Risk-averse mitigation decisions under an unpredictable climate system*

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May 30, 2011

Abstract

Risk aversion plays a central role in decisions in the face of uncertainties, and climate change mitigation should be no exception. However, the interlinkage of risk aversion and climate change uncertainties has been hardly investigated numerically in part due to computational difficulties of stochastic optimization. In this paper, we apply numerical techniques of stochastic optimization to the economic modeling of climate change with the aim to model decision preferences of a risk-conscious agent in the face of unpredictable climate change. The model underlines the critical role played by the risk aversion parameter in determining the effects of uncertainties on mitigation, not only in level but also in sign.

JEL Classification: C63, Q54, D81

Keywords: climate change and uncertainties, stochastic control, climate policy

*We are grateful to Ken Judd for his valuable advise and support. We thank Paul Fackler, Gernot Klepper, Sonja Peterson, Martin Quaas, Till Requate and Karl Schmedders for their helpful suggestions and review of earlier drafts. Ole Wilms provided excellent research support. We greatfully acknowledge the financial support provided by the German Research Foundation (the "Future Ocean" Cluster of Excellence program) for Daiju Narita and by the Zürcher Universitätsverein and the Ecoscientia Foundation for Thomas S. Lontzek.

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1 Introduction

Uncertainty has been regarded as a key issue in the economics of climate change (for reviews, see Heal and Kriström (2002), Peterson (2006) and Pindyck (2007)), as the climate system involves highly unknown, and to some extent inherently random, long-term behavior. Much of the discussions by economists regarding this issue have been about the meaning of quasi-option value in climate change,[1] and little has been investigated about how the risk aversion by the economic agent influences optimal mitigation in the face of climate uncertainties. As Heal and Kriström (2002) sketch with their simple model, the risk aversion can be a key determinant for optimal mitigation decisions. Their simple model lacks a representation of economic growth, and a more comprehensive model should identify a more precise picture on the issue. However, the previous analyses of integrated assessment models incorporating uncertainty do not address this question explicitly. Many of those studies (e.g. Peck and Teisberg (1993), Nordhaus (1994), Nordhaus (2008), Scott et al. (1999), Pizer (1999)) are simply not able to provide any insights on risk aversion because of a constraint of their methodological choice (the Monte-Carlo approach). Others, (e.g., Peck and Teisberg (1993), Nordhaus and Popp (1999), Bosetti et al. (2009)) where risk-averse decisions are in principle incorporated, utilize a very simple representation of uncertainty (often in a binary form) to specifically answer different research questions (the effect of learning, etc.) and also do not discuss risk aversion directly.

In this study, we address the question of risk aversion and climate uncertainties by performing numerical stochastic optimization of a climate-economy system with climate randomness represented as a Brownian motion. Stochastic dynamic optimization has an established body of analytical model studies (in the field of environmental and resource economics, e.g.: Arrow and Chang (1982) and Tsur and Zemel (1998)), but has been generally considered difficult in finding numerical solutions. Recently, however, standardized techniques are developed (e.g. Judd (1998)), and some simple models are now able to be solved readily.

[1]Major studies on this question include the following: Arrow and Fisher (1974), Henry (1974), Kolstad (1996b), Ulph and Ulph (1997), Heal et al. (1998), Pindyck (2000), Gollier et al. (2000), and Fisher and Narain (2003).

Our modeling approach is different from the previous economic modeling studies of climate change in two important ways. The first difference is methodological: In our modeling framework, the risk-conscious agent makes decisions in anticipation of the future climate system that is uncertain but is also partly influenced by the agent's own decisions. In other words, the agent faces a decision making problem with risk that is partly endogenous. This feature is not captured in the above Monte-Carlo studies of integrated assessment models, which treat system variability uncertainties as external to decision makers considerations, while the endogeneity of risk decisions does not have much significance under a binary or other simplified uncertainty representations adopted in the other studies. This difference gives our model an advantage in examining the effects of climate uncertainty with a variety of risk preferences. The second concerns the focus of analysis: Their models investigate parameter uncertainties, that is, lack of knowledge on key parameters (e.g., the damage coefficient), whereas in our model, uncertainties come from the variability of the system itself, which follows a process that is not predictable yet affected by the decision makers actions (i.e. Brownian motion). Note that our assumption of climate uncertainty deals with the fluctuations of the climate system itself and thus somewhat differs from that taken by most studies in the economics of climate change, which concerns the parameter levels and is to be resolved through human learning. But our modeling choice is also consistent with the current scientific understanding that the climatic system embodies some highly non-linear, in part inherently unpredictable mechanisms,[2] and should provide a perspective complementary to the previous studies.

Our analysis is composed of two parts. In a first step we perform a stochastic perturbation analysis (Judd (1998)) of the model. The advantage of perturbation methods is the fact that they produce an asymptotically exact solution to the mitigation policy function near the deterministic steady state. However, perturbation methods are limited in the range of their validity.[3] Therefore, to complement our analysis, we also solve the model by using projection methods (see Judd (1992) for an introduction of projection

[2]It is well known that the climate system is subject to the so-called butterfly effect, which enables minuscule changes in the system to influence large-scale patterns. Also, random disruptions of the climate system are evidenced by paleoclimatic records of abrupt temperature changes, e.g., NRC (2002).

[3]see Judd and Sy-Ming (1997) for a discussion of the range of validity of perturbation methods.

methods to economic studies). Despite a lower guarantee of accuracy than perturbation methods the advantage of projection methods is their ability to examine the optimal policy functions over a more global range of states.

Our results show that the effects of uncertainty are different with different levels of agent's risk aversion. Indeed, the effects of stochasticity differ even in sign as to mitigation with varying parameters: introduction of stochasticity may increase or decrease mitigation depending on parameter settings, in other words, uncertainties of climatic trends may induce peoples precautionary emission reduction but also may drive away money from abatement.

We proceed as follows: In section 2 we describe the model framework. In section 3 we perform a stochastic perturbation analysis. In section 4 we conduct an analysis by using the projection method. Section 5 concludes.

2 The Model

Consider an economy where total output Y is a function of the capital stock K, with $Y_K > 0$ and $Y_{KK} < 0$. The production process generates emissions $\epsilon \cdot Y$, where ϵ denotes the emissions coefficient of output. With additional expenditure, the amount of emissions is reduced; m represents the fraction of carbon emissions which is under control, i.e. not emitted in the atmosphere. Consequently, the atmospheric stock of carbon S evolves with

$$dS = (\epsilon \cdot Y(K) \cdot (1 - m) - \beta \cdot S)dt \quad (1)$$

where β is the constant removal rate of atmospheric carbon into the ocean. At this point we assume that the atmospheric stock of carbon causes a rise in the level of global mean temperature. Let $T(S)$ be the increase of global mean temperature from the pre-industrial level with $T_S > 0$ and $T_{SS} \leq 0$. We assume that rising levels of global mean temperature cause damage to output and the damage is subject to randomness. Denote the damage by $D(T, \eta)$ with η being a scaling factor of the temperature's impact on damage: we assume $D_T, D_\eta > 0$, $D_{TT}, D_{\eta\eta} > 0$, $D_{T,\eta} > 0$ and $D(T, 0) = D(0, \eta) = 1$.

