficient condition for inducing the monetary policy-maker of an open economy to deviate from a strategy of strict markup stabilization and contemplate some (optimal) degree of exchange rate stabilization. We focus on the optimal setting of policy both in the case of firms setting prices one period in advance and in a gradual fashion subject to adjustment costs. While the first setup allows us to analytically highlight home bias as an independent source of equilibrium markup variability, the second setup allows to study the effects of future expectations on the optimal policy problem and the effect of home bias on optimal inflation volatility. The latter, in particular, is shown to be related to the degree of trade openness in a U-shaped fashion, whereas exchange rate volatility is monotonically decreasing in openness.

Keywords. Openness, Optimal Monetary Policy, Ramsey Planner, Home Bias, Sticky Prices.

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1 Introduction

A central issue in the recent open-economy New Keynesian literature is whether exchange rate stabilization should be part of a central bank’s monetary policy strategy. This is also the key feature that induces fundamental differences in the design of optimal policy between closed and open economies.\(^1\)

This paper studies optimal monetary policy in a small open economy characterized by home bias in consumption. In our context, the presence of home bias is the key factor generating endogenous real exchange rate fluctuations. Hence, despite the fact that, in the absence of any impediment to trade, the law of one price holds continuously at the level of each individual good, equilibrium deviations from PPP are feasible. In addition, our economy features goods markets characterized by imperfect competition and nominal rigidities, and complete markets for internationally traded state contingent securities.

We refrain from providing a theory of home bias in this context, but rather we model it as a primitive feature of our economic environment. Importantly, the presence of home bias in consumption is a fundamental characteristic of international trade data. For instance, Obstfeld and Rogoff (2003) list home bias in trade as one of the six major puzzles in international macroeconomics. Our interest here is in studying the effects of home bias on the optimal setting of monetary and exchange rate policy.

We study monetary policy both in the case in which firms set prices one period in advance as well as in the case in which prices are set gradually subject to adjustment costs. While the former static setup permits an analytical inspection of the main forces that drive the behavior of the markup under the optimal policy, the latter setup (intrinsically dynamic) emphasize the impact of future expectations on the optimal policy problem and in particular on the equilibrium volatility of inflation.

Our analysis makes two main contributions. First, we highlight that home bias in consumption is an independent condition inducing the monetary policy-maker of an open economy to deviate from an inward-looking strategy of strict markup stabilization, and thus contemplate some (optimal) degree of exchange rate stabilization. This differs from the popular Friedman (1956) prescription, derived for instance in Devereux and Engel (2003),

according to which, in the presence of price stickiness, exchange rate movements should be instrumental to have the economy replicate the allocation under purely flexible prices.

In the absence of home bias (i.e., with PPP holding), a motive for deviating from strict markup stabilization generally lies in the possibility of strategically affecting the terms of trade (the relative price of imports). A terms of trade variation, by altering domestic residents’ purchasing power, affects consumption for any given level of output (labor effort). In the efficient allocation of an open economy, then, the planner can improve upon the constant-markup allocation prevailing under flexible prices. Previous contributions - such as Corsetti and Pesenti (2001), Sutherland (2002), Benigno and Benigno (2003) - have shown that this terms-of-trade motive for optimal markup variability depends on the underlying specification of the utility function. In particular, it vanishes if either of two conditions holds: (i) the intra-temporal elasticity of substitution between domestic and imported goods is unitary; (ii) that same elasticity coincides with the inter-temporal elasticity of substitution in consumption.

This paper suggests that, in the presence of home bias, the conditions for markup stability to be constrained-efficient are more restrictive: in particular, a unitary elasticity of substitution (condition (i) above) ceases to be sufficient for markup stability to be constrained-optimal (whereas sufficiency of condition (ii) still holds). Under home bias, in fact, variations in the terms of trade induce also variations in the real exchange rate (i.e., the relative price of the consumption basket), which, in turn, affect domestic consumption via international risk-sharing (for any given level of foreign consumption). At the margin, and relative to an allocation with constant markup, this endows the policymaker with a complementary channel to affect consumption which is absent in the baseline case with PPP.

Our second contribution has a more methodological flavor. We suggest that optimal monetary policy in a small open economy can be usefully characterized by applying a Ramsey-type analysis. In the classic approach to the study of optimal policy in dynamic economies (Ramsey (1927), Atkinson and Stiglitz (1976), Lucas and Stokey (1983), Chari, Christiano and Kehoe (1991)), and in a typical public finance spirit, a Ramsey planner maximizes household’s welfare subject to a resource constraint, to the constraints describing the equilibrium in the private sector economy, and via an explicit consideration of all the distortions that characterize both the long-run and the cyclical behavior of the economy. Recently there has been a resurgence of interest for a Ramsey-type approach in dynamic general equi-
librium models with monopolistic competition and nominal rigidities. Examples include, in
the context of closed economy models, Adao et al. (2003), Khan et al. (2003), Schmitt-Grohé
and Uribe (2004) and Siu (2004). However, most of the welfare analysis of monetary policy
in the recent literature builds on a linear-quadratic approximation approach in the spirit of
Rotemberg and Woodford (1997), Woodford (2003) and Benigno and Woodford (2004). A
Ramsey-type approach has featured even more limited applications to the recent growing
literature of New Keynesian open economy models.

The remainder of the paper is as follows. Section 2 describes the economic environment.
Section 3 illustrates the details of the optimal monetary policy problem under pre-set prices.
Section 4 extends the analysis to forward-looking price setting. Section 5 concludes.

2 The Model

The world economy consists of two economic entities, a small economy and a rest of the world.
Preferences feature home bias in consumption. Each economy is populated by infinitely-lived
agents. The total measure of the world economy is normalized to unity, with Home and
Foreign having measure $n$ and $(1 - n)$ respectively. To characterize the small economy case
we resort to a ”limit-case” approach, as in Galí and Monacelli (2002), Sutherland (2005), De
Fiore and Liu (2005) and De Paoli (2004). This consists in modelling the domestic economy
as small in size relative to the rest of the world, whose equilibrium dynamics are akin to the
one of a standard closed economy.

2.1 Domestic Households

Consumption preferences in the Home economy are described by the following composite
index of domestic and imported bundles of goods:

$$C_t \equiv \left[ \left( 1 - \gamma \right)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta}{\eta-1}} + \gamma^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta}{\eta-1}} \right]^{\frac{\eta-1}{\eta}}$$  \hspace{1cm} (1)$$

where $\eta > 0$ is the elasticity of substitution between domestic and foreign goods, and $\gamma \equiv
(1 - n)\alpha$ denotes the weight of imported goods in Home consumption basket. This weight

\[\text{Schmitt-Grohé and Uribe (2004) and Siu (2004), in particular, analyze the more general issue of the optimal joint determination of monetary and fiscal policy.}\]

\[\text{For an application of a Ramsey-type analysis in the context of a two-country model, see, under sticky prices, Faia and Monacelli (2003), and, under flexible prices, Arsenau (2004). For applications employing a so-called linear-quadratic approach, see Benigno and Benigno (2004) and De Paoli (2004).}\]
depends on \((1 - n)\), the relative size of Foreign, and on \(\alpha\), the degree of trade openness of Home. In an analogous manner, preferences in Foreign can be described as:

\[
C_t^* \equiv [(1 - \gamma^*)\frac{1}{n} C_{F,t}^{\frac{n-1}{n}} + (\gamma^*)\frac{1}{n} C_{H,t}^{\frac{n-1}{n}}]^{\frac{n}{n-1}}
\]

where \(\gamma^* \equiv n\alpha^*\). We assume home bias in consumption, which entails:

\[
(1 - \gamma) = (1 - (1 - n)\alpha) > \gamma^* = n\alpha^*
\]

Notice that in the symmetric case of \(\alpha = \alpha^*\) (and regardless of the relative size assumption), as well as in the limiting case \(n \to 0\), home bias requires \(\alpha < 1\). The same argument holds exactly for consumption preferences in Foreign.\(^4\)

Each consumption bundle \(C_{H,t}\) and \(C_{F,t}\) is composed of imperfectly substitutable varieties (with elasticity of substitution \(\varepsilon > 1\)). Optimal allocation of expenditure within each variety of goods yields:

\[
C_{H,t}(i) = \frac{1}{n} \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t}; \quad C_{F,t}(i) = \frac{1}{1 - n} \left( \frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t}
\]

where \(C_{H,t} \equiv \left( \frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n [C_{H,t}(i)^{\frac{1}{1-\varepsilon}} di]^\frac{1}{1-\varepsilon}\) and \(C_{F,t} \equiv \left( \frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^n [C_{F,t}(i)^{\frac{1}{1-\varepsilon}} di]^\frac{1}{1-\varepsilon}\).

