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Keywords: Fiscal rule, fiscal reaction function, deficits, stability, instability.

JEL classification: E62, E63, H60.

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Fiscal targeting rules and macroeconomic stability under distortionary taxation

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Abstract

While European countries have engaged in a debate about fiscal policy rules, little is known about the ability of these rules to ensure stable debt and output paths when taxes are distortionary, particularly in a small open economy. In this situation, it turns out that the interaction between a fiscal rule and output may affect whether or not fiscal policy is stabilizing, or "passive", in equilibrium. For instance, under moderate debt-multiplier combinations, a debt-GDP targeting rule can result in instability, while a debt-level targeting rule, irrespective of GDP, can result in stability. A primary deficit target may result in instability for the debt but stability for output, while a total deficit target can result in stability for both debt and output. A fiscal reaction function similar to those found in the macro literature may result in stability for certain parameter values, so long as the response of fiscal policy to the past debt level is strong enough to overcome the interactions among fiscal policy, output, and interest rates. Furthermore, under certain conditions, optimal policy mimics a fiscal reaction function with a moderate degree of business cycle stabilization policy.

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1 Introduction

Within the EMU, ongoing events have touched off a fresh debate about fiscal rules. Proponents of fiscal rules such as the Fiscal Compact (European Council, 2012), the German Debt Brake, or a fiscal reaction like that proposed by Snower, Burmeister, and Seidel (2011) argue that such rules may help to ensure stability in the public debt, in the sense of avoiding explosive dynamics.\footnote{In the fiscal reaction function of Snower et al. (2011), the primary surplus may respond to the output gap or to past debt levels. Formulated this way, a fiscal reaction function is similar in spirit to a monetary reaction function. See Taylor (2000), Auerbach (2002), and Galí and Perotti (2003) for some early examples of fiscal reaction functions.} Such an outcome is of particular relevance within the EMU given that the European Central Bank sets a common monetary policy for the entire union, with a mandate of price stability. Given this mandate, using the terminology of Leeper (1991), individual countries face a particular type of "active" monetary policy regime. In order to support this "active" monetary policy regime, a fiscal rule should encourage a "passive" fiscal policy regime, or one which ensures government solvency without having to resort to inflation or default. However, some of the more stringent of the proposed rules have come under critique with respect to their implied time paths of fiscal policy aggregates—see Barnes, Davidsson, and Rawdanowicz (2012) and Creel, Hubert, and Saraceno (2013) on the Fiscal Compact and Truger and Will (2013) on the German debt brake. These authors point out that some particular rules are likely to encourage procyclical fiscal policy, which is an issue if fiscal policy also desires to stabilize output in addition to stabilizing the debt.

A tradeoff between stabilizing output and stabilizing the debt is likely if fiscal policy operates with a multiplier effect, for instance, due to distortionary taxes or Keynesian consumption behavior. However, despite the quantitative policy simulations of Creel et al. (2011), not much is known about the theoretical conditions underlying this tradeoff under different types of fiscal rules. The current study helps to fill this gap by analyzing which types of fiscal rules stabilize output and the public debt in the type of situation faced by an EMU country, modeled as a small open economy with distortionary taxation under a monetary union. The rules in question are a debt-GDP target, a debt target, a primary deficit target, a total deficit target, and a fiscal reaction function. It turns out that the ability for a fiscal rule to ensure stability is sensitive to the presence of a multiplier, such that for reasonable parameter values, a debt-GDP target may destabilize both debt and output. Furthermore, the presence of a multiplier affects the conditions under which a fiscal reaction function may stabilize debt and output. In light of these results, it is important to take multiplier effects into account when analyzing the ability of a fiscal rule to actually stabilize the public debt.
These findings are based on intuition from a simple small open-economy model with distortionary taxation, and where fiscal policy may also have a "confidence" effect on perceived default risk and hence on interest rates. Assuming an exogenous path for prices, the model can then be expressed in its reduced form by three equations: a multiplier equation which links the primary surplus with output, a law of motion for the public debt, and a fiscal rule. In a technical sense, the question of stability depends strongly upon the exact way in which a fiscal rule is specified, and how that fiscal rule interacts with the other two equations, particularly the multiplier equation.

For example, the conditions for stability under a debt-GDP targeting rule are actually quite stringent, requiring a debt-multiplier combination less than about one half. The problem with such a target at moderate debt ratios lies in that a debt-GDP target encourages stop-and-go fiscal policy, given the ways in which debt (a stock) and GDP (a flow) are linked over time. The problem with such a target at higher debt ratios lies in that a round of fiscal consolidation will reduce both the public debt and GDP, and the overall effect of consolidation on the debt-GDP ratio depends on which of these effects predominates. Given that these problems emerge because GDP shows up in the debt-GDP ratio, one solution would be to implement a debt targeting rule that does not rely upon GDP. As one might predict, this type of rule does not suffer from the pathologies suffered by a debt-GDP targeting rule. As a result of all of this, a debt-GDP targeting rule might not be able to ensure passive fiscal policy in equilibrium, while a debt-targeting rule will always be able to ensure passive fiscal policy in equilibrium.

As with a debt rule, details related to the design of a deficit rule may ultimately determine whether or not fiscal policy is passive in equilibrium. Here, the results of Sargent and Wallace (1981) and the subsequent monetary literature come into play. For reasons discussed in that literature, a primary deficit target does not ensure passive fiscal policy, since that type of targeting rule does not ensure fiscal solvency in all states of the world. However, a total deficit target can ensure passive fiscal policy, so long as the trend nominal growth rate of the economy is positive. This situation is another situation that illustrates the role that seemingly minor differences in the design of a fiscal rule can play in whether or not that rule promotes passive fiscal policy. This time, however, it is the trend path of nominal output that delivers these results, rather than the presence of a multiplier.

In addition to affecting the stability properties of simple debt and deficit rules, the presence of a multiplier also affects the stability properties of a fiscal reaction function, whereby the primary surplus responds to the output gap ("stabilization policy") and to past debt
levels and other shocks to solvency ("consolidation policy"). For a fiscal reaction function, consolidation policy needs to be strong enough to overcome any "confidence" effect of fiscal policy on interest rates as well as a "clawback" effect which results from the interaction between stabilization policy and the multiplier. A confidence effect may emerge when a high debt level feeds into higher perceived default risk, for instance, when a country faces a fiscal limit. The result on the "clawback" effect appears to be new. These results point toward a tradeoff between a strong degree of stabilization policy and a leisurely degree of consolidation policy. Adding this tradeoff generalizes the results of Bohn (1998) and the rest of the consolidation literature to a situation with a multiplier.

These results are also of practical interest since an optimal policy exercise indicates that when business cycles are driven by a "labor wedge", an optimal fiscal policy rule might take the form of a fiscal reaction function, possibly with stronger stabilization and consolidation policies than are observed in past data. This optimal policy exercise indicates that the "tax-smoothing" (or policy-smoothing) results of Barro (1979), Chari, Christiano, and Kehoe (1994), Schmitt-Grohé and Uribe (2004), Benigno and Woodford (2006), Kirsanova and Wren-Lewis (2012), and others is operative when it comes to determining the optimal strength of consolidation policy (which includes the pre-emptive management of fiscal crises), while the tax smoothing result is not operative in terms of stabilization policy. This breakdown of the tax-smoothing result also occurs in the analysis of Arseneau and Chugh (2008), and this breakdown seems to hinge on the role of the labor wedge in driving inefficient business cycles. This line of reasoning for stabilization policy stands in addition to the reasons given by Galí and Monacelli (2008) and Ferrero (2009) to undertake stabilization policy in a small open economy in a monetary union, where a country is subject to country-specific shocks. Taken together, it appears that a well-specified fiscal reaction function has a number of attractive properties with respect to stability and optimality, in contrast with some of the other rules which are intended to ensure passive fiscal policy.

These findings on what determines a passive fiscal policy are driven by the presence of distortionary taxes. It is worth pointing out that similar types of findings show up elsewhere in the fiscal-monetary literature, although that literature has typically focused on the issue of determinacy in the inflation rate under particular monetary regimes, in a closed economy. For instance, Schmitt-Grohé and Uribe (1997), Linnemann (2006), Schabert and von Thadden (2009), and Koruzumi (2011) all discuss cases where distortionary taxation affects whether or not an apparently "active" or "passive" interest rate rule (i.e. a rule where interest rates adjust in response to inflation by more or less than one-for-one, respectively) and
an apparently "active" or "passive" fiscal policy rule together can interact to produce a determinate inflation rate. Leeper and Yun (2006) also discuss situations where the presence of distortionary taxation does or does not affect the ways in which monetary and fiscal policy interact. Altogether, though the literature has focused more on the issue of price level determination—especially under an active fiscal policy rule—the literature has also shown that the presence of distortionary taxation can affect the interactions between monetary and fiscal policy rules in terms of stability and determinacy. In light of this literature, further work can help to exactly quantify the effects of the choice of monetary policy regime, the degree to which an economy is closed or open, or the effects of other types of modeling choices. Nonetheless, the model presented here offers some basic intuition as to why some types of fiscal policy regimes are more likely to result in instability than others, and why an apparently passive fiscal policy regime might not always result in passive fiscal policy in equilibrium.