For the rest of the paper we assume that η is stochastic with

$$d\eta = (\theta \cdot (\bar{\eta} - \eta))dt + \sigma dB \tag{2}$$

i.e, the damage coefficient follows an Ornstein-Uhlenbeck process, the continuous time equivalent of a mean-reverting AR(1) process.[4] The mean of η is denoted by $\bar{\eta}$ and θ is the strength of mean reversion. For the diffusion, we assume $B \sim (0, \sigma^2)$. Furthermore, the output balance condition reads

$$\frac{Y(K)}{D(T(S), \eta)} = I + c + M(m, Y(K)) \tag{3}$$

The left-hand side of (3) is the net output inclusive of damage. The net output is in balance with the sum of the following: (i) consumption c ; (ii) $M(m, Y(K))$, the mitigation costs with $M_m > 0$, $M_{mm} > 0$, $M_Y > 0$, $M_{mY} > 0$ and $M_{YY} = 0$; (iii) capital accumulation via investment I. The stock of capital K evolves according to

$$dK = (I - \delta \cdot K)dt \tag{4}$$

where δ is the capital depreciation rate. Our purpose is to investigate the dynamically optimal choice of consumption, mitigation and capital investment given uncertainty about the temperature's impact on damage to gross output. To this end, we formulate the problem from the social planner's perspective. Given the uncertainty over η, the social planner maximizes the expected present value welfare.

$$\max_{c_t > 0, 0 \leq m_t \leq 1} E \int_0^\infty e^{-\rho t} \left[U(c_t) \right] dt \tag{5}$$

[4]Note that our focus is randomness of the climate system itself rather than knowledge uncertainty (as stated in the Introduction) The choice of the Ornstein-Uhlenbeck process has an advantage from a computational standpoint, too. We choose the Ornstein-Uhlenbeck process for numerical reasons. Without it, the stochastic process would be non-stationary and therefore move outside the η grid. This would require imposing boundary conditions on η. Our alternative formulation solves the non-stationarity problem and, provided the mean reversion is relatively strong at the grid bounds (compared to the diffusion force), the stochastic process will remain within the grid. From a purely climatic perspective mean reversion implies that if $\bar{\eta} = 1$ and a high damage is observed today (i.e: $\eta = 1.5$), tomorrow's η of 1.4 is more likely than 1.6. This feature is reasonable as it is a representation of fluctuations of climate trend, rather than learning uncertainty.

subject to (1)-(4) and $S(0) = S_0$, $K(0) = K_0$ and $\eta(0) = \eta_0$. Here we assume a standard CRRA form of utility that satisfies $U_c > 0$ and $U_{cc} < 0$.[5] To solve (5) we perform stochastic control, the continuous time version of stochastic programming. The corresponding Hamilton-Jacobi-Bellman (HJB) equation is [6]

$$\begin{aligned} 0 = \max_{c>0, 0\leq m\leq 1} \{U(c) &+ V_S(S,K,\eta)(\epsilon \cdot Y(K) \cdot (1-m) - \beta \cdot S) \\ &+ V_K(S,K,\eta)(\frac{Y(K)}{D(\eta,S))} - c - M(m, Y(K)) - \delta \cdot K) \\ &+ V_\eta(S,K,\eta)(\theta \cdot (\bar{\eta} - \eta)) \\ &+ \frac{1}{2}\sigma^2 V_{\eta\eta}(S,K,\eta) - \rho V(S,K,\eta)\} \end{aligned} \quad (6)$$

where $V(S,K,\eta)$ is the value function. A solution to (6) requires finding a value function and policy functions $c(S,K,\eta)$ and $m(S,K,\eta)$ which constitute explicit control rules. The first-order conditions for c and m are

$$U_c = V_K(S,K,\eta) \quad (7)$$

$$M_m(m, Y(K) = -\frac{V_S(S,K,\eta) \cdot \epsilon \cdot Y(K)}{V_K(S,K,\eta)} \quad (8)$$

Equation (7) states that the marginal utility from consumption should be equal to the derivative of the value function with respect to capital, i.e. the shadow price of capital. From (8) it can be easily seen that $V_S \leq 0$. The optimal choice of m, the mitigation rate, thus positively depends on the shadow price of atmospheric carbon (in absolute terms) and instant emissions. It negatively depends on the shadow price of capital.

A closed form solution to (6)-(8) could be obtained by applying specific function

[5] In favor of comparability with the existing literature, we adopt the standard framework of CRRA utility, in which risk aversion, preferences for intertemporal substitution and inequality aversion are represented by the same parameter (in our case α). However, we are aware that the validness of this framework is debated. For example, it is indicated that the formulation of risk aversion as used in the CRRA utility is not applicable to the case in which risk aversion concerns many commodities (Kihilstrom and Mirman (1974)). Also, a utility formulation to disentangle risk aversion and preferences for intertemporal substitution exists (Epstein and Zin (1989)). See Atkinson et al. (2009) for an analysis of this issue in the climate change context.

[6] Notice that by setting up the maximization problem as in (6), we do not restrict capital investments I to be non-negative. In fact, for some areas of the state and parameter space optimal investment is negative.

forms to Y, D, M, T and U and using an intelligent guess for the value function $V(S, K, \eta)$. However, due to the dimension of the state space and the nonlinearities of the functional forms we are not able to derive a closed form solution. Reflecting this fact, here we derive solutions by using two alternative methods, which are complementary to each other. In the next section we conduct stochastic perturbation analysis to determine an asymptotically exact solution to the mitigation policy function near the deterministic steady state. In section 4, we determine the value function and the policy functions numerically and present a more detailed analysis.

3 Stochastic Perturbation Analysis

A stochastic perturbation analysis finds closed form solutions that are valid around a steady state. The basic idea of stochastic perturbation analysis is that for small variances, the dynamical system will stay close to the deterministic steady state. Formally, let $m(S, K, \eta, \sigma)$ be the mitigation policy when the system is in the state (S, K, η) and the variance is $v = \sigma^2$. If $v = 0$ then the system will converge to the steady state $(S_{ss}, K_{ss}, \eta_{ss})$. For values of (S, K, η) close to the steady state $(S_{ss}, K_{ss}, \eta_{ss})$, the mitigation policy can be approximated by the second-order Taylor series:

$$
\begin{aligned}
m(S, K, \eta, v) &= m(S_{ss}, K_{ss}, \eta_{ss}, 0) + (\eta - \eta_{ss}) \frac{\partial m(S_{ss}, K_{ss}, \eta_{ss}, 0)}{\partial \eta} \\
&+ (K - K_{ss}) \frac{\partial m(S_{ss}, K_{ss}, \eta_{ss}, 0)}{\partial K} + (S - S_{ss}) \frac{\partial m(S_{ss}, K_{ss}, \eta_{ss}, 0)}{\partial S} \\
&+ v \frac{\partial m(S_{ss}, K_{ss}, \eta_{ss}, 0)}{\partial v} + 0.5(\eta - \eta_{ss})^2 \frac{\partial^2 m(S_{ss}, K_{ss}, \eta_{ss}, 0)}{\partial^2 \eta} \\
&+ (K - K_{ss})(\eta - \eta_{ss}) \frac{\partial^2 m(S_{ss}, K_{ss}, \eta_{ss}, 0)}{\partial \eta \partial K} + 0.5(K - K_{ss})^2 \frac{\partial^2 m(S_{ss}, K_{ss}, \eta_{ss}, 0)}{\partial^2 K} \\
&+ (S - S_{ss})(\eta - \eta_{ss}) \frac{\partial^2 m(S_{ss}, K_{ss}, \eta_{ss}, 0)}{\partial \eta \partial S} \\
&+ (S - S_{ss})(K - K_{ss}) \frac{\partial^2 m(S_{ss}, K_{ss}, \eta_{ss}, 0)}{\partial S \partial K} + 0.5(S - S_{ss})^2 \frac{\partial^2 m(S_{ss}, K_{ss}, \eta_{ss}, 0)}{\partial^2 S}
\end{aligned}
$$

