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Productivity Shocks and the New Keynesian Phillips Curve: Evidence from US and Euro Area *

Gene Ambrocio† and Tae-Seok Jang†

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Abstract

This paper seeks to understand dynamics of inflation and marginal cost (labor share) in models that account for the inclusion of productivity shocks in standard New Keynesian Phillips Curve (NKPC). The question of interest is on the empirical importance of and whether productivity shocks shift the Phillips curve using U.S. and Euro area data. Highlighting the inclusion of productivity growth, we employ a hybrid model specification augmented with a productivity term. The model is estimated using the Generalized Method of Moments (GMM) following Gali and Gertler (1999). Our main finding is that a simple extension of the baseline and hybrid models using more recent data (2006:Q4 for the US and 2005:Q4 for the Euro area) yield less convincing results than the previous literature. Furthermore, our estimation results provide some support for the inclusion of productivity growth particularly for the US. We conclude that a better understanding of the inflation-unemployment tradeoff requires accounting for shifts in the Phillips Curve due to productivity shocks.

JEL Classification: E24, E31, J3

Keywords: New Keynesian, Phillips Curve, Productivity Growth, GMM

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1 Introduction

Much of the recent literature on the relationship between real economic activity and inflation has focused on the New Keynesian Phillips Curve (NKPC). It departs from the "old" Phillips curve in that it is forward-looking and has more rigorous micro-foundations. A stream of literature beginning with the paper by Gali and Gertler (1999) utilize labor’s share of income as a proxy for real marginal cost and thus the driver for inflation. In attempts to better fit the data, the "workhorse" Phillips curve, with a forward-looking term for inflation expectations and a demand-side pressures component (real marginal cost or excess demand) are commonly modified into hybrid models featuring a backward-looking (lagged-inflation) component. These models have been empirically tested with some success as in Gali and Gertler (1999) and Gali et al. (2001) for US and EU data respectively. Rudd and Whelan (2005, 2007) also tested these models but have contentions with regard to the degree of forward- as against backward-looking behavior exhibited in the empirical results.

However, more recent extensions of these models incorporating updated data have been less successful. In this light, there seems to be a need to return to frameworks which incorporate supply-side factors. In particular, this paper is concerned with testing the NKPC with a productivity shock, in effect unifying two streams of literature - one which proposes the use of unit labor costs and another which emphasizes the importance of supply side factors in inflation dynamics by including productivity shocks. The objective of this paper is to empirically verify whether a simple "hybrid" specification featuring productivity shocks using a modified version of Gali and Gertler (1999) works well with the data. This framework, in a similar vein as Gordon’s Triangle (1997)\(^1\) and Blanchard and Gali (2007), incorporates supply-side inflationary pressures.

A cursory review of inflation and unemployment data does suggest some shifting in the Phillips curve from supply side pressures (Figure 1). Following the series of oil price shocks in the 60s and 70s, inflation in the U.S. took a turn towards a gradual slowdown in the 1980s. Thereafter, despite a brief resurgence at the turn of the decade, inflation and unemployment continue to trend downwards into the "Goldilocks Economy" of the latter 1990s (the New Economy). On the other hand, looking at productivity growth, one can also picture a story of a slowdown in productivity growth up until the 1980s after which the U.S. economy experienced several surges in productivity growth before spiraling down shortly after the beginning of the new millennium. A less convincing but similar story is also apparent in the Euro area. After the supply shocks of the 1970s, we also see a drop in inflation in the 1980s despite moderate unemployment.

\(^1\)Gordon’s "Triangle" framework suggest a Phillips curve with three components: lagged inflation (inertia), supply shocks, and demand shocks. This empirical specification is open to several theoretical interpretations. For instance, both adaptive expectations and rational expectations arguments may be used to explain for the presence of the lagged inflation term.
The rest of the paper proceeds as follows: Section 2 provides a model of the NKPC with productivity shocks. Sections 3 follows with a brief discussion on the labor share based Phillips curve. Section 4 presents the results of estimating the NKPC with productivity shock using the Generalized Method of Moments (GMM). Finally, Section 5 concludes the paper with a summary of our findings.

