Monetary Policy as an Optimum Currency Area Criterion*

Dominik Groll
Kiel Institute for the World Economy

The costs and benefits of moving from a flexible exchange rate regime to a monetary union depend critically on the conduct of monetary policy. In particular, whether countries are better off in one or the other currency regime is sensitive not only to the choice of the variables that monetary policy targets but also to the strength of the response to these target variables. In addition to being an optimum currency area (OCA) criterion itself, monetary policy can modify the nature of traditional OCA criteria, such as the degree of trade openness.

JEL Codes: F33, F41, E52.

1. Introduction

Over the decades since its initiation by Mundell (1961), the optimum currency area (OCA) theory has identified numerous criteria that are considered important in determining whether countries benefit from monetary unification. Traditional OCA criteria include the degree of labor mobility, price and wage flexibility, trade openness, the incidence of asymmetric shocks, country size, the similarity of economic structures, the degree of product diversification, and the degree of fiscal integration.

However, one criterion has received hardly any attention, although it is critical for the welfare implications of monetary unification: the conduct of monetary policy. In particular, I show that whether countries are better off under a flexible exchange rate regime or a monetary union is sensitive not only to the choice of the variables that monetary policy targets but also to the strength of the response

*Author e-mail: dominik.groll@ifw-kiel.de. Phone: +49 431 8814 266.

1Excellent surveys of the OCA literature are Mongelli (2002), Dellas and Tavlas (2009), Beetsma and Giuliodori (2010), and De Grauwe (2012).
to these target variables. When monetary policy in each country responds to inflation aggressively or implements a high degree of interest rate smoothing, forming a monetary union, where the common monetary authority continues to follow the same policy, tends to make countries worse off in terms of welfare by reducing macroeconomic stability. By contrast, when monetary policy responds to inflation only modestly or implements a low degree of interest rate smoothing, forming a monetary union with exactly the same monetary policy tends to make countries better off. Furthermore, monetary unification is beneficial when monetary policy responds to output, whereas it is costly when monetary policy responds to the output gap. And finally, it is important whether countries respond to the nominal exchange rate and whether they do so in a coordinated or uncoordinated way. Monetary unification is generally beneficial in the latter case, but not in the former case.

I show that monetary policy, in addition to being an OCA criterion itself, has the potential to modify the nature of traditional OCA criteria, such as the degree of trade openness. Whether the likelihood of a monetary union being beneficial increases with the degree of trade openness, as proposed by the vast bulk of OCA studies, depends critically on whether independent monetary policy targets producer price inflation or consumer price inflation. In the former case, it is also possible that the likelihood of a monetary union being beneficial decreases with the degree of trade openness.

The conduct of monetary policy matters for the welfare implications of monetary unification for two reasons. First, monetary policy determines to what extent a flexible nominal exchange rate fosters or hampers macroeconomic stabilization, even if monetary policy does not target the nominal exchange rate explicitly. A flexible nominal exchange rate renders monetary policy more powerful in the sense that monetary policy affects all welfare-relevant variables directly. By contrast, in a monetary union, the influence of monetary policy is limited by the fixed exchange rate, especially with respect to international relative prices such as the terms of trade. However, the fact that monetary policy is more powerful under a flexible exchange rate regime is a double-edged sword. When the interest rate policy happens to move the nominal exchange rate in the “right” direction, forming a monetary union generally—not always (see second reason)—reduces macroeconomic stability and welfare by
eliminating the *stabilizing* effects of the nominal exchange rate. By contrast, when the interest rate policy happens to move the nominal exchange rate in the “wrong” direction, forming a monetary union increases macroeconomic stability and welfare by eliminating the *destabilizing* effects of the nominal exchange rate. Importantly, which policies move the exchange rate in which direction is anything but obvious.

The second reason for monetary policy being an OCA criterion is the existence of a benefit that is inherent to monetary unions (Groll and Monacelli 2020). This renders a monetary union beneficial even for interest rate policies that move the nominal exchange rate in the right direction, e.g., a modest response to inflation. While constraining monetary policy to some extent, the fixed exchange rate has the advantage of stabilizing private-sector expectations about future inflation and thereby stabilizing actual inflation. This can overcompensate for the cost of inefficient fluctuations in international relative prices, which are also due to the fixed exchange rate.

With few exceptions, these conclusions are not to any important degree sensitive to the price-setting assumption (producer-currency pricing versus local-currency pricing) or the type of shocks (productivity shocks versus cost-push shocks). However, local-currency pricing and cost-push shocks—individually as well as jointly—tend to increase the likelihood that countries benefit from monetary unification. Compared with producer-currency pricing, local-currency pricing renders monetary unification more favorable because the benefit of exchange rate flexibility in the presence of nominal price rigidity—and therefore the cost of fixing the exchange rate—is considerably smaller. Under local-currency pricing, import prices no longer fluctuate one-to-one with the exchange rate but are as sticky as domestic prices. Therefore, exchange rate flexibility no longer facilitates the desirable adjustment in international relative goods prices. Compared with productivity shocks, cost-push shocks render monetary unification more favorable because the inherent benefit of monetary unions mentioned above is stronger under these circumstances. Cost-push shocks induce (possibly additional) tradeoffs for monetary policy in stabilizing different welfare-relevant variables. The bigger these tradeoffs are, the greater is the benefit of stabilizing private-sector expectations about future inflation.
1.1 Contribution to the Literature

The idea that monetary policy is an important OCA criterion has been touched upon in the literature at best only indirectly. There are two basic arguments: According to the “credibility” argument, a country that is unable to withstand the temptation to induce surprise inflations in a discretionary way suffers from both a higher level and a higher instability of inflation.\footnote{In practice, there are a number of reasons to pursue such a policy. In times with a private or public debt overhang, monetary policy might let the inflation rate overshoot the inflation target for a prolonged period of time, with the aim of reducing the real debt burden and lowering borrowing costs. In times of high unemployment, this might seem attractive because it reduces real wages in the presence of fixed-term nominal wage contracts, thereby increasing the demand for labor.} Joining or forming a monetary union can compensate for such a lack of commitment, thereby reducing the long-run level of inflation (Giavazzi and Pagano 1988; Alesina and Barro 2002; Chari, Dovis, and Kehoe 2020) and increasing the stability of inflation (Cook and Devereux 2016; Groll and Monacelli 2020).\footnote{Note that the “currency union” in Alesina and Barro (2002) refers to a situation where a client country unilaterally adopts the currency of an anchor country—a situation also known as dollarization. Nevertheless, the benefit of eliminating an inflation bias also exists if the client and anchor country form a monetary union where the common monetary policy inherits the credibility of the anchor country. The “advantage of tying one’s hands” in Giavazzi and Pagano (1988) follows the same logic, while referring to the former European Monetary System (1979–99).}

According to the “competitive devaluations” argument, high and variable inflation arises when two countries with competing monetary policies try to strategically manipulate the real exchange rate or the terms of trade in their own favor. If the two countries form a monetary union, competitive devaluations are no longer possible and inflation is both lower (Cooley and Quadrini 2003) and more stable (Pappa 2004).

Without explicitly making the point, these contributions show en passant that monetary policy is an important OCA criterion. That is, whether countries are better off with flexible exchange rates or in a monetary union depends on whether their monetary authorities credibly commit to future policies (commitment versus discretion) and whether they coordinate their policies (coordination versus competition). In this paper, I broaden the perspective by looking...
through the lens of practical interest rate rules, thereby highlighting
the importance of two different dimensions of monetary policy: the
choice of the target variables that monetary policy responds to and
the strength of the response to these variables. This enables me to
describe the implications of a wide variety of interest rate policies
reflecting the diversity of monetary policy in practice. It is impor-
tant to realize that monetary policy represents an OCA criterion not
only under these suboptimal interest rate rules but also under opti-
mal monetary policy. This is shown by the studies mentioned above,
which are all based on some form of optimal monetary policy.

The remainder of this paper is organized as follows. Section 2
briefly outlines the structure of the model. Section 3 shows how dif-
ferent interest rate policies lead to different welfare rankings between
a monetary union and a flexible exchange rate regime. Section 4
shows how different interest rate policies change the nature of the
traditional OCA criterion of trade openness. Section 5 concludes.

2. Model

The model I use is a standard two-country New Keynesian dynamic
stochastic general equilibrium (DSGE) model, and thus I provide
only a very brief description. The model features two currency
regimes:

(i) A monetary union (MU) regime: Both countries share the
same currency. A common monetary policy governs the com-
mon nominal interest rate.

(ii) A flexible exchange rate (FX) regime: Each country maintains
its national currency and conducts its own, independent mon-
etary policy. Nominal interest rates are country specific. The
nominal exchange rate between the two currencies is flexible.

The FX version of the model, including the microfounded, qua-
dratic welfare measure, is described in Corsetti, Dedola, and Leduc
(2011). The MU version of the model is largely identical (see,
e.g., Benigno 2004). The model economy features two countries of
equal size (labeled $H$ and $F$) with trade in consumption goods.
The consumption baskets are allowed to differ among countries, so
purchasing power parity does not necessarily hold. International asset markets are complete, i.e., risk sharing is perfect across countries. Producers act in an environment of monopolistic competition. The only factor of production is labor, which is immobile between countries. The only rigidity is the nominal price rigidity in the spirit of Calvo (1983).

Under the FX regime, the baseline model assumes “producer-currency pricing.” Prices are set in the currency of the producer’s country. The price of imports expressed in domestic currency fluctuates one-to-one with the nominal exchange rate. Thus, the law of one price holds and exchange rate pass-through to import prices is complete. This implies that import prices are not sticky even though prices for domestically produced goods are sticky.

In order to check whether the main results are sensitive to the price-setting assumption, the case of “local-currency pricing” is also considered. Under local-currency pricing, prices are set in the currency of the destination market, i.e., in domestic currency if the good is sold domestically, and in foreign currency if the good is sold abroad. This implies that not only prices for domestically produced goods but also import prices are sticky. As a result, exchange rate pass-through is incomplete, and fluctuations in the nominal exchange rate lead to temporary deviations from the law of one price.

Under the MU regime, local-currency pricing is literally impossible, as both countries share one common currency. While other forms of price discrimination are clearly conceivable within a monetary union, modeling them is beyond the scope of this paper. Thus, the law of one price is assumed to always hold under the MU regime.

2.1 Model Equations

The equations of the complete log-linearized model are shown below (for the full derivation, see appendixes A and B). Deviations of the logarithm of a variable $X_t$ from its steady state are denoted by $\hat{X}_t$.

\[^{4}\text{To the best of my knowledge, price discrimination within a monetary union has not yet been modeled in the New Keynesian open-economy macroeconomics literature. Interestingly, there is empirical evidence supporting the idea that the law of one price holds within a monetary union but not outside (see, e.g., Cavallo, Neiman, and Rigobon 2014).}\]
Table 1. Variables

| \( C_t, C^*_t \) | Consumption in Country \( H \) and \( F \), Respectively |
| \( Y_{H,t}, Y^*_{F,t} \) | Output in Country \( H \) and \( F \), Respectively |
| \( \pi_{H,t} \) | Producer Price Inflation in Country \( H \) in Country \( H \)'s Currency |
| \( \pi^*_{H,t} \) | Producer Price Inflation in Country \( H \) in Country \( F \)'s Currency |
| \( \pi_{F,t} \) | Producer Price Inflation in Country \( F \) in Country \( H \)'s Currency |
| \( \pi^*_{F,t} \) | Producer Price Inflation in Country \( F \) in Country \( F \)'s Currency |
| \( \pi_t, \pi^*_t \) | Consumer Price Inflation in Country \( H \) and \( F \), Respectively |
| \( \pi_t^{MU} \) | Union-wide Inflation (Average of Country-Specific Inflation) |
| \( R_t, R^*_t \) | Nominal Interest Rate in Country \( H \) and \( F \), Respectively |
| \( R_t^{MU} \) | Nominal Interest Rate in Monetary Union |
| \( T_t \) | Terms of Trade |
| \( S_t \) | Nominal Exchange Rate |
| \( Q_t \) | Real Exchange Rate |
| \( \Delta_t \) | Deviation from the Law of One Price |
| \( \zeta_{Y,t}, \zeta^*_{Y,t} \) | Productivity Shock in Country \( H \) and \( F \), Respectively |
| \( \zeta_{C,t}, \zeta^*_{C,t} \) | Consumption Preference Shock in Country \( H \) and \( F \), Respectively |
| \( \mu^H_t, \mu^F_t \) | Cost-Push (or Markup) Shock in Country \( H \) and \( F \), Respectively |

Table 2. Parameters and Baseline Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>1/6</td>
</tr>
<tr>
<td>Inverse of Elasticity of Intertemporal Substitution in Consumption</td>
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</tr>
<tr>
<td>( \beta )</td>
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<tr>
<td>Discount Factor</td>
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<tr>
<td>( \eta )</td>
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<tr>
<td>Inverse of Elasticity of Producing the Differentiated Good</td>
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<td>( \epsilon_{wy} )</td>
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<tr>
<td>Production Elasticity of Average Real Wage</td>
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</tr>
<tr>
<td>( \gamma )</td>
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</tr>
<tr>
<td>Labor Income Share</td>
<td></td>
</tr>
<tr>
<td>( a )</td>
<td>0.75</td>
</tr>
<tr>
<td>Home Bias/Degree of Trade Openness</td>
<td></td>
</tr>
<tr>
<td>( \sigma )</td>
<td>7.66</td>
</tr>
<tr>
<td>Elasticity of Substitution between Differentiated Goods within Countries</td>
<td></td>
</tr>
<tr>
<td>( \theta )</td>
<td>2</td>
</tr>
<tr>
<td>Elasticity of Substitution between Goods across Countries</td>
<td></td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.75</td>
</tr>
<tr>
<td>Probability of Not Being Able to Reset the Price</td>
<td></td>
</tr>
</tbody>
</table>

if prices are sticky and by \( \tilde{X}^b_t \) if prices are flexible and markups are neutralized (efficient allocation). The variables and parameters are defined in tables 1 and 2, respectively.

