Monetary Policy as an Optimum Currency Area Criterion

Dominik Groll

Abstract: Whether countries benefit from forming a monetary union depends critically on the way monetary policy is conducted. This is mainly because monetary policy determines whether and to what extent a flexible nominal exchange rate fosters or hampers macroeconomic stabilization, even if monetary policy does not target the nominal exchange rate explicitly.

Keywords: Monetary union; macroeconomic stabilization; welfare analysis; optimum currency area theory; trade openness.

JEL classification: F33, F41, E52.

Dominik Groll
Kiel Institute for the World Economy
24100 Kiel, Germany
Telephone: +49 431 8814 266
E-mail: dominik.groll@ifw-kiel.de
Whether countries benefit from forming a monetary union depends critically on the way monetary policy is conducted. This is mainly because monetary policy determines whether and to what extent a flexible nominal exchange rate fosters or hampers macroeconomic stabilization, even if monetary policy does not target the nominal exchange rate explicitly.

Keywords: Monetary union; macroeconomic stabilization; welfare analysis; optimum currency area theory; trade openness.

JEL classification: F33, F41, E52.

* E-mail: dominik.groll@ifw-kiel.de. Phone: +49 431 8814 266.
1 Introduction

Over the decades since its initiation by Mundell (1961), the Optimum Currency Area (OCA) literature has identified numerous criteria that are considered important in determining whether countries benefit from forming a monetary union. These OCA criteria typically comprise the economic characteristics of the countries involved, such as the degree of labor mobility, the degree of price and wage flexibility, or the degree of trade openness. \[^1\]

In this study, I argue that one OCA criterion has not yet been emphasized sufficiently, although it is absolutely critical for the welfare implications of monetary unification: the conduct of monetary policy. In particular, using a standard two-country New Keynesian dynamic stochastic general equilibrium (DSGE) model, I systematically demonstrate how and why different types of rule-based interest rate policies lead to different welfare rankings between a monetary union and a flexible exchange rate regime. These interest rate policies differ in terms of both the choice of the target variables and the strength of the response to these variables. Both dimensions can be decisive in determining whether countries benefit from forming a monetary union. \[^2\]

When monetary policy in each country responds to inflation aggressively or implements a high degree of interest rate smoothing, forming a monetary union, where the common monetary policy continues to follow the same interest rate policy, tends to make countries worse off in terms of welfare by decreasing macroeconomic stability. However, when monetary policy responds to inflation only modestly or implements a low degree of interest rate smoothing, forming a monetary union tends to make countries better off by increasing macroeconomic stability. When monetary policy responds to output, forming a monetary union is almost always beneficial.

The conduct of monetary policy is not only an independent OCA criterion in itself, but it can also modify the nature of other OCA criteria, such as the degree of trade openness. Whether an increase in the degree of trade openness increases the likelihood of a monetary union being beneficial, as proposed by the vast bulk of previous OCA studies, depends critically on whether monetary policy targets producer price inflation or consumer price inflation. If monetary policy targets producer price inflation, it is also possible that an increase in the degree of trade openness decreases the likelihood of a monetary union being beneficial.

The most important reason why monetary policy is crucial for the welfare implications of monetary unification is as follows. Monetary policy determines whether and to what extent a flexible nominal exchange rate fosters or hampers macroeconomic stabilization, even if monetary policy does not target the nominal exchange rate explicitly. The flexibility of the nominal exchange rate renders monetary policy more potent under a flexible exchange rate regime in the sense that monetary policy affects all welfare-relevant variables directly. By contrast, in a monetary union, the influence of monetary policy is limited, especially with respect to relative prices such as the terms of trade and the real exchange rate.

---

1 Summarizing the vast OCA literature is beyond the scope of this study. Excellent surveys are Mongelli (2002), Dellas and Tavlas (2009), Beetsma and Giuliodori (2010), and De Grauwe (2012).

2 The present study builds on my previous research (Groll, 2013) and elaborates on the importance of monetary policy for the welfare implications of monetary unification. Also, the present study employs a more general model that allows, among other things, for trade imbalances, deviations from purchasing power parity (i.e., real exchange rate fluctuations), and more variation in the conduct of monetary policy.
The fact that monetary policy is more potent under a flexible exchange rate regime is a double-edged sword. When the interest rate policy is specified such that the nominal exchange rate moves in the “right” direction (which will be defined precisely), forming a monetary union generally reduces macroeconomic stability and thus welfare, because it eliminates the stabilizing effects of the nominal exchange rate. This is especially true if monetary policy can implement the optimal (i.e., welfare-maximizing) policy, as is assumed frequently in the literature.

However, the information that is required to implement such an optimal policy is usually not available in practice. Even the specification of a simple interest rate rule that is expected to move the nominal exchange rate in the right direction is not always obvious ex ante. For example, an interest rate policy that induces the nominal exchange rate to move in the right direction in a rather closed economy can induce it to move in the wrong direction in a very open economy.

When the interest rate policy is specified such that the nominal exchange rate moves in the wrong direction, forming a monetary union increases macroeconomic stability and thus welfare, because it eliminates the destabilizing effects of the nominal exchange rate. Essentially, the nominal exchange rate does not compensate for monetary policy mistakes, but instead it reinforces them. In this sense, a monetary union provides a protective mechanism against monetary policy mistakes.

Certain interest rate policies render a monetary union beneficial, although they push the nominal exchange rate in the right direction in a flexible exchange rate regime. In these cases, countries benefit from monetary unification due to higher inflation stability. One example is a very modest response to inflation. In practice, there are many temptations to implement such a lax policy. In times with a (private or public) debt overhang, monetary policy might let the inflation rate overshoot the inflation target for a prolonged period of time, with the aim of reducing the real debt burden and lowering borrowing costs. In times of high unemployment, allowing inflation to temporarily overshoot the target might also seem attractive, with the aim of reducing real wages in the presence of fixed-term nominal wage contracts, thereby increasing the demand for labor.

These temptations lead to a related problem, which is also less severe in a monetary union. A country that is unable to credibly commit itself to withstand these types of temptations suffers from a persistently high level of inflation. If it forms a monetary union with a country that does not suffer from such commitment problems, it benefits from a lower level of inflation by eliminating the so-called inflation bias (Alesina and Barro, 2002).

However, this point differs from the point made in the present study in at least two important respects. First, the benefit of eliminating an inflation bias is related to the level of inflation, whereas the benefit described in the present study is related to the stability of inflation. Second, the benefit of eliminating an inflation bias is not inherent to a monetary union because it is only obtained if the monetary policy after monetary unification is more credible than that before monetary unification. By contrast, the

---

3 That is, the inflation coefficient in the interest rate rule is above but close to one.

4 Note that the “currency union” in Alesina and Barro (2002) refers to a situation where a client country unilaterally adopts the currency of an anchor country—a situation also known as dollarization. Nevertheless, the benefit of eliminating an inflation bias also exists if the client and anchor country form a monetary union where the common monetary policy inherits the credibility of the anchor country. In Giavazzi and Pagano (1988), the “advantage of tying one’s hands” follows the same logic, although they refer to the former European Monetary System (1979–1999).
benefit described in the present study is inherent to a monetary union because it is obtained even if the monetary policy after monetary unification continues to make the same mistake it made before monetary unification. This is because the same policy mistake is less harmful in a monetary union.

One reason for the inability to withstand the temptations mentioned above is a lack of “discipline and institutions that can provide a firm domestic commitment to a monetary policy that is dedicated to price stability” (Alesina and Barro 2001, p. 382). It seems very likely that such weak institutions not only lead to a high level of inflation, but also to a poor stabilization of inflation. If this is the case, the trade-off that countries face when forming a monetary union between relinquishing national monetary policy as a macroeconomic stabilization device and gaining credibility to reduce undesired inflation, as stressed by Alesina and Barro (2002), does not exist.

The remainder of this paper is organized as follows. Section 2 briefly outlines the structure of the model. Section 3 shows how different interest rate policies lead to different welfare rankings between a monetary union and a flexible exchange rate regime. Section 4 shows how different interest rate policies change the nature of the traditional OCA criterion of “trade openness.” Section 5 concludes this study.

2 Model

The model I use is a standard two-country New Keynesian DSGE model, and thus I provide only a very brief description. The model features two international monetary regimes:

1. A monetary union (MU) regime: Both countries share the same currency. A common monetary policy governs the common nominal interest rate.

2. A flexible exchange rate (FX) regime: Each country maintains its national currency and conducts its own, independent monetary policy. Nominal interest rates are country-specific. The nominal exchange rate between the two currencies is flexible.

The FX version of the model, including the microfounded, quadratic welfare measure, is described in Corsetti, Dedola, and Leduc (2011). The MU version of the model is largely identical (see, e.g., Benigno, 2004). The model economy features two countries of equal size (labeled $H$ and $F$), where there is trade in consumption goods (as opposed to trade in intermediate goods). The consumption baskets are allowed to differ among countries, so purchasing power parity does not necessarily hold. International asset markets are complete, i.e., risk sharing is perfect across countries. Producers act in an environment of monopolistic competition. The only factor of production is labor, which is immobile between countries. The only rigidity is the nominal price rigidity in the spirit of Calvo (1983).

Under the FX regime, prices are set in the currency of the producer’s country (“producer currency pricing”), i.e., the producer does not discriminate the price between

5 Indeed, empirical evidence suggests that countries with higher levels of inflation also tend to experience a higher variability of inflation (early examples are Okun 1971; Taylor 1981). The latter study also presents a theoretical model where a “policy under-reaction” to inflation causes high and variable inflation. Similarly, Friedman (1977) establishes a link between weak institutions and high and variable inflation.
countries. The law of one price holds and exchange rate pass-through is complete. Complete asset markets imply that uncovered interest parity holds.

In both regimes, monetary policy is conducted via Taylor-type interest rate rules. Importantly, I assume that monetary policy is not able to observe the flexible-price equilibrium of the economy, particularly the flexible-price interest rate and flexible-price output, because this information is usually not available in practice. Thus, monetary policy responds to inflation (either producer price inflation or consumer price inflation) and to output (deviation from the steady state), rather than to the output gap (deviation from flexible-price output).

2.1 Model equations

The equations of the complete log-linearized model are shown below (for the full derivation, see Appendices A and B). Deviations of the logarithm of a variable $X_t$ from its steady state are denoted by $\hat{X}_t$ if prices are sticky and by $\tilde{X}_{fb}^t$ if prices are flexible and markups are neutralized (efficient allocation). The variables and parameters are defined in Tables 1 and 2 respectively.

