Endogenous Growth, Skill Obsolescence and Fiscal Multipliers

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No. 2184 May 2021
ABSTRACT

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We analyze the effects of government spending in a New-Keynesian model with search and matching frictions featuring endogenous growth through learning-by-doing and skill loss from long-term unemployment. We show that medium-run and long-run output and unemployment multipliers are much larger compared to the standard model that abstracts from endogenous growth and skill loss. In our model the aggregate effect of a temporary fiscal stimulus is amplified via the skill loss channel through lower training costs. Via the learning-by-doing channel, it leads to hysteresis in human capital accumulation and thereby output. These results hold for alternative forms of fiscal financing (lump-sum tax, distortionary tax and government debt) as well as alternative labor market institutions (US and Europe).

Keywords: Sovereign default, debt restructuring, international financial architecture, creditor Coordination

JEL: E24; E52

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1 Introduction

The big and persistent drop in GDP during the Great Recession and the sluggish recovery thereafter have revived the old debate about hysteresis in business cycles and the role of macroeconomic policy in reducing the long-term damage from big recessions so as to regain lost output in the aftermath of the recession. We contribute to this debate by analyzing the effects of government spending in a model with endogenous growth through learning-by-doing, search-and-matching frictions and skill-loss from long-term unemployment. We show that medium-run and long-run output and unemployment multipliers are much larger compared to the standard model that abstracts from endogenous growth and skill loss. In our model the aggregate effect of a temporary fiscal stimulus is amplified by the skill loss channel through lower training costs. Via the learning-by-doing channel, higher employment leads to hysteresis in human capital accumulation and thereby output. These results hold for alternative forms of fiscal financing (lumpsum tax, distortionary tax and government debt) as well as labor market institutions (US and Europe).

Our results are significant because, until recently, the main paradigm in business cycle macro was that the long-run trend in productivity is given and exogenous to business cycle fluctuations. Thus, the typical business cycle models assumed an exogenous long-run growth path and only modeled the fluctuations around this trend. By construction, temporary business cycle shocks and temporary policy interventions could not affect the long-run trend and had only temporary consequences on output.

The conventional paradigm has recently been questioned by the large and persistent decrease in GDP during and after the Great Recession across the advanced economies. For example, Figure 1 shows that, even a decade after the end of the recession US GDP failed to return to the pre-crisis trend. The notion of hysteresis, that temporary demand and supply shocks can have permanent effects, is confirmed by recent empirical evidence (see, e.g., Cerra, Fatas, and Saxena (2020) for a recent survey of the literature).

The effects of hysteresis also have important implications for macroeconomic policy. If macroeconomic stabilization policy can reduce the depth of a downturn, it seems rea-
sonable that it can also reduce the permanent scarring effects of recessions. In other words, by reducing output hysteresis, temporary macroeconomic stabilization policy can have permanent positive effects on output. Again this is confirmed by recent empirical evidence as discussed in Cerra, Fatas, and Saxena (2020).

A case in point is the development of fiscal policy and potential GDP in advanced economies in the aftermath of the Great Recession. The large fiscal stimulus packages implemented during the Great Recession left countries with a large burden of fiscal debt. Many dealt with this through fiscal consolidation already early in the recovery. Fatas and Summers (2018) empirically analyze this episode and find a strong correlation between fiscal consolidations and revisions to potential GDP, i.e., those countries that improved their fiscal balances more strongly, experienced larger downwards revisions in their long-run growth prospects.

The analysis in this paper builds on the model developed in Lechthaler and Tesfaselassie (2020) and extends the framework to incorporate a fiscal sector. In the model unemployed workers and firms with vacancies are brought together by a matching function in the spirit of Diamond, Mortensen and Pissarides (see, e.g., Pissarides, 2000). Relative to the standard New Keynesian variants of the model (see, e.g., Walsh, 2005), the model has two additional features: i) skill loss from long-term unemployment that necessitates

Figure 1: US real GDP vs pre-recession trend. Source: FRED
(re)training-investments from the firms hiring these workers; ii) a learning-by-doing mechanism by which higher employment leads to a higher level of human capital. Lechthaler and Tesfaselassie (2020) have shown that the interplay of both factors is able to generate large and persistent fluctuations in output and provides a potential explanation for the missing-(dis-)inflation puzzle.\footnote{The missing-(dis-)inflation puzzle refers to the surprisingly weak response of inflation during both the Great Recession and the ensuing recovery.}

For our baseline scenario, we calibrate the model to broadly match important stylized facts of the US labor market and analyze the effects of temporary shocks to government consumption, financed by a lump sum tax. As is common in the literature, the spending shock raises aggregate demand, and thus employment, output and inflation. However, the novel features of our model imply important deviations from the standard approach. First, the presence of skill loss raises the fiscal multiplier. The increase in aggregate demand that follows the fiscal stimulus makes it easier for unemployed workers to find a job. This lowers the share of long-term unemployed in the pool of searching workers, and thus reduces the training costs of hiring firms, effectively lowering the expected hiring cost. This in turn raises the incentives to post vacancies even further leading to a beneficial feedback loop between lower training costs, stronger hiring, and lower long-term unemployment.

Second, the presence of endogenous growth implies that temporary fiscal policy shocks have permanent level effects. The increase in employment enhances learning-by-doing and thus leads temporarily to stronger growth in human capital and productivity. In the long run, the growth rate is not affected, but human capital is permanently at a higher level than without the stimulus, and thus the economy will grow on a higher trajectory.

Both factors complement each other to generate output and unemployment multipliers that are substantially larger than in standard models both in the short run and especially in the long run, and that are closer to the recent empirical evidence as, e.g., in Monacelli, Perotti, and Trigari (2010). Interestingly, even though multipliers are larger, the increase in inflation is smaller. This is the case because both mechanisms dampen the increase in
the real marginal cost that follows the fiscal stimulus.

Our paper connects to two distinct literatures—on the one hand the literature on fiscal multipliers and on the other hand the literature on endogenous growth and output hysteresis. The voluminous literature on fiscal multiplier in recent years was led by the massive government stimulus packages undertaken during the Great Recession. Most of this work is done within the standard real business cycle (RBC) or New-Keynesian framework (as surveyed in, e.g., Hall, 2009) and thus abstracts from search and matching frictions in the labor market, from endogenous growth or from hysteresis effects in the labor market.

Recent papers that include labor market frictions in their analysis of fiscal multipliers are Monacelli, Perotti, and Trigari (2010), Campolmi, Faia, and Winkler (2011) or Faia, Lechthaler, and Merkl (2013). Monacelli, Perotti, and Trigari (2010) analyze output and unemployment multipliers in a model with search and matching frictions and price rigidity and show that multipliers in their model are much smaller than what they find in their empirical analysis. Campolmi, Faia, and Winkler (2011) use a model with search and matching frictions and endogenous participation to show that hiring subsidies, contrary to increases in government spending, deliver large multipliers, even with distortionary taxation. Faia, Lechthaler, and Merkl (2013) use a labor-selection model and find that multipliers are small but positive for government spending and large for hiring subsidies and short-time work. None of these paper considers endogenous growth as we do.