2.1.1 Sticky-Price Model under the FX Regime

**Producer-Currency Pricing.** Under sticky prices, the model equations for the FX regime and producer-currency pricing are given by
\[ E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} \left( \hat{R}_t - E_t \pi_{t+1} + E_t \zeta_{C,t+1} - \hat{\zeta}_{C,t} \right) \]  

(1)

\[ \hat{Q}_t = \rho \left( \hat{C}_t - \hat{C}_t^* \right) + \left( \hat{\zeta}_{C,t}^* - \hat{\zeta}_{C,t} \right) \]  

(2)

\[ E_t \Delta \hat{S}_{t+1} = \hat{R}_t - \hat{R}_t^* \]  

(3)

\[ \hat{Q}_t = (2a - 1) \hat{T}_t \]  

(4)

\[ \hat{Y}_{H,t} = 2a(1 - a)\theta \hat{T}_t + a \hat{C}_t + (1 - a) \hat{C}_t^* \]  

(5)

\[ \hat{Y}_{F,t} = -2a(1 - a)\theta \hat{T}_t + (1 - a) \hat{C}_t + a \hat{C}_t^* \]  

(6)

\[ \pi_{H,t} = (\rho + \eta)k \left( \hat{Y}_{H,t} - \tilde{Y}_{H,t}^{fb} \right) - 2a(1 - a)(\rho \theta - 1)k \left( \hat{T}_t - \tilde{T}_t^{fb} \right) \]

\[ + k\hat{\mu}_t^H + \beta E_t \pi_{H,t+1} \]  

(7)

\[ \pi_{F,t}^* = (\rho + \eta)k \left( \hat{Y}_{F,t} - \tilde{Y}_{F,t}^{fb} \right) + 2a(1 - a)(\rho \theta - 1)k \left( \hat{T}_t - \tilde{T}_t^{fb} \right) \]

\[ + k\hat{\mu}_t^F + \beta E_t \pi_{F,t+1} \]  

(8)

\[ \hat{T}_t = \hat{T}_{t-1} + \pi_{F,t}^* - \pi_{H,t} + \Delta \hat{S}_t \]  

(9)

\[ \pi_t = a \pi_{H,t} + (1 - a)(\pi_{F,t}^* + \Delta \hat{S}_t) \]  

(10)

\[ \pi_t^* = (1 - a)(\pi_{H,t} - \Delta \hat{S}_t) + a \pi_{F,t}^* \]  

(11)

where

\[ k = \frac{(1 - \alpha \beta)(1 - \alpha)}{\alpha} \frac{1}{1 + \sigma \eta}. \]  

(12)

Monetary policy in each country can respond to some measure of inflation, to some measure of output, and to the nominal exchange rate, and it can engage in interest rate smoothing. The specific functional forms of the interest rate rules will be shown in section 3.

**Local-Currency Pricing.** Under sticky prices, the model equations for the FX regime and local-currency pricing are given by

\[ E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} \left( \hat{R}_t - E_t \pi_{t+1} + E_t \zeta_{C,t+1} - \hat{\zeta}_{C,t} \right) \]  

(13)
\[ \dot{Q}_t = \rho \left( \hat{C}_t - \hat{C}_t^* \right) + \left( \hat{\zeta}_{C,t} - \hat{\zeta}_{C,t} \right) \]  
\( E_t \Delta \hat{S}_{t+1} = \hat{R}_t - \hat{R}_t^* \)  
\[ \dot{Q}_t = (2a - 1)\hat{T}_t + 2a\hat{\Delta}_t \]  
\[ \hat{Y}_{H,t} = 2a(1 - a)\theta(\hat{T}_t + \hat{\Delta}_t) + a\hat{C}_t + (1 - a)\hat{C}_t^* \]  
\[ \hat{Y}_{F,t} = -2a(1 - a)\theta(\hat{T}_t + \hat{\Delta}_t) + (1 - a)\hat{C}_t + a\hat{C}_t^* \]  
\[ \pi_{H,t} = (\rho + \eta)k \left( \dot{Y}_{H,t} - \dot{Y}_{H,t}^{fb} \right) \]  
\[ - (1 - a)k \left[ 2a(\rho\theta - 1) \left( \hat{T}_t - \hat{T}_t^{fb} + \hat{\Delta}_t \right) - \hat{\Delta}_t \right] \]  
\[ + k\hat{\mu}_t^H + \beta E_t \pi_{H,t+1} \]  
\[ \pi_{F,t} = (\rho + \eta)k \left( \dot{Y}_{F,t} - \dot{Y}_{F,t}^{fb} \right) \]  
\[ + (1 - a)k \left[ 2a(\rho\theta - 1) \left( \hat{T}_t - \hat{T}_t^{fb} + \hat{\Delta}_t \right) - \hat{\Delta}_t \right] + k\hat{\Delta}_t \]  
\[ + k\hat{\mu}_t^F + \beta E_t \pi_{F,t+1} \]  
\[ \pi_{F,t}^* = (\rho + \eta)k \left( \dot{Y}_{F,t} - \dot{Y}_{F,t}^{fb} \right) \]  
\[ + (1 - a)k \left[ 2a(\rho\theta - 1) \left( \hat{T}_t - \hat{T}_t^{fb} + \hat{\Delta}_t \right) - \hat{\Delta}_t \right] \]  
\[ + k\hat{\mu}_t^F + \beta E_t \pi_{F,t+1}^* \]  
\[ \hat{T}_t = \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t}^* - \Delta \hat{S}_t \]  
\[ \pi_t = a\pi_{H,t} + (1 - a)\pi_{F,t} \]  
\[ \pi_t^* = (1 - a)\pi_{H,t}^* + a\pi_{F,t}^* \]  
\[ \hat{\Delta}_t = \hat{\Delta}_{t-1} + \Delta S_t + \pi_{H,t}^* - \pi_{H,t}. \]
2.1.2 Sticky-Price Model under the MU Regime

Under sticky prices, the model equations for the MU regime are given by

\[ E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} \left( \hat{R}_{t}^{MU} - E_t \pi_{t+1} + E_t \hat{\xi}_{C,t+1} - \hat{\zeta}_{C,t} \right) \]  \hspace{1cm} (27)

\[ \hat{Q}_t = \rho \left( \hat{C}_t - \hat{C}^*_t \right) + \left( \hat{\zeta}^*_C - \hat{\zeta}_{C,t} \right) \]  \hspace{1cm} (28)

\[ \hat{Q}_t = (2a - 1) \hat{T}_t \]  \hspace{1cm} (29)

\[ \hat{Y}_{H,t} = 2a(1 - a)\theta \hat{T}_t + a \hat{C}_t + (1 - a)\hat{C}^*_t \]  \hspace{1cm} (30)

\[ \hat{Y}_{F,t} = -2a(1 - a)\theta \hat{T}_t + (1 - a)\hat{C}_t + a\hat{C}^*_t \]  \hspace{1cm} (31)

\[ \pi_{H,t} = (\rho + \eta)k \left( \hat{Y}_{H,t} - \tilde{Y}_{fb,H,t} \right) - 2a(1 - a)(\rho \theta - 1)k \left( \hat{T}_t - \tilde{T}_{fb,t} \right) + k\hat{\mu}^H_t + \beta E_t \pi_{H,t+1} \]  \hspace{1cm} (32)

\[ \pi^*_{F,t} = (\rho + \eta)k \left( \hat{Y}_{F,t} - \tilde{Y}_{fb,F,t} \right) + 2a(1 - a)(\rho \theta - 1)k \left( \hat{T}_t - \tilde{T}_{fb,t} \right) + k\hat{\mu}^F_t + \beta E_t \pi^*_{F,t+1} \]  \hspace{1cm} (33)

\[ \hat{T}_t = \hat{T}_{t-1} + \pi^*_{F,t} - \pi_{H,t} \]  \hspace{1cm} (34)

\[ \pi_t = a \pi_{H,t} + (1 - a) \pi^*_{F,t} \]  \hspace{1cm} (35)

\[ \pi^*_t = (1 - a) \pi_{H,t} + a \pi^*_F \]  \hspace{1cm} (36)

The common monetary policy responds to union-wide variables, i.e., to cross-country averages. The specific functional forms of the interest rate rule will be shown in section 3.

Note that whether the common monetary policy responds to producer price inflation or consumer price inflation does not make a difference in this model, given that the two countries are of equal size. Using equations (35) and (36), it is straightforward to show that the average of consumer price inflation rates is equal to the average of producer price inflation rates:

\[ \frac{\pi_t + \pi^*_t}{2} = \frac{\pi_{H,t} + \pi^*_{F,t}}{2} \equiv \pi_t^{MU}. \]  \hspace{1cm} (37)
2.1.3 Efficient Allocation

The following equations describe the first-best \((fb)\) or efficient allocation, where prices are fully flexible, where the law of one price holds, and where markups are neutralized at all times with an appropriate subsidy \((\mu_i = 0)\). This efficient allocation provides a useful benchmark for assessing the welfare implications of the two currency regimes.

The efficient output in each country is given by

\[
(\rho + \eta)\tilde{Y}_{H,t}^{fb} = 2a(1 - a)(\rho \theta - 1)\tilde{T}^{fb}_t
- (1 - a) \left( \tilde{\zeta}_{C,t} - \tilde{\zeta}_{C,t}^* \right) + \tilde{\zeta}_{C,t} + \eta \tilde{\zeta}_{Y,t} \tag{38}
\]

\[
(\rho + \eta)\tilde{Y}_{F,t}^{fb} = -2a(1 - a)(\rho \theta - 1)\tilde{T}^{fb}_t
+ (1 - a) \left( \tilde{\zeta}_{C,t} - \tilde{\zeta}_{C,t}^* \right) + \tilde{\zeta}_{C,t}^* + \eta \tilde{\zeta}_{Y,t}^*. \tag{39}
\]

The efficient terms of trade can be written as

\[
[4a(1 - a)\rho \theta + (2a - 1)^2]\tilde{T}^{fb}_t = \rho \left( \tilde{Y}_{H,t}^{fb} - \tilde{Y}_{F,t}^{fb} \right)
- (2a - 1) \left( \tilde{\zeta}_{C,t} - \tilde{\zeta}_{C,t}^* \right). \tag{40}
\]

2.2 Model Description

**Producer-Currency Pricing.** Consumption growth is described by standard Euler equations, which are given by equations (1) and (27) in the case of country \(H\). The difference between these two Euler equations is that the nominal interest rate is country specific under the FX regime, whereas it is common to both countries under the MU regime. The risk-sharing condition, which describes the link between consumption across countries, is identical across regimes and it is given by (2) and (28), respectively. Purchasing power parity does not hold at all times, i.e., the real exchange rate is not constant, unless consumption and consumption preference shocks are perfectly correlated across countries. Under the FX regime, perfect risk sharing implies that the uncovered interest parity (3) holds, i.e., the expected change in the nominal exchange rate corresponds
to the interest rate differential across countries. This equation is obsolete under the MU regime because both countries share the same currency and a common nominal interest rate.

The link between the real exchange rate and the terms of trade is described by equations (4) and (29), respectively. Accordingly, the correlation between the real exchange rate and the terms of trade can be positive, zero, or negative, depending on the degree of trade openness between the two countries. Aggregate demand in each country depends on consumption in both countries and the terms of trade and is given by equations (5), (6), (30), and (31), respectively. The country-specific New Keynesian Phillips curves are also identical across regimes and they are given by (7), (8), (32), and (33), respectively. In contrast to a closed-economy framework, not only the output gap but also the terms-of-trade gap (the difference between the sticky price and the efficient terms of trade) affect producer price inflation. I follow much of the related literature in modeling cost-push shocks in an ad hoc way as exogenous fluctuations in the markup $\mu_t$ induced by time-varying taxes.

The terms-of-trade identity is given by equation (9) under the FX regime and by equation (34) under the MU regime, which differ due to the presence of the nominal exchange rate in the former. Equations (10), (11), (35), and (36) describe the relationship between the consumer price inflation rate and the producer price inflation rates in each country. Likewise, these equations only differ across regimes in terms of the presence of the nominal exchange rate.

Under flexible prices, monetary policy is neutral and real variables are driven only by productivity shocks and consumption preference shocks. Thus, the efficient allocation, which is given by equations (38) through (40), is the same under both currency regimes.

**Local-Currency Pricing.** The Euler consumption equation (13), the risk-sharing condition (14), and the uncovered interest parity condition (15) are identical to the case of producer-currency pricing. The real exchange rate is still linked to the terms of trade,

\footnote{Combining the Euler consumption equation for country $H$, the risk-sharing condition, and the uncovered interest parity condition yields the Euler consumption equation for country $F$, which is therefore redundant. Alternatively, the model can be specified by including both country-specific Euler consumption equations and the risk-sharing condition, while omitting the uncovered interest parity condition.}
but it is now also linked to the deviation from the law of one price (equation (16)). Since the countries are assumed to be symmetric, the deviation from the law of one price is identical across countries ($\Delta_{H,t} = \Delta_{F,t} = \Delta_t$). The aggregate demand equations (17) and (18) as well as the four New Keynesian Phillips curves (19) through (22) contain the deviation from the law of one price as well. The terms-of-trade identity (23) and the definitions of the CPI inflation rates (24) and (25) are different from the case of producer-currency pricing, since the law of one price does not hold under local-currency pricing. Finally, equation (26) defines the deviation from the law of one price, expressed in first differences.

2.3 Welfare Loss Function

The welfare analysis follows the logic of the familiar linear-quadratic approach, where the log-linear model equations are used to evaluate a quadratic welfare loss measure (Woodford 2003). The joint welfare loss function is given by the discounted value of a weighted average across countries of the average utility flow of agents using a second-order Taylor-series expansion. It is assumed that the distortion induced by monopolistic competition is offset by an appropriate subsidy, thereby ensuring efficiency in the steady state.

Producer-Currency Pricing. The welfare loss function in the case of producer-currency pricing is given by (see Corsetti, Dedola, and Leduc 2011):

$$W_t = -\frac{1}{2} \left( (\rho + \eta) \text{var} \left( \hat{Y}_{H,t} - \tilde{Y}_{H,t}^{fb} \right) + (\rho + \eta) \text{var} \left( \hat{Y}_{F,t} - \tilde{Y}_{F,t}^{fb} \right) \right)$$

$$- \frac{2a(1-a)(\rho \theta - 1)\rho}{4a(1-a)\rho \theta + (2a - 1)^2} \text{var} \left[ \left( \hat{Y}_{H,t} - \tilde{Y}_{H,t}^{fb} \right) - \left( \hat{Y}_{F,t} - \tilde{Y}_{F,t}^{fb} \right) \right]$$

$$+ \frac{\sigma}{\kappa} \left[ \text{var} \pi_{H,t} + \text{var} \pi_{F,t}^* \right] + t.i.p. + O(||\xi||^3).$$

Computing country-specific welfare would complicate the calculations significantly because more accurate approximations of the nonlinear model equations would be necessary (Benigno and Woodford 2005), which is beyond the scope of this paper. As long as the countries are symmetric, a gain in joint welfare always implies a gain for both countries. There is only one case where asymmetric countries are considered (section 3.3).
The weights in front of each component of the welfare loss function are functions of the deep parameters of the model. The term \textit{t.i.p.} contains all the terms that are independent of monetary policy and the currency regime. The term $O(||\xi||^3)$ contains third- and higher-order terms, which can be neglected provided that the model equations are log-linear, i.e., first-order approximations of the nonlinear equilibrium conditions.