<table>
<thead>
<tr>
<th>$C_t, \ast C_t$</th>
<th>Consumption in country $H$ and $F$, respectively</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_{H,t}, \ast Y_{F,t}$</td>
<td>Output in country $H$ and $F$, respectively</td>
</tr>
<tr>
<td>$\pi_{H,t}, \pi_{F,t}^*$</td>
<td>Producer price inflation in country $H$ and $F$, respectively</td>
</tr>
<tr>
<td>$\pi_t, \pi_t^*$</td>
<td>Consumer price inflation in country $H$ and $F$, respectively</td>
</tr>
<tr>
<td>$\pi_{MU}$</td>
<td>Union-wide inflation (average of country-specific inflation)</td>
</tr>
<tr>
<td>$R_t, \ast R_t$</td>
<td>Nominal interest rate in country $H$ and $F$, respectively</td>
</tr>
<tr>
<td>$R_{MU}$</td>
<td>Nominal interest rate in monetary union</td>
</tr>
<tr>
<td>$T_t$</td>
<td>Terms of trade</td>
</tr>
<tr>
<td>$S_t$</td>
<td>Nominal exchange rate</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>Real exchange rate</td>
</tr>
<tr>
<td>$\zeta_{Y,t}, \ast \zeta_{Y,t}$</td>
<td>Productivity shock in country $H$ and $F$, respectively</td>
</tr>
<tr>
<td>$\zeta_{C,t}, \ast \zeta_{C,t}$</td>
<td>Consumption preference shock in country $H$ and $F$, respectively</td>
</tr>
<tr>
<td>$\mu_t, \ast \mu_t$</td>
<td>Cost-push (or markup) shock in country $H$ and $F$, respectively</td>
</tr>
</tbody>
</table>

Table 1: Variables
2.1 Model equations

| $\rho$ | Inverse of elasticity of intertemporal substitution in consumption |
| $\beta$ | Discount factor |
| $\eta$ | Inverse of elasticity of producing the differentiated good |
| $a$ | Home bias/degree of trade openness |
| $\sigma$ | Elasticity of substitution between differentiated goods within countries |
| $\theta$ | Elasticity of substitution between goods across countries |
| $\alpha'$ | Probability of not being able to reset the price in country $i = H, F$ |
| $\phi_{\pi}$ | Inflation coefficient in interest rate rule |
| $\phi_Y$ | Output coefficient in interest rate rule |
| $\phi_R$ | Interest rate smoothing coefficient in interest rate rule |
| $\rho_i$ | Persistence of productivity shock in country $i = H, F$ |
| $k_{1Y}^*$ | $k_{1Y}^* = \frac{(1-\alpha'\beta)(1-\alpha')}{\alpha'\eta}$ |
| $k_{1Y}^*$ | $k_{1Y}^* = \frac{(1-\alpha'\beta)(1-\alpha')}{\alpha'\eta}$ |
| $k_{1Y}^*$ | $k_{1Y}^* = \frac{(1-\alpha'\beta)(1-\alpha')}{\alpha'\eta}$ |

Table 2: Parameters

2.1.1 Sticky-price equilibrium under the FX regime

Under sticky prices, the model equations for the FX regime are given by

$$E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} (\hat{R}_t - E_t \pi_{t+1} + E_t \hat{C}_{C,t+1} - \hat{C}_{C,t})$$  \hspace{1cm} (2.1)

$$\hat{Q}_t = \rho (\hat{C}_t - \hat{C}_t^*) + (\hat{C}_{C,t}^* - \hat{C}_{C,t})$$  \hspace{1cm} (2.2)

$$E_t \Delta \hat{S}_{t+1} = \hat{R}_t - \hat{R}_t^*$$  \hspace{1cm} (2.3)

$$\hat{Q}_t = (2a - 1) \hat{T}_t$$  \hspace{1cm} (2.4)

$$\hat{Y}_{H,t} = 2a(1-a)\theta \hat{T}_t + a \hat{C}_t + (1-a)\hat{C}_t^*$$  \hspace{1cm} (2.5)

$$\hat{Y}_{F,t} = -2a(1-a)\theta \hat{T}_t + (1-a)\hat{C}_t + a \hat{C}_t^*$$  \hspace{1cm} (2.6)

$$\pi_{H,t} = k_{1Y}^H (\hat{Y}_{H,t} - \hat{Y}_{H,t}^{fb}) - 2a(1-a)k_{\mu}^H (\hat{T}_t - \hat{T}_t^{fb}) + k_{\mu}^H \mu \hat{H} + \beta E_t \pi_{H,t+1}$$  \hspace{1cm} (2.7)

$$\pi_{F,t} = k_{1Y}^F (\hat{Y}_{F,t} - \hat{Y}_{F,t}^{fb}) + 2a(1-a)k_{\mu}^F (\hat{T}_t - \hat{T}_t^{fb}) + k_{\mu}^F \hat{F} + \beta E_t \pi_{F,t+1}$$  \hspace{1cm} (2.8)

$$\hat{T}_t = \hat{T}_{t-1} + \pi_{H,t}^* - \pi_{H,t} + \Delta \hat{S}_t$$  \hspace{1cm} (2.9)

$$\pi_t = a \pi_{H,t} + (1-a)(\pi_{F,t}^* + \Delta \hat{S}_t)$$  \hspace{1cm} (2.10)

$$\pi_t^* = (1-a)(\pi_{H,t}^* - \Delta \hat{S}_t) + a \pi_{F,t}^*$$  \hspace{1cm} (2.11)

Following standard Taylor-type interest rate rules, monetary policy in each country responds to output and some measure of inflation. If monetary policy responds to producer price inflation, the interest rate rules for each country are given by

$$\hat{R}_t = \phi_R \hat{R}_{t-1} + (1-\phi_R)(\phi_{\pi} \pi_{H,t} + \phi_Y \hat{Y}_{H,t})$$  \hspace{1cm} (2.12)

$$\hat{R}_t^* = \phi_R \hat{R}_{t-1}^* + (1-\phi_R)(\phi_{\pi} \pi_{H,t}^* + \phi_Y \hat{Y}_{F,t})$$  \hspace{1cm} (2.13)

If monetary policy responds to consumer price inflation, the interest rate rules are given by

$$\hat{R}_t = \phi_R \hat{R}_{t-1} + (1-\phi_R)(\phi_{\pi} \pi_t + \phi_Y \hat{Y}_{H,t})$$  \hspace{1cm} (2.14)

$$\hat{R}_t^* = \phi_R \hat{R}_{t-1}^* + (1-\phi_R)(\phi_{\pi} \pi_t^* + \phi_Y \hat{Y}_{F,t})$$  \hspace{1cm} (2.15)
2.1.2 Sticky-price equilibrium under the MU regime

Under sticky prices, the model equations for the MU regime are given by

\[ E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} \left( \hat{R}^{MU}_t - E_t \pi_{t+1} + E_t \hat{C}_{t+1} - \hat{C}_{t+1} \right) \]  

(2.16)

\[ \hat{Q}_t = \rho \left( \hat{C}_t - \hat{C}_t^* \right) + (\hat{\pi}^*_C - \hat{\pi}_C) \]  

(2.17)

\[ \hat{Q}_t = (2a - 1) \hat{T}_t \]  

(2.18)

\[ \hat{Y}_{H,t} = 2a(1 - a) \theta \hat{T}_t + a \hat{C}_t + (1 - a) \hat{C}_t^* \]  

(2.19)

\[ \hat{Y}_{E,t} = -2a(1 - a) \theta \hat{T}_t + (1 - a) \hat{C}_t + a \hat{C}_t^* \]  

(2.20)

\[ \pi_{H,t}^* = k_F^H \left( \hat{Y}_{H,t} - \hat{Y}_{H,t}^f \right) - 2a(1 - a) k_F^H \left( \hat{T}_t - \hat{T}_t^f \right) + k_H^H \hat{\mu}_t^H + \beta E_t \pi_{H,t+1} \]  

(2.21)

\[ \pi_{E,t}^* = k_F^E \left( \hat{Y}_{E,t} - \hat{Y}_{E,t}^f \right) + 2a(1 - a) k_F^E \left( \hat{T}_t - \hat{T}_t^f \right) + k_H^E \hat{\mu}_t^E + \beta E_t \pi_{E,t+1} \]  

(2.22)

\[ \hat{T}_t = \hat{T}_{t-1} + \pi_{E,t}^* - \pi_{H,t} \]  

(2.23)

\[ \pi_t = a \pi_{H,t} + (1 - a) \pi_{E,t}^* \]  

(2.24)

\[ \pi_t^* = (1 - a) \pi_{H,t} + a \pi_{E,t}^* . \]  

(2.25)

The common monetary policy responds to union-wide inflation (average country-specific inflation) and union-wide output (average country-specific output). However, whether the common monetary policy responds to producer price inflation or consumer price inflation does not make a difference. Using equations (2.24) and (2.25), it is straightforward to show that the average of consumer price inflation rates is equal to the average of producer price inflation rates:

\[ \frac{\pi_t + \pi_t^*}{2} = \frac{\pi_{H,t} + \pi_{E,t}^*}{2} = \pi_t^{MU} . \]  

(2.26)

Accordingly, the interest rate rule of the common monetary policy can be written in either case as

\[ \hat{R}^{MU}_t = \phi_R \hat{R}_{t-1}^{MU} + (1 - \phi_R) \left( \phi_{\pi} \pi_t^{MU} + \phi_{\pi} \frac{\hat{Y}_{H,t} + \hat{Y}_{E,t}}{2} \right) . \]  

(2.27)

2.1.3 Efficient allocation under both regimes

The following equations describe the first-best (fb) or efficient allocation, where prices are fully flexible and markups are neutralized at all times with an appropriate subsidy (\( \mu_t^i = 0 \)). This efficient allocation provides a useful benchmark for assessing the welfare implications of the two international monetary regimes.

The efficient output in each country is given by

\[ (\rho + \eta) \hat{Y}_{H,t}^{fb} = 2a(1 - a)(\rho \theta - 1) \hat{T}_{t}^{fb} \]

\[ - (1 - a) (\hat{\pi}_{C,t} - \hat{\pi}_{C,t}^*) + \hat{\pi}_{Y,t}^* \]  

(2.28)

\[ (\rho + \eta) \hat{Y}_{E,t}^{fb} = -2a(1 - a)(\rho \theta - 1) \hat{T}_{t}^{fb} \]

\[ + (1 - a) (\hat{\pi}_{C,t} - \hat{\pi}_{C,t}^*) + \hat{\pi}_{Y,t}^* . \]  

(2.29)

The efficient terms of trade can be written as

\[ [4a(1 - a) \rho \theta + (2a - 1)^2] \hat{T}_{t}^{fb} = \rho \left( \hat{Y}_{H,t}^{fb} - \hat{Y}_{E,t}^{fb} \right) - (2a - 1) (\hat{\pi}_{C,t} - \hat{\pi}_{C,t}^*) . \]  

(2.30)
2.2 Model description

Consumption is described by standard Euler equations, which are given by equations (2.1) and (2.16) in the case of country $H$. The difference between these two Euler equations is that the nominal interest rate is country-specific under the FX regime, whereas it is common to both countries under the MU regime. The risk sharing condition, which describes the link between consumption across countries, is identical across regimes and it is given by (2.2) and (2.17), respectively. Purchasing power parity does not hold, i.e., the real exchange rate is not constant, unless consumption and consumption preference shocks are perfectly correlated across countries. Under the FX regime, perfect risk sharing implies that the uncovered interest parity (2.3) holds, i.e., the expected change in the nominal exchange rate corresponds to the interest rate differential across countries. This equation is obsolete under the MU regime because both countries share the same currency and a common nominal interest rate.