As the above formulation shows, one explicitly defines the optimal mitigation policy to depend also on variance v. The mitigation function is computed as follows: First, one computes the known deterministic steady state solution. The partial derivatives, $\frac{\partial m}{\partial(\cdot)}$ are obtained analytically by solving systems of linear equations of first- and second-order expansion around the known deterministic steady state. Then, one builds a second-order Taylor series expansion around the known deterministic solution and $\sqrt{v}$.[7] Gaspar and Judd (1997) and Judd (1996) describe the mathematical foundations for the validity of this approximation and how to compute the partial derivatives, built on Fleming (1971) and Magill (1977).

In order to find a closed form solution we need to specify the model equations and parameters. We choose the following functional forms for our analysis:[8] $Y(K) = A \cdot K^{\nu}$, $D(\eta, T(S)) = 1 + \kappa \cdot \eta \cdot T(S)^2$, $T(S) = \tau \cdot (S - S_{PI})$, $M(m, Y(K)) = \psi \cdot \epsilon \cdot Y(K) \cdot m^2$ and $U(c) = \frac{c^{1-\alpha}}{1-\alpha}$. In addition, we choose the following parameter values for the basic model: $A = 1$, $S_{PI} = 400$, $\beta = 0.01$, $\delta = 0.1$, $\bar{\eta} = 1$, $\epsilon = 0.1$, $\kappa = 0.005$, $\theta = 0.1$, $\nu = 0.75$, $\tau = 0.003$, $\rho = 0.01$ and $\psi = 2$.

Given the simplified model structure, a rigorous calibration warranting a perfect match with empirical data is not feasible. Despite this difficulty, we choose the levels of parameters to make the simulation as realistic as possible. Here, we give reasonings behind our parameter choices. The level of damage coefficient κ corresponds to a world GDP loss of around 3 percent with a doubling of carbon dioxide from the pre-industrial level. This level is similar to DICEs (Nordhaus (2008)) and lies within the range of levels adopted by other major integrated assessment models. The combination of τ and S_{PI} is set to agreement with the current scientific consensus that the increase of the global average surface temperature from the preindustrial time to the present is in the order of 1 degree Celsius and the doubling of carbon dioxide from the pre-industrial level would most likely result in an increase of the global average temperature by 2 to 3 degrees Celsius. Computational demand necessitates us to adopt a relatively high level of ψ (mitigation cost parameter), a parameter whose estimation is subject to a great deal of speculation. We choose a ψ corresponding to the cost of 20% GDP to realize 100% mitigation. This level is significantly higher than the level implied by DICEs assumption of a backstop price at USD 1,200 per ton CO2 (corresponding to approximately a 5% world GDP loss for a 100% emission reduction). On the other hand, however, a different set of evidence suggests that our choice may rather be optimistic:

[7] Note that $\frac{\partial m(S_{ss}, K_{ss}, \eta_{ss}, 0)}{\partial v}$ is the crucial derivative which denotes the effect of stochasticity on mitigation.

[8] with $A, \nu, \kappa, \tau, \epsilon, \psi, S_{PI}, \alpha > 0$.

many technology-oriented integrated assessment models estimate that a deep cut of emissions (stabilization of greenhouse gases at 450ppm CO2e) is infeasible even with worlds best effort (Clark et al. (2009)). The GDP-emission conversion factor ϵ reflects the current world GDP and carbon dioxide emissions data and is nearly equal to the DICE models. Other economic parameters are set to yield general economic trends in the model that are as close as possible to those of the actual economy.

With these parameter values we numerically compute the deterministic steady state and obtain $\tilde{S} = 1551.56$, $\tilde{K} = 1188.62$. Assuming $\alpha = 2$ we find that the mitigation policy is approximately

$$\begin{aligned} m = {} & m_{ss} \\ & + 2.36 \cdot 10^{-4}(K - K_{ss}) + 1.43 \cdot 10^{-8}(K - K_{ss})^2 + 1.02 \cdot 10^{-4}(S - S_{ss}) \\ & + 8.18 \cdot 10^{-8}(K - K_{ss})(S - S_{ss}) - 2.04 \cdot 10^{-8}(S - Sss)^2 + 5.97 \cdot 10^{-2}(\eta - \eta_{ss}) \\ & + 4.15 \cdot 10^{-5}(K - K_{ss})(\eta - \eta_{ss}) + 1.58 \cdot 10^{-5}(S - S_{ss})(\eta - \eta_{ss}) \\ & + \ 7.71 \cdot 10^{-3}(\eta - \eta_{ss})^2 + 0.78v \end{aligned}$$

The key derivative for us is how v affects mitigation policy. We find that $\frac{\partial m}{\partial v} = 0.78$. Therefore, mitigation increases as the variance increases if the current state (S, K, η) is around to the steady state $(S_{ss}, K_{ss}, \eta_{ss})$. The same analysis is performed for different levels of α, the risk aversion parameter. Table 1 shows how optimal mitigation policy

$\alpha = .5$	$\alpha = 2$	$\alpha = 5$
$\frac{\partial m}{\partial v} = 0.33$	$\frac{\partial m}{\partial v} = 0.78$	$\frac{\partial m}{\partial v} = 1.43$

Table 1: The effect of uncertainty (variance) on optimal mitigation

is affected by rising uncertainty for different levels of risk aversion. We observe the risk aversion has a significantly positive effect on the effect of uncertainty on optimal mitigation policy. However, we also note that this effect is weaker with lower levels of risk aversion. It is important to keep in mind that these results are exact in the near proximity of the steady state but do not reliably apply for the entire state space. In order of investigate the effect described above at different areas of the state space we solve the model numerically using projection methods.

