Does Money Matter for the Identification of Monetary Policy Shocks: A DSGE Perspective.[1]

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Abstract

This paper investigates how the identification assumptions of monetary policy shocks modify the inference in a standard DSGE model. Considering SVAR models in which either the interest rate is predetermined for money or these two monetary variables are simultaneously determined, two DSGE models are estimated by Minimum Distance Estimation. We emphasize that real balance effects are necessary to replicate the high persistence implied by the simultaneity assumption. In addition, the estimated monetary policy rule is strongly sensitive to the identification scheme. This suggests that the way money is introduced in the identification scheme is not neutral for the estimation of DSGE models.

Keywords: SVAR model; DSGE model; Non recursive identification; Money

JEL Codes: E41, E52, C52.

Résumé

Dans cet article, nous analysons l'effet des restrictions d'identification d'un choc de politique monétaire sur l'inférence d'un modèle DSGE standard. Nous considérons deux modèles SVAR qui différèrent par leur schéma d'identification : soit le taux d'intérêt est prédéterminé par rapport à la monnaie ou ces deux variables sont simultanément déterminées. Puis, nous estimons deux modèles DSGE compatibles avec le modèle SVAR associé par la méthode de minimisation de la distance des fonctions de réponse à un choc de politique monétaire. Nous montrons que les effets d'encaisses réelles sont nécessaires dans le modèle DSGE afin de répliquer la forte persistance induite par l'hypothèse de simultanéité. Par ailleurs, l'estimation de la règle de politique monétaire est sensible au schéma d'identification adopté. Ces résultats suggèrent que la façon dont la monnaie est introduite dans le schéma d'identification n'est pas neutre dans l'estimation d'un modèle DSGE.

Mots-clefs : Modèle SVAR, modèle DSGE, identification non récursive, monnaie.

Codes JEL : E31, C51.

Non-technical summary

In the literature, the effects of a monetary policy shock are currently examined by estimating Structural VAR models (SVAR), either from an purely empirical point of view, or they serve as a support to estimate a Dynamic Stochastic General Equilibrium (DSGE) model by Minimum Distance Estimation (MDE). This approach consists in matching as closely as possible the impulse response functions of a set of variables to a monetary policy shock, computed from a empirical SVAR model. The estimation of the SVAR model requires to impose identification restrictions so as to insulate the effects of monetary policy shocks. Most papers resort to the *recursive identification strategy* which imposes that certain private sector variables respond with a lag to the monetary policy shock rather than some informative variables respond immediately to the shock. In this case, the introduction of a monetary aggregate in a SVAR requires to make a choice: either the interest elasticity for money demand or the interest elasticity for money supply is constrained to be zero. In this paper, we depart from the traditional recursive identification strategy by assuming that none of these elasticities are constrained. This approach named *non recursive identification strategy* – in line with Leeper and Roush (2003) – means that the interest rate and money growth are simultaneously determined, on the contrary to the assumption made in the recursive identification strategy. We focus on the estimation of a DSGE model by MDE in order to investigate how the choice of the identification strategy in the SVAR model can change inference about this structural model.

We consider two SVAR models which differ in the restrictions imposed to identify the monetary policy shock. In a first specification, we assume that the interest rate is predetermined for money growth and in a second specification, we assume that these two variables are simultaneously determined. Then, we build two DSGE models so that the theoretical and empirical models have identical timing restrictions and they share similar monetary policy specifications. Finally, we estimate the model's parameters so as to minimize the distance between the model-based and the SVAR-based impulse response functions.

Our results emphasize that a standard DSGE model, which embodies reasonable degrees of rigidities, is able to replicate the stronger persistence implied by the simultaneity assumption between the interest rate and money growth. In addition, we show that the estimated monetary

policy rule in the DSGE model is deeply changed by the identification scheme. This paper also highlights that the real balance effect helps to precisely estimate this relationship between the interest rate and money in the monetary policy rule. Indeed, this effect can be viewed as an important monetary transmission channel which modify money's dynamics. Finally, the taste parameters and the degrees of nominal rigidities are not strongly changed by the identification scheme. Consequently, we show that the way to insert money in the identification scheme is not neutral on the inference about DSGE models when we are interested on the transmission of monetary policy shocks.

Résumé non technique

Les effets d'un choc de politique monétaire sont souvent étudiés dans la littérature à travers l'utilisation de modèles VAR structurels (SVAR), soit d'un point de vue purement empirique, ou comme support pour l'estimation d'un modèle d'équilibre général dynamique et stochastique (DSGE) par la méthode de minimisation de la distance (MDE). Cette dernière approche consiste à répliquer au mieux les fonctions de réponse d'un ensemble de variables à un choc de politique monétaire, calculées à l'aide du modèle SVAR. L'estimation du modèle SVAR nécessite l'existence de restrictions d'identification permettant d'isoler les effets du choc de politique monétaire. La plupart des papiers ont recours à la *stratégie d'identification récursive* qui impose que les variables du secteur privé répondent une période après le choc de politique monétaire tandis que certaines variables informatives peuvent répondre immédiatement au choc. Dans ce cas, l'introduction de l'agrégat monétaire dans le modèle SVAR nécessite de faire un choix : soit l'élasticité du taux d'intérêt à une variation de la demande de monnaie ou l'élasticité du taux d'intérêt à une variation de l'offre de monnaie doit être contrainte. Dans cet article, nous nous éloignons de la stratégie d'identification récursive en supposant qu'aucune de ces élasticités n'est contrainte. Cette approche, nommée *l'identification non récursive* – utilisée par Leeper et Roush (2003) – implique que le taux d'intérêt et le taux de croissance de la monnaie sont simultanément déterminés, contrairement à l'hypothèse faite dans une identification récursive. Nous nous focalisons sur l'estimation d'un modèle DSGE par MDE afin d'analyser comment le choix de la stratégie d'identification dans le modèle SVAR joue sur l'inférence du modèle DSGE.