The link between the real exchange rate and the terms of trade is described by equations (2.4) and (2.18), respectively. Accordingly, the correlation between the real exchange rate and the terms of trade can be positive, zero, or negative, depending on the degree of trade openness between the two countries. Aggregate demand in each country depends on consumption in both countries and the terms of trade and is given by equations (2.5), (2.6), (2.19), and (2.20), respectively. The country-specific New Keynesian Phillips curves are also identical across regimes and they are given by (2.7), (2.8), (2.21), and (2.22), respectively. In contrast to a closed-economy framework, not only the output gap but also the terms-of-trade gap (the difference between the sticky-price and the efficient terms of trade) affect producer price inflation.

The terms-of-trade identity is given by equation (2.9) under the FX regime and by equation (2.23) under the MU regime, which differ due to the presence of the nominal exchange rate in the former. Equations (2.10), (2.11), (2.24), and (2.25) describe the relationship between the consumer price inflation rate and the producer price inflation rates in each country. Likewise, these equations only differ across regimes in terms of the presence of the nominal exchange rate.

Monetary policy is conducted via Taylor-type interest rate rules, according to which it responds to output and some measure of inflation. Under the FX regime, the interest rate rules for each country are given by equations (2.12) and (2.13) if monetary policy responds to producer price inflation, and by equations (2.14) and (2.15) if monetary policy responds to consumer price inflation. Under the MU regime, whether the common monetary policy responds to producer price inflation or consumer price inflation does not make a difference. In both cases, the interest rate rule is given by equation (2.27).

Under flexible prices, monetary policy is neutral and real variables are driven only by productivity shocks and consumption preference shocks. Thus, the efficient alloca-

---

6 Combining the Euler consumption equation for country $H$, the risk sharing condition, and the uncovered interest parity condition yields the Euler consumption equation for country $F$, which is therefore redundant. Alternatively, the model can be specified by including both country-specific Euler consumption equations and the risk sharing condition, while omitting the uncovered interest parity condition. The model specification I employ includes the uncovered interest parity condition because it more clearly illustrates the economic mechanisms that underlie the results of this study.

7 Throughout this study, I only consider interest rate rules with a contemporaneous response to output and inflation. However, forward-looking and backward-looking interest rate rules do not change the main conclusions of this study.
tion, which is given by equations (2.28) through (2.30), is the same under both international monetary regimes.

2.3 Welfare loss function

The welfare analysis follows the logic of the familiar linear-quadratic approach, where log-linear model equations are used to evaluate a quadratic welfare loss measure (Woodford, 2003). The world welfare loss function is given by the discounted value of a weighted average across countries of the average utility flow of agents using a second-order Taylor series expansion. It is assumed that the distortion induced by monopolistic competition is offset by an appropriate subsidy, thereby ensuring efficiency in the steady state. Thus,

\[
W_t = -\frac{1}{2} \left( (\rho + \eta) \text{var}(\hat{Y}_{H,t} - \hat{Y}_{H,t}^{fb}) + (\rho + \eta) \text{var}(\hat{Y}_{F,t} - \hat{Y}_{F,t}^{fb}) 
- 2a(1-a)\frac{\rho\theta - 1}{\rho} \left[ 4a(1-a)\rho\theta + (2a - 1)^2 \right] \text{var}(\hat{T}_t - \hat{T}_t^{fb}) 
+ \sigma(1 + \sigma\eta) \frac{\alpha^H}{(1 - \alpha^H)(1 - \alpha^H\beta)} \text{var} \pi_{H,t} 
+ \sigma(1 + \sigma\eta) \frac{\alpha^F}{(1 - \alpha^F)(1 - \alpha^F\beta)} \text{var} \pi_{F,t}^* \right) 
+ t.i.p. + O(\|\xi\|^3).
\] (2.31)

As in the closed economy, the welfare loss depends on the producer price inflation rate and the output gap. In the open economy, the welfare loss also depends on the terms-of-trade gap. Intuitively, when the terms of trade deviate from their efficient level, the resulting allocation of production across countries is inefficient due to the presence of price stickiness. The weights in front of each component of the welfare loss function are functions of the deep parameters of the model. The term \(t.i.p\) contains all the terms that are independent of monetary policy and the international monetary regime. The term \(O(\|\xi\|^3)\) contains third and higher order terms, which can be neglected provided that the model equations are log-linear, i.e., first-order approximations of the non-linear equilibrium conditions.

2.4 Calibration

Unless stated otherwise, the parameters of the model are calibrated to the values displayed in Table 3 (see also Benigno, 2004). For the sake of simplicity, the two countries are assumed to be symmetric. A value of 0.99 for the discount factor \(\beta\) implies a steady-state real interest rate of around 4.1 percent annually. A value of 7.66 for the elasticity of substitution between differentiated goods \(\sigma\) implies a steady-state markup of prices over marginal costs of 15 percent. A value of 0.75 for the probability of not being able to reset the price \(\alpha\) implies an average duration of price contracts of 4 quarters.

\(^8\) Computing country-specific welfare would complicate the calculations significantly because more accurate approximations of the non-linear model equations would be necessary (Benigno and Woodford, 2005), which is beyond the scope of this study.
### Table 3: Baseline calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>1/6</td>
<td>Inverse of elasticity of intertemporal substitution in consumption</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.67</td>
<td>Inverse of elasticity of producing the differentiated good</td>
</tr>
<tr>
<td>$\epsilon_{wy}$</td>
<td>0.5</td>
<td>Production elasticity of average real wage</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.75</td>
<td>Labor income share</td>
</tr>
<tr>
<td>$a$</td>
<td>0.75</td>
<td>Home bias/degree of trade openness</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>7.66</td>
<td>Elasticity of substitution between differentiated goods within countries</td>
</tr>
<tr>
<td>$\theta$</td>
<td>2</td>
<td>Elasticity of substitution between goods across countries</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>0.75</td>
<td>Probability of not being able to reset the price in country $i = H, F$</td>
</tr>
<tr>
<td>$\phi_{\pi}$</td>
<td>1.5</td>
<td>Inflation coefficient in interest rate rule</td>
</tr>
<tr>
<td>$\phi_Y$</td>
<td>0</td>
<td>Output coefficient in interest rate rule</td>
</tr>
<tr>
<td>$\phi_R$</td>
<td>0</td>
<td>Interest rate smoothing coefficient in interest rate rule</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>0.9</td>
<td>Persistence of productivity shock in country $i = H, F$</td>
</tr>
</tbody>
</table>

The degree of trade openness $a$ is calibrated to 0.75, which corresponds to a steady-state share of home-produced goods in the consumption basket of 75 percent in each country (i.e., a home bias in consumption) and a steady-state trade-to-GDP ratio of 50 percent. This roughly equals the average trade-to-GDP ratio across OECD countries. Following Rotemberg and Woodford (1998) and Benigno (2004), the inverse of the elasticity of producing the differentiated good $\eta$ is calculated as

$$\eta = \epsilon_{wy} - \rho + \frac{1 - \gamma}{\gamma},$$

(2.32)

where $\epsilon_{wy}$ denotes the elasticity of the average real wage with respect to production and $\gamma$ denotes the labor income share.

Under the baseline calibration, monetary policy responds to inflation ($\phi_{\pi} = 1.5$), but it does not respond to output ($\phi_Y = 0$) and does not engage in interest rate smoothing ($\phi_R = 0$). All interest rate rule coefficients are assumed to be identical across countries and regimes. Finally, the persistence of the productivity shock is 0.9 in each country and the cross-country correlation is zero. I abstract from consumption preference shocks and cost-push shocks because they do not change the main conclusions of this study.

### 3 Monetary policy as an OCA criterion

In the following, I use the theoretical model described in the previous section to show that the conduct of monetary policy is a critical criterion for the welfare implications of the MU regime relative to the FX regime. The conduct of monetary policy is modified in two dimensions: (1) the coefficients in the interest rate rules that determine the response of monetary policy to inflation (Section 3.1), to output (Section 3.2), and to past realizations of the interest rate (Section 3.3); and (2) the measure of inflation to which monetary policy responds, i.e., producer price inflation (henceforth PPI inflation targeting) or consumer price inflation (henceforth CPI inflation targeting).

---

9 The trade-to-GDP ratio expressed in percent is given by $2(1 - a) \times 100$. 

---
3.1 Response to inflation

The aggressiveness of monetary policy in its response to inflation is an important criterion that determines whether countries are better off under the MU regime or under the FX regime (Figure 1, upper left panel). If the response to inflation is relatively modest (i.e., low values of $\phi_{\pi}$), the two countries are better off under the MU regime. If the response to inflation is relatively strong, the two countries are better off under the FX regime. The threshold value of $\phi_{\pi}$ beyond which the FX regime becomes superior in terms of welfare depends on whether monetary policy targets PPI inflation or CPI inflation. Under CPI inflation targeting, the threshold value for $\phi_{\pi}$ is lower than that under PPI inflation targeting, i.e., the two countries are less likely to be better off under the MU regime.

![Figure 1: World welfare loss and variances of welfare-relevant variables as a function of the inflation coefficient ($\phi_{\pi}$)](image)

The welfare ranking between the MU and the FX regime is driven almost exclusively by the variance of PPI inflation, which exhibits the same pattern with respect to $\phi_{\pi}$ as the welfare loss (Figure 1, lower right panel). This is because agents attach by far the highest weight to inflation, which is traditionally the case in microfounded welfare measures derived from New Keynesian models. Accordingly, the cost of a higher variance of the output gap and of the terms-of-trade gap under the MU regime (Figure 1, upper right and lower left panel) may be outweighed by the benefit of a

---

10 Under the MU regime, PPI inflation targeting and CPI inflation targeting are the same policy (recall equation 2.27).

11 Although the welfare loss depends on the output gap and the PPI inflation rate of both countries, Figure 1 shows only one of each because the variances are identical in both countries due to the assumption of symmetric countries.
lower variance of PPI inflation. This is the case for low values of $\phi_\pi$, i.e., a relatively modest response of monetary policy to inflation.\footnote{A brief explanation of why PPI inflation may be more stable under the MU regime is given in Section \ref{sec:3.3}. A detailed explanation is given in Groll (2013).}

The two countries are better off under the FX regime for a sufficiently strong response of monetary policy to inflation (either PPI or CPI inflation) because monetary policy is more potent under the FX regime than under the MU regime. This becomes clear if the number of policy instruments is compared to the number of welfare-relevant distortions in the economy. Under the FX regime, there are as many policy instruments as distortions in the economy. These distortions are due to monopolistic competition and to sticky prices.\footnote{Both distortions are common to the closed-economy framework (see, e.g., Woodford (2003) for details).}

The distortion due to monopolistic competition induces an inefficiently low level of aggregate output. This distortion can be eliminated by an appropriate subsidy in each country. The distortion due to sticky prices induces inefficient markup fluctuations, which lead to inefficiently low or high levels of aggregate output, and an inefficient dispersion of prices in the presence of inflation, which causes an inefficient dispersion of output across the producers of differentiated goods within each country. This distortion can be mitigated or even eliminated by monetary policy in each country by using the nominal interest rate to reduce the fluctuations of inflation around zero as far as possible.