2 The economic environment

2.1 A simple small open-economy model

2.1.1 Consumer behavior

In the model, consumers live in a small open economy, and they have access to a complete set of financial assets which index all aggregate states, including all actions of the government. However, consumers cannot hedge against their labor supply decisions. Each country is arbitrarily small and takes prices, global interest rates, and global economic conditions as given. In contrast with much of the New Keynesian literature on monetary policy, nominal prices are set by an external monetary authority and are exogenous, following a constant path (without loss of generality). Furthermore, labor and output markets are competitive. Consumers consume a consumption basket $C_t$; they supply hours of labor $H_t$ which earns a gross wage $W_t$; and they pay a tax rate $T_t$ on labor income. In addition to this tax, the household receives a time-varying exogenous subsidy on post-tax labor income at a gross rate $V_t$ which equals zero in the steady state; this subsidy resembles the "markup shocks" commonly found in the New Keynesian literature or the "labor wedges" of Chari, Kehoe, and McGrattan (2007). This subsidy is funded every period by a lump-sum levy $\varphi_t$ which is levied by the head of household.
Within this setup, consumers seek to maximize the present discounted value of utility subject to a sequence of budget constraints. A complete $J_t$-by-one array of asset holdings $A_t$ is available over all $J_t$ possible states of the world, at a vector of ex-ante prices $P_t$. Each asset $j$ pays off one consumption unit in state of the world $j$ in the following period, and pays off nothing otherwise. The ex-post realization of the state of the world is given by $\sigma_t$, and the ex-post vector of asset prices is therefore given by an indicator function $1_{\sigma_t=j}$ of size $J_{t-1}$-by-one, which consists of zeros except for the one element where $\sigma_t = j$. Taking asset holdings into account, the consumers’ budget constraint is given by:

$$C_t + P_t'A_t = 1_{\sigma_t=j}A_{t-1} + W_tH_t(1 - T_t + V_t) - \varphi_t.$$ Subject to this constraint, consumers seek to maximize the objective function:

$$\Xi_t = E_t \sum_{i=0}^{\infty} \beta^i \left[ \ln(C_{t+i}) - \frac{\phi H_t^{1+1/\chi}}{1 + 1/\chi} - \lambda^C_{t+i} \left( C_{t+i} + P_{t+i}'A_{t+i} - 1_{\sigma_{t+i}=j}A_{t+i-1} \right. \right.$$

$$- W_{t+i}H_t(1 - T_{t+i} + V_{t+i}) + \varphi_{t+i} \left. \right).$$

(1)

Completeness in asset markets and separability in preferences imply perfect worldwide risk sharing in consumption, so that $C_t$ in equilibrium is the same across all countries. The resulting first-order conditions for consumption and labor, respectively, imply that $\lambda^C_t = \frac{1}{C_t}$ and $\phi H_t^{1/\chi} = \lambda^C_t W_t(1 - T_t + V_t)$.

### 2.1.2 Producer behavior and market clearing

Producers produce output $Y_t$ competitively according to a linear production technology $Y_t = Z_tH_t$, with an exogenous rate of productivity $Z_t$ which increases smoothly at a constant nominal rate $g$. Firms pay a wage $W_t$, and profit maximization ensures that $W_t = Z_t$. Assuming market clearing and combining the consumers’ and producers’ first-order conditions yields an expression for output as a function of tax rates, the labor wedge, and productivity, such that:

$$\phi \left( \frac{Y_t}{Z_t} \right)^{1/\chi} = \frac{Z_t}{C_t}(1 - T_t + V_t).$$

(2)
2.1.3 Government behavior

On the government side, fiscal authorities finance an exogenous stream of government spending $G_t$ which also increases smoothly at a constant nominal rate $g$, using only distortionary taxes. In this case the primary surplus is given by the relationship:

$$S_t = T_t Y_t - G_t.$$  (3)

The law of motion for the end-of-period real public debt stock $B_t$ given the previous end-of-period debt stock, no default, an exogenous risk-free global nominal interest rate $r^G_{t-1}$, an exogenous inflation rate $\pi_t$, and a real primary balance (or primary surplus) $S_t$, takes the form:

$$B_t = \frac{(1 + r^G_{t-1})}{(1 + \pi_t)} \Gamma \left( \frac{B_{t-1}}{Y_{t-1}} \right) B_{t-1} - S_t.$$  (4)

The (possibly time-varying) default risk function $\Gamma(B_{t-1}/Y_{t-1})$ gives the effect of past debt levels on interest rates (the "confidence effect"), following the setup of Bonam and Lukkezen (2013). This function, which is increasing in $B_{t-1}/Y_{t-1}$, represents the premium that lenders require to lend to countries which are at risk of running into a stochastic, exogenous "debt limit", the probability of which is increasing in debt levels. This setup is a simplification of the setup of Davig, Leeper, and Walker (2011) and Bi (2012), in which countries are at risk of running into a stochastic, endogenous "debt limit". Given that solving for an endogenous debt limit typically requires a full nonlinear solution to the model, the setup given here parsimoniously captures some of the effects that debt levels might have on investor confidence and hence on interest rates, within a tractable modeling setup.

2.2 A representation as a three-equation multiplier model

The theoretical model has a reduced-form representation as a three-equation multiplier model consisting of three equations—a multiplier equation, a law of motion for the public debt, and a fiscal rule. The first equation comes from linearizing and combining equations (2) and (3). To see this, it is first necessary to linearize the model. Bars denote steady states or smooth trends. The percent deviation of output from its trend is given by $y_t = (Y_t - \bar{Y}_t)/\bar{Y}_t$; the arithmetic deviation of the tax rate from its trend is given by $\tau_t = T_t - \bar{T}$; the arithmetic deviation of the primary surplus level from its trend is given by $s_t = (S_t - \bar{S}_t)/\bar{Y}_t$; and the
arithmetic deviation of the labor wedge from its trend is given simply by \( v_t = V_t \). Linearizing (2) and (3) around the steady state yields the first-order approximations:

\[
y_t = \frac{\chi}{1 - \bar{T}} (-\tau_t + v_t),
\]

and

\[
s_t = \tau_t + \bar{T} y_t,
\]

respectively. Substituting (6) into (5) and solving for \( y_t \) gives \( y_t \) as a function of \( s_t \) and the labor wedge \( v_t \) such that:

\[
y_t = -\frac{\chi}{1 - \bar{T} - \chi \bar{T}} (s_t - v_t),
\]

which can be expressed as the reduced form:

\[
y_t = y_t^* - m s_t,
\]

for a multiplier \( m = \chi / [1 - \bar{T} - \chi \bar{T}] \) and an exogenous output gap shifter \( y_t^* = m v_t \). That the production side of this particular model has as its reduced form a simple multiplier equation like (8) makes it relatively simple to discuss the stability properties of different fiscal regimes in an intuitive way using multiplier-based thinking.

The second equation in the reduced-form model is simply the law of motion for the public debt. Letting \( b_t \) equal \( B_t / Y_t \) given a local debt ratio \( b \), letting \( r \) equal a local nominal interest rate which may in part be a function of \( \Gamma(b) \), and assuming a functional form for \( \Gamma(B_t/Y_t) \) given by \( \exp(\gamma^*_t + \gamma B_{t-1}/Y_{t-1}) \), the debt path then obeys the following law of motion to a first-order approximation:

\[
b_t = \frac{(1 + r)}{(1 + g)} (1 + \gamma b) b_{t-1} + \frac{(1 + r)}{(1 + g)} b \gamma^*_t - s_t,
\]

The term \( \gamma^*_t \) is an exogenous level shifter for sovereign interest rates, or the exogenous part of the sovereign yield spread. This spread is large during a fiscal crisis. Additionally, the presence of \( \gamma b \) in this equation implies the possibility of different dynamics for the public debt at high debt levels versus low debt levels, when the conduct of fiscal policy itself feeds back into the confidence of investors.

The third equation in the reduced-form model is a fiscal rule specified by the fiscal poli-
cymaker, which sets $b_t$ or $s_t$ as a function of other variables. This equation captures the notion that fiscal authorities act with some target in mind, and the interactions between this equation and the other two equations are the object of the current exercise. This simple three-equation system is meant to give some basic intuition as to which fiscal targeting regimes are likely to result in a stable path for the real economy, just as the simple three-equation system discussed by Woodford (2003) and others can deliver intuition as to the effects of different monetary policy regimes on the real economy. In all of what follows, it is assumed that $r$ equals or exceeds $g$ and that $\gamma$ and $b$ are weakly greater than zero.