Optimal allocation of expenditure between domestic and foreign bundles yields:

\[
C_{H,t} = (1 - \gamma) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t; \quad C_{F,t} = \gamma \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t
\]

where

\[
P_t \equiv [(1 - \gamma)P_{H,t}^{1-\eta} + \gamma P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}}
\]

is the CPI index.

We assume, both within and across countries, the existence of complete markets for state-contingent claims expressed in units of domestic currency. Let \(h^t = \{h_0, ..., h_t\}\) denote the history of events up to date \(t\), where \(h_t\) is the event realization at date \(t\). The date 0 probability of observing history \(h^t\) is given by \(\rho(h^t)\). The initial state \(h^0\) is given so that \(\rho(h^0) = 1\).

\(^4\)Home bias in Foreign preferences requires \((1 - \gamma^*) > \gamma\). This implies \((1 - \alpha) > n(\alpha^* - \alpha)\), which can be rewritten as (3).
Agents maximize the following expected discounted sum of utilities over possible paths of consumption and labor:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U (C_t, N_t) \right\}$$  \hspace{1cm} (7)

where $E_0 \{ \}$ denotes the mathematical expectations operator conditional on $h_0$, and $N_t$ is labor hours.\(^5\) The function $U(\bullet)$ features typical regularity conditions and is assumed to be separable in its arguments. To insure their consumption pattern against random shocks at time $t$ households spend $\nu_{t+1,t} B_{t+1}$ in nominal state contingent securities, where $\nu_{t,t+1} \equiv \nu(h_{t+1}|h_t)$ is the period-$t$ price of a claim to one unit of domestic currency in state $h_{t+1}$ divided by the probability of occurrence of that state. Each asset in the portfolio $B_{t+1}$ pays one unit of domestic currency at time $t + 1$ and in state $h_{t+1}$.

By considering the optimal expenditure conditions (4) and (5), the sequence of budget constraints assumes the following form:

$$P_t C_t + \sum_{h_{t+1}} \nu_{t+1,t} B_{t+1} \leq W_t N_t + \tau_t + B_t + \int_{0}^{1} \Gamma_t(i)$$  \hspace{1cm} (8)

where $\tau_t$ are government net transfers of domestic currency and $\Gamma_t(i)$ are the profits of monopolistic firm $i$, whose shares are owned by the domestic residents.\(^6\)

The representative household chooses processes $\{C_t, N_t\}_{t=0}^{\infty}$ and bonds $\{B_{t+1}\}_{t=0}^{\infty}$ taking as given the set of processes $\{P_t, W_t, \nu_{t+1,t}\}_{t=0}^{\infty}$ and the initial wealth $B_0$ so as to maximize (7) subject to (8). For any given state of the world, the following set of efficiency conditions must hold:

$$U_{c,t} = -U_{n,t}$$  \hspace{1cm} (9)

$$\beta \frac{P_t}{P_{t+1}} \frac{U_{c,t+1}}{U_{c,t}} = \nu_{t,t+1}$$  \hspace{1cm} (10)

$$\lim_{j \to \infty} E_t \{ \nu_{t,t+j} B_{t+j} \} = 0$$  \hspace{1cm} (11)

where $U_{j,t}$ defines the first order derivative of utility with respect to its argument $j = C, N$. Equation (9) equates the CPI-based real wage to the marginal rate of substitution between

\(^5\)Hence the expression for lifetime utility is equivalent to writing $\sum_{t=0}^{\infty} \sum_{h_t} \beta^t U (C(h_t), N(h_t)) \rho(h_t)$, where $\rho(h_t) = \rho(h_t|h_0)$.

\(^6\)Each domestic household owns an equal share of the domestic monopolistic firms. We abstract from international trade in shares.
consumption and labor. Equation (10) describes a set of asset pricing conditions for each possible state $h^{t+1}$. Along with (8) holding with equality, optimality requires that the first order conditions (9), (10), and the no-Ponzi game condition (11), are simultaneously satisfied.

Taking conditional expectations of equation (10) a gross nominal interest rate (or return on the corresponding riskless one-period bond) can be defined as:

$$R_t = E_t \{ \nu_{t+1} \}^{-1} = \left[ \beta E_t \left\{ \frac{P_t}{P_{t+1}} \frac{U_{c,t+1}}{U_{c,t}} \right\} \right]^{-1}$$

Equation (12) takes the form of a familiar consumption Euler equation. Notice that, following large part of the recent literature, we do not introduce money explicitly, but rather think of it as playing the role of nominal unit of account.\(^7\)

### 2.2 Law of One Price, Foreign Demand, Terms of Trade and the Real Exchange Rate

We assume throughout that the law of one price holds, implying that $P_{F,t}(i) = E_t P_{F,t}^*(i)$ for all $i \in [0,1]$, where $E_t$ is the nominal exchange rate, i.e., the price of foreign currency in terms of home currency, and $P_{F,t}^*(i)$ is the price of foreign good $i$ denominated in foreign currency. Importantly, the holding of the law of one price does not necessarily imply that PPP holds, unless we make the further restrictive assumption of absence of home bias.

Foreign demand for domestic variety $i$ must satisfy:

$$C_{H,t}(i) = \frac{1}{n} \left( \frac{P_{H,t}^*(i)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t}^*$$

$$= \frac{1}{n} \left( \frac{P_{H,t}^*(i)}{P_{H,t}^*} \right)^{-\varepsilon} \gamma^* \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-\eta} C_t^*$$

The terms of trade is the relative price of imported goods:

$$S_t = \frac{P_{F,t}}{P_{H,t}}$$

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\(^7\)See Woodford (2003a), chapter 3. Thus the present model may be viewed as approximating the limiting case of a money-in-the-utility model in which the weight of real balances in the utility function is arbitrarily close to zero.
while the real exchange rate is defined as $Q_t \equiv \frac{E_t P_t^*}{P_t}$. The terms of trade can be related to the CPI-PPI ratio as follows

$$\frac{P_t}{P_{H,t}} = [(1 - \gamma) + \gamma S_t^{1-\eta}] \frac{1}{1-\eta} \equiv g(S_t)$$

(15)

with $g_{s,t} \equiv \frac{\partial g(S_t)}{\partial S_t} > 0$.

Notice that the terms of trade and the real exchange rate are linked through the following expression:

$$Q_t = S_t \frac{P_t^*}{P_{F,t}^*} \left( \frac{P_t}{P_{H,t}} \right)^{-1}$$

(16)

$$= S_t g^*(S_t) \equiv q(S_t)$$

where

$$\frac{P_t^*}{P_{F,t}^*} = [(1 - \gamma^*) + \gamma^* S_t^{\eta-1}] \frac{1}{1-\eta} \equiv g^*(S_t)$$

(17)

with $q_{s,t} \equiv \frac{\partial q(S_t)}{\partial S_t} > 0$ and $g^*_{s,t} \equiv \frac{\partial g^*(S_t)}{\partial S_t} < 0$.

### 2.3 Risk-Sharing

Under complete markets for state contingent assets, the efficiency condition for bonds’ holdings by residents in Foreign reads:

$$\beta \frac{P_t^* \xi_t}{P_{t+1}^* \xi_{t+1}} \frac{U_{c,t+1}^*}{U_{c,t}^*} = \nu_{t,t+1}$$

(18)

Taking conditional expectations of (18) and defining the foreign nominal interest rate $R_t^* \equiv \left( E_t \left\{ \nu_{t,t+1} \frac{\xi_{t+1}}{\xi_t} \right\} \right)^{-1}$ one can write:

$$R_t^* = \left[ \beta E_t \left\{ \frac{P_t^*}{P_{t+1}^*} \frac{U_{c,t+1}^*}{U_{c,t}^*} \right\} \right]^{-1}$$

(19)

Equating (10) with (18), and iterating, yields a condition linking the real exchange rate to the ratio of the marginal utilities of consumption across countries (all in levels):

$$\frac{U_{c,t}^*}{U_{c,t}} = \frac{E_t P_t^*}{P_t} = Q_t = q(S_t)$$

(20)
where $\kappa \equiv \frac{E_0 \rho_t u_{t,0} \rho_{u,t,0}}{P_{0,t} u_{t,0}}$. In the following, we assume that the initial distribution of wealth is implemented in such a way that $\kappa = 1$. Equation (20) is a typical condition that emerges in the presence of international asset markets where households engage in risk-sharing via the trading of state contingent securities. Such a trading allows the agents in the two countries to equalize their respective intertemporal budget constraints.\footnote{It is easy to show that, if the risk-sharing trading of assets at time zero corresponds to the two agents equalizing their respective intertemporal budget constraints, necessarily $\kappa = 1$ (see Devereux and Engel (2003), Faia and Monacelli (2004)). As a consequence of complete markets, whether the same asset trading is undertaken at time zero or sequentially is irrelevant for the specification of the equilibrium.}

### 2.4 Production and Price Setting

Each monopolistic firm $i$ in Home produces a homogenous good according to the production function:

$$Y_t(i) = A_t F(N_t(i))$$  \hfill (21)

where $A_t$ is a labor productivity shifter (common across firms) and $F(\bullet)$ is a homogeneous function with $F_n_t \equiv \frac{\partial F(\cdot)}{\partial N_t} > 0$. The cost minimizing choice of labor input implies:

$$\frac{W_t}{P_{H,t}(i)} = \frac{MC_t}{P_{H,t}(i)} A_t F_n_t$$  \hfill (22)

where $MC$ denotes the nominal marginal cost. Notice that, since households supply a homogenous type of labor, the nominal wage and the marginal cost are common across firms.