Closer to our paper Engler and Tervala (2018) use a New-Keynesian model and show that recessions are deeper and more persistent, and that the fiscal output multiplier is much larger in the presence of learning-by-doing than in traditional models. Unlike us Engler and Tervala (2018) assume a perfectly competitive labor market, and thus neither consider unemployment multipliers nor the skill loss effect that complements the endogenous growth channel.

Blanchard (2017) and Cerra, Fatas, and Saxena (2020) provide recent surveys of the literature on output hysteresis and endogenous growth in business cycle models. Follow-
ing Stadler (1990), a prominent approach to model hysteresis is the learning-by-doing mechanism in which human capital and thus productivity depends on employment. For example, Chang, Gomes, and Schorfheide (2002) show that this mechanism improves the performance of an RBC-model with respect to output and hours worked. Jordà, Singh, and Taylor (2020) analyze the effect of monetary policy shocks in the presence of hysteresis driven by learning-by-doing, and Engler and Tervala (2018) analyze similar channels but focusing on fiscal policy. All these papers use a similar learning-by-doing mechanism as we do, but assume a perfectly competitive labor market.

Likewise, the issue of skill loss during unemployment has received more attention following the persistence of unemployment during the Great Recession. Esteban-Pretel and Faraglia (2010) analyze skill loss during unemployment in a New-Keynesian model and show that the skill loss mechanism helps to explain the magnitude of the response of unemployment to monetary shocks. Acharya, Bengui, Dogra, and Wee (2018) analyze monetary policy in a model with a zero-lower bound constraint and hysteresis effects whereby skill loss generates multiplicity of steady-state unemployment. Walentin and Westermark (2018) quantify the importance of human capital dynamics and job mismatch in slowing down the recovery from the Great Recession. They find that the increase in unemployment during 2007-2009 had long-lasting effects through the skill loss it induced, mainly in terms of increased unemployment and reduced GDP. None of these studies considers endogenous growth as we do.

The remainder of the paper is organized as follows. In section 2 we present the baseline model, which incorporates endogenous growth and skill loss from long-term unemployment into the standard New-Keynesian model featuring labor market frictions. We also derive the key aggregate relationships as well as setting out the assumptions regarding monetary and fiscal policy rules. In section 3 we discuss the model calibration based on the US labor market and present simulation results using impulse response functions and report both short-run and long-run fiscal multipliers. We also discuss the transmission

\[2\]Esteban-Pretel and Faraglia (2010) and Acharya, Bengui, Dogra, and Wee (2018) assume that new hires are equally productive as existing workers once a fixed training cost to ‘upgrade’ the human capital of new hires has been paid.
mechanism of fiscal policy so as to aid the intuition behind our main results. In section 4 we discuss sensitivity of our quantitative and qualitative results under alternative assumptions about fiscal policy rules and an alternative model calibration based on the European labor market. Section 5 gives a summary and concluding remarks.

2 The model framework

Following Lechthaler and Tesfaselassie (2020) endogenous growth is assumed to arise from learning-by-doing externalities in human capital accumulation. In particular, the growth of human capital depends on aggregate employment and thus on the business cycle. As is common in the endogenous growth literature the change in human capital is linear in the level of human capital. It is the absence of diminishing returns in human capital accumulation that allows the model to generate sustained growth. Importantly, a temporary decline in the rate of productivity growth implies permanently lower levels of aggregate human capital and aggregate output. Furthermore, we assume that long-term unemployed workers experience skill obsolescence and thus need training before becoming productive at a new job.

These features are embedded into a two sector New-Keynesian model with search and matching unemployment, as pioneered by Walsh (2005). Firms in the wholesale sector face labor search and matching frictions, combine raw labor and human capital to produce output and sell their output to the retail sector in a perfectly competitive market. Retail firms transform the wholesale good into a differentiated final good, which is sold in a monopolistically competitive market. Retail firms set prices under Calvo-type nominal price staggering. Each household consists of a continuum of employed and unemployed (and searching) workers who pool their income.\(^3\)

The fiscal authority follows a balanced budget every period and finances its expenditure on final good with lump-sum taxes as well as distortionary labor income taxes. Monetary

\(^3\)As is well-known locating labor market frictions and nominal price rigidities in different sectors as well as income pooling by workers make the model tractable.
policy follows a simple Taylor-type rule.

2.1 Labor market and human capital dynamics

We start by describing the aggregate relationships in the labor market within the wholesale sector and the endogeneity of aggregate human capital dynamics. The size of the labor force is normalized to one. At the beginning of each period a fraction $\delta$ of previously employed workers are separated from their jobs. These unemployed workers immediately engage in job search. As a result aggregate employment evolves according to the dynamic equation

$$N_t = (1 - \delta)N_{t-1} + M_t,$$

where $M_t$ is the number of newly formed matches in period $t$, which become productive immediately. Moreover, the number of searching workers in period $t$ is given by

$$S_t = 1 - N_{t-1} + \delta N_{t-1} = 1 - (1 - \delta)N_{t-1},$$

and the unemployment rate after hiring has taken place is $u_t = 1 - N_t$.

The number of newly created matches, $M_t$, is determined by a constant returns-to-scale matching function, with the number of searching workers, and the number of posted vacancies, $V_t$, as its arguments

$$M_t = \mu S_t^\alpha V_t^{1-\alpha},$$

where $\mu > 0$ is a scale parameter describing the efficiency of the labor market and $\alpha > 0$ is the elasticity of the matching function. Dividing equation (3) by $V_t$ and defining labor market tightness as $\theta_t \equiv V_t/S_t$ we can write the vacancy filling rate as

$$q(\theta_t) \equiv \frac{M_t}{V_t} = \mu \theta_t^{-\alpha}.$$

Learning-by-doing as a driver of endogenous growth is introduced in a standard way: higher aggregate economic activity (higher aggregate employment) imposes a positive ex-
ternality on the accumulation of aggregate human capital due to enhanced opportunities of learning-by-doing. Let $H_t$ denote aggregate human capital in the economy, which can have the interpretation of aggregate knowledge. Its dynamic development is given by

$$H_{t+1} = (1 - \delta_H)H_t + BN_tH_t,$$

where $\delta_H$ is the depreciation rate of human capital and $B > 0$ is a scale parameter. One can rewrite equation (5) in terms of the gross growth rate of human capital

$$\Gamma_{H,t+1} = \frac{H_{t+1}}{H_t} = 1 - \delta_H + BN_t,$$

which shows that a fall in aggregate employment today leads to a fall in future productivity growth.

### 2.2 Households

There is a representative household with a continuum of members over the unit interval. The utility function of the household features external habit persistence

$$U_t = \frac{(C_t - h_p\bar{C}_{t-1})^{1-\sigma} - 1}{1 - \sigma},$$

where $\sigma > 0$, $0 \leq h_p \leq 1$ and $\bar{C}_t$ represents aggregate consumption, which in equilibrium is equal to $C_t$. The presence of consumption habits play a key role in generating a boom accompanied by a disinflation in response to a positive news shock, a pattern consistent with the data Christiano, Ilut, Motto, and Rostagno (see, e.g., 2010). It also helps generate government spending multipliers of similar magnitudes as those in the literature (e.g., Monacelli, Perotti, and Trigari (2010)).