4 Stochastic Projection Analysis

4.1 Application of the Chebyshev Collocation Method

As mentioned above, validity of a perturbation analysis is limited in the neighborhood of a steady state. In the climate change context, this is a problematic property because climate policy is normally discussed for periods well before a steady state is reached. Projection methods can provide complementary insights to the ones we obtain from a perturbation analysis. A projection analysis necessitates approximation of equations. In this light, we use the Chebyshev collocation method.[9] From the first-order conditions (7) and (8) we can obtain explicit solutions for the optimal stochastic control of c and m as functions of the state variables.

$$\tilde{c} = \Gamma_U^{-1}(V_K(S, K, \eta)) \tag{9}$$

$$\tilde{m} = \Gamma_M^{-1}\left(-\frac{V_S(S, K, \eta) \cdot \epsilon \cdot Y(K)}{V_K(S, K, \eta)}\right) \tag{10}$$

Inserting (9) and (10) into (6) we obtain the concentrated HJB equation in terms of the value function and its derivatives with respect to the states. Thus, the concentrated HJB equation is three-dimensional in S, K and η and reads

$$\begin{aligned} 0 &= V_S(S, K, \eta)(\epsilon \cdot Y(K) \cdot \left(1 - \Gamma_M^{-1}\left(-\frac{V_S(S, K, \eta) \cdot \epsilon \cdot Y(K)}{V_K(S, K, \eta)}\right)\right) - \beta \cdot S) \\ &+ V_K(S, K, \eta)(1 + \frac{Y(K)}{D(\eta, S))} - \Gamma_U^{-1}(V_K(S, K, \eta)) + \frac{V_S(S, K, \eta) \cdot \epsilon \cdot Y(K)}{V_K(S, K, \eta)} - \delta \cdot K) \\ &+ V_\eta(S, K, \eta)(\theta \cdot (\bar{\eta} - \eta)) + \frac{1}{2}\sigma^2 V_{\eta\eta}(S, K, \eta) - \rho V(S, K, \eta) \end{aligned} \tag{11}$$

Equation (11) constitutes a nonlinear second-order partial differential equation which can be solved numerically using projection methods. For that purpose, we approximate the value function with the Chebyshev collocation method which approximates the solution to (11) with a linear combination of Chebyshev polynomial basis functions. The

[9] Judd (1998) describes computational applications of this method in detail.

approximated value function is

$$\tilde{V}(S,K,\eta) = \sum_{i=1}^{n_i}\sum_{j=1}^{n_j}\sum_{k=1}^{n_k} g_{ijk}T_i(x_S)T_j(x_K)T_k(x_\eta)$$

and $T_i(x_S), T_j(x_K)$ and $T_k(x_\eta)$ are n_i, n_j, n_k-degree Chebyshev polynomials which are evaluated at the states with x_S, x_K, x_η being the mapping $[S_{\min}, S_{\max}] \times [K_{\min}, K_{\max}] \times [\eta_{\min}, \eta_{\max}] \longmapsto [-1,1] \times [-1,1] \times [-1,1]$. The collocation coefficients g_{ijk} are then estimated in order to deliver a good approximation of (11).

4.2 Results of projection analysis

The formulations of approximation described in 4-1 allow us to compute solutions of the model that are valid for a broad range of states. Here, we derive numerical solutions by using the same parameterization and functional forms we have used in the perturbation analysis of section 3. We set up the projection grid by discretizing the spate space around the steady state. We choose $S \in [800, 3500]$, $K \in [500, 3000]$ and $\eta \in [0, 2]$. The Chebyshev polynomials are of degree 10 in all states i.e.: $n_i, n_j, n_k = 10$. Figure 1

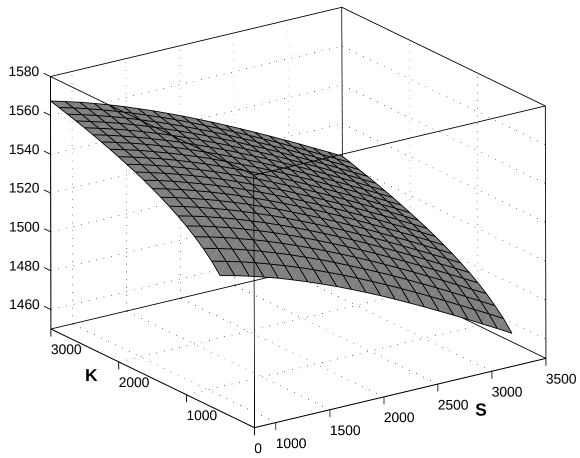

Figure 1: Value function

illustrates the value function for the stochastic case ($\sigma = .05$) in the $S - K$ grid.[10] For this simulation we assume α, the degree of risk aversion to be .9. Later in the analysis we will cover a range for α between .025 and 3. The value function is concave and smooth. It increases with larger volumes of the capital stock and decreases with rising atmospheric

[10]For the graphical presentation of the results we choose $\eta = 1$ unless stated otherwise.

carbon concentrations.[11] Figure 2 displays the shadow values of the atmospheric

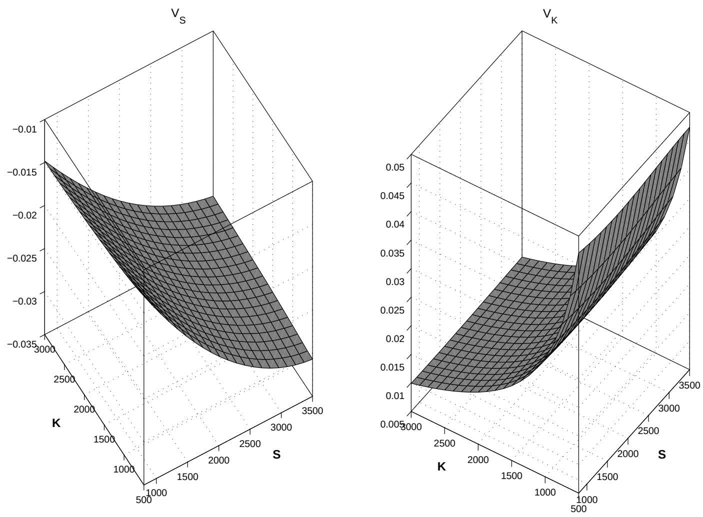

Figure 2: Shadow prices of atmospheric carbon stock (λ_S) and capital stock (λ_K)

carbon stock (V_S) and the capital stock (V_K). Notice that V_S is negative over the entire state space - an intuitive result, since rising temperature levels are proportional to the atmospheric carbon stock. This fact also explains why V_S decreases with rising levels of the carbon stock while it is rather invariant to changes in capital. An analogous picture is obtained for V_K, the shadow value of the capital stock (right plot in Figure 2). It is positive over the entire state space and decreases with rising levels of the capital stock. Figure 3 maps the policy functions for consumption c, mitigation m and investment I into the $K - S$ space, again for the stochastic case. The optimal consumption policy rule follows the Euler equation which sets equal marginal utility to the shadow price of capital. Consumption thus increases with the level of capital.

The mitigation policy generally replicates the tendency that most integrated assessment models exhibit, i.e., both carbon stock and capital accumulation increase enhances mitigation (e.g., Nordhaus, 1994). Notice that for a constant level of K, a higher atmospheric carbon concentration generates more damage to output, and that less output is available to be divided between consumption, mitigation and investment. Also, for any

[11] Notice that low levels of the capital stock imply low levels of gross output. This in turn results in low emissions. On the other hand, lower output volumes are available for consumption, investment and mitigation. Furthermore, for any level of capital a higher S invokes more damage and consequently less net output while the level of gross output is unchanged.