Nous considérons deux modèles SVAR qui diffèrent dans les restrictions imposées pour identifier le choc de politique monétaire. Dans une première spécification, nous supposons que le taux d'intérêt est prédéterminé par rapport au taux de croissance de la monnaie. Dans une seconde spécification, nous supposons que ces deux variables sont simultanément déterminées. Nous construisons ensuite deux modèles DSGE de sorte que les modèles théorique et empirique aient les mêmes restrictions temporelles et qu'ils partagent la même spécification en termes de règle monétaire. Enfin, nous estimons les paramètres du modèle de façon à minimiser la distance entre les fonctions de réponse empiriques et théoriques.

Nos résultats montrent qu'un modèle DSGE standard, composé de rigidités nominales, est capable de répliquer la forte persistance induite par l'hypothèse de simultanéité entre le taux d'intérêt et le taux de croissance de la monnaie. De plus, nous montrons que l'estimation de la règle de politique monétaire dans le modèle DSGE dépend du schéma d'identification. Cet article met aussi en évidence l'importance de l'effet d'encaisses réelles qui aident à estimer précisément la relation entre le taux d'intérêt et la monnaie dans la règle de politique monétaire. En effet, cet effet peut être vu comme un canal de transmission de la politique monétaire important pouvant modifier la dynamique de la monnaie. Enfin, les paramètres de préférence et les degrés de rigidités nominales ne sont pas fortement affectés par le schéma d'identification. Par conséquent, nous montrons que la façon d'insérer la monnaie dans le schéma d'identification n'est pas neutre sur l'inférence d'un modèle DSGE lorsque nous nous intéressons aux canaux de transmission de la politique monétaire.

1 Introduction

Recent years have witnessed a resurgence of interest in developing Dynamic Stochastic General Equilibrium (DSGE) models so as to deepen our knowledge of the transmission of monetary policy shocks. Following the contributions by Rotemberg and Woodford (1997, 1999) and Christiano et al. (2005), it has now become standard practice to confront DSGE models to the predictions of monetary Structural Vector AutoRegressive (SVAR) models. In particular, an increasing list of authors resort to the Minimum Distance Estimation (MDE), which consists of picking the DSGE parameters that best reproduce the empirical impulse response functions drawn from the monetary SVAR model.[1]

The implementation of the MDE methodology requires to impose identification restrictions on the SVAR model so as to insulate the effects of monetary policy shocks. Following Christiano et al. (2005), most papers resort to the *recursive identification strategy* – namely the Cholesky decomposition. It is usually assumed that certain private sector variables respond with a lag to the monetary policy shock, usually depicted by an exogenous variation of the interest rate. On the contrary, some informative variables are assumed to respond immediately after the shock. As a consequence, when the interest rate and money are combined into a SVAR model, the Cholesky decomposition requires to make a choice: is money predetermined for the interest rate or not? If the answer is yes, the money demand is forced to be interest inelastic. This implies that no direct role is assigned to money in the transmission of the shock. In the opposite case, the interest rate cannot directly respond to monetary disturbances since it is assumed to be inelastic to money supply.

Departing from this *recursive identification strategy*, Leeper and Roush (2003) freely estimate the interest elasticities of supply and demand for money, implying that the interest rate and money are now simultaneously determined. Under this assumption, their key findings are a rise of the degree of inertia exhibited by inflation and a rise in the magnitude of the responses of output and consumption to the monetary policy shock. In addition, standard overidentifying restrictions tests suggest that the data favor the simultaneity specification between the interest rate and

[1] A non exhaustive list of contributions includes Amato and Laubach (2003), Trigari (2004), Altig et al. (2005), Boivin and Giannoni (2006), Hüelsewig et al. (2006), Meier and Müller (2006), DiCecio and Nelson (2007), Carrillo et al. (2007).

money rather than the Cholesky-type decomposition. Thus, the omission of this simultaneity in the identification strategy might result in a misspecification of the SVAR model.

In light of these findings, one may legitimately wonder how inference about a DSGE model, estimated by MDE, is changed when we resort to the *non recursive identification strategy*, proposed by Leeper and Roush (2003), instead of the standard Cholesky decomposition, implemented in the CEE-based model.[2] This is the question under study in this paper. Particularly, we address three specific issues: (i) Can the CEE-based model replicate the increased amount of persistence in inflation, without relying on unreasonable degrees of nominal rigidity? (ii) What are the consequences on the deep parameters of adopting a *non recursive identification strategy* in the SVAR model? (iii) How does this identification strategy impact on the theoretical representation of monetary policy?

To answer to these questions, we proceed in two steps. First, we consider two SVAR models which differ in the restrictions imposed to identify the monetary policy shock. In a first specification, we assume that all the macroeconomic variables are predetermined for the interest rate, except money growth, which roughly corresponds to the Cholesky decomposition. In a second specification, we follow Leeper and Roush (2003) by assuming that money growth and the interest rate are simultaneously determined, which is to say that the interest elasticities of supply and demand for money are unconstrained. We estimate these quarterly SVAR models on a set of U.S. variables over the sample 1959Q2-2004Q4.

