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JEL classification: E24, E32, J63, J64

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An Incentive Theory of Matching*

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Abstract

This paper presents a theory of the labor market matching process in terms of incentive-based, two-sided search among heterogeneous agents. The matching process is decomposed into its two component stages: the contact stage, in which job searchers make contact with employers and the selection stage, in which they decide whether to match. We construct a theoretical model explaining two-sided selection through microeconomic incentives. Firms face adjustment costs in responding to heterogeneous variations in the characteristics of workers and jobs. Matches and separations are described through firms' job offer and firing decisions and workers' job acceptance and quit decisions. Our calibrated model for the U.S. can account for important empirical regularities, such as the large volatilities of labor market variables, that the conventional matching model cannot.

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1 Introduction

The mainstream literature on labor market search views the number of unemployed job searchers and vacancies as inputs into a matching process, whose outcome is the number of

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hired workers. The matching function, meant as a summary description of this matching process, is assumed to be stable (e.g. Pissarides, 2000). This paper takes a fresh look at the matching process by analyzing it explicitly in terms of its two component stages: (i) the *contact stage*, in which job searchers make contact with employers who have vacancies and (ii) the *selection stage*, in which both potential employers and job searchers gain some information about one another and decide whether to match. We will show that while some contributions to the search literature do acknowledge these two stages as separate decision making processes, the full implications of this distinction for labor market dynamics have thus far not been worked out. We address this issue by constructing a theoretical model of two-sided selection among heterogeneous firms and workers, calibrating this model for the U.S. economy, and showing that it can account for empirical regularities (such as the Shimer puzzle, i.e., the inability of the standard matching model to generate sufficiently large volatilities of the job-finding rate and the unemployment) that have eluded the conventional matching models.

The contact and selection stages are distinct in practice. In the contact stage, the job searchers and potential employers have relatively little information about one another,[1] so that workers and vacancies each appear relatively homogeneous (as assumed in conventional matching functions). At this stage, workers and firms are engaged in a process of "outreach," i.e. reaching out to people who were hitherto unknown. In the selection stage, the two parties exchange enough information about one another to permit them to decide whether to consumate the match. On the basis of this additional information, workers and vacancies appear as more heterogenous. At this stage, workers and firms are in the process of assessing the "match suitability." The labor market frictions relevant to the contact stage are search costs; the frictions relevant to the selection stage are hiring costs for the firm and job acceptance costs for the worker.[2] The outcome of the contact stage is an interview; the outcome of the selection stage is a hire or a rejection. A job searcher who makes contact with a potential employer becomes an applicant; an applicant who is selected becomes an entrant to the firm's workforce.

The search literature thus far has either ignored the distinction between contact and selection, or distinguished between them only in a very rudimentary way. Specifically, the matching function has been interpreted in two ways. In the first, traditional interpretation (e.g. Pissarides, 2000, chapter 1), the matching function describes the outcome of both

[1] The information is limited to what can be gleaned from the vacancy ads, CVs, and other information available before the job interview.

[2] For analytical simplicity and calibration tractability, the latter costs are not considered in the model below.

contact and selection, explaining how a given job searchers and vacancies lead to new hires.[3] We call this the "encompassing matching function," since it encompasses both the contact and selection stages.

In the second interpretation, the matching function covers only the contact stage, and thus we call it the "contact function."[4] A recent generation of matching models, where workers and firms are first matched through a contact function and then decide whether to continue or to sever the contact in response to productivity perturbations, can be interpreted in this vein.[5] In these "productivity perturbation models," however, the distinction between contact and selection is unsatisfactory for two reasons. First, when these models are calibrated, the calibration relates unemployment to new hires, not to contacts (such as interviews).[6] Second, these models invariably assume that the proportion of interviews that do not lead to hiring is equal to the proportion of currently employed workers who separate from their jobs. But in practice interviews fail far more frequently than existing employment relationships, and the job finding rate is much smaller than the retention rate.[7] This indicates that we need to distinguish between the breaking of contacts and the breaking of selection (i.e. deselection of employees).

Our analysis addresses these difficulties and has the following features. First, it distinguishes sharply between contact and selection. The contact process is described by a contact function, whereas the selection process is the outcome of two-sided search among heterogeneous agents. Second, selection is modeled analogously to deselection, i.e. the breaking of existing employment relationships. Since the selection and deselection of employees in our analysis is derived from incentives facing firms and workers, we call our approach an "incentive theory of matching." Third, the match-specific shocks that give rise to selection and deselection are not just productivity perturbations, but shocks to both firms' profitability

[3] Pissarides (2000, p. 3-4) claims that the matching function summarizes "heterogeneities, frictions and information imperfections" and represents "the implications of the costly trading process without the need to make the heterogeneities and the other features that give rise to it explicit." This accords with many other explanations of the matching function, such as that of Petrongolo and Pissarides (2001, p. 390): "The attraction of the matching function is that it enables the modeling of frictions in otherwise conventional models, with a minimum of added complexity. Frictions derive from information imperfections about potential trading partners, heterogeneities..."

[4] Models that offer microfoundations of the matching process fall into this category, since they investigate the probability that randomly searching, homogeneous workers and homogeneous firms (or, more generally, buyers and sellers) find one another (Burdett, Shi and Wright, 2001, Montgomery, 1991, and others). In the contact stage, after all, the specific characteristics of the matched partners are not known and thus these partners can be considered homogeneous.

[5] See, for example, the models of Mortensen and Pissarides (1994), den Haan, Ramey and Watson (2000).

[6] Calibrating with respect to contacts would require vast new data sets on formal and informal meetings between searching workers and searching employers and these data sets are not currently available.

[7] In the U.S., the average monthly job-finding rate is 0.45) and the average retention rate is around 0.97, see e.g. Shimer (2005).

and workers' disutility of work. Thus the matching rate in our model is not the same as the job offer rate (as in conventional search models), but depends on both the firms' job offer rate and the workers' job acceptance rate. Fourth, the making and breaking of matches in our model are influenced by hiring and firing costs.[8] These costs drive a wedge between the job-finding and the retention rate, so that the proportion of contacts that lead to new hires is less than the proportion of incumbent workers that are retained.

In the context of a simple incentive model, we show that the encompassing matching function is unable to replicate the behavior of our incentive theory. Next, we calibrate an extended incentive model for the U.S. economy and show that it can account for some important empirical regularities that the conventional matching model cannot. First, our model generates labor market volatilities that are close to what can be found in the empirical data, specifically for the unemployment rate, the job finding rate and the separation rate. This is remarkable, as we do not rely on any form of real wage rigidity. The standard calibration of the conventional matching model[9] (with exogenous or endogenous separations) is unable to generate these high volatilities of labor market variables (see Shimer, 2005). Second, our model can account for a negative correlation between job creation and job destruction. And third, our model generates a strong negative correlation between vacancies and unemployment (i.e., the Beveridge curve correlation). The standard calibrations of the matching model, with endogenous job destruction (see Krause and Lubik, 2007), cannot account for these last two stylized facts.[10]

Intuitively, the reason our model is more successful than the conventional matching model at replicating the stylized facts above is that macroeconomic shocks are propagated differently. In the conventional matching models, the employment effect of a change in aggregate productivity depends on the change in new hires generated by the matching function, and this matching function exhibits diminishing returns (i.e. a declining marginal product of matches with respect to unemployment and vacancies). In our incentive model, the adjustments are made on a different margin. Since the agents in our model face heterogeneous

[8] The hiring costs are not to be confused with vacancy posting costs, since the vacancy posting costs are incurred before the contact is made, whereas the hiring costs are incurred after the contact.

[9] The "standard" calibration of the model excludes rigid wages and small surplus calibrations. Although the rigid wage version of the search and matching model can also generate higher volatilities (Hall, 2005), it implies that counterfactual prediction that wages are acyclical. Thus we do not make this assumption here. We also do not rely on Hagedorn and Manvoskii's (2008) small surplus calibration, in which the average unemployed worker is basically indifferent between working and not working. In the calibrated version of our model, the current period's utility of an average unemployed is only about 60% of the utility of an employed.

[10] The search and matching model with exogenous job destruction actually has a strong negative correlation between the job finding rate and the unemployment (see Shimer, 2005). However, there is an intensive debate in the literature whether separations are exogenous or not (see, for example, Hall, 2006, and Fujita and Ramey, 2009, for opposing views). Separations are endogenous in our analysis.

match-specific shocks, a change in aggregate productivity affects the range of match-specific shocks over which firms are willing to make job offers and workers are willing to accept these offers. Since aggregate productive shocks are autocorrelated, they can have a substantial leverage effect on the expected present value of profit generated by newly hired workers and incumbent workers, and thereby a strong effect on the hiring and separation thresholds. In short, whereas an aggregate productivity shock affects employment via the matching function in the conventional matching models, it affects employment via the mass of the distribution of match-specific shocks at which job-offer decisions and job-acceptance decisions are made. This explains why our incentive model is more successful than the conventional matching model in generating the observed high volatilities of the unemployment rate, the job finding rate. The other stylized facts can be understood intuitively along the same lines.

Finally, we take a first step towards examining the relative importance of the contact and selection stages of matching in accounting for the stylized facts above. We show that the greater the role played by a conventional contact function in determining matches, the less the calibrated model is able to replicate the above stylized facts. We take this to be preliminary, indirect evidence that the selection process (rather than the contact process) must play a major role in generating the observed labor market dynamics. Obviously, more empirical evidence is required to shed light on this issue. We expect the question of "outreach" versus "match suitability" to be an important issue for future research.

