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Alessio J. G. Brown, Christian Merkl and Dennis J. Snower

Abstract: This paper sheds new light on the effects of the minimum wage on employment from a two-sided theoretical perspective, in which firms' job offer and workers' job acceptance decisions are disentangled. Minimum wages reduce job offer incentives and increase job acceptance incentives. We show that sufficiently low minimum wages may do no harm to employment, since their job-offer disincentives are countervailed by their job-acceptance incentives.

Keywords: Minimum wage, labor market, employment, unemployment, job offer, job acceptance
JEL codes: J3, J6, J2

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1 Introduction

This paper provides a new theoretical explanation for the following empirical regularities: (i) Minimum wages that are "low" (close to the wage without government intervention, for the relevant labor service) have negligible or even positive employment effects. (ii) Minimum wages that are "high" have negative employment effects.[1] Many theoretical explanations of the employment effects of minimum wages have focused on the demand side of the labor market, with firms' employment decisions playing the central role in determining employment (e.g. the monopsony theory of Manning 2003). Our paper provides an alternative, observationally distinct, model of how minimum wages affect employment, based on a two-sided labor market flow model which makes both firms' job offer and workers' job acceptance decisions explicit. We show analytically that larger wages depress firms' job offer rates, but raise workers' acceptance rates. Under sufficiently low minimum wages, the latter effect may dominate the former.

The paper is organized as follows. In Section 2 we present a dynamic model of two-sided selection in terms of optimizing decisions of firms and workers. Section 3 provides comparative statics on the employment effect of minimum wages and explores the intuition underlying our results. In Section 4 we parametrize the model and present numerical results. Section 5 concludes.

2 The Model

We use the dynamic incentive model by Brown, Merkl and Snower (2014) containing two-sided selection in the labor market. In the context of conventional calibrations, this model fares better than the standard matching model in reproducing the volatilities of major labor market variables.[2] The sequence of decisions in our model may be summarized as follows. First, once a contact between workers and firms has been made, two types of heterogeneous match-specific idiosyncratic shocks are revealed. Firms learn about different suitability of workers, workers learn about the disagreeability of work. Second, firms make their job offer decisions and the households make their job acceptance decisions, based on the realization of the idiosyncratic shocks and anticipating the wage. Because the wage is set after the employment decisions, the match-specific idiosyncratic random shocks are already sunk when the wage is set (for a similar assumption see Pissarides 2009). Thus, wages do not to depend on the idiosyncratic shocks, i.e. we focus on those cases where the exogenously set minimum

[1] These empirical regularities arise from a combination of studies. In countries where minimum wage are low (relative to the median wage), they are often found to have no negative or even positive effects on employment (e.g. Card and Krueger 1994 or Dube et al. 2010). By contrast, a minimum wage may have strong negative effects in countries where it is "high," such as in France (Abowd et al. 2000). Similar results arise for sectoral minimum wages within countries (see e.g. König and Möller 2009 for the effects of the minimum wage in the construction industry in East and West Germany).

[2] To focus on the contribution of this paper we make the following simplifying assumptions: separations are completely exogenous, and we do not consider aggregate uncertainty.

wage is binding and thus, has an effect on labor market outcomes.

We assume that the profit generated by a particular worker at a new match[3] is subject to a match-specific random shock ε_t in period t, which is meant to capture idiosyncratic variations in workers' suitability for the available jobs. The random shock ε_t is positive and *iid* across workers, with a stable probability density function $G_\varepsilon(\varepsilon_t)$. Let the corresponding cumulative distribution be $J_\varepsilon(\varepsilon_t)$.[4] In each period of analysis, a new value of ε_t is realized for each worker. The average productivity of each worker is a, the wage is w[5], the unemployment benefits are b and the hiring cost is h.

The firm maximizes the present value of its expected profit, with a time discount factor δ. The profit generated by an entrant (a newly hired worker), after the random cost term ε_t is observed, is

$$\pi_t^E = a_t - \varepsilon_t - w_t - h + (1-\sigma)\,\delta E_t \pi_{t+1}^I, \tag{1}$$

where the superscript "E" stands for entrant and

$$\pi_{t+1}^I = a_{t+1} - w_{t+1} + \delta E_t (1-\sigma)\,\pi_{t+1}^I, \tag{2}$$

where the superscript "I" stands for an incumbent worker, δ is the time discount factor and σ is the exogenous separation rate.