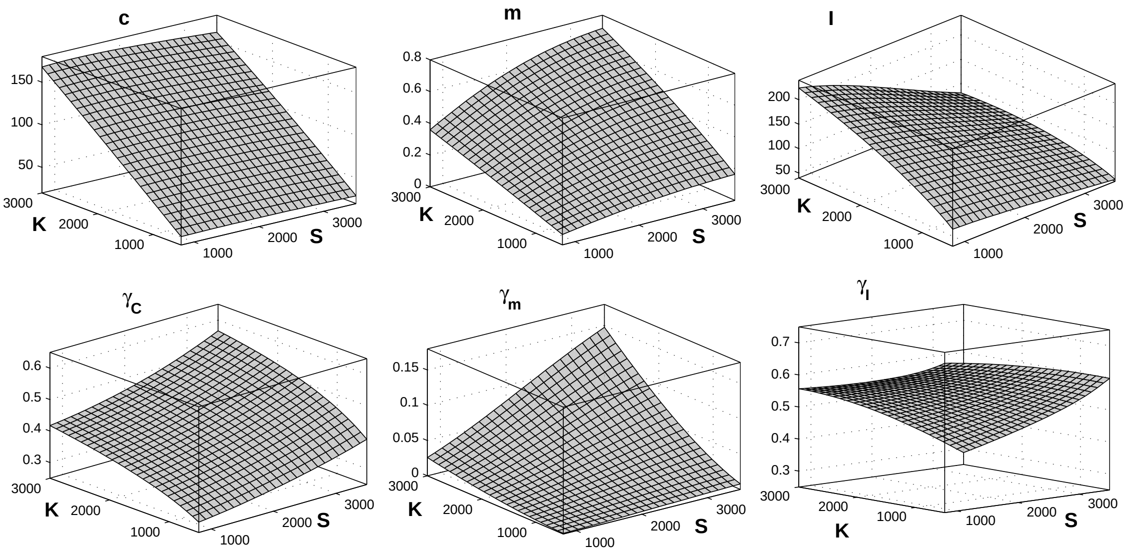

Figure 3: Policy functions for consumption (c), mitigation (m), investment(I) and shares of net output spent on c, m and I.

level of K optimal mitigation increases with higher values of S. Since consumption is constant, capital investments must decrease in order to balance the economy's budget (Equation 3). This behavior is shown in the lower left plot of Figure 3. On the other hand, higher levels of the capital stock invoke more investment. The lower right plot in Figure 3 displays the shares of net output[12] spent on consumption, mitigation and investment which we define as $\gamma_M = \frac{M(m)}{Y^{net}}$, $\gamma_C = \frac{C}{Y^{net}}$ and $\gamma_I = \frac{I}{Y^{net}}$ respectively.[13] We observe that γ_C and γ_M both follow the same pattern. They increase with higher levels of capital and atmospheric carbon stock. However, while the share of net output spent on consumption ranges from 25% (low K, low S) to 60% (high K, high S), a lower fractions of net output is used for mitigation. Its share ranges from 1% (low K, low S) to 15% (high K, high S). On the contrary, the policy function for investment implies lower investment values for rising levels of capital and atmospheric carbon stock.

In order to shed more light on the effect of uncertainty on the distribution of net output over consumption, mitigation and investment Figure 4 displays the absolute change in γ_C, γ_M and γ_I when including uncertainty. We define $\Delta g_M = \gamma_M(\sigma =$

[12]Net output is defined as $Y^{net} = \frac{Y}{D}$.

[13]Consumption, mitigation and investment are defined in units of output. It holds that: $\gamma_M + \gamma_C + \gamma_I = 1$.

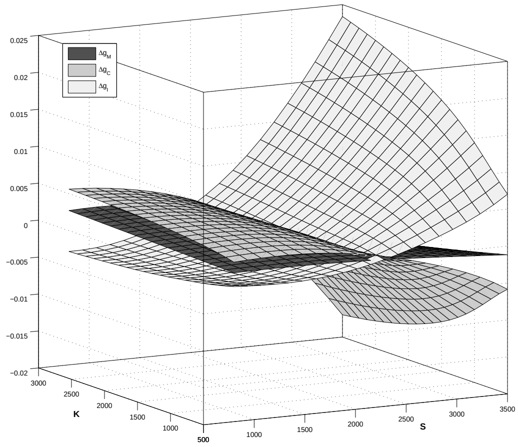

Figure 4: Difference in the shares of net output spent on mitigation, investment, and consumption $(\Delta g_M, \Delta g_I, \Delta g_C)$ when uncertainty is included.

$0.05)-\gamma_M(\sigma = 0)$, $\Delta g_C = \gamma_C(\sigma = 0.05)-\gamma_C(\sigma = 0)$ and $\Delta g_I = \gamma_I(\sigma = 0.05)-\gamma_I(\sigma = 0)$. Three important points can be made: 1) For low values of the atmospheric carbon stock, uncertainty leads to higher mitigation and consumption while it lowers capital accumulation. 2) The previous effect is reversed for high values of the atmospheric carbon stock. When uncertainty is included, a larger share of net output is spent on capital accumulation while a lower share of net output is used for consumption and mitigation. 3) The impact of uncertainty on reallocating net output optimally among c, m and I decreases (lower amplitude) with smaller levels of the capital stock. The latter effect mirrors the fact that a low value of the capital stock limits the freedom of action to adapt to stochasticity. Meanwhile, as for the absolute level of impact of stochasticity on mitigation, the figure shows that the effect remains well less than 1%, although the deviation of the realized time path can be much greater in response to a random pattern of climate (see also Section 4.3 for intuitions about this point).

From Figure 4 it becomes clear that if the carbon content in the atmosphere is large, uncertainty about damage to output induces a reallocation of net output towards capital services. A striking feature of this result is that in the latter case mitigation is reduced. Note that the apparent difference between this result and the one in perturbation analysis comes from the fact that the focus of the perturbation analysis is only the vicinity of the steady state. To obtain more insights on the effect of stochasticity on mitigation, we examine the optimal levels of mitigation with varying levels of risk aversion and uncertainty (Figure 5). We show nine contour plots in the $\alpha - \sigma$ space for different levels of S and K. Again, the absolute differences that stochasticity causes on mitigation are small, but general qualitative features are noteworthy. They exhibit three major patterns. (1) The general tendency is that high capital and high carbon stock result in high mitigation. A large carbon stock corresponds to a large emission reduction, while a