By contrast, under the MU regime, there are less policy instruments than distortions. First, monetary policy sets the nominal interest rate for both countries and thus it can no longer target inflation in each country separately, thereby losing one policy instrument. Second, in addition to the two distortions described above, the \textit{de facto} fixed nominal exchange rate combined with sticky prices induces an additional distortion into the economy, namely an intrinsic inertia in the terms of trade (Benigno 2004; Pappa 2004).\footnote{Intrinsic inertia is defined as follows: Consider a one-off (i.e., non-persistent) productivity shock in one country. Under the MU regime, several periods are required before the terms of trade return to the steady state after the shock has vanished. Thus, the terms of trade are said to be intrinsically persistent or inertial. Under the FX regime, the terms of trade return to the steady state immediately after the shock has vanished. In this case, the terms of trade are not intrinsically inertial.} This causes an inefficient dispersion of aggregate output across the two countries.

Given that there are as many policy instruments as distortions under the FX regime but less policy instruments than distortions under the MU regime, monetary policy is more potent under the FX regime, which is shown clearly in Figure 1. The “leverage” of monetary policy is higher under the FX regime than under the MU regime in the sense that a given increase in the aggressiveness of monetary policy towards inflation (measured by $\phi_\pi$) leads to a larger reduction in the variance of each welfare-relevant variable. In fact, under PPI inflation targeting, monetary policy can reduce the variances of all welfare-relevant variables to zero (if $\phi_\pi \to \infty$). This is impossible under the MU regime.\footnote{See Groll (2013) for the analytical proof of the special case where $a = 1/2$ and $\theta = 1$. The proof in the case with no restrictions on $a$ and $\theta$ is completely analogous.}
countries face the same nominal interest rate, any interest rate adjustment by the common monetary policy has the same initial effect on both countries. If the degree of price stickiness is identical across the two countries, such an interest rate adjustment propagates through both economies in exactly the same way. In these circumstances, the influence of monetary policy on the terms of trade is zero. If the degree of price stickiness is not identical across the two countries, the influence of monetary policy on the terms of trade is not exactly zero, but it is close to zero.

3.2 Response to output

As shown above, monetary policy is more potent under the FX regime than under the MU regime due to the flexibility of the nominal exchange rate. However, there is a flipside to this greater potency, which becomes clear when considering the case where monetary policy responds to output in addition to inflation.

If monetary policy responds to output in addition to inflation, the two countries are generally better off under the MU regime (Figure 2, upper left panel). Moreover, the welfare gain of the MU regime relative to the FX regime increases with the strength of the response to output (measured by $\phi_Y$). Beyond a certain value for $\phi_Y$, all welfare-relevant variables are more stable under the MU regime (Figure 2, upper right, lower left, and lower right panel). All of this holds irrespective of whether monetary policy targets PPI or CPI inflation rates. Under the FX regime, these two policies are virtually identical (in the figure, the ‘FX PPI’ lines and the ‘FX CPI’ lines basically coincide). Under the MU regime, they are identical anyway.

The key to understanding these results is the role played by the nominal exchange rate in stabilizing the terms-of-trade gap. Combining equations (2.28) to (2.30) and focusing on productivity shocks in country $H$ yields the following relationship between the efficient terms of trade $\tilde{T}^{fb}_t$ and the productivity shock $\hat{\zeta}_Y,t$:

$$\tilde{T}^{fb}_t = \frac{\rho \eta}{4a(1-a)\rho(1+\eta \theta)} + \frac{\rho + \eta}{2a - 1} \hat{\zeta}_Y,t.$$  \hspace{1cm} (3.1)

The term in front of the productivity shock is unambiguously positive. Accordingly, the terms of trade would increase in response to a positive productivity shock in country $H$ if prices were perfectly flexible. However, because prices are sticky, the increase in the terms of trade is actually smaller. In these circumstances, an increase in the nominal exchange rate would help to close the gap between the actual response of the terms of trade and its efficient counterpart, thereby stabilizing the terms-of-trade gap and reducing the welfare loss. But whether the nominal exchange rate stabilizes or destabilizes the terms-of-trade gap depends crucially on the conduct of monetary policy and, in particular, on whether and how strongly monetary policy responds to output. This is because the nominal exchange rate is linked directly via the uncovered interest parity condition (2.3) to the interest rates governed by monetary policy in each country.

---

16 The basic conclusions of this section also hold if monetary policy responds to the change in output instead. The corresponding graphs are available upon request.

17 Importantly, this link also exists when asset markets are incomplete. In this case, the uncovered interest parity condition merely contains an additional term that describes the net asset positions of countries.
3.2 Response to output

Figure 2: World welfare loss and variances of welfare-relevant variables as a function of the output coefficient ($\phi_Y$)

If monetary policy does not respond to output ($\phi_Y = 0$), the impact response of the nominal exchange rate to a positive productivity shock in country $H$ is positive, i.e., country $H$’s currency depreciates (Figure 3, dashed line)\(^\text{18}\). Thus, the nominal exchange rate pushes the sticky-price terms of trade in the same direction as the efficient terms of trade, thereby stabilizing the terms-of-trade gap to some extent. This holds under both PPI and CPI inflation targeting\(^\text{19}\).

However, as monetary policy starts to respond to output and becomes more aggressive towards output (i.e., $\phi_Y$ becomes positive and increases), the impact response of the nominal exchange rate becomes smaller and it is already negative for very small values of $\phi_Y$ (Figure 3, solid and dotted line)\(^\text{20}\). A negative impact response means that...

\(^{18}\)The degree of price stickiness was set low ($\alpha = 0.2$) to ensure that the differences in the impulse responses are clearly visible. The differences for higher degrees of price stickiness are smaller, but they are qualitatively the same.

\(^{19}\)Under CPI inflation targeting, this is sensitive to the degree of trade openness $a$, which is analyzed in detail in Section 4.

\(^{20}\)The threshold value of $\phi_Y$ at which the impact response of the nominal exchange rate becomes negative also depends on other model parameters. Under PPI inflation targeting, it is feasible to derive the exact analytical expression, i.e., the impact response is negative if $\phi_Y > -\frac{(\phi_\pi - 1)\rho}{4a(1-a)\rho^2 + 2(2a-1)\rho^3}$. This expression is obtained in the following way (for details, see Groll, 2013, who presents a completely analogous derivation for a special case of the model). Derive the coefficient $c_3$ of the recursive law of motion for the reduced system of equations given by $\hat{T}_t = b_1 \hat{T}_{t-1} + c_1 \hat{\eta}_t^{fb} + \hat{\pi}_t^f - \hat{\pi}_t^H = b_2 \hat{T}_{t-1} + c_2 \hat{\eta}_t^{fb}$, and $\Delta \hat{S}_t = b_3 \hat{T}_{t-1} + c_3 \hat{\eta}_t^{fb}$, using the method of undetermined coefficients. Recall that $\hat{\eta}_t^{fb}$ is a function of the productivity shock $\hat{\zeta}_t^{Y}$, see equation (3.1). Then, the impact response of the nominal exchange rate to the productivity shock is given by the coefficient $c_3$, which is negative if $\phi_Y$ is larger than the expression given above. This procedure involves the solution of a full quadratic equation, but apply-
the nominal exchange rate destabilizes the terms-of-trade gap by pushing the sticky-price terms of trade away from the efficient terms of trade. As a result, not only the terms-of-trade gap, but also the output gap and the PPI inflation rate are destabilized, thereby increasing the welfare loss. In these circumstances, a fixed nominal exchange rate would make the countries better off because this is neither destabilizing nor stabilizing.

Importantly, the nominal exchange rate only amplifies a detrimental effect that is already present, but it does not cause the detrimental effect. In a closed economy, a response of monetary policy to output is also detrimental to welfare (see, e.g., Gali, 2008, Ch. 4.4). It is not the deviation of output from the steady state that is welfare-relevant, it is the deviation from the flexible-price counterpart (i.e., the output gap). For example, a positive productivity shock in country $H$ induces an increase in output, but a decrease in the output gap, because the increase in output is lower than the increase in flexible-price output. A welfare-oriented response of monetary policy would require a reduction in the interest rate due to the negative output gap. Instead, monetary policy raises the interest rate due to the rise in output. This is detrimental to welfare.

For these reasons, a response of monetary policy to output is detrimental under both the FX regime and the MU regime (in Figure 2, the welfare loss increases in $\phi_Y$ under both regimes). However, and this is of the utmost importance, the detrimental effect is larger under the FX regime due to amplification by the nominal exchange rate. The same “bad” policy is more harmful under the FX regime. As described in the previous section, monetary policy is more potent under the FX regime than under the MU regime in terms of macroeconomic stabilization because of the flexibility of the nominal exchange rate. The flipside of this is that monetary policy can do more harm in terms of macroeconomic stabilization when it is not conducted properly. Essentially, the nominal exchange rate does not compensate for monetary policy mistakes, it re-

Figure 3: Impulse response of the change in the nominal exchange rate ($\Delta \hat{S}_t$) to a positive one-off productivity shock in country $H$ for three different values of the output coefficient ($\phi_Y$), with $\alpha = 0.2$. Left panel: PPI inflation targeting. Right panel: CPI inflation targeting.

---

This derivation is too cumbersome and its solution is too complex to be useful, which is why I chose a numerical approach.
3.3 Interest rate smoothing

Finally, whether countries are better off under the MU regime or under the FX regime also depends on the degree of interest rate smoothing implemented by monetary policy, which is particularly true under PPI inflation targeting (Figure 4, solid and dashed lines). Starting with very low degrees of interest rate smoothing (i.e., low values of $\phi_R$), the two countries are better off under the MU regime. As the degree of interest rate smoothing increases (i.e., an increase in $\phi_R$), the welfare loss decreases faster under the FX regime than under the MU regime. At some point, the welfare ranking changes and the two countries are better off under the FX regime.

![Figure 4: World welfare loss and variances of welfare-relevant variables as a function of the interest rate smoothing coefficient ($\phi_R$)](image)

As described in Section 3.1, the MU regime entails the cost of higher instability of both the output gap and the terms-of-trade gap, but the benefit of higher stability of the PPI inflation rate (except for high values of $\phi_R$). The higher stability of the PPI inflation rate under the MU regime is explained as follows (see Groll, 2013, for details). The nominal exchange rate is fixed under the MU regime. As a result, the terms of trade exhibit an inertial or history-dependent behavior, even if monetary policy does not smooth interest rates. Since the terms of trade are an important determinant of
Marginal costs, this history dependence has the advantage of stabilizing private-sector expectations about future PPI inflation, thereby stabilizing actual PPI inflation.\textsuperscript{21}

If monetary policy targets PPI inflation and does not smooth interest rates under the FX regime, there is no such history dependence. This regime suffers from a kind of stabilization bias. As a result, PPI inflation is less stable under the FX regime. However, if monetary policy starts to smooth interest rates, it induces history dependence into the economy, with the same advantageous effect on inflation expectations. This effect strengthens as the degree of interest rate smoothing increases. If the degree of interest rate smoothing is sufficiently high, PPI inflation is more stable under the FX regime.

Under CPI inflation targeting, the degree of interest rate smoothing does not have such an important effect on the welfare ranking between the MU and FX regime (Figure 4, solid and dotted lines).\textsuperscript{22} This is because, like the MU regime, the FX regime also features history dependence, even if monetary policy does not smooth interest rates. As a result, engaging in interest rate smoothing, thereby inducing greater history dependence into the economy, does not change the relative welfare performance of the FX and MU regimes dramatically.