As in the closed economy, the welfare loss depends on the producer price inflation rate and the output gap. In the open economy, the welfare loss also depends on the output gap differential across countries. If the output gap differential is different from zero, the allocation of production \textit{across} countries is inefficient. Importantly, under producer-currency pricing, the output gap differential and the terms-of-trade gap are two sides of the same coin. To see this, combine equation (40) with its analogous sticky-price counterpart to obtain

\begin{equation}
\left(\hat{Y}_{H,t} - \hat{Y}_{H,t}^{fb}\right) - \left(\hat{Y}_{F,t} - \hat{Y}_{F,t}^{fb}\right) = \frac{4a(1-a)\rho \theta + (2a-1)^2}{\rho} \left(\hat{T}_t - \hat{T}_t^{fb}\right).
\end{equation}

Thus, stabilizing the output gap differential automatically stabilizes the terms-of-trade gap, and vice versa. And the welfare loss function above can be expressed in terms of the terms-of-trade gap instead of the output gap differential, which I will make use of in the analysis.

**Local-Currency Pricing.** The welfare loss function in the case of local-currency pricing is given by (see Corsetti, Dedola, and Leduc 2011):

\begin{equation}
W_t = -\frac{1}{2} \left( (\rho + \eta) \text{var} \left(\hat{Y}_{H,t} - \hat{Y}_{H,t}^{fb}\right) + (\rho + \eta) \text{var} \left(\hat{Y}_{F,t} - \hat{Y}_{F,t}^{fb}\right) \right.
- \frac{2a(1-a)(\rho \theta - 1)\rho}{4a(1-a)\rho \theta + (2a-1)^2} \text{var} \left[ \left(\hat{Y}_{H,t} - \hat{Y}_{H,t}^{fb}\right) - \left(\hat{Y}_{F,t} - \hat{Y}_{F,t}^{fb}\right) \right]
+ \frac{2a(1-a)\theta}{4a(1-a)\rho \theta + (2a-1)^2} \text{var} \hat{\Delta}_t
+ \frac{\sigma}{k} \left( a \text{var} \pi_{H,t} + (1-a) \text{var} \pi_{H,t}^* \right).
\end{equation}
\[ + a \text{var } \pi^*_F,t + (1 - a) \text{var } \pi_F,t \]
\[ + t.i.p. + O(\|\xi\|^3). \quad (43) \]

Compared with the case of producer-currency pricing, the welfare loss function under local-currency pricing contains additional terms. First, it depends on the deviation from the law one price. Second, it depends on all four producer price inflation rates. Importantly, under local-currency pricing, the output gap differential and the terms-of-trade gap are no longer two sides of the same coin. Following the same steps as before yields

\[
\left( \hat{Y}_{H,t} - \tilde{Y}^{fb}_{H,t} \right) - \left( \hat{Y}_{F,t} - \tilde{Y}^{fb}_{F,t} \right) = \frac{4a(1-a)\rho \theta + (2a-1)^2}{\rho} \left( \hat{T}_t - \tilde{T}^{fb}_t \right) + \frac{4a(1-a)\rho \theta + 2a(2a-1)}{\rho} \hat{\Delta}_t. \quad (44)
\]

Thus, stabilizing the output gap differential does not automatically stabilize the terms-of-trade gap, and vice versa, because of potential deviations from the law of one price.

### 2.4 Calibration

Unless stated otherwise, the parameters of the model are calibrated to the values displayed in table 2 (see also Benigno 2004). For the sake of simplicity, the two countries are assumed to be symmetric. A value of 0.99 for the discount factor \( \beta \) implies a steady-state real interest rate of around 4.1 percent annually. A value of 7.66 for the elasticity of substitution between differentiated goods \( \sigma \) implies a steady-state markup of prices over marginal costs of 15 percent. A

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7Recall that inflation rates are relevant for welfare losses because they imply inefficient price dispersion in the presence of staggered price setting. Thus, the reason the welfare loss function under producer-currency pricing only contains two inflation rates is that the dispersion of prices of, e.g., domestically produced goods is identical in domestic and foreign currency (Engel 2011). It is not because the inflation rate for one good in different currencies is identical, which generally it is not.
value of 0.75 for the probability of not being able to reset the price \( \alpha \) implies an average duration of price contracts of four quarters.

The degree of trade openness \( a \) is calibrated to 0.75, which corresponds to a steady-state share of home-produced goods in the consumption basket of 75 percent in each country (i.e., a home bias in consumption) and a steady-state trade-to-GDP ratio of 50 percent. This roughly equals the average trade-to-GDP ratio across OECD countries. Following Rotemberg and Woodford (1998) and Benigno (2004), the inverse of the elasticity of producing the differentiated good \( \eta \) is calculated as

\[
\eta = \epsilon_{wy} - \rho + \frac{1 - \gamma}{\gamma}, \tag{45}
\]

where \( \epsilon_{wy} \) denotes the elasticity of the average real wage with respect to production and \( \gamma \) denotes the labor income share.

With the exception of the exchange rate coefficient \( \phi_S \), all interest rate rule coefficients are assumed to be identical across countries and regimes. Finally, the persistence of shocks is set to 0.9 in each country, and the cross-country correlation of shocks is zero.

3. Monetary Policy as an OCA Criterion

In the following, I use the theoretical model described in the previous section to show that the conduct of monetary policy is a critical criterion for the welfare implications of monetary unification. The conduct of monetary policy can differ with respect to the coefficients in the interest rate rules that determine the response of monetary policy to inflation (section 3.1), to output (section 3.2), to the nominal exchange rate (section 3.3), and to past realizations of the interest rate (section 3.4). In addition, the conduct of monetary policy can differ with respect to the target variables themselves. Monetary policy can respond to producer price inflation (henceforth PPI inflation targeting) or consumer price inflation (henceforth CPI inflation targeting), and it can respond to output (deviation from steady state) or the output gap (deviation from efficient allocation).

The baseline results are shown for producer-currency pricing and productivity shocks. In addition, I discuss the cases of local-currency

\[ \text{The steady-state trade-to-GDP ratio in percent is given by } 2(1 - a) \times 100. \]
pricing and cost-push shocks to stress that the results are not to any important degree sensitive to these modeling choices.

3.1 Response to Inflation

Under the FX regime and PPI inflation targeting, the interest rate rules for both countries are given by

\[ \hat{R}_t = \phi_\pi \pi_{H,t} \]  
\[ \hat{R}_t^* = \phi_\pi \pi_{F,t}^*. \]  

(46) (47)

Under CPI inflation targeting, they take the following form:

\[ \hat{R}_t = \phi_\pi \pi_t \]  
\[ \hat{R}_t^* = \phi_\pi \pi_t^*. \]  

(48) (49)

Under the MU regime, the interest rate rule of the common monetary policy is the same under PPI and CPI inflation targeting (recall equation (37)):

\[ \hat{R}_t^{MU} = \phi_\pi \pi_{MU}^t. \]  

(50)

Producer-Currency Pricing and Productivity Shocks.

The aggressiveness of monetary policy in its response to inflation has a determining influence on whether countries are better off under the MU regime or under the FX regime (figure 1). If the response to inflation is relatively modest (i.e., low values of \( \phi_\pi \)), the two countries are better off under the MU regime. If the response to inflation is relatively strong, the two countries are better off under the FX regime. The threshold value of \( \phi_\pi \) beyond which the FX regime becomes superior depends on the measure of inflation monetary policy responds to. Under CPI inflation targeting, the threshold value for \( \phi_\pi \) is lower than under PPI inflation targeting.

The welfare ranking between the MU and the FX regime is driven by the inflation component, which exhibits the same pattern with respect to \( \phi_\pi \) as the welfare loss (figure 2, lower right panel).\(^9\) This is

\(^9\)Although the welfare loss depends on the output gap and the PPI inflation rate of both countries, figure 2 shows only one of each because the variances are identical due to the assumption of symmetric countries.
because agents attach by far the highest weight to inflation, which is traditionally the case in microfounded welfare measures derived from New Keynesian models. Accordingly, the cost of a higher variance of the output gap and of the terms-of-trade gap—or, equivalently, of the output gap differential (recall equation (42))—under the MU regime (figure 2, upper right and lower left panel) can be outweighed by the benefit of a lower variance of PPI inflation. This is the case for low values of $\phi_\pi$, i.e., a relatively modest response of monetary policy to inflation.

The reason why the two countries are better off under the FX regime for a sufficiently strong response of monetary policy to inflation is predominantly related to the effectiveness of monetary policy.

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[^10]: Under the baseline calibration, the coefficients in front of the inflation rate, the output gap, and the terms-of-trade gap in the welfare loss function are 555.98, 0.83, and 0.75, respectively.
This becomes clear by comparing the number of policy instruments with the number of welfare-relevant distortions in the economy.

Under the FX regime, there are as many policy instruments as distortions in the two-country world (four). The distortions are due to monopolistic competition and to sticky prices in each country. The distortion due to monopolistic competition induces an inefficiently low level of aggregate output. This distortion can be eliminated by an appropriate subsidy in each country. The distortion due to sticky prices induces inefficient markup fluctuations, which lead

\[11\] Both distortions are common to the closed-economy framework (see, e.g., Woodford 2003 for details).
to inefficiently low or high levels of aggregate output, and an inefficient dispersion of prices in the presence of inflation, which causes an inefficient dispersion of output across the producers of differentiated goods *within* each country. This distortion can be mitigated or even eliminated by monetary policy in each country by using the nominal interest rate to reduce the fluctuations of inflation around zero as far as possible.

By contrast, under the MU regime, there are fewer policy instruments (three) than distortions (five) in the two-country world. First, monetary policy sets the nominal interest rate for both countries and thus it can no longer target inflation in each country separately, thereby losing one policy instrument. Second, the combination of the fixed nominal exchange rate with sticky prices induces an additional distortion, namely an intrinsic inertia in the terms of trade (Benigno 2004; Pappa 2004; Groll and Monacelli 2020).\(^{12}\) This causes an inefficient dispersion of aggregate output *across* countries.

Given that there are as many policy instruments as distortions under the FX regime but fewer policy instruments than distortions under the MU regime, monetary policy is more effective under the FX regime, which shows up clearly in figure 2. The “leverage” of monetary policy is higher under the FX regime than under the MU regime in the sense that a given increase in the aggressiveness of monetary policy toward inflation (measured by \(\phi_{\pi}\)) leads to a larger reduction in the variance of each welfare-relevant variable. In fact, under PPI inflation targeting, monetary policy can reduce the variances of all welfare-relevant variables to zero \((\phi_{\pi} \rightarrow \infty)\). This is impossible under the MU regime.\(^{13}\)

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\(^{12}\)Intrinsic inertia is defined as follows: Consider a one-off (i.e., nonpersistent) productivity shock in one country. Under the MU regime, several periods are required before the terms of trade return to the steady state after the shock has vanished. The terms of trade are said to be intrinsically persistent or inertial. Under the FX regime, the terms of trade return to the steady state immediately after the shock has vanished. In this case, the terms of trade are not intrinsically inertial.

\(^{13}\)See Groll (2013) for the analytical proof in the special case where \(a = 1/2\) and \(\theta = 1\). The proof in the case of no restrictions on \(a\) and \(\theta\) is completely analogous.
The limitations of monetary policy under the MU regime apply in particular to the terms-of-trade gap or, equivalently, to the output gap differential (figure 2, lower left panel). Monetary policy has no effect whatsoever on the terms of trade and thus on the terms-of-trade gap. Since both countries face the same nominal interest rate, any interest rate adjustment by the common monetary policy has the same initial effect on both countries. If the degree of price stickiness is identical across the two countries, an interest rate adjustment propagates through both economies in exactly the same way, and the influence of monetary policy on the terms of trade is zero. If the degree of price stickiness were not identical across the two countries, the influence of monetary policy on the terms of trade would not be zero, but would still be very small.

Despite those limitations of monetary policy, countries can be better off under the MU regime, as is the case for a relatively modest response of monetary policy to inflation. Paradoxically, the intrinsic inertia in the terms of trade due to the fixed exchange rate can also be beneficial, as is explained in detail in Groll and Monacelli (2020). In short, the inertia in the terms of trade has the advantage of stabilizing private-sector expectations about future inflation and thereby stabilizing actual inflation. This can overcompensate for the cost of inefficient terms-of-trade fluctuations, which are also induced by the fixed exchange rate. I will refer to this “inherent benefit of monetary unions” a number of times throughout the paper.

Robustness. Under either local-currency pricing or cost-push shocks, it continues to hold that the countries are better off under the MU regime if monetary policy responds to inflation modestly (see appendix C, figure C.1). However, the threshold value of $\phi_\pi$ beyond which the FX regime becomes superior is generally higher compared with the case of producer-currency pricing or productivity shocks. Thus, the MU regime is more likely to be superior. If local-currency pricing and cost-push shocks concur, the MU regime is superior irrespective of the aggressiveness of monetary policy toward inflation.

Compared with productivity shocks, cost-push shocks render the MU regime more favorable because the inherent benefit of monetary unions mentioned above is stronger under these circumstances. Cost-push shocks induce (possibly additional) tradeoffs for monetary policy in stabilizing different welfare-relevant variables. The bigger
these tradeoffs are, the greater is the benefit of stabilizing private-sector expectations about future inflation. This benefit is inherent to the MU regime due to the fixed exchange rate.\footnote{The logic is completely analogous to the gains of optimal monetary policy under commitment. These gains also operate through expectations and are increasing in the severity of the tradeoffs faced by monetary policy (Woodford 2003).}

Compared with producer-currency pricing, local-currency pricing renders the MU regime more favorable because the benefit of exchange rate flexibility in the presence of nominal price rigidity—and therefore the cost of fixing the exchange rate—is considerably smaller. Under local-currency pricing, import prices no longer fluctuate one-to-one with the exchange rate but are as sticky as domestic prices. Therefore, exchange rate flexibility no longer facilitates the desirable adjustment in international relative prices of goods ($\hat{T}_t + \hat{\Delta}_t$) in response to country-specific shocks (Devereux and Engel 2003; Corsetti, Dedola, and Leduc 2011; Engel 2011). There are more distortions than policy instruments, namely two sticky prices versus one interest rate within each country. As a result, monetary policy is less effective under local-currency pricing. Nevertheless, a case for flexible exchange rates remains even if there is no expenditure-switching effect of the exchange rate: Exchange rate flexibility facilitates the desirable adjustment in the real exchange rate ($\hat{Q}_t$), accommodating the efficient response of aggregate consumption across countries (Duarte and Obstfeld 2008). This explains why countries can be better off under the FX regime even under local-currency pricing.