The rest of the paper is organized as follows. In Section 2 we present a simple model two-sided selection in terms of optimizing decisions. Section 3 shows that this simple incentive model cannot be replicated by a conventional search model with an encompassing matching function. Section 4 presents an extended incentive model, which is calibrated in Section 5. Section 6 presents the numerical results and Section 7 concludes.

2 A Simple Incentive Model

To set the stage, we begin by constructing a particularly simple model of the incentive theory of matching, based on heterogeneous match-specific shocks. We make the standard assumption that unemployed workers search for jobs, while employed workers do not. Our model has the following sequence of labor market decisions. First, vacancies are posted. Second, the realized values of the match-specific shocks are revealed. Third, the firms make their hiring decisions and the households make their job acceptance decisions. To keep our analysis particularly simple, we assume that every searcher makes contact with a vacancy each period.

This assumption is equivalent to a trivial contact function: $C_t = U_{t-1}$, where C_t is the number of contacts (interviews) made in period t and U_{t-1} is the number of unemployed job searchers in the previous period. In addition, we assume in this section that the real wage w is exogenously given.[11] Finally, to provide a maximally transparent comparison of our incentive model and the standard matching model, we assume that workers and firms are myopic (i.e. their rates of time discount are 100%).[12]

2.1 The Firm's Behavior

We assume that the profit generated by a particular worker at a particular job is subject to a match-specific random shock ε_t in period t, which is meant to capture idiosyncratic variations in workers' suitability for the available jobs.[13] For example, workers in a particular skill group and sector may exhibit heterogeneous profitabilities due to random variations in their state of health, levels of concentration, and mobility costs, or to random variations in firms' operating costs, screening, training, and monitoring costs, and so on. The random shock ε_t is positive and iid across workers, with a stable probability density function $G_\varepsilon(\varepsilon_t)$, known to the firm.[14] Let the corresponding cumulative distribution be $J_\varepsilon(\varepsilon_t)$[15].

In each period of analysis a new value of ε_t is realized for each worker.

The average productivity of each worker is a, a positive constant. The hiring cost h per worker is also a constant. The hiring cost includes the administrative costs, screening costs, retraining costs, and relocation costs, as well as the basic instruction, mentoring and on-the-job training costs that are required to integrate the worker in the firm's workforce. The profit generated by an entrant (a newly hired worker) is

$$\pi_t^E = a - \varepsilon_t - w - h, \tag{1}$$

where the superscript "E" stands for "entrant" and w is the real wage.

The firm's "job offer incentive" (its payoff from hiring a worker) is the difference between its gross profit[16] from hiring an entrant worker $(a - w - h)$ and its profit from not doing (namely, zero):

$$\nu^E = a - w - h. \tag{2}$$

[11] This assumption is relaxed in the next section, where wages are determined through bargaining.

[12] This assumption is also relaxed in the next section.

[13] Since each worker draws from the same distribution of random shocks, ε_{it}, we omit the subscript i for notational simplicity.

[14] Our analysis can of course be extended straightforwardly to shocks with AR and MA components.

[15] Specifically the cumulative distribution at the point ν is $J_\varepsilon(\nu) = \int_{-\infty}^{\nu} G_\varepsilon(\varepsilon_t)\, d\varepsilon_t$.

[16] This "gross" profit is the expected profit generated by hiring an unemployed worker, without taking the match-specific shock ε_t into account.

The firm offers this job to a worker whenever that worker generates positive profit: $\varepsilon_t < \nu^E$. Thus, the job offer rate is

$$\eta = J_\varepsilon \left(\nu^E\right). \tag{3}$$

The firm's "retention incentive" (its payoff from retaining a worker) is the difference between its gross profit from retaining a worker is $(a - w)$ and the (negative) profit from firing that worker:

$$\nu^I = a - w + f, \tag{4}$$

where the superscript "I" stands for the incumbent employee who has been retained, and f is the firing cost per worker, assumed constant. The firm with a filled job will fire an incumbent worker whenever she generates negative profit: $\varepsilon_t > \nu^I$. Thus the firing rate is:

$$\phi = 1 - J_\varepsilon \left(\nu^I\right). \tag{5}$$

Note that due to the hiring and firing costs, the retention incentive exceeds the job offer incentive ($\nu^I > \nu^E$) and thus the retention rate exceeds the job offer rate ($(1 - \phi) > \eta$).

2.2 The Worker's Behavior

The worker faces a discrete choice of whether or not to work. If she works, her disutility of work effort is e_t, a random variable, which is iid, with a stable probability density function $G_e(e_t)$, known to the worker. The corresponding cumulative distribution is $J_e(e_t)$. The random variable captures match-specific heterogeneities in the disagreeability of work, due to such factors as idiosyncratic reactions to particular workplaces or variations in the qualities of these workplaces. The worker's utility is linear in consumption and work effort. She consumes all her income. If she is unemployed, her utility is $\Omega^U = b$, a constant. If she is employed, her utility is $\Omega_t^N = w - e_t$.[17]

A worker's "work incentive" (her payoff from choosing to work) is the difference between her gross utility[18] from working (w) and her utility from not working (b):

$$\iota = (w - b). \tag{6}$$

Assuming that $w > b$ and letting $E(e_t) = 0$, all unemployed workers have an ex ante incentive

[17] Observe that on the firm's side, we distinguish between entrants (E) and incumbent workers (I); whereas on the workers' side, we distinguish between employed (N) and unemployed (U) workers. The rationale for these two distinctions is that the firm can hire two types of workers (entrants and incumbents), whereas the worker can be in two states (employment and unemployment).

[18] This "gross" utility is the expected utility generated by employment, without taking the match-specific shock e into account.

to seek work.

An unemployed worker will accept a job offer whenever $e_t < \iota$. This means that the job acceptance rate is

$$\alpha = J_e(\iota). \tag{7}$$

Along the same lines, an employed worker will decide to quit when $e_t > \iota$. This means that the quit rate is

$$\chi = 1 - J_e(\iota). \tag{8}$$

Note that, for simplicity, we have assumed that the job acceptance rate is identical to the job retention rate ($\alpha = 1 - \chi$). When unemployed workers face costs of adjusting to employment (e.g. buying a car to get to work, or psychic costs of changing one's daily routine) or when employed workers face costs of adjusting to unemployment (e.g. building networks of friends with potential job contacts, psychic costs of adjusting to joblessness), then the job acceptance rate would fall short of the job retention rate.[19]

2.3 Match and Separation Probabilities

An unemployed worker gets a job when two conditions are fulfilled: (i) she receives a job offer *and* (ii) she accepts that offer. Thus the *match probability* (μ) is the product of the job offer rate (η) and the job acceptance rate (α):

$$\mu = \eta\alpha. \tag{9}$$

An employee separates from her job when at least one of two conditions is satisfied: (i) she is fired *or* (ii) she quits. Thus the *separation probability* is

$$\sigma = \phi + \chi - \phi\chi. \tag{10}$$

[19] Specifically, for example, the unemployed worker's job acceptance incentive could be expressed as $\iota^U = w - b - \xi^U$, where ξ^U is the cost of adjusting to employment, and the incumbent worker's job retention incentive could be expressed as $\iota^N = w - b + \xi^N$, where ξ^N is the cost of adjusting to unemployment. Then the job acceptance rate becomes $\alpha = C_e\left(\iota^U\right)$, the job retention rate becomes $C_e\left(\iota^N\right)$ so that the quit rate becomes $\chi = 1 - C_e\left(\iota^N\right)$.

2.4 Vacancies

Vacancies are posted before ε_t is realized. As in the conventional search literature, we assume free entry of firms, so that the number of vacancies is determined by a zero-profit condition.[20] Let V be the number of vacancies posted, κ be the cost of posting a vacancy, and U_{t-1} be the number of unemployed in the previous period. If $V_t \geq U_{t-1}$, then the probability that a vacancy is filled is $(U_{t-1}/V_t)\,\mu_t$, i.e. the probability of a contact times the probability that the contact leads to a match. The expected profit per match is $\left(a - w - h - E_t\left(\varepsilon_t|\varepsilon_t < \nu^E\right)\right)$

Thus the zero-profit condition for posting vacancies is

$$\mu\left(U_{t-1}/V_t\right)\left(a - w - h - E_t\left(\varepsilon_t|\varepsilon_t < \nu^E\right)\right) - \kappa = 0, \tag{11}$$

Thus the equilibrium number of vacancies is

$$V_t^* = \left(a - w - h - E_t\left(\varepsilon_t|\varepsilon_t < \nu^E\right)\right)\frac{\mu U_{t-1}}{\kappa} \tag{12}$$

If $V_t < U_{t-1}$, then all vacancies are filled and if the expected profit from posting a vacancy is $E_t(\pi_t) = \mu\left(a - w - h - E_t\left(\varepsilon_t|\varepsilon_t < \nu^E\right)\right) - \kappa > 0$, then the firm continues to post vacancies until $V^* = U_{t-1}$ (for which eq (12) applies).

2.5 The Labor Market Equilibrium

Given that all unemployed people are job searchers and assuming that each job searcher makes one contact per period, the number of unemployed workers who get jobs in period t is μU_{t-1}, where U_{t-1} is the number of unemployed in the previous period.[21] The number of employed people who separate from their jobs in period t is σN_{t-1}, where N_{t-1} is the number of employed in the previous period.