## 2 The Model

Following Gali (2008), we derive the NKPC with productivity shocks. Our model applies some modifications in the supply side. It emphasizes the effect of productivity shocks on inflation dynamics.

### 2.1 The demand block

We assume aggregated households with time separable preferences and a constant discount factor $\beta$. Expected present discounted value of utility is given by:

$$U_t(C, N) \equiv E_t \sum_{j=0}^{\infty} \beta^j \left[ \frac{C_{t+j}^{1-\gamma}}{1 - \gamma} - \frac{N_{t+j}^{1+\phi}}{1 + \phi} \right]$$

Figure 1: Inflation/unemployment and productivity growth in the US (upper panels) and Euro area (lower panels)
where $C$ is composite consumption and $N$ is employment. The composite consumption good that enter the household’s utility function is defined as:

$$C_t = \left[ \int_0^1 \frac{\theta-1}{\theta \cdot c_t^j \cdot d_j} \right]^{\frac{\theta-1}{\theta}} , \quad \theta > 1$$ (2)

where $\theta$ is the price elasticity of demand for the individual goods. From cost minimization of buying $c_t$, demand for good $j$ is as follows:

$$C_t(j) = \left( \frac{p_t^*(j)}{p_t} \right)^{-\theta} C_t$$ (3)

where $p_t$ is the aggregate price level and $p_t^*(j)$ the optimal reset price of good $j$.

### 2.2 The supply block

Each generic firm $j$ produces a single differentiated good (monopolistic competition). A firm has the following diminishing returns production function:

$$Y_t(j) = A_t N_t(j)^{1-\alpha}$$ (4)

where $A_t$ denotes productivity at time, $t$. Then the firm maximizes the expected discount profit:

$$\max_{p_t^*(j)} E_t \sum_{i=0}^{\infty} (\beta \theta p)^i \left[ p_t^*(j) Y_{t+i}(j) - TC_{t+i}(j) \right]$$ (5)

where $TC_{t+i}(j) = MC_{t+i}(j) Y_{t+i}(j)$. And $Y_{t+i}(j) = \left( \frac{p_t^*(j)}{p_t} \right)^{-\theta} Y_{t+i}$ in the equilibrium. Following Calvo, we assume that firms reset prices with a probability of $1 - \theta_p$ in each period, independently of the time elapsed since the last adjustment. Then the maximization leads to the optimal reset price as a function of expected discount marginal cost and mark-up:

$$p_t^*(j) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{i=0}^{\infty} (\beta \theta p)^i MC_{t+i}(j) Y_{t+i}(j)}{E_t \sum_{i=0}^{\infty} (\beta \theta p)^i Y_{t+i}(j)}$$ (6)
We log-linearize equation (6) around the steady state, then the optimal reset price is given by:

\[
\tilde{p}_t^*(j) = (1 - \beta \theta_p) E_t \sum_{i=0}^{\infty} (\beta \theta_p)^i \tilde{mc}_{t+i}(j) \tag{7}
\]

where \(\tilde{p}_t^*(j)\) and \(\tilde{mc}_{t+i}(j)\) are log-linearized optimal reset price and marginal cost of individual firm, j. Firms cost minimization results in the following marginal cost equation:

\[
MC_t(j) = \frac{W_t}{1 - \alpha} \frac{N_t(j)}{Y_t} \left[ \left( \frac{p_t^*(j)}{p_t} \right)^{-\theta} Y_t \right]^{\frac{\alpha}{1 - \alpha}} \tag{8}
\]

Denote the average marginal cost as \(\frac{W_t}{1 - \alpha} \frac{N_t}{Y_t} = MC_t^{AV}\) and by the log-linearization, we get the following equation:

\[
\tilde{mc}_t(j) = \tilde{mc}_t^{AV} - \frac{\theta \alpha}{1 - \alpha} (\tilde{p}_t^*(j) - \tilde{p}_t) - \frac{a_t}{1 - \alpha} \tag{9}
\]