### 3.2 Response to Output

In this subsection, monetary policy responds not only to inflation but also to output (deviation from steady state) or to the output gap (deviation from efficient allocation). Under the FX regime, if monetary policy targets output, the interest rate rules for both countries are given by

\begin{align*}
\hat{R}_t &= \phi_\pi \pi_{H_t} + \phi_Y \hat{Y}_{H_t} , \\
\hat{R}_t^* &= \phi_\pi \pi_{F_t}^* + \phi_Y \hat{Y}_{F_t}.
\end{align*}
If monetary policy targets the output gap, they take the following form:

\[
\hat{R}_t = \phi_\pi \pi_{H,t} + \phi_Y \left( \hat{Y}_{H,t} - \hat{Y}_{H,t}^{fb} \right)
\] (53)

\[
\hat{R}_t^* = \phi_\pi \pi_{*,t}^* + \phi_Y \left( \hat{Y}_{F,t} - \hat{Y}_{F,t}^{fb} \right).
\] (54)

Under the MU regime, if the common monetary policy responds to output, the interest rate rule is given by

\[
\hat{R}_{t}^{MU} = \phi_\pi \pi_t^{MU} + \phi_Y \frac{\hat{Y}_{H,t} + \hat{Y}_{F,t}}{2}.
\] (55)

If it responds to the output gap, it is given by

\[
\hat{R}_{t}^{MU} = \phi_\pi \pi_t^{MU} + \phi_Y \frac{\left( \hat{Y}_{H,t} - \hat{Y}_{H,t}^{fb} \right) + \left( \hat{Y}_{F,t} - \hat{Y}_{F,t}^{fb} \right)}{2}.
\] (56)

In all of these cases, the inflation coefficient \( \phi_\pi \) is set to 1.5. As the difference between PPI and CPI inflation targeting is very small in this context, only the results under PPI inflation targeting are reported.

**Producer-Currency Pricing and Productivity Shocks.**

Whether countries are better off under the MU regime or under the FX regime depends crucially on whether monetary policy responds to output (deviation from steady state) or the output gap (deviation from efficient allocation). If monetary policy responds to output, the two countries are better off under the MU regime (figure 3, left panel). By contrast, if monetary policy responds to the output gap, the two countries are better off under the FX regime (figure 3, right panel). As before, the driving factor is the inflation component.\(^{15}\)

The key to understanding these results is the role played by the nominal exchange rate in stabilizing the terms-of-trade gap. Combining equations (38) to (40) and focusing on productivity shocks in country \( H \) yields the following relationship between the efficient terms of trade \( \hat{T}_{t}^{fb} \) and the productivity shock \( \hat{\zeta}_{Y,t} \):

\[
\hat{T}_{t}^{fb} = \frac{\rho \eta}{4a(1-a)\rho(1+\eta\theta) + (\rho + \eta)(2a-1)^2} \hat{\zeta}_{Y,t}.
\] (57)

\(^{15}\)Not shown, but available upon request.
The term in front of the productivity shock is unambiguously positive. Accordingly, the terms of trade would increase in response to a positive productivity shock in country $H$ if prices were perfectly flexible. However, because prices are sticky, the actual increase in the terms of trade is smaller. In these circumstances, an increase in the nominal exchange rate would help to close the gap between the actual response of the terms of trade and its efficient counterpart, thereby stabilizing the terms-of-trade gap and reducing the welfare loss. But whether the nominal exchange rate stabilizes or destabilizes the terms-of-trade gap depends crucially on whether monetary policy responds to output or the output gap. This is because the nominal exchange rate is directly linked to the interest rates governed by monetary policy via the uncovered interest parity condition (3).

If monetary policy responds to neither output nor the output gap ($\phi_Y = 0$), the impact response of the nominal exchange rate to a positive productivity shock in country $H$ is positive, i.e., country $H$’s currency depreciates (figure 4).\(^{16}\) Thus, the nominal exchange

\(^{16}\) The degree of price stickiness was set to a low value ($\alpha = 0.2$) to ensure that the differences in the impulse responses are clearly visible. The differences for higher degrees of price stickiness are smaller but qualitatively the same.
Figure 4. Impulse Response of the Change in the Nominal Exchange Rate ($\Delta \hat{S}_t$) to a Positive One-Off Productivity Shock in Country $H$ for Three Different Values of the Output Coefficient ($\phi_Y$), with $\alpha = 0.2$, under Producer-Currency Pricing

Notes: Left panel: Response to output ($\hat{Y}_t$). Right panel: Response to output gap ($\hat{Y}_t - \hat{Y}_t^{fr}$).

rate pushes the sticky-price terms of trade in the same direction as the efficient terms of trade, thereby stabilizing the terms-of-trade gap to some extent. If monetary policy responds to the output gap, the positive impact response of the nominal exchange rate becomes greater as $\phi_Y$ increases (figure 4, right panel). The stabilizing effect increases accordingly, further reducing the terms-of-trade gap and reducing the welfare loss. Since this stabilizing mechanism is absent under the MU regime, the countries are better off under the FX regime.

By contrast, as monetary policy starts to respond to output, the impact response of the nominal exchange rate first becomes smaller and then negative for already very small values of $\phi_Y$ (figure 4, left panel). A negative impact response means that the nominal exchange rate destabilizes the terms-of-trade gap by pushing the sticky-price terms of trade away from the efficient terms of trade. As a result, not only the terms-of-trade gap but also the output gap and the PPI inflation rate are destabilized, thereby increasing the welfare loss. Under these circumstances, the countries are better off with a fixed exchange rate.
Importantly, the nominal exchange rate amplifies a detrimental effect that is already present; it does not cause the detrimental effect. In a closed economy, a response of monetary policy to output is also detrimental to welfare (see, e.g., Galí 2015, chapter 4.4). It is not the deviation of output from the steady state that is welfare relevant; it is the deviation from the efficient counterpart (output gap). A positive productivity shock in country $H$ induces an increase in output but a decrease in the output gap, because the increase in output is lower than the increase in efficient output. A welfare-oriented response of monetary policy would require a reduction in the interest rate due to the negative output gap. Instead, monetary policy raises the interest rate due to the rise in output.

For these reasons, a response of monetary policy to output is detrimental under both the FX regime and the MU regime (in figure 3, left panel, the welfare loss increases in $\phi_Y$ under both regimes). However, the detrimental effect is larger under the FX regime due to the amplification by the nominal exchange rate. As described above, monetary policy is more effective under the FX regime than under the MU regime in terms of macroeconomic stabilization because of the flexibility of the nominal exchange rate. The flipside of this is that monetary policy can do more harm when it is not conducted properly. Essentially, the nominal exchange rate does not compensate for monetary policy mistakes; it reinforces them. In this sense, the MU regime provides a protective mechanism against monetary policy mistakes.

**Robustness.** Under local-currency pricing, it continues to hold that monetary unification is beneficial when monetary policy responds to output, and costly when it responds to the output gap (see appendix C, figure C.2). However, the welfare loss differences between the two currency regimes are smaller than under producer-currency pricing. This is because exchange rate flexibility is less beneficial under local-currency pricing due to the missing effect on international relative goods prices, which reduces the effectiveness of monetary policy under the FX regime (see above).

Under cost-push shocks, the situation is a little different than under productivity shocks. Both the response to output and the response to the output gap are detrimental to welfare, and both policies render the MU regime superior to the FX regime (see appendix C, figure C.3). This is primarily due to the inherent benefit of
monetary unions, which is much stronger under cost-push shocks (see above).

As in a closed economy, there is no difference between targeting output and targeting the output gap because the two variables are identical under cost-push shocks (the efficient allocation is unaffected). Cost-push shocks move output/the output gap and inflation in opposite directions. Given this tradeoff, responding more aggressively to output/the output gap automatically reduces the response to inflation. As a result, the output gap becomes more stable, but inflation becomes less stable. This reduces welfare, as agents attach a higher weight to inflation. For this reason, a response of monetary policy to output/the output gap is detrimental to welfare in the presence of cost-push shocks.

This continues to hold in the open economy under both the FX and MU regime. However, while the stabilizing effect on the output gap—and now in addition on the output gap differential—is smaller under the MU regime, the destabilizing effect on inflation is also smaller under the MU regime. Both effects are due to the fixed exchange rate. While hampering the stabilization of output gaps due to inefficient fluctuations of international relative prices, the fixed exchange rate has the advantage of stabilizing private-sector expectations about future inflation and thereby actual inflation. Due to the higher weight of inflation stability, the MU regime turns out to be superior in terms of welfare if monetary policy targets output/the output gap in the presence of cost-push shocks.

3.3 Response to Nominal Exchange Rate

In this subsection, monetary policy responds to inflation and the nominal exchange rate. I distinguish between unilateral exchange rate targeting, where only one of the two countries responds to the exchange rate, and bilateral exchange rate targeting, where both countries respond to the exchange rate symmetrically. Under unilateral exchange rate targeting and PPI inflation targeting, the interest rate rules for both countries are given by

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17 See, e.g., Calvo and Reinhart (2002) for empirical estimates on the number of countries that target the exchange rate.
\begin{align*}
\hat{R}_t &= \phi_t \pi_{H,t} + \frac{\phi_S}{1 - \phi_S} \hat{S}_t \quad (58) \\
\hat{R}_t^* &= \phi_t \pi_{F,t}^* \quad (59)
\end{align*}

Under CPI inflation targeting, they are given by

\begin{align*}
\hat{R}_t &= \phi_t \pi_t + \frac{\phi_S}{1 - \phi_S} \hat{S}_t \\
\hat{R}_t^* &= \phi_t \pi_t^* \quad (60)
\end{align*}

\begin{align*}
\hat{R}_t &= \phi_t \pi_{H,t} + \frac{\phi_S}{1 - \phi_S} \hat{S}_t \\
\hat{R}_t^* &= \phi_t \pi_{F,t}^* - \frac{\phi_S}{1 - \phi_S} \hat{S}_t \quad (61)
\end{align*}

Under bilateral exchange rate targeting and PPI inflation targeting, the interest rate rules for both countries are given by

\begin{align*}
\hat{R}_t &= \phi_t \pi_{H,t} + \frac{\phi_S}{1 - \phi_S} \hat{S}_t \\
\hat{R}_t^* &= \phi_t \pi_{F,t}^* - \frac{\phi_S}{1 - \phi_S} \hat{S}_t \quad (62)
\end{align*}

\begin{align*}
\hat{R}_t &= \phi_t \pi_t + \frac{\phi_S}{1 - \phi_S} \hat{S}_t \\
\hat{R}_t^* &= \phi_t \pi_t^* - \frac{\phi_S}{1 - \phi_S} \hat{S}_t \quad (63)
\end{align*}

Under CPI inflation targeting, they are given by

\begin{align*}
\hat{R}_t &= \phi_t \pi_t + \frac{\phi_S}{1 - \phi_S} \hat{S}_t \\
\hat{R}_t^* &= \phi_t \pi_t^* - \frac{\phi_S}{1 - \phi_S} \hat{S}_t \quad (64)
\end{align*}

\begin{align*}
\hat{R}_t &= \phi_t \pi_{H,t} + \frac{\phi_S}{1 - \phi_S} \hat{S}_t \\
\hat{R}_t^* &= \phi_t \pi_{F,t}^* - \frac{\phi_S}{1 - \phi_S} \hat{S}_t \quad (65)
\end{align*}

The coefficient $\phi_S \in [0, 1)$ governs the strength of the response to the exchange rate. It ranges from a regime of full exchange rate flexibility ($\phi_S = 0$) to a fixed exchange rate regime ($\phi_S \to 1$) with hybrid regimes in between (Galí and Monacelli 2016).

Under the MU regime, the interest rate rule is given by

$$\hat{R}_t^{MU} = \phi_t \pi_t^{MU} \quad (66)$$

In all of these cases, the inflation coefficient $\phi_t$ is set to 1.5.

**Producer-Currency Pricing and Productivity Shocks.** Whether countries are better off under the MU regime or under the FX regime is not only sensitive to the degree to which countries respond to the nominal exchange rate but also, and more importantly, to whether the exchange rate targeting regime is carried out
Figure 5. Welfare Loss as a Function of the Exchange Rate Coefficient ($\phi_S$) under Producer-Currency Pricing and Productivity Shocks

Notes: Left panel: Unilateral exchange rate targeting. Right panel: Bilateral exchange rate targeting.

unilaterally or bilaterally$^{18}$ Under unilateral exchange rate targeting, the countries are generally better off under the MU regime (figure 5, left panel)$^{19}$ An exception is the case where monetary policy targets CPI inflation and responds to the exchange rate only very modestly. By contrast, under bilateral exchange rate targeting, the countries are generally worse off under the MU regime (figure 5, right panel). Here, the difference between CPI and PPI inflation targeting is small.

The principal reason for the different welfare implications of the unilateral and the bilateral exchange rate targeting regime vis-à-vis the MU regime are coordination gains. Consider the limiting case of a fixed exchange rate ($\phi_S \rightarrow 1$). Although the exchange rate is fixed under both unilateral and bilateral exchange rate targeting

\footnote{Both for simplicity and comparability with other sections, I continue to use the term “FX regime,” although, clearly, targeting the exchange rate does not implement a regime in which the nominal exchange rate is perfectly flexible.}

\footnote{In this particular case, it is possible that one country suffers a welfare loss, which is overcompensated by the other country’s welfare gain. In all other welfare comparisons in this paper, a gain in joint welfare always implies a gain for both countries, due to the symmetry in country characteristics as well as in interest rate rules.}
as well as under the MU regime, only the bilateral fixed exchange rate regime yields the same welfare as the MU regime. These two regimes are in fact identical in every respect. This is because both regimes implement the fixed exchange rate in a coordinated way. The MU regime represents a coordinated fixed exchange rate regime by construction. The bilateral fixed exchange rate regime implies coordination because both countries respond to the exchange rate symmetrically.