The change in employment is $\Delta N_t = N_t - N_{t-1} = \mu U_{t-1} - \sigma N_{t-1}$. Let L be the labor force, while $n = N/L$ and $u = U/L$ be rates of employment and unemployment, respectively, so that $u_t = 1 - n_t$. Normalizing the labor force to unity, equilibrium employment n_t may be described by the following employment equation:

$$n_t = \mu + (1 - \mu - \sigma)\, n_{t-1}, \tag{13}$$

where the degree of employment persistence, measured by the parameter $(1 - \mu - \sigma)$, de-

[20] Note that we do not have to specify the number of firms, as they face constant returns (there is only an ex-post heterogeneity, once there are particular worker-firm pairs).

[21] All other variables (without subscripts) refer to the current period.

pends inversely on the matching rate and separation rate.[22]

Substituting $u_t = 1 - n_t$ into the vacancy equation (12), we obtain the equilibrium number of vacanices. Observe that vacancies play no allocative role in our model: the number of vacancies has no effect on employment and unemployment. The reason is that vacancies do not influence the number of contacts made by a given number of unemployed job searchers, since we have assumed that the number of contacts is equal to the number of job searchers. In this context, vacancies are simply an "attention-seeking device:" the greater the number of vacancies that a firm posts for a given job, the greater the number of job applicants it attracts relative to other firms. The greater is the aggregate number of vacancies, the lower is the probability that they will be filled by a given number of job searchers and in the labor market equilibrium, the aggregate number of vacancies has no effect on aggregate employment.

3 The Encompassing Matching-Function as Description of Contact and Selection

Assuming the incentive model above to be the "true" description of the labor market, we ask whether it is possible to specify a corresponding model with a stable encompassing matching function – to be called the "encompassing matching model" – so as to replicate the behavior of the incentive model, for any values of the model's macroeconomic and policy parameters. We show that, in general, this is not possible. For such replication to occur, the encompassing matching function generally needs to shift in response to macroeconomic and policy changes.[23]

[22] An alternative interpretation of the persistence parameter is given by $1 - \sigma - \mu = (1 - \phi)(1 - \chi) - c\eta\alpha$, where $(1 - \phi)(1 - \chi)$, the product of the incumbents' retention rate and staying rate, is the incumbents' survival rate. Thus the persistence parameter is the difference between the incumbents' survival rate and the unemployed workers' match probability.

[23] Various authors (e.g. Lagos, 2000) have noted that policies which affect labor market heterogeneities (e.g. retraining programs), frictions (e.g. job counselling) and information imperfections (e.g. job exchanges) may naturally be expected to influence the matching function. In short, there is no reason to believe that the matching function is invariant with respect to labor market policies that are designed to improve the matching process. Our analysis shows, however, that the Lucas critique extends well beyond such policies. Several empirical studies indicate instabilities of the matching function. Often a negative time trend is found when estimating the search and matching function, thus casting doubt on the stability through time (Blanchard and Diamond, 1989, for the United States, and Fahr and Sunde, 2001, 2004, for Germany).

3.1 The Encompassing Matching Model

We now specify an encompassing matching model, which is the counterpart to the incentive model above, with an encompassing matching function. Let the matching function be

$$C_t = C_t\left(U_{t-1}, V_t\right). \tag{14}$$

This function satisfies the standard conditions: $C_i' > 0$, $C_{ii}' < 0$, $i = U, V$; $C_t\left(U_{t-1}, 0\right) = C_t\left(0, V_t\right) = 0$; and there are constant returns to scale: $\Theta C_t\left(U_{t-1}, V_t\right) = C_t\left(\Theta U_{t-1}, \Theta V_t\right)$ where Θ is any positive constant.

Let $\theta_t = V_t/U_{t-1}$ denote labor market tightness, so that $q_t\left(\theta_t\right) = C_t\left(U_{t-1}/V_t, 1\right)$ is the probability that a job is matched with a worker, and $\theta q\left(\theta\right)$ is the probability that a worker is matched by a job. Along the lines of the simple labor market matching models, we assume that jobs are destroyed at an exogenous rate λ, $0 < \lambda < 1$. Then the change in the employment rate is[24] $\Delta n_t = \theta q\left(\theta\right)\left(1 - n_{t-1}\right) - \lambda n_{t-1}$, implying the following employment dynamics equation:

$$n_t = \theta q\left(\theta_t\right) + \left(1 - \theta q\left(\theta\right) - \lambda\right) n_{t-1}. \tag{15}$$

Vacancies are posted until the expected profit is reduced to zero: $a - w = \frac{\kappa}{q_t(\theta_t)}$, where κ is a vacancy posting cost, $\kappa/q_t\left(\theta_t\right)$ is the expected vacancy posting cost per worker. Expressing this zero-profit condition in terms of labor market tightness:

$$\theta = g\left(\frac{\kappa}{a - w}\right), \tag{16}$$

where $g = q^{-1}$.

The equilibrium employment rate n is obtained by substituting the zero-profit condition (16) into the employment dynamics equation (15).

3.2 Equivalence Condition

In order for the two models to be comparable, let the exogenous wage w be identical in both models and suppose that the separation rate σ in the incentive model is a constant equal to the job destruction rate λ in the conventional matching model. Then the two models are observationally equivalent when $\theta q\left(\theta\right) + \left(1 - \theta q\left(\theta\right) - \sigma\right) n_{t-1} = \mu + \left(1 - \mu - \sigma\right) n_{t-1}$, so that

$$\theta q\left(\theta\right) = \mu, \tag{17}$$

[24] To keep this model comparable with our the simple incentive model above, we assume (without loss of generality) the same timing in both models. Matches are not destroyed in the match period and they become immediately productive.

which we call the "equivalence condition." It may be expressed as

$$\frac{\kappa}{a-w} g\left(\frac{\kappa}{a-w}\right) = J_{\varepsilon}\left(a-w-h\right) J_{e}\left(w-b\right). \tag{18}$$

For given cumulative distributions J_{ε} and J_e, can a matching function g be found, so that the condition (18) holds for any values of the parameters which the incentive model and the conventional matching model have in common? To answer this question, we differentiate condition (18) with respect to productivity a,

$$-\frac{\kappa}{\left(a-w\right)^2}\left[\frac{\kappa}{a-w} g' + g\right] = J_{\varepsilon}' J_e. \tag{19}$$

Differentiating with respect to w,

$$\frac{\kappa}{\left(a-w\right)^2}\left[\frac{\kappa}{a-w} g' + g\right] = -J_{\varepsilon}' J_e + J_{\varepsilon} J_e'. \tag{20}$$

Differentiating with respect to κ

$$\frac{\kappa}{a-w} g'\left(\frac{\kappa}{a-w}\right) + g\left(\frac{\kappa}{a-w}\right) = 0 \tag{21}$$

Conditions (19) and (20) are mutually exclusive, unless the slope of the cumulative distribution of e is $J_e' = 0$. The latter implies that the underlying density is $G_e = 0$. This occurs when there exist no households with a marginal disutility of effort e over the relevant range. (It is on this account that the number of households that accept jobs is not affected by $(w-b)$.) A distribution of e with zero mass is indeed a special case; it amounts to excluding the possibility of heterogeneous workers in our model.

Conditions (19) and (21) are mutually exclusive, unless the slope of the cumulative distribution of ε_t is $J_{\varepsilon}' = 0$, which implies that the underlying density is $G_{\varepsilon} = 0$. This occurs when there exist no jobs with workplace heterogeneities ε_t over the relevant range, i.e. the possibility of heterogeneous profitabilities is excluded.

In short, there exists no functional form for g such that condition (18) always holds - for any given cumulative distributions J_{ε} and J_e, and for any values of the parameters common to the two models - unless heterogeneities on the firm and households side are absent. Thus the encompassing matching model is not observationally equivalent to the incentive model with heterogenous firms and households. If the incentive model above is assumed to be the "true" model of the labor market, then the standard matching model can reproduce the "true" employment effects of variations in all the relevant parameters – the wage w, productivity a, the hiring cost h, or the leisure utility b – *only* if we assume that the matching function is modified whenever these parameters are changed.

It is clear that this non-equivalence also applies to more complicated models (such as the one in the next section), for the underlying idea is quite general: For any encompassing matching function - specified independently of the optimizing decisions relevant to the selection process - it is always possible to construct a microfounded macro model that systematically fools this matching function.

The basic intuition underlying this result is straightforward. Although it is often claimed that the matching function is analogous to a production function, an important difference stands out. A firm's production function captures the portfolio of available technologies, and these are indeed often invariant with respect to many policy and macroeconomic variations. By contrast, the encompassing matching function summarizes the market activity generated by the decisions of firms and workers, responding to their individual incentives to create jobs, and these incentives are in general *not* invariant with respect to policy and macroeconomic variations. On the contrary, policy changes and macroeconomic shocks usually affect firms' incentives to offer jobs and workers' incentives to accept them. Thus the relation between new hires and match inputs is mediated by these policy and macroeconomic variations. On this account, the encompassing matching function may be expected to change when these variations occur. This calls into question the usefulness of the encompassing matching function for forecasting or policy analysis.

In short, our analysis suggests that an encompassing matching function is not suitable for describing the selection process, and should be replaced by a microfounded model in which new hires are explained in terms of the optimizing decisions of firms and workers in the presence of heterogeneous contacts.[25]

4 A Dynamic Incentive Model

We now relax several restrictive assumptions of the incentive model above – that households and firms are myopic, wages are exogenous, productivity is constant, each searcher finds a distinct vacancy – in order to examine the relative performance of the incentive model and the standard matching model in accounting for well-known stylized facts. In the context of conventional calibrations, we will show that the incentive model fares better than the standard matching model in reproducing the volatilities of major labor market variables.