We assume that prices are determined one period in advance. There is no international price discrimination. Each producer $i$ chooses the price $P_{H,t}(i)$ to satisfy local and foreign demand and to maximize expected discounted nominal profits:

$$E_{t-1} \{ \nu_{t-1,t} [P_{H,t}(i)Y_t(i) - W_t N_t(i)] \}$$

subject to

$$Y_t(i) \leq \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} Y_t$$  \hfill (23)

and (21), where $Y_t(i)$ is total demand for variety $i$ and $Y_t$ is world aggregate demand. By using (23) and (21) we can rewrite the profit function:
\[ \Gamma_t(i) = \left\{ \nu_{t-1,t} \left[ \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{1-\varepsilon} P_{H,t}Y_i - W_t \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \frac{Y_i}{A_t} \right] \right\} \]

where \( h(\bullet) \equiv F^{-1} \left( \frac{Y_i(i)}{A_t} \right) = F^{-1} \left( P_{H,t}(i), P_{H,t}, A_t, Y_t \right) = N_t(i). \)

The first order condition with respect to \( P_{H,t}(i) \) reads:

\[ E_{t-1} \left\{ (1 - \varepsilon) \frac{P_{H,t}(i)}{P_{H,t}} Y_i + \varepsilon \frac{W_t}{P_{H,t}} h_{p,t} \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon-1} \frac{Y_i}{A_t} \right\} = 0 \tag{24} \]

where \( h_{p,t} \equiv \frac{\partial h(i)}{\partial P_{H,t}(i)} \). Notice that, in the case of linear technology \( Y_t(i) = A_t N_t(i) \), we have \( h_{p,t} = 1 \) for all \( t \). In general, recall that \( \frac{\partial h(i)}{\partial P_{H,t}(i)} = (\frac{\partial F}{\partial P_{H,t}(i)})^{-1} \).

Dividing through by \( \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \), writing the product wage as \( \frac{W_t}{P_t} = \frac{W_t}{P_t} g(S_t) \) and using (10) we obtain:

\[ \frac{\beta P_{t-1}}{U_{c,t-1}} E_{t-1} \left\{ \frac{U_{c,t} Y_t}{P_t} \left[ \frac{P_{H,t}(i)}{P_{H,t}} - \frac{W_t}{P_t} g(S_t) \left( \frac{\varepsilon}{\varepsilon - 1} \right) \right] \right\} = 0 \tag{25} \]

### 2.5 Symmetric Equilibrium in a Small Open Economy

Market clearing for domestic variety \( i \) must satisfy:

\[ Y_t(i) = n C_{H,t}(i) + (1 - n) C^*_t(i) \tag{26} \]

\[ = \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \left[ (1 - \gamma) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \frac{(1 - n)}{n} \gamma^* \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C^*_t \right] \]

\[ = \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \left[ (1 - (1 - n)\alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + (1 - n)\alpha^* \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C^*_t \right] \]

In a symmetric equilibrium, each domestic producer charges the same price and produces the same level of output, so that \( P_{H,t}(i) = P_{H,t}, N_t(i) = N_t \) and \( Y_t(i) = Y_t \) for all \( i \) and \( t \).

Next, we restrict our attention to the limiting case of a small economy. This implies that the relative size of Home is negligible relative to the rest of the world, i.e., \( n \to 0 \). It follows that Foreign is an aggregate economy whose equilibrium dynamics are exogenous.
from the viewpoint of the small economy and approximately closed to trade. Notice that this assumption further implies $P^*_{F,t} = P^*_t$, which in turn implies $g^*(S_t) = 1$. Hence, from (16), we have the following expression for the real exchange rate:

$$q(S_t) = \frac{S_t}{g(S_t)}$$  \hspace{1cm} (27)

We further assume symmetric degree of home bias across countries, which requires $\alpha = \alpha^*$. Hence we can finally write (26) as:

$$Y_t = g(S_t)^\eta \left[ (1 - \alpha)C_t + \alpha q(S_t)^\eta C^*_t \right]$$

$$= (1 - \alpha)g(S_t)^\eta C_t + \alpha S^*_t C^*_t$$  \hspace{1cm} (28)

In equilibrium, and using (9) to replace the real wage, the price setting condition can be written

$$E_{t-1} \left\{ \left[ \frac{A_t F(N_t)}{g(S_t)} \frac{U_{c,t}}{g(S_t)} + \mu U_{n,t} \omega(N_t) \right] \right\} = 0$$  \hspace{1cm} (29)

where $\omega(N_t) \equiv \frac{F(N_t)}{F_{n,t}}$ and $\mu \equiv \frac{\varepsilon}{\varepsilon - 1}$ is the steady-state level of the markup. Notice that, in obtaining (29), we used the fact that $P_{H,t}$ is predetermined from the viewpoint of time $t$.

#### 2.6 Flexible Prices

In the particular case of fully flexible prices, equation (29) simplifies to:

$$\Phi_t \equiv -\frac{U_{n,t}}{A_t F_{n,t}} g(S_t) = \mu^{-1} \equiv \Phi^f_t$$  \hspace{1cm} (30)

where $\Phi_t$ is the equilibrium expression for the real marginal cost (or inverse of the markup). Hence, with flexible prices, each firm would optimally choose to replicate a constant markup. The open-economy dimension explicitly affects the markup via the presence of the relative price $g(S_t)$, which is positively related to the terms of trade.

9The small economy assumption allows us to abstract from any strategic interaction in the conduct of policy. See Faia and Monacelli (2004) for an analysis of cooperative and non-cooperative monetary policy in a two-country world.
2.6.1 Equilibrium with Pre-set Prices

To understand the nature of the equilibrium and the role of monetary policy under pre-set prices, notice that, from (20), we can write the terms of trade as a function of domestic and foreign consumption, \( S_t = S(C_t, C_t^*) \). In turn, substituting into (28), output can be expressed as

\[
Y_t = Y(S(C_t, C_t^*), C_t^*) = \tilde{Y}(C_t, C_t^*).
\]

Therefore consumption is a function of output and foreign consumption:

\[
C_t = C(Y_t, C_t^*),
\]

and equivalently the terms of trade:

\[
S_t = S(C(Y_t, C_t^*), C_t^*) = \tilde{S}(Y_t, C_t^*).\]

Substituting for \( C_t \) and \( S_t \) into (29), and for \( N_t \) from (21), one can write the following implicit relation:

\[
E_{t-1} \left\{ \left[ \frac{Y_t}{\mu Y_{n,t}} \right] + \mu U_{n,t}(Y_t, A_t) \omega(Y_t, A_t) \right\} = 0
\]

In order for (31) to uniquely pin down the price level \( P_{H,t} \), one needs a specification of monetary policy. We can suppose that the level of nominal spending \( Y_t \equiv P_{H,t}Y_t \) is implicitly selected by a given path of the nominal interest rate, so that \( Y_t = \mathcal{Y}(R_t) \). Hence (31) can be rewritten:

\[
E_{t-1} \left\{ \left[ \frac{\mathcal{Y}(R_t)}{P_{H,t}} \frac{U_c(Y_t, C_t^*)}{g(Y_t, C_t^*)} + \mu U_{n,t}(Y_t, A_t) \omega(Y_t, A_t) \right] \right\} = 0
\]

In (32), for any exogenous process \( \{C_t^*, A_t\} \), the ex-post realization of the interest rate policy \( \{R_t\} \) (and therefore the ex-post realization of \( \mathcal{Y}_t \)) determines the price level \( P_{H,t} \). In turn, given \( \mathcal{Y}_t \) and \( P_{H,t} \), we can pin down \( Y_t \), all the remaining real variables, and the nominal exchange rate. Given \( P_{H,t}, S_t = \tilde{S}(Y_t, C_t^*) \) and therefore \( g(S_t) \), we can compute the CPI level from the definition \( P_t = P_{H,t} g(S_t) \). Given \( S_t, P_{H,t} \) and the foreign price level \( P_t^* \), we can finally derive the equilibrium nominal exchange rate as \( E_t = \frac{S_t P_{H,t}}{P_t^*} \).