### 4 Monetary policy and trade openness

The conduct of monetary policy is not only an independent OCA criterion in itself, as illustrated in Section 3, but it can also modify the nature of other OCA criteria. This is demonstrated in the following using the degree of trade openness as an example. But first, I briefly summarize how the relationship between trade openness and the costs and benefits of a monetary union has been described in previous studies.

#### 4.1 Trade openness in OCA theory

The degree of trade openness or trade integration is one of the oldest and most prominent OCA criteria. Most studies have established a positive link between trade openness and the likelihood of a monetary union being beneficial. More precisely, the more open economies are, the smaller are the costs and the larger are the benefits associated with monetary unification.

McKinnon (1963) first proposed trade openness as an OCA criterion. He argued that with an increasingly open economy, the effects of exchange rate fluctuations on consumer prices via import prices become greater, thereby making it more difficult for monetary policy to maintain (consumer) price stability. Thus, the costs of giving up monetary independence decrease with the degree of trade openness.

One of the main costs attributed to monetary unification is the loss of the ability to react to asymmetric (i.e., country-specific) shocks via monetary policy and the nominal exchange rate. However, there are conflicting views on whether the incidence of country-specific shocks decreases or increases with the degree of trade openness. This

\textsuperscript{21}The assumptions in Groll (2013) are somewhat more restrictive ($a = 1/2$ and $\theta = 1$), but this benefit is also found in the more general model employed in the present study (i.e., no restrictions on $a$ and $\theta$).

\textsuperscript{22}Under the baseline calibration, the welfare ranking does not change with the degree of interest rate smoothing. However, parameter constellations exist where this is the case, e.g., if $\phi_\pi = 1.2$ instead of 1.5.
4.2 Monetary policy and the nature of trade openness as an OCA criterion

depends on whether trade between countries is characterized predominantly by intra-industry trade or inter-industry trade. In the former case, industry-specific shocks affect countries symmetrically, thus an increase in the degree of trade openness reduces the cost of giving up monetary independence (see Ch. 6.2 in the ‘One Market, One Money’ report by the European Commission in [Emerson et al. 1992]). In the latter case, industry-specific shocks affect countries asymmetrically, thus an increase in the degree of trade openness raises the cost of giving up monetary independence (Krugman, 1991, p. 82).

The benefits traditionally associated with monetary unification are usually considered to increase with the degree of trade openness, such as the elimination of transaction costs when exchanging currencies, the increase in price transparency across countries, or the elimination of exchange rate risk (e.g., De Grauwe, 2012, Ch. 3.8). The latter point is also made by Kollmann (2004) using a New Keynesian DSGE model similar to that employed in the present study. He concludes that if the nominal exchange rate is subject to shocks, a monetary union is welfare-improving because the cost of giving up monetary independence is overcompensated by the benefit of eliminating exchange rate risk. This benefit increases with the degree of trade openness because exchange rate shocks are more harmful to macroeconomic stability as economies become more open.

Another benefit of a monetary union also increases with the degree of trade openness, as shown by Pappa (2004) using a similar model. Compared to a flexible exchange rate regime where the monetary authorities do not cooperate to maximize domestic welfare, forming a monetary union eliminates the possibility of strategic terms-of-trade manipulations. This benefit increases with the degree of trade openness because terms-of-trade movements have larger effects on macroeconomic stability as economies become more open.

4.2 Monetary policy and the nature of trade openness as an OCA criterion

The preceding, brief overview shows that OCA theory mainly establishes a favorable relationship between the degree of trade openness and the costs and benefits of a monetary union. Next, I show that this relationship is highly sensitive to the way monetary policy is conducted.

The influence of monetary policy on the nature of the degree of trade openness as an OCA criterion is particularly clear when distinguishing between PPI inflation targeting and CPI inflation targeting. First consider the case of PPI inflation targeting (Figure 5, solid and dashed lines). Two observations are noteworthy. First, under both the MU and the FX regime, the relationship between the welfare loss (and each of its components) and the degree of trade openness is symmetric around a trade-to-GDP ratio of 100 percent ($a = 1/2$). Second, the two countries are better off under the FX regime if they are either relatively closed ($a$ close to one) or very open to trade ($a$ close to zero), but better off under the MU regime for intermediate values. Thus, the likelihood of the MU regime being beneficial increases initially and then decreases with the degree of trade openness.

$^{23}$ See De Grauwe (2012, Ch. 2.1) for a more detailed description and assessment of the “European Commission view” and the “Krugman view.”
The relationship between trade openness and the welfare ranking between the MU and FX regime changes considerably if monetary policy targets CPI inflation rates instead of PPI inflation rates (Figure 5, solid and dotted lines). First, the relationship between the welfare loss (and each of its components) and the degree of trade openness is no longer symmetric under the FX regime. Second, the two countries are better off under the FX regime for trade-to-GDP ratios between 0 and 100 percent (\( \frac{1}{2} < a \leq 1 \)) and better off under the MU regime for ratios between 100 and 200 percent (\( 0 \leq a < \frac{1}{2} \)). Thus, the likelihood of the MU regime being beneficial increases with the degree of trade openness.

Again, the key to understanding these results is the role played by the nominal exchange rate in stabilizing the terms-of-trade gap. Consider a positive productivity shock in country \( H \). Recall that the efficient terms of trade unambiguously increase on impact (see equation 3.1), thus an increase in the nominal exchange rate would help to stabilize the terms-of-trade gap, thereby reducing the welfare loss. But whether the nominal exchange rate stabilizes or destabilizes the terms-of-trade gap depends crucially on the conduct of monetary policy and, in particular, on whether monetary policy targets PPI or CPI inflation rates.

Under PPI inflation targeting, the impact response of the nominal exchange rate is positive irrespective of the degree of trade openness, i.e., country \( H \)’s currency depreciates (Figure 6, left panel). Thus, the nominal exchange rate pushes the sticky-price terms of trade in the same direction as the efficient terms of trade, thereby stabilizing

\[24\text{ Recall that there is no difference between PPI and CPI inflation targeting under the MU regime.} \]

\[25\text{ The degree of price stickiness was set low (} \alpha = 0.2 \text{) to ensure that the differences in the impulse responses are clearly visible. The differences are smaller for higher degrees of price stickiness, but they are qualitatively the same.} \]
4.2 Monetary policy and the nature of trade openness as an OCA criterion

the terms-of-trade gap to some extent. Note that the response of the nominal exchange rate is identical for $a = 0.25$ and $a = 0.75$, which explains the symmetric patterns visible in Figure 5.

![Figure 5: Impulse response of the change in the nominal exchange rate ($\Delta \hat{S}_t$) to a positive one-off productivity shock in country $H$ for three different degrees of trade openness ($a$), with $\alpha = 0.2$. Left panel: PPI inflation targeting. Right panel: CPI inflation targeting.](image)

By contrast, under CPI inflation targeting, the impact response of the nominal exchange rate is positive if the two countries have a trade-to-GDP ratio below 100 percent ($a > 1/2$), but negative if it is above 100 percent ($a < 1/2$) (Figure 6, right panel). Thus, the nominal exchange rate helps to stabilize the terms-of-trade gap only in the first case. In the latter case, the nominal exchange rate actually destabilizes the terms-of-trade gap by pushing the sticky-price terms of trade away from the efficient terms of trade.

This is robust with respect to the other deep parameters of the economy (except with respect to the interest rate rule coefficients, see Section 3). To see this, insert the country-specific interest rate rules (2.14) and (2.15) with $\phi_Y = 0$ and $\phi_R = 0$ together with the definitions of the CPI inflation rates (2.10) and (2.11) and the terms-of-trade identity (2.9) into the uncovered interest parity condition (2.3) to obtain

$$\Delta \hat{S}_t = (2a - 1)\Delta \hat{T}_t + \frac{1}{\phi_\pi} E_t \Delta \hat{S}_{t+1}. \quad (4.1)$$

Solving forward yields

$$\Delta \hat{S}_t = (2a - 1)E_t \sum_{k=0}^{\infty} \left( \frac{1}{\phi_\pi} \right)^k \Delta \hat{T}_{t+k}. \quad (4.2)$$

In particular, it does not make a difference whether $\rho \theta$ is smaller than, equal to, or larger than 1, although this condition has important macroeconomic implications. For example, it determines whether the cross-country correlation of output is positive, zero, or negative (see, e.g., Corsetti, Dedola, and Leduc 2011 for details). Also, if it is zero ($\rho \theta = 1$), the terms-of-trade gap vanishes from the welfare loss function (2.31).
Accordingly, the current change in the nominal exchange rate depends on the discounted sum of current and expected future changes in the terms of trade. Importantly, this relationship is unambiguously positive if \( a > \frac{1}{2} \), but negative if \( a < \frac{1}{2} \).

Under PPI inflation targeting, the analogous equations are given by

\[
\Delta \hat{S}_t = \Delta \hat{T}_t + \frac{1}{\phi_\pi} E_t \Delta \hat{S}_{t+1} 
\]

(4.3)

and

\[
\Delta \hat{S}_t = E_t \sum_{k=0}^{\infty} \left( \frac{1}{\phi_\pi} \right)^k \Delta \hat{T}_{t+k}.
\]

(4.4)

In contrast to the PPI inflation targeting case, the relationship between the current change in the nominal exchange rate and the discounted sum of current and expected future changes in the terms of trade is always positive, regardless of the degree of trade openness \( a \).

The nominal exchange rate can be destabilizing under CPI inflation targeting for the following reason. If the trade-to-GDP ratio is above 100 percent (\( a < \frac{1}{2} \)), consumer prices in one country are determined mainly by producer prices in the other country because consumers consume more imported goods than home-produced goods. If monetary policy targets consumer prices, interest rate adjustments in one country are triggered mainly by producer price changes in the other country. This pushes the nominal exchange rate, which depends on the interest rate differential between the two countries, in the wrong direction, i.e., away from the efficient terms of trade. As a result, the welfare-relevant terms-of-trade gap is destabilized by the nominal exchange rate. In these circumstances, a fixed nominal exchange rate would make the countries better off because this is neither destabilizing nor stabilizing. For this reason, the countries are better off under the MU regime for \( a < \frac{1}{2} \).

In the special case of a trade-to-GDP ratio of exactly 100 percent (\( a = \frac{1}{2} \)), the two countries are indifferent in terms of welfare between the FX and the MU regime under CPI inflation targeting. Ultimately, this is because the nominal exchange rate is constant under both regimes. Under the MU regime, the nominal exchange rate is fixed by construction. Under the FX regime, it is fixed by coincidence. That is, by targeting CPI inflation rates, the two countries unintentionally implement a symmetric fixed exchange rate regime. This is because consumer price changes and thus interest rate adjustments are identical in the two countries.