Plugging equation (9) into (7) leads to the following optimal reset price:

\[
\tilde{p}_t^*(j) = \frac{(1 - \alpha)(1 - \beta \theta_p)}{1 + \alpha(\theta - 1)} E_t \sum_{i=0}^{\infty} (\beta \theta_p)^i \left( \tilde{mc}_{t+i}^{AV} + \frac{\theta \alpha}{1 - \alpha} \tilde{p}_{t+i} - \frac{a_t}{1 - \alpha} \right) \tag{10}
\]

With Calvo pricing, the forward looking Phillips curve with productivity is derived as follows (see Appendix for the details):

\[
\tilde{\pi}_t = \beta E_t \tilde{\pi}_{t+1} + \frac{(1 - \beta \theta_p)(1 - \theta_p)(1 - \alpha)}{\theta_p(1 + \alpha(\theta - 1))} \tilde{mc}_t^{realAV} - \frac{(1 - \beta \theta_p)(1 - \theta_p)}{\theta_p(1 + \alpha(\theta - 1))} a_t \tag{11}
\]
3 The Labor Share Based NKPC

By and large, empirical work on the NKPC uses either real marginal cost or excess demand as the source of (demand-side) inflationary pressures\(^2\). Largely due to the inability of earlier models of the NKPC to accurately depict inflation dynamics using output gap measures a stream of empirical work utilize labor’s share of income as a proxy for real marginal cost. One interpretation is that marginal cost is more closely correlated with inflation than the output gap.\(^3\) Behind these assertions are the observed inertia in both inflation and labor’s share of income which is not present in estimates of the output gap and is complicated by the difficulties that arise in the estimation of potential output necessary for determining the output gap.

However, in response to criticisms that a purely forward-looking Phillips curve does not capture inflation inertia in a more structural manner, hybrid versions of the NKPC have been put forward and empirically tested. These allow for inflation dynamics to have both a backward- and forward-looking component.

In this specification, some fraction of inflation is attributed to past inflation and the rest to expectations of future inflation. Empirical results from estimations of such hybrid models have polarized research into those that suggest that a purely forward-looking NKPC is at least a good first approximation of inflation dynamics as in Gali and Gertler (1999) for the US, Gali et al. (2001) for the Euro area, and Muto (2008) for Japan. against those that propose that the quantitative fit of such hybrid versions of NKPCs are largely due to the backward-looking component (Rudd and Whelan 2005, 2007). Other works have also shown mixed evidence for a host of country cases (Jondeau and Bihan 2005).

Previous exercises at estimating the labor share based Phillips curve using GMM have had varied results (Table 1)\(^4\). It should be noted that for models which use the labor share as driving force, three basic sets of instruments are used with varying lags of either two or four periods. These studies empirically validate the use of labor share as the driving force for inflation in both the US and EU-wide data. However, a simple extension of the baseline and hybrid model using more recent data (2006:Q4 for the US and 2005:Q4 for the Euro area) yield less convincing results.


\(^3\)However, Neiss and Nelson (2002) claim that marginal cost and the output gap are closely relate once the output gap is measured in a manner consistent with theory.

\(^4\)All estimations use quarterly data on logged GDP deflator for the U.S. and Euro area unless otherwise stated. Reported results for hybrid models are for unrestricted inflation parameters (not necessary to sum to unity). The variable \(s_t\) refers to labor share, \(u_t\) is unemployment, and \(v_t\) is producer prices.
### Table 1: Selected GMM estimates on U.S. and Euro Area