By contrast, under a unilateral fixed exchange rate regime (one-sided peg), only one of the two countries ensures that the exchange rate is fixed, while the other country can choose its interest rate policy independently. Since fixing the exchange rate requires the country-specific interest rates to be perfectly aligned at all times, the pegging country must always follow the other country’s interest rate adjustments, which severely restricts its ability to respond to country-specific variables, like in this case domestic inflation. Under these circumstances, a coordination of monetary policies to implement the fixed exchange rate raises overall macroeconomic stability and therefore welfare. Monetary unification provides such a coordination device (Cooley and Quadrini 2003; Pappa 2004).

Note that in this model the benefit of monetary unification compared with a unilateral fixed exchange rate regime does not derive from a credibility gain. By abstracting from speculative attacks, it is implicitly assumed that the fixed exchange rate is perfectly credible under both regimes. In reality, of course, a monetary union provides a much more credible fixed exchange rate regime than an exchange rate peg, due to the much greater costs of leaving or dissolving a monetary union (see, e.g., Eichengreen 1993). This credibility gain adds to the coordination gain described above.

**Robustness.** Under local-currency pricing or under cost-push shocks, the results are qualitatively very similar (see appendix C, figures C.4–C.6). The MU regime tends to be superior to unilateral exchange rate targeting but inferior to bilateral exchange rate targeting. Again, local-currency pricing and cost-push shocks work in favor of the MU regime, for the reasons explained above. Notably, under cost-push shocks, the MU regime and the bilateral exchange rate targeting regime are nearly identical for most values of the exchange rate coefficient.
3.4 Interest Rate Smoothing

Finally, in this subsection, monetary policy engages in interest rate smoothing. Under the FX regime and PPI inflation targeting, the interest rate rules for both countries are given by

\[
\hat{R}_t = \phi R \hat{R}_{t-1} + (1 - \phi R)\phi \pi_{H,t} \\
\hat{R}_t^* = \phi R \hat{R}_{t-1}^* + (1 - \phi R)\phi \pi_{F,t}^*.
\]

(67) (68)

Under CPI inflation targeting, they take the following form:

\[
\hat{R}_t = \phi R \hat{R}_{t-1} + (1 - \phi R)\phi \pi_t \\
\hat{R}_t^* = \phi R \hat{R}_{t-1}^* + (1 - \phi R)\phi \pi_t^*.
\]

(69) (70)

Under the MU regime, the interest rate rule of the common monetary policy is given by:

\[
\hat{R}_{t}^{MU} = \phi R \hat{R}_{t-1}^{MU} + (1 - \phi R)\phi \pi_{t}^{MU}.
\]

(71)

In all of these cases, the inflation coefficient \(\phi \pi\) is set to 1.5.

**Producer-Currency Pricing and Productivity Shocks.** Whether countries are better off under the MU regime or under the FX regime depends on the degree of interest rate smoothing implemented by monetary policy, which is particularly true under PPI inflation targeting (figure 6, solid blue and dashed red line).\(^{20}\)

Starting with very low degrees of interest rate smoothing (i.e., low values of \(\phi R\)), the two countries are better off under the MU regime. As the degree of interest rate smoothing increases, the welfare loss decreases faster under the FX regime than under the MU regime. At some point, the welfare ranking changes and the two countries are better off under the FX regime.

As described in section 3.1, the MU regime entails the cost of higher instability of both the output gap and the terms-of-trade gap, but the benefit of higher stability of the PPI inflation rate. This is again due to the mechanism mentioned earlier: As the nominal exchange rate is fixed and prices are sticky, the terms of trade

\(^{20}\)For color versions of the figures, see the paper on the IJCB website (http://www.ijcb.org).
exhibit an inertial or history-dependent behavior, even if monetary policy does not smooth interest rates. This history dependence has the advantage of stabilizing private-sector expectations about future inflation and thereby actual inflation.

If monetary policy does not smooth interest rates under the FX regime, there is no such history dependence. The regime suffers from a kind of stabilization bias. As a result, PPI inflation is less stable under the FX regime. However, if monetary policy starts to smooth interest rates, it induces history dependence into the economy, with the same advantageous effect on inflation expectations. This effect strengthens as the degree of interest rate smoothing increases. If the degree of interest rate smoothing is sufficiently high, PPI inflation is more stable under the FX regime.

Under CPI inflation targeting, the degree of interest rate smoothing does not have such an important effect on the welfare ranking between the MU and FX regime (figure 6, solid blue and dotted red line). This is because, like the MU regime, the FX regime under CPI
inflation targeting features history dependence even if monetary policy does not smooth interest rates. As a result, engaging in interest rate smoothing, thereby inducing greater history dependence into the economy, does not change the relative welfare performance of the FX and MU regimes dramatically.  

**Robustness.** The results continue to hold under the combination of local-currency pricing and productivity shocks (see appendix C, figure C.7). As before, the differences in welfare losses between the MU and the FX regime are smaller because exchange rate flexibility is less beneficial under local-currency pricing due to the missing effect on international relative goods prices.

Under cost-push shocks, while the welfare performance of the MU regime relative to the FX regime continues to deteriorate with the degree of interest rate smoothing under PPI inflation targeting, there is no longer a change in the ranking, at least under the baseline calibration. For very high degrees of interest rate smoothing, the welfare loss is basically identical under both currency regimes. Under CPI inflation targeting, the degree of interest rate smoothing continues to have a much more limited influence on the welfare implications of monetary unification, as was the case under productivity shocks. But since cost-push shocks work in its favor, the likelihood of monetary unification being beneficial is higher than under productivity shocks.

### 4. Monetary Policy and Trade Openness

The conduct of monetary policy is not only an independent OCA criterion by itself, as illustrated in section 3, but it can also modify the nature of other OCA criteria. This is demonstrated in the following using the degree of trade openness as an example. But first, I briefly summarize how the relationship between trade openness and the costs and benefits of a monetary union is seen in the literature.

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21 As evident from figure 6, the welfare ranking does not change at all with the degree of interest rate smoothing under CPI inflation targeting. For other parameter constellations, however, this is the case, e.g., if $\phi_\pi = 1.2$ instead of 1.5.
4.1 Trade Openness in OCA Theory

The degree of trade openness or trade integration is one of the oldest and most prominent OCA criteria. Most studies have established a positive link between trade openness and the likelihood of a monetary union being beneficial. More precisely, the more open economies are, the smaller are the costs and the larger are the benefits associated with monetary unification.

McKinnon (1963) first proposed trade openness as an OCA criterion. He argued that with an increasingly open economy, the effects of exchange rate fluctuations on consumer prices via import prices become greater, thereby making it more difficult for monetary policy to maintain (consumer) price stability. Thus, the costs of giving up monetary independence decrease with the degree of trade openness.

One of the main costs attributed to monetary unification is the loss of the ability to react to asymmetric (i.e., country-specific) shocks via monetary policy and the nominal exchange rate. However, there are conflicting views on whether the incidence of country-specific shocks decreases or increases with the degree of trade openness. This depends on whether trade between countries is characterized predominantly by intra-industry trade or inter-industry trade. In the former case, industry-specific shocks affect countries symmetrically, thus an increase in the degree of trade openness reduces the cost of giving up monetary independence (Emerson et al. 1992, chapter 6.2). In the latter case, industry-specific shocks affect countries asymmetrically, thus an increase in the degree of trade openness raises the cost of giving up monetary independence (Krugman 1991, p. 82).²²

The benefits traditionally associated with monetary unification are usually considered to increase with the degree of trade openness, such as the elimination of transaction costs when exchanging currencies, the increase in price transparency across countries, or the elimination of exchange rate risk (e.g., De Grauwe 2012, chapter 3.8). The latter point is also made by Kollmann (2004) using a New Keynesian DSGE model similar to that employed in the present study. He concludes that if the nominal exchange rate is subject to shocks,

²²See De Grauwe (2012, chapter 2.1) for a more detailed description and assessment of the “European Commission view” and the “Krugman view.”
a monetary union is welfare improving because the cost of giving up monetary independence is overcompensated by the benefit of eliminating exchange rate risk. This benefit increases with the degree of trade openness because exchange rate shocks are more harmful to macroeconomic stability as economies become more open.

Furthermore, the coordination gains associated with monetary unification also increase with the degree of trade openness, as shown by Pappa (2004) using a similar model. Compared with a flexible exchange rate regime where the monetary authorities do not cooperate to maximize welfare, forming a monetary union eliminates the possibility of strategic terms-of-trade manipulations. This benefit increases with the degree of trade openness because terms-of-trade movements have larger effects on macroeconomic stability as economies become more open.

4.2 Monetary Policy and the Nature of Trade Openness as an OCA Criterion

The preceding overview shows that OCA theory mainly establishes a favorable relationship between the degree of trade openness and the costs and benefits of a monetary union. As shown next, this is highly sensitive to the way monetary policy is conducted.

In what follows, the interest rate rules are given by equations (46) through (50), with the inflation coefficient $\phi_{\pi}$ set to 1.5 in all cases.

**Producer-Currency Pricing and Productivity Shocks.**

The influence of monetary policy on the nature of the degree of trade openness as an OCA criterion is particularly clear when distinguishing between PPI and CPI inflation targeting. First consider the case of PPI inflation targeting (figure 7, solid blue and dashed red line). Two observations are noteworthy. First, under both the MU and the FX regime, the relationship between the welfare loss and the degree of trade openness is symmetric around a trade-to-GDP ratio of 100 percent ($a = 1/2$). Second, the two countries are better off under the FX regime if they are either relatively closed ($a$ close to one) or very open to trade ($a$ close to zero), but better off under the MU regime for intermediate values. Thus, the likelihood of the MU regime being beneficial first increases and then decreases with the degree of trade openness.
The relationship between trade openness and the welfare ranking between the MU and FX regime changes considerably if monetary policy targets CPI inflation rates instead of PPI inflation rates (figure 7, solid blue and dotted red line). First, the relationship between the welfare loss and the degree of trade openness is no longer symmetric under the FX regime. Second, the two countries are better off under the FX regime for trade-to-GDP ratios between 0 and 100 percent \((0 < a < \frac{1}{2})\) and better off under the MU regime for ratios between 100 and 200 percent \((0 \leq a < \frac{1}{2})\). Thus, the likelihood of the MU regime being beneficial increases with the degree of trade openness.

The key to understanding these results is again the role played by the nominal exchange rate in stabilizing the terms-of-trade gap. Consider a positive productivity shock in country \(H\). Recall that

\[\text{Recall that there is no difference between PPI and CPI inflation targeting under the MU regime.}\]
Figure 8. Impulse Response of the Change in the Nominal Exchange Rate ($\Delta \hat{S}_t$) to a Positive One-Off Productivity Shock in Country $H$ for Three Different Degrees of Trade Openness ($\alpha$), with $\alpha = 0.2$, under Producer-Currency Pricing

Notes: Left panel: PPI inflation targeting. Right panel: CPI inflation targeting.

the efficient terms of trade unambiguously increase on impact (see equation (57)), thus an increase in the nominal exchange rate would help to stabilize the terms-of-trade gap, thereby reducing the welfare loss. But whether the nominal exchange rate stabilizes or destabilizes the terms-of-trade gap depends crucially on whether monetary policy targets PPI or CPI inflation.

Under PPI inflation targeting, the impact response of the nominal exchange rate is positive irrespective of the degree of trade openness, i.e., country $H$’s currency depreciates (figure 8, left panel). Thus, the nominal exchange rate pushes the sticky-price terms of trade in the same direction as the efficient terms of trade, thereby stabilizing the terms-of-trade gap to some extent. Note that the response of the nominal exchange rate is identical for $\alpha = 0.25$ and $\alpha = 0.75$, which explains the symmetric pattern visible in figure 7.

By contrast, under CPI inflation targeting, the impact response of the nominal exchange rate is positive if the two countries have a trade-to-GDP ratio below 100 percent ($\alpha > 1/2$), but negative if

\[ \text{Note: The degree of price stickiness was set to a low value (}\alpha = 0.2\text{) to ensure that the differences in the impulse responses are clearly visible. The differences for higher degrees of price stickiness are smaller but qualitatively the same.} \]
it is above 100 percent ($a < 1/2$) (figure 8, right panel). Thus, the nominal exchange rate helps to stabilize the terms-of-trade gap only in the first case. In the latter case, the nominal exchange rate actually destabilizes the terms-of-trade gap by pushing the sticky-price terms of trade away from the efficient terms of trade.

This is robust with respect to the other deep parameters of the economy. To see this, insert the country-specific interest rate rules (48) and (49) together with the definitions of the CPI inflation rates (10) and (11) and the terms-of-trade identity (9) into the uncovered interest parity condition (3) to obtain

$$\Delta \hat{S}_t = (2a - 1) \Delta \hat{T}_t + \frac{1}{\phi_\pi} E_t \Delta \hat{S}_{t+1}.$$  

(72)

Solving forward yields

$$\Delta \hat{S}_t = (2a - 1) E_t \sum_{k=0}^{\infty} \left( \frac{1}{\phi_\pi} \right)^k \Delta \hat{T}_{t+k}.$$  

(73)

Accordingly, the current change in the nominal exchange rate depends on the discounted sum of current and expected future changes in the terms of trade. Importantly, this relationship is positive if $a > 1/2$, but negative if $a < 1/2$.

Under PPI inflation targeting, the analogous equations are given by

$$\Delta \hat{S}_t = \Delta \hat{T}_t + \frac{1}{\phi_\pi} E_t \Delta \hat{S}_{t+1}$$  

(74)

and

$$\Delta \hat{S}_t = E_t \sum_{k=0}^{\infty} \left( \frac{1}{\phi_\pi} \right)^k \Delta \hat{T}_{t+k}.$$  

(75)

---

$^{25}$In particular, it does not make a difference whether $\rho_\theta$ is smaller than, equal to, or larger than 1, although this condition has important macroeconomic implications. For example, it determines whether the cross-country correlation of output is positive, zero, or negative (see, e.g., Corsetti, Dedola, and Leduc 2011 for details). Also, if it is zero ($\rho_\theta = 1$), the terms-of-trade gap vanishes from the welfare loss function (41).
In contrast to the CPI inflation targeting case, the relationship between the current change in the nominal exchange rate and the discounted sum of current and expected future changes in the terms of trade is always positive, regardless of the degree of trade openness $a$.\footnote{Note that equation (73) and equation (75) are equivalent if $a = 1$. In this case, there is no difference between PPI and CPI inflation targeting. This is because the consumer price index equals the producer price index if $a = 1$; see equations (10) and (11).}

The intuition behind the fact that the nominal exchange rate can be destabilizing under CPI inflation targeting is the following. If the trade-to-GDP ratio is above 100 percent ($a < 1/2$), consumer prices in one country are determined mainly by producer prices in the other country because consumers consume more imported goods than home-produced goods. If monetary policy targets consumer prices, interest rate adjustments in one country are triggered mainly by producer price changes in the other country. This pushes the nominal exchange rate, which depends on the interest rate differential between the two countries, away from the efficient terms of trade. As a result, the welfare-relevant terms-of-trade gap is destabilized by the nominal exchange rate. Under these circumstances, a fixed exchange rate would make the countries better off because this is neither destabilizing nor stabilizing. For this reason, the countries are better off under the MU regime for $a < 1/2$.

