Price Dynamics and Production Lags

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Abstract: This paper provides a new explanation of why inflation is sluggish in response to aggregate demand shocks and why aggregate output changes as result of such shocks. We argue that these phenomena are related to lags between inputs and outputs in the production process, “production lags” for short. The broad intuition is that production activities in a modern economy are interconnected through complex input-output relations, with production lags within individual firms, and that it takes considerable time for cost and price changes to penetrate the entire input-output system. Our analysis provides a rationale for a prolonged inverse relation between inflation and unemployment. The paper suggests that the interaction of inflation persistence and unemployment persistence may offer a possible explanation of high and prolonged European unemployment.

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A main ambition in macroeconomic theory has been to explain why inflation is sluggish in response to aggregate product demand shocks and why aggregate output changes as result of such shocks. We argue in this paper that these phenomena are related to lags between inputs and outputs in the production process, “production lags” for short. The broad intuition is that production activities in a modern economy are interconnected through complex input-output relations, with production lags within individual firms, and that it takes considerable time for cost and price changes to penetrate the entire input-output system.

Our analysis indicates that production lags are not sufficient to generate real effects of monetary shocks if it is assumed that the real interest rate is completely unaffected by the rate of inflation. To get real effects in our model, it is also necessary to assume that nominal interest rates adjust only gradually to inflation. The assumption of sluggish adjustment of nominal interest rates is also a highly realistic one. There has been a strong negative correlation between inflation and real interest rates in many countries over much of the 20th century — not only on a yearly basis but also for periods as long as five or ten years (Homer and Sylla (1991), Ibbotson (1989), and national price statistics, as well as Mishkin (1993)). Although this phenomenon is not yet well understood, conceivable explanations are that many long-term loan contracts are expressed in nominal terms and that central banks often have nominal interest rate targets.

Of course, temporary real effects of product demand shocks will emerge in all intertemporal models in which nominal interest rates adjust only gradually to inflation. But the main point of our analysis is that when the nominal interest rate is sluggish, inflation dynamics depends on production lags. The associated movements in unemployment generate a downward-sloping short- and medium-run Phillips curve.

Our theory is independent of the prevailing theories of price sluggishness, such as the menu-cost and wage-price staggering theories. Unlike the menu cost theory, we provide an explanation for why prices in practice are often changed frequently in the same direction but not by sufficiently large amounts to obviate the need for significant quantity adjustments. Our theory is observationally distinct from the wage-price staggering theory, because in our theory price inertia depends on technologically given production lags, whereas in the staggering theory it depends on the length of staggered
contract periods. Whereas contract periods often change in response to inflation, production lags are less likely to do so.

At first sight, our model appears similar to those of Blanchard (1987) and Basu (1995) in that it involves intermediate goods; but in contrast to Blanchard, it is choice theoretical and relates price dynamics to production lags (rather than price-wage contracts), and in contrast to Basu, it is dynamic and unrelated to menu costs. Our model differs also from the heuristic argument of Gordon (1990) which in fact implies that price inertia is generated by systematic expectational errors in connection with the inability of firms to predict correctly costs changes that arise within a complex input-output system.

But why develop a new explanation for sluggish aggregate prices when celebrated theories already exist in this field? It has turned out that these theories suffer from serious problems. As shown by Caplin and Spulber (1987), though menu costs reduce firms’ frequency of price change, they don’t necessarily generate aggregate price sluggishness, because there is no reason why infrequent, large changes of individual product prices should lead to more aggregate price sluggishness than frequent, small changes. Furthermore, menu costs do not provide a convincing rationale for inertia in employment and unemployment, if - as in generally the case - the costs of adjusting prices are dwarfed by the costs of adjusting employment. Moreover, while the wage-price staggering theory does not as yet rest on optimizing choice-theoretic foundations, our theory does. Whereas our theory takes account of how inflation affects real and nominal interest rates and thereby influences the discounting mechanism that links wages and prices through time, the existing wage-price staggering models do not.²

I. The Basics

Our model is meant to be a short-hand representation of an economy in which goods are produced through input-output chains in which outputs are lagged behind inputs. In this underlying vision of economic activity, the current profit-maximizing price of each output is a markup over the lagged price of its associated input. Consequently, final product demand shocks (initiated, say, by monetary shocks) give rise to a succession of price responses, rippling their way through the input-output system. During this prolonged adjustment process, the product demand shocks have real effects, generating a short- and medium-run inflation-unemployment trade-off.