As mentioned above, the nominal exchange rate stabilizes the terms-of-trade gap under PPI inflation targeting regardless of the degree of trade openness. Hence, one would expect that this would always make the FX regime superior to the MU regime, where the nominal exchange rate is fixed. However, as is shown in Figure 5, higher instability of the terms-of-trade gap (and the output gap) may be outweighed by higher stability of the PPI inflation rate. The reason why the PPI inflation rate may be more stable under the MU regime was explained briefly in Section 3.3 and is explained in detail in Groll (2013). In short, because the nominal exchange rate is fixed under the

\[\text{Note that equation (4.2) and equation (4.4) are equivalent if } a = 1. \text{ In this case, there is no difference between PPI and CPI inflation targeting. This is because the consumer price index equals the producer price index if } a = 1, \text{ see equations (2.10) and (2.11).}\]

\[\text{According to equation (4.3), } \Delta \hat{S}_t = 0 \text{ if } a = 1/2.\]

\[\text{Note that an asymmetric fixed exchange rate regime (i.e., one country pegs its currency to the other country’s currency) is generally not equivalent to a monetary union regime.}\]
5 Conclusion

MU regime, the terms of trade exhibit an inertial or history-dependent behavior. Since the terms of trade are an important determinant of marginal costs, this history dependence has the advantage of stabilizing private-sector expectations about future PPI inflation, thereby stabilizing actual PPI inflation.

5 Conclusion

The main point of this study is that monetary policy is one of the most important OCA criteria. That is, whether countries benefit from forming a monetary union depends critically on the way monetary policy is conducted. This point has not yet been emphasized sufficiently in previous OCA studies, if at all. In this study, I demonstrated systematically how and why different types of rule-based interest rate policies lead to different welfare rankings between a monetary union and a flexible exchange rate regime.

In general, when countries are unable or unwilling to implement monetary policy optimally, for whatever reason, they are likely to benefit from forming a monetary union. This is especially true when countries tend to make monetary policy mistakes. In all these circumstances, forming a monetary union entails the benefit of higher macroeconomic stability, particularly inflation stability.

This study only presented results based on productivity shocks and contemporaneous interest rate rules, but all of these conclusions are still valid under consumption preference shocks and cost-push shocks, as well as under backward-looking and forward-looking interest rate rules.
References


A Flexible exchange rate regime

This appendix contains the full derivation of the model under the flexible exchange rate regime (based on Corsetti, Dedola, and Leduc, 2011). The world, which consists of two countries labeled $H$ and $F$, is populated by a continuum of agents on the interval $[0, 1]$. The population on the segment $[0, n)$ lives in country $H$, the population on the segment $[n, 1]$ lives in country $F$. Thus, $n$ measures the population size as a fraction of world population. An agent is both consumer and producer. He produces a single differentiated good and consumes all the goods produced in both countries.

A.1 Consumer problem

Agent $j$ in country $H$ derives positive utility from consumption $C^j$ and negative utility from producing the differentiated good $y(h)$. The present discounted value of lifetime utility $U^j$ is given by:

$$U^j = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \zeta_{C,t}^{1-\rho - 1} - \zeta_{Y,t}^{-1} y_t(h)^{1+\eta} \right].$$  \hspace{1cm} (A.1)

$E$ denotes the expectations operator, $\beta$ the discount factor, $\rho$ the inverse of the intertemporal elasticity of substitution in consumption, and $\eta$ the inverse of the elasticity of producing the differentiated good. $\zeta_{C,t}$ and $\zeta_{Y,t}$ denote shocks to productivity and to preferences in consumption, respectively. These shocks are common to all agents living in country $H$.

The agent consumes both a bundle of differentiated goods from country $H$ and from country $F$ according to the following Constant-Elasticity-of-Substitution (CES) aggregator:

$$C^j = \left[ a^{\frac{1}{\sigma}} C^j_{H,t}^{\frac{\sigma-1}{\sigma}} + (1-a)^{\frac{1}{\sigma}} C^j_{F,t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$ \hspace{1cm} (A.2)

where the bundles of differentiated goods are given by aggregators according to Dixit and Stiglitz (1977):

$$C^j_{H,t} = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c^j_l(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}},$$  \hspace{1cm} (A.3)

$$C^j_{F,t} = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c^j_l(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}}.$$

These preferences imply (1) that the elasticity of substitution between differentiated goods $c^j_l$ from one country is $\sigma$, which is assumed to be greater than one and equal across countries, (2) that the elasticity of substitution between the bundles of goods from the two countries $C^j_{H,t}$ and $C^j_{F,t}$ is $\theta$, which is assumed to be greater than zero and

\[30\] In Corsetti, Dedola, and Leduc (2011), the agent derives utility also from the liquidity services of holding money. I abstract from money in the utility function, since monetary policy is conducted via interest rate rules.

\[31\] The parameter $\eta$ is equivalent to the inverse of the Frisch elasticity of labor supply.
equal across countries, and (3) that the steady-state share of imported goods in overall consumption expenditures is $1 - a$. If $a > 1/2$, the agent consumes more goods from the country the agent lives in than from the other country, i.e., the agent has a home bias in consumption. This home bias is assumed to be symmetric across countries. Thus, the CES aggregator for an agent $j$ living in country $F$ is given by

$$C_{jt}^* = \left[ (1 - a) \frac{1}{\theta} C_{ht}^* \frac{\sigma+1}{\sigma} + a \frac{1}{\theta} C_{ft}^* \frac{\sigma+1}{\sigma} \right]^\frac{\sigma}{\sigma+1}. \quad (A.4)$$

Accordingly, the consumer price index (CPI) in country $H$ expressed in country $H$’s currency is given by

$$P_t = \left[ a P_{ht, t}^{1-\theta} + (1 - a) P_{ft, t}^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad (A.5)$$

where the producer price indices (PPI) for the bundles of differentiated goods expressed in country $H$’s currency are defined by

$$P_{ht, t} = \left[ \frac{1}{n} \int_0^h p_t(h)^{1-\sigma} dh \right]^{\frac{1}{1-\sigma}}, \quad (A.6)$$

$$P_{ft, t} = \left[ \frac{1}{1-n} \int_n^1 p_t(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}}.$$

In their role as producers, agents charge one price for their good irrespective of whether the good is sold in their country or is exported to the other country (no price discrimination), setting the price in their country’s currency (producer currency pricing). Furthermore, exporting does not entail transportation costs. These assumptions imply that the law of one price holds, i.e., a single differentiated good has the same price in both countries if expressed in the same currency, and that exchange rate pass-through is complete:

$$p_t(h) = S_t p_t^*(h), \quad p_t(f) = S_t p_t^*(f), \quad (A.7)$$

where $p_t(h)$ denotes the price of a differentiated good $y(h)$ produced in country $H$ denominated in country $H$’s currency, $p_t^*(h)$ denotes the price of the same good $y(h)$ denominated in country $F$’s currency, $p_t(f)$ denotes the price of a differentiated good $y(f)$ produced in country $F$ denominated in country $H$’s currency, $p_t^*(f)$ denotes the price of the same good $y(f)$ denominated in country $F$’s currency, and $S_t$ is the nominal exchange rate defined as the price of country $F$’s currency in terms of country $H$’s currency. Given equations (A.6), it is straightforward to show that the law of one price for each differentiated good translates into the law of one price for each bundle of goods:

$$P_{ht, t} = S_t P_{ht, t}^*, \quad P_{ft, t} = S_t P_{ft, t}^*. \quad (A.8)$$

The CPI in country $F$ expressed in country $F$’s currency is given by

$$P_t^* = \left[ (1 - a) P_{ht, t}^{*1-\theta} + a P_{ft, t}^{*1-\theta} \right]^{\frac{1}{1-\theta}}. \quad (A.9)$$

In general, the law of one price does not translate into purchasing power parity. Thus, the real exchange rate, defined as the ratio of country-specific consumer prices

$$Q_t = \frac{S_t P_t^*}{P_t}. \quad (A.10)$$
A Flexible exchange rate regime

adjusts in response to changing economic conditions. Purchasing power parity \( (Q_t = 1) \) only holds if the consumption baskets are identical across countries \( (a = 1/2) \).

Another international relative price of interest are the terms of trade, defined from the perspective of country \( H \) as the ratio of the price of imported goods to the price of exported goods:

\[
T_t = \frac{S_t P_{F,t}^*}{P_{H,t}}.
\]  
(A.11)

Agent \( j \) in country \( H \) takes three decisions with respect to his consumption choices. First, he decides on the overall level of consumption \( \sum C_{j,t} \).

Second, given \( C_{j,t} \) the agent optimally allocates expenditures between the bundles of differentiated goods \( C_{H,t} \) and \( C_{F,t} \) by minimizing total expenditure \( P_t C_{j,t} \) subject to the CES aggregator (A.2). As a result, demand for these bundles is given by

\[
C_{H,t} = a \left( \frac{P_{H,t}}{P_t} \right)^{\frac{\theta}{n}} C_{j,t} \quad \text{and} \quad C_{F,t} = (1 - a) \left( \frac{P_{F,t}^*}{P_t} \right)^{\frac{\theta}{n}} C_{j,t}.
\]  
(A.12)

Third, given \( C_{H,t} \) and \( C_{F,t} \) the agent optimally allocates expenditures between the differentiated goods by minimizing \( P_{H,t} C_{H,t} \) and \( P_{F,t} C_{F,t} \) subject to equations (A.3). This yields

\[
c_j(h) = \frac{1}{n} \left( \frac{p_{t}(h)}{P_{H,t}} \right)^{-\sigma} C_{H,t} \quad \text{and} \quad c_j(f) = \frac{1}{1 - n} \left( \frac{p_{t}(f)}{P_{F,t}} \right)^{-\sigma} C_{F,t}.
\]  
(A.13)

Combining (A.12) and (A.13) yields

\[
c_j(h) = a \left(\frac{P_{H,t}}{P_t}\right)^{-\sigma} \left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_{j,t}
\]  
(A.14)

\[
c_j(f) = (1 - a) \left(\frac{P_{F,t}^*}{P_t}\right)^{-\sigma} \left(\frac{P_{F,t}^*}{P_t}\right)^{-\theta} C_{j,t}.
\]

Analogously, demand equations for an agent \( j \) in country \( F \) are given by

\[
c_j^*(h) = \frac{1 - a}{n} \left(\frac{p_{t}(h)}{P_{F,t}^*}\right)^{-\sigma} \left(\frac{p_{F,t}^*}{P_t}\right)^{-\theta} C_{j,t}^*
\]  
(A.15)

\[
c_j^*(f) = a \frac{1}{1 - n} \left(\frac{p_{t}(f)}{P_{F,t}^*}\right)^{-\sigma} \left(\frac{P_{F,t}^*}{P_t}\right)^{-\theta} C_{j,t}^*.
\]

Market clearing for the differentiated goods \( y_t(h) \) and \( y_t(f) \) requires

\[
y_t(h) = \int_0^n c_j(h) \, dj + \int_1^n c_j^*(h) \, dj
\]  
(A.16)

\[
y_t(f) = \int_0^n c_j(f) \, dj + \int_1^n c_j^*(f) \, dj.
\]

\[32 \text{ As shown below, } C_{j,t}^* \text{ is determined by the usual Euler consumption equation.}\]
A.1 Consumer problem

Using equations (A.14) and (A.15), world demand for the differentiated goods can be expressed as

\[ y_t(h) = \frac{1}{n} \left( \frac{p_t(h)}{P_{Ht}} \right)^{-\sigma} \left( \frac{P_{Ht}}{P_t} \right)^{-\theta} \left[ aC_t + (1 - a)Q^H_t C_t^* \right] \]

\[ y_t(f) = \frac{1}{1 - n} \left( \frac{p_t(f)}{P_{Ft}} \right)^{-\sigma} \left( \frac{P_{Ft}}{P_t} \right)^{-\theta} \left[ (1 - a)C_t + aQ^H_t C_t^* \right], \]

where \( C_t \equiv \int_0^n C^j_t d_j \) and \( C_t^* \equiv \int_1^n C^j_t^* d_j \).