3 Optimal Monetary Policy

Optimal policy is determined by a monetary authority that, under commitment, maximizes the discounted sum of utility of the representative agent under the constraints that characterize the competitive economy. As in the classical literature on optimal taxation (Chari, Christiano and Kehoe (1991)) or more recently in the monetary policy closed-economy analysis of Adao et al. (2003) and Khan et al. (2003), the policy problem takes the form of an
allocation problem, in which the government can be thought of choosing directly a feasible allocation subject to those constraints that summarize the competitive equilibrium.

3.1 The Efficient Allocation

We begin by characterizing the efficient allocation from the viewpoint of the small open economy’s social planner. For the sake of comparability, and in order to isolate the specific impact of openness and home bias on the nature of the optimal policy problem (see more below), it is convenient to restrict our attention to standard isoelastic preferences. Thus, in the following, we assume:

\[ U(C, N) = \frac{1}{1 - \sigma} C^{1-\sigma} - \frac{1}{1 + \zeta} N^{1+\zeta} \]  

which entails \(-\frac{U_{CC}C_t}{U_{c,t}} = \sigma\) and \(-\frac{U_{NN}N_t}{U_{n,t}} = \zeta\), where \(\zeta\) and \(\sigma\) are both constant. We use these assumptions to rewrite constraints (20) and (28), and define by \(\beta^t\varphi(h^t)\) and \(\beta^t\chi(h^t)\) the Lagrange multipliers on the feasibility constraint (28), and the risk-sharing constraint (20) respectively. Then the social planner’s problem can be described in terms of the following Lagrangian:

\[
\begin{align*}
\max_{\{C_t, S_t, N_t\}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{1 - \sigma} C_t^{1-\sigma} - \frac{1}{1 + \zeta} N_t^{1+\zeta} \right\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \varphi(h^t) \left( A_t F(N_t) - (1 - \alpha) g(S_t)^\eta C_t - \alpha S_t^\eta C_t^* \right) \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \chi(h^t) \left( C_t - q(S_t)^\frac{1}{2} C_t^* \right)
\end{align*}
\]

After substituting for \(C_t\) from constraint (20), first order conditions with respect to \(S_t\) and \(N_t\) yield (see Appendix A for more details):

\[ 1 - \alpha = \varphi(h^t) C_t^\sigma g(S_t) H(S_t) \]  

\[ -N_t^\zeta + \varphi(h^t) A_t F_{n,t} = 0 \]

where
Combining (35) and (36) yields the following expression for the real marginal cost (under our assumed preferences):

\[
H(S_t) = 1 - \alpha \left\{ 1 - \sigma \left[ \eta q(S_t)^{\eta - \frac{1}{2}} + (1 - \alpha) \left( \eta - \frac{1}{\sigma} \right) q(S_t)^{1-\eta} \right] \right\}
\]

Equation (37) shows that, in the efficient allocation, the marginal cost (inverse markup) must be time-varying. In fact, by resorting to variations in international relative prices (terms of trade and/or real exchange rate), the social planner can improve upon the flexible-price allocation, which instead requires a constant markup (see equation (30)). Put differently, variations in the degree of external competitiveness can affect the ratio between the marginal rate of substitution between consumption and labor and the marginal product of labor (with that ratio being instead constant and equal to 1 in a closed economy \((\alpha = 0)\)). The reason is related to the general nature of openness: variations in relative prices can affect consumption for any given level of output (and therefore labor effort). To better illustrate this point, one can combine the risk-sharing condition (20) with the resource constraint (28), obtaining:

\[
Y_t = C_t K(S_t)
\]

where

\[
K(S_t) = g(S_t)^{\eta} \left[ (1 - \alpha) + \alpha q(S_t)^{\eta - \frac{1}{2}} \right]
\]

can be defined as the open-economy output-consumption wedge. Thus, via the wedge \(K(S_t)\), and for any given level of domestic output, variations in both the terms of trade and the real exchange rate can affect the level of consumption. The fact that also variations in the real exchange rate impact on the output-consumption wedge is a specific implication of the presence of home bias: at the margin, a variation in the terms of trade produces also a variation in the real exchange rate via the risk-sharing condition (20), thereby altering domestic consumption for any given level of foreign consumption. This additional link between relative prices and consumption is absent in the baseline PPP case.

\footnote{Notice that the efficient allocation is well defined only for values of \(\alpha\) strictly < 1. The reason is simple. In the case \(\alpha = 1\) the consumption basket of the small economy would exactly coincide with the one of the rest of the world, and hence would be completely determined by world output (labor). In this case, the small economy’s social planner would optimally choose to set domestic labor equal to zero, which would also imply a zero optimal markup.}
3.2 Constrained-Efficient Allocation under Pre-Set Prices

Next we characterize the constrained-efficient (Ramsey) allocation under pre-set prices. We continue to assume isoelastic preferences. This is the assumption under which Adao et al. (2003) - in the context of a closed economy with pre-set prices - show that the constant-markup allocation is consistent with the constrained optimum.\(^\text{11}\)

In our cashless economy, the minimal set of constraints that are relevant for the Ramsey allocation problem are equations (20), (28), and (29). Let us additionally define by \(\beta^t \lambda(h^{t-1})\) the Lagrange multiplier on the price implementability constraint (29). Notice that \(\lambda(h^{t-1})\) depends on the history of events up to period \(t - 1\) and is therefore time-invariant as of time \(t\).

The constrained-efficient allocation for the small open economy’s planner can be characterized in terms of the following Lagrangian:

\[
\begin{align*}
\text{Max}_{(C_t, S_t, N_t)} & \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{1-\sigma} C_{t-1}^{1-\sigma} - \frac{1}{1+\zeta} N_{t+1}^{1+\zeta} \right\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \lambda(h^{t-1}) \left\{ \frac{C_{t-1}^{1-\sigma}}{g(S_t)} A_t F(N_t) - \mu N_t^\zeta \omega(N_t) \right\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \varphi(h^t) \left( A_t F(N_t) - (1-\alpha)g(S_t)^\eta C_t - \alpha S_t^\eta C_t^* \right) \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \chi(h^t) \left( C_t - q(S_t)^{1/2} C_t^* \right)
\end{align*}
\]

To simplify the analysis it is convenient to substitute for \(C_t\) from constraint (20). After defining \(\omega_{n,t} = \frac{\partial \omega(N_t)}{\partial N_t}\), first order conditions with respect to \(S_t\) and \(N_t\) can be written:

\[
1 - \alpha = \lambda(h^{t-1}) \sigma K(S_t)g(S_t)^{1-\eta} + \varphi(h^t)C_t^\sigma g(S_t)H(S_t) \tag{39}
\]

\[
-N_t^\zeta + \lambda(h^{t-1}) \left\{ \frac{C_{t-1}^{1-\sigma}}{S_t} A_t F_{n,t} - \mu N_t^\zeta \left( \frac{\omega(N_t)}{N_t} + \omega_{n,t} \right) \right\} + \varphi(h^t) A_t F_{n,t} = 0 \tag{40}
\]

\(^{11}\)Adao et al. (2003) emphasize, however, the non-generality of the constant markup result in the presence of variable expenditure components (such as government purchases) and/or some form of non-isoelastic preferences. Hence, in order to bias our results more in favor of markup stability, we have abstracted from the presence of government expenditure shocks.
Our goal is to establish under what conditions replicating a constant-markup allocation coincides with the constrained optimum. Recalling equation (30), this corresponds to determining whether the planner problem can sustain the term

\[
\Phi_t \equiv \frac{U_{n,t} g(S_t)}{A_{t} P_{n,t} U_{c,t}} = \frac{N_t^\xi C_t^\gamma g(S_t)}{A_{t} P_{n,t}}
\]
as a constant (across time and states).

Let us assume (without loss of generality) that the function \(F(\bullet)\) specifies to:

\[
F(N_t) = N_t^\xi \quad \xi \leq 1 \quad (41)
\]

which implies \(\omega(N_t) = \frac{N_t}{\xi}\) and \(\omega_{n,t} = \frac{1}{\xi}\).