<table>
<thead>
<tr>
<th>Authors</th>
<th>Model and Estimation results</th>
<th>Instrument list</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gali and Gertler(1999)</strong></td>
<td>US</td>
<td><strong>US Baseline model:</strong> ( \pi_t = 0.023s_t + 0.942\pi_{t+1} ) ( \pi_t = 0.037s_t + 0.682\pi_{t+1} + 0.252\pi_{t-1} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>US Hybrid model:</strong> ( \pi_t = 0.25s_t + 0.924\pi_{t+1} ) ( \pi_t = 0.26s_t + 0.61\pi_{t+1} + 0.339\pi_{t-1} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Euro Baseline model:</strong> ( \pi_t = 0.088s_t + 0.914\pi_{t+1} ) ( \pi_t = 0.018s_t + 0.877\pi_{t+1} + 0.025\pi_{t-1} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Euro Hybrid model:</strong> ( \pi_t = 0.009s_t + 0.677\pi_{t+1} + 0.318\pi_{t-1} ) ( \pi_t = 0.000s_t + 0.608\pi_{t+1} + 0.387\pi_{t-1} )</td>
</tr>
<tr>
<td><strong>Jondeau and Bihan(2005)</strong></td>
<td>US</td>
<td><strong>US Baseline model:</strong> ( \pi_t = 0.01 \sum_{i=1}^{12} (0.891)^i s_{t+1} ) ( + 0.951^3 (\pi_{t+13} - \pi_{t+12-i} + 0.793\pi_{t-1}) ) ( + 0.95^3 (\pi_{t+13} - \pi_{t+12-i} + 0.752\pi_{t-1}) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>US Restricted ( \beta ):</strong> ( \pi_t = 0.042s_t + 0.475\pi_{t+1} + 0.486\pi_{t-1} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>US Instrument set 1:</strong> ( \pi_t = 0.032s_t + 0.535\pi_{t+1} + 0.44\pi_{t-1} ) ( \pi_t = 0.036s_t + 0.516\pi_{t+1} + 0.452\pi_{t-1} )</td>
</tr>
<tr>
<td><strong>Whelan(2007)</strong></td>
<td></td>
<td><strong>US Instrument set 2:</strong> ( \pi_t = 0.000s_t + 0.608\pi_{t+1} + 0.387\pi_{t-1} ) ( \pi_t = 0.0229s_t + 0.840\pi_{t+1} )</td>
</tr>
<tr>
<td><strong>Lawless and Whelan(2007)</strong></td>
<td>Euro</td>
<td><strong>Restricted ( \beta ):</strong> ( + 0.95^3 (\pi_{t+13} - \pi_{t+12-i} + 0.752\pi_{t-1}) )</td>
</tr>
</tbody>
</table>

\(^a\)Output gap estimates are derived from time-trend and its square.
In the GMM estimation, the moment conditions are derived from the orthogonality condition under rational expectations. The error in the forecast of $\pi_{t+1}$ is uncorrelated with information dated $t$ and earlier. There are 2 or 3 parameters to be estimated with the set of instruments ($z_t$) from Gali and Gertler (1999), Gali et al. (2001), and Rudd and Whelan (2007):

$$E_t[(\pi_t - \beta E_t\pi_{t+1} - \lambda s_t)z_t] = 0 \quad (12)$$

For the instrument set of U.S and Euro area data, we present the results using Rudd and Whelan (Set 3 in Table 1) \(^5\). Note here that inflation is in terms of the total implicit GDP deflator (index 2000 = 100) and the labor income share is used as a proxy for deflated unit labor cost (nonfarm business). Wage inflation is computed from compensation per hour in the nonfarm business sector (BLS, 1992=100). All data are quarterly for the US over the period 1960:1 - 2006:4. Data for the EU-wide regressions are taken from the updated Area Wide Model (Fagan et al. 2001) following the previous papers. We use the same variable definitions as in Gali et al. (2001). The figures begin on 1970:Q1 and have been updated up to 2005:Q4. In the baseline forward looking Phillips curve, the estimates for the U.S and Euro area is as follows:

**U.S:**

$$\pi_t = 0.9905 E_t\{\pi_{t+1}\} + 0.0125 s_t$$

(0.0108) (0.0212)

**Euro Area:**

$$\pi_t = 1.0059 E_t\{\pi_{t+1}\} + 0.0029 s_t$$

(0.0127) (0.0076)

Clearly, the baseline model does not fit the updated data set for both the U.S and Euro area.

$$\pi_t = \alpha \pi_{t-1} + \beta E_t\pi_{t+1} + \lambda s_t \quad (13)$$

However, the hybrid version of the above model provides a better fit when the instrument set is modified to four lags for the labor income share and retaining the lag lengths for the other instruments (see Table 2). Therefore we test our hypothesis with a hybrid Phillips curve (equation 13) by adding a productivity shock in the next section.