### 2.3 A definition of stability

The analysis which follows relies upon a particular definition of stability, in the sense of paths of debt and output levels which, absent default, are nonexplosive. This emphasis on stability is in contrast with much of the monetary policy literature, which emphasizes instability. For instance, in the three-equation New Keynesian sticky price model, the presence of an adequate number of explosive roots ensures the uniqueness of equilibrium output, interest rates, and inflation, while too many stable roots result in indeterminacy. This is because that model is purely forward-looking, and so it is necessary to solve for current allocations as the present value of current and expected future shocks. By contrast, the model presented above is backward-looking with a state variable given by $b_t$, and that state variable should follow a nonexplosive law of motion, in order to ensure "passive" fiscal policy. In order to analyze the stability of the public debt and the output gap under a given fiscal targeting regime, therefore, it is necessary to first adopt a formal definition of stability which is equivalent to a situation where debt and output follow a nonexplosive law of motion. Two definitions of stability make sense in this context–global stability and local stability.

**Definition 1** A system consisting of $\{X_t\}$ is defined as globally stable if and only if the law of motion governing the sequence $\{X_t\}$ satisfies the condition $\lim_{T \to \infty} \theta^T \{X_T\} = \{0\}$ for any and all $\theta$ such that $|\theta| < 1$.

While nonlinearity is an important issue in fiscal policy–particularly in the "fiscal limits" literature–it is difficult to fully evaluate nonlinear models for global stability, particularly large, nonlinear forward-looking models. Since it is difficult to analyze such models for global stability, it is useful to follow the New Keynesian literature and look at local stability in linear models.
**Definition 2** Let \( \{x_t\} \) equal a local linear approximation to \( \{X_t - \bar{X}\} \) for some \( \bar{X} \). A system characterized by \( \{X_t\} \) or, equivalently, \( \{x_t\} \), is defined as locally stable if and only if the law of motion governing the sequence \( \{x_t\} \) satisfies the condition \( \lim_{T \to \infty} \theta^T \{X_T\} = \{0\} \) for any and all \( \theta \) such that \( |\theta| < 1 \).

Both definitions of stability are related to the general notion of sustainability discussed by Bohn (2007) and others, for which a sufficient condition would be any finite order of integration of \( \{X_t\} \) or \( \{x_t\} \). In addition, these definitions of stability expand upon the idea of sustainability to take output stability into account. Stability in the sense discussed here is sufficient, but not strictly necessary, to satisfy most transversality conditions of the sort found in macroeconomic models. In what follows, it will remain as a maintained assumption that the exogenous stochastic sequence \( \{y_t^*\} \) or, equivalently, \( \{\nu_t\} \), is locally stable. In addition, it is assumed that \( \{\gamma_t^*\} \) is locally stable. By manipulating the model under this maintained set of assumptions, it is possible to derive conditions under which the sequence \( \{b_t, y_t\} \) is locally stable for several cases which correspond with different fiscal policy regimes.

## 3 The local instability of output and debt under strict debt-GDP targeting

### 3.1 Main result

The first case is a situation where fiscal authorities follow a strict debt-GDP targeting regime. A strict debt-GDP targeting regime is defined as a regime which sets the debt-GDP ratio to a target \( B_t^* = B_t/y_t \) for an exogenously given \( B_t^* \). An example of a strict debt-GDP targeting regime would be a stylized version of the "1/20" rule in the euro area based on the Fiscal Compact, whereby euro area governments are required to reduce the gap between their debt ratio and the 60% cutoff by one-twentieth of that gap per year, given already-realized values of the debt ratio. It turns out that if any strict debt-level targeting regime

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2For instance, the transversality condition governing \( \{X_t\} \) in many macroeconomic models would also be satisfied when \( \{X_t\} \) grows at a rate less than the rate of interest. The notions of stability discussed here rule out mild explosions of this sort.

3The actual "1/20" rule depends on additional leads and lags of the debt ratio, in conjunction with the other targets laid out by Stability and Growth Pact, but it is useful to think of this rule as requiring the debt-GDP ratio to follow a predetermined path based on its values at the time of the Fiscal Compact.
were to be strictly enforced, under reasonable parameter values for a number of countries in the Eurozone, such a regime could potentially lead to local instability in output and debt.

To analyze this case locally and to show these results, it is first necessary to write the debt-target as obeying:

$$b_t^* = b_t - by_t,$$  \hspace{1cm} (10)

to a first-order approximation, for an exogenous, locally stable sequence \(\{b_t^*\}\). Substituting this approximation into (9) gives:

$$b_t^* = \frac{(1 + r)}{(1 + g)} (1 + \gamma b) b_{t-1}^* + \frac{(1 + r)}{(1 + g)} (1 + \gamma b) by_{t-1} + \frac{(1 + r)}{(1 + g)} b\gamma_t^* - by_t - s_t,$$  \hspace{1cm} (11)

Solving (11) for \(s_t\) and then substituting this condition into the multiplier relationship (8) yields an expression for output such that:

$$y_t = y_t^* + m \left( b_t^* - \frac{(1 + r)}{(1 + g)} (1 + \gamma b) b_{t-1}^* - \frac{(1 + r)}{(1 + g)} (1 + \gamma b) by_{t-1} - \frac{(1 + r)}{(1 + g)} b\gamma_t^* + by_t \right).$$  \hspace{1cm} (12)

Next, solving this equation for \(y_t\) gives the law of motion for output, such that:

$$y_t = \frac{1}{1 - mb} \left[ y_t^* + m \left( b_t^* - \frac{(1 + r)}{(1 + g)} (1 + \gamma b) b_{t-1}^* - \frac{(1 + r)}{(1 + g)} (1 + \gamma b) by_{t-1} - \frac{(1 + r)}{(1 + g)} b\gamma_t^* \right) \right].$$  \hspace{1cm} (13)

By examining the local dynamics implied by equation (13), it is possible to demonstrate the following proposition:

**Proposition 1** Given an exogenous, locally stable sequence \(\{b_t^*\}\) which is not identically zero and given the linearized model (8), (9), and (10), the sequence \(\{b_t, y_t\}\) is locally stable at \(b\) if and only if

$$-\frac{m(1+r)}{(1+g)} \leq 1,$$

or equivalently, if \(mb \leq \frac{1}{(1+g)(1+\gamma b)+1} \).
To demonstrate the first part of this proposition, the exogeneity and local stability of \( \{b_t^*\} \) would imply that equation (13) could be decomposed into a feedback component multiplying \( y_{t-1} \) and a locally stable, exogenous, composite shifter given by \( e_t^y \), such that:

\[
y_t = \frac{-m(1+r)(1+\gamma b) b}{1-mb} y_{t-1} + e_t^y.
\]  

(14)

The main issue lies in the conditions that determine the local stability of \( y_t \). By inspecting equation (14), it becomes readily apparent that given a locally stable \( e_t^y \), \( y_t \) is locally stable if and only if its feedback coefficient satisfies

\[
-1 \leq \frac{-m(1+r)(1+\gamma b) b}{1-mb} \leq 1.
\]

Furthermore, given a locally stable \( \{b_t^*\} \), \( b_t \) is locally stable if and only if \( y_t \) is locally stable, since the debt-GDP targeting regime requires that \( b_t = b_t^* + by_t \). Therefore, the system \( \{b_t, y_t\} \) is locally stable if and only if

\[
-1 \leq \frac{-m(1+r)(1+\gamma b) b}{1-mb} \leq 1,
\]

under a strict debt-GDP targeting regime.

To demonstrate the second part of the proposition, there are three interesting, mutually exclusive, and exhaustive cases to consider. In the first case when \( mb < 1 \), multiplying this condition through by \( 1 - mb \) implies that local stability for \( y_t \) holds if and only if

\[
mb - 1 \leq -m(1+r)(1+\gamma b) b \leq 1 - mb.
\]

Adding \( m(1+r)(1+\gamma b) b \) to all portions of this condition gives the condition

\[
mb \left( \frac{(1+r)}{(1+g)} (1 + \gamma b) + 1 \right) - 1 \leq 0 \leq 1 + mb \left( \frac{(1+r)}{(1+g)} (1 + \gamma b) - 1 \right),
\]

which is equivalent to the condition

\[
mb \leq \frac{1}{\frac{(1+r)}{(1+g)} (1 + \gamma b) + 1}.
\]

In the second case when \( mb = 1 \), the system admits no possible value for \( y_t \) consistent with the desired path of fiscal policy. In the third case when \( mb > 1 \), then multiplying the stability condition through by \( 1 - mb \) implies that local stability for \( y_t \) holds if and only if

\[
mb - 1 \geq -m(1+r)(1+\gamma b) b \geq 1 - mb.
\]