Substituting for \(\varphi(h^t)\) from (40) into (39), and rearranging, one obtains (after some algebra):

\[
\Phi_t = \Lambda^{-1} \left\{ \lambda + \frac{(1 - \alpha) - \lambda \sigma \left( 1 - \alpha + \alpha q(S_t)^{\gamma - \frac{1}{\gamma}} \right)}{H(S_t)} \right\} \equiv \Phi_t^{ce} \quad (42)
\]

where \(\Phi_t^{ce}\) denotes the real marginal cost in the constrained-efficient allocation, \(\lambda\) is compact notation for \(\lambda(h^{t-1})\), and \(\Lambda \equiv 1 + \lambda \mu \left( \frac{1+\xi}{\xi} \right)\) is a time-invariant term (under our assumed preferences).

From (42) we infer that, in the constrained-efficient allocation, variations in international relative prices are the only source of variation in \(\Phi_t^{ce}\). Thus, in an open economy with preset prices, and even under isoelastic preferences, a constant mark-up is inconsistent with constrained efficiency, unless shocks are perfectly correlated across countries.

The intuition for this result lies in the key feature that differentiates the setting of monetary policy in an open economy from its closed economy counterpart: namely, and as already hinted above, the ability of affecting the level of consumption for any given level of output (labor effort). In the presence of sticky prices, this stems from monetary (exchange rate) policy exerting a leverage on the terms of trade. Hence the domestic policymaker, at the margin and relative to an allocation with constant markup, has an incentive to use the variability in the terms of trade to improve upon the flexible-price allocation.

The presence of home bias offers additional insights. In fact, a variation in the terms of trade produces also a variation in the real exchange rate via the risk-sharing condition, thereby affecting domestic consumption through a complementary channel. As we will demonstrate below, this additional real exchange rate channel entails that the conditions
under which markup stability is constrained-optimal are more restrictive. To better illustrate this point, we now discuss a series of particular cases nested by the general result in (42).

Closed Economy  In a closed economy $\alpha = 0$, and therefore $H(S_t) = 1$. This implies:

$$\Phi_t = \frac{1 + \lambda(1 - \sigma)}{\Lambda} = \Phi$$

for all $t$. Hence, in a closed economy, and under isoelastic preferences, a constant-markup policy always coincides with a constrained optimum. This result is consistent with the one in Adao et al. (2003), as well as with a broad class of contributions in the New Keynesian optimal monetary policy literature. The intuition is well understood. Under commitment, the monetary authority cannot neutralize the average distortion stemming from the presence of market power (a strategy that would involve trying to systematically raise output above its potential level), but can indeed succeed in placing the economy in the constrained-efficient allocation by neutralizing the distortion stemming from the inefficient adjustment of prices.

Open Economy: the Role of Home Bias  In an open economy ($\alpha > 0$), the recent literature has shown that replicating a constant-markup allocation can be constrained-efficient if either of two conditions holds: (i) the elasticity of substitution between domestic and foreign goods is unitary ($\eta = 1$), or (ii) $\eta = \sigma^{-1}$.

That literature, however, has almost invariably assumed absence of home bias (Benigno and Benigno (2003), Corsetti and Pesenti (2003)). Matters are different in the case of home bias. Consider case (i). With $\eta = 1$, the real marginal cost expression reads:

$$\Phi_t = \Lambda^{-1} \left\{ \lambda + \frac{(1 - \alpha) - \lambda \sigma \left(1 - \alpha + \alpha q(S_t)^{1 - \frac{1}{\sigma}}\right)}{(1 - \alpha) \left(1 + \alpha \sigma \left(1 - \frac{1}{\sigma}\right)\right) + \alpha \sigma q(S_t)^{1 - \frac{1}{\sigma}}} \right\}$$

Hence, with home bias, a unitary elasticity of substitution between domestic and imported goods ceases to be a sufficient condition for a constant-markup allocation to be constrained optimal. Intuitively, while a unitary elasticity neutralizes the terms-of-trade channel affecting the output-consumption wedge $K(S_t)$, it cannot neutralize the real exchange rate channel. Thus, an alternative interpretation of our results is that, in an open economy with home bias.

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12 See Woodford (2003), Clarida et al. (1999), Khan et al. (2003).
bias, the conditions under which markup stability emerges as constrained-optimal are more restrictive than the ones in the baseline PPP case.

This result differentiates our analysis from the one in Benigno and Benigno (2003) who assume PPP (as well as a linear-quadratic approach in the analysis of the optimal policy) and find that the assumption \( \eta = 1 \) is indeed a sufficient condition for markup stability to be constrained-optimal. In particular, with PPP, \( \eta = 1 \) is a knife-edge case in which the income effect of a variation in the terms of trade (which affects consumption for any given level of output) is exactly balanced by a corresponding substitution effect (which induces an expenditure switching between domestic and foreign goods).

From (42) it is clear that the only specification of preferences under which a constantmarkup policy is constrained-optimal is \( \eta = \sigma^{-1} \) (or, a fortiori, \( \eta = \sigma = 1 \)). In that case, in fact, we have \( H(S_t) = 1 \) (exactly like in a closed economy), which implies:

\[
\Phi_t^{ce} = \frac{(1 - \alpha) + \lambda \left( 1 - \frac{1}{\eta} \right)}{\Lambda} \equiv \Phi_t^{ce}_{\eta=\sigma^{-1}}
\]

Under this particular preference specification, there is no room for monetary policy to use relative prices to improve upon the flexible-price allocation, for the efficient allocation requires the markup to be time-invariant. From (37), in fact, we have \( \Phi^e_t = (1 - \alpha) \). This result is consistent with the one in Clarida et al. (2002) and Benigno and Benigno (2003), who show that, under the assumption \( \eta = \sigma^{-1} \), monetary policy is completely inward-looking: namely, it is concerned only with stabilizing the domestic marginal cost, in a way isomorphic to its closed-economy counterpart.

### 3.3 Gradual Price Adjustment

So far we have assumed that prices are set one period in advance. However, the most recent literature on the analysis of optimal policy typically embeds fully dynamic forms of price setting in the standard New Keynesian framework. We assume that changing output prices is subject to some cost. We follow Rotemberg (1982) and model the cost of adjusting prices for each domestic firm \( i \) equal to:

\[
\frac{\vartheta}{2} \left( \frac{P_{H,t}(i)}{P_{H,t-1}(i)} - 1 \right)^2
\]

17
where the parameter $\vartheta$ measures the degree of price stickiness. The higher $\vartheta$ the more sluggish is the adjustment of nominal prices. If $\vartheta = 0$, prices are flexible.

The cost of price adjustment renders the domestic producer’s pricing problem dynamic. Each producer chooses the price $P_{H,t}(i)$ of variety $i$ to maximize expected nominal discounted profits:

$$E_t \left\{ \sum_{t=0}^{\infty} \nu_{0,t} \left[ P_{H,t}(i) Y_t(i) - W_t N_t(i) - \frac{\vartheta}{2} \left( \frac{P_{H,t}(i)}{P_{H,t-1}(i)} - 1 \right)^2 P_{H,t} \right] \right\}$$

subject to (21) and (23). In (47), $\nu_{0,t}$ is the time-zero price of one unit of domestic currency to be delivered in time $t$.

The first order condition of the above problem reads:

$$\nu_{0,t} \left\{ (1 - \varepsilon) \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} Y_t + \varepsilon \frac{W_t}{P_{H,t}} h_{p,t} \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon - 1} Y_t \right\}$$

$$= \nu_{0,t} P_{H,t} \vartheta \left( \frac{P_{H,t}(i)}{P_{H,t-1}(i)} - 1 \right) \frac{1}{P_{H,t-1}(i)} - E_t \left\{ \nu_{0,t+1} P_{H,t+1} \vartheta \left( \frac{P_{H,t+1}(i)}{P_{H,t}(i)} - 1 \right) \frac{P_{H,t+1}(i)}{P_{H,t}(i)^2} \right\}$$

where, again, $h_{p,t} \equiv \frac{\partial h(.)}{\partial P_{H,t}(i)}$. Dividing all terms by $\nu_{0,t}$ and imposing a symmetric equilibrium (which implies $P_{H,t}(i) = P_{H,t}$ for all $i$ and $t$) we can rewrite:

$$\pi_{H,t}(\pi_{H,t} - 1) = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{g(S_t)}{g(S_{t+1})} \pi_{H,t+1}(\pi_{H,t+1} - 1) \right\}$$

$$+ \varepsilon Y_t \frac{W_t}{\vartheta} \frac{g(S_t)}{A_t F_{n,t}} - \varepsilon - 1 \right)$$

where $\pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$, and where we have used the fact that, from (10), $\frac{\nu_{0,t+1}}{\nu_{0,t}} = \frac{U_{c,t+1}}{U_{c,t}} \frac{g(S_t)}{g(S_{t+1})}$. The above equation has the form of a non-linear forward-looking New-Keynesian Phillips curve. Notice that the openness dimension affects the form of the Phillips curve via movements in the terms of trade. The latter affect both the form of the stochastic discount factor $\frac{U_{c,t+1}}{U_{c,t}} \frac{g(S_t)}{g(S_{t+1})}$ and the marginal cost expression $\frac{W_t}{A_t F_{n,t}}$.