Second, by using these two SVAR models, we estimate by MDE the structural parameters of a standard CEE-based model. We consider two DSGE models since the theoretical and empirical SVAR models must have identical timing restrictions and they also have to share similar monetary policy specifications. Especially, in the two theoretical models, the private sector variables are predetermined for the monetary policy shock. In addition, in each DSGE model, we specify an interest rate rule which closely corresponds to the monetary policy representation of the SVAR model.[3] Finally, we estimate the model's parameters so as to minimize the distance

[2] We mean by CEE-based model, the fully fledged model proposed by Christiano, Einchenbaum and Evans (2005). This framework has become a benchmark in the literature when it comes to understanding the effects of monetary policy shocks. It features a set of frictions, namely habit formation, nominal rigidities on prices and wages, investment adjustment costs and variable capital utilization in order to reproduce the persistence properties of key macroeconomic variables.

[3] Precisely, the interest elasticity to money supply is estimated in the DSGE associated to the SVAR model with the simultaneity assumption. It is constrainted to zero in the Cholesky specification.

between the model-based and the SVAR-based impulse response functions.

Our results emphasize that a standard DSGE model, which embodies reasonable degrees of rigidities, is able to replicate the stronger persistence implied by the simultaneity assumption between the interest rate and money growth. In addition, we show that the estimated monetary policy rule in the DSGE model is deeply changed by the identification scheme. Indeed, it corresponds to the standard Taylor rule when we resort to the Cholesky decomposition. However, as soon as we make the simultaneity assumption, the interest elasticity to money supply in the monetary policy rule is high and significant. This paper also highlights that the real balance effect helps to precisely estimate the relationship between the interest rate and money in the monetary policy rule. Indeed, this effect can be viewed as an important monetary transmission channel which modifies money's dynamics. Due to the simultaneity assumption, this effect on money's dynamics thereby impact on those of the interest rate. Finally, the taste parameters and the degrees of nominal rigidities are not strongly changed by the identification scheme. Consequently, we show that the way to insert money in the identification scheme is not neutral on the inference about DSGE models when we are interested on the transmission of monetary policy shocks.

The remainder is as follows. Section 2 expounds the SVAR models. Section 3 presents the theoretical model. Section 4 presents the estimation strategy. Section 5 discusses the estimation results. The last section briefly concludes.

2 Money in a SVAR model

We estimate two empirical SVAR models by identifying monetary policy shocks with a non recursive identification strategy. We compare two identification schemes, based on the simultaneity (or not) between the interest rate and money growth, in order to state whether the monetary aggregate provides information useful to identify monetary policy shocks. First, we introduce the estimation method and second, we describe the identification strategy and the results.

2.1 Estimation Method

Before identifying the monetary policy shock in the SVAR model, we estimate the canonical VAR(p) model[4]

$$x_t = \phi_1 x_{t-1} + ... + \phi_p x_{t-p} + \varepsilon_t, \tag{1}$$

where x_t is an $(n \times 1)$ vector of data, p is the maximum lag and we assume that $\varepsilon_t \sim \text{iid}(0, \Sigma)$, where Σ is a symmetric positive definite matrix. We use U.S. quarterly data over the sample 1959Q2-2004Q4.[5] Let us define x_t, the data vector

$$x_t = (\log(y_t), \quad \log(i_t), \quad \log(c_t), \quad \pi_t^w, \quad \pi_t, \quad R_t, \quad \Delta \log(m_t), \quad \log(crb_t))', \tag{2}$$

where y_t is real output, i_t is real investment, c_t is real consumption expenditures, π_t^w is wage inflation, π_t is inflation, R_t is the Fed Fund rate, $\Delta \log(m_t)$ is the growth rate of M2 and crb_t is commodity prices.[6] The variables in the SVAR model have been selected so as to be consistent with the theoretical model used in this paper. However, the main results concerning the empirical impulse responses are not modified by this choice (Leeper and Roush, 2003). In addition, due to the convergence issues related to the non recursive identification strategy, we limit the size of the SVAR model. Consequently, we do not use as many variables as some authors who follow the MDE approach (Altig et al., 2005 or Christiano et al., 2005, for instance). Finally, minimization of the Hannan-Quinn information criterion yields $p = 4$.

In order to identify monetary policy shocks, we require some identification restrictions. Following Amisano and Giannini (1997), we can express the relation between the reduced form residuals, ε_t, and the structural innovations, η_t, using the linear combination

$$A\varepsilon_t = B\eta_t, \tag{3}$$

where A and B are non singular matrices. We assume that $\text{diag}(A) = \mathbf{1}$ where $\mathbf{1}$ is a n dimensional vector of ones and B is a diagonal matrix with $\text{diag}(B) > 0$. In addition, we

[4] A detailed technical appendix is available upon request.

[5] In the technical appendix, we proceed to a subsample analysis. None of the results are affected by this analysis.

[6] A detailed description of the data sources and construction is provided in the appendix.

assume that the structural innovations are normally distributed, such that $\eta_t \sim \mathrm{N}(0, \mathrm{I}_n)$.[7]

We seek to identify the monetary policy shock by restricting parameters on matrices A and B, the remaining free parameters having to be estimated. Following Lütkepohl (2005), we stack the free parameters of A and B in vectors denoted by γ_A and γ_B, respectively, and we estimate these parameters by Full Information Maximum Likelihood (FIML) subject to the identification restrictions. In addition, we fulfil the order condition by imposing no more than $n(n-1)/2$ free parameters. Finally, we check the rank condition in order to guarantee global identification[8]. Under local identification, Lütkepohl (2005) shows that the FIML estimators $\hat{\gamma}_A$ and $\hat{\gamma}_B$ are consistent and asymptotically normally distributed.