Specifically, we extend the simple model above by

[25] While various other authors have modeled the matching process without resorting to a matching function (e.g. Hall, 1977, Lagos, 2000, Shimer, 2007, and others), our analysis explicitly focuses on two-sided search (i.e. search by both workers and firms at the same time).

- including aggregate risk: the average aggregate productivity parameter a is now subject to random productivity shocks;
- allowing for rates of time discount that are less than 100%, so that workers and firms become intertemporal optimizers;
- introducing wage determination through bargaining; and
- allowing the number of contacts to depend on both the number of job searchers and the number of vacancies.

The first extension enables us to simulate productivity shocks as done in Hall (2005), Shimer (2005) and numerous other papers and to make our framework quantitatively comparable to the matching theory. The second and third extensions provide a richer depiction of the determinants of employment and wages. The fourth enables us to include a non-trivial contact function.

In the context of our extended model, the sequence of decisions may be summarized as follows. First, vacancies are posted. Second, the aggregate productivity shock and the idiosyncratic shocks are revealed. Third, the wage is set through bargaining. Fourth, the firms make their hiring and firing decisions and the households make their job acceptance and refusal decisions, taking the wage and the realization of the aggregate and idiosyncratic shocks as given. We proceed to consider these decisions in reverse order.

4.1 The Firm's Behavior

The firm maximizes the present value of its expected profit, with a time discount factor δ.

4.1.1 The Firing Decision

The expected present value of profit generated by an incumbent employee, after the random profitability term ε_t is observed, is

$$E_t\left(\pi_t^I\right) = \left(a_t - w_t - \varepsilon_t\right) + \delta E_t\left(\left(1-\sigma_{t+1}\right)\pi_{t+1}^I - \phi_{t+1} f\right), \tag{22}$$

where δ is the time discount factor, a_t is the incumbent employee's productivity, and

$$E_t\left(\pi_{t+1}^I\right) = E_t\left[\left(a_{t+1} - w_{t+1} - E\left(\varepsilon_{t+1}\middle|\left(\varepsilon_{t+1} < \nu_{t+1}^I\right)\right) + \delta\left(1-\sigma_{t+2}\right)\pi_{t+2}^I\right) - \phi_{t+2} f\right]. \tag{23}$$

$E\left(\varepsilon_{t+1}|\left(\varepsilon_{t+1}<\nu_t^I\right)\right)$ is the expectation of the random term ε_{t+1}, conditional on this shock falling short of the *incumbent employee's retention incentive* ν_t^I, which is defined as

$$\nu_t^I = a_t - w_t + \delta E_t\left(\left(1-\sigma_{t+1}\right)\pi_{t+1}^I - \phi_{t+1}f\right) + f, \tag{24}$$

i.e. the retention incentive is the difference between the gross expected profit from retaining the employed worker $\left(a_t - w_t + \delta E_t\left(\left(1-\sigma_{t+1}\right)\pi_{t+1}^I - \phi_{t+1}f\right)\right)$ and the expected profit from firing her $(-f)$.

An incumbent worker is fired in period t when the realized value of the random cost ε_t is greater than the incumbent worker's employment incentive: $\varepsilon_t > \nu_t^I$. Since the cumulative distribution of ε_t is $J_\varepsilon\left(\nu_t^I\right)$, the employed worker's firing rate is

$$\phi_t = 1 - J_\varepsilon\left(\nu_t^I\right). \tag{25}$$

4.1.2 The Job Offer Decision

The expected present value of profit generated by an entrant, given that a contact has been made and the random cost ε_t has been observed, is

$$E_t\left(\pi_t^E\right) = a_t - w_t - \varepsilon_t - h + \delta E_t\left(\left(1-\sigma_{t+1}\right)\pi_{t+1}^I - \phi_{t+1}f\right). \tag{26}$$

We define the firm's expected *job offer incentive* ν_t^E as the difference between the gross expected profit from a hired worker $\left(a_t - w_t - h + \delta E_t\left(\left(1-\sigma_{t+1}\right)\pi_{t+1}^I - \phi_{t+1}f\right)\right)$ and the profit from not hiring him (i.e. zero):

$$\nu_t^E = a_t - w_t - h + \delta E_t\left(\left(1-\sigma_{t+1}\right)\pi_{t+1}^I - \phi_{t+1}f\right) \tag{27}$$

A job is offered when $\nu_t^E > \varepsilon_t$. Thus the job offer rate is

$$\eta_t = J_\varepsilon\left(\nu_t^E\right). \tag{28}$$

4.2 The Worker's Behavior

The incumbent worker's expected present value of utility from working is

$$E_t\left(\Omega_t^N\right) = w_t - e_t + \delta E_t\left(\left(1-\sigma_{t+1}\right)\Omega_{t+1}^N + \sigma_{t+1}\Omega_{t+1}^U\right), \tag{29}$$

where $E_t\left(\Omega_{t+1}^N\right)$ is the expected present value of utility of the following period (before the realized value of the shock e_{t+1} is known):

$$E_t\left(\Omega_{t+1}^N\right) = E_t\left(w_{t+1} - E_t\left(e_{t+1}|e_{t+1} < \iota_{t+1}\right) + \delta\left(\left(1-\sigma_{t+2}\right)\Omega_{t+2}^N + \sigma_{t+2}\Omega_{t+2}^U\right)\right). \tag{30}$$

The expected present value utility from unemployment is

$$E_t\left(\Omega_t^U\right) = b_t + \delta E_t\left(\mu_{t+1}\Omega_{t+1}^N + \left(1-\mu_{t+1}\right)\Omega_{t+1}^U\right). \tag{31}$$

An unemployed worker's expected "work incentive" ι_t is the expected gross difference[26] between these two utility streams:

$$\iota_t = E_t\left(\Omega_t^N - \Omega_t^U\right). \tag{32}$$

Thus the unemployed accepts a job offer when $e_t < \iota_t$.Consequently, the job acceptance rate is

$$\alpha_t = J_e\left(\iota_t\right). \tag{33}$$

The incumbent worker decides to quit his job when the present value of becoming unemployed exceeds the present value of remaining employed $\left(E_t\left(\Omega_t^N\right) - e_t < E_t\left(\Omega_t^U\right)\right)$, so that his expected work incentive is lower than the utility cost $e_t > E_t\left(\Omega_t^N - \Omega_t^U\right) = \iota_t$. Thus the quit rate is

$$\chi_t = 1 - J_e\left(\iota_t\right). \tag{34}$$

4.3 Employment

Let C_t be the number of contacts made in period t and c_t be an unemployed worker's contact probability:

$$c_t = C_t/U_{t-1}.$$

Then the match probability is the product of the contact, matching and acceptance probabilities:

$$\mu_t = c_t\eta_t\alpha_t. \tag{35}$$

As in the one-period incentive model, the separation probability is

$$\sigma_t = \phi_t + \chi_t - \phi_t\chi_t, \tag{36}$$

[26] "Gross" means that the utility shock e_t is not taken into account.

and the associated employment dynamics equation is

$$n_t = \mu_t + (1 - \sigma_t - \mu_t)\, n_{t-1}. \tag{37}$$

4.4 Wage Determination

We now endogenize the real wage through bargaining. We assume here that wage bargaining takes place before the job offer, acceptance, firing and quit decisions are made. Our aim is to formulate a wage determination model that is (i) simple and tractable, (ii) comparable to the wage bargaining process in the conventional matching models and (iii) able to reproduce the stylized fact that wages are as volatile as productivity. Accordingly, we assume that the incumbent workers and entrants receive the same wage w_t,[27] determined through Nash bargaining between the firm and its median incumbent worker. The median worker faces no risk of dismissal, as he is at the middle of the ε distribution. These assumptions satisfy the three aims above, because (i) the simplify the analysis by allowing the employment rate to depend on the wage, but not vice versa, (ii) the Nash bargaining between the firm and the median incumbent is comparable to the wage bargaining in the conventional matching models, and (iii) the negotiated wage turns out to be as volatile as productivity. It can be shown that individualistic wage bargaining leads to results similar to those here – in particular, to comparably high labor market volatilies (see Appendix A.1) – suggesting that our results are not driven by our particularly simple bargaining model.[28]

The wage bargain takes place in each period of analysis. In the current period t, under bargaining agreement, the median incumbent worker receives the wage w_t incurs effort cost e^M and the firm receives the expected profit $\left(a_t - w_t - \varepsilon^M\right)$ in each period t. Thus the expected present value of the median incumbent worker's utility $E(\Omega_t^M)$ under bargaining agreement is

$$E_t(\Omega_t^M) = w_t - e^M + \delta E_t \left((1 - \sigma_{t+1})\, \Omega_{t+1}^N + \sigma_{t+1} \Omega_{t+1}^U\right). \tag{38}$$

The expected present value of firm's returns under bargaining agreement are

[27] This assumption also implies that an increase in wages leads to a fall in employment. This employment effect can of course also be generated when incumbent workers and entrants have different wages. For example, Lindbeck and Snower (2001) provide a variety of reasons why entrants do not receive their reservation wage and thus a rise in incumbent workers' wages is not met a counterveiling fall in entrant wages, and thus a rise in incumbent workers' wage lead to a fall in employment. In the context of a Markov model, Diaz-Vazquez and Snower (2003) show that incumbent workers' wages are inversely related to aggregate employment even when entrants receive their reservation wages.