Substituting (9), and the symmetric equilibrium version of (21), which implies $A_t F(N_t(i)) = A_t F(N_t)$ for all $i$, we can write (49) in terms of real allocations only:

---

For a log-linear Phillips curve derived in the context of the so called Calvo-Yun model, see Woodford (2003a) and Gali and Gertler (1999).
\[
\pi_{H,t} \left( \pi_{H,t} - 1 \right) = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} g(S_t) \pi_{H,t+1} \left( \pi_{H,t+1} - 1 \right) \right\} + \frac{\varepsilon}{\theta} \frac{A_t F(N_t)}{U_{c,t} A_t F_{n,t}} g(S_t) - \frac{\varepsilon - 1}{\varepsilon} \tag{50}
\]

Equation (50) is a modified Phillips curve equation suitable for the policy allocation problem to be analyzed below. Notice that, under this form of price setting, it is unfeasible to eliminate the condition for the evolution of inflation from the minimal set of conditions that summarize the competitive equilibrium.

To complete the set of restrictions that will be relevant for the optimal policy problem, notice that the resource constraint will now comprise a price adjustment cost factor, and therefore will read:

\[
Y_t = (1 - \alpha) g(S_t) C_t + \alpha S_t^{\theta} C_t^* + \frac{\theta}{2} (\pi_{H,t} - 1)^2 \tag{51}
\]

### 3.3.1 Equilibrium with Gradual Price Adjustment

We can now describe the competitive equilibrium in the economy with quadratic costs of changing prices. For any given policy sequence \( \{R_t\} \) and exogenous processes \( \{C_t, A_t\} \), a recursive competitive equilibrium with quadratic costs of changing prices is a sequence \( \{C_t, S_t, N_t, \pi_{H,t}\} \) solving (12), (20), (50) and (51).

### 3.4 Optimal Monetary Policy with Gradual Price Adjustment

The presence of the forward-looking pricing condition (50) alters the form of the policy problem in a fundamental way. Once again, we assume that planner in the small economy can resort to commitment, and that the preference specification is of the form (33).

Let us define by \( \{\lambda_{p,t}, \lambda_{f,t}, \lambda_{r,t}\}_{t=0}^\infty \) a sequence of Lagrange multipliers on constraints (50), (51) and (20) respectively. The planner’s problem can then be characterized as follows:

\[
Max \{C_t, N_t, S_t, \pi_{H,t}\} \quad E_0 \left\{ \sum_{t=0}^\infty \beta^t \left\{ \frac{1}{1 - \sigma} C_t^{1 - \sigma} - \frac{1}{1 + \zeta} N_t^{1 - \zeta} \right\} \right\} \tag{52}
\]
As a result of the constraint (50) exhibiting future expectations of control variables, the maximization problem in (52) is intrinsically non-recursive. As first emphasized in Kydland and Prescott (1980), and then developed in Marcet and Marimon (1999), a formal way to rewrite the same problem in a recursive stationary form is to enlarge the planner’s state space with additional (pseudo) costate variables. In our particular case, the enlarged state space is composed by the vector \((A_t, Z_t)\), where \(Z_t\) is a new costate variable with law of motion \(Z_{t+1} = p_t\). The costate \(Z_t\) bears the crucial meaning of tracking, along the dynamics, the value to the planner of committing to the pre-announced policy plan.

For any given processes \(\{A_t, C_t^*\}\), first order efficiency conditions with respect to \(\pi_{H,t}\), \(C_t, S_t, N_t\) for \(t > 0\) read:

\[
C_t^{-\sigma} \frac{g(S_t)}{\pi_{H,t} - 1} (\lambda_{p,t} - \lambda_{p,t-1}) = \lambda_{f,t} \vartheta (\pi_{H,t} - 1) \quad (53)
\]

\[
0 = C_t^{-\sigma} \frac{\pi_{H,t} - 1}{g(S_t)} C_t^{-\sigma - 1} (\lambda_{p,t} - \lambda_{p,t-1})
= -\sigma \lambda_{p,t} \left( \frac{\vartheta - 1}{\vartheta} \right) A_t F(N_t) \frac{g(S_t)}{g(S_t)} C_t^{-\sigma - 1} - \lambda_{f,t} \left( 1 - \alpha \right) g(S_t)^{\eta} - \sigma \lambda_{r,t} C_t^{-\sigma - 1} q(S_t) \quad (54)
\]

\[
C_t^{-\sigma} \left( -g(S_t)^{\eta - 2} g_{s,t} \right) \left[ (\pi_{H,t} - 1) \pi_{H,t} (\lambda_{p,t} - \lambda_{p,t-1}) + \lambda_{p,t} \left( \frac{\vartheta - 1}{\vartheta} \right) A_t F(N_t) \right] = \lambda_{f,t} \left[ (1 - \alpha) \eta g(S_t)^{\eta - 1} g_{s,t} C_t + \eta S_t^{\eta - 1} C_t^* \right] + \sigma \lambda_{r,t} C_t^{-\sigma} q_{s,t} \quad (55)
\]

\[15\] See the Appendix for a derivation of the recursive problem. If one or more constraints featured expectations extending more than one period in the future, the set of costate variables would be enlarged accordingly (see Marcet and Marimon (1999)).
\[ 1 + \lambda_{p,t} \left\{ \frac{\varepsilon}{\vartheta} \left( \frac{\omega(N_t)}{N_t} + \omega_{n,t} \right) + (\varepsilon - 1) \frac{A_t F(N_t)}{N_t} \frac{C_t^{-\sigma}}{g(S_t)} \right\} + \lambda_{f,t} \frac{A_t F_{n,t}}{N_t^\kappa} = 0 \]  

(56)

where \( g_{s,t} = \frac{\partial g(S_t)}{\partial S_t} \), \( q_{s,t} = \frac{\partial q(S_t)}{\partial S_t} \), and we recall that \( \omega(N_t) \equiv \frac{F(N_t)}{F_{n,t}} \).

The system (53)-(56) is recursive in the state space \((A_t, Z_t)\) for \( t > 0 \). As in Khan et al. (2003), to avoid a typical non-recursivity problem at time \( t = 0 \), we assume that the initial value of the multiplier \( \lambda_{p,-1} \) is set at the steady-state value implicit in the system (53)-(56). We refer to the steady-state version of equations (53)-(56) as the deterministic Ramsey steady state. We proceed to analyze its properties below.

### 3.4.1 Ramsey Steady State

To determine the long-run inflation rate associated to the optimal policy problem above, one needs to solve the steady-state version of the set of efficiency conditions (53)-(56).\(^{16}\) In that steady-state, we have \( \lambda_{p,t} = \lambda_{p,t-1} \). Hence condition (53) immediately implies:

\[ \lambda_f \vartheta (\pi_H - 1) = 0 \]  

(57)

Since \( \lambda_f > 0 \) (the resource constraint must hold with equality) and \( \vartheta > 0 \) (we are not imposing a priori that the steady state coincides with the flexible price allocation), in turn (57) must imply \( \pi_H = 1 \). Hence the Ramsey planner would like to generate an average (net) inflation rate of zero. The intuition for why the long-run optimal inflation rate is zero is simple. Under commitment, the planner cannot resort to ex-post inflation as a device for eliminating the inefficiency related to market power in the goods market. Hence the planner aims at choosing that rate of inflation that minimizes the cost of adjusting prices and is summarized by the quadratic term \( \frac{\vartheta}{2} (\pi_H - 1)^2 \).