Household consumption $C_t$ is a Dixit-Stiglitz composite of a continuum of differentiated goods $C_t = \left(\int_0^1 C_{k,t}^{1/\mu_p} dk\right)^{\mu_p}$ where each differentiated good is indexed by $k$, $\mu_p = \frac{\epsilon}{\epsilon - 1}$ and $\epsilon$ is the elasticity of substitution between goods. Optimal consumption allocation across goods gives the demand equation $C_{k,t} = \left(\frac{P_{k,t}}{P_t}\right)^{-\epsilon} C_t$ where $P_t = \left(\int_0^1 P_{k,t}^{1-\epsilon} dk\right)^{\frac{1}{1-\epsilon}}$ is the price index.
In a given period a fraction $N_t$ of household members are employed by firms and earn a nominal wage $W_t$. The rest earn nominal unemployment benefits of $P_tu_bH_t$, $u_b > 0$, and search for work. As common in the literature, we assume that the income is pooled within the household so that unemployed workers do not face lower consumption than employed workers. The household maximizes the lifetime utility $E_t \sum_{i=0}^{\infty} \beta^i U_{t+i}$, where $\beta$ is the subjective discount factor. The household’s budget constraint is

$$P_tC_t + B_t = (1 - \tau^n_t)W_tN_t + P_tu_bH_t(1 - N_t) + R_{t-1}B_{t-1} + D_t - P_tT_t,$$ (8)

where $R_t$ is the nominal interest rate on bond holdings $B_t$, $D_t$ is aggregate nominal profit from ownership of firms, $\tau^n_t$ is a wage tax and $T_t$ is real lump-sum taxes.

It is straightforward to derive the familiar consumption Euler equation

$$1 = E_t \left( Q_{t,t+1} \frac{R_t}{\Pi_{t+1}} \right),$$ (9)

where $\Pi_t = P_t/P_{t-1}$ is gross inflation rate and $Q_{t,t+1}$ is the household’s stochastic discount factor. Using the utility function (7) we rewrite $Q_{t,t+1}$ in stationary variables,

$$Q_{t,t+1} \equiv \beta \left( \frac{C_{t+1} - h_p C_t}{C_t - h_p C_{t-1}} \right)^{-\sigma} \equiv \beta \left( \frac{\Gamma_{H,t+1}c_{t+1} - h_p c_t}{c_t - h_p \Gamma_{H,t}^{-1}c_{t-1}} \right)^{-\sigma},$$ (10)

where $c_t = C_t/H_t$ is stationary.

### 2.3 Firms

Next we describe the structure of the intermediate goods sector followed by the final goods sector.

#### 2.3.1 Intermediate goods sector

Firms in the intermediate goods sector face standard search and matching frictions as well as frictions related to skill obsolescence and associated training costs incurred for
skill upgrading. There is an unlimited number of potential entrants that need to post a vacancy at real cost $H_t \kappa$ to have the chance to find a worker and enter the market. In addition, potential entrants anticipate to pay training costs if the matched worker needs skill upgrade. Following Pissarides (2009), Acharya, Bengui, Dogra, and Wee (2018) and Lechthaler and Tesfaselassie (2020) training costs are assumed to be sunk at the time of wage bargaining.

At the vacancy creation stage the expected training cost per hired worker $TC_t$ is given by

$$TC_t = \frac{[1 - \theta_{t-1} q(\theta_{t-1})]u_{t-2}}{S_t} (\chi H_t),$$

where the term $(1 - \theta_{t-1} q(\theta_{t-1}))u_{t-2}$ is the number of job seekers in period $t$ whose last job was in period $t-3$ or earlier. This term divided by $S_t$ thus represents the probability that a firm matches with a job seeker who as of period $t$ had been unemployed for at least two periods (where a period represents a quarter), and thus needs to upgrade the worker’s skill at a cost equal to $\chi H_t$. By contrast, a searching worker in period $t$ whose last job was in period $t-2$ or $t-1$ (i.e., had been unemployed for at most one period) does not need skill upgrade. These two types of workers maybe differentiated as long-term unemployed vs. short-term unemployed.

Note that we can rewrite the definition of job seekers, as given in equation (2), in term of the mass of short-term and long-term unemployed

$$S_t = \delta N_{t-1} + \left[1 - \theta_{t-1} q(\theta_{t-1})\right]\delta N_{t-2} + [1 - \theta_{t-1} q(\theta_{t-1})]u_{t-2},$$

A detailed discussion of the standard search and matching model can be found in, e.g., Pissarides (2000).

The presence of $H_t$ ensures that along the balanced growth path the vacancy posting cost grows at the same rate as aggregate labor productivity. Without this assumption vacancies would overtime converge towards infinity and unemployment towards zero, since the ratio of vacancy creation costs to labor productivity would converge towards zero. A similar argument applies to the training cost to be defined further below.

Pissarides (2009) argues that "the attractive feature of making them sunk, ..., is that they can be interpreted as a component of the cost of frictions that characterize search models, so they are an alternative way of calibrating frictions to the conventional proportional [vacancy posting] costs." Unlike Pissarides (2009) training costs are not fixed but depend on the share of long-term unemployment in the pool of job searchers. As a result training costs are a predetermined variable, as in Acharya, Bengui, Dogra, and Wee (2018).
where the last term represents the pool of long-term unemployed and the sum of the first two terms represents the pool of short-term unemployed. The ratio of short-term unemployed $STU_t$ to long-term unemployed $LTU_t$ is

$$\frac{STU_t}{LTU_t} = \frac{\delta N_{t-1} + [1 - \theta_{t-1}q(\theta_{t-1})]u_{t-2}}{\delta N_{t-2} + [1 - \theta_{t-1}q(\theta_{t-1})]u_{t-2}} = \frac{1}{\left[1 - \theta_{t-1}q(\theta_{t-1})\right]} \frac{\delta N_{t-1}}{u_{t-2}} + \frac{\delta N_{t-2}}{u_{t-2}}.$$  \hspace{1cm} (13)

Equation (13) shows that an adverse shock in period $t-1$ that lowers employment $N_{t-1}$ and the job-finding rate $\theta_{t-1}q(\theta_{t-1})$ also increases the share of long-term unemployment in total job seekers in period $t$ and thus the expected training cost, as given in equation (11).

Each firm can employ only one worker and produces with aggregate human capital $H_t$. Since training costs are sunk, new and continuing workers receive the same wage rate. Let $J_t$ denote the value of an existing match. The value of a vacancy is then given by $q(\theta_t)(J_t - TC_t - \kappa H_t)$. Free entry of firms drives down the value of a vacancy to zero so that

$$\kappa H_t = q(\theta_t)(J_t - TC_t)$$  \hspace{1cm} (14)

which is the standard vacancy creation condition, adjusted for the presence of a training cost and a balanced growth path. The cost of posting a vacancy equals the net benefit of posting a vacancy, the potential profits that can be earned in case the search for a worker was successful. If the cost of posting a vacancy were lower than the expected profit of posting a vacancy, new vacancies would be posted, lowering the vacancy filling rate and thereby expected profits until the incentive to post further vacancies vanishes. Likewise, an increase in the training cost has similar effects on the incentive to post vacancies. But crucially, the training cost depends on the probability that a new hire comes from the long-term unemployed who need skill upgrading.