Similar to (A.3), aggregate output in the two countries can be expressed as

\[ Y_{Ht} = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n y_t(h)^{\frac{\sigma - 1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma - 1}} \]

\[ Y_{Ft} = \left[ \left( \frac{1}{1 - n} \right)^{\frac{1}{\sigma}} \int_1^n y_t(f)^{\frac{\sigma - 1}{\sigma}} df \right]^{\frac{\sigma}{\sigma - 1}}. \]

Inserting equations (A.17) into the previous equations finally yields aggregate demand

\[ Y_{Ht} = \left( \frac{P_{Ht}}{P_t} \right)^{-\theta} \left[ aC_t + (1 - a)Q^H_t C_t^* \right] \]

\[ Y_{Ft} = \left( \frac{P_{Ft}}{P_t} \right)^{-\theta} \left[ (1 - a)C_t + aQ^H_t C_t^* \right]. \]

Asset markets are assumed to be complete within and across countries. Agents can insure against all possible states of nature by holding a portfolio of state-contingent, one-period securities whose real value (denominated in units of the consumption-based price index) is denoted by \( B^H_j \) and whose vector of prices is denoted by \( q^H \). In addition, agents can trade in a non-contingent, one-period bond whose nominal value (denominated in country \( H \)'s currency) is denoted by \( B^j \) and whose nominal interest rate is denoted by \( R_t \). The intertemporal budget constraint of agent \( j \) in country \( H \) is then given by

\[ C_t^j + q^H_t B^H_t + \frac{B^j_t}{P_t(1 + R_t)} = B^H_{t-1} + \frac{B^j_{t-1}}{P_t} + (1 - \tau^H_t) \frac{p_t(h) y_t(h)}{P_t}, \]

The agent’s income stems also from sales revenues \( p_t(h) y_t(h) \) net of a proportional, country-specific tax \( \tau_t^H \).

All contingent securities and non-contingent bonds are assumed to be in zero supply in the initial period, so \( B^H_0 = B^j_0 = 0 \) for all \( j \). Together with the facts that, within countries, agents have identical preferences and that asset markets are complete, this implies perfect risk sharing of consumption within each country. Therefore, it is possible to analyze the consumer problem from the viewpoint of the representative agent of country \( H \) and country \( F \).

The tax will turn out to be a subsidy to exactly offset the distortion caused by monopolistic competition.
A Flexible exchange rate regime

The representative agent in country $H$ maximizes his lifetime utility (A.1) subject to the budget constraint (A.20). By combining the resulting first order conditions with respect to consumption and bond holdings, the usual Euler consumption equation is then given by

$$U_C(C_t, \zeta_{C,t}) = (1 + R_t) \beta E_t \left\{ U_C(C_{t+1}, \zeta_{C,t+1}) \frac{P_t}{P_{t+1}} \right\}. \quad (A.21)$$

The Euler consumption equation for the representative agent in country $F$ is obtained analogously and given by

$$U_C(C_t^*, \zeta_{C,t}^*) = (1 + R_t^*) \beta E_t \left\{ U_C(C_{t+1}^*, \zeta_{C,t+1}^*) \frac{P_t^*}{P_{t+1}^*} \right\}. \quad (A.22)$$

Complete asset markets across countries leads to price equalization in the state-contingent securities (expressed in country $H$’s currency), implying the following risk sharing condition:

$$\beta \frac{U_C(C_{t+1}, \zeta_{C,t})}{U_C(C_t, \zeta_{C,t})} \frac{P_t}{P_{t+1}} = \beta \frac{U_C(C_{t+1}^*, \zeta_{C,t}^*)}{U_C(C_t^*, \zeta_{C,t}^*)} \frac{S_t P_t^*}{S_{t+1} P_{t+1}^*} \quad (A.23)$$

Inserting the country-specific Euler consumption equations yields the uncovered interest parity condition, according to which the expected change in the nominal exchange rate corresponds to the ratio of the country-specific interest rates:

$$E_t \Delta S_{t+1} = \frac{1 + R_t}{1 + R_t^*}. \quad (A.24)$$

Assuming net foreign asset positions to be initially symmetric and applying the definition of the real exchange rate (A.10), the risk sharing condition takes the following form:

$$Q_t = \left( \frac{C_t^*}{C_t} \right)^{-\rho} \frac{\zeta_{C,t}^*}{\zeta_{C,t}}. \quad (A.25)$$

A.2 Producer problem

In their role as producers, agents act in an environment of monopolistic competition, in which they dispose of some degree of market power. Furthermore, prices are sticky in the sense that the agent is able to change his price in a given period with a fixed probability, as in Calvo (1983). The probability of being able to change the price may differ across countries and is given by $1 - \alpha^i$ for $i = H, F$.

Agent $j$ in country $H$ maximizes expected, discounted profits by choosing the price $\tilde{p}_t(h)$ taking into account that demand depends on the chosen price and that the price may remain unchanged for some periods. Formally, the agent maximizes

$$E_t \sum_{k=0}^{\infty} (\alpha^H)^k \left[ \lambda_{t+k} (1 - \tau^H_{t+k}) \tilde{p}_t(h) \tilde{y}_{t,t+k}(h) - V(\tilde{y}_{t,t+k}(h), \zeta_{Y,t+k}) \right] \quad (A.26)$$

subject to the demand function

$$\tilde{y}_{t,t+k}(h) = \frac{1}{n} \left( \frac{\tilde{p}_t(h)}{P_{H,t+k}} \right)^{-\sigma} \left( \frac{P_{H,t+k}}{P_{t+k}} \right)^{-\theta} \left[ aC_{t+k} + (1 - a)Q_{t+k}^\theta C_{t+k}^* \right] \quad (A.27)$$
where $\tilde{y}_{t,t+k}(h)$ denotes total demand of good $h$ at time $t+k$ if the price $\tilde{p}_t(h)$ prevails. Profits are expressed in utility units. Therefore, nominal sales revenues net of taxes $(1 - \tau^h_t) \tilde{p}_t(h) \tilde{y}_{t,t+k}(h)$ are converted into utility units using the marginal utility of nominal revenues $\lambda_{t+k} = \frac{U_C(C_{t+k}, \zeta_{C_{t+k}})}{p_{t+k}}$. The cost of production expressed in utility units is given by the function $V = \zeta^{-\eta \gamma(h)^{1+\eta}}_{Y,t} \frac{1}{1+\eta}$.

The first order condition yields the optimal price

$$\hat{p}_t(h) = \frac{\sigma}{(\sigma - 1)(1 - \tau^H_t)} \frac{E_t \sum_{k=0}^{\infty} (\alpha^H \beta)^k V_y(\tilde{y}_{t,t+k}(h), \zeta_{Y,t+k}) \tilde{y}_{t,t+k}(h)}{H E_t \sum_{k=0}^{\infty} (\alpha^H \beta)^k \lambda_{t+k} \tilde{y}_{t,t+k}(h)},$$

(A.28)

where $V_y$ denotes the derivative of function $V$ with respect to output $\tilde{y}(h)$. All agents that live in the same country and are able to reset their price in a certain period will set the same price, since they share identical preferences (function $V$) and face the same demand curves, which depend only on aggregate variables such as $P_s, P^*, S, C$ and $C^*$, and the common elasticities of substitution $\sigma$ and $\theta$. Hence, in a given period, a fraction $1 - \alpha^i$ of agents will set the same optimal price, while for a fraction $\alpha^i$ of agents the price from the previous period remains effective:

$$P_{H,t} = [\alpha^H P_{H,t-1}^{1-\sigma} + (1 - \alpha^H) \hat{p}_t(h)]^{1-\sigma}$$

(A.29)

$$P_{F,t}^* = [\alpha^F P_{F,t-1}^{1-\sigma} + (1 - \alpha^F) \hat{p}_t^*(f)]^{1-\sigma}.$$  

When prices are flexible, the optimal-price equation (A.28) for country $H$ simplifies to

$$\frac{P_{H,t}}{\hat{p}_t} = \frac{\sigma}{(\sigma - 1)(1 - \tau^H_t)} \frac{V_y(y_t(h), \zeta_{Y,t})}{U_C(C_t, \zeta_{C,t})},$$

(A.30)

and for country $F$ to

$$\frac{P_{F,t}^*}{\hat{p}_t^*} = \frac{\sigma}{(\sigma - 1)(1 - \tau^F_t)} \frac{V_y(y_t(f), \zeta_{Y,t}^*)}{U_C(C_t^*, \zeta_{C,t}^*)}.$$  

(A.31)

Moreover, variations in the marginal disutility of production of one country relative to the other country are reflected in relative variations in the terms of trade and the real exchange rate. Dividing the previous two equations yields

$$\frac{T_t}{Q_t} = \frac{1 - \tau^H_t}{1 - \tau^F_t} \frac{V_y(y_t(f), \zeta_{Y,t})}{V_y(y_t(h), \zeta_{Y,t})}. $$

(A.32)

The markup that agents in country $i = H, F$ are able to charge is defined as

$$\mu^i_t \equiv \frac{\sigma}{(\sigma - 1)(1 - \tau^i_t)}.$$  

(A.33)

### A.3 Log-linearization

Deviations of the logarithm of a variable $X_t$ from its steady state are denoted by $\tilde{X}_t$ if prices are sticky and by $\tilde{X}_t^{fb}$ if prices are flexible and markups are neutralized (efficient allocation).
A.3.1 Sticky-price equilibrium

Under sticky prices, the system of equations is given by

\[
E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} \left( \hat{R}_t - E_t \pi_{t+1} + E_t \hat{C}_{C,t+1} - \hat{C}_{C,t} \right) \tag{A.34}
\]

\[
\hat{Q}_t = \rho (\hat{C}_t - \hat{C}_t^*) + (\hat{C}_{C,t}^* - \hat{C}_{C,t}) \tag{A.35}
\]

\[
E_t \Delta \hat{S}_{t+1} = \hat{R}_t - \hat{R}_t^* \tag{A.36}
\]

\[
\hat{Q}_t = (2a - 1) \hat{T}_t \tag{A.37}
\]

\[
\hat{Y}_{H,t} = 2a(1 - a) \theta \hat{T}_t + a \hat{C}_t + (1 - a) \hat{C}_t^* \tag{A.38}
\]

\[
\hat{Y}_{F,t} = -2a(1 - a) \theta \hat{T}_t + (1 - a) \hat{C}_t + a \hat{C}_t^* \tag{A.39}
\]

\[
\pi_{H,t} = k_Y H \left( \hat{Y}_{H,t} - \hat{Y}_{H,t}^{fb} \right) - 2a(1 - a) k_T^H \left( \hat{T}_t - \hat{T}_t^{fb} \right) + k_{\mu}^H \mu_t^H + \beta E_t \pi_{H,t+1} \tag{A.40}
\]

\[
\pi_{F,t}^* = k_Y^F \left( \hat{Y}_{F,t} - \hat{Y}_{F,t}^{fb} \right) + 2a(1 - a) k_T^F \left( \hat{T}_t - \hat{T}_t^{fb} \right) + k_{\mu}^F \mu_t^F + \beta E_t \pi_{F,t+1}^* \tag{A.41}
\]

\[
\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t} + \Delta \hat{S}_t \tag{A.42}
\]

\[
\pi_t = a \pi_{H,t} + (1 - a)(\pi_{F,t} - \Delta \hat{S}_t) \tag{A.43}
\]

\[
\pi_t^* = (1 - a)(\pi_{H,t} - \Delta \hat{S}_t) + a \pi_{F,t}^* \tag{A.44}
\]

Equation (A.34) is the log-linear approximation of the Euler consumption equation (A.21), where \( \pi_t = \ln(P_t/P_{t-1}) \). Equation (A.35) is the log-linear approximation of the risk sharing condition (A.25), and equation (A.36) is the log-linear approximation of the uncovered interest parity condition (A.24).\(^{34}\) Equation (A.37) describes the link between the real exchange rate and the terms of trade and is obtained by combining the log-linearized definition of the real exchange rate (A.10) with the log-linearized expressions for the country-specific CPI’s (A.5) and (A.9), applying the law of one price. Equations (A.38) and (A.39) are obtained by log-linearizing the aggregate demand equations (A.19), using equation (A.37) to eliminate the real exchange rate.