In our simplified analytical representation of this vision, we consider an economy that produces a fixed number ($F$) of differentiated, nondurable final goods, one by each firm. We let each firm in our model be vertically integrated, so that the entire input-output chain leading to its final good lies within the firm. In this context, final product
prices are markups over wages paid in earlier stages of production. Assuming all firms face symmetric demand and cost conditions, we can restrict our analysis to just one firm.

II. The Firm’s Decision

For expositional simplicity, we consider the following rudimentary production chain. In the initial stage of production, the first intermediate good \( Q_{0,t} \) is produced instantaneously in period \( t \) by means of labor \( W_t Q_{0,t} = A_0 N_t^{b_0} \). Then this intermediate good is used to produce another intermediate good, with a lag. This second intermediate good produces yet another intermediate good with a lag, and so on, for \( T \) stages of production \( Q_{j+1,t} = A_j Q_{j+1,t-1}^{b_j} \), for \( j = 1, \ldots, T \), and \( A_j \) and \( B_j \) are constants (\( A_j > 0, 0 < B_j < 1 \)). Thus the production relation between the final good and the labor input may be expressed as \( Q_{T,t} = \Gamma_0 N_t^{\Gamma_1} \), where \( Q_{T,t} \) is denoted by \( Q_{T,t} \), \( \Gamma_0 \equiv \Pi_{j=0}^{T} A_j \), and \( \Gamma_1 \equiv \Pi_{j=0}^{T} B_j \). Whereas \( t \) denotes the period of analysis, \( T \) is the total production period (the time between the initial labor input and the final output).

For simplicity, let the final demand for the firm's output have the following constant-elasticity form: \( Q_{T+1,t} = \Lambda (M_{T+1,t} / P_{T+1,t}^{\frac{1}{\gamma}}) \left( P_{T+1,t} / P_{T+1,t}^{\frac{1}{\gamma}} \right)^{\gamma} \), where \( P_{T+1,t} \) is the price of the final good, \( P_{T+1,t} \) is the aggregate price level, and \( M_{T+1,t} \) is the money supply. (In what follows, underlined variables stand for macroeconomic aggregates; Greek letters stand for parameters.)

At time period \( t \), each firm takes the nominal wage \( W_t \) as predetermined, decides to employ \( N_t \) of labor, and plans to produce \( Q_{T+1,t} \) of output and sell it at the price \( P_{T+1,t} \). The firm’s objective is to maximize the present value of its stream of profits. Since the firm’s decision problem contains no intertemporal links extending beyond the production period (from period \( t \) to \( t+T \)), it is not necessary for the firm to anticipate events lying more than \( T \) periods in the future. Let \( R_{T+1,t} \) be the average real interest factor per period of analysis from period \( t \) to \( t+T \), so that \( R_{T+1,t} \) is the interest factor for the entire production period. Let \( P_{T+1,t}^{E} \) be the firm’s period- \( t \) point expectation of the period-(\( t+T \)) price level. Then the firm’s decision can be reduced to the maximization of

\[
\frac{1}{R_{T+1,t}} \frac{P_{T+1,t}^{E}}{P_{T+1,t}^{E}} Q_{T+1,t}^{\gamma} W_t^{\frac{1}{\gamma}} N_t, \text{ subject to the production function } (\text{relating the final good to the labor input}) \text{ and the expected product demand function.}
\]
The first-order condition of the optimization problem above implies that the discounted real marginal revenue product of labor is equal to the real wage:

\[
\frac{1}{R_i^{t+T}} \frac{P_{i,t+T}}{P_{e,i,t+T}} H_\Gamma \Gamma_i N_i^{t+T-1} = \frac{W_i}{P_{i,t}}
\]

where \( H = 1 - \frac{1}{\eta} \) is Lerner's index of monopoly power.