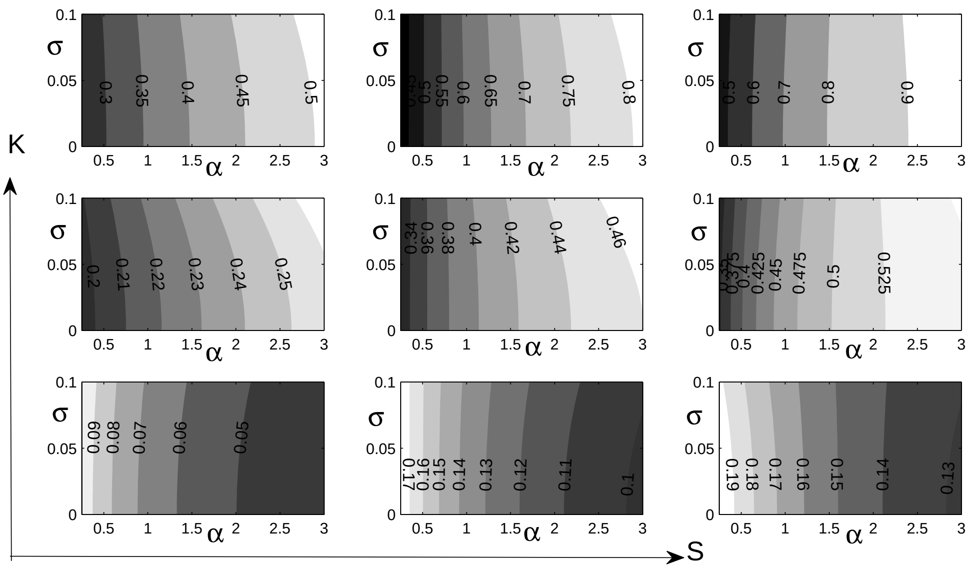

Figure 5: Optimal mitigation - Contour plots in the $\alpha - \sigma$ space for combinations of S and K. $S \in 800, 2150, 3500$ and $K \in 500, 1750, 3000$, with (S,K)=(800,500) in the lower left subplot.

large capital stock is linked to a low capital return and thus a diversion of resource from investment to mitigation. (2) The risk aversion is a very influential parameter on the level of mitigation. Interestingly however, the risk aversion exerts different effects on the control level depending on the level of capital.[14] For moderate or high levels of capital, more risk aversion leads to more abatement. Mitigation decreases with higher levels of risk aversion when the capital level is low. This is because a risk averse agent prefers capital investment over mitigation when the return to capital is relatively high (i.e., for low levels of capital), In other words, capital investment facilitates inter-temporal income smoothing more effectively than mitigation does. (3) When the level of risk

[14]Note that this feature is in contrast with the result of a simple but similar analysis carried out by Heal and Kriström (2002). With a simple model with a constant income flow, they show that high risk aversion always raises precaution against climate change. As explained below, the opportunity of capital investment and income growth has a critical meaning for the behavior of our model and also explains the difference from Heal and Kriström's.

aversion is low, the effect of stochasticity on mitigation crucially depends on the size of the carbon stock. Higher uncertainty leads to higher mitigation with a low S but lower emission reduction with a high S. In other words, with a low risk aversion, the agent rather prefers consumption over mitigation when climate mitigation needs considerable effort. Among the above findings, the ambiguity regarding the effect of uncertainty and the influence of risk aversion on this ambiguity are consistent with the findings by Gollier et al. (2000) and Lange and Treich (2008) obtained from simple analytical models. However, this feature has never been tested in a more realistic model of climate change that embodies feedback mechanisms with the carbon and capital stocks. In fact, this ambiguity is a persistent feature of our model, and in some cases it is clearly identifiable from the patterns. Figure 6 is a contour plot for low K and high S when the climate

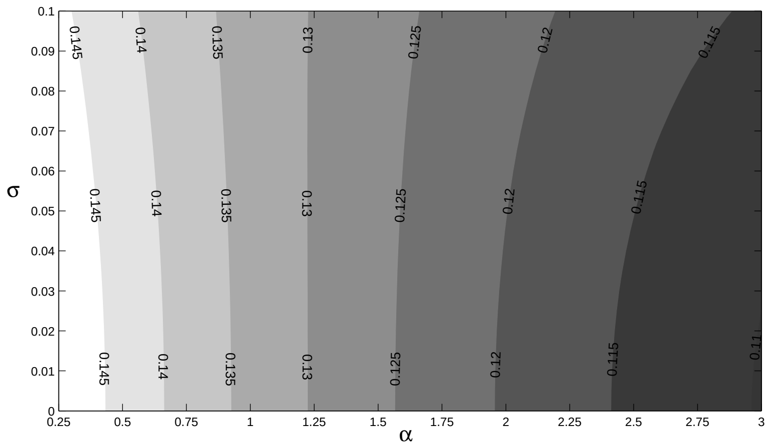

Figure 6: Optimal mitigation - Contour plot in the $\alpha - \sigma$ space for high S and low K, i.e. (S,K)=(3500,500).

change damage coefficient is set low ($\eta = 0.5$). It clearly shows that the effect of uncertainty on the sign of change in mitigation even depends on the level of risk aversion. With a low risk aversion, uncertainty decreases emission reduction, whereas it increases emission reduction with a high risk aversion.[15]

An interesting point regarding these patterns is that uncertainty may in fact reduce the optimal level of mitigation. The general pattern (reduction of mitigation tends to occur with a low risk aversion) is consistent with the finding by Gollier et al. (2000)

[15]Fisher and Narain (2003) also discuss cases in which the risk aversion parameter plays a decisive role in determining the effect of climate change uncertainty, although the context is different (the effect of capital convertibility).

that the precautionary effect, the effect due to the shape of the utility function, can work negatively for mitigation when risk aversion if low. Besides, the negative effect of uncertainty on mitigation could be also attributed to the structure of irreversibility our model possesses. Previous studies already clarified that if investment in abatement involves sunk costs, uncertainty in stock pollution can either enhance or decrease abatement, dependent on the parameter choice (e.g., Pindyck (2000)). This is because both the installation of abatement equipment and the pollution stock have irreversibility. Our model does not have an explicit representation of sunk investment on abatement, but there is a similar, though indirect, mechanism at work. Abatement costs (flow) are subtracted from the output, and thus reduce either consumption, capital investment, or both, if the output is unchanged. Capital produces a continuous flow of income from the time of investment onwards, and foregone capital investment due to excessive abatement therefore sets irreversibility in the other direction. This argument could be paraphrased as follows: The presence of uncertainty leads to increasing abatement of stock pollution because one cannot reduce the pollution stock later in case the pollution damage is greater than expected. Since the risk goes in both directions, a similar argument holds for the other direction. If we overspend our resource on abatement, capital investment could be decreased. Lower investment leads to lower capital accumulation. By the time we realize the overspending on abatement, the accumulated abatement cannot be converted into capital, and one cannot recover the income flow that capital would bring about if our resource was allocated in investment, not abatement. The relative significance between capital returns on one hand and climate damage on the other hand determines the dynamics to either of the two directions.

4.3 Monte-Carlo Analysis of the Optimal Solution

From the solution of the dynamic programming problem in section 4.1 we obtained the value function and the optimal policy functions for mitigation and consumption. Given these results we perform 10,000 optimal Monte-Carlo runs[16] for a selected initial state.