Equations (A.40) and (A.41) represent the New Keynesian Phillips curves for country \( H \) and country \( F \) respectively, where \( \pi_{H,t} = \ln(P_{H,t}/P_{H,t-1}) \) and \( \pi_{F,t}^* = \ln(P_{F,t}^*/P_{F,t-1}^*) \). They are derived by combining the log-linear approximation of the optimal price (A.28) with the log-linear approximation of (A.29) for each country separately. The parameters in front of the output gap (\( \hat{Y}_{i,t} - \hat{Y}_{i,t}^{fb} \)), the terms-of-trade gap (\( \hat{T}_t - \hat{T}_t^{fb} \)), and the markup \( \mu_t^i \) are defined as follows (for \( i = H, F \)):

\[
k_Y^i = \frac{(1 - \alpha' \beta)(1 - \alpha')}{\alpha'} \left( \frac{\rho + \eta}{1 + \sigma \eta} \right) \tag{A.45}
\]

\[
k_T^i = \frac{(1 - \alpha' \beta)(1 - \alpha')}{\alpha'} \left( \frac{\rho \theta - 1}{1 + \sigma \eta} \right) \tag{A.46}
\]

\[
k_{\mu}^i = \frac{(1 - \alpha' \beta)(1 - \alpha')}{\alpha'} \left( \frac{1}{1 + \sigma \eta} \right) \tag{A.47}
\]

\(^{34}\) Alternatively, the model could be specified by including both country-specific Euler consumption equations next to the risk sharing condition, while omitting the uncovered interest parity condition.
Equation (A.42) is the log-linear approximation of the terms of trade (A.11), expressed in first differences. Equations (A.43) and (A.44) are the log-linear approximations of the country-specific CPI's (A.5) and (A.9), applying the law of one price and expressed in first differences.

The model is closed by a Taylor-type interest rate rule for each country, according to which monetary policy responds to inflation and output with coefficients $\phi_\pi$ and $\phi_Y$ and engages in interest rate smoothing with coefficient $\phi_R$. Monetary policy responds either to PPI inflation or to CPI inflation. If monetary policy responds to PPI inflation, the interest rate rules are given by

$$\hat{R}_t = \phi_R \hat{R}_{t-1} + (1 - \phi_R)(\phi_\pi \pi_H, t + \phi_Y \hat{Y}_H, t)$$

$$\hat{R}_t^* = \phi_R \hat{R}_{t-1}^* + (1 - \phi_R)(\phi_\pi \pi_H^*, t + \phi_Y \hat{Y}_F, t).$$

If monetary policy responds to CPI inflation, the interest rate rules are given by

$$\hat{R}_t = \phi_R \hat{R}_{t-1} + (1 - \phi_R)(\phi_\pi \pi_t + \phi_Y \hat{Y}_H, t)$$

$$\hat{R}_t^* = \phi_R \hat{R}_{t-1}^* + (1 - \phi_R)(\phi_\pi \pi_t^* + \phi_Y \hat{Y}_F, t).$$

### A.3.2 Efficient allocation

The first-best (fb) or efficient allocation describes the equilibrium in which prices are fully flexible and in which markups are neutralized at all times with an appropriate subsidy ($\mu_i^t = 0$). This efficient allocation provides a useful benchmark in order to assess the welfare implications of the two international monetary regimes.

Accordingly, efficient output in each country is given by

$$(\rho + \eta)\tilde{Y}_{f,t}^H = 2a(1 - a)(\rho \theta - 1)\tilde{Y}_{f,t}^H$$

$$- (1 - a)(\hat{\zeta}_{C,t} - \hat{\zeta}_{C,t}^*) + \hat{\zeta}_{C,t} + \eta \hat{\zeta}_{Y,t}$$

$$(\rho + \eta)\tilde{Y}_{f,t}^F = -2a(1 - a)(\rho \theta - 1)\tilde{Y}_{f,t}^F$$

$$+ (1 - a)(\hat{\zeta}_{C,t} - \hat{\zeta}_{C,t}^*) + \hat{\zeta}_{C,t}^* + \eta \hat{\zeta}_{Y,t}^*.$$

The efficient terms of trade can be written as

$$[4a(1 - a)\rho \theta + (2a - 1)^2]\tilde{Y}_{f,t}^H = \rho (\tilde{Y}_{H,t} - \tilde{Y}_{F,t}) - (2a - 1)(\hat{\zeta}_{C,t} - \hat{\zeta}_{C,t}^*).$$

The first equation is obtained by combining the risk sharing condition (A.35), equation (A.37), and the aggregate demand equation (A.38), all of which hold under flexible prices as well, with the log-linear approximation of the optimal-price equation (A.30). The second equation is derived completely analogously. The third equation is derived by subtracting the country-specific aggregate demand equations (A.38) and (A.39) from each other and by using the risk sharing condition (A.35) and equation (A.37) to eliminate country-specific consumption and the real exchange rate.
B Monetary union regime

The main difference of the monetary union regime compared to the flexible exchange rate regime, of course, is that the two countries share one currency and that the common monetary policy sets one union-wide nominal interest rate. Notwithstanding, the model structure is to a large extent identical (see, e.g., Benigno, 2004).

Under flexible prices, monetary policy is neutral, so that real variables are only driven by fundamental shocks. Thus, the efficient allocation is independent of the international monetary regime. Therefore, the behavior of efficient output and the efficient terms of trade under the monetary union regime is also described by equations (A.52) and (A.53).

Under sticky prices, the system of equations is given by

\[
\begin{align*}
E_t \hat{C}_{t+1} &= \hat{C}_t + \frac{1}{\phi} \left( \hat{R}^{MU}_t - E_t \pi_{t+1} + E_t \hat{\zeta}_{C,t+1} - \hat{\zeta}_C \right) \\
\hat{\zeta}_t &= \rho \left( \hat{C}_t - \tilde{C}_t \right) + \left( \hat{\zeta}^C_{C,t} - \hat{\zeta}_C \right) \\
\hat{Q}_t &= (2a - 1) \hat{F}_t \\
\hat{Y}_{H,t} &= 2a(1-a)\hat{F}_t + a\hat{C}_t + (1-a)\hat{C}^*_t \\
\hat{Y}_{F,t} &= -2a(1-a)\hat{T}_t + (1-a)\hat{C}_t + a\hat{C}^*_t \\
\pi_{H,t} &= k^H_{Y} \left( \hat{Y}_{H,t} - \hat{Y}^{f}_{H,t} \right) - 2a(1-a)k^H_{T} \left( \hat{T}_t - \hat{T}^{f}_{t} \right) + k^H_{\mu} \hat{\mu}_{t} + \beta E_t \pi_{H,t+1} \\
\pi_{F,t}^* &= k^F_{Y} \left( \hat{Y}_{F,t} - \hat{Y}^{f}_{F,t} \right) + 2a(1-a)k^F_{T} \left( \hat{T}_t - \hat{T}^{f}_{t} \right) + k^F_{\mu} \hat{\mu}_{t} + \beta E_t \pi_{F,t+1} \\
\hat{F}_t &= \hat{F}_{t-1} + \pi_{F,t}^* - \pi_{H,t} \\
\pi_{t} &= a\pi_{H,t} + (1-a)\pi_{F,t}^* \\
\pi_{t}^* &= (1-a)\pi_{H,t} + a\pi_{F,t}^*.
\end{align*}
\]

The Euler consumption equation (B.1) differs from the one under the flexible exchange rate regime only in that the nominal interest rate is given by the union-wide interest rate \( \hat{R}^{MU}_t \). Nonetheless, real interest rates \( \hat{R}^{MU}_t - E_t \pi_{t+1} \) are generally country-specific, since CPI inflation rates usually differ across countries. Since the two countries form a monetary union, the uncovered interest parity condition is obsolete and the nominal exchange rate disappears from all relevant equations.

The model is closed by a Taylor-type interest rate rule, according to which the common monetary policy responds to union-wide inflation (average of country-specific inflation) and union-wide output (average of country-specific output) with coefficients \( \phi_{\pi} \) and \( \phi_{Y} \) and engages in interest rate smoothing with coefficient \( \phi_{R} \). If monetary policy responds to PPI inflation, the interest rate rule is given by

\[
\hat{R}^{MU}_t = \phi_{R} \hat{R}^{MU}_{t-1} + (1 - \phi_{R}) \left( \phi_{\pi} \frac{\pi_{H,t} + \pi_{F,t}^*}{2} + \phi_{Y} \frac{\hat{Y}_{H,t} + \hat{Y}_{F,t}}{2} \right). \tag{B.11}
\]

If monetary policy responds to CPI inflation, the interest rate rule is given by

\[
\hat{R}^{MU}_t = \phi_{R} \hat{R}^{MU}_{t-1} + (1 - \phi_{R}) \left( \phi_{\pi} \frac{\pi_{t} + \pi_{t}^*}{2} + \phi_{Y} \frac{\hat{Y}_{H,t} + \hat{Y}_{F,t}}{2} \right). \tag{B.12}
\]
Using equations (B.9) and (B.10), it is straightforward to show that the average of CPI inflation rates is equal to the average of PPI inflation rates:

\[ \frac{\pi_t + \pi_t^*}{2} = \frac{\pi_{H,t} + \pi_{F,t}^*}{2} \equiv \pi_{MU}^t. \]  \hspace{1cm} (B.13)

As a result, the interest rate rule is the same irrespective of whether the common monetary policy responds to CPI or PPI inflation:

\[ \hat{R}_{MU}^t = \phi_R \hat{R}_{MU}^{t-1} + (1 - \phi_R) \left( \phi_{\pi} \pi_{MU}^t + \phi_{Y} \frac{\hat{Y}_{H,t} + \hat{Y}_{F,t}}{2} \right). \]  \hspace{1cm} (B.14)

This implies that the behavior of all variables and the resulting welfare losses are the same irrespective of whether the common monetary policy responds to CPI or PPI inflation.