2.2 Identification Strategy and Results

2.2.1 Identification of Monetary Policy Shocks

Leeper and Roush (2003) question the recursive identification strategy that is widely used in the literature. Indeed, the Cholesky decomposition requires extreme assumptions about the interest elasticities of money supply and money demand which in turn may imply a misspecification of the SVAR model. Following these authors, we investigate whether identification restrictions impact on the responses of key macroeconomics variables. Precisely, we highlight whether the contemporaneous interactions between money growth and the interest rate can change the dynamics of the variables. Table 1 describes the identification restrictions on matrix A for the two specifications.[9] In the first panel of table 1, the interest rate is predetermined for money growth. In the second panel, the interest rate and money growth are simultaneously determined.

First, in each identification scheme, we assume that output, investment, consumption, wage inflation and inflation respond only to their own contemporaneous disturbances. This specification means that these variables are determined before the realization of monetary policy shocks. Second, in an attempt to mitigate the so-called price puzzle, we use the commodity price index in order to take additional information about future inflation into account (Sims,

[7] As explained by Lutkepöhl (2005), the Gaussian distribution is assumed for computational convenience. Indeed, as usual, the FIML estimators will be consistent and asymptotically Normal without the Gaussian assumption, as soon as the structural innovations are independent and identically distributed.

[8] A complete description of global identification in SVAR models is proposed by Christiano et al. (1999).

[9] We focus here on identification restrictions on matrix A. Indeed we restrict matrix B such that it is a positive diagonal matrix where the parameters of the diagonal are free parameters included in the vector γ_B.

1992, Leeper and Roush, 2003). We assume that the commodity price index responds to contemporaneous disturbances of all the variables. In doing so, we stress the informative nature of this variable which captures economic news. Finally, the money demand function is expressed in its traditional form: the monetary aggregate responds to contemporaneous disturbances of consumption, prices and the interest rate. This assumption implies that the interest elasticity of money demand is finite and has to be estimated.

Finally, let us focus on the sixth line of matrix A which corresponds to the identification of monetary policy shocks. Formally, we can identify monetary policy shocks with the disturbance term in the following equation

$$R_t = f(\Omega_t) + \sigma_R \eta_t^R,$$

where $\sigma_R \eta_t^R$ is a monetary policy shock, $f(\cdot)$ is a linear function that represents the monetary authority's feedback rule and Ω_t is the monetary authority's information set.[10] In table 1, we compare two identification schemes which differ in terms of Ω_t.

- The first panel of table 1 corresponds to scheme $\mathcal{B}$: the interest rate, as well as the private sector variables, are predetermined for money growth. In addition, we assume that the interest rate responds to contemporaneous disturbances of output and inflation. This identification assumption is close to the Cholesky decomposition which is widely used in the literature (Kim, 2000; Amato and Laubach, 2003; Christiano et al., 2005). In this case, the interest elasticity of money supply is infinite, which might be viewed as an extreme assumption, as argued by Leeper and Roush (2003).

- Contrary to the Cholesky-type identification, in the last panel of table 1, we assume that the interest rate and money growth are simultaneously determined (scheme $\mathcal{C}$). This means that the interest elasticity of money supply has to be freely estimated and we assume that the interest rate responds only to contemporaneous money growth disturbances. This simultaneity assumption has been used by Christiano et al. (1997), Leeper and Roush (2003) and Sims and Zha (2006). Its advantage is that it offers the possibility of distinguishing

[10] Some authors also propose to measure the monetary policy instrument with non borrowed reserves (Eichenbaum, 1992) or money base (Poole, 1970). However, following a large part of the literature, our monetary policy rule features a short term interest rate as monetary authorities' instrument (Clarida et al., 2000; Giannoni and Woodford, 2004).

money demand disturbances from monetary policy shocks through the interest elasticities of money demand and money supply.

2.2.2 Comparison of the Impulse Response Functions

Figures 1 reports the Impulse Response Functions (IRFs), over 20 quarters, of the key variables in x_t, to an exogenous increase in the interest rate which corresponds to a monetary contraction in each scheme.[11] The overall pattern of the IRFs is not greatly altered by a change in identification restrictions. However, the difference between the two schemes is mainly reflected in the extra persistence and the magnitude of the impulse responses. Indeed, as soon as the interest rate and money growth are simultaneously determined, the recession is deeper. However, unlike Leeper and Roush (2003), the reduction of the price puzzle is not so clear.[12] The magnitude of the response of wage inflation is also altered by the identification scheme. Indeed, its response is stronger and much more persistent when we assume simultaneity but its size is smaller compared with the other variables. In addition, the impact response of the interest rate is smaller in scheme $\mathcal{C}$ and its IRF displays much more persistence. Finally, we can point out that the impact response of money growth is stronger when we assume simultaneity between the interest rate and money growth.[13] These differences in the IRFs of the interest rate and money growth highlight the key role of the identification assumptions of monetary policy shocks.[14]

These results confirm that a rise of degree of inertia exhibited by inflation and the magnitude the responses of output, consumption and investment to a monetary policy shock, when we depart from the restrictions made in the Cholesky decomposition.[15] Therefore, we will investigate in the next section whether a CEE-based model is able to replicate this stronger degree of persistence without relying unreasonable degrees of nominal rigidity. In addition, we will highlight how the estimation of deep parameters and the monetary policy representation are modified with respect

[11] The confidence intervals of these IRFs are given in figures 2 and 3.