Active firms in this sector face a perfectly competitive output market. Let $P^I_t$ denote the nominal market price and $p^I_t \equiv P^I_t / P_t$ the real market price. Then the value of a filled job
is defined as

$$J_t = a_t H_t p_t^I - w_t + (1 - \delta) E_t \{ Q_{t,t+1} J_{t+1} \}$$

where $a_t$ is total factor productivity (TFP) and $w_t = W_t / P_t$ is real wage. The value of a firm consists of contemporaneous profits plus the expected future value of the match discounted by the appropriate discount factor. Combining equations (14) and (15) the vacancy creation condition can be written as

$$\frac{\kappa H_t}{q(\theta_t)} + TC_t = a_t H_t p_t^I - w_t + (1 - \delta) E_t \left\{ Q_{t,t+1} \left( \frac{\kappa H_{t+1}}{q(\theta_{t+1})} + TC_{t+1} \right) \right\}$$

where $\kappa H_t / q(\theta_t)$ is the expected vacancy posting cost per hired worker, and thus $\kappa H_t / q(\theta_t) + TC_t$ is expected cost of hiring a worker. Equation (16) says that in equilibrium the expected cost of hiring a worker must equal the contemporaneous profits generated by a worker plus the discounted savings in future hiring costs. A positive productivity shock increases the match surplus, which induces more job creation until market tightness $\theta_t$ rises and the probability of filling a job $q(\theta_t)$ falls sufficiently so as to keep the value of a vacant job at zero. It is important to note that the training cost is a predetermined endogenous variable ($TC_t$ is given as of period $t$ but responds to shocks with a one-period lag).

Dividing equation (16) by the growing labor productivity $H_t$ we get a stationary version of the vacancy creation condition

$$\frac{\kappa}{q(\theta_t)} + tc_t = a_t H_t p_t^I - \frac{w_t}{H_t} + (1 - \delta) \beta E_t \left\{ \left( \frac{c_{t+1}}{c_t} \right)^{-1} \left( \frac{\kappa}{q(\theta_{t+1})} + tc_{t+1} \right) \right\}$$

where $tc_t \equiv TC_t / H_t$. From equation (17) we see that in response to a positive TPF-shock (i.e., higher values of $a_t$), firms post more vacancies. As in the standard search and matching model, the resulting increase in labor market tightness increases the average duration of vacancies, and thus raises the expected hiring cost. However, since vacancy posting costs are only part of the hiring cost, and since the other part, training costs, are predetermined the total expected hiring cost does not increase in proportion to the increase in the average duration of vacancies. Thus the presence of sunk training costs
has an amplification effect on vacancy creation and market tightness.

Note also that, as aggregate vacancies rise, both unemployment and the future share of long-term unemployment in total unemployment fall. The latter implies that firms expect future training costs, \( tc_{t+1} \), to decline. This effect alone reduces the continuation value of a match and thus lowers the incentive to post a vacancy.

The wage rate is set under the standard assumption of Nash bargaining. Moreover, as remarked above, wage bargaining is assumed to happen after training costs have been paid, so that new and continuing workers receive the same wage rate. The real value to the household of an employed worker is given by

\[
V^e_t = (1 - \tau^n_t)w_t + E_t \left\{ Q_{t,t+1} \left[ (1 - \delta (1 - \theta_{t+1}q(\theta_{t+1}))V^e_{t+1} + \delta (1 - \theta_{t+1}q(\theta_{t+1}))V^u_{t+1} ) \right] \right\} 
\]  

(18)

where \( \theta_{t+1}q(\theta_{t+1}) = M_{t+1}/S_{t+1} \) is an unemployed worker’s job finding rate. The corresponding real value of an unemployed worker is given by

\[
V^u_t = u_b H_t + E_t \left\{ Q_{t,t+1} \left[ \theta_{t+1}q(\theta_{t+1})V^e_{t+1} + (1 - \theta_{t+1}q(\theta_{t+1}))V^u_{t+1} \right] \right\}
\]  

(19)

Thus the household surplus from an employment relationship is given by

\[
S^h_t = V^e_t - V^u_t = (1 - \tau^n_t)w_t - u_b H_t + (1 - \delta)E_t \left\{ Q_{t,t+1} (1 - \theta_{t+1}q(\theta_{t+1}))S^h_{t+1} \right\}
\]  

(20)

Given that in equilibrium the value of a vacancy is zero, the firm’s surplus is equal to \( J_t \). Under Nash bargaining the optimal surplus sharing rule is given by \( S^h_t = [(1 - \nu)/\nu](1 - \tau^n_t)J_t \), where \( \nu \) is the bargaining power of the firm and \( J_t \) satisfies equation (14). Using the surplus sharing rule to substitute out \( S^h_t \) in equation (20) and in turn using equation (16) to substitute out \( \kappa/q(\theta_t) \) gives, after rearranging, the wage setting equation

\[
w_t = \frac{\nu}{1 - \tau^n_t}u_b H_t + (1 - \nu)a_tHp_t^l \\
+ (1 - \nu)(1 - \delta)E_t \left\{ Q_{t,t+1} \left( 1 - (1 - f_{t+1}) \frac{1 - \tau^n_{t+1}}{1 - \tau^n_t} \right) \left( \frac{\kappa H_{t+1}}{q(\theta_{t+1})} + TC_{t+1} \right) \right\}
\]  

(21)
where $f_{t+1} = \theta_{t+1} q(\theta_{t+1})$. Equation (21) can be rewritten in stationary form as

$$w^d_t = \frac{\nu}{1 - \tau^*_{t} u_t} + (1 - \nu) a_{t} p^f_t$$

$$+ (1 - \nu)(1 - \delta) \beta E_t \left\{ \frac{c_t}{c_{t+1}} \left( 1 - \frac{\tau^*_{t+1}}{1 - \tau^*_{t}} \right) \left( \frac{\kappa}{q(\theta_{t+1})} + t c_{t+1} \right) \right\}$$

where $w^d_t \equiv w_t / H_t$.

2.3.2 Final goods sector

Each firm $k$ in the final goods sector produces a differentiated final good using a linear technology $Y_{k,t} = Y^f_{k,t}$ implying that the firm’s real marginal cost, $m_{c_k,t}$, is given by $p^f_t$. Price setting is subject to Calvo-type price staggering, where $\omega$ is the fraction of firms whose prices are fixed in any given period. Let $P_{k,t}$ denote firm $k$’s output price. Each firm $k$ maximizes lifetime profit $E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+i} (P_{k,t}/P_{t+i} - p^*_t) Y_{k,t+i}$ subject to the total demand for good $k$, $Y_{k,t+i} = (P_{k,t}/P_{t+i})^{-\epsilon} Y_{t+i}$, where $Y_{t+i} = C_{t+i} + G_{t+i} + H_{t+i} \kappa V_{t+i} + TC_{t+i} q(\theta_{t+i}) V_{t+i}$ is total aggregate demand that includes consumption, government spending $G_t$, the vacancy posting costs and training costs. The resulting optimal price is

$$p^*_t = \mu_p \frac{E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+i} p^f_t Y_{t+i} \left( \frac{P_{t+i}}{P_{t}} \right)^{\epsilon} \left( \frac{P_{t+i}}{P_{t}} \right)^{\epsilon-1}}{E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+i} Y_{t+i} \left( \frac{P_{t+i}}{P_{t}} \right)^{\epsilon-1}},$$

where $p^*_t \equiv P^*_t / P_t$, $y_t = Y_t / H_t$ and $\mu_p$ is the price markup in the absence of price staggering. Endogenous growth feeds back into optimal pricing through two counteracting effects. Lower expected growth implies a lower discount rate (higher stochastic discount factor) but also lower expected future demand growth.