[16]It should be noted that each Monte-Carlo run is based on the optimal solution of the stochastic model, i.e.: each underlying Monte-Carlo run represents an optimal path with stochasticity, in other words, under a diffusion process. Thus, our "ex-post" Monte-Carlo analysis which is based on stochastic optimization internalizing uncertainty differs substantially from the "ex-ante" Monte-Carlo approach to

The Monte-Carlo simulations provide more intuitions about the general behavior of our model and also visual evidence that supports the validity of our modeling approach. The resulting distributional statistics are reported in Figure 7. We report 1) the mean of all 10,000 runs at each point in time, 2) the outer envelopes of the runs, i.e. the minimum and maximum over all runs at each point in time (i.e.: all 10,000 simulation runs are within the shaded area), and 3) the 25% (75%) quartiles, i.e.: at each period the 2500th lowest (highest) value obtained. All simulations start at $(S, K, \eta) = (800, 500, 1)$ and run for 300 time periods. The upper-left plot of Figure 7 illustrates the distribution of the

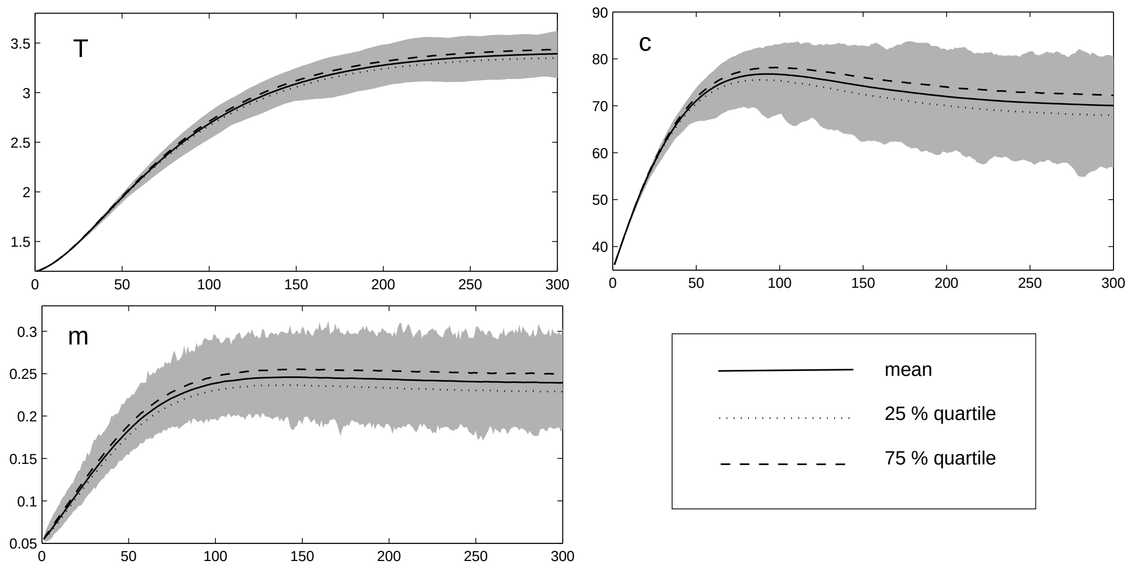

Figure 7: Statistical results from 10,000 Monte-Carlo runs

optimal level of temperature increase (T). We observe that for the first 50-75 time steps of the simulation interval there is little difference in the level of temperature increase among the runs. This feature reflects the fact that atmospheric carbon levels and consequently global temperature are mainly determined by past emissions and thus exhibit a slow response to stochasticity. After about 100 periods, when the temperature is near its deterministic steady state level, we observe a min-max range of the Monte-Carlo runs of roughly 0.5 degree points. The Monte-Carlo distribution for consumption (c) is depicted in the upper-right plot of Figure 7. After about 50 time periods consumption is near its deterministic steady-state level and the distribution range of the simulation runs

uncertainty which is common to some IAM and critically discussed in the Introduction.

widens. Looking at the 25% and 75% quartiles we can state that that in 50% percent of the simulation runs consumption can deviate by roughly 10% of its steady-state level. A similar characteristic is observed in the distribution plot for mitigation (m: the lower plot in Figure 7). Here again, in 50% of the simulation runs the optimal mitigation policy paths show a variation in size by more than 10%. The variation among all 10,000 runs however is as large as 18 percentage points (0.3-0.18). This variation is about 0.5 degree for temperature and about 25% for consumption.

The Monte-Carlo analysis also provides an illustration regarding the effect of risk aversion. Figure 8 shows the mean levels of mitigation (Figure 8a) and the differences between the 10% and 90% quantiles (Figure 8b) with different levels of risk aversion. The parameter specifications other than risk aversion levels are identical to those for Figure 7. Figure 8a indicates a long-run convergence of mitigation levels for the cases

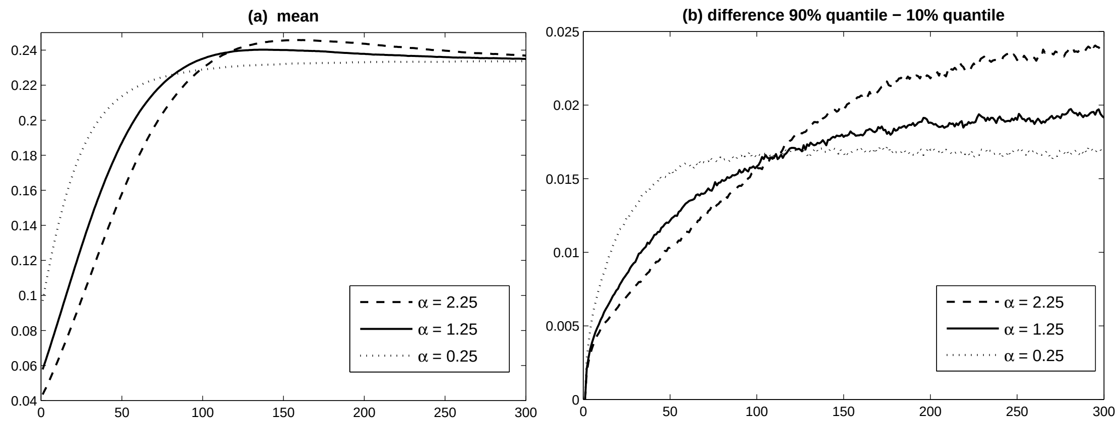

Figure 8: Mitigation policy - statistical results from 10,000 Monte-Carlo runs

of different risk aversion levels, after relatively high mitigation levels in initial periods with low risk aversion corresponding to high consumption and low investment (i.e., low consumption smoothing). By contrast, the differences in quantiles shown in Figure 8b, amounting to roughly one tenth of the mean values, exhibit significantly high long-run levels with high risk aversion - in other words, stochasticity has greater influence on the mitigation trends under higher risk aversion. This effect of high risk aversion is consistent with the results shown in 4.2. It is worth noting that the effect is invisible in the pure comparison of the mean levels shown in Figure 8a. In sum, this Monte-Carlo analysis underlines the findings from the previous sections; namely that uncertainty about the climate system has an significant impact on the optimal control policies and the climate system itself.

5 Concluding Remarks

We carried out a numerical stochastic optimization in the context of climate change. We applied standardized numerical techniques of stochastic optimization to the climate change issue with uncertainty about the climate system. A novelty of this study is that we directly performed stochastic dynamic optimization, rather than reproducing randomness by conducting a large number of simulation runs, to see changes of key determinants of climate policy. An advantage of our stochastic optimization approach over previous climate-economy simulation studies is that the model internalizes agent's preference about risk in optimization. Our analysis covers a large range of the parameter space, in particular the degree of risk aversion and the level of uncertainty. We identify regions of the state space for which higher levels of uncertainty or risk aversion result in different optimality rules for mitigation.