The firm's "job offer incentive" (its payoff from hiring a worker) is the difference between its gross profit[6] from hiring an entrant worker and its profit from not doing so (namely, zero):

$$\nu_t^E = a_t - \varepsilon_t - w_t - h + (1-\sigma)\,\delta E_t \pi_{t+1}^I. \tag{3}$$

The firm offers this job to a worker whenever that worker generates positive profit: $\varepsilon_t < \nu^E$. Thus, the job offer rate is

$$\eta_t = J_\varepsilon\left(\nu_t^E\right). \tag{4}$$

The worker faces a discrete choice of whether or not to work. Her idiosyncratic disutility of work effort at a given job is e_t, a random variable, which is iid, with a stable probability density function $G_e(e_t)$, known to the worker. The corresponding cumulative distribution is $J_e(e_t)$. The worker's utility is linear in consumption and work effort. She consumes all her income.

[3] The Brown et al. (2013) model views matching and separation as analogous phenomena. The match-specific shocks give rise to both the making and the breaking of employment relationships. Since in line with the focus of this paper separations are exogenous, the match-specific shocks are only relevant at the making of the employment relationship, whereby consequently we will only include them in the first period of a new match.

[4] Specifically the cumulative distribution at the point ν is $J_\varepsilon(\nu) = \int_{-\infty}^{\nu} G_\varepsilon(\varepsilon_t)\, d\varepsilon_t$.

[5] The wage may be determined by bargaining or posting. For our purpose, we do not have to take a stance on the nature of the wage determination mechanism. Instead, we analyze the effects of an exogenous increase of the wage.

[6] This "gross" profit is the expected profit generated by hiring an unemployed worker, without taking the match-specific shock ε_t into account.

The incumbent employed worker's expected present value of utility from working $\Omega_t^N(e_t)$ for a given work effort e is

$$\Omega_t^N = w_t - e_t + \delta E_t \left((1-\sigma) \Omega_{t+1}^N + \sigma \Omega_{t+1}^U \right), \tag{5}$$

where $E_t \left(\Omega_{t+1}^N \right)$ is the expected present value of utility of the following period (before the realized value of the shock e_{t+1} is known):

$$\Omega_{t+1}^N = E_t \left(w_{t+1} + \delta \left((1-\sigma) \Omega_{t+2}^N + \sigma \Omega_{t+2}^U \right) \right). \tag{6}$$

The expected present value utility from unemployment is

$$\Omega_t^U = b + \delta E_t \left(\mu_{t+1} \Omega_{t+1}^N + \left(1 - \mu_{t+1} \right) \Omega_{t+1}^U \right). \tag{7}$$

An unemployed worker's expected "work incentive" ι_t is the expected gross difference[7] between these two utility streams:

$$\iota_t = \Omega_t^N - \Omega_t^U, \tag{8}$$

which is

$$\iota_t = w_t - b + \delta E_t \left(\left(1 - \sigma - \mu_{t+1} \right) \Omega_{t+1}^N - \left(1 - \sigma - \mu_{t+1} \right) \Omega_{t+1}^U \right). \tag{9}$$

Thus the unemployed accepts a job offer when $e_t < \iota_t$. Consequently, the job acceptance rate is

$$\alpha_t = J_e \left(\iota_t \right). \tag{10}$$

The change in employment is the difference between the number of hires and the number of fires. The number of hires depends the job offer probability and the job acceptance probability (contacts are assumed to be made with probability one). Thus the match probability (μ_t) is the product of the job offer probability (η_t) and the job acceptance probability (α_t):

$$\mu_t = \eta_t \alpha_t. \tag{11}$$

The resulting employment dynamics equation is

$$n_t = \mu_t + (1 - \sigma - \mu_t) n_{t-1}. \tag{12}$$

3 Comparative Statics and Intuition

We now proceed to analyze the effect of a minimum wage on the firm's job offer and the worker's acceptance decision and thereby, on employment. The firm's job offer incentive (Eq. 3) and job offer rate (Eq. 4) in the steady state are

$$\nu_t^E = \frac{a - w}{1 - \delta (1 - \sigma)} - h \tag{13}$$

[7] "Gross" means that the utility shock e_t is not taken into account.

and

$$\eta = J_{\varepsilon}\left(\frac{a-w}{1-\delta\left(1-\sigma\right)}-h\right), \tag{14}$$

respectively.