Equation (23) can be rewritten as

$$p^*_t = \mu_p \frac{F_{n,t}}{F_{d,t}},$$

where $F_{n,t}$ and $F_{d,t}$ are auxiliary variables given by

$$F_{n,t} = p^f_t y_t c^{-1}_t + \omega \beta \left( \frac{c_{t+1}}{c_t} \right)^{-1} \Pi_{t+1} F_{n,t+1}, \quad (25)$$
and

\[ F_{d,t} = y_t c_t^{\epsilon - 1} + \omega \beta \left( \frac{c_{t+1}}{c_t} \right)^{-1} \Pi_{t+1}^{\epsilon - 1} F_{d,t+1}. \]  

(26)

Under Calvo-type price staggering the aggregate price index can be rewritten as

\[ 1 = (1 - \omega) p_t^{\epsilon(1-\epsilon)} + \omega \Pi_t^{\epsilon - 1}. \]  

(27)

Aggregating both sides of the market clearing condition for the intermediate good and using the demand equation for the final good \( k \) leads to a relationship between aggregate final output \( y_t \) and intermediate good output \( y'_I \),

\[ y'_I = \Delta_t y_t, \]  

(28)

where \( \Delta_t = \int_0^1 (P_{k,t}/P_t)^{-1} df \) is a measure of price dispersion, which can be rewritten as

\[ \Delta_t = (1 - \omega) p_t^{\epsilon(1-\epsilon)} + \omega \Pi_t^{\epsilon - 1} \Delta_{t-1}. \]  

(29)

As aggregate output in the intermediate good sector is equal to aggregate employment, Eq. (28) can be rewritten as

\[ N_t = \Delta_t y_t. \]  

(30)

### 2.4 Monetary and fiscal policies

Closing the model requires specification of monetary and fiscal policies. As in Lechthaler and Tesfaselassie (2020) the central bank is assumed to follow a simple policy rule by adjusting the nominal interest rate in response to deviations of inflation and output growth from their respective target levels, \( \Pi \) and \( \Gamma_Y \), where the latter is equal to steady state output growth consistent with steady state inflation.

\[ \frac{R_t}{\bar{R}} = \left( \frac{\Pi_t}{\Pi} \right)^{\phi_{\pi}} \left( \frac{Y_t/Y_{t-1}}{\Gamma_Y} \right)^{\phi_{\gamma}} \]  

(31)
where $\phi_\pi, \phi_y > 0$. Regarding the presence of output growth in the policy rule, Barsky and Sims (2009) show that the disinflationary nature of news shocks found in the data contradicts the implications of the standard New Keynesian model augmented with the standard policy rule that responds to the output gap. They show that with a policy rule that responds to output growth the model does better at fitting the empirical evidence.

For our benchmark simulations we follow the standard approach in assuming that the government follows a balanced budget and finances its stimulus using lump-sum taxes (see, e.g., Bilbiie, 2011). In section 4 we also consider cases where the government must finance a fraction of its expenditure using distortionary labor income taxes. The government budget identity in stationary form is given by

$$g_t + \kappa V_t + u_b(1 - N_t) = \tau^n w_t N_t + \tau_t$$

(32)

where $\tau_t = T_t/H_t$. The left-hand side of the equation is the sum of government outlays while the right-hand side of the equation is the sum of government sources of income. As is standard, government spending follows an AR(1) process

$$g_t = \rho g_{t-1} + \epsilon_t$$

(33)

where the innovation $\epsilon_t$ is i.i.d. with zero mean and constant variance.

Finally, the aggregate resource constraint in stationary form is given by

$$y_t = c_t + g_t + \kappa V_t + tc_t q(\theta_t) V_t.$$  

(34)

where the last term represents aggregate training costs.

### 3 Main results

#### 3.1 Calibration

The model is calibrated similar to Lechthaler and Tesfaselassie (2020) but allowing for the presence of the government sector. Table 1 provides the details. The annualized steady
### Table 1: Parameter configuration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calibrated values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>subjective discount factor</td>
</tr>
<tr>
<td>$\omega$</td>
<td>fraction of non-optimizing firms</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>elasticity of substitution between final goods</td>
</tr>
<tr>
<td>$\Gamma_H$</td>
<td>steady state growth</td>
</tr>
<tr>
<td>$h_p$</td>
<td>degree of habit persistence</td>
</tr>
<tr>
<td>$\delta_H$</td>
<td>human capital depreciation rate</td>
</tr>
<tr>
<td>$\delta$</td>
<td>job separation rate</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>elasticity of the matching function</td>
</tr>
<tr>
<td>$\nu$</td>
<td>firm’s share of surplus</td>
</tr>
<tr>
<td>$u_b$</td>
<td>unemployment benefit</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>vacancy posting cost</td>
</tr>
<tr>
<td>$\chi$</td>
<td>training cost</td>
</tr>
<tr>
<td>$B$</td>
<td>learning-by-doing coefficient</td>
</tr>
<tr>
<td>$\phi_p$</td>
<td>inflation coefficient</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>output growth coefficient</td>
</tr>
<tr>
<td>$g$</td>
<td>steady state government spending</td>
</tr>
<tr>
<td>$\tau^n$</td>
<td>steady state labor tax</td>
</tr>
</tbody>
</table>

State growth rate (respectively, inflation rate) of the economy is set at 3% (respectively, 2%). The steady state government-to-output ratio is set to zero so as to avoid the issue that a shock that permanently affects output also permanently affects government spending if the government-to-output ratio is to remain constant. It also aids comparability to the standard model with exogenous growth in which temporary shocks never affect the long-run growth trajectory of government spending. For similar reasons, we set the steady state labor tax equal to zero. In section 4 we provide robustness with respect to these assumptions.

The elasticity of the matching function $\alpha$ is set at 0.5, and the job separation rate $\delta$ is set at 0.1, values that are common in the literature (see, e.g., Pissarides (2009)). We impose the Hosios condition for efficiency in the absence of sunk training costs and learning-by-doing externalities, so that the firm’s share of surplus $\nu$ is equal to the elasticity of the matching function with respect to vacancies $1 - \alpha$.\(^8\) The scale parameter in the matching function $\mu$ is set such that the steady state job-finding rate is 0.7, as in Blanchard and Gali.

---

\(^8\)Note that in our model the Hosios condition is not sufficient for efficiency due to additional externalities related to endogenous growth and training.
(2010), and the steady state job-filling rate is 0.9, as in Andolfatto (1996) and Arsenau and Chugh (2012). The chosen values for the job-finding rate and the job separation rate, as well as the definition of job seekers, imply a steady state unemployment rate \( u \) of 0.04 and a steady state employment rate \( N \) of 0.96.