Let capital letters stand for levels and small letters represent logs (with the exception of the unemployment rate, which will not be specified in logs). Then the firm's labor demand relation becomes:

\[
\kappa_1 = \left( \ln(1 - \Gamma_1) \right) + \left( \ln(1 - \Gamma_2) \right) + \left( \ln(1 - \Gamma_3) \right) + \left( \ln(1 - \Gamma_4) \right) + \left( \ln(1 - \Gamma_5) \right) - \beta_1, \quad \beta_2, \quad \beta_3, \quad \beta_4, \quad \beta_5
\]

where \( \kappa_1 \) is a constant (and similarly, below, \( \kappa_j, \forall j \) are also constants) and \( p_{i,t+T} \) is the log of the firm's expected price level. In words, labor demand \( n_t \) depends on the real wage \( w_{t+T} - p_{i,t} \), relative price \( p_{i,t+T} - p_{i,t+T}^E \), the average real interest rate \( r_{t+T} \) over the production period, and the length of that production period \( T \).

III. The Labor Market Equilibrium

Let the labor force be constant: \( l = (in \ logs) \). Then the unemployment rate (not in logs) may be approximated by \( u_l = l - n \).

By the Fisher equation, let \( i_{t+T} = r_{t+T} + \pi_{t+T}^E \), where \( k_{t+T} \) and \( r_{t+T} \) are the logs of the one-period nominal and real interest factors (respectively) and

\[
\pi_{i,t+T}^E = \left( p_{i,t+T}^E - p_{i,t} \right) / T
\]

is the one-period inflation rate expected by the firms. For simplicity, let the long-run real interest rate be constant \( r_f = \bar{r} \). We assume, plausibly, that the nominal interest rate adjusts gradually to the inflation rate:

\[
i_{t+T} = \bar{r} + \left( 1 - \alpha \right) \alpha^T \pi_{i,t+T}^E,
\]

where \( 0 < \alpha < 1 \) in the short and medium run and \( \alpha = 0 \) in the long run. Thus the real interest rate is given by the following interest dynamics equation:

\[
r_{t+T} = \bar{r} - \alpha \pi_{i,t+T}^E.
\]

Substituting the interest dynamics equation into the firm's labor demand equation, aggregating over all firms \( (n = \log f) \), where \( f \) is the log of the number of firms), expressing employment in terms of unemployment \( (u = l - n) \), and letting \( p_{t+T} = p_{t+T}^E \) (since all firms face symmetric conditions), we obtain an aggregate labor demand equation, expressed in terms of unemployment:

\[
u_l = \kappa_2 + \frac{1}{1 - \Gamma_3} \left( \ln(1 - \Gamma_4) \ln(1 - \Gamma_5) \right) - \frac{\alpha T}{1 - \Gamma_3} \pi_{i,t+T}^E
\]
where $\pi_{t,T}^E = \left( p_{t+T}^E - p_{t+T} \right) / T$ is the average inflation rate over the production period.

The initial labor demand equation (1) is depicted by the LD curve in Fig. 1c.

In order to concentrate our attention on the implications of production lags for price dynamics and the inflation-unemployment trade-off, we ignore the possibility of lags in wage adjustment to the price level and, instead, consider a simple, standard wage setting equation in which the real wage expected by the households depends inversely on the unemployment rate:

$$w_t - p_{t-1,t}^e = b_0 - b_1 u_t$$

where $p_{t-1,t}^e$ is the households’ period-(t-1) expectation of the period-t price level, and $b_1 > 0$. This wage setting equation, depicted by the WS curve in Fig. 1c, can be generated by a variety of labor market models; for example, in both the efficiency wage models (where firm’s seek to discourage shirking, quitting, and low-productivity applicants) and in monopoly union models, it is the households’ (rather than the firms’) price-level expectations that are relevant for wage setting.$^5$