Differentiating with respect to the wage yields

$$\frac{\partial \eta}{\partial w} = -\frac{1}{1-\delta\left(1-\sigma\right)}J_{\varepsilon}'. \tag{15}$$

Thus, higher wages depress the job offer rate. So when a minimum wage is introduced (or rises), firms make job offers only to workers with sufficiently low idiosyncratic costs.

Analogously, the worker's work incentive (Eq. 8) and job acceptance rate (Eq. 10) in the steady state are

$$\iota = \frac{w-b}{1-\delta\left(1-\sigma-\mu\right)}, \tag{16}$$

and

$$\alpha = J_{e}\left(\frac{w-b}{1-\delta\left(1-\sigma-\mu\right)}\right), \tag{17}$$

respectively.

Differentiating the job acceptance rate with respect to wage yields[8]

$$\frac{\partial \alpha}{\partial w} = J_{e}'\frac{\left(1-\delta\left(1-\sigma-\mu\right)\right)-\left(w-b\right)\delta\left(\frac{\partial \eta}{\partial w}\alpha\right)}{\left(1-\delta\left(1-\sigma-\mu\right)\right)^{2}+\left(w-b\right)\delta\eta} \tag{18}$$

This expression is positive: higher wages increase the job acceptance rate.[9] The reason is that workers with a comparatively large idiosyncratic disutility shock, who were previously disinclined to accept work, are now willing to accept it because the higher wage raises the value of work relative to unemployment.

By the matching rate Eq. (11), an increase in the minimum wage accordingly has two countervailing effects, one on the job offer rate ($\frac{\partial \eta}{\partial w} < 0$) and one on the job acceptance rate ($\frac{\partial \alpha}{\partial w} > 0$):

$$\frac{\partial \mu}{\partial w} = \frac{\partial \eta}{\partial w}\alpha + \eta\frac{\partial \alpha}{\partial w}. \tag{19}$$

Which of these two effects dominate is an empirical issue.[10]

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[8] See Appendix 6.1 for details.

[9] The reason is that the denominator is positive, as well as the numerator (since $\frac{\partial \eta}{\partial w} < 0$).

[10] Disentangling the household and firm decisions on the separation margin will imply analoguous effects of firing and quits, namely that a binding mimimum wage will increase firings but decrease quits with ambuguous results. We do not focus on this analoguous mechanism here, since our aim is to establish this mechanism, further quantitative investigations are left for future research.

4 Parametrization and Numerical Analysis

We now show that for minimum wages that are sufficiently low, their expansionary effect on the job acceptance rate may dominate their contractionary effect on the job offer rate. However, for minimum wages that are sufficiently high, the contractionary effect on the job offer rate dominates.

To address this issue, we begin with the following parametrization. For choosing steady state targets for the low wage sector, we use Blau and Robins' (1990) evidence for average offers per contact and acceptances per contact and per offer.[11] Accordingly, we set the match probability μ, which is the probability that a unemployed worker finds a new job within one period, to 12%, the job offer rate to 17%, and equation 11 then yields a job acceptance rate of 71%.[12] The unemployment rate $u = 1 - n$ is set to 8.96% (as in Cairo and Cajner 2011). According to the employment dynamics equation, we obtain an exogenous separation rate of 1.2%.