Adding \( m(1+r)(1+\gamma b) b \) to all portions of this condition gives the condition

\[
mb \left( \frac{(1+r)}{(1+g)} (1 + \gamma b) + 1 \right) - 1 \geq 0 \geq 1 + mb \left( \frac{(1+r)}{(1+g)} (1 + \gamma b) - 1 \right),
\]

which is impossible. Aggregating these three cases together, the local stability of \( b_t^* \) implies that \( y_t \) is locally stable and hence \( \{b_t, y_t\} \) is locally stable if and only if

\[
mb \leq \frac{1}{\frac{(1+r)}{(1+g)} (1 + \gamma b) + 1}.
\]

3.2 Intuition: the role of GDP

There is clear economic intuition behind this set of results. For small debt-multiplier combinations where

\[
mb \leq \frac{1}{\frac{(1+r)}{(1+g)} (1 + \gamma b) + 1}
\]

(which is just under one half for realistic values of \( b, r, g, \) and \( \gamma \)), it is possible to target a path for the debt-GDP ratio while maintaining a stable output path. In fact, when the fiscal multiplier is zero, consolidation toward a given debt-GDP path can be undertaken painlessly. As the multiplier \( m \) starts to increase, the negative coe-
icient governing (14) implies that fiscal authorities must engage in a degree of stop-and-go fiscal policy in order to maintain the debt-GDP ratio on a given path. This is because the debt-GDP ratio contains a debt component and a GDP component, and fiscal policy works at cross purposes with respect to these aggregates. When fiscal policy acts to shrink the debt level through spending cuts or tax increases, output also falls, and it can take a large fiscal consolidation to shrink the debt-GDP ratio by a small amount. Once that round of consolidation is finished, fiscal authorities find themselves at the beginning of the following period with a much-reduced stock of debt in levels but with no corresponding downward pressure on GDP, which is a flow. However, a low level of debt combined with a normal level of GDP would then cause the debt-GDP ratio to undershoot its target. In order to maintain a constant debt-GDP ratio, therefore, fiscal authorities would have to then engage in a fiscal expansion in order to find a debt level and a GDP level which satisfy the debt-GDP ratio target. So long as debt-multiplier combinations are not too high, these oscillations dampen over time and $y_t$ and $b_t$ revert toward trend. For a stylized country like Germany with a growth-adjusted real interest rate of two percent, no effect of debt on interest rates, a local debt-GDP ratio of about 75%, and a moderate multiplier of 0.6, this coefficient would equal approximately 0.46, which is below but near the critical value of 0.495. If such a country were to implement something like the "1/20" rule under these circumstances, it could do so but at the cost of engaging in a significant degree of stop-and-go fiscal policy.

There is a point at which this type of stop-and-go policy becomes explosive. Once the debt-multiplier combination reaches a point where $mb$ exceeds the critical value, the oscillations in $y_t$ become so violent that $y_t$ explodes. This happens when the coefficient on the feedback equation (14) goes below negative one, which is a realistic possibility for some of the more highly-indebted European countries. For a stylized country like a pre-crisis Italy with a growth-adjusted real interest rate of two percent, a debt-GDP ratio of over 100%, and a moderate multiplier of 0.6, this coefficient would equal approximately 0.61, which is above the 0.495 cutoff. If such a country were to attempt to implement something like the "1/20" rule under these circumstances, it would face violent explosive fluctuations in output and in the level of debt but not in the debt-GDP ratio.

At even larger debt-multiplier combinations, a different set of effects kicks in, whereby fiscal consolidation automatically increases the debt-GDP ratio in the short run, rather than decreasing the debt-GDP ratio. This happens as $mb$ crosses one. First of all, as $mb$ hits one, fiscal policy works at such cross purposes with respect to the debt level and GDP level that no fiscal policy actions can support a debt-GDP ratio consistent with the target. Every time
fiscal authorities adjust fiscal policy, output adjusts to such an extent that the debt-GDP ratio does not move at all. Once $mb$ exceeds one, the way to reduce the debt-GDP ratio in the short run is through expansionary fiscal policy. This strategy, however, still results in explosions. The intuition for this is straightforward. While a fiscal expansion in this situation can reduce the debt-GDP ratio in the short run, a fiscal expansion creates a higher debt stock for the future, which necessitates yet more expansionary fiscal policy to target the debt ratio, and so on. In this case, the debt level and output level have to explode in opposite directions in order to keep the debt-GDP ratio from rising.

This set of results differs somewhat from the results of Creel et al. (2013), who obtain stability for a roughly similar set of parameter values. Creel et al. model the “1/20” rule of the Fiscal Compact in a manner similar to that above. However, they assume a country-specific independent monetary policy regime which follows a Taylor rule, and they also assume a closed economy. Creel et al. obtain stability in the sense of nonexplosiveness for a wider set of parameter values, but they also find that the Fiscal Compact is expected to have adverse effects on output during the short to medium run. That these results differ from the current results suggests that the monetary policy regime and the assumption of an open economy could play some role in determining which fiscal policy regimes are “passive” in equilibrium. More work is needed to disentangle which of these effects are due to an independent monetary policy, and which are due to the assumption of an open economy.

Altogether, in summary, the analytical evidence suggests that a binding debt-GDP ratio target can destabilize both GDP and debt levels, even for moderate combinations of the debt-GDP ratio and the fiscal multiplier. Even at low values for $mb$, supporting a debt-GDP ratio target could require fiscal authorities to engage in stop-and-go fiscal policy. This set of problems occurs for the simple reason that the debt-GDP ratio contains both a debt portion and a GDP portion. Since fiscal policy works at cross purposes with respect to debt and GDP, it may take large fluctuations in fiscal policy to substantially affect the debt-GDP ratio in the short run, in situations where that is possible at all.
4 The local stability of output and debt under strict debt-level targeting

Given that the problems with a debt-GDP targeting regime emanate from the GDP portion of the debt-GDP ratio, one simple solution would be to institute a debt-level targeting regime instead. A strict debt-level targeting regime is defined as a regime which sets the debt-potential GDP ratio to a target $B_t^{**} = B_t / \bar{Y}_t$ for an exogenously given $B_t^{**}$. It turns out that if any strict debt-level targeting regime were to be strictly enforced, this regime would lead to local stability in output and debt under realistic parameter values.

To analyze this case locally, it is first necessary to write the debt target as obeying:

$$b_t^{**} = b_t,$$  \hspace{1cm} (15)

for an exogenous, locally stable sequence $\{b_t^{**}\}$. Substituting this target into (9) gives:

$$b_t^{**} = \left(1 + \frac{r}{1 + g}\right) (1 + \gamma b) b_{t-1}^{**} + \left(1 + \frac{r}{1 + g}\right) b\gamma_t^{*} - s_t,$$ \hspace{1cm} (16)

Solving for $s_t$ and then substituting (16) into the multiplier relationship (8) yields an expression for output such that:

$$y_t = y_t^{*} + m \left(b_t^{**} - \left(1 + \frac{r}{1 + g}\right) (1 + \gamma b) b_{t-1}^{**} - \left(1 + \frac{r}{1 + g}\right) b\gamma_t^{*}\right).$$ \hspace{1cm} (17)

By examining equation (17), it is possible to demonstrate the following proposition:

**Proposition 2** Given an exogenous, locally stable sequence $\{b_t^{**}\}$ which is not necessarily identically zero and given the linearized model (8), (9), and (15), the sequence $\{b_t, y_t\}$ is always locally stable.

To demonstrate this proposition, the local stability of $\{b_t^{**}\}$ straightforwardly implies the local stability of $b_t$ because of equation (15). Furthermore, equation (17) implies that the same holds true for the output gap $y_t$, since the right hand side of that equation is locally stable. Therefore, the sequence $\{b_t, y_t\}$ is locally stable under a strict debt-level targeting regime.

As one might suspect, the targeting of a debt level rather than a debt-GDP ratio eliminates
the problems that might occur with respect to stability when GDP adjusts too strongly in response to fiscal policy actions. This result serves to show that seemingly small changes in the operating target for fiscal policymakers can have drastic consequences for the stability of debt and output paths. While such a stability result says nothing about optimality—and, in fact, much of the fiscal policy literature suggests that such a rule would not be optimal—the stability result suggests that removing GDP from the fiscal rule may help to ensure stability and passive fiscal policy, as a technical matter.

5 The local stability of output and the local instability of debt under strict primary deficit targeting

Another possible targeting regime is for fiscal policy to target primary deficits (or, equivalently, primary surpluses) rather than debt levels. This idea can be motivated by the point that, apart from the "1/20" rule, the Stability and Growth Pact contains rules with regard to the magnitude of government deficits as a share of GDP. Motivated by these types of targets, a strict primary deficit targeting regime (or, equivalently, a primary surplus targeting regime) is defined as a regime which sets the primary surplus-potential GDP ratio to a target \( S_t^* = S_t/Y_t \) for an exogenously given \( S_t^* \). It turns out that if a strict primary deficit targeting regime of this type were to be enforced, this regime would lead to local stability in output but local instability in debt levels under realistic parameter values. This is a well-known result in the literature which dates at least to Sargent and Wallace (1981) and Leeper (1991), but it deserves to be repeated here since it stands somewhat in contrast with the result on debt-level targeting.