In the special case of a trade-to-GDP ratio of exactly 100 percent ($a = 1/2$), the two countries are indifferent between the FX and the MU regime under CPI inflation targeting. This is because the nominal exchange rate is constant under both regimes.\footnote{According to equation (73), $\Delta \hat{S}_t = 0$ if $a = 1/2$.} Under the MU regime, the nominal exchange rate is fixed by construction. Under the FX regime, it is fixed by coincidence. That is, by targeting CPI inflation rates, the two countries unintentionally implement a symmetric fixed exchange rate regime. This is because consumer price changes and thus interest rate adjustments are identical in the two countries.

Lastly, as shown, under PPI inflation targeting the nominal exchange rate stabilizes the terms-of-trade gap regardless of the degree of trade openness. Nonetheless, for a broad range of degrees of
trade openness, the FX regime is inferior to the MU regime, where the nominal exchange rate is fixed (recall figure 7). This is again due to the inherent benefit of monetary unions explained earlier: As the nominal exchange rate is fixed and prices are sticky, the terms of trade exhibit an inertial or history-dependent behavior. This history dependence has the advantage of stabilizing private-sector expectations about future inflation and thereby actual inflation. This benefit weakens as the degree of trade openness becomes either very low \((a \to 1)\) or very high \((a \to 0)\). In the extreme cases, consumers consume only one of the two internationally traded goods. The relative price (terms of trade) becomes irrelevant for price setters, and the terms of trade no longer affect inflation. As a result, the inertia in the terms of trade no longer has a stabilizing effect on inflation.

Robustness. The conclusion that the nature of trade openness as an OCA criterion differs markedly between PPI and CPI inflation targeting is robust to local-currency pricing or cost-push shocks, though how that difference specifically looks varies from case to case (see appendix C, figure C.8). The only exception to this conclusion results if local-currency pricing and productivity shocks concur. In that case, there is no difference between PPI and CPI inflation targeting in the sense that the likelihood of the MU regime being beneficial is lowest under either very closed or very open economies.

5. Conclusion

The costs and benefits of moving from a flexible exchange rate regime to a monetary union depend critically on the conduct of monetary policy. Whether countries are better off in one or the other currency regime is sensitive not only to the choice of the variables that monetary policy targets but also to the strength of the response to these target variables. In particular, when monetary policy in each country responds to inflation aggressively or implements a high degree of interest rate smoothing, forming a monetary union, where the common monetary authority continues to follow the same policy, tends

\(^{28}\)Note how the terms of trade vanish from the New Keynesian Phillips curves (7) and (8) if \(a = 0\) or \(a = 1\).
to make countries worse off in terms of welfare by reducing macroeconomic stability. By contrast, when monetary policy responds to inflation only modestly or implements a low degree of interest rate smoothing, forming a monetary union tends to make countries better off. Furthermore, monetary unification is beneficial when monetary policy responds to output, whereas it is costly when monetary policy responds to the output gap. And finally, it is important whether countries respond to the nominal exchange rate and whether they do so in a coordinated or uncoordinated way. In the latter case, monetary unification is generally beneficial, whereas it is costly in the former case.

In addition to being an OCA criterion itself, monetary policy has the potential to modify the nature of traditional OCA criteria, such as the degree of trade openness. Whether the likelihood of a monetary union being beneficial increases with the degree of trade openness, as proposed by the vast bulk of OCA studies, depends critically on whether monetary policy targets producer price inflation or consumer price inflation. In the former case, it is possible that the likelihood of a monetary union being beneficial decreases with the degree of trade openness.

With few exceptions, these conclusions are not to any important degree sensitive to the price-setting assumption (producer-currency pricing versus local-currency pricing) or the type of shocks (productivity shocks versus cost-push shocks). However, local-currency pricing and cost-push shocks—individually as well as jointly—tend to increase the likelihood that countries benefit from monetary unification.

**Appendix A. Flexible Exchange Rate Regime**

This appendix contains the full derivation of the model under the flexible exchange rate regime for producer-currency pricing and local-currency pricing, respectively (based on Corsetti, Dedola, and Leduc 2011). The world, which consists of two countries labeled $H$ and $F$, is populated by a continuum of agents on the interval $[0, 1]$. The population on the segment $[0, n)$ lives in country $H$; the population on the segment $[n, 1]$ lives in country $F$. Thus, $n$ measures the population size as a fraction of world population. An agent is both
consumer and producer. He produces a single differentiated good and consumes all the goods produced in both countries.

A.1 Consumer Problem

Agent $j$ in country $H$ derives positive utility from consumption $C_j$ and negative utility from producing the differentiated good $y(h)$. The present discounted value of lifetime utility $U^j$ is given by\(^{29}\)

$$U^j = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \zeta_{C,t} C_t^{1-\rho} - \frac{1}{1-\rho} - \zeta_{Y,t} y_t(h)^{1+\eta} \right]. \tag{A.1}$$

$E$ denotes the expectations operator, $\beta$ the discount factor, $\rho$ the inverse of the intertemporal elasticity of substitution in consumption, and $\eta$ the inverse of the elasticity of producing the differentiated good.\(^{30}\) $\zeta_{Y,t}$ and $\zeta_{C,t}$ denote shocks to productivity and to preferences in consumption, respectively. These shocks are common to all agents living in country $H$.

Consumption Preferences. The agent consumes a bundle of differentiated goods both from country $H$ and from country $F$ according to the following constant-elasticity-of-substitution (CES) aggregator:

$$C_t = \left[ a^{\frac{1}{\sigma}} C_{H,t}^{\frac{\rho}{\sigma}} + (1-a) a^{\frac{1}{\sigma}} C_{F,t}^{\frac{\rho}{\sigma}} \right]^{\frac{\sigma}{\rho-1}}, \tag{A.2}$$

where the bundles of differentiated goods are given by aggregators according to Dixit and Stiglitz (1977):

$$C_{H,t} = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c_t(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}} \tag{A.3}$$

$$C_{F,t} = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_1^n c_t(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}}.$$

\(^{29}\)In Corsetti, Dedola, and Leduc (2011), the agent derives utility also from the liquidity services of holding money. I abstract from money in the utility function, since monetary policy is conducted via interest rate rules.

\(^{30}\)The parameter $\eta$ is equivalent to the inverse of the Frisch elasticity of labor supply.
These preferences imply (i) that the elasticity of substitution between differentiated goods $c^j_t$ from one country is $\sigma$, which is assumed to be greater than one and equal across countries, (ii) that the elasticity of substitution between the bundles of goods from the two countries $C_{H,t}$ and $C_{F,t}$ is $\theta$, which is assumed to be greater than zero and equal across countries, and (iii) that the steady-state share of imported goods in overall consumption expenditures is $1 - a$. If $a > 1/2$, the agent consumes more goods from the country the agent lives in than from the other country, i.e., the agent has a home bias in consumption. This home bias is assumed to be symmetric across countries. Thus, the CES aggregator for an agent $j$ living in country $F$ is given by

$$C^j_t^* = \left[ (1 - a)^{\frac{1}{\theta}} C^j_{H,t}^{\frac{\theta-1}{\sigma}} + a^{\frac{1}{\theta}} C^j_{F,t}^{\frac{\theta-1}{\sigma}} \right]^{\frac{\theta}{\theta-1}}. \quad (A.4)$$

Accordingly, the consumer price index (CPI) in country $H$ expressed in country $H$’s currency is given by

$$P_t = \left[ a P_{H,t}^{1-\theta} + (1 - a) P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad (A.5)$$

where the producer price indexes (PPI) for the bundles of differentiated goods expressed in country $H$’s currency are defined by

$$P_{H,t} = \left[ \frac{1}{n} \int_0^n p_t(h)^{1-\sigma} dh \right]^{\frac{1}{1-\sigma}} \quad (A.6)$$

$$P_{F,t} = \left[ \frac{1}{1-n} \int_n^1 p_t(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}}.$$

The CPI in country $F$ expressed in country $F$’s currency is given by

$$P^*_t = \left[ (1 - a) P^*_{H,t}^{1-\theta} + a P^*_{F,t}^{1-\theta} \right]^{\frac{1}{1-\sigma}}. \quad (A.7)$$

**Producer-Currency Pricing.** In their role as producers, agents charge one price for their good irrespective of whether the good is sold in their country or is exported to the other country, setting the price in their country’s currency. Furthermore, exporting
does not entail transportation costs. These assumptions imply that the law of one price holds, i.e., a single differentiated good has the same price in both countries if expressed in the same currency, and that exchange rate pass-through is complete:

$$p_t(h) = S_t p^*_t(h), \quad p_t(f) = S_t p^*_t(f), \quad (A.8)$$

where $p_t(h)$ denotes the price of a differentiated good $y(h)$ produced in country $H$ denominated in country $H$’s currency, $p^*_t(h)$ denotes the price of the same good $y(h)$ denominated in country $F$’s currency, $p_t(f)$ denotes the price of a differentiated good $y(f)$ produced in country $F$ denominated in country $H$’s currency, $p^*_t(f)$ denotes the price of the same good $y(f)$ denominated in country $F$’s currency, and $S_t$ is the nominal exchange rate defined as the price of country $F$’s currency in terms of country $H$’s currency. Given equations (A.6), it is straightforward to show that the law of one price for each differentiated good translates into the law of one price for each bundle of goods:

$$P_{H,t} = S_t P^*_{H,t}, \quad P_{F,t} = S_t P^*_{F,t}. \quad (A.9)$$

In general, the law of one price does not translate into purchasing power parity. Thus, the real exchange rate, defined as the ratio of country-specific consumer prices

$$Q_t = \frac{S_t p^*_t}{P_t}, \quad (A.10)$$

adjusts in response to changing economic conditions. Purchasing power parity ($Q_t = 1$) only holds if the consumption baskets are identical across countries ($a = 1/2$).

Another international relative price of interest are the terms of trade, defined from the perspective of country $H$ as the ratio of the price of imported goods to the price of exported goods:

$$T_t = \frac{P_{F,t}}{S_t P^*_{H,t}}. \quad (A.11)$$

Under producer-currency pricing, where the law of one price holds, the terms of trade can be expressed as

$$T_t = \frac{S_t p^*_t}{P_{H,t}}. \quad (A.12)$$
Agent $j$ in country $H$ takes three decisions with respect to his consumption choices. First, he decides on the overall level of consumption $C^j_t$. Second, given $C^j_t$, the agent optimally allocates expenditures between the bundles of differentiated goods $C^j_{H,t}$ and $C^j_{F,t}$ by minimizing total expenditure $P_tC^j_t$ subject to the CES aggregator (A.2). As a result, demand for these bundles is given by

$$C^j_{H,t} = a \left( \frac{P_{H,t}}{P_t} \right)^{-\theta} C^j_t, \quad C^j_{F,t} = (1-a) \left( \frac{P_{F,t}}{P_t} \right)^{-\theta} C^j_t.$$  

(A.13)

Third, given $C^j_{H,t}$ and $C^j_{F,t}$, the agent optimally allocates expenditures between the differentiated goods by minimizing $P_{H,t}C^j_{H,t}$ and $P_{F,t}C^j_{F,t}$ subject to equations (A.3). This yields

$$c^j_t(h) = \frac{1}{n} \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\sigma} C^j_{H,t}, \quad c^j_t(f) = \frac{1}{1-n} \left( \frac{p_t(f)}{P_{F,t}} \right)^{-\sigma} C^j_{F,t}.$$  

(A.14)

Combining (A.13) and (A.14) yields

$$c^j_t(h) = \frac{a}{n} \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\sigma} \left( \frac{P_{H,t}}{P_t} \right)^{-\theta} C^j_t, \quad c^j_t(f) = \frac{1-a}{1-n} \left( \frac{p_t(f)}{P_{F,t}} \right)^{-\sigma} \left( \frac{P_{F,t}}{P_t} \right)^{-\theta} C^j_t.$$  

(A.15)

Analogously, demand equations for an agent $j$ in country $F$ are given by

$$c^j_t(h) = \frac{1-a}{n} \left( \frac{p^*_t(h)}{P^*_{H,t}} \right)^{-\sigma} \left( \frac{P^*_{H,t}}{P^*_t} \right)^{-\theta} C^j_t, \quad c^j_t(f) = \frac{a}{1-n} \left( \frac{p^*_t(f)}{P^*_{F,t}} \right)^{-\sigma} \left( \frac{P^*_{F,t}}{P^*_t} \right)^{-\theta} C^j_t.$$  

(A.16)
Market clearing for the differentiated goods $y_t(h)$ and $y_t(f)$ requires

$$y_t(h) = \int_0^n c^i_t(h) dj + \int_n^1 c^*_t(h) dj$$

(A.17)

$$y_t(f) = \int_0^n c^i_t(f) dj + \int_n^1 c^*_t(f) dj.$$ 

Using equations (A.15) and (A.16), world demand for the differentiated goods can be expressed as

$$y_t(h) = \frac{1}{n} \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\sigma} \left( \frac{P_{H,t}}{P_t} \right)^{-\theta} \left[ aC_t + (1 - a)Q^\theta_t C^*_t \right]$$

(A.18)

$$y_t(f) = \frac{1}{1 - n} \left( \frac{p_t(f)}{P_{F,t}} \right)^{-\sigma} \left( \frac{P_{F,t}}{P_t} \right)^{-\theta} \left[ (1 - a)C_t + aQ^\theta_t C^*_t \right],$$

where $C_t = \int_0^n c^i_t dj$ and $C^*_t = \int_n^1 c^*_t dj$.