[12] This difference may results from the data construction (Leeper and Roush, 2003, use the price level rather than inflation) and the data frequency (they use monthly data rather than quarterly data).

[13] This result is also confirmed by Smets (2003) on euro area data.

[14] These SVAR models are not just identified. We thus report overidentifying restrictions tests (LR tests and Schwarz criterion minimization) in the technical appendix. We obtain similar results to Leeper and Roush (2003): scheme $\mathcal{C}$ is favored by the data, compared to scheme $\mathcal{B}$.

[15] In the technical appendix, we provide an interesting result: omitting money in a SVAR model identified with the Cholesky decomposition is not harmful. Indeed, the empirical IRFs obtained in scheme $\mathcal{B}$ are very close to those obtained if we exclude money from the SVAR model. This means that the inclusion of money in the empirical model has a very small effect on the variables' dynamics if we assume a recursive decomposition.

to the identification assumptions made in the SVAR model.

3 The Theoretical Model

In this section, we describe the theoretical model based on Christiano et al. (2005).[16] We build a framework in which the timing of events is consistent with the previous identification schemes. This means that all the optimization decisions of households and firms are made before the realization of the monetary policy shock, except households' decisions concerning asset and money holdings which are made at the same period. This specification implies that production, investment, consumption, prices and wages decisions are predetermined for monetary variables.

3.1 Production Side and Price Setting

In the first sector, the final good d_t is produced in a competitive market by combining a continuum of intermediate goods indexed by $\varsigma \in [0, 1]$. In addition, we use two assumptions which are known to increase the degree of strategic complementarities between price-setting firms. Firstly, we assume that the aggregate demand for final good is decomposed between a consumption good (y_t) and a material good (x_t), which both are produced by combining the same intermediate goods, and which have the same nominal price P_t. Secondly, the final good is produced through a production function characterized by a variable elasticity (Kimball, 1995).

In the second sector, monopolistic firms produce the intermediate goods $\varsigma \in [0, 1]$. Each firm ς is the sole producer of intermediate good ς. We assume that monopolist ς uses labor ($n_t(\varsigma)$), capital ($k_t(\varsigma)$) and material goods ($x_t(\varsigma)$) as inputs in order to produce $d_t(\varsigma)$. In addition, following Calvo (1983), we assume that in each period of time, a monopolistic firm can reoptimize its price with probability $1 - \alpha_p$, irrespective of the elapsed time since it last revised its price. If the firm cannot reoptimize its price, the latter is completely indexed to past inflation.

Standard manipulations yields the loglinearized new Phillips curve

$$\hat{\pi}_t - \hat{\pi}_{t-1} = \frac{(1-\alpha_p)(1-\beta\alpha_p)}{\alpha_p(1+\theta_p\epsilon_\mu)} \mathrm{E}_{t-1}\{\hat{s}_t\} + \beta \mathrm{E}_{t-1}\{\hat{\pi}_{t+1} - \hat{\pi}_t\}, \tag{4}$$

[16] We present here the loglinear version of the model. The details are provided in the appendix and the calculations are given in the technical appendix available upon request.

where $\hat{\pi}_t$ is the logdeviation of gross inflation, π_t, around its steady state, $\hat{s}_t$ is the logdeviation of the real marginal cost, s_t.[17] In addition, θ_p is the steady state elasticity of demand for a producer of intermediate good and ϵ_μ is the elasticity of the varying markup $\mu_p\left(d_t\left(\varsigma\right)/d_t\right)$, evaluated at the steady state. Finally, $\beta \in (0,1)$ is the subjective discount factor.

The aggregation of inputs is defined by $\int_0^1 n_t(\varsigma)\mathrm{d}\varsigma = \ell_t$, and $\int_0^1 k_t(\varsigma)\mathrm{d}\varsigma = u_t k_t$, where u_t is the utilization rate of capital. Following Christiano et al. (2005), we assume for convenience, that capital accumulation and utilization decisions are made by households. The loglinearized version of the real factor demands can be expressed as

$$(1-\mu_p s_x)\hat{w}_t = \hat{s}_t + \phi(1-\mu_p s_x)(\hat{k}_t + \hat{u}_t - \hat{\ell}_t), \tag{5}$$

$$(1-\mu_p s_x)\hat{r}_t^k = \hat{s}_t + (\phi-1)(1-\mu_p s_x)(\hat{k}_t + \hat{u}_t - \hat{\ell}_t), \tag{6}$$

where μ_p is the steady state price markup, s_x is the share of material goods in gross output and $0 < \phi < 1$ is the elasticity of value added with respect to capital. Finally, w_t is the real wage and r_t^k is the real rental rate of physical capital.

3.2 Households' Decisions

The economy is inhabited by differentiated households indexed by $\upsilon \in [0,1]$, each of which is endowed with a specific labor type. The typical household υ seeks to maximize his lifetime utility subject to the budget constraint and the law of capital accumulation.[18] Solving the household's optimization program yields the following behavioral equations expressed in their loglinearized version.

First, the risk free bond equation is given by

$$\hat{\lambda}_t - \hat{R}_t = \mathrm{E}_t\{\hat{\lambda}_{t+1} - \hat{\pi}_{t+1}\}, \tag{7}$$

where $\hat{\lambda}_t$ is marginal utility of wealth and $\hat{R}_t$ is the gross nominal interest rate of the economy.

[17] Afterwards, $\hat{x}_t$ denotes the logdeviation of x_t, around its steady state, where x_t is a generic variable of the model.

[18] The detailed optimization program is provided in the appendix.