One may wonder why the openness dimension does not apparently exert any influence on the desired optimal long-run inflation rate. In light of our analysis above, the desire of adjusting the terms of trade and/or the real exchange rate (under home bias) has been

\(^{16}\)To develop an analogy with the Ramsey-Cass-Koopmans model, this amounts to computing the modified golden rule steady state. This per se contrasts with the golden rule inflation rate, which would correspond to the one that maximizes households’ instantaneous utility under the requirement that the planner is constrained to choose only among constant allocations. In dynamic economies with discounted utility the two concepts of long-run optimal policy do not coincide. See King and Wolman (1999) and Khan et al. (2003) for a closed-economy analysis on this point. See Faia and Monacelli (2004) for additional discussion in the context of a two-country model.
shown to be a sufficient motive for inducing the planner to deviate from choosing a constant markup allocation. However, these considerations can drive the planner’s behavior only in the presence of equilibrium fluctuations (as induced by country-specific shocks) around the same long-run steady state. It is only in the presence of such shocks that variations in (international) relative prices are efficiently calibrated to implement the optimal allocation. In other words, under commitment, the planner cannot on average resort to movements in inflation to alter the relative purchasing power of domestic residents. Thus, under commitment, the desire to influence the terms of trade and/or the real exchange rate shapes the optimal policy behavior only outside the long-run steady state.

3.5 Dynamics under the Optimal Policy and the Effect of Home Bias

In this section we study the equilibrium dynamics under the optimal policy in response to productivity shocks. In particular, our goal is to assess the extent to which home bias affects the optimal volatility of inflation. In conducting our analysis we specialize the production technology to be $Y_t = A_t N_t$. The time unit is meant to be quarters. The discount factor $\beta$ is equal to 0.99. The degree of risk aversion $\sigma$ is 1 (which implies log-utility), the inverse elasticity of labor supply $\zeta$ is equal to 3, which is a common value in the real business cycle literature. The literature is largely polarized on the likely value of the elasticity $\eta$. Obstfeld and Rogoff (2000) summarize the related micro empirical-trade literature, which suggests values in the range [8, 10] (see also Anderson and Van Wincoop (2004)). The current New Open Macroeconomics literature usually adopts much lower values, in the range [1, 2]. However, there seems to be some consensus that the value of $\eta$ lies above unity. A recent series of studies employing Bayesian estimation of fully structural DSGE open macro models seems to support a range for $\eta$ between 1.5 and 2: see, for instance, Justiniano and Preston (2006), De Walque, Smets and Wouters (2005), Rabanal and Tuesta (2005). Importantly, many normative results in the literature hinge on the assumed value of this parameter.\(^{17}\)

In order to parameterize the degree of price stickiness, we observe that, by log-linearizing equation (50) around a zero-inflation steady-state, we can obtain an elasticity of inflation to the real marginal cost (normalized by the steady-state level of output)\(^{18}\) that takes the

\(^{17}\)See Pappa (2004), Sutherland (2002), Benigno and Benigno (2003).

\(^{18}\)To produce a slope coefficient directly comparable to the empirical literature on the New Keynesian Phillips curve this elasticity needs to be normalized by the level of output when the price adjustment cost
form $\frac{1}{1-\delta}$. This permits a direct comparison with empirical studies on the New Keynesian Phillips curve such as Galí and Gertler (1999) and Sbordone (2002) using a Calvo approach. In those studies, the slope coefficient of the log-linear Phillips curve can be expressed as $\frac{(1-\delta)(1-\beta\delta)}{\delta}$, where $\delta$ is the probability of not resetting the price in any given period. For any given value of $\varepsilon$, which entails a choice on the steady-state level of the markup, we can thus build a mapping between the frequency of price adjustment in the Calvo model $\frac{1}{1-\delta}$ and the degree of price stickiness $\vartheta$ in the Rotemberg setup. Traditionally, the sticky price literature has been considering a frequency of four quarters as a realistic value. Recently, Bils and Klenow (2004) argue that the observed frequency of price adjustment is much higher in the U.S., and in the order of two quarters. In their comprehensive study on Europe (which includes small open economies such as Belgium and Spain), Angeloni et al. (2005) find evidence of lower frequency of price adjustment, and in the order of four quarters. Hence we parameterize $\frac{1}{1-\delta} = 4$, which implies $\delta = 0.75$. Setting the elasticity $\varepsilon$ equal to 7.5, which implies a steady-state markup of 15 percent, the resulting stickiness parameter satisfies $\vartheta = \frac{\delta(\varepsilon-1)}{(1-\delta)(1-\beta\delta)} = 75$.

As a benchmark, we set the share of foreign imported goods in the domestic consumption basket (degree of openness) to a value of 0.4. However, we will conduct a series of sensitivity experiments on the value of this parameter. Finally (log) productivity is assumed to follow an autoregressive process:

$$\log(A_t) = \rho^a \log(A_{t-1}) + \varepsilon_t^a$$

Following King and Rebelo (1999), we set $\rho^a = 0.95$ and the volatility of the iid component $\varepsilon_t^a$ equal to $\sigma^\varepsilon = 0.0056$.

Our solution strategy consists in deriving a log-linear approximation of the Ramsey equilibrium conditions (53)-(56) around the deterministic Ramsey steady state.

### 3.5.1 Responses to Productivity Shocks and the Effect of Varying Openness

*Figure 1* depicts impulse responses of the domestic price level, the nominal exchange rate, the CPI level and the real exchange rate to a one percent rise in *Home productivity* under the Ramsey policy. All responses are compared for alternative values of the elasticity of substitution $\eta$. Furthermore, monetary policy in the rest of the world is assumed to be factor is not explicitly proportional to output, as assumed here.
conducted in terms of strict inflation targeting, so that $\pi_{F,t}^* = 1$ for all $t$.

The figure shows, as expected, that only in the particular case of $\eta = 1$ stability of the producer price level is part of the optimal response to the shock (recall that we have assumed $\sigma = 1$). In response to higher productivity, the equilibrium adjustment requires an increase in the demand of domestic goods relative to foreign goods (to match the initial rise in supply). This is achieved by means of a terms of trade and real exchange rate depreciation (as well as via a depreciation of the nominal exchange rate). Notice that the response of the nominal exchange rate (and in turn of the real exchange rate) is magnified for lower values of $\eta$. In fact, the lower the elasticity of substitution between goods, the larger the nominal exchange rate adjustment (depreciation in this case) necessary to bring about the necessary expenditure switching from foreign to domestic goods. Notice also that the price level is stationary under the Ramsey allocation. This is reminiscent of the history dependence feature of optimal policy emphasized in the same recent closed economy literature.\textsuperscript{19} In turn, stationarity of the price level, coupled with stationarity of the terms of trade (which is a feature of this economy under complete markets), generates the mean reverting behavior of the nominal exchange rate. Finally, the CPI level rises in response to the shock (with the effect being inversely proportional to $\eta$), due to the CPI being a convex combination of the response of the domestic producer price and of the nominal exchange rate.

Figure 2 displays impulse responses to the same domestic productivity rise for alternative values of openness (inverse degree of home bias). Intuitively, the size of the response of the real exchange rate is decreasing in $\alpha$, for the limit case of $\alpha \to 1$ corresponds to the one in which PPP holds. As a result, the required nominal depreciation is also decreasing in $\alpha$, suggesting a role of home bias in enhancing exchange rate volatility under the optimal policy.

Figure 3 displays the effects of varying openness (inverse degree of home bias) on the volatility of inflation, the terms of trade and the real exchange rate under the optimal policy. All values are expressed in percent terms. For this simulation exercise we extend the set of shocks to include a foreign output shock. Hence we assume that $Y_t^* = C_t^* = \rho^* Y_{t-1}^* + \varepsilon_t^*$, and estimate an AR(1) process for HP-filtered U.S. (log) output over the period 1956:1-2005:4, obtaining $\rho^* = 0.85$ and $\sigma_{t}^* = 0.0083$.

Notice, first, that optimal (domestic) inflation volatility is U-shaped in the degree of

\textsuperscript{19}See Woodford (2003b).
trade openness. The largest inflation volatility is obtained for intermediate values of openness. The intuition for this result is simple. Recall that, in our framework, the limit case of absence of home bias corresponds to \( \alpha \rightarrow 1 \). This is a limit case in the sense that, for \( \alpha \) approaching 1, the consumption basket of the small economy tends to coincide with the one of the rest of world (which per se corresponds to an approximately closed economy). Thus, from the point of view of the optimal markup policy, the two limit cases of \( \alpha \rightarrow 0 \) (no trade openness) and \( \alpha \rightarrow 1 \) (PPP, or absence of home bias) tend to mimic the situation of a closed economy. In that particular case, as already discussed above, a large (and related) closed economy literature has pointed out that the optimal policy prescription coincides with strict price stabilization.