Substituting the wage equation (2) into the aggregate labor demand equation (1), we obtain a labor market equilibrium condition:

$$u_t = \kappa_3 - \left[ \frac{\alpha T}{1 - \Gamma_1 + b_1} + \frac{1}{1 - \Gamma_1 + b_1} \left( \alpha T e_{t+T}^f - e_{t-1}^h \right) \right]$$

where $\pi_{t,T} = \left( p_{t+T} - p_{t+T} \right) / T$ is the average inflation rate over the production period, $\varepsilon_{t,T}^f = \pi_{t,T} - \pi_{t+T}^E$ is the firms’ expectational error regarding inflation, and $\varepsilon_{t-1}^h = \pi_t - \pi_{t-1}^e$ is the households’ expectational error. Condition (3) represents a short-, medium-, and long-run "Phillips curve." In the long run, when the nominal interest rate has adjusted fully to inflation ($\alpha = 0$, so that the real interest rate returns to its initial level), unemployment is at its long-run equilibrium rate ($u_t = \kappa_3$). In the medium run, when the nominal interest rate is sluggish ($\alpha > 0$), there is an intertemporal trade-off between inflation ($\pi_{t,T}$) and unemployment ($u_t$), depicted by the PC curve in Fig. 1a. And in the short and medium run, the economy may be off this medium-run Phillips curve on account of firms’ and households’ expectational errors ($\varepsilon_{t,T}^f$ and $\varepsilon_{t-1}^h$).

The greater the length of the aggregate production period (T), the flatter is the medium-run Phillips curve (i.e. the greater the change in current unemployment associated with a given change in future inflation). Observe the complementarity between the production period length (T) and the sluggishness of the nominal interest rate ($\alpha$). The
more sluggish is the nominal interest rate (i.e. the greater \( \alpha \)), the more will an increase the length of the production period (\( T \)) flatten the Phillips curve.

**IV. Price Dynamics**

To close our model, we include the product market clearing condition:

\[ q^s_{t+T} = q^d_{t+T}. \]

In logs, product supply is \( q^s_{t+T} = \gamma_0 + \Gamma_i n_t \) and product demand is

\[ q^d_{t+T} = \lambda + p (n_{t+T} - p_{t+T} \hat{i}) \text{ when } p_{t+T} = p_{t+T}. \]

Equating this supply and demand, aggregating over all final goods, and expressing aggregate employment \( \hat{Q} \) in terms of the unemployment rate \( (\hat{u}) \), we obtain the following *product market equilibrium condition*:

\[ \hat{u}_t = \kappa + \frac{\rho}{\Gamma_i} \hat{d}_{t+T} - p_{t+T} \hat{i} \tag{4} \]

Let the money supply grow at rate \( \mu \). Combining the product market equilibrium condition (4) with the labor market equilibrium condition (3), we obtain the general-equilibrium inflation *dynamics function*:

\[ \pi_{t+T} = a \pi_t + b \hat{u} - \alpha a \pi_t + a e^{t} + c e^{h}, \]

where

\[ a = \frac{\alpha T_i}{\alpha T_i + \rho \Gamma_i + b}, \]

\[ c = a / \alpha T \hat{\zeta}. \]

The coefficient \( a \) may be called the “inflation persistence coefficient,” for the greater is \( a \), the more current inflation depends on past inflation. Thus, when production takes time \( (T>0) \) and the nominal interest rate is sluggish \( (\alpha > 0) \), expectational errors have prolonged effects on inflation, production and employment. In the case when inflation is fully anticipated by the firms and households, the inflation dynamics function becomes:

\[ \pi_{t+T} = a \pi_t + b \hat{u} - \alpha a \pi_t \tag{5} \]

The intuition underlying the inflation dynamics function (5) may be clarified as follows. Suppose that initially the economy is at its long-run equilibrium, with unemployment at its long-run equilibrium rate \( \hat{u}_0 = \kappa \) and \( \pi_0 = \mu \), illustrated by point \( A_0 \) in Figs. 1. Next suppose that, in period \( t=0 \), the money growth rate is increased to \( \mu' \). As firms raise prices in response to the new money growth rate, there is an increase in the average inflation rate for the production period (extending from \( t=0 \) to \( t=T \)), and since the nominal interest rate is sluggish, the real interest rate falls over the medium run. Thus firms demand more labor (for any given real wage) and so the aggregate labor demand curve (1) shifts upwards, from \( LD_0 \) to \( LD_1 \) in Fig. 1c. Consequently, the labor market equilibrium moves from point \( A_0 \) to \( A_1 \) in Fig. 1c, and unemployment falls beneath its long-run equilibrium rate (from \( u_0 \) to \( u_1 \)).
The fall in unemployment and the rise in average inflation is depicted by an upward movement along the medium-run Phillips curve $PC$, from point $A_0$ to $A_1$ in Fig. 1a. This Phillips curve is given by the labor market equilibrium condition (3) in the absence of expectational errors: $$u_t = \kappa - \frac{\alpha T}{\Gamma_1 + b_1} \kappa_{t+T}.$$ The initial product market equilibrium condition (4), first differenced as $$u_t = u_{t-1} - \frac{b}{\Gamma_1} \left( \mu - \pi_{t+1} \right),$$ is depicted by the upward-sloping curve $QE_0$ in Fig. 1a.