\(^5\)We also tested the model using the original instrument sets of Gali and Gertler(1999) and Gali et. al(2001), but the parameter estimates have unreasonable ranges.
Table 2: GMM estimates of Hybrid Phillips curve

<table>
<thead>
<tr>
<th></th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(\lambda)</th>
<th>J-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.3637</td>
<td>0.6316</td>
<td>0.0264</td>
<td>1.205</td>
</tr>
<tr>
<td></td>
<td>(0.0921)</td>
<td>(0.0906)</td>
<td>(0.0128)</td>
<td></td>
</tr>
<tr>
<td>Euro Area</td>
<td>0.4233</td>
<td>0.5707</td>
<td>0.0110</td>
<td>2.084</td>
</tr>
<tr>
<td></td>
<td>(0.2497)</td>
<td>(0.2477)</td>
<td>(0.0085)</td>
<td></td>
</tr>
</tbody>
</table>

4 Shifting the Phillips Curve with Productivity Shocks

As earlier stated, the 'basic' specification has been criticized for not accounting for supply-side inflationary pressures and real rigidities which may be shifting the NKPC and producing a downward bias on parameter estimates of the (demand-side) driving force for inflation. Figure 2 depicts a similar story as the inflation-unemployment dynamics described in the introduction. Again, the early 1980s and late 1990s exhibit falling or steady inflation in the face of rising unit labor costs in the US. The relationship seems to hold more for the Euro area where there seems to be a consistent comovement of inflation and unit labor cost. Such shifts in the Phillips curve is attributed to productivity growth.

One line of research has highlighted the adaptive process of agents’ expectation formation in inflation-unemployment dynamics (Ball and Moffitt 2001, Jiri 2004). In particular, Ball and Moffit (2001) utilize a wage aspiration framework wherein additional wage inflationary pressure arises from the difference between productivity growth and workers aspiration for wage changes in response to past real wages and thus generating shifts of the Phillips curve with changes in productivity growth. As an alternative to Ball and Moffit’s (2001) framework, Karanassou and Sala (2008) show that productivity growth shifts the long-run Phillips curve where productivity growth factors into the mark-up and price-setting decision of firms.

Such specifications allow for the inclusion of productivity growth as a driver for inflation dynamics. Drawing on the benchmark hybrid NKPC specification of Gali and Gertler (1999), and taking into account the model presented in section 2, a simple hybrid extension is one with four terms:
lagged inflation, inflation expectations, labor share, and productivity growth as the supply-side shock.

In order to investigate the impact of productivity shock, we estimate an augmented Phillips curve. The following six instrument variables \((z_t)\) are used as the moment conditions in GMM: a constant, four lags of the labor share, and two lags each of inflation, wage inflation, output gap and the productivity shock. In the estimation, the following hybrid Phillips curve specification is employed where \(g\) is the growth rate of output per hour (nonfarm business, 1992=100):

\[
\pi_t = \alpha \pi_{t-1} + \beta E_t \pi_{t+1} + \lambda s_t + \delta g_t
\]  

(14)

Table 3: GMM estimates of Phillips curve with productivity shock

<table>
<thead>
<tr>
<th></th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(\lambda)</th>
<th>(\delta)</th>
<th>J-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.4944</td>
<td>0.4957</td>
<td>0.0177</td>
<td>-0.0158</td>
<td>2.896</td>
</tr>
<tr>
<td></td>
<td>(0.1007)</td>
<td>(0.0998)</td>
<td>(0.0122)</td>
<td>(0.0079)</td>
<td></td>
</tr>
<tr>
<td>Euro Area</td>
<td>0.1760</td>
<td>0.8481</td>
<td>0.0005</td>
<td>-0.0668</td>
<td>2.954</td>
</tr>
<tr>
<td></td>
<td>(0.4070)</td>
<td>(0.4247)</td>
<td>(0.0110)</td>
<td>(0.0618)</td>
<td></td>
</tr>
</tbody>
</table>