Similar to (A.3), aggregate output in the two countries can be expressed as

$$Y_{H,t} = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n y_t(h) \right]^{\frac{\sigma}{\sigma - 1}} \left( \frac{1}{\sigma} \int_0^n y_t(h) \right)^{\frac{\sigma - 1}{\sigma - 1}}$$

(A.19)

$$Y_{F,t} = \left[ \left( \frac{1}{1 - n} \right)^{\frac{1}{\sigma}} \int_n^1 y_t(f) \right]^{\frac{\sigma}{\sigma - 1}} \left( \frac{1}{\sigma} \int_n^1 y_t(f) \right)^{\frac{\sigma - 1}{\sigma - 1}}.$$

Inserting equations (A.18) into the previous equations finally yields aggregate demand

$$Y_{H,t} = \left( \frac{P_{H,t}}{P_t} \right)^{-\theta} \left[ aC_t + (1 - a)Q^\theta_t C^*_t \right]$$

(A.20)

$$Y_{F,t} = \left( \frac{P_{F,t}}{P_t} \right)^{-\theta} \left[ (1 - a)C_t + aQ^\theta_t C^*_t \right].$$

**Local-Currency Pricing.** Under local-currency pricing, agents set the price for their good in the currency of the destination market, i.e., in their country’s currency if the good is sold domestically,
and in the other country’s currency if the good is sold abroad. When able to change prices in a given period, the agent will reset in both currencies. Since this is not possible every period, subsequent fluctuations in the nominal exchange rate lead to temporary deviations from the law of one price.

Following the same steps as before, aggregate demand under local-currency pricing is given by

\[
Y_{H,t} = \left( \frac{P_{H,t}}{P_t} \right)^{-\theta} \left[ aC_t + (1 - a) \left( \frac{Q_t}{\Delta_{H,t}} \right)^{\theta} C_t^* \right] \tag{A.21}
\]

\[
Y_{F,t} = \left( \frac{P_{F,t}}{P_t} \right)^{-\theta} \left[ (1 - a)C_t + a \left( \frac{Q_t}{\Delta_{F,t}} \right)^{\theta} C_t^* \right],
\]

where

\[
\Delta_{H,t} = \frac{S_t P_{H,t}^*}{P_{H,t}}, \quad \Delta_{F,t} = \frac{S_t P_{F,t}^*}{P_{F,t}} \tag{A.22}
\]

represent the deviation from the law of one price for each good.

**Cross-Country Risk Sharing.** Asset markets are assumed to be complete within and across countries. Agents can insure against all possible states of nature by holding a portfolio of state-contingent, one-period securities whose real value (denominated in units of the consumption-based price index) is denoted by $B_{H,j}^t$ and whose vector of prices is denoted by $q_t^H$. In addition, agents can trade in a noncontingent, one-period bond whose nominal value (denominated in country $H$’s currency) is denoted by $B_t^j$ and whose nominal interest rate is denoted by $R_t$. The intertemporal budget constraint of agent $j$ in country $H$ is then given by

\[
C_t^j + q_t^H B_t^{H,j} + \frac{B_t^j}{P_t (1 + R_t)} = B_{t-1}^{H,j} + \frac{B_{t-1}^j}{P_t} + (1 - \tau_t^H) \frac{p_t(h) y_t(h)}{P_t} \tag{A.23}
\]

The agent’s income stems also from sales revenues $p_t(h) y_t(h)$ net of a proportional, country-specific tax $\tau_t^H$.

\[31\] The tax will turn out to be a subsidy to exactly offset the distortion caused by monopolistic competition.
All contingent securities and noncontingent bonds are assumed to be in zero supply in the initial period, so $B_H^j = B_0 = 0$ for all $j$. Together with the facts that, within countries, agents have identical preferences and that asset markets are complete, this implies perfect risk sharing of consumption within each country. Therefore, it is possible to analyze the consumer problem from the viewpoint of the representative agent of country $H$ and country $F$.

The representative agent in country $H$ maximizes his lifetime utility (A.1) subject to the budget constraint (A.23). By combining the resulting first-order conditions with respect to consumption and bond holdings, the usual Euler consumption equation is then given by

$$ U_C(C_t, \zeta_{C,t}) = (1 + R_t) \beta E_t \left\{ U_C(C_{t+1}, \zeta_{C,t+1}) \frac{P_t}{P_{t+1}} \right\}. \quad (A.24) $$

The Euler consumption equation for the representative agent in country $F$ is obtained analogously and given by

$$ U_C(C^*_t, \zeta^*_{C,t}) = (1 + R^*_t) \beta E_t \left\{ U_C(C^*_{t+1}, \zeta^*_{C,t+1}) \frac{P^*_t}{P^*_{t+1}} \right\}. \quad (A.25) $$

Complete asset markets across countries leads to price equalization in the state-contingent securities (expressed in country $H$’s currency), implying the following risk-sharing condition:

$$ \beta \frac{U_C(C_{t+1}, \zeta_{C,t+1})}{U_C(C_t, \zeta_{C,t})} \frac{P_t}{P_{t+1}} = \beta \frac{U_C(C^*_{t+1}, \zeta^*_{C,t+1})}{U_C(C^*_t, \zeta^*_{C,t})} \frac{P^*_t}{P^*_{t+1}}. \quad (A.26) $$

Inserting the country-specific Euler consumption equations yields the uncovered interest parity condition, according to which the expected change in the nominal exchange rate corresponds to the ratio of the country-specific interest rates:

$$ E_t \Delta S_{t+1} = \frac{1 + R_t}{1 + R^*_t}. \quad (A.27) $$

Assuming net foreign asset positions to be initially symmetric and applying the definition of the real exchange rate (A.10), the risk-sharing condition takes the following form:

$$ Q_t = \left( \frac{C^*_t}{C_t} \right)^{-\rho} \frac{\zeta^*_{C,t}}{\zeta_{C,t}}. \quad (A.28) $$
A.2 Producer Problem

In their role as producers, agents act in an environment of monopolistic competition, in which they dispose of some degree of market power. Furthermore, prices are sticky in the sense that the agent is able to change the price in a given period with a fixed probability, as in (A.10). The probability of being able to change the price is identical across countries and given by $1 - \alpha$.

**Producer-Currency Pricing.** Agent $j$ in country $H$ maximizes expected, discounted profits by choosing the price $\tilde{p}_t(h)$ taking into account that demand depends on the chosen price and that the price may remain unchanged for some periods in the domestic market. In the foreign market, the price in foreign currency fluctuates with the nominal exchange rate, so that the law of one price always holds (complete exchange rate pass-through). Formally, the agent maximizes

$$E_t \sum_{k=0}^{\infty} (\alpha \beta)^k \left[ \lambda_{t+k} (1 - \tau_H^t) \tilde{p}_t(h) \tilde{y}_{t,t+k}(h) - V(\tilde{y}_{t,t+k}(h), \zeta_{Y,t+k}) \right]$$

(A.29)

subject to the demand function

$$\tilde{y}_{t,t+k}(h) = \frac{1}{n} \left( \frac{\tilde{p}_t(h)}{P_{H,t+k}} \right)^{-\sigma} \left( \frac{P_{H,t+k}}{P_{t+k}} \right)^{-\theta}$$

$$\times \left[ aC_{t+k} + (1 - a)Q_{t+k}^\theta C^*_t \right],$$

(A.30)

where $\tilde{y}_{t,t+k}(h)$ denotes total demand of good $h$ at time $t + k$ if the price $\tilde{p}_t(h)$ prevails. Profits are expressed in utility units. Therefore, nominal sales revenues net of taxes $(1 - \tau_H^t) \tilde{p}_t(h) \tilde{y}_{t,t+k}(h)$ are converted into utility units using the marginal utility of nominal revenues $\lambda_{t+k} = \frac{U_C(C_{t+k}, \zeta_C, t+k)}{P_{t+k}}$. The cost of production expressed in utility units is given by the function $V = \zeta_{Y,t}^{1+\eta} \tilde{y}_{t}^{\eta}$. The first-order condition yields the optimal price

$$\tilde{p}_t(h) = \frac{E_t \sum_{k=0}^{\infty} (\alpha \beta)^k \frac{\sigma}{(\sigma - 1)(1 - \tau_H^t)} V_y(\tilde{y}_{t,t+k}(h), \zeta_{Y,t+k}) \tilde{y}_{t,t+k}(h)}{E_t \sum_{k=0}^{\infty} (\alpha \beta)^k \lambda_{t+k} \tilde{y}_{t,t+k}(h)},$$

(A.31)
where \(V_y\) denotes the derivative of function \(V\) with respect to output \(\tilde{y}(h)\). All agents that live in the same country and are able to reset their price in a certain period will set the same price, since they share identical preferences (function \(V\)) and face the same demand curves, which depend only on aggregate variables such as \(P_H, P, P^*, S, C,\) and \(C^*\), and the common elasticities of substitution \(\sigma\) and \(\theta\). Hence, in a given period, a fraction \(1 - \alpha\) of agents will set the same optimal price, while for a fraction \(\alpha\) of agents the price from the previous period remains effective:

\[
P_{H,t} = \left[ \alpha P_{H,t-1}^{1-\sigma} + (1 - \alpha)\tilde{p}_t(h) \right]^{\frac{1}{1-\sigma}}
\]

\[
P_{F,t}^* = \left[ \alpha P_{F,t-1}^{1-\sigma} + (1 - \alpha)\tilde{p}_t^*(f) \right]^{\frac{1}{1-\sigma}}.
\]

When prices are flexible, the optimal-price equation (A.31) for country \(H\) simplifies to

\[
\frac{P_{H,t}}{P_t} = \frac{\sigma}{(\sigma - 1)(1 - \tau_t^H)} \frac{V_y(y_t(h), \zeta_{Y,t})}{U_C(C_t, \zeta_{C,t})},
\]

and for country \(F\) to

\[
\frac{P_{F,t}^*}{P_t^*} = \frac{\sigma}{(\sigma - 1)(1 - \tau_t^F)} \frac{V_y(y_t(f), \zeta_{Y,t}^*)}{U_C(C_t^*, \zeta_{C,t}^*)}.
\]

The markup that agents in country \(i = H, F\) are able to charge is defined as

\[
\mu_t^i \equiv \frac{\sigma}{(\sigma - 1)(1 - \tau_t^i)}.
\]

**Local-Currency Pricing.** In contrast to producer-currency pricing, agent \(j\) in country \(H\) sets two prices to maximize expected, discounted profits: the price in country \(H\’s\) currency for sales in country \(H\) and the price in country \(F\’s\) currency for sales in country \(F\). Formally, the agent maximizes

\[
E_t \sum_{k=0}^{\infty} (\alpha \beta)^k \left[ \lambda_{t+k} (1 - \tau_t^H) [\tilde{p}_t(h)\tilde{c}_{t,t+k}(h) + S_{t+k}\tilde{p}_t^*(h)\tilde{c}_{t,t+k}^*(h)] - V(\tilde{y}_{t,t+k}(h), \zeta_{Y,t+k}) \right]
\]

(A.36)
subject to the demand functions

\[
\tilde{c}_{t,t+k}(h) = \frac{1}{n} \left( \frac{\tilde{p}_t(h)}{P_{H,t+k}} \right)^{-\sigma} \left( \frac{P_{H,t+k}}{P_{t+k}} \right)^{-\theta} aC_{t+k} \tag{A.37}
\]

\[
\tilde{c}^*_t, t+k(h) = \frac{1}{n} \left( \frac{\tilde{p}^*_t(h)}{P^*_{H,t+k}} \right)^{-\sigma} \left( \frac{P^*_{H,t+k}}{P^*_{t+k}} \right)^{-\theta} (1-a)C^*_{t+k}.
\]

The first-order conditions yield the two optimal prices

\[
\tilde{p}_t(h) = \frac{E_t \sum_{k=0}^\infty (\alpha \beta)^k \mu_{t+k}^H V_y(\tilde{y}_{t,t+k}(h), \zeta_{Y,t+k}) \tilde{c}_{t,t+k}(h)}{E_t \sum_{k=0}^\infty (\alpha \beta)^k \lambda_{t+k} \tilde{c}_{t,t+k}(h)} \tag{A.38}
\]

\[
\tilde{p}^*_t(h) = \frac{E_t \sum_{k=0}^\infty (\alpha \beta)^k \mu_{t+k}^H V_y(\tilde{y}^*_{t,t+k}(h), \zeta_{Y,t+k}) \tilde{c}^*_{t,t+k}(h)}{E_t \sum_{k=0}^\infty (\alpha \beta)^k \lambda_{t+k} S_{t+k} \tilde{c}^*_{t,t+k}(h)}.
\]

Together with the two optimal prices of agent \( j \) in country \( F \), which are derived analogously, there are four sticky producer prices, compared with two in the case of producer-currency pricing. The corresponding producer price indexes are given by

\[
P_{H,t} = \left[ \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha) \tilde{p}_t(h)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{A.39}
\]

\[
P^*_{H,t} = \left[ \alpha P^*_{H,t-1}^{1-\sigma} + (1-\alpha) \tilde{p}^*_t(h)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}
\]

\[
P_{F,t} = \left[ \alpha P_{F,t-1}^{1-\sigma} + (1-\alpha) \tilde{p}_t(f)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}
\]

\[
P^*_{F,t} = \left[ \alpha P^*_{F,t-1}^{1-\sigma} + (1-\alpha) \tilde{p}^*_t(f)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}.
\]

A.3 Log-Linearization

Deviations of the logarithm of a variable \( X_t \) from its steady state are denoted by \( \tilde{X}_t \) if prices are sticky and by \( \tilde{X}^f_t \) if prices are flexible and markups are neutralized (efficient allocation).
A.3.1 Sticky-Price Equilibrium under Producer-Currency Pricing

Under sticky prices and producer-currency pricing, the system of equations is given by

\[ E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} \left( \hat{R}_t - E_t \pi_t + E_t \hat{\zeta}_{C,t+1} - \hat{\zeta}_{C,t} \right) \]  
(A.40)

\[ \hat{Q}_t = \rho \left( \hat{C}_t - \hat{C}_t^* \right) + \left( \hat{\zeta}_{C,t}^* - \hat{\zeta}_{C,t} \right) \]  
(A.41)

\[ E_t \Delta \hat{S}_{t+1} = \hat{R}_t - \hat{R}_t^* \]  
(A.42)

\[ \hat{Q}_t = (2a - 1) \hat{T}_t \]  
(A.43)

\[ \tilde{Y}_{H,t} = 2a(1 - a) \theta \hat{T}_t + a \hat{C}_t + (1 - a) \hat{C}_t^* \]  
(A.44)

\[ \tilde{Y}_{F,t} = -2a(1 - a) \theta \hat{T}_t + (1 - a) \hat{C}_t + a \hat{C}_t^* \]  
(A.45)

\[ \pi_{H,t} = (\rho + \eta)k \left( \tilde{Y}_{H,t} - \tilde{Y}_{H,t}^{fb} \right) - 2a(1 - a)(\rho \theta - 1)k \left( \hat{T}_t - \hat{T}_t^{fb} \right) \]
\[ + k \hat{\mu}_t^H + \beta E_t \pi_{H,t+1} \]  
(A.46)

\[ \pi_{F,t}^* = (\rho + \eta)k \left( \tilde{Y}_{F,t} - \tilde{Y}_{F,t}^{fb} \right) + 2a(1 - a)(\rho \theta - 1)k \left( \hat{T}_t - \hat{T}_t^{fb} \right) \]
\[ + k \hat{\mu}_t^F + \beta E_t \pi_{F,t+1}^* \]  
(A.47)

\[ \hat{T}_t = \hat{T}_{t-1} + \pi_{F,t}^* - \pi_{H,t} + \Delta \hat{S}_t \]  
(A.48)

\[ \pi_t = a \pi_{H,t} + (1 - a)(\pi_{F,t}^* + \Delta \hat{S}_t) \]  
(A.49)

\[ \pi_t^* = (1 - a)(\pi_{H,t} - \Delta \hat{S}_t) + a \pi_{F,t}^*. \]  
(A.50)

Equation (A.40) is the log-linear approximation of the Euler consumption equation (A.24), where \( \pi_t = \ln(P_t/P_{t-1}) \). Equation (A.41) is the log-linear approximation of the risk-sharing condition (A.28), and equation (A.42) is the log-linear approximation of the uncovered interest parity condition (A.27).\(^{32}\) Equation (A.43) describes

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\(^{32}\)Alternatively, the model could be specified by including both country-specific Euler consumption equations next to the risk-sharing condition, while omitting the uncovered interest parity condition.
the link between the real exchange rate and the terms of trade and is obtained by combining the log-linearized definition of the real exchange rate (A.10) with the log-linearized expressions for the country-specific CPIs (A.5) and (A.7), applying the law of one price. Equations (A.44) and (A.45) are obtained by log-linearizing the aggregate demand equations (A.20), using equation (A.43) to eliminate the real exchange rate.