[28] Note also that under indivdualistic bargaining, the household and firm decisions can not be disentangled, which complicates the exposition of the intuition underlying our results. .

$$E_t\left(\pi_t^M\right) = \left(a_t - w_t - \varepsilon^M\right) + \delta E_t\left(\left(1-\sigma_{t+1}\right)\pi_{t+1}^I - \phi_{t+1} f\right).^{29} \tag{39}$$

Under disagreement in bargaining, the incumbent worker's fallback income is d, which can be conceived as financial support from family and friends, strike pay out of a union fund, or other forms of support. The firm's fallback profit is $-z$, a constant. Assuming that disagreement in the current period does not affect future returns, the present value of utility under disagreement for the incumbent worker is

$$E\left(\widetilde{\Omega}_t^M\right) = d + \delta E_t\left(\left(1-\sigma_{t+1}\right)\Omega_{t+1}^N + \sigma_{t+1}\Omega_{t+1}^U\right), \tag{40}$$

and the present value of profit under disagreement for the firm is

$$E\left(\widetilde{\pi}_t^M\right) = -z + \delta E_t\left(\left(1-\sigma_{t+1}\right)\pi_{t+1}^I - \phi_{t+1} f\right). \tag{41}$$

The incumbent worker's bargaining surplus is

$$\begin{aligned} E_t\left(\Omega_t^M\right) - E_t\left(\widetilde{\Omega}_t^M\right) &= w_t - e^M + \delta E_t\left(\left(1-\sigma_{t+1}\right)\Omega_{t+1}^N + \sigma_{t+1}\Omega_{t+1}^U\right) \\ &\quad - d - \delta E_t\left(\left(1-\sigma_{t+1}\right)\Omega_{t+1}^N + \sigma_{t+1}\Omega_{t+1}^U\right) \\ &= w_t - d - e^M, \end{aligned} \tag{42}$$

and the firm's surplus is

$$\begin{aligned} E_t\left(\pi_t^M\right) - E_t\left(\widetilde{\pi}_t^M\right) &= \left(a - w_t - \varepsilon^M\right) + \delta E_t\left(\left(1-\sigma_{t+1}\right)\pi_{t+1}^I - \phi_{t+1} f\right) - \\ &\quad E_t\left(-z + \delta\left(\left(1-\sigma_{t+1}\right)\pi_{t+1}^I - \phi_{t+1} f\right)\right) \\ &= a_t - w_t - \varepsilon^M + z. \end{aligned} \tag{43}$$

The negotiated wage maximizes the Nash product (Λ):

$$\Lambda = \left(w_t - e^M - d\right)^\gamma \left(a_t - w_t + z - \varepsilon^M\right)^{1-\gamma}, \tag{44}$$

where γ represents the bargaining strength of the incumbent worker relative to the firm. Thus the negotiated wage is

$$w_t = \gamma\left(a_t + z - \varepsilon^M\right) + (1-\gamma)\left(e^M + d\right). \tag{45}$$

4.5 Contacts

We let the contact function have the standard Cobb-Douglas form:

$$C_t = \zeta U_{t-1}^{\lambda} V_t^{1-\lambda}, \tag{46}$$

where λ is the contact elasticity and ζ contact efficiency. As in the traditional search models, the number of vacancies V_t^* is determined through a zero-profit condition:

$$a_t - w_t - E\left(\varepsilon_t \middle| \left(\varepsilon_t < \nu_t^I\right)\right) - h - \frac{\kappa}{\zeta\left(\frac{U_{t-1}}{V_t^*}\right)^{\lambda}\eta_t\alpha_t} + \delta E_t\left(\left(1-\sigma_{t+1}\right)\pi_{t+1}^I - \phi_{t+1}f\right) = 0, \tag{47}$$

where the vacancy posting cost is κ and the expected vacancy posting cost[30] is $\frac{\kappa V_t}{\eta_t\alpha_t U_{t-1}}$.

4.6 The Labor Market Equilibrium

The labor market equilibrium is the solution of the system comprising the following equations:

- *Incentives:* the incumbent worker retention incentive ν_t^I (eq. 24), the job offer incentive ν_t^E (eq. 27) and the work incentive ι_t (eq. 32).
- *Employment decisions:* the firing rate ϕ_t (eq. 25) and the job offer rate η_t (eq. 28).
- *Work decisions:* the job acceptance rate α_t (eq. 33) and the quit rate χ_t (eq. 34).
- *Contacts and vacancies*: the contact function C_t (eq. 46) and the number of vacancies V^* (eq.47).
- *Match and separation probabilities:* the match probability μ_t (eq. 35) and the separation probability σ_t (eq. 36).
- *Employment and wage:* the employment level N_t (eq. 37) and the negotiated wage w_t (eq. 45).

[30] Note that

$$\frac{\kappa V_t}{\mu_t U_{t-1}} = \frac{\kappa V_t}{c_t\eta_t\alpha_t U_{t-1}} = \frac{\kappa V_t}{C_t\eta_t\alpha_t} = \frac{\kappa}{\zeta\left(\frac{U_{t-1}}{V_t}\right)^{\lambda}\eta_t\alpha_t}$$

.

5 Calibration

We now calibrate our incentive model for the US economy. The calibration is done on a monthly basis. The simulation results are aggregated to quarterly frequency to make them comparable to the empirical data, as for example in Shimer (2005).

Our monthly discount factor $\delta = \frac{1}{1+r}$ is consistent with an annual real interest rate of 4 percent. We normalize the average productivity (a) to 1. As in Hall (2005) and Shimer (2005), we set b by applying a replacement rate of $\beta = 40\%$ of the wage. For simplicity, we set $d = b$. As commonly found in the literature we adopt a bargaining power parameter of $\gamma = 0.5$.

In this section, we begin with the simplifying assumption, which will be relaxed later, that $\lambda = 1$, so that each unemployed worker makes one distinct contact in each period: $C_t = U_{t-1}$, implying a contact rate of unity: $c_t = 1$. (In the next section, this assumption is relaced by the standard Cobb-Douglas contact function, with $0 < \lambda < 1$.) Furthermore, we normalize ζ to unity. The vacancy posting costs κ of 0.19 are chosen to satisfy the zero-profit condition.

Dolfin (2006) shows that an average U.S. worker spends 203 hours in training activities during her first three months of employment, while other employees spend around 146 training her. In line with this evidence, we set hiring costs, h, to 130% of the monthly productivity.[31]

The literature does not provide reliable direct estimates of the magnitude of US firing costs. Thus, we assess these costs indirectly. For this purpose, note that Belot et al. (2007) provide index measures of employment protection for regular jobs in the US and UK, and that Bentolila and Bertola (1990) provide estimates of the average magnitude of UK firing costs on a yearly basis.[32] Assuming that the index measures of employment protection are proportional to the estimates of the magnitude of firing costs, we multiply the magnitude of the UK firing costs by the ratio of the US to the UK employment protection indeces to derive a rough estimate of the magnitude of US firing costs. Accordingly, the magnitude of monthly US firing costs, relative to productivity, is 0.08. The same exercise based on other industrialized countries (France, Germany and Italy), however, yields higher estimates of US firing costs. Thus we choose a value of 0.1 for our baseline calibration, but provide a robustness analysis for other values in Appendix A.2.[33] For simplicity, we set the firm's

[31] We only take the direct training costs into account. We divide 203 by 8, thereby obtaining 25.4 working days. Assuming 20 working days per month, this yields 1.3 months.

[32] We take averages over the time periods provided by these authors.

[33] Specifically, we provide simulation results for firing costs calculated relative to the UK, $f = 0.08$, and as an upper bound we choose $f = 0.2$.

fallback profit $-z$ equal to $-f$.[34]

We assume that the random profitability term ε_t and the utility shock e_t have cumulative distributions given by logistic functions with scale factors s_ε and s_e and expected values $\bar{\varepsilon}$ and $\bar{e}$, respectively.[35] We calibrate our model such that it replicates the stylized fact that wages are as volatile as productivity.[36] This is achieved by setting $\bar{e} = 0.17$. Thereby our calibration excludes the possibility that our results are driven by real wage rigidity.

After having set all our other parameter values, we set the remaining three free distributional parameters (the average operating costs $\bar{\varepsilon}$, the scale factor of the cumulative distribution of s_ε, the scale factor of the cumulative distribution of s_e) to replicate three steady state labor market flow rates: the job acceptance rate α, the job offer rate η, and the firing rate ϕ. These steady state values are calibrated as follows. The match probability μ, which is the probability for a worker to find a new job within one period, is calibrated to 45%[37], as in Shimer (2005) and Hagedorn and Manvoskii (2008). The unemployment rate u is calibrated to 5.6% (as in Shimer, 2005). According to our employment dynamics equation (13) steady state unemployment is $u = \frac{\mu}{\mu+\sigma}$ which implies a separation rate of 2.68%. Based on Hall (2006), who shows that fires and quits have approximately the same share in separation, we assume firings to account for 50% of the separations, namely $\phi = 1.34\%$. Eq. (36) then yields the quit rate of $\chi = 1.36\%$. Since α is equal to $1 - \chi$, the job acceptance rate is set at 98.64%. Recalling that $\mu = c\alpha\eta$ and that we have assumed $c = 1$, the implied job offer rate η is 45.6%.We determine the number of vacancies by assuming a market tightness equal to 1.