To analyze a deficit target locally, it is first necessary to write the deficit target as obeying the targeting rule:

\[
s_t^* = s_t, \tag{18}
\]

for an exogenous, locally stable sequence \( \{s_t^*\} \). Substituting this target into (9) gives:

\[
b_t = \frac{(1 + r)}{(1 + g)} (1 + \gamma b) b_{t-1} + \frac{(1 + r)}{(1 + g)} b_t^* - s_t^*. \tag{19}
\]
Substituting the target into (8) also yields an expression for output, such that:

\[ y_t = y_t^* - ms_t^*. \]  

(20)

It is relatively simple to see that equations (19) and (20) imply locally stable dynamics for output but locally unstable dynamics for debt levels, which leads to the next proposition:

**Proposition 3** Given an exogenous, locally stable sequence \( \{s_t^*\} \) which is not identically zero and given the linearized model (8), (9), and (18), the sequence \( \{b_t\} \) is always locally unstable, although the sequence \( \{y_t\} \) is always locally stable.

To demonstrate this proposition, it is self-evident from equation (19) that \( b_t \) features explosive dynamics even when \( \{s_t^*\} \) is locally stable, and hence \( \{b_t\} \) is not locally stable. However, \( \{y_t\} \) by itself is locally stable since the starred components of (20) are locally stable by assumption. This set of results is well-known from the monetary policy literature, which states that iterating (19) forward would imply that a primary deficit target itself does not enforce the government’s budget constraint. Rather, the government must resort toward surprise inflation to deflate away part of the debt. Given that none of these things occurs in the model (where monetary policy is "active" by assumption), a primary deficit target of this type can result in serious problems with stability in debt levels.

6 The local stability of output and debt under strict total deficit targeting

Given that a strict primary deficit target fails to ensure stability in debt levels because interest payments give rise to unstable debt dynamics, it is worth investigating the effects that a total deficit target would have on stability. It turns out that a total deficit target results in stable paths for debt and output, so long as nominal growth is positive. This result suggests that fiscal rules that target total deficits, rather than primary deficits, stand a better chance at ensuring stability, although the optimality properties of such a rule may still be open to question.

A strict total deficit targeting regime is defined as a regime which sets the total surplus-
potential GDP ratio to a target given by:

\[ S_t^{**} = \left[ S_t - \left( \frac{(1 + r_t^G)}{(1 + \pi_t)} \Gamma \left( B_{t-1} \frac{B_{t-1}}{Y_{t-1}} \right) - 1 \right) B_{t-1} \right] / \bar{Y}_t, \]  

(21)

for an exogenously given \( S_t^{**} \). To analyze the deficit target locally, it is first necessary to linearize the deficit target for an exogenous, locally stable sequence \( \{s_t^{**}\} \), such that:

\[ s_t^{**} = s_t - \frac{r}{1 + g} b_{t-1} - \frac{1 + r}{1 + g} (\gamma b b_{t-1} + b r_t^*). \]  

(22)

Substituting this target into (9) and doing some algebra yields:

\[ b_t = \frac{1}{1 + g} b_{t-1} - s_t^{**}. \]  

(23)

Substituting the target into (8) also yields an expression for output, such that:

\[ y_t = y_t^* - m \left[ s_t^{**} + \frac{r}{1 + g} b_{t-1} + \frac{1 + r}{1 + g} (\gamma b b_{t-1} + b r_t^*) \right]. \]  

(24)

It is relatively simple to see that equations (23) and (24) imply locally stable dynamics for output and debt levels when nominal growth is positive, which leads to the next proposition:

**Proposition 4** Given an exogenous, locally stable sequence \( \{s_t^{**}\} \) which is not identically zero and given the linearized model consisting of the equations (8), (9), and (22), the sequence \( \{b_t, y_t\} \) is always locally stable if and only if \( g \geq 0 \).

To demonstrate this proposition, it is self-evident from equation (23) that when \( \{s_t^*\} \) is locally stable, \( b_t \) is locally stable if and only if nominal growth \( g \geq 0 \). Furthermore, \( \{y_t\} \) is locally stable if and only if \( \{b_t\} \) is locally stable given the structure of (24). Therefore, the joint sequence \( \{b_t, y_t\} \) is locally stable if and only if \( g \geq 0 \).

This finding is interesting in two respects. First of all, this finding implies that some of the total deficit targets laid out in the Stability and Growth Pact as well as in national fiscal rules might actually result in stable paths for debt and output, although is it important to keep in mind that stability does not imply optimality. Secondly, the seemingly minor differences between a primary deficit target and a total deficit target are in fact relatively important. As is the case with a debt targeting regime, the exact form that a deficit targeting regime takes can determine whether or not debt and output follow a stable path.
7 The conditions for the local stability of debt and output under a fiscal reaction function

7.1 Main result

While certain types of debt and deficit targets may result in stable paths for debt and output, these targets do not necessarily capture the tradeoffs that policymakers make between stabilization policy and consolidation policy. One way to deal with these tradeoffs would be to model the behavior of fiscal authorities as following a fiscal reaction function. This way of modeling the behavior of fiscal authorities is in keeping with much of the theoretical macro literature and is also in keeping with much of the empirical literature. Here, the specification of the fiscal reaction function mirrors the basic forms given by Galí and Perotti (2003), Snower, Burmeister, and Seidel (2011), Reicher (2012), Plödt and Reicher (2014), and many others. The one novel addition here is a term $l \left[ \exp(\gamma_t^*) - 1 \right]$ which allows for fiscal policymakers to preemptively react to fiscal crises as they happen.

The baseline fiscal reaction function takes the form:

$$
\frac{S_t}{Y_t} = k + a \left( \frac{\bar{Y}}{Y_t} - 1 \right) + c \left( \frac{B_{t-1}}{Y_{t-1}} - b \right) + l \left[ \exp(\gamma_t^*) - 1 \right] + e_t^*,
$$

for a sustainable primary surplus ratio given by $k$ and a mean zero, locally stable, exogenous fiscal policy shifter $e_t^*$. The coefficient $a$ represents the allowable influence of output on the primary surplus including, but not limited to, automatic stabilizers (stabilization policy). The coefficient $c$ represents the rate at which the primary surpluses increases in response to past deviations of the debt ratio from its long-run target (consolidation policy). The coefficient $l$ represents the rate at which fiscal authorities directly respond to shocks to the yield spread, in a form of preemptive consolidation policy.

As with the other targets, an analysis of the local stability of this target requires taking a first order approximation. Assuming that $k$ is constant, equation (25) implies, to a first order approximation, that:

$$
s_t = ay_t + cb_{t-1} + l\gamma_t^* + e_t^*.
$$
Substituting (26) into the multiplier relationship (8) and then solving for $y_t$ yields:

$$y_t = \frac{1}{1 + ma} \left[ y_t^* - m(cb_{t-1} + l\gamma_t^* + \epsilon_t^*) \right].$$  \hspace{1cm} (27)

Substituting these two expressions into (9) yields:

$$b_t = \frac{(1 + r)}{(1 + g)} (1 + \gamma b) b_{t-1} + \frac{(1 + r)}{(1 + g)} b\gamma_t^* - \frac{a}{1 + ma} \left[ y_t^* - m(cb_{t-1} + l\gamma_t^* + \epsilon_t^*) \right] - cb_{t-1} - \epsilon_t^*,$$ \hspace{1cm} (28)

which can be analyzed for its local stability properties. After some light algebra, this law of motion has the representation:

$$b_t = \left[ \frac{(1 + r)}{(1 + g)} (1 + \gamma b) - \frac{c}{1 + ma} \right] b_{t-1} + e_t^b,$$ \hspace{1cm} (29)

for some locally stable process $e_t^b$. This representation leads to a generalized and modified version of a well-known stability result:

**Proposition 5** Given a locally stable sequence $e_t^b$ and given the linearized model (8), (9), and (26), the sequence $\{b_t, y_t\}$ is locally stable if and only if $\left[ \frac{(1 + r)}{(1 + g)} (1 + \gamma b) - \frac{c}{1 + ma} \right] \leq 1$.

To demonstrate this proposition, the necessary and sufficient conditions underlying the local stability of $\{b_t\}$ are readily apparent from inspecting the feedback coefficient on (29), which implies that $\{b_t\}$ is locally stable if and only if $\left[ \frac{(1 + r)}{(1 + g)} (1 + \gamma b) - \frac{c}{1 + ma} \right] \leq 1$. Furthermore, $\{y_t\}$ is also locally stable if and only if $\{b_t\}$ is locally stable, based on (27). Therefore, the sequence $\{b_t, y_t\}$ is locally stable if and only if $\left[ \frac{(1 + r)}{(1 + g)} (1 + \gamma b) - \frac{c}{1 + ma} \right] \leq 1$.