The increase in money growth from $\mu$ to $\mu'$ shifts this curve upwards from $QE_0$ to $QE_1$ in Fig. 1a. Thus the average inflation rate (for the production period from 0 to $T-1$) rises. But since the nominal interest rate is sluggish, firms raise their period-$T$ prices less than proportionately to the money supply. The sluggish price level in period $t=T$ then affects the period-$T$ wage, and the resulting wage inertia, in turn, induces further inertia in the average inflation rate for the production period from $T$ to $2T$, and so on. In this way, the average inflation rate gradually approaches the new money growth rate $\mu'$. This time path of inflation is depicted in Fig. 1b. The increase in the money supply shifts the inflation dynamics curve (Eq. (5)) upwards from $ID_0$ to $ID_1$, and inflation rises from point $A_0$ to $A_1$, and subsequently increases further in the direction of the arrows.

In the long run, once the nominal interest rate has adjusted fully to the inflation rate ($\alpha = 0$), the inflation dynamics line becomes horizontal at point $A_2$ in Fig. 1b (since $a = 0$ in Eq. (5)); the Phillips curve turns vertical at $u_0 = \kappa$ in Fig. 1a (by Eq. (3)); and the labor demand curve returns to $LD_0$ (by Eq. (1)).

In this way our analysis provides links among three common macroeconomic relations: the labor market equilibrium (described by the intersection between the labor demand curve and the wage setting curve), the inflation-unemployment trade-off, and the inflation dynamics function. The flatter is the wage setting curve (the less the real wage responds to unemployment, i.e. the lower is $b_1$ in Eq. (2)) and the flatter the labor demand curve (the slower the rate of diminishing returns to labor, i.e. the greater is $\Gamma_1$), the flatter will be the medium-run Phillips curve and more slowly will the economy travel along this curve. An increase in the length of the production period ($T$) and in nominal interest sluggishness ($\alpha$) will also flatten the medium-run Phillips curve and induce the economy to move slowly along it.
V. Concluding Thoughts

While the simple model above is based on a linear wage setting curve (2), this curve is commonly considered to be nonlinear in practice. It is widely held that real wages are relatively unresponsive to changes in unemployment rates when these rates are high, but very responsive when the unemployment rates are low.\(^{10}\) In our analysis, such a nonlinear wage setting curve implies a nonlinear Phillips curve: the lower the unemployment rate, the steeper the Phillips curve becomes.\(^{11}\) As the economy approaches full employment, both the wage-setting curve and the Phillips curve may become very steep, possibly vertical, even in the short and medium run.

Furthermore, various mechanisms may make wage setting asymmetric, with real wages rising more steeply in upturns than they fall in downturns.\(^{12}\) Then, in the context of our analysis, the Phillips curve is steeper (viz, changes in unemployment are associated with larger changes in inflation) in business upswings and than in downswings.

Our analysis is motivated by the observation that, given the macroeconomic experiences of recent decades, in particular in Western Europe, the notion of a vertical long run Phillips curve appears to be an *incomplete* description of longer-term movements in unemployment. To argue that the high, prolonged rates of European unemployment from the early 1980s are *solely* the outcome of a shift in the natural rate of unemployment from about 3 to about 10 percent is not plausible, in our judgment, since no really convincing explanations have been given for such shifts. (Structural changes that may have caused such shifts — higher marginal tax wedges, stiffer labor market legislation and more generous welfare state benefits — occurred already in the 1960s and early 1970s.) To assert that such shifts have occurred just because European unemployment has remained high, makes the vertical long-run Phillips curve a tautologous explanation of prolonged unemployment, ruling out other possible explanations.