As expected, the optimal volatility of the real exchange rate is decreasing in \( \alpha \), for the limit case of \( \alpha \rightarrow 1 \) corresponds to the one in which PPP holds. As openness increases, the optimal policy prescribes also enhanced smoothing of the nominal exchange rate. In fact, since higher values of \( \alpha \) correspond to smaller degrees of home bias, the real exchange motive for nominal exchange rate adjustment is dampened relative to the necessity of inducing an adjustment in the terms of trade. However, this effect does not vanish when the environment approaches the PPP case. In the PPP case, in fact, the equilibrium adjustment still requires a depreciation of the terms of trade.

Notice that optimal inflation volatility remains, at the peak, quite low. However, for a sufficiently high degree of home bias, the volatility of the real exchange rate becomes sizeable and is above 4%. In a sample of industrialized small open economies, the average volatility of the real exchange rate in the post-Bretton-Woods era is about 4.86% (Monacelli (2004)). Thus we conclude that, under the optimal policy, high home bias can be a potentially vigorous source of nominal and real exchange rate volatility.

4 Conclusions

An important strand of the recent open-economy New Keynesian literature has focused on the issue of optimal monetary and exchange rate policy. However, most contributions have remained largely disconnected from the traditional Ramsey-type approach that has characterized the optimal monetary and fiscal policy literature of closed economy (typically flexible-price) models.

This paper studies optimal monetary policy in a small open economy with nominal
rigidities and home bias in trade. Specific to our approach is a Ramsey-type analysis of the optimal policy problem. In this context, home bias in consumption emerges as an independent factor contributing to deviations from the typical closed-economy paradigm of strict markup stabilization. In this respect, and given that home bias is a prominent feature of international trade data, the nature of optimal monetary policy in an open economy emerges as fundamentally different from the one of a closed economy.

Our analysis lends itself to several possible extensions. First, and within the same Ramsey-type approach, one may explore the role of alternative sources of real exchange rate volatility, such as deviations from the law of one price induced either by stickiness in import prices or by the presence of distributions costs (Burstein et al. (2003), Corsetti and Dedola (2004)). Second, one may observe that home bias is a fundamental feature of international trade data not only in consumption but also in equities (Engel and Matsumoto (2005)). The extension of our setup to the analysis of optimal exchange rate policy with a simultaneous presence of home bias in goods and equities is an interesting avenue for future research.
A The Social Planner Problem

In this Appendix we derive in more detail the social planner problem that leads to condition (37) and to the expression for $H(S_t)$. Let $q_t$ and $g_t$ be compact notation for $q(S_t)$ and $g(S_t)$ respectively. After substituting (20) into (28), the Lagrangian can be specified as follows:

$$
Max_{C_t, s_t, N_t} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{1-\sigma} \left( \frac{S_t}{g(S_t)} \right)^{\frac{1-\sigma}{\sigma}} (C_t^*)^{1-\sigma} - \frac{1}{1+\zeta} N_t^{1+\zeta} \right\} + E_0 \sum_{t=0}^{\infty} \beta^t \varphi(h^t) \left( A_t F(N_t) - (1-\alpha)g_t^\eta \left( \frac{S_t}{g_t} \right)^{\frac{1}{\eta}} C_t^* - \alpha S_t^\eta C_t^* \right)
$$

The first order condition with respect to $S_t$ reads:

$$
0 = C_t^{-\sigma} \frac{1}{\sigma} \left( \frac{S_t}{g_t} \right)^{\frac{1}{\eta}-1} q_{s,t} - \varphi(h^t) (1-\alpha) \left[ \left( \eta - \frac{1}{\sigma} \right) g_t^{-\frac{1}{\eta}-1} q_{s,t} S_t^\eta + \frac{1}{\sigma} S_t^{\frac{1}{\eta}-1} S_t^\eta - \frac{1}{\eta} S_t^\eta \right] (59)
$$

The first order condition with respect to $N_t$ reads:

$$
-N_t^\zeta + \varphi(h^t) A_t F_{n,t} = 0 (60)
$$

It is useful to notice that:

$$
q_{s,t} = (1-\alpha) \left( \frac{S_t}{g_t} \right)^{-2} (61)
$$

and that:

$$
g_{s,t} = \alpha q_t^{-\eta} (62)
$$

Combining (59) and (60), and using (61) and (62), we can write:

$$
\frac{1}{\sigma} q_t^{\frac{1}{\eta}-1} q_{s,t} = \frac{N_t^\zeta C_t^\sigma}{A_t F_{n,t}} D(S_t) (63)
$$

where

$$
D(S_t) \equiv (1-\alpha) \left( \eta - \frac{1}{\sigma} \right) g_t^{-\frac{1}{\eta}-1} \alpha \left( \frac{g_t}{S_t} \right)^{\eta} S_t^{\frac{1}{\eta}} + \frac{1}{\sigma} S_t^{\frac{1}{\eta}-1} g_t^{-\frac{1}{2}} S_t^\eta + \alpha \eta S_t^{\eta-1}
$$
Notice that we can rewrite
\[
D(S_t) \equiv (1 - \alpha) g_t^{\eta - 1} \left[ \left( \eta - \frac{1}{\sigma} \right) \alpha q_t^{\frac{1}{\sigma} - \eta} + \frac{1}{\sigma} \right] + \alpha \eta S_t^{\eta - 1}
\]
Using (61) and (62), equation (63) reads:
\[
\frac{1 - \alpha}{\sigma} q_t^{\frac{1}{\sigma} - \eta} S_t^{1 - \eta} = \Phi_t D(S_t)
\]
Notice that:
\[
q_t^{\eta - \frac{1}{\sigma}} D(S_t) = \alpha q_t^{\eta - \frac{1}{\sigma}} + \alpha (1 - \alpha) q_t^{1 - \eta} \left( \eta - \frac{1}{\sigma} \right) + \frac{1 - \alpha}{\sigma}
\]
Hence we can finally write (64) as:
\[
\Phi_t = \frac{1 - \alpha}{H(S_t)}
\]
where
\[
H(S_t) \equiv 1 - \alpha \left\{ 1 - \sigma \left[ \eta q_t^{\eta - \frac{1}{\sigma}} + (1 - \alpha) \left( \eta - \frac{1}{\sigma} \right) q_t^{1 - \eta} \right] \right\}
\]
which is the expression used in the text. A conceptually similar derivation applies in the case of the Ramsey problem.

B The Recursive Lagrangian Problem

Let us define the following policy functional:
\[
\mathcal{W}(C_t, N_t, Z_t, \pi_{\text{H},t}, S_t) \equiv U(C_t, N_t) - Z_t \left( \frac{U_{c,t}}{g(S_t)} \pi_{\text{H},t} (\pi_{\text{H},t} - 1) \right)
\]
where $Z_t$ is a new costate variable. We can then write the optimal policy plan in the following form:
\[
\text{Max}_{\{C_t, \pi_{\text{H},t}, N_t, S_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \mathcal{W}(C_t, N_t, Z_t)
\]
\[
+ E_0 \sum_{t=0}^{\infty} \beta^t \lambda_{p,t} \left[ \frac{U_{c,t}}{g(S_t)} \left( \pi_{H,t} (\pi_{H,t} - 1) - \frac{\varepsilon}{\vartheta} \frac{A_t F(N_t)}{U_{c,t}} \frac{g(S_t) - (\varepsilon - 1)}{\vartheta} \right) \right] \\
+ E_0 \sum_{t=0}^{\infty} \beta^t \lambda_{f,t} \left[ A_t F(N_t) - (1 - \alpha) g(S_t)^\vartheta C_t - \alpha S_t^\vartheta C_t^* - \frac{\vartheta}{2} (\pi_{H,t} - 1)^2 \right] \\
+ E_0 \sum_{t=0}^{\infty} \beta^t \lambda_{r,t} \left( U_{c,t} q(S_t) - U_{c,t}^* \right)
\]

with law of motion for the new costate:

\[ Z_{t+1} = \lambda_{p,t} \]

Following Marcet and Marimon (1999), one can show that this maximization program is saddle-point stationary in the amplified state space \( \{ A_t, Z_t \} \). Our strategy consists in setting the initial value \( Z_0 \) as:

\[ Z_0 = \bar{Z} \]

where \( \bar{Z} \) is the value of \( Z_t \) in the deterministic Ramsey steady state.
References


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Figure 1: Impulse Responses to a Rise in Home Productivity: Effect of Varying the Elasticity of Substitution $\eta$ ($\sigma = 1$).
Figure 2: Impulse Responses to a Rise in Home Productivity: Effect of Varying Openness $\alpha$ (Inverse Degree of Home Bias).
Figure 3: Volatility under the Ramsey Policy: Effect of Varying Openness $\alpha$ (Inverse Degree of Home Bias).