We target a steady state ratio of training costs to vacancy posting costs equal to 0.3, which is at the lower end of values considered in Pissarides (2009).\(^9\) The training cost parameter \( \chi \) and the cost of posting a vacancy \( \kappa \) are set consistent with the resulting steady state solution of the model. The scale parameter in the human capital accumulation equation \( B \) is consistent with the steady state annualized growth rate and the steady state employment rate. The human capital depreciation rate \( \delta_H \) is set at 0.019, as in Jones, Manuelli, and Stachetti (2000). For the model simulation, the autocorrelation coefficients \( \rho_g \) is set equal to the standard value of 0.9.

Following Blanchard and Galí (2010) the steady state aggregate hiring costs (i.e., the sum of vacancy posting and training costs) represent one percent of steady state aggregate output. Given the parameters and steady state targets set as above the implied value of the unemployment benefit parameter \( u_b \) is 0.75 (the corresponding replacement rate is 0.91).

### 3.2 Government spending multipliers

Figure 2 shows the impulse response functions for output, productivity, the inflation rate, the nominal interest rate, unemployment, and the share of unskilled workers in total job seekers, to a government spending shock equal to 1% of GDP. The impulse response named ‘output hysteresis’ shows the gap between actual output and output in the absence of the stimulus, expressed as a percentage of the latter. The impulse response named ‘productivity hysteresis’ is defined analogously. The solid line represents our baseline model with endogenous growth and skill obsoles-

\(^9\) We think the chosen value is reasonable, as Pissarides considers fixed matching costs that may also include "costs of finding out about the qualities of the particular worker, of interviews, and of negotiating with her".
cence from unemployment while the dashed line shows the standard model with exogenous growth and no skill obsolescence (basically the standard search and matching model with nominal rigidities).

In both models, an increase in government spending stimulates aggregate demand, which in turn raises the demand for workers, so that firms post more vacancies, workers find it easier to find a job, and unemployment goes down. In the baseline model (solid line) output and productivity increase more strongly while unemployment decreases more strongly than in the standard model (dashed line). On the one hand, increased hiring activity leads to a reduction in the share of long-term unemployed among all job-seekers. In our baseline model, this leads to a reduction in hiring costs because fewer workers need retraining. This pushes up the incentives to post vacancies even further, lowers marginal cost and thus leads to a stronger increase in employment and production, setting off a virtuous circle. On the other hand, the expansion in employment leads to stronger productivity growth through learning-by-doing, which again induces a stronger expansion in employment and production in our baseline model. Furthermore, the level of output is permanently higher, a hysteresis-like phenomenon. The reason is that the gain in human capital is permanent so that productivity rises to a permanently higher level (although the long-run growth rate is not affected). By contrast, in the standard model (dashed line), the rise in output above trend is only temporary, as output returns back to the trend level that is exogenous in the standard model.

The rise in aggregate demand leads also to an increase in inflation due to the rise in the real marginal cost of production. While this is a standard result, in the baseline model inflation increases by less, and is less persistent, despite output and employment increasing more strongly than in the standard model. The reason is that the increase in productivity and the decrease in training costs imply that the marginal cost rises less strongly than in the standard model. Thus, unlike traditional business cycle models, our model implies a flatter Phillips curve,\textsuperscript{10} a result that is in line with the recent empirical evidence in Hazell, Herreño, Nakamura, and Steinsson (2020) who show that the Phillips

\textsuperscript{10}See also the related discussion in Lechthaler and Tesfasellassie (2020).
curve is indeed flatter than previously thought. As a consequence of the weaker reaction of inflation in the baseline model, the nominal interest rate also rises by less, and is less persistent than in the standard model.

The positive permanent effects of government spending are especially relevant in light of recent evidence that recessions can have permanent negative effects (see Cerra, Fatas, and Saxena (2020) for a recent survey). Our results suggest that fiscal stimulus can offset the permanent negative effects of recessions, at least partially. This has direct bearings on the forceful reaction of fiscal policy to the deep recession following the outbreak of the global COVID-19 pandemic. Also note that the positive permanent effects of government spending are in line with recent empirical evidence on government spending in Fatas and Summers (2018) and Gechert, Horn, and Paetz (2019). They are also in line with Ball, Mankiw, and Nordhaus (1999) and Ball (2009) who argue that hysteresis also works in response to positive shocks and not only in recessions, and with Jordà, Singh, and Taylor (2020) who find persistent effects for monetary policy shocks.

Turning next to government spending multipliers, Table 2 compares different measures of fiscal output multipliers in the baseline model with those in the standard model. The
The impact multiplier is calculated as the change in output in the impact period divided by the corresponding change in stimulus \((Y_0 - Y_0^0)/(G_0 - G_0^0)\), where \(Y_0^0\) denotes output in the impact period in the absence of the fiscal stimulus and likewise for \(G_0^0\). For calculating long-run multipliers we use two different approaches. The common practice in fiscal policy simulations is to calculate the long-run multiplier as the ratio of the sum of the discounted values of output changes to the corresponding values of the stimulus changes over a certain horizon \(j\), \(\sum_{t=0}^{j} Q^t(Y_t - Y_0^t)/\left(\sum_{t=0}^{j} Q^t(G_t - G_0^t)\right)\), where \(Q\) is the relevant discount factor. However, empirical analyses typically focus on shorter horizons for cumulative effects and therefore ignore discounting. Thus we report multipliers on the two-years and four-years horizons without discounting (columns three and four) and a long-run multiplier with discounting (column five). Following Monacelli, Perotti, and Trigari (2010) we also report unemployment multipliers in Table 3 defined as \(\sum_{t=0}^{j} (u_t - u_0^t)/\left(\sum_{t=0}^{j} (G_t - G_0^t)\right)\).

\[\text{Table 2: Output multipliers} \]

<table>
<thead>
<tr>
<th>Model</th>
<th>Impact</th>
<th>Two years</th>
<th>Four years</th>
<th>Long-run (discounted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.95</td>
<td>0.98</td>
<td>1.17</td>
<td>2.50</td>
</tr>
<tr>
<td>Exogenous growth &amp; no training cost</td>
<td>0.81</td>
<td>0.55</td>
<td>0.45</td>
<td>0.41</td>
</tr>
</tbody>
</table>

\[\text{Table 3: Unemployment multipliers} \]

<table>
<thead>
<tr>
<th>Model</th>
<th>Impact</th>
<th>Two years</th>
<th>Four years</th>
<th>Long-run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>-0.94</td>
<td>-0.88</td>
<td>-0.94</td>
<td>-1.04</td>
</tr>
<tr>
<td>Exogenous growth &amp; no training cost</td>
<td>-0.80</td>
<td>-0.54</td>
<td>-0.45</td>
<td>-0.38</td>
</tr>
</tbody>
</table>

The baseline model yields a larger impact-multiplier than the standard model (without endogenous growth and training costs), but the difference is rather small. However, the longer the time horizon considered, the larger the difference between both models. In fact, while in the standard model the multiplier decreases for longer horizons, it increases in the baseline model. This is because most of the effect is front-loaded by construction in the standard model, whereas the permanent increase in human capital in the baseline model implies permanent output effects that last longer than the stimulus. The

\[\text{\footnote{We set } j = 1000 \text{ (i.e., 1000 quarters) for the long-run multiplier.}}\]
unemployment multipliers are also larger in the baseline model compared to the standard model. The difference in the long-run multipliers is substantial but smaller than for the output multipliers because in contrast to output unemployment is not permanently affected (our model features hysteresis in output but not in employment).