The standard deviation of the idiosyncratic productivity shock is of significant importance for the aggregate dynamics, since the lower it is, the stronger are the reactions to productivity in our model. Our calibration strategy yields a standard deviation of the idiosyncratic productivity shock of $\sigma_\varepsilon = 0.57$, which is narrower than values commonly used in the literature (e.g. Den Haan et al., 2000, choose $\sigma_\varepsilon = 0.1$, Krause and Lubik, 2007, use $\sigma_\varepsilon = 0.12$ and Mortensen and Pissarides, 1994, choose $\sigma_\varepsilon = 0.0375$). Using this conservative value we bias the dynamics against our model and this ensures that our volatilities are not driven by an unrealistically small standard deviation of the idiosyncratic productivity

[34] Here we implicitly assume that during disagreement the incumbent worker imposes the maximal cost on the firm short of inducing dismissal.

[35] The cumulative logistic distribution is very close to the cumulative normal distribution.

[36] See Hornstein et al. (2005).

[37] Note: In our model the worker finding rate (i.e., the probability of a firm to find a new worker) and the job finding rate (i.e., the probability of a worker to find a new) are the same.

shock.[38]

We normalize the autocorrelation (ρ_a) of the aggregate productivity shock and normalize the standard error such that we obtain the empirical values for the autocorrelation and the volatility of productivity in the model simulation below. Table 2 summarizes our calibrated parameter values.

Variable	In Words	Steady State Value
u	unemployment rate	0.056
μ	match probability	0.450
c	contact rate	1
η	hiring/job offer rate	0.456
σ	separation rate	0.0268
ϕ	firing rate	0.0134
$\chi = 1 - \alpha$	job quit rate	0.0136
θ	market tightness	1

Table 1: Steady state values for a contact function $C_t = U_{t-1}$, i.e. $\lambda = 1$ and $\zeta = 1$

Parameter	In Words	Value
a	productivity	1
β	replacement rate $\frac{b}{w}$, $\frac{d}{w}$	0.4
f	firing cost	0.1
h	hiring cost	1.3
ζ	contact efficiency	1
λ	contact elasticity	1
κ	vacancy posting cost	0.188
γ	workers' bargaining strength	0.5
r	discount factor	0.997
$-z$	firm's fallback profit	-0.1
$\bar{e}$	average value of leisure	0.17
$\bar{\varepsilon}$	average operating costs	0.51
s_ε	scale factor of the cumulative distribution of ε_t	0.313
s_e	scale factor of the cumulative distribution of e_t	0.058
ρ_a	autocorrelation of the aggregate productivity shock	0.975
ϖ_a	standard error of the aggregate productivity shock	0.007

Table 2: Parameter values for a contact function $C_t = U_{t-1}$, i.e. $\lambda = 1$ and $\zeta = 1$.

[38] While the standard deviation of the idiosynchratic utility schock is indeed small, $\sigma_e = 0.1$, under our calibration the household acts countercyclical, which counteracts and dampens the effect of the idiosynchratic productivity schock on unemployment.

6 Description of Results

6.1 Labor Market Volatilies

Costain and Reiter (2008) and Shimer (2005) show that the conventional calibration of the matching model is unable to replicate the volatility of the job finding rate, the unemployment rate, and other labor market variables in response to productivity shocks. Table 3 shows that the empirical volatilities for the United States (from 1951-2003, HP filtered data with smoothing parameter 100,000, as calculated by Shimer) are far greater than the corresponding volatilities in response to productivity shocks, as generated by the simulation of the conventional matching model (in its standard calibration, as calculated by Shimer).

Empirical Labor Market Statistics by Shimer (2005), from 1951-2003						
Volatilities	U. Rate	Match. R.	Sep. Rate	Vac.	M. Tight.	Prod.
Standard deviation	0.19	0.12	0.08	0.20	0.38	0.02
Relative to productivity	**9.5**	**5.9**	**3.8**	**10.1**	**19.1**	**1**
Quarterly autocorr.	0.94	0.91	0.73	0.94	0.94	0.88
Correlations	U,V	JCR, JDR				
	-0.89	-0.36				
Labor Market Statistics by Shimer's (2005) Search and Matching Model						
Volatilities	U. Rate	Match. R.	Sep. Rate	Vac.	M. Tight.	Prod.
Standard deviation	0.01	0.01	-	0.03	0.04	0.02
Relative to productivity	**0.5**	**0.5**	-	**1.4**	**1.8**	**1**
Quarterly autocorr.	0.94	0.88	-	0.84	0.88	0.88
Correlations	U,V	JCR, JDR				
	-0.93	-				

Table 3: Empirical labor market statistics and those generated by the search and matching model from Shimer (2005).[39]

To compare our model with the conventional matching theory, we use our baseline calibration (with robustness checks in the Appendix B) to simulate our model for 200 quarters (i.e. 600 months). We repeat this exercise 1000 times and report the average of the macroeco-

[39] The correlation between job creation and job destruction is not available in Shimer (2005). The empirical correlation is taken from Krause and Lubik (2007).

nomic volatilities (HP filtered simulated data with smoothing parameter 100,000) in Table 4.[40]

Standard Calibration						
Volatilities	U. Rate	Match. R.	Sep. Rate	Vac.	M. Tight.	Prod.
Standard deviation	0.18	0.18	0.01	0.09	0.31	0.02
Relative to productivity	**8.8**	**9.2**	**0.5**	**4.6**	**15.5**	**1**
Quarterly autocorr.	0.91	0.87	0.88	0.56	0.85	0.88
Correlations	U, V	JCR, JDR				
	-0.71	-0.06				

Table 4: Labor market statistics generated by the incentive model of matching

The differences between our model and the conventional matching model are striking. Our model can generate the high macroeconomic volatilities found in the data. Our results are all the more remarkable, as we do not neither have to resort to Hall's (2005) real wage rigidity assumption nor to Hagedorn and Manovskii's (2008) small surplus calibration.

Specifically, the more rigid the wage in the conventional matching model (Hall, 2005), the greater the share of productivity variations that is captured by the firm and thus the greater the volatility of vacancies. However, there is evidence against the rigid-wage hypothesis both from the microeconometric and the macro perspective. Haefke et al. (2008) infer that wages for newly created jobs (i.e., those modeled in the matching model) are completely flexible on a microeconomic level. Hornstein et al. (2005) show that wages are roughly as volatile as the labor productivity on a macroeconomic level. By contrast, our model generates high labor market volatilities, even though it replicates the stylized fact that wages are as volatile as productivity.

Hagedorn and Manovskii (2008) choose a small-surplus calibration to resolve the volatility puzzle of the matching model. Under this calibration, aggregate profits are only a very small share of the overall production in the steady state, so that a positive productivity shock sharply increases the relative profits. This gives a large incentive to firms to post more vacancies (due to the free entry condition). Consequently, all labor market variables become volatile. This type of calibration has several shortcomings. Besides the unrealistically low profit share, the utility value of unemployment is extremely high and workers' bargaining power is very low in the calibration. Therefore workers are almost indifferent between working and not working. We do not need to rely on any of these mechanisms in our calibration. As

[40] The correlation between job destruction and job creation is calculated as the correlation between the separation rate and the proportion of new jobs $\mu_t u_{t-1}/n_{t-1}$.

noted, we assume that worker's bargaining power is 50 percent. Furthermore, the average worker's disutility of labor and unemployment benefits make up only 60 percent of the current wage. As a consequence, the average worker is nowhere near indifferent between unemployment and employment.

6.2 Correlations

Our model features several additional advantages compared to the conventional matching framework. Krause and Lubik (2007) show that the matching model with endogenous job destruction and flexible wages cannot generate a negative correlation between the between job destruction and job creation (as in the data). In our model, the correlation between job creation and job destruction (namely, -0.06) has the appropriate sign.

Further, in a matching model with endogenous job destruction and flexible wages, the negative correlation between vacancies and unemployment disappears (i.e., the dynamic Beveridge Curve). In our simulation, we obtain a strong negative correlation between vacancies and unemployment (namely, -0.71).[41]

7 Outreach versus Suitability Assessment

In this section we seek to shed light on the roles of outreach (of the contact stage) and match suitability assessment (of the selection stage) in generating the well-known stylized facts described above. The outreach phenomenon – jobless workers and workerless employers reaching out to one another without knowing the identity of the other party – is captured by the contact function in our model; whereas the suitability assessment phenomenon – having made contact, the two parties examine whether a match would be suitable (i.e. would generate a positive surplus) – is captured by two-sided optimization over heterogeneous agents. As noted, the conventional search models with an encompassing matching function focus solely on outreach (as all job searchers and all vacancies are treated as homogeneous). In the models with exogenous separations (e.g. Shimer, 2005), suitability is invariant over the business cycle (since a constant fraction of contacts is broken in each period, equal to the fraction of employment relationships that is broken). Finally, in the productivity-perturbation models with endogenous separations (e.g. den Haan, et al, 2000, Krause and Lubik, 2007), suitability varies over the cycle, but (as discussed) these models have the

[41] We calculate the dynamic Beveridge curve as the correlation between contemporaneous unemployment and vacancies. If we used the lagged unemployment, we would obtain a correlation of -0.61, i.e., there remains a strong negative relationship. Many empirical studies use the contemporaneous correlation. Therefore, we do the same to be in line with the data. The reality might lie somewhere in between.

counterfactual implication that the hiring probability after a contact is made is equal to the retention probability. In practice, however, the retention probability is substantially larger than the hiring probability, and thus the selection decisions underlying the suitability of incumbents differ from the selection decisions underlying the suitability of entrants. Our model can account for this difference, since it includes linear hiring and firing costs that drive a wedge between the hiring probability and the retention probability.