This result generalizes a well-known result in the literature (see, for instance, Bohn (1998)) in two ways. That result, derived under the assumption of a zero multiplier ($m = 0$) and no "confidence effect" ($\gamma = 0$), states that the sequence $\{b_t, y_t\}$ is locally stable at $b$ if and only if $\left[ \frac{(1 + r)}{(1 + g)} - c \right] \leq 1$. By contrast, when there is a meaningful cyclical response of fiscal policy to the output gap given by $a$ in the presence of multiplier effects given by $m$, a tradeoff appears between this cyclical response and the consolidating response of fiscal policy to the debt given by $c$. Furthermore, the presence of confidence effects, given by $\gamma$, requires a stronger consolidation policy at high debt levels than otherwise would be the case. Finally,
this required degree of consolidation policy does not depend on \( l \).

### 7.2 The "clawback" and "confidence" effects, and some magnitudes

There is intuition behind why these results differ from previous results from the literature. The first difference—the effect of \( \alpha \) on the stability of the economy—results from a "clawback" effect caused by automatic stabilizers in the presence of a fiscal multiplier, which can be seen from solving equation (28) for the total effect of \( \epsilon_t^* \) on \( b_t \). The mechanism behind this clawback effect is as follows. First, when fiscal authorities act to increase the primary surplus in response to a high debt level, the multiplier effects of this action cause a fall in output. Then, when output falls, the primary surplus partly decreases in a manner governed by \( \alpha \), which again affects output, and so on. The total size of the intervention net of this "clawback" effect equals \( \frac{1}{1+ma} \) times the size of the original intervention. Therefore, it is necessary to have a stronger initial intervention if fiscal authorities wish to aim for a particular speed of debt reduction, relative to a situation without a "clawback" effect.

The second difference—the effect of \( \gamma \) on stability of the economy—results from a "confidence" effect caused by debt levels. High debt levels result in a fall in confidence with respect to the solvency of the government, which causes a future increase in debt levels because of higher interest rates. Consolidation efforts must therefore outrun the gradual deterioration in confidence if the debt level were to be stabilized, and this issue becomes more acute at high debt levels since a large debt stock is more sensitive to changes in interest rates caused by changes in confidence.

To analyze the stability properties of fiscal policy as it is actually practiced, it is necessary to turn to evidence from the data. Fortunately, estimates exist as to which coefficients in a fiscal response function might provide a reasonable match with past data. The estimates of Plödt and Reicher (2014) for the Eurozone and Reicher (2014) for the OECD indicate that a reasonable coefficient estimate for \( c \) for most countries would fall in the 0.05 to 0.07 range, while a reasonable estimate for \( \alpha \) would fall in the 0.4 to 0.5 range. Under a multiplier \( m \) of 0.6 and a cyclical coefficient \( \alpha \) of 0.5, the "clawback" coefficient \( \frac{1}{1+ma} \) would equal \( \frac{1}{1.3} \), or about 0.77. It would therefore take an original fiscal intervention of 1.3% of potential GDP to reduce the primary deficit by 1% of potential GDP at the outset. Using a coefficient value of \( c \) equal to 0.05, the coefficient \( \frac{c}{1+ma} \) net of clawback would equal approximately 0.038, which is somewhat above but near most estimates of the growth-adjusted real interest rate.
Estimates of $\gamma$ and $l$ are difficult to come by, although Plödt and Reicher (2014) provide some evidence of an increased speed of consolidation policy at high debt levels for Eurozone governments. Taken together, these values would result in slow consolidation in the debt-GDP ratio over time, relative to its initial state. Nonetheless, this rate of consolidation would be sufficient to ensure local stability in debt and output levels.

Altogether, these results taken together suggest that a flexible fiscal reaction function which responds adequately to past debt could result in stable paths for the debt-GDP ratio and output under reasonable conditions. These conditions require that systematic fiscal consolidation (the $c$ coefficient) be strong enough to overcome "clawback" and "confidence" effects. This finding is not of mere theoretical curiosity, since it turns out that these main principles guide the design of optimal fiscal policy, which under certain cases, follows a fiscal reaction function of this type.

8 Optimal fiscal policy

8.1 The optimal fiscal policy problem

As the previous results from the literature on labor wedges would tend to suggest, optimal fiscal policy in this model requires fiscal policymakers to make a tradeoff between using tax policy to offset fluctuations in the labor wedge $V_t$ and using tax policy to stabilize the public debt. Optimal fiscal policy here is derived using a Ramsey approach. To instead take a second-order approximation to the objective function $\Xi_t$, following Rotemberg and Woodford (1997) and Woodford (2003), would run into two potential complications. First of all, there is no guarantee of a non-distorted steady state since the steady-state tax rate is greater than zero in most countries. Secondly, as Benigno and Woodford (2012) point out, the possibility of a nonlinear fiscal multiplier implies that the constraints faced by the policymaker are not necessarily linear. Instead of constructing a stand-in quadratic approximation which corrects for the nonlinearity of these constraints, it is more straightforward to solve directly for the optimal policy path using the full solution to the model as a set of constraints.

In this case, the goal of fiscal policymakers is to maximize the objective function $\Omega_t$ given
by:

\[
\Omega_t = E_t \sum_{i=0}^{\infty} \beta^i \left[ \ln(C_{t+i}) - \frac{\phi H_{t+i}^{1+1/\chi}}{1 + 1/\chi} - \Lambda^H_{t+i} \left( \frac{\phi H_{t+i}^{1/\chi}}{C_{t+i}} - \frac{Z_{t+i}}{C_{t+i}} (1 - T_{t+i} + V_{t+i}) \right) \right.
\]

\[
- \frac{\Lambda^B_{t+i}}{Z_{t+i}} \left( B_{t+i} - (1 - D_{t+i}) \frac{(1 + r_{t+i-1}^G)}{(1 + \pi_{t+i})} \Gamma \left( \frac{B_{t+i-1}}{Y_{t+i-1}} \right) B_{t+i-1} + T_{t+i} Z_{t+i} H_{t+i} \right.
\]

\[
- G_{t+i} \right) \right],
\]

(30)

with respect to hours \(H_t\), the tax rate \(T_t\), and bonds \(B_t\). The first constraint multiplied by \(\Lambda^H_t\) (an implementability constraint) gives the requirement that economic equilibrium satisfy the economic plans of workers and firms—this constraint expresses hours as a function of taxes \(T_t\) and the driving process \(V_t\). The second constraint multiplied by \(\frac{\Lambda^B_t}{Z_t}\) gives the law of motion for the public debt, with an indicator variable \(D_t\) which indicates default. It is assumed that the probability of default is correctly priced into \(\Lambda^B_t\) such that \(E_t(1 + D_{t+1}) \Gamma \left( \frac{B_{t+1}}{Y_{t+1}} \right) = 1\). For the sake of simplicity, it assumed here that default is costless; an interesting extension would involve making default costly, in which case optimal policy would possibly seek to avoid high debt levels.

First-order conditions for the policymakers’ problem are given by:

\[-\phi H_{t}^{1/\chi} - \frac{\phi^H_{t}}{\chi} H_{t}^{1/\chi - 1} - \Lambda^B_H T_t = 0; \]

(31)

\[- \Lambda^H_t \frac{Z_t}{C_t} - \Lambda^B_t H_t = 0; \]

(32)

and, to a first order approximation:

\[- \frac{\Lambda^B_t}{Z_t} + E_t \beta^1 \frac{\Lambda^B_{t+1}}{Z_{t+1}} \frac{1 + \pi_{t+1}}{1 + \pi_{t+1}} = 0. \]

(33)

Substituting (32) into (31) to eliminate \(\Lambda^H_t\) gives:

\[-\phi H_{t}^{1/\chi} + \Lambda^B_t \frac{C_t}{Z_t} \frac{\phi}{\chi} H_{t}^{1/\chi} - \Lambda^B_t T_t = 0. \]

(34)

Substituting the implementability condition into this expression to eliminate \(H_t\) then gives,
after some algebra:

\[-\frac{Z_t}{C_t}(1 - T_t + V_t) + \Lambda_t^B \left( \frac{1}{\chi}(1 - T_t + V_t) - T_t \right) = 0. \quad (35)\]