Second, the loglinearized version of consumption FOCs is given by

$$\mathrm{E}_{t-1}\{(1-\beta b)\,\sigma\hat{\lambda}_t\} = \mathrm{E}_{t-1}\{\beta b\hat{c}_{t+1} - (1+\beta b^2)\hat{c}_t + b\hat{c}_{t-1} + \tau(\hat{m}_t - \beta b\hat{m}_{t+1})\}, \tag{8}$$

where $\hat{c}_t$ is household's consumption, $\hat{m}_t$ denotes real cash balances and $b \in (0,1)$ is the consumption habit parameter. Let σ denote the curvature of the utility function with respect to c_t. In addition, τ measures the real balance effects upon aggregate demand, such that

$$\tau = \sigma\chi, \quad \text{where} \quad \chi = \frac{\mathbb{U}_{cm}m}{\mathbb{U}_c}.$$

Here, $\mathbb{U}_m$ and $\mathbb{U}_c$ denote the steady state value of the derivative of the utility function with respect to m_t and c_t, respectively. In our model, we assume that broad monetary aggregate facilitates transactions which implies that real balances and consumption expenditures are complement ($\mathbb{U}_{cm} > 0$). Therefore, τ measures to what extent a change in real money balances – caused for instance by a variation of the interest rate – affects household's consumption. Thereafter, this parameter will be essential to investigate money's role in the monetary policy transmission.

Third, the money demand condition is given by

$$\hat{m}_t = \eta_c(\hat{c}_t - b\hat{c}_{t-1}) - \eta_R\hat{R}_t, \tag{9}$$

where η_c measures the consumption elasticity of money demand and η_R measures the interest semi-elasticity of money demand. In addition, we assume that the representative household stands next to the satiation level of money, which implies that $\mathbb{U}_m$ tends to zero and then the structural parameters are bound by

$$(1-\beta b)\eta_c = \eta_R\chi\nu, \quad \text{where } \nu \equiv \frac{c}{m}. \tag{10}$$

Fourth, the Euler equation on capital is given by

$$\mathrm{E}_{t-1}\{\hat{\lambda}_{t+1} + [1-\beta\,(1-\delta)]\,\hat{r}^k_{t+1} + (1-\delta)\beta\hat{p}_{k,t+1} - \hat{\lambda}_t - \hat{p}_{k,t}\} = 0, \tag{11}$$

where $\hat{p}_{k,t}$ can be interpreted as the shadow price of installed capital and δ is the depreciation rate of capital.

Fifth, the household's capital utilization decision is given by

$$\sigma_a^{-1}\mathrm{E}_{t-1}\{\hat{r}_t^k\} = \mathrm{E}_{t-1}\{\hat{u}_t\}, \tag{12}$$

where σ_a^{-1} is the elasticity of capital utilization with respect to the rental rate of capital.

Finally, the evolution of investment is given by

$$\mathrm{E}_{t-1}\{(\hat{\imath}_t - \hat{\imath}_{t-1}) - \beta(\hat{\imath}_{t+1} - \hat{\imath}_t) - \varkappa^{-1}\hat{p}_{k,t}\} = 0, \tag{13}$$

where $\hat{\imath}_t$ is investment and $\varkappa^{-1}$ is the elasticity of investment with respect to the current price of installed capital.

Finally, the loglinearized version of the law of motion for capital is defined by

$$\hat{k}_{t+1} = (1-\delta)\hat{k}_t + \delta\hat{\imath}_t. \tag{14}$$

3.3 Households' Wage Setting

Now, we focus on type-υ household's labor supply decisions. Following Erceg et al. (2000), we assume for convenience that a set of differentiated labor inputs, indexed by υ, are aggregated into a single labor index ℓ_t by competitive firms, which will be referred to labor intermediaries in the sequel. They produce the aggregate labor input according to a CES technology. It is assumed that, at each point in time, only a fraction $1-\alpha_w$ of the households can set a new wage, which will remain fixed until the next time period the household is drawn to reset its wage. The remaining households completely index their wages on past inflation. Standard manipulations yield the loglinearized version of the wage setting equation

$$\hat{\pi}_t^w - \hat{\pi}_{t-1} = \frac{(1-\alpha_w)(1-\beta\alpha_w)}{\alpha_w(1+\omega_w\theta_w)}\mathrm{E}_{t-1}\{\omega_w\hat{\ell}_t - \hat{\lambda}_t - \hat{w}_t\} + \beta\mathrm{E}_{t-1}\{\hat{\pi}_{t+1}^w - \hat{\pi}_t\} \tag{15}$$

where $\hat{\pi}_t^w$ denotes wage inflation, $\theta_w > 1$ is the elasticity of substitution between any two labor types and ω_w is the elasticity of labor disutility.

3.4 Monetary Authorities

The timing of events in the previous DSGE framework is consistent with that assumed in the previous two SVAR models. However, these empirical models differ with regard to the assumed simultaneity between the interest rate and money growth. Consequently, we consider two monetary policy representations in the DSGE model which are specified in order to be consistent with the monetary policy specification of the associated SVAR model.