Next, with reference to the empirical literature, we consider a plausible range of of labor demand elasticities [-1,-0,25][13] and labor supply elasticities [0.1, 0.6][14]. In the context of our model, we use the steady state employment equation ($n = \frac{\mu}{\mu+\sigma}$) to calculate the labor demand and labor supply elasticities, by holding the household-side and firm-side employment activities constant, respectively.[15]

	Labor Demand Elasticity			
Labor Supply Elasticity	**-0.25**	**-0.5**	**-0.75**	**-1**
0.6	14.3%	7.1%	-	-
0.5	14.3%	7.1%	-	-
0.4	14.3%	7.1%	-	-
0.3	14.3%	-	-	-
0.2	14.3%	-	-	-
0.1	-	-	-	-

Table 1: Maximum wage increase without job losses under different labor supply and labor demand elasticities.

Table 1 shows the largest minimum wage that does not reduce employment, for different combinations of the labor supply and labor demand elasticities. Under the lowest labor demand elasticity (-0.25), for most labor supply elasticities wage increases of up to 14.3% above the wage without government intervention are possible without job losses, i.e. with positive employment effects. The

[11] The data used by the authors is from the Employment Opportunity Pilot Projects (EOPP) baseline household survey. We use the values for unemployed workers.

[12] The value by Blau and Robins is 67%,

[13] See e.g. Falk and Köbel (2001) or Slaughter (2001).

[14] See Bargain et al. (2011) and (2012). The latter publication also highlights higher labor supply responses in low-income groups, which generally are those affected by minimum wages.

[15] See Appendix 6.2 for details.

number shrinks to 7.1% for a labor demand elasticity that is twice as large. Furthermore, observe that a minimum wage without job losses is only possible for a smaller range of supply elasticities.

Intuitively, a larger labor demand elasticity leads to a quantitatively stronger reaction of the job offer rate. When the job offer reaction is sufficiently large (i.e. for demand elasticities of -0.75 and -1), the job acceptance effect cannot compensate for this (under conventional labor supply elasticities). For lower labor demand elasticities, the job acceptance effect is dominant for small minimum wage increases. But after some moderate increase of the minimum wage, the job acceptance rate (which is calibrated to 71%) reaches its upper bound of 100%. Thus, the job acceptance effect is no longer at work and the job offer effect starts dominating.[16] In other words, the labor supply elasticity does not matter any more, because further increases of the job acceptance rate (due to the minimum wage) are impossible. Note, however, that the quantitative response is different for wage increases below the threshold. With a labor demand elasticity of -0.25, a wage increase of 5% leads, for example, to an employment increase of 1.8%, 1.5% and 1.0% with a labor supply elastsicity of 0.6, 0.5 and 0.4 respectively.

While a more detailed empirical investigation is required in the future, our analysis shows that minimum wages increases up to 14% are conceivable without job losses. This is is a similar magnitude to the minimum wage increases analyzed in Card and Krueger (1994).

5 Conclusion

We present a new channel for the analysis of minimum wages. Our model, which disentangles household and firm decisions, complements the existing literature by outlining a mechanism that is absent in standard search and matching models. We show analytically that larger wages depress firms' job offer rates, but raise workers' acceptance rates. Under moderate minimum wages, the latter effect may dominate the former. Obviously, there are other channels that prevent negative effects of a moderate minimum wage (e.g. monopsony power). However, our numerical analysis illustrates that our job acceptance effect alone is quantitatively meaningful. Thus, it is certainly of interest for future research to combine different theoretical effects and to disentangle the job offer and job acceptance effects in labor market flow data.

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[16] Interestingly, for a given labor demand elasticity, the largest mimimum wage that does not reduce employment is the same for several different labor supply elasticities. At the threshold, the job acceptance rate has hit its upper bound of 100% in all cases.