The results show that the effects of uncertainty are indeed different with different levels of risk aversion by the agent. A main finding is that the effects of stochasticity on mitigation differ even in sign with varying parameters: Uncertainty may increase or decrease mitigation depending on parameter settings, in other words, uncertainties of climatic trends may induce people's precautionary emission reduction but also draw away money from abatement. Our results show some consistency with the somewhat counterintuitive argument by Nordhaus (2008) that the risk of high climate change could carry a negative risk premium – high climate change could in fact be outweighed by enhanced wealth. It should be noted, however, that in our case, reduction in mitigation due to uncertainty appears only with certain combinations of the wealth, the level of atmospheric carbon dioxide concentrations, and importantly, the level of risk aversion. Meanwhile, the model shows that the absolute level of impact of stochasticity on mitigation could be small at least from the standpoint of the current policy planning, although it could be magnified in the future in response to random climatic patterns.

This paper's conclusions stress the fact that from a decision making standpoint, the meaning of climate change uncertainty cannot be formulated purely scientifically but is fundamentally intertwined with human risk perceptions. In policy discussions, we would need a more precise conceptualization of uncertainties than, for instance, the way phrased in the United Nations Framework Convention on Climate Change's objective to "prevent dangerous anthropogenic interference with the climate system." As our model implies, the policy meaning of climate uncertainty (i.e., what constitutes "dangerous" interference) is shaped both by the mechanisms of the natural system and by our perception (risk aversion tendency).

As a final note, while our model highlights some important features of uncertainties and climate change, the simulations are admittedly simple for explaining the complex phenomena of climate change. A more comprehensive numerical stochastic model, perhaps with uncertainties in technological change and global business cycle in addition to climate indicators, would allow us to conduct a complete sensitivity analysis of parameters. In particular, the lack of technological change in our model might have resulted in somewhat pessimistic patterns of mitigation as well as in consumption in comparison to the ones might be achieved in reality with autonomous technological change. Impacts of uncertainties about large discrete shocks, a feature that could be represented with a jump process, should also be a future research question.

References

Arrow, K.J. , and Chang, S. (1982) 'Optimal pricing, use, and exploration of uncertain natural resource stocks.' *Journal of Environmental Economics and Management* 9, 1–10

Arrow, K.J. , and Fisher, A.C. (1974) 'Environmental preservation, uncertainty, and irreversibility.' *Quarterly Journal of Economics* 88(2), 312–319

Atkinson, G. , Dietz, S. , Helgeson, J. , Hepburn, C. , and Soelen, H. (2009) 'Siblings, not triplets: Social preferences for risk, inequality and time in discounting climate change.' *Economics: The Open-Access Open-Assessment E-Journal*

Bosetti, V. , Carraro, C. , Sgobbi, A. , and Tavoni, M. (2009) 'Delayed action and uncertain stabilization targets: How much will climate policy cost?' *Climatic Change* 96, 299–312

Clark, L.E. , Edmonds, J.A. , Krey, B.H. , Richels, R.G. , Rose, S. , and Tavoni, M. (2009) 'International climate policy architectures: overview of the emf 22 international scenarios.' *Energy Economics* 31(2), 64–81

Epstein, L.G. , and Zin, S.E. (1989) 'Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework.' *Econometrica* 57(4), 937–969

Fisher, A.C. , and Narain, U. (2003) 'Global warming, endogenous risk, and irreversibility.' *Environmental and Resource Economics* 25, 395–416

Fleming, W. (1971) 'Stochastic control for small noise intensities.' *SIAM Journal of Control* 9(3), 1025–1042

Gaspar, L. , and Judd, K. L. (1997) 'Solving large-scale rational expectations models.' *Macroeconomic Dynamics* 1(1), 45–75

Gollier, C. , Jullien, B. , and Treich, N. (2000) 'Scientific progress and irreversibility: An economic interpretation of the precautionary principle.' *Journal of Public Economics* 75, 229–253

Heal, G. , and Kriström, B. (2002) 'Uncertainty and climate change.' *Environmental and Resource Economics* 22, 3–39

Heal, G. , Chichilnisky, G. , and Beltratti, A. (1998) 'Sustainability: Dynamics and uncertainty.' pp. 257–275

Henry, C (1974) 'Investment decisions under uncertainty: The irreversibility effect.' *American Economic Review* 64(6), 1006–1012

Judd, K. L. (1992) 'Projection methods for solving aggregate growth models.' *Journal of Economic Theory* 59, 410–452

_ (1996) *Approximation, Perturbation, and Projection Methods in Economic Analysis* Handbook of Computational Economics (North Holland)

_ (1998) *Numerical Methods in Economics* (The MIT Press)

Judd, K. L. , and Sy-Ming, G. (1997) 'Asymptotic methods for aggregate growth models.' *Journal of Economic Dynamics and Control* 21(6), 1025–1042

Kihilstrom, R.E. , and Mirman, L.J. (1974) 'Risk aversion with many commodities.' *Journal of Economic Theory* 8(4), 361–388

Kolstad, C.D. (1996b) 'Fundamental irreversibilities in stock externalities.' *Journal of Public Economics* 60, 221–233

Lange, A. , and Treich, N. (2008) 'Uncertainty, learning and ambiguity in economic models on climate policy: Some classical results and new directions.' *Climatic Change* 89, 7–21

Magill, M. J. P. (1977) 'A local analysis of n-sector capital accumulation under uncertainty.' *Journal of Economic Theory* 15, 211–219

Nordhaus, W.D. (1994) 'Managing the global commons.' *Cambridge, MA: MIT Press*

_ (2008) *A Question of Balance: Weighing the Options on Global Warming Policies* (Yale University Press, New Haven, CT)

Nordhaus, W.D. , and Popp, D. (1999) 'What is the value of scientific knowledge? an application to global warming using the price model.' *The Energy Journal* 18(1), 1–45

NRC (2002) *Abrupt Climate Change: Inevitable Surprises* (National Research Council, Washington, DC: National Academy Press)

Peck, S.C. , and Teisberg, T.J. (1993) 'Global warming uncertainties and value of information: An analysis using ceta.' *Resource and Energy Economics* 15, 71–97

Peterson, S. (2006) 'Uncertainty and economic analysis of climate change: A survey of approaches and findings.' *Environmental Modeling and Assessment* 11(1), 1–17

Pindyck, R.S. (2000) 'Irreversibilities and the timing of environmental policy.' *Resource and Energy Economics* 22, 233–259

_ (2007) 'Uncertainty in environmental economics.' *Review of Environmental Economics and Policy* 1(1), 46–65

Pizer, W.A. (1999) ‘The optimal choice of climate change policy in the presence of uncertainty.’ *Resource and Energy Economics* 21, 255–287

Scott, M.J. , Sands, R.D. , Edmonds, J. , Liebetrau, A.M. , and Engel, D.W. (1999) ‘Uncertainty in integrated assessment models: Modeling with minicam 1.0.’ *Energy Policy* pp. 885–879

Tsur, Y. , and Zemel, A. (1998) ‘Pollution control in an uncertain environment.’ *Journal of Economic Dynamics and Control* 22, 967–975

Ulph, A. , and Ulph, D. (1997) ‘Global warming, irreversibility and learning.’ *The Economic Journal* 107(442), 636–650