First, let us consider a monetary policy rule on the form

$$\hat{R}_t = \rho_i \hat{R}_{t-1} + [(1-\rho_i)a_\pi]\hat{\pi}_t + [(1-\rho_i)a_y]\hat{y}_t + \epsilon_t, \tag{16}$$

where ϵ_t is a serially uncorrelated monetary policy shock such that $\epsilon_t \sim \text{iid}\,(0, \sigma_\epsilon)$. In addition, ρ_i measures the speed of adjustment of the interest rate to its steady state level and a_π and a_y measure the sensitivity of the interest rate to current inflation and output, respectively. This Taylor rule corresponds to the identification assumption which has been made in scheme $\mathcal{B}$. Indeed, money demand decisions are made after observing the monetary policy shock and the interest rate is money inelastic. Consequently, this DSGE model can be viewed as our benchmark case. Indeed, the monetary policy shock has been identified with the usual Cholesky decomposition and we estimate a standard DSGE model specified with the Taylor rule. In this case, money can only have a role through the real balance effect since it does not appear in the interest rate rule. If the real balance effects are negligible ($\chi = 0$) – as proposed by some authors (Woodford, 2003; McCallum, 2001) – introducing money in the model only aims at determining the quantity of money that the monetary authorities have to supply in order to clear the monetary market.

Second, we assume an alternative monetary policy rule

$$\hat{R}_t = \rho_i \hat{R}_{t-1} + [(1-\rho_i)a_m]\Delta\hat{m}_t + [(1-\rho_i)a_\pi]\hat{\pi}_{t-1} + [(1-\rho_i)a_y]\hat{y}_{t-1} + \epsilon_t. \tag{17}$$

This reaction function is consistent with scheme $\mathcal{C}$ in the previous section. Particularly, money and the interest rate are determined simultaneously since households make their money and bonds acquisition at the same period as the monetary shock and the interest rate responds to

contemporaneous variations in money growth ($\Delta\hat{m}_t$) and lagged values of inflation and production. In doing so, we seek to deal with a potential misspecification resulting from the extreme usual assumption that the elasticity of the interest rate to money is nil ($a_m = 0$). As a result, parameters a_m, η_R and τ – which are directly related to money supply and money demand – are essential components to measure the degree of simultaneity between the interest rate and money growth in our model. The evaluation of this simultaneity specification allows us to highlight whether the way money is introduced in the empirical and theoretical models impacts on the overall dynamics of the economy.

3.5 Model's Summary

The theoretical model can be summarized by equations (4), (5), (6), (7), (8), (9), (11), (12), (13), (14), (15), the monetary policy rules (16) or (17), together with the resources constraint and aggregate production constraint

$$[1-\beta(1-\delta)-\delta\beta\phi]\,\hat{c}_t+\delta\beta\phi\hat{\imath}_t+[1-\beta(1-\delta)]\,\phi\hat{u}_t=[1-\beta(1-\delta)]\,\hat{y}_t, \tag{18}$$

$$(1-\mu_p s_x)\hat{y}_t=\mu_p(1-s_x)[\phi(\hat{u}_t+\hat{k}_t)+(1-\phi)\hat{\ell}_t], \tag{19}$$

and $\hat{\pi}_t^w=\hat{w}_t-\hat{w}_{t-1}+\hat{\pi}_t$.

4 Model Estimation

4.1 Calibration

We partition the model parameters into two groups. The first one collects the parameters which we calibrate prior to estimation. These include parameters given by first order moments in the data, as well as parameters that cannot be separately identified. Let $\psi^c = (\beta,\phi,\theta_p,s_x,\epsilon_\mu,\theta_w,\omega_w,\delta,\nu)'$ denote the vector of calibrated parameters, whose values are reported in table 2. We choose $\beta = 0.99$ as is conventional in the literature for models confronted with quarterly data. As usual in the literature, we set the elasticity of output to capital to $\phi = 0.36$. In addition, we calibrate θ_p, s_x and ϵ_μ since these parameters cannot be identified, if we seek to estimate the degree of price rigidities, α_p (equation (4)). We set $\theta_p = 11$, implying a

steady state markup charged by intermediate goods producers of 10%, as proposed by Leith and Malley (2005). In addition, we set $s_x = 0.55$ implying that $\mu_p s_x = 0.6$, as suggested by Woodford (2003). Finally, we set $\epsilon_\mu = 0.2$ which implies that a 2% increase in relative prices results in a 24% decline in demand. This level of convexity of the demand function is reasonable.[19] For the same reason as before, we calibrate θ_w, which cannot be identified if we estimate the degree of wage rigidities, α_w, in equation (15). Therefore, we set $\theta_w = 11$, which is close to the value obtained by Kim (2000). We set $\omega_w = 1$, which implies a quadratic disutility of labor. In addition, we set the conventional value $\delta = 0.025$ which implies an annual rate of depreciation on capital of 10%. Finally, we use actual data to calibrate the steady state value of money's velocity, ν, such that $\nu = 1.15$.

4.2 Estimation Strategy

The second set of model's parameters is estimated by MDE. Let ψ denote the vector of estimated parameters

$$\psi = (\sigma,\ \chi,\ \eta_c,\ \eta_R,\ \alpha_p,\ \alpha_w,\ b,\ \kappa,\ \sigma_a,\ \rho_i,\ a_p,\ a_y,\ a_m,\ \sigma_\epsilon)'.$$

In the first section, we estimated different SVAR models on output, investment, consumption, wage inflation, inflation, the Fed Fund rate, money growth and commodity prices from 1959Q2 to 2004Q4. Let us recall that the vector of data, x_t, is given by vector (2).

We compute the empirical $(n \times 1)$ vector of dynamic responses of the variables to a monetary policy shock j periods ago, denoted by Φ_j such that[20]

$$\Phi_j = \frac{\partial x_{t+j}}{\partial \eta_t^i},$$

and we define Φ, such that

$$\Phi = \text{vec}(\Phi_0, ..., \Phi_h),$$

where the vec $(\cdot)$ operator transforms an $(n \times m)$ matrix into an $(nm \times 1)$ vector by stacking the columns of the original matrix and h is the final horizon. In our case, recall that $h = 20$

[19] A complete discussion about the curvature of the demand function is proposed in Chari et al. (2000) and Eichenbaum and Fisher (2004).