Our results are similar to Engler and Tervala (2018), who also study fiscal stimulus in a model with endogenous growth through learning-by-doing, but without labor market frictions and training costs. The output multipliers of the baseline model are also broadly in line with recent results from the empirical literature: e.g., Monacelli, Perotti, and Trigari (2010), IMF (2012), Auerbach and Gorodnichenko (2012) find cumulative output multipliers in the range of 1 to 1.5 over the range of two to five years, while Hall (2009) argues that the multiplier lies between 0.5 and 1. By contrast, the multipliers of the standard model are way too small (but in line with other papers using models without endogenous growth). Finally, in contrast to the benchmark model, the baseline model with endogenous growth is able to replicate the relatively large difference between output and unemployment multipliers reported by Monacelli, Perotti, and Trigari (2010), although the unemployment multipliers appear a bit too high compared to the 0.43 over two years reported in Monacelli, Perotti, and Trigari (2010).

3.3 Endogenous growth vs. training costs

As noted above, our baseline model features two separate but closely related deviations from the standard model, endogenous growth based on human capital and learning-by-doing and training costs related to the skill loss of long-term unemployed workers.\footnote{As shown in Lechthaler and Tesfaselassie (2020) both features are necessary to yield impulse responses that are broadly consistent with the recent empirical findings on the fall in productivity growth after deep recessions and the observed relative stability of inflation despite the pronounced fall in GDP during the Great Recession.} In order to see the role of each in isolation Figure 3 additionally shows the impulse responses of a model in which one of these features is shut down.

The dot-dash line in Figure 3 shows impulse responses when only the training cost channel is present, while the dotted line shows impulse responses when only the endogenous
growth channel is present. Three observations can be made from this figure. First, the training cost channel is key in the amplification of the rise in output and productivity, and the fall in unemployment, but not for output hysteresis. Second, the endogenous growth channel is responsible for output and productivity hysteresis, but is irrelevant for the dynamics of unemployment. Third, the long-run effect of endogenous growth is larger in the presence of training costs (solid line vs. dot-dash line) than in the absence of training costs (dotted line vs. dashed line), suggesting complementarity between the two channels.

Table 4 compares the short-run and the long-run government spending multipliers in the baseline model and the two limiting cases, namely, the case with exogenous growth and the case with no training costs. For ease of comparison we also show the standard model (exogenous growth and no training costs).

The table confirms the lessons drawn from figure 3. The presence of endogenous growth alone has virtually no effect for the short-run multiplier (0.82 vs. 0.81), while the presence of training costs alone increases the short-run multiplier considerably (0.93 vs. 0.81). Even though the presence of training costs does not imply output hysteresis, the long-run
multiplier when only training costs are present is slightly larger than the corresponding multiplier when only endogenous growth is present. This arises due to the larger effects on output in the immediate periods after the realization of the government spending shock (dot-dash line vs. dotted line of Figure 3). Again, the strong complementarity of both features is confirmed, by the large difference in the multiplier between the baseline model and both limiting cases (2.5 vs 1.02 and 1.0).

4 Sensitivity analysis

4.1 Distortionary taxes and debt financing

Our analysis thus far is based on the standard assumption of a balanced budget with lump-sum taxes financing government spending, and steady state government spending equal to zero, so that in the baseline model government spending, unlike output, is transitory. We now consider a more detailed government sector with positive government spending in steady state, and where the government levies a distortionary labor tax, $\tau_w$, to generate income, but can use borrowing to smooth taxation over time. Similar to Monacelli, Perotti, and Trigari (2010) and Leeper, Plante, and Traum (2010) the fiscal instruments of the government are assumed to follow simple rules that respond to

<table>
<thead>
<tr>
<th>Model</th>
<th>Short-run</th>
<th>Long-run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.95</td>
<td>2.50</td>
</tr>
<tr>
<td>Exogenous growth</td>
<td>0.93</td>
<td>1.02</td>
</tr>
<tr>
<td>No training cost</td>
<td>0.82</td>
<td>1.0</td>
</tr>
<tr>
<td>Exogenous growth &amp; no training cost</td>
<td>0.81</td>
<td>0.41</td>
</tr>
</tbody>
</table>
government debt and assure that the ratio of government debt-to-GDP is stationary:  

\[ \log \left( \frac{g_t}{g} \right) = \rho_g \log \left( \frac{g_{t-1}}{g} \right) - \phi_g \log \left( \frac{E_t b_{g,t+1}}{b_g} \right) + \epsilon_t \]  

(35)

\[ \log \left( \frac{\tau^w_t}{\tau^w} \right) = \phi_{\tau^w} \log \left( \frac{E_t b_{g,t+1}}{b_g} \right) \]  

(36)

where \( b_g \) is detrended government debt. Following Leeper, Plante, and Traum (2010) we target a steady state labor tax rate equal to \( \tau^w = 22\% \), a ratio of steady state government spending to GDP of \( g/y = 9\% \) and a steady state ratio of government debt to GDP to \( b_g/y = 34\% \). Given this calibration, the government’s steady state income from the labor tax exceeds its expenses on government spending, interest, and unemployment benefits. As in Leeper, Plante, and Traum (2010) this surplus is distributed via a lump-sum transfer \( \tau \) that also follows a fiscal rule:

\[ \log \left( \frac{\tau_t}{\tau} \right) = -\phi_\tau \log \left( \frac{E_t b_{g,t+1}}{b_g} \right) \]  

(37)

The fiscal-rule parameters are also calibrated in line with Leeper, Plante, and Traum (2010)’s estimation results: \( \phi_g = 0.23 \), \( \phi_{\tau^w} = 0.049 \), and \( \phi_\tau = 0.5 \), thus the labor tax reacts relatively little to changes in government debt. Finally, the government’s budget constraint is given by

\[ b_{gt} + \tau^w_t u_t^d N_t = u_t u_t + g_t + R_{t-1} b_{gt-1} \]  

(38)

The impulse response functions for an increase in government spending under this setup are shown in Figure 4. For comparison the figure also shows the impulse response functions for the baseline model with lump-sum taxation, recalibrated to use the same value of steady state government spending, \( g/y = 9\% \). As in Monacelli, Perotti, and Trigari (2010) we generate the impulse response functions such that the impact effects of government spending on output are identical.

\[^{13}\text{In contrast to Leeper, Plante, and Traum (2010) our fiscal rules react to expected debt rather than past debt because the latter would imply, an implausible decrease in the tax rate on impact. The reason is that, given the labor tax, the rise in inflation following the fiscal stimulus reduces real debt in the impact period. While the fiscal rule with expected debt implies a rise in the tax rate, the fiscal multipliers are almost identical whether expected debt or past debt enter the fiscal rules.}\]
The dynamics of aggregate variables following a positive shock to government spending are altered by the presence of the distortionary labor tax. Since the increase in government spending raises government debt, the distortionary labor tax also increases, albeit only slowly because of the slow rise in government debt. The increase in government debt also implies, via the government spending rule, a faster reduction in government spending relative to the case with lump-sum taxation. Both of these factors imply a smaller effect of the stimulus on output, productivity and unemployment, but primarily in the medium run, because the fiscal rules are slow to adapt. Moreover, with a distortionary labor tax the permanent effects of the fiscal stimulus on output and productivity is somewhat muted. Nevertheless, the main conclusions of our analysis are robust to the use of distortionary taxation.