By contrast, our analysis suggests that the interaction of inflation persistence and unemployment *persistence* may offer a more plausible explanation of high and prolonged European unemployment. The existing literature on unemployment persistence is in real terms, based on mechanisms such as loss of skills, firms’ stigmatization of the long-term unemployed, reduced search intensities as the unemployment spells lengthen, insider-outsider effects, changes in social norms, etc. But to explain the movement of
unemployment alongside inflation, one needs to relate unemployment persistence to nominal magnitudes, including inflation persistence. Wage and price wage staggering is one well-known rationale for such a relation. Our analysis indicates that the combination of production lags and gradual adjustment of nominal interest rates comprises another, potentially complementary, rationale.

If high, prolonged European unemployment is ascribed solely to shifts in the natural rate, then demand management policies have no useful role to play in reducing unemployment. But when prolonged movements in unemployment arise from the interaction between real and nominal persistence then, in the aftermath of severe recessions, unemployment may remain above its long-run equilibrium for a long time and, during that time, demand management policies (e.g. monetary policy) may be effective in reducing unemployment. In this sense, our analysis suggests a potentially useful role for demand management in the medium run, lasting as long as the nominal interest rate is sluggish, that is, for periods up to five years or even longer.

Moreover, our analysis indicates that the effectiveness of demand management is likely to depend on the implementation of supply-side policies. After all, supply-side policies – such as those lowering barriers to the entry and exit of firms, reducing insider power, facilitating job search and worker mobility, etc. – may be expected to flatten the wage setting curve (i.e. reduce $b_1$ in Eq. (2)) and thereby flatten the Phillips curve (i.e. make unemployment more responsive to changes in inflation in Eq. (3)). In our model above, a flatter Phillips curve, in turn, leads to increased price inertia (i.e. a higher inflation persistence coefficient in Eq. (5)) and consequently to greater influence of demand management on production and employment.
Figures 1: Price Dynamics and the Phillips Curve
References
See, for example, Fisher (1925) and Tobin (1975).

These problems exist for both the old and new generation of wage-price staggering models. (The latter are surveyed by Roberts (1997), Goodfriend and King (1997), and others).

For instance, \( N_t \) and \( n_t \) are the level and log (respectively) of the firm’s employment, and similarly for the pairs \( \Gamma_0, \gamma_0 \), \( P_t, p_t \), \( \zeta \), and so on.

Alternatively, \( p_{t+T} - w_t = -\kappa - t + \Gamma_1 \Gamma_t - \Gamma_2 \Gamma_{t+T} - \Gamma_3 \Gamma_T = -\kappa - t + \Gamma_1 \Gamma_t - T_{t+T} \), i.e. the mark-up of the \((t + T)\)-period price over the \( t \)-period wage depends on employment, the nominal interest rate, and the length of the production period.

See, for instance, Layard, Nickell, and Jackman (1991), Lindbeck and Snower (1990), and Manning (1993)). Since the labor force is assumed constant in our model, the level of employment is tied to the employment rate.

The mark-up equation of footnote 4 indicates how nominal interest rate sluggishness is translated in price sluggishness.

The long-run inflation dynamics line and the long-run Phillips curve are not depicted in Figs. 1.

The first is in real terms, the third is in nominal terms, and the second links the two.

In other words, the more responsive will unemployment \( u_t \) be to given changes in inflation \( \pi_{t+T} \) (in Eq. (3)), and the greater will be the inflation inertia coefficient \( a \) in Eq. (5)).

It is not necessary to assume that the long-run wage setting curve is vertical in order to ensure that wages rise with productivity over the long run. Instead, the wage setting function could shift upwards in response to productivity increases (i.e. \( b_0 \) may depend on the productivity level).

In our model, the greater is \( b_1 \), the steeper the Phillips curve. In other words, the larger is the rise in inflation associated with a unit decrease in unemployment.

Examples are insider market power and discouraged worker effects. (See, for example, Caruth and Oswald (1987) and Lindbeck and Snower (1988, ch.11).