As before, it is easier to work with the linearized model when undertaking simulations. Linearizing (35) as well as (33) around the steady state growth path yields the system:

\[\lambda_t^B = -\zeta_T T_t + \zeta_V V_t, \quad (36)\]

and:

\[-\partial E_t \lambda_{t+1}^B = -\lambda_t^B, \quad (37)\]

where \(\zeta_T = \left[ \frac{Z_t}{C_t} + \Lambda_t^B \left( \frac{1}{\chi} + 1 \right) \right] / \left[ \frac{1}{\chi}(1 - \bar{T}) - \bar{T} \right] \); \(\zeta_V = \left[ \frac{Z_t}{C_t} + \Lambda_t^B \left( \frac{1}{\chi} \right) \right] / \left[ \frac{1}{\chi}(1 - \bar{T}) - \bar{T} \right] \); and \(\Lambda_t^B = \left[ \frac{Z_t}{C_t}(1 - T_t) \right] / \left[ \frac{1}{\chi}(1 - \bar{T}) - \bar{T} \right] \). In general, it is reasonable to assume that \(\zeta_T\) and \(\zeta_V\) are strictly positive so long as a country sits on the left side of the Laffer Curve, the maximum-revenue point of which is given at \(\frac{1}{\chi}(1 - \bar{T}) - \bar{T} = 0\). Based on this system, substituting (36) into (37) gives the composite forward-looking difference equation governing the law of motion for taxes, such that:

\[E_t(\zeta_T T_{t+1} - \zeta_V V_{t+1}) = \zeta_T T_t - \zeta_V V_t. \quad (38)\]

8.2 Optimal fiscal policy as a tax rule, when \(v_t\) and \(\gamma_t^*\) each follow an AR(1)

8.2.1 The derivation of optimal policy

While equation (38) by itself does not have much content, results emerge when putting some structure onto \(v_t\) and \(\gamma_t^*\). In particular, when \(v_t\) follows a first-order autoregressive process with a persistence coefficient \(\rho_v\) and when \(\gamma_t^*\) follows a first-order autoregressive process with a persistence coefficient \(\rho_\gamma\), then optimal fiscal policy follows a tax rule, such that tax rates respond to the current labor wedge, the solvency shock, and past debt levels. To solve for the optimal fiscal policy rule, it is conjectured that optimal fiscal policy would follow a rule
of the form:

$$\tau_t = dv_t + fb_{t-1} + h\gamma_t^*, \quad (39)$$

for some response coefficients $d$, $f$, and $h$. The coefficients of this rule can be derived using the method of undetermined coefficients. To find these coefficients requires a fair bit of algebra, which follows.

First, substituting the primary surplus (6) into the law of motion for the debt (9) gives the latter law of motion as a function of tax rates and output, such that:

$$b_t = \frac{(1 + r)}{(1 + g)} (1 + \gamma b) b_{t-1} + \frac{(1 + r)}{(1 + g)} b\gamma_t^* - \tau_t - Ty_t, \quad (40)$$

and substituting the output equation (5) in turn gives the law of motion as a function of tax rates and the labor wedge, such that:

$$b_t = \frac{(1 + r)}{(1 + g)} (1 + \gamma b) b_{t-1} + \frac{(1 + r)}{(1 + g)} b\gamma_t^* - \frac{1 - T(1 + \chi)}{1 - T} \tau_t - \frac{T\chi}{1 - T} v_t. \quad (41)$$

Then, substituting the conjectured tax rule (39) into the left-hand side of (38) gives:

$$\zeta_T (E_t dv_{t+1} + fb_t + h\gamma_t^*) - \zeta_V \partial E_t v_{t+1} = \zeta_T \tau_t - \zeta_V v_t. \quad (42)$$

Then, substituting the laws of motion for $v_t$ and $\gamma_t^*$ into this expression and then dividing everything by $\zeta_T$ gives this expression as a function of variables known at time $t$, such that:

$$\left( d\rho_v + \frac{\zeta_V}{\zeta_T} (1 - \rho_v) \right) v_t + fb_t + h\rho_v \gamma_t^* = \tau_t. \quad (43)$$

Then, substituting the law of motion for $b_t$ given by (41) into this expression yields an expression which links current taxes, the current labor wedge, past debt, and the solvency shock, such that:

$$\tau_t = \left( d\rho_v + \frac{\zeta_V}{\zeta_T} (1 - \rho_v) \right) v_t + f \left( \frac{(1 + r)}{(1 + g)} (1 + \gamma b) b_{t-1} + \frac{(1 + r)}{(1 + g)} b\gamma_t^* \right)$$

$$- \frac{1 - T(1 + \chi)}{1 - T} \tau_t - \frac{T\chi}{1 - T} v_t + h\rho_v \gamma_t^*. \quad (44)$$
Rearranging and solving for the tax rate gives:

\[
\tau_t = \frac{d\rho_v + \xi_v (1 - \rho_v) - f \frac{T_x}{1 - \gamma} v_t}{1 + f \frac{1 - T(1 + \chi)}{1 - \gamma}} + \frac{f (1 + r) (1 + \gamma b)}{1 + f \frac{1 - T(1 + \chi)}{1 - \gamma} b_{t-1}} + \frac{f (1 + r) b + h \rho_{\gamma}}{1 + f \frac{1 - T(1 + \chi)}{1 - \gamma} \gamma^*_t},
\]

which takes the form conjectured for the tax rule. Therefore, solving for the tax rule coefficients \( f, h, \) and \( d \) requires solving the nonlinear system given by:

\[
f = \frac{f \frac{(1 + r)}{1 + g} (1 + \gamma b)}{1 + f \frac{1 - T(1 + \chi)}{1 - \gamma}};
\]

\[
h = \frac{f \frac{(1 + r)}{1 + g} b + h \rho_{\gamma}}{1 + f \frac{1 - T(1 + \chi)}{1 - \gamma}};
\]

and

\[
d = \frac{d\rho_v + \xi_v (1 - \rho_v) - f \frac{T_x}{1 - \gamma} v_t}{1 + f \frac{1 - T(1 + \chi)}{1 - \gamma}}.
\]

The possible solutions for \( f \) are given by the solutions to the equation:

\[
Q f^2 + (1 - R) f = 0,
\]

where \( Q = \frac{1 - T(1 + \chi)}{1 - \gamma} \) and \( R = \frac{(1 + r)}{1 + g} (1 + \gamma b) \). The stable solution to this equation is given by:

\[
f = \frac{R - 1}{Q}.
\]

The solution for \( h \), conditional on the solution for \( f \), is given by:

\[
h = \frac{f \frac{(1 + r)}{1 + g} b}{1 - \rho_{\gamma} + fQ};
\]

The solution for \( d \), conditional on the solution for \( f \), is given by:

\[
d = \frac{\xi_v (1 - \rho_v) - f \frac{T_x}{1 - \gamma} v_t}{1 - \rho_v + fQ}.
\]
8.2.2 Quantitative implications of optimal policy

Despite the fairly large amount of algebra needed to arrive at these solutions, these solutions are rather intuitive with respect to the coefficients which govern optimal policy. Looking at consolidation policy, for a special case, in an economy without steady-state debt, government spending, taxes, or confidence effects such that $\bar{T} = 0$ and $Q = 1$, no matter what the value of $\chi$, the optimal value of $f$ equals $R - 1$, which is exactly the minimal amount of fiscal policy consolidation needed to keep the public debt from exploding. The solution to $h$, meanwhile, is zero. This particular value for $h$ follows from the fact that a government faced with an exogenous shock to its financing costs does not worry about that shock when it does not have any debt. More generally, as debt levels rise, $h$ will be small and increasing in $\rho_\gamma$, which suggests that fiscal authorities will wish to smooth out the effects of these shocks over time. Altogether, these results suggest that fiscal authorities should follow a fiscal rule which allows for a relatively modest degree of consolidation policy—both backward-looking and preemptive—in line with the tax-smoothing results from the optimal policy literature.

The results with respect to stabilization policy are less clear-cut, and these results depend greatly on the persistence of business cycles. When business cycles are highly persistent (in an extreme case, when $\rho_v = 1$), in this stylized setting, optimal policy keeps tax rates constant in response to the business cycle ($d = 0$), which is in line with the tax-smoothing literature, and which is also in line with the approximate acyclicity of tax rates in industrialized countries as estimated by Reicher (2014) for the OECD and Végh and Vuletin (2012) for a larger panel of countries. Adding in a steady-state tax distortion such that $\bar{T} > 0$ complicates things somewhat, but the basic intuition remains unchanged. In this situation, $Q$ falls below one and $d$ takes on a negative value when $\rho_v$ is large. When governments face a budget constraint under a distorted steady state, they should partially accommodate long-lived business cycle shocks (shocks to $v_t$), and they should also engage in preemptive consolidation in order to maintain government solvency.

These extreme results do not hold for a more typical case, in which the positive portions of the numerator and denominator of (52) dominate. For a stylized country with government spending equal to 40% of potential GDP, a debt ratio of 60% of GDP, a labor supply elasticity $\chi$ of 1, a world growth-adjusted real interest rate of 0.02, a steady-state debt ratio of 60% of potential GDP, an annual business cycle persistence coefficient $\rho_v$ of 0.8, an annual solvency shock persistence coefficient $\rho_\gamma$ of 0.5, and no confidence effects such that $\gamma = 0$, these parameters would yield an optimal value of $f$ of 0.067 and an optimal value of $h$ of
0.079. Reducing $\chi$ to 0.5 would reduce the optimal value of $f$ to 0.031 and the optimal value of $h$ to 0.036. Based on these parameter values, a typical case requires a more aggressive consolidation policy than the idealized cases discussed above. Furthermore, the required degree of consolidation policy is roughly in line with past data.