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6 Appendix: Analytical Derivations

6.1 Differentiation of the Job Acceptance Rate with Respect to the Wage

Derivation of Equation 18:

Differentiating the employment incentive with respect to the wage yields

$$\frac{\partial \iota}{\partial w} = \frac{\left(1-\delta\left(1-\sigma-\mu\right)\right)-\left(w-b\right)\delta\frac{\partial\mu}{\partial w}}{\left(1-\delta\left(1-\sigma-\mu\right)\right)^2}, \tag{20}$$

given that

$$\frac{\partial \mu}{\partial w} = \frac{\partial \eta}{\partial w}\alpha + \eta\frac{\partial \alpha}{\partial w},$$

this yields

$$\frac{\partial \iota}{\partial w} = \frac{\left(1-\delta\left(1-\sigma-\mu\right)\right)-\left(w-b\right)\delta\left(\frac{\partial\eta}{\partial w}\alpha+\eta\frac{\partial\alpha}{\partial w}\right)}{\left(1-\delta\left(1-\sigma-\mu\right)\right)^2}. \tag{21}$$

Thus,

$$\frac{\partial \alpha}{\partial w} = J_e'\frac{\partial \iota}{\partial w} = J_e'\frac{\left(1-\delta\left(1-\sigma-\mu\right)\right)-\left(w-b\right)\delta\left(\frac{\partial\eta}{\partial w}\alpha+\eta\frac{\partial\alpha}{\partial w}\right)}{\left(1-\delta\left(1-\sigma-\mu\right)\right)^2} \tag{22}$$

$$\frac{\partial \alpha}{\partial w}\left(1+\frac{\left(w-b\right)\delta\eta}{\left(1-\delta\left(1-\sigma-\mu\right)\right)^2}\right) = J_e'\frac{\left(1-\delta\left(1-\sigma-\mu\right)\right)-\left(w-b\right)\delta\left(\frac{\partial\eta}{\partial w}\alpha\right)}{\left(1-\delta\left(1-\sigma-\mu\right)\right)^2} \tag{23}$$

$$\begin{aligned}\frac{\partial \alpha}{\partial w} &= J_e'\frac{\frac{\left(1-\delta\left(1-\sigma-\mu\right)\right)-\left(w-b\right)\delta\left(\frac{\partial\eta}{\partial w}\alpha\right)}{\left(1-\delta\left(1-\sigma-\mu\right)\right)^2}}{1+\frac{\left(w-b\right)\delta\eta}{\left(1-\delta\left(1-\sigma-\mu\right)\right)^2}} && (24)\\ &= J_e'\frac{\left(1-\delta\left(1-\sigma-\mu\right)\right)-\left(w-b\right)\delta\left(\frac{\partial\eta}{\partial w}\alpha\right)}{\left(1-\delta\left(1-\sigma-\mu\right)\right)^2+\left(w-b\right)\delta\eta} && (25)\end{aligned}$$

6.2 Calculation of Elasticities in the Parametrization and Numerical Analysis

$$
\begin{aligned}
\frac{\partial n}{\partial w}\frac{w}{n} &= \frac{\partial \frac{\mu}{\mu+\sigma}}{\partial w}\frac{w}{n} = \frac{\partial \frac{\eta\alpha}{\alpha\eta+\sigma}}{\partial w}\frac{w}{n} \\
&= \frac{\left(\frac{\partial\eta}{\partial w}\alpha + \frac{\partial\alpha}{\partial w}\eta\right)(\mu+\sigma) - \mu\left(\frac{\partial\eta}{\partial w}\alpha + \frac{\partial\alpha}{\partial w}\eta\right)}{(\mu+\sigma)^2}\frac{w}{n} \\
&= \frac{\left(\frac{\partial\eta}{\partial w}\alpha + \frac{\partial\alpha}{\partial w}\eta\right)\sigma}{(\mu+\sigma)^2}\frac{w}{n}.
\end{aligned}
$$

For e.g. deriving the labor demand elasticity, we keep the household side constant, i.e. $\frac{\partial\alpha}{\partial w} = 0$. Thus:

$$
\begin{aligned}
\frac{\partial n}{\partial w}\frac{w}{n} &= \frac{\frac{\partial\eta}{\partial w}\alpha\sigma}{(\mu+\sigma)^2}\frac{w}{n} \\
&= \frac{\frac{\partial\eta}{\partial w}\alpha\sigma}{(\mu+\sigma)}\frac{w}{\mu} \\
&= \frac{\frac{\partial\eta}{\partial w}\sigma}{(\mu+\sigma)}\frac{w}{\eta} \\
&= \frac{\partial\eta}{\partial w}\frac{w}{\eta}u.
\end{aligned}
$$