The results indicate that productivity shocks shift the Phillips curve downwards but it does not improve on the earlier estimates with regard to the parameter for labor share (Table 3). In the US, we find that productivity shocks have a significant impact and that alternative proxies for real marginal cost may be warranted \(^8\). On the other hand, the same cannot be said for the Euro area as both parameter estimates for the labor share and productivity shock are insignificant. From the GMM estimation results, there is evidence that productivity shocks account for some of the change

\(^8\)We tried to estimate the model with output gap instead of labor income share, but its estimates do not have any significant result.
in the inflation rate, at least in the US.

However, these results point to further investigation with regard to the interaction between real marginal cost (with labor share as a proxy) and productivity shocks beyond the theoretical derivation outlined in section 2. The above exercise lends support to criticisms made on the GMM estimation procedure predominantly used in estimating forward-looking models. As Rudd and Whelan (2007) point out, the estimation of the NKPC as generally done in the literature are sensitive to choice of instruments. Our results also verify that the estimated slope of the Phillips curve can be upward biased and insignificant with the inclusion of productivity shocks.9

5 Conclusion

Our results show that the empirical fit of unit labor cost-based Phillips curves seem to have worsened with the addition of new data. Notwithstanding the shortcomings of the GMM estimation procedure we followed, our empirical tests imply that productivity shocks may indeed be a relevant term using US and Euro area inflation data. This is in line with the stream of research proposing a Phillips curve that is shifting with supply shocks over time and provides some explanation for the inflation-unemployment patterns in the New Economy of the roaring 90s in the US [Eller and Gordon 2002]. However, given the worse fit for the Euro area, the impact of productivity shocks appear limited in explaining inflation dynamics. The staggered price-setting scheme for European inflation seems to require the inclusion of other factors, for instance labor market rigidities.

In future works, these results may be further verified via the estimation of these empirical models using the multivariate VAR approach as in [Rudd and Whelan 2006]. Another extension is to proceed in the direction of more systemic representations of inflation dynamics in Kalman filter approaches which may account for a time varying natural rate of unemployment. Furthermore, these type of models could be applied to other countries’ inflation-unemployment data. Though, undoubtedly only a modest contribution, continuing on this line of research should lead to more precise estimates of the inflation-unemployment relationship as embodied in the New Keynesian Phillips curve. Finally, this exercise provides some impetus for further developments of theoretical models incorporating productivity growth in inflation dynamics.

9An alternative estimation procedure utilizing Maximum Likelihood estimation of a model which utilizes VAR-model based forecasts of the forward-looking component have also been more recently applied to the NKPC [Rudd and Whelan 2005, Tillman 2003, Jondeau and Bihan 2003, Muto 2007].
References


Appendix  Derivation of NKPC with Productivity Shock

From equation (10) outlining the optimal reset price:

\[
\tilde{p}_t(j) = \frac{(1 - \alpha)(1 - \beta \theta_p)}{1 + \alpha(\theta - 1)} \mathcal{E}_t \sum_{i=0}^{\infty} (\beta \theta_p)^i \left( \hat{m}_{t+i}^{AV} + \frac{\theta \alpha}{1 - \alpha} \tilde{p}_{t+i} - \frac{a_t}{1 - \alpha} \right) \tag{15}
\]