As is the case with consolidation policy, the optimal degree of stabilization policy is also somewhat higher under a more typical, realistic case. For these parameter values, optimal fiscal policy requires tax rates to rise with output. Assuming a labor supply elasticity $\chi$ of 1 under these same parameter values would yield an optimal cyclical coefficient $d$ for tax rates of about 0.30. Reducing $\chi$ to 0.5 would raise the optimal value of $d$ to 0.65. While the data suggest that tax rates are approximately acyclical in most industrialized countries, optimal policy may call for tax rates to be fairly procyclical when the economy is distorted and business cycles are not highly persistent. This finding is at odds with the tax-smoothing literature but is instead more in line with the literature that features a "labor wedge", such as the results of Arseneau and Chugh (2008). Given the importance of labor wedges in the accounting exercise of Chari, Kehoe, and McGrattan (2007), this finding suggests that a stronger degree of anticyclical stabilization policy may possibly be warranted, relative to what is observed in past data.

These results are for a case without a confidence effect. When countries run into the confidence effect at high debt levels, these coefficients change somewhat. A country in this type of situation might have a debt stock equal to 60% and a value of $\gamma$ of 0.004, in the case where $\chi$ equals 1. These coefficients are not well-estimated in the data, and so this exercise could be thought of as a thought experiment. This particular country would wish to consolidate at a rate of $f = 0.084$ and $h = 0.098$, and this country would wish for taxes to respond to output at a rate of $d = 0.24$. If the same country had a debt stock equal to 110% of potential GDP, this country would wish to consolidate at a faster rate of $f = 0.114$ and $h = 0.241$, and this country would wish for taxes to respond to output at a rate of $d = 0.12$. These results suggest that the presence of a confidence effect would imply that fiscal authorities should consolidate more quickly than they otherwise would in response to past debt levels and to solvency shocks, while engaging in less anticyclical policy. These results are in line with the theoretical results of Linnemann and Schabert (2010) and Bi (2012), who provide the same intuition behind why fiscal authorities might wish to consolidate more quickly at high debt levels. Such an increased speed of consolidation has some support in the data as well. In particular, the estimates of Plödt and Reicher (2014) indicate that European countries tend to consolidate at a faster rate at high debt levels than at low debt levels—possibly
even before the Maastricht Treaty—and this result is what would be predicted by a model of optimal fiscal policy in the presence of a confidence effect.

### 8.3 A tax rule as an adaptive fiscal reaction function

While the discussion of optimal fiscal policy has so far focused on tax rules, the optimal tax rule (39) can also be represented as an adaptive fiscal reaction function in the form of (26). This representation makes it possible to talk about the tax rule within the class of more general fiscal reaction functions, and to discuss stability within the context of those fiscal reaction functions. To see this equivalence, substituting the tax rule (39) into the surplus equation (6) gives:

\[
s_t = dv_t + fb_{t-1} + h\gamma_t^* + \bar{T}y_t,
\]  

Substituting the multiplier relationship (8) and then solving for \(s_t\) gives:

\[
s_t = \frac{\bar{T} + \frac{d}{1-d}y_t}{1-d} + \frac{f}{1-d}b_{t-1} + \frac{h}{1-d}\gamma_t^*,
\]

which is in the form given by the fiscal reaction function (26) where \(a = \frac{\bar{T} + \frac{d}{1-d}}{1-d}; c = \frac{f}{1-d};\) and \(l = \frac{h}{1-d}.

In the example of the industrialized country given in the previous section, in the absence of a confidence effect, the optimal coefficient \(a\) of the primary deficit on output would equal 0.67, and the optimal consolidation coefficient \(c\) would equal 0.096. These reactions of the primary surplus to output and to the public debt are somewhat stronger than the estimates of Mendoza and Ostry (2008) for a panel of industrialized countries, Reicher (2014) for the OECD, and Plödt and Reicher (2014) for the euro area. Furthermore, the optimal coefficient \(c\) is again large enough to exactly satisfy the stability condition \(\left| \frac{(1+r)}{(1+g)} (1 + \gamma b) - \frac{c}{1+ma} \right| \leq 1.\)

This result is the same result as those of Benigno and Woodford (2003) and Schmitt-Grohé and Uribe (2004), whereby debt follows a random walk. As with these previous results, countries wish to consolidate only to the extent that consolidation is necessary to maintain solvency, and no more. This result comes with one caveat: in the presence of a confidence effect, this condition no longer holds exactly since the debt level itself has an effect on fiscal solvency; countries will wish to engage in somewhat less anticyclical policy and will wish to consolidate somewhat more quickly than would otherwise be the case.

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Altogether, the results from an optimal fiscal policy exercise are broadly in line with the behavior of real-life fiscal authorities, with one possible exception. These results suggest that for reasonable parameter values, optimal fiscal policy resembles a fiscal reaction function with coefficients on output and on debt that are somewhat higher than those found in the data. Optimal fiscal policy features tax rates which respond neutrally or positively to output, and a primary surplus which responds somewhat more positively to output, with a response of both aggregates to past debt levels strong enough to overcome the "clawback" and "confidence" effects. These are all features of fiscal policy as it appears to be conducted in industrialized countries, although the results on optimal policy suggest that industrialized countries may have room to engage in stronger anticyclical policy than is currently the case. Even so, this latter result appears to be sensitive to the degree to which business cycle fluctuations are persistent as well as to the strength of the confidence effect.

9 Conclusion

While the choice of a short-run fiscal target might seemingly be innocuous, such a choice could conceivably lead, based on a simple model of a small open economy with distortionary taxation, to unstable paths for output and the public debt, and inadvertently, to "active" fiscal policy in equilibrium. The main reason for this is that when a fiscal rule contains output or nominal variables, the distinction between "active" and "passive" fiscal policy becomes an equilibrium construct. For instance, a debt-GDP target such as the "1/20" rule can potentially lead to instability for realistic debt-multiplier combinations, for the simple reason that fiscal policy works at cross purposes with respect to debt and GDP. This result implies, in practical terms, that European countries might face some difficulties in their attempts to implement the Fiscal Compact. There are some fiscal targets which do not have this problem—for instance, a simple debt target which does not take actual GDP into account. As with a debt target, the exact design of a deficit target can result in either instability or stability, with a primary deficit target likely to result in instability and a total deficit target more likely to result in stability. Alternatively, an adaptive fiscal reaction function could allow for automatic stabilizers and consolidation to work in tandem, although it is important to take into account the potential "clawback" effect of automatic stabilizers as well as the "confidence" effect of high debt when choosing the parameters which govern the responses of the primary surplus to output and to past debt levels.
In addition to being easy to analyze, an adaptive fiscal reaction function has several attractive theoretical properties. An optimal policy exercise implies that given some restrictions on the process driving business cycle shocks, fiscal authorities should follow a fiscal reaction function similar to those discussed in the literature. For reasonable parameter values, this reaction function features moderately procyclical tax rates, a strongly procyclical primary surplus, and a weak to moderate response of fiscal policy to the past debt levels and current solvency, which increases in the presence of a confidence effect. Estimates from the empirical literature find that fiscal policy in industrialized countries appears to follow this basic pattern, with the exception that tax rates on average appear to be acyclical in the data. Altogether, the theoretical evidence supports the idea that fiscal policy should be allowed to follow a fiscal reaction function rather than a strict set of debt-GDP targets, and that a reasonably calibrated fiscal reaction function is more likely to ensure "passive" fiscal policy than some of these other targets.

All of these results are based on a simple model which can be derived from optimizing behavior in a small open economy, where distortionary taxes are the only fiscal policy instrument. Despite its simplicity, such a model delivers a surprising amount of intuition as to which types of fiscal policy targets might better stabilize output, tax rates, and the public debt. Nonetheless, using a more complex model might help researchers to better understand in a quantitative sense which types of fiscal rules may have better consequences for welfare while avoiding adverse dynamics in the public debt. For instance, the model of Bohn (1992) features productive public spending; the model of Galí and Monacelli (2008) contains a more developed foreign sector; the models of Linnemann and Schabert (2010) and Bi (2012) endogenize the confidence effect in a nonlinear setting; and the model of McKay and Reis (2013) allows for fiscal policy to affect the distribution of income and consumption across households. Additionally, there are likely to be implementation lags, asymmetric preferences between policymakers and households, or a basic reluctance for fiscal policymakers to adjust tax rates. Nonetheless, despite the simplicity of the three-equation model, that model appears to offer a strong degree of intuition as to which types of fiscal policy regimes are likely to be "passive" in equilibrium, and the model also describes certain aspects of past fiscal policy behavior relatively well.
References


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