In the calibration of the previous section, the outreach phenomenon had a very limited role to play in explaining labor market activity, since we assumed the contact elasticity to be $\lambda = 1$ and the contact efficiency $\zeta = 1$, implying the trivial contact function $C_t = U_{t-1}$. In this case, as noted, vacancies are simply an attention-seeking device and play no allocational role. Specifically, observe that since the contact probability is $c_t = C_t/U_{t-1} = 1$, the employment dynamics equation (37) reduces to $n_t = \eta_t \alpha_t + (1 - \sigma_t - \eta_t \alpha_t)\, n_{t-1}$, where the hiring probability η_t, the job acceptance probability α_t, and the separation probability σ_t are all independent of the number of vacancies.

To examine how the labor market volatilities depend on the exogenous contact probability, we perform the calibration above for different values of the contact probability c_t by varying the contact efficiency ζ in the trivial contact function, i.e. for a contact elasticity of $\lambda = 1$. The results are given in Table 5.[42]

	Volatilities (std. dev. relative to productivity)					**Correlations**	
$c = \zeta$	U. Rate	Match. R.	Sep. Rate	Vac.	M. Tight.	U,V	JCR,JDR
1	8.9	9.2	0.5	4.6	15.5	-0.71	-0.06
0.8	11.5	7.7	3.4	3.6	15.1	-0.42	0.64
0.6	21.3	5.0	11.4	4.5	13.5	0.49	0.88

Table 5: Labor market statistics for various contact rates c.

Note that as the contact probability falls, the elasticity of vacancies with respect to productivity declines, since vacancies are less likely to lead to hires.[43] Consequently, the

[42] The following calibration values differ with respect to the values in Section 5 as follows:

Param.	$c = \zeta$	
	0.9	0.7
η	0.507	0.652
s_ε	0.328	0.381
$\bar{\varepsilon}$	0.503	0.494
κ	0.206	0.278

Table 6: Difference to the standard calibration for different contact rates c.

[43] The correlation between unemployment and vacancies becomes positive with c=0.6. Thus, although the standard deviation increases, vacancies move in the wrong direction.

Beveridge Curve relation weakens, in the sense that negative shocks which raise unemployment do not depress vacancies as much (and conversely from positive shocks). As firms and households use the hiring and job acceptance margins less intensively to vary employment (due to the lower contact probability), they compensate by using the firing and quit margins more intensively. Thus, the variability of the separation rate rises.

When the contact rate falls, the models generates smaller amplification effects for the matching rate. However, the volatility of the unemployment rate depends on the matching-rate volatility and the separation-rate volatility. Since the matching-rate volatility declines and the separation-rate volatility rises as the contact probability falls, the joint effect on the unemployment-rate volatility is ambiguous. Table 5 shows, however, that the effect of the separation rate dominates, so that the volatility of the unemployment rate rises. But it can be seen that the volatility of the separation rate becomes implausibly large for low contact rates.

Finally, we assume that the contact elasticity is $0 < \lambda < 1$, [44] as is standard in the conventional search models. In this context, vacancies have an allocative role: Variations in aggregate vacancies lead to variations in the contact probability (ceteris paribus) and thereby affect aggregate employment. Table 6 shows the results of our calibration for various values of the contact elasticity.[45]

		Volatilities (std. dev. relative to productivity)					**Correlations**	
$c^* = \zeta$	λ	U rate	Match. rate	Sep. rate	Vac.	M. Tight.	U,V	JCR,JDR
0.8	0.75	7.1	8.9	0.8	5.4	14.9	-0.73	0.53
0.8	0.5	3.8	10.3	5.1	8.2	14.6	-0.87	0.98
0.6	0.75	11.2	6.3	4.3	3.1	12.5	-0.15	0.75
0.6	0.5	6.0	7.5	0.9	4.9	11.6	-0.76	0.60

Table 7: Labor market statistics for various steady state contact rates c^* and contact elasticities λ

When λ is smaller than 1, vacancies become allocationally relevant in our model. Observe that a fall in the contact elasticity, λ, leads to a stronger Beveridge Curve relation. The reason is that as the contact elasticity, λ, declines, vacancies become more important relative to unemployment in generating contacts and thus the responsiveness of vacancies

[44] Recall eq. 46, where contacts are given by $C_t = \zeta U_{t-1}^{\lambda} V_t^{1-\lambda}$.

[45] The following calibration values differ with respect to the values in Table 6 as follows:

Param.	$c = \zeta$			
	0.9		0.7	
	λ=0.75	λ=0.5	λ=0.75	λ=0.5
κ	0.314	0.254	1.157	0.567

Table 8: Differences to the previous calibration for different contact rates c and matching elasticities λ.

to productivity shocks rises. Not surprisingly, the variability of the unemployment rate falls in response to a decline in the contact elasticity. The effect of the contact elasticity on the variability of the matching rate is ambiguous: the reduced variability of unemployment and the increased variability of vacancies pull in opposite directions with respect to the matching rate.

Note that a reduction of the contact elasticity, λ, generates counterfactual correlations between labor market variables, for example the matching rate and the separation rate exhibit an even stronger positive correlation. While this is a well-known feature of matching model with endogenous separations (see, e.g., Krause and Lubik, 2007), in our model this results from the countercyclical behaviour of the household-side, which overcompensates the firm side on the separations margin.

Furthermore, note that as the contact elasticity falls beneath unity, the matching function plays an increasingly important role in determining the variability of unemployment, vacancies, matching, and separations. When the contact elasticity is equal to unity, the two distinctive features of the matching function - (i) the complementarity between unemployment and vacancies in generating matches and (ii) the diminishing marginal product of unemployment (declining effectiveness of unemployment to generate matches as unemployment increases, ceteris paribus) and the diminishing marginal product of vacancies - do not come into play. As the contact elasticity drops from unity towards 0.5, these two features become increasingly important, and the suitability of workers and firms become correspondingly less important. The diminishing marginal products help explain why the volatilities of unemployment decline when the contact elasticity falls.

As the observed volatilities of these variables are replicated more closely by the model when the contact elasticity is unity, this result suggests that the conventional contact function – in terms of its two distinctive features above – does not play a major role in explaining the labor market volatilities above. In short, it is suitability assessment, rather than outreach, that is responsible for our model's success in replicating labor market volatilities. This consideration helps justify our focus on the selection stage of matching – with incentive-based, two-sided search among heterogeneous agents – rather than on the contact stage, as in the conventional search literature.

8 Conclusion

This paper has presented a theory of labor market matching that distinguishes sharply between contacts and selection. The selection takes place in the presence of frictions, heterogeneous jobs and heterogeneous workers. Our empirical results suggest that selection has a particularly important role to play in accounting for the observed labor market volatilities.

Our theory replaces the traditional, encompassing matching function by a contact function combined with optimizing, incentive-based, two-sided selection decisions. Our analysis indicates that the encompassing matching function is vulnerable to the Lucas critique, since it is not stable with respect to changes in policy and macroeconomic variables. Thus its use for policy analysis and prediction becomes problematic.

The basic idea that motivates our incentive theory of matching is that the matching and separation probabilities can be understood in terms of job offer, job acceptance, firing, and quit probabilities, which may be derived from the optimizing decisions of firms and workers. These optimizing decisions – in the presence of heterogeneous workers and jobs, as well as costs of adjustment – explain why some job-seeking workers remain unemployed and some vacant jobs remain unfilled. We have shown that, even on the basis of our radically simplifying assumptions, our calibrated incentive model can account for various important empirical regularities that have eluded the conventional matching models. In particular, our model comes close to generating the empirically observed volatilities of the unemployment rate, vacancies, the job finding rate and the separation rate. Furthermore, our model can also account for the observed strong negative correlations between the job finding rate and the unemployment rate, and between vacancies and unemployment.

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A Appendix

A.1 Labour Market Volatilities with Individualistic Bargaining

This Appendix shows that the median worker baraining does not drive the volatilities. We derive the job creation, job destruction conditions and the wages. We then present the resulting volatilities for this model.

A.1.1 Job Creation Condition

Assuming individualistic bargaining a firm will hire a worker i when the following condition holds

$$\varepsilon_{t,i} < a_t - w^E_{t,i}\left(e_{t,i}\right) - h + \delta E_t \left(1 - \sigma_{t+1}\right) \pi^I_{t+1} - \delta \sigma_{t+1} f, \tag{48}$$

where the superscript N refers to an employed worker.

We assume that the marginal entrant receives her reservation wage. This reservation wage, for any given $e_{t,i}$, is

$$w^E_{t,i} = b + e_{t,i} - \delta E_t \begin{pmatrix} (1-\sigma_{t+1})\,\Omega^N_{t+1} - (1-\sigma_{t+1}-\mu_{t+1})\,\Omega^U_{t+1} \\ -\mu_{t+1} E_t\left(\Omega^{FE}_{t+1}\right) \end{pmatrix}, \tag{49}$$

where the superscripts U refers to an unemployed worker and FE to a future entrant, namely a worker who will be an entrant in the next period.