With Calvo pricing, \( \tilde{p}_t = (1 - \theta_p) \tilde{p}_t^C + \theta_p \tilde{p}_{t-1} \). Therefore:

\[
\tilde{p}_t = \frac{\tilde{p}_t - \theta_p \tilde{p}_{t-1}}{1 - \theta_p} \tag{16}
\]

and equating with the first equation:

\[
\tilde{p}_t - \theta_p \tilde{p}_{t-1} = \frac{(1 - \alpha)(1 - \beta \theta_p)}{1 + \alpha(\epsilon_p - 1)} E_t \sum_{i=0}^{\infty} (\beta \theta_p)^i \left( \hat{m}_{t+i}^{AV} + \frac{\theta \alpha}{1 - \alpha} \tilde{p}_{t+i} - \frac{a_t}{1 - \alpha} \right) \tag{17}
\]

or

\[
\tilde{p}_t - \theta_p \tilde{p}_{t-1} = \frac{(1 - \theta_p)(1 - \alpha)(1 - \beta \theta_p)}{1 + \alpha(\epsilon_p - 1)} E_t \sum_{i=0}^{\infty} (\beta \theta_p)^i \left( \hat{m}_{t+i}^{AV} + \frac{\theta \alpha}{1 - \alpha} \tilde{p}_{t+i} - \frac{a_t}{1 - \alpha} \right) \tag{18}
\]

and

\[
E_t \{ \tilde{p}_t - \theta_p \tilde{p}_{t-1} \} = \frac{(1 - \theta_p)(1 - \alpha)(1 - \beta \theta_p)}{1 + \alpha(\epsilon_p - 1)} E_t \sum_{i=0}^{\infty} (\beta \theta_p)^i \left( \hat{m}_{t+i}^{AV} + \frac{\theta \alpha}{1 - \alpha} \tilde{p}_{t+i} - \frac{a_t}{1 - \alpha} \right) \tag{19}
\]

Multiplying the last equation with \( \beta \theta_p \) and subtracting the result from the preceding equation yields:

\[
(\tilde{p}_t - \theta_p \tilde{p}_{t-1}) - \theta_p E_t \{ \tilde{p}_t - \theta_p \tilde{p}_{t-1} \} = \frac{(1 - \theta_p)(1 - \alpha)(1 - \beta \theta_p)}{1 + \alpha(\epsilon_p - 1)} \left\{ M_{Ct}^{AV} + \frac{\epsilon_p - x}{1 - \alpha} \tilde{p}_t - \frac{1}{1 - \alpha} a_t \right\} \tag{20}
\]

Thus:

\[
\theta_p \tilde{p}_t - \theta_p \tilde{p}_{t-1} - \theta_p E_t \{ \tilde{p}_{t+1} - \tilde{p}_t \} = \frac{(1 - \theta_p)(1 - \alpha)(1 - \beta \theta_p)}{1 + \alpha(\epsilon_p - 1)} M_{CrealAV} - \frac{(1 - \theta_p)(1 - \beta \theta_p)}{1 + \alpha(\epsilon_p - 1)} a_t \tag{21}
\]
\[
\theta_p \tilde{\pi}_t - \beta \theta_p E_t \tilde{\pi}_{t+1} = \left( 1 - \theta_p \right) \left( 1 - \alpha \right) \frac{\text{MC}_{\text{realAV}}}{1 + \alpha (\epsilon_p - 1)} - \left( 1 - \theta_p \right) \left( 1 - \beta \theta_p \right) \frac{\text{MC}_{\text{realAV}}}{1 + \alpha (\epsilon_p - 1)} a_t
\]  

Or

Finally, rearranging terms will give us the NKPC with productivity shocks:

\[
\tilde{\pi}_t = \beta E_t \tilde{\pi}_{t+1} + \frac{\left( 1 - \beta \theta_p \right) \left( 1 - \theta_p \right) \left( 1 - \alpha \right)}{\theta_p (1 + \alpha (\epsilon_p - 1))} \frac{\text{MC}_{\text{realAV}}}{\theta_p (1 + \alpha (\epsilon_p - 1))} - \frac{\left( 1 - \beta \theta_p \right) \left( 1 - \theta_p \right)}{\theta_p (1 + \alpha (\epsilon_p - 1))} a_t
\]