Figure 4: Impulse responses to an increase in government spending of 1% of GDP in the presence of distortionary tax and government debt.

Table 5 compares the short-run and the long-run government spending multipliers in the baseline model (with only the lump-sum tax) and the alternative case with distortionary taxes. It can be seen, that distortionary taxation somewhat reduces the medium and long-
term multipliers (e.g., the long-run output multiplier is 2.2 whereas the corresponding one under lumpsum taxation is 2.35).\(^{14}\) Table 5 is also informative as to the effects of a permanent shock to government spending financed with lump-sum taxes. It can be seen that a higher level of government spending increases short-term multipliers but decreases long-term multipliers somewhat.

<table>
<thead>
<tr>
<th>Table 5: Fiscal multipliers with distortionary taxation (&quot;*&quot;: discounted)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model</strong></td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td><strong>Output mult.</strong></td>
</tr>
<tr>
<td>Lump-sum tax</td>
</tr>
<tr>
<td>Distortionary tax and debt</td>
</tr>
<tr>
<td><strong>Unemployment mult.</strong></td>
</tr>
<tr>
<td>Lump-sum tax</td>
</tr>
<tr>
<td>Distortionary tax and debt</td>
</tr>
</tbody>
</table>

4.2 Calibration based on European labor market

The baseline calibration reflects the characteristics of the US labor market. Here we consider alternative calibrations based on the European labor market. The targeted steady state job-filling rate is 0.7, as in Christoffel, Kuester, and Linzert (2009). Following Blanchard and Galí (2010) (i) the steady state unemployment rate is set at 10% and the job destruction rate at 0.04, so that the implied steady job-finding rate is 0.26, which is much lower than in the US calibration and (ii) the steady state ratio of hiring costs to output is identical to the US calibration, 0.01. The scale parameter in the human capital accumulation equation \(B\) is set at 0.029, which is consistent with the steady state annualized productivity growth and the steady state employment rate.

Regarding calibration of the training and vacancy posting costs, we examine two alternative approaches. In the first calibration, we target the same steady state ratio of training costs to vacancy posting costs as in the baseline case (i.e., 0.3). Then as before the unemployment benefits parameter \(u_b\), the training cost parameter \(\chi\) and the cost of posting

\(^{14}\) The difference appears bigger in figure 4, but note that, with distortionary taxation not only the output effect but also the stimulus is smaller, i.e., both the numerator and the denominator of the cumulative multiplier become smaller.
a vacancy $\kappa$ are set consistent with the steady state solution of the model. The implied values are, respectively, 0.75, 0.11 and 0.13 (thus $\chi$ is smaller while $\kappa$ is larger than the counterpart in the US calibration—0.26 and 0.07). In the second calibration, we fix the training cost parameter as in the baseline ($\chi = 0.26$) and calibrate the vacancy posting cost $\kappa$ consistent with the steady state solution of the model, which gives $\kappa = 0.08$.

Figure 5 shows the impulse responses of key variables to a fiscal stimulus under the two European calibrations, namely, the first calibration (dashed line) and the second calibration (dot-dashed line), as well as the baseline US calibration (solid line, replicated from Figure 2). Qualitatively, the three calibrations imply broadly similar patterns, including the presence of output and productivity hysteresis. Moreover, in quantitative terms, in both calibrations the impact effect on output, productivity and unemployment of fiscal stimulus are very similar. The difference between them lies in the longer run dynamics, as output and productivity hysteresis are weaker under the European calibration, more so under the first calibration (dashed line). This is because the decrease in unemployment is less persistent, which leads to a dampening of the learning-by-doing externality on productivity growth and long-run level of output.
Table 6: Output and unemployment multipliers ("*": discounted)

<table>
<thead>
<tr>
<th>Model</th>
<th>Impact</th>
<th>Two years</th>
<th>Four years</th>
<th>Long-run</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Output mult.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline (US calibration)</td>
<td>0.95</td>
<td>0.98</td>
<td>1.17</td>
<td>2.50*</td>
</tr>
<tr>
<td>European calibration ($\chi = 0.11, \kappa = 0.13$)</td>
<td>0.96</td>
<td>0.64</td>
<td>0.63</td>
<td>1.13*</td>
</tr>
<tr>
<td>European calibration ($\chi = 0.25, \kappa = 0.08$)</td>
<td>1.01</td>
<td>0.84</td>
<td>0.94</td>
<td>1.85*</td>
</tr>
<tr>
<td><strong>Unemployment mult.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline (US calibration)</td>
<td>-0.94</td>
<td>-0.88</td>
<td>-0.94</td>
<td>-1.04</td>
</tr>
<tr>
<td>European calibration ($\chi = 0.11, \kappa = 0.13$)</td>
<td>-0.95</td>
<td>-0.57</td>
<td>-0.49</td>
<td>-0.43</td>
</tr>
<tr>
<td>European calibration ($\chi = 0.25, \kappa = 0.08$)</td>
<td>-1.00</td>
<td>-0.75</td>
<td>-0.74</td>
<td>-0.74</td>
</tr>
</tbody>
</table>

Table 6 shows the short-run and long-run fiscal multipliers under the European calibration. For ease of comparison we also show the corresponding multipliers under the baseline US calibration (taken from the first rows of table 2 and table 3).

Consistent with the dynamics of aggregate variables shown in Figure 5 we see that (i) the impact output and unemployment multipliers are roughly the same under the alternative calibrations, (ii) as the time horizon increases the difference in the corresponding multipliers widens with the long-run multipliers under the European calibrations significantly lower than under the US calibration. For e.g., under the European calibrations the output multipliers are 1.13 (first calibration) and 1.85, while under the US calibration it is 2.50.

5 Concluding remarks

In this paper we analyze the effects of government spending in a New-Keynesian model with search and matching frictions featuring endogenous human capital accumulation and skill loss from long-term unemployment. The novel features of our model complement each other to generate medium-run and long-run output and unemployment multipliers are much larger compared to the standard model that abstracts from endogenous growth and skill loss. These results hold for alternative forms of fiscal financing as well as labor market institutions.
Our main results have implications for fiscal policy responses to the ongoing COVID-19 global pandemic that led to even sharper downturns than the Great Recession in many countries. Interestingly, various tools have been used by different governments in dealing with the economic consequences of the pandemic. For instance, while the US relied heavily on financial support for unemployed workers, many countries in Europe used short-time work schemes in order to preserve employment relationships. From the perspective of our model fiscal stimulus is crucial for supporting economic activity, maintaining human capital and thereby avoiding the permanent scars from deep recessions. Our model also suggests lower concern about surging inflation than more traditional business cycle models.

References