There will be a job for a new worker whenever the following condition holds

$$\begin{aligned} \varepsilon_{t,i} + e_{t,i} &< a_t - h + \delta E_t (1-\sigma_{t+1})\,\pi^I_{t+1} - \sigma_{t+1} f - b \\ &+ \delta E_t \begin{pmatrix} (1-\sigma_{t+1})\,\Omega^N_{t+1} - (1-\sigma_{t+1}-\mu_{t+1})\,\Omega^U_{t+1} \\ -\mu_{t+1} E_t\left(\Omega^{FE}_{t+1}\right) \end{pmatrix}, \end{aligned} \tag{50}$$

i.e., the sum of the disutility and the operating cost shock are smaller than firm's present value of profits and household's expected present value of utility from working (not taking the current wage into account). We define

$$\begin{aligned} \upsilon^C_t &= a_t - h + \delta E_t (1-\sigma_{t+1})\,\pi^I_{t+1} - \sigma_{t+1} f - b \\ &+ \delta E_t \begin{pmatrix} (1-\sigma_{t+1})\,\Omega^N_{t+1} - (1-\sigma_{t+1}-\mu_{t+1})\,\Omega^U_{t+1} \\ -\mu_{t+1} E_t\left(\Omega^{FE}_{t+1}\right) \end{pmatrix} \end{aligned} \tag{51}$$

Thus,

$$\varepsilon_{t,i} + e_{t,i} < \upsilon^C_{t,i} \tag{52}$$

$$P\left(\varepsilon_{t,i} + e_{t,i} \leq \upsilon^C_{t,i}\right) = \int_{-\infty}^{\infty} \int_{-\infty}^{\upsilon^C - e} f(\varepsilon, e)\, d\varepsilon de \tag{53}$$

$$P\left(\varepsilon_{t,i} + e_{t,i} \leq \upsilon^C_{t,i}\right) = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\upsilon^C - e} f(\varepsilon)\, d\varepsilon \right] f(e)\, de \tag{54}$$

A.1.2 Job Destruction Condition

$$e_{t,i}+\varepsilon_{t,i}>\left(a_t+\delta E_t\left(1-\sigma_{t+1}\right)\pi^I_{t+1}-\sigma_{t+1}f+f\right)-b \\ +\delta E_t\left(\begin{array}{c}\left(1-\sigma_{t+1}\right)\Omega^N_{t+1}-\left(1-\sigma_{t+1}-\mu_{t+1}\right)\Omega^U_{t+1} \\ -\mu_{t+1}E_t\left(\Omega^{FE}_{t+1}\right)\end{array}\right) \tag{55}$$

$$e_{t,i}+\varepsilon_{t,i}>\upsilon^D_{t,i} \tag{56}$$

$$P\left(\varepsilon_{t,i}+e_{t,i}\succeq\upsilon^D_{t,i}\right)=1-\int_{-\infty}^{\infty}\int_{-\infty}^{\upsilon^D-e}f\left(\varepsilon,e\right)d\varepsilon de \tag{57}$$

$$P\left(\varepsilon_{t,i}+e_{t,i}\succeq\upsilon^D_{t,i}\right)=1-\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\upsilon^D-e}f\left(\varepsilon\right)d\varepsilon\right]f\left(e\right)de \tag{58}$$

A.1.3 Conditional Expectations

Conditional expected value of the operating costs

$$\begin{aligned}E\left[e_{t,i}|\varepsilon_{t,i}+e_{t,i}<\upsilon_{t,i}\right]&=\int_{-\infty}^{\infty}\int_{-\infty}^{\upsilon-e}ef\left(\varepsilon,e\right)d\varepsilon de \\ &=\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\upsilon-e}f\left(\varepsilon\right)d\varepsilon\right)ef\left(e\right)de\end{aligned} \tag{59}$$

$$\begin{aligned}E\left[\varepsilon_{t,i}|\varepsilon_{t,i}+e_{t,i}<\upsilon_{t,i}\right]&=\int_{-\infty}^{\infty}\int_{-\infty}^{\upsilon-e}ef\left(\varepsilon,e\right)d\varepsilon de \\ &=\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\upsilon-e}f\left(\varepsilon\right)d\varepsilon\right)ef\left(e\right)de\end{aligned} \tag{60}$$

A.1.4 Expected Future Values

$$E_t\left(\pi^I_{t+1}\right)=E_t\left[a_{t+1}-w^I_{t+1}-E\left[\varepsilon_{t,i}|\varepsilon_{t,i}+e_{t,i}<\upsilon^D_{t,i}\right]+\delta\left(1-\sigma_{t+2}\right)\pi^I_{t+2}-\sigma_{t+2}f\right]. \tag{61}$$

$$E_t\left(\Omega^N_{t+1}\right)=E_t\left[w^I_{t+1}-E\left[e_{t,i}|\varepsilon_{t,i}+e_{t,i}<\upsilon^D_{t,i}\right]+\delta\left(\left(1-\sigma_{t+2}\right)\Omega^N_{t+2}+\sigma_{t+2}\Omega^U_{t+2}\right)\right]. \tag{62}$$

$$E_t\left(\Omega_t^U\right) = b + \delta E_t\left(\mu_{t+1}\Omega_{t+1}^{FE} + \left(1-\mu_{t+1}\right)\Omega_{t+1}^U\right). \tag{63}$$

$$E_t\left(\Omega_{t+1}^{FE}\right) = E_t\left(w_{t+1}^{FE} - E\left[e_{t,i}|\varepsilon_{t,i}+e_{t,i}<\upsilon_{t,i}^C\right] + \delta\left(\left(1-\sigma_{t+2}\right)\Omega_{t+2}^N + \sigma_{t+2}\Omega_{t+2}^U\right)\right). \tag{64}$$

A.1.5 Wage Bargaining

Expected wage of a retained incumbent worker:

$$w_t^I\left(\varepsilon_{t,i},e_{t,i}\right) = \gamma\left(a_t - E\left[\varepsilon_{t,i}|\varepsilon_{t,i}+e_{t,i}<\upsilon_{t,i}^D\right] + \delta\left(E_t\left(1-\sigma_{t+1}\right)\pi_{t+1}^I - \sigma_{t+1}f\right) + f\right) + \tag{65}$$
$$\left(1-\gamma\right)\left(\begin{array}{c} b + E\left[e_{t,i}|\varepsilon_{t,i}+e_{t,i}<\upsilon_{t,i}^D\right] \\ -\delta E_t\left(\begin{array}{c}\left(1-\sigma_{t+1}\right)\Omega_{t+1}^N - \left(1-\sigma_{t+1}-\mu_{t+1}\right)\Omega_{t+1}^U \\ -\mu_{t+1}E_t\left(\Omega_{t+1}^{FE}\right)\end{array}\right)\end{array}\right)$$

Expected wage of a future entrant:

$$w_t^{FE}\left(\varepsilon_{t,i},e_{t,i}\right) = \gamma\left(a_t - E\left[\varepsilon_{t,i}|\varepsilon_{t,i}+e_{t,i}<\upsilon_{t,i}^C\right] - h + \delta\left(E_t\left(1-\sigma_{t+1}\right)\pi_{t+1}^I - \sigma_{t+1}f\right)\right) + \tag{66}$$
$$\left(1-\gamma\right)\left(\begin{array}{c} b + E\left[e_{t,i}|\varepsilon_{t,i}+e_{t,i}<\upsilon_{t,i}^C\right] \\ -\delta E_t\left(\begin{array}{c}\left(1-\sigma_{t+1}\right)\Omega_{t+1}^N - \left(1-\sigma_{t+1}-\mu_{t+1}\right)\Omega_{t+1}^U \\ -\mu_{t+1}E_t\left(\Omega_{t+1}^{FE}\right)\end{array}\right)\end{array}\right)$$

A.1.6 Results for Indivdualistic Bargaining

Table 9 shows the results under individualistic bargaining. The model also generates large volatilities under individualistic bargaining (i.e., our main results do not depend on the chosen bargaining regime. Admittely, the separation rate is somewhat too volatile. However, if we use completely exogenous separations[46], the magnitudes are fairly close to the empirical data. The choice of the appropriate bargaining regime and the issue of completely endogenous versus partly or completely exogenous separations is certainly an interesting topic for future research.

[46] In order to keep the parametrizations comparable, we keep all other model components as exogenous constants (e.g., the linear firing costs).

	U. Rate	Match. Rate	Sep. Rate	Product.
Volatilities for Individualistic Bargaining				
Standard deviation	0.56	0.11	0.30	0.02
Relative to productivity	**26.7**	**5.3**	**14.4**	**1**
Quarterly autocorrelation	0.83	0.88	0.85	0.88
Volatilities for Individualistic Bargaining (Exogenous Separations)				
Standard deviation	0.10	0.11	-	0.02
Relative to productivity	**4.8**	**5.2**	-	**1**
Quarterly autocorrelation	0.91	0.88	-	0.88

Table 9: Labor market volatilities for individualistic bargaining.

A.2 Robustness

The following table provides a robustness analysis (same bargaining assumption as in Tables 5 and 7) of the labor market volatilities implied by our model for values of the firing cost $f = 0.08$ and $f = 0.20$.

	U. Rate	Match. Rate	Sep. Rate	Product.
Volatilities for $f = 0.08$				
Standard deviation	0.24	0.20	0.03	0.02
Relative to productivity	**11.8**	**10.1**	**1.6**	**1**
Quarterly autocorrelation	0.91	0.87	0.88	0.88
Volatilities for $f = 0.2$				
Standard deviation	0.09	0.14	0.04	0.02
Relative to productivity	**4.4**	**7.1**	**1.9**	**1**
Quarterly autocorrelation	0.90	0.87	0.88	0.88

Table 10: Robustness analysis for the labor market volatilities Implied for firing cost values of $f = 0.08$ and $f = 0.20$.