Equations (A.46) and (A.47) represent the New Keynesian Phillips curves for country $H$ and country $F$, respectively, where $\pi_{H,t} = \ln(P_{H,t}/P_{H,t-1})$ and $\pi_{F,t}^* = \ln(P_{F,t}^*/P_{F,t-1}^*)$. They are derived by combining the log-linear approximation of the optimal price (A.31) with the log-linear approximation of (A.32) for each country separately. The parameter $k$ is defined as

$$k = \frac{(1 - \alpha \beta)(1 - \alpha)}{\alpha} \frac{1}{1 + \sigma \eta}. \quad (A.51)$$

Equation (A.48) is the log-linear approximation of the terms of trade (A.12), expressed in first differences. Equations (A.49) and (A.50) are the log-linear approximations of the country-specific CPIs (A.5) and (A.7), applying the law of one price and expressed in first differences.

**A.3.2 Sticky-Price Equilibrium under Local-Currency Pricing**

Under sticky prices and local-currency pricing, the system of equations is given by

$$E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} \left( \hat{R}_t - E_t \pi_{t+1} + E_t \hat{\zeta}_{C,t+1} - \hat{\zeta}_{C,t} \right) \quad (A.52)$$

$$\hat{Q}_t = \rho \left( \hat{C}_t - \hat{C}_t^* \right) + \left( \hat{\zeta}_{C,t}^* - \hat{\zeta}_{C,t} \right) \quad (A.53)$$

$$E_t \Delta \hat{S}_{t+1} = \hat{R}_t - \hat{R}_t^* \quad (A.54)$$

$$\hat{Q}_t = (2a - 1) \hat{T}_t + 2a \hat{\Delta}_t \quad (A.55)$$

$$\hat{Y}_{H,t} = 2a(1 - a) \theta (\hat{T}_t + \hat{\Delta}_t) + a \hat{C}_t + (1 - a) \hat{C}_t^* \quad (A.56)$$

$$\hat{Y}_{F,t} = -2a(1 - a) \theta (\hat{T}_t + \hat{\Delta}_t) + (1 - a) \hat{C}_t + a \hat{C}_t^* \quad (A.57)$$
\[ \pi_{H,t} = (\rho + \eta)k\left(\hat{Y}_{H,t} - \hat{Y}_{H,t}^{fb}\right) \]
\[ - (1 - a)k\left[2a(\rho \theta - 1)\left(\hat{T}_t - \hat{T}_t^{fb} + \hat{\Delta}_t\right) - \hat{\Delta}_t\right] \]
\[ + k\hat{\mu}_H + \beta E_t\pi_{H,t+1} \quad (A.58) \]
\[ \pi^*_{H,t} = (\rho + \eta)k\left(\hat{Y}_{H,t} - \hat{Y}_{H,t}^{fb}\right) \]
\[ - (1 - a)k\left[2a(\rho \theta - 1)\left(\hat{T}_t - \hat{T}_t^{fb} + \hat{\Delta}_t\right) - \hat{\Delta}_t\right] - k\hat{\Delta}_t \]
\[ + k\hat{\mu}_H + \beta E_t\pi^*_{H,t+1} \quad (A.59) \]
\[ \pi_{F,t} = (\rho + \eta)k\left(\hat{Y}_{F,t} - \hat{Y}_{F,t}^{fb}\right) \]
\[ + (1 - a)k\left[2a(\rho \theta - 1)\left(\hat{T}_t - \hat{T}_t^{fb} + \hat{\Delta}_t\right) - \hat{\Delta}_t\right] + k\hat{\Delta}_t \]
\[ + k\hat{\mu}_F + \beta E_t\pi_{F,t+1} \quad (A.60) \]
\[ \pi^*_{F,t} = (\rho + \eta)k\left(\hat{Y}_{F,t} - \hat{Y}_{F,t}^{fb}\right) \]
\[ + (1 - a)k\left[2a(\rho \theta - 1)\left(\hat{T}_t - \hat{T}_t^{fb} + \hat{\Delta}_t\right) - \hat{\Delta}_t\right] \]
\[ + k\hat{\mu}_F + \beta E_t\pi^*_{F,t+1} \quad (A.61) \]
\[ \hat{T}_t = \hat{T}_{t-1} + \pi_{F,t} - \pi^*_{H,t} - \Delta \hat{S}_t \quad (A.62) \]
\[ \pi_t = a\pi_{H,t} + (1 - a)\pi_{F,t} \quad (A.63) \]
\[ \pi^*_t = (1 - a)\pi^*_H + a\pi^*_F \quad (A.64) \]
\[ \hat{\Delta}_t = \hat{\Delta}_{t-1} + \Delta S_t + \pi^*_H - \pi_{H,t}. \quad (A.65) \]

The Euler consumption equation, the risk-sharing condition, and the uncovered interest parity condition are identical to the case of producer-currency pricing. The real exchange rate is still linked to the terms of trade, but now also to the deviation from the law of one price (equation (A.55)). Since the countries are assumed to be symmetric, the deviation from the law of one price is identical across countries (\(\hat{\Delta}_{H,t} = \hat{\Delta}_{F,t} = \hat{\Delta}_t\)). The aggregate demand equations (A.56) and (A.57) as well as the four New Keynesian Phillips curves (A.58) through (A.61) contain the deviation from the law of one price as well. The terms-of-trade identity (A.62) and the definitions
of the CPI inflation rates (A.63) and (A.64) are different from the case of producer-currency pricing, since the law of one price does not hold under local-currency pricing. Finally, equation (A.65) is the log-linear approximation of the definition of the deviation from the law of one price (A.22), expressed in first differences.

**A.3.3 Efficient Allocation**

The first-best \((fb)\) or efficient allocation describes the equilibrium in which prices are fully flexible, in which the law of one price holds, and in which markups are neutralized at all times with an appropriate subsidy \((\mu^i_t = 0)\). This efficient allocation provides a useful benchmark in order to assess the welfare implications of the two currency regimes.

Accordingly, efficient output in each country is given by

\[
(\rho + \eta)\tilde{Y}^{fb}_H, t = 2a(1 - a)(\rho \theta - 1)\tilde{T}^{fb}_t
\]

\[
- (1 - a) \left( \hat{\zeta}_{C, t} - \hat{\zeta}^*_{C, t} \right) + \hat{\zeta}_{C, t} + \eta \hat{\zeta}_{Y, t}
\]

\[
(\rho + \eta)\tilde{Y}^{fb}_F, t = - 2a(1 - a)(\rho \theta - 1)\tilde{T}^{fb}_t
\]

\[
+ (1 - a) \left( \hat{\zeta}_{C, t} - \hat{\zeta}^*_{C, t} \right) + \hat{\zeta}^*_{C, t} + \eta \hat{\zeta}^*_{Y, t}.
\]

The efficient terms of trade can be written as

\[
[4a(1 - a)\rho \theta + (2a - 1)^2]\tilde{T}^{fb}_t = \rho \left( \tilde{Y}^{fb}_H, t - \tilde{Y}^{fb}_F, t \right)
\]

\[
- (2a - 1) \left( \hat{\zeta}_{C, t} - \hat{\zeta}^*_{C, t} \right).
\]

The first equation is obtained by combining the risk-sharing condition (A.41), equation (A.43), and the aggregate demand equation (A.44), all of which hold under flexible prices as well, with the log-linear approximation of the optimal-price equation (A.33). The second equation is derived completely analogously. The third equation is derived by subtracting the country-specific aggregate demand equations (A.44) and (A.45) from each other and by using the risk-sharing condition (A.41) and equation (A.43) to eliminate country-specific consumption and the real exchange rate.
Appendix B. Monetary Union Regime

The main difference of the monetary union regime compared with the flexible exchange rate regime, of course, is that the two countries share one currency and that the common monetary policy sets one union-wide nominal interest rate. Notwithstanding, the model structure is to a large extent identical (see, e.g., Benigno 2004). However, the type of local-currency pricing that was specified under the flexible exchange rate regime is impossible in a monetary union, since both countries share one currency. Accordingly, the law of one price is assumed to always hold under the monetary union regime.

Under flexible prices, monetary policy is neutral, so that real variables are only driven by fundamental shocks. Thus, the efficient allocation is independent of the currency regime. Therefore, the behavior of efficient output and the efficient terms of trade under the monetary union regime is also described by equations (A.66) and (A.67).

Under sticky prices, the system of equations is given by

\[ E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} \left( \hat{R}^M_{t} - E_t \pi_{t+1} + E_t \hat{\xi}_{C,t+1} - \hat{\xi}_{C,t} \right) \]  
\[ \hat{Q}_t = \rho \left( \hat{C}_t - \hat{C}_t^* \right) + \left( \hat{\xi}_{C,t} - \hat{\xi}_{C,t} \right) \]  
\[ \hat{Y}_{H,t} = 2a(1-a)\theta \hat{T}_t + a \hat{C}_t + (1-a)\hat{C}_t^* \]  
\[ \hat{Y}_{F,t} = -2a(1-a)\theta \hat{T}_t + (1-a)\hat{C}_t + a\hat{C}_t^* \]  
\[ \pi_{H,t} = (\rho + \eta)k \left( \hat{Y}_{H,t} - \hat{Y}_{H,t}^f \right) - 2a(1-a)(\rho\theta - 1)k \left( \hat{T}_t - \hat{T}_{t}^f \right) \]  
\[ \pi_{F,t} = (\rho + \eta)k \left( \hat{Y}_{F,t} - \hat{Y}_{F,t}^f \right) + 2a(1-a)(\rho\theta - 1)k \left( \hat{T}_t - \hat{T}_{t}^f \right) \]  
\[ \hat{T}_t = \hat{T}_{t-1} + \pi_{F,t}^* - \pi_{H,t} \]  
\[ \pi_t = a\pi_{H,t} + (1-a)\pi_{F,t}^* \]  
\[ \pi_{t}^* = (1-a)\pi_{H,t} + a\pi_{F,t}^*. \]
The Euler consumption equation (B.1) differs from the one under the flexible exchange rate regime only in that the nominal interest rate is given by the union-wide interest rate $\hat{R}_{t}^{MU}$. Nonetheless, real interest rates $\hat{R}_{t}^{MU} - E_{t} \pi_{t+1}$ are generally country specific, since CPI inflation rates usually differ across countries. Since the two countries form a monetary union, the uncovered interest parity condition is obsolete and the nominal exchange rate disappears from all relevant equations.

Appendix C. Robustness of Results

Figure C.1. Welfare Loss as a Function of the Inflation Coefficient ($\phi_{\pi}$) for Different Combinations of Producer-Currency Pricing (PCP), Local-Currency Pricing (LCP), Productivity Shocks, and Cost-Push Shocks
Figure C.2. Welfare Loss as a Function of the Output Coefficient ($\phi_Y$) under Local-Currency Pricing and Productivity Shocks

Notes: Left panel: Response to output ($\hat{Y}_t$). Right panel: Response to output gap ($\hat{Y}_t - \tilde{Y}_{fb}$).

Figure C.3. Welfare Loss as a Function of the Output Coefficient ($\phi_Y$) under Cost-Push Shocks

Notes: Left panel: Producer-currency pricing. Right panel: Local-currency pricing Under cost-push shocks, there is no difference between targeting output and targeting the output gap.
Figure C.4. Welfare Loss as a Function of the Exchange Rate Coefficient ($\phi_S$) under Local-Currency Pricing and Productivity Shocks

Notes: Left panel: Unilateral exchange rate targeting. Right panel: Bilateral exchange rate targeting.

Figure C.5. Welfare Loss as a Function of the Exchange Rate Coefficient ($\phi_S$) under Producer-Currency Pricing and Cost-Push Shocks

Notes: Left panel: Unilateral exchange rate targeting. Right panel: Bilateral exchange rate targeting.
Figure C.6. Welfare Loss as a Function of the Exchange Rate Coefficient ($\phi_S$) under Local-Currency Pricing and Cost-Push Shocks

Notes: Left panel: Unilateral exchange rate targeting. Right panel: Bilateral exchange rate targeting.

Figure C.7. Welfare Loss as a Function of the Interest Rate Smoothing Coefficient ($\phi_R$) for Different Combinations of Producer-Currency Pricing (PCP), Local-Currency Pricing (LCP), Productivity Shocks, and Cost-Push Shocks
Figure C.8. Welfare Loss as a Function of the Trade Openness ($a$) for Different Combinations of Producer-Currency Pricing (PCP), Local-Currency Pricing (LCP), Productivity Shocks, and Cost-Push Shocks

References