[20] Since the commodity prices variable, crb_t, doesn't have any counterpart in the DSGE model, this variable is removed from x_t for the MDE procedure.

quarters.[21] Let $\hat{\Phi}_T$ denote the empirical estimate of Φ, resulting from the estimated SVAR model where T is the sample size. As shown in Lütkepohl (2005)

$$\sqrt{T}(\hat{\Phi}_T - \Phi) \xrightarrow{d} \mathrm{N}\left(0, \Sigma_\Phi\right),$$

where Σ_Φ depends on the VAR parameters and matrices $\hat{A}$ and $\hat{B}$. In a second step, we compute the theoretical counterparts of vector Φ, denoted by $\Phi^m(\psi^c, \psi)$, obtained from the theoretical system which has been solved with the AIM algorithm (Anderson and Moore, 1985). Finally, we search for estimated values of ψ, denoted by $\hat{\psi}_T$, which fulfil

$$\hat{\psi}_T = \arg\min_{\psi \in \Psi} \; [\Phi^m(\psi^c, \psi) - \hat{\Phi}_T]' W_T [\Phi^m(\psi^c, \psi) - \hat{\Phi}_T], \tag{20}$$

where W_T is a diagonal matrix with the inverse of the asymptotic variances of each element of $\hat{\Phi}_T$ along the diagonal. Following Christiano et al. (2005), we compute the standard errors of the estimated parameters by using the asymptotic delta function method applied to the first order condition associated with (20).

5 Simultaneity between Money and the Interest Rate: Some Results

In the first section, we showed that simultaneity between the interest rate and money growth changes the magnitude and the persistence of the empirical responses of key variables. The purpose of this section is to investigate whether the standard CEE-based model is able to replicate these features and how the inference about the DSGE model is affected by the identification assumptions of the monetary policy shock.

5.1 Comparison of Empirical and Theoretical IRF

Figures 2 and 3 plot model-based and SVAR-based impulse responses, as well as the 90% asymptotic confidence intervals of the latter, when the SVAR model is identified with schemes $\mathcal{B}$

[21] Since some variables are predetermined for monetary policy shocks, the corresponding lines are zero in vector Φ_0. Thus, we remove these lines from vector Φ before estimation.

and $\mathcal{C}$, respectively.[22] The comparison of the empirical and theoretical IRFs helps us to assess the models' ability to replicate the main features of the economy's dynamics after a monetary contraction.[23]

In figure 2, the interest rate is predetermined for money growth (scheme $\mathcal{B}$). In this case, the responses of output, investment, consumption and wage inflation are well replicated. Furthermore, the model is also able to reproduce the responses of the interest rate and money growth. However, the model has some difficulty in reproducing the response of inflation. We may suggest that the reproduction of the magnitude of the response of inflation may be tricky for the model because of the length of the price puzzle: the response of inflation is positive for five quarters. In figure 3, the interest rate responds to current money growth disturbances and lagged output and prices disturbances (scheme $\mathcal{C}$). The goodness-of-fit of the theoretical model seems to be slightly improved, compared with the other scheme. Indeed, the model is able to perfectly reproduce the responses of the interest rate and money growth. In addition, the reproduction of the hump-shaped response of inflation is better, although the price puzzle does not disappear in the data. This means that the theoretical model is better able to generate inflation persistence which might be due to the rise in the persistence of the response of the interest rate in comparison with scheme $\mathcal{B}$.

Therefore, we show that a standard fully-fledged DSGE model is able to match the extra persistence resulting from the simultaneity assumption between the interest rate and money growth, as soon as it is built in order to closely correspond to the SVAR model. Now, we wonder whether this goodness-of-fit of the model is obtained through unreasonable estimated parameter values and we also investigate how the monetary policy representation is affected in the DSGE model with respect to the identification restrictions.

[22] Recall that the theoretical model is suitable for each identification scheme. Particularly, we estimate the model's parameters using monetary policy rule (16) when we implement the MDE on the SVAR identified with scheme $\mathcal{B}$. In the same way, we use monetary policy rule (17) when we refer to scheme $\mathcal{C}$ for estimation.

[23] We resort to bootstrap techniques to reveal the distribution of the minimum distance. Recall that since the weighting matrix W_T is not optimal, this statistic is not distributed as a χ^2 with $\dim(\hat{\Phi}_T) - \dim(\hat{\psi}_T)$ degrees of freedom. Bootstrapping the minimum distance allows us to circumvent this difficulty. In schemes $\mathcal{B}$ and $\mathcal{C}$, we generate 300 bootstrap replications of the SVAR model. For each replication, we reestimate the parameters of the DSGE model and we compute the value of the minimum distance. Then, the bootstraped distribution of this distance allows us to deduce a p-value for the overidentification test. With this methodology, we can check whether these DSGE model pass the overidentification test implied by our choice of moments. We obtain that the p-value of the overidentification test statistic is equal to 16.53% in scheme $\mathcal{B}$ and 48.41% in scheme $\mathcal{C}$. This means that the two DSGE models are not rejected by the data.

5.2 Estimation Results

The first two columns of table 3 report the estimated parameters for identification schemes $\mathcal{B}$ and $\mathcal{C}$. In a first step, we tried to estimate all the parameters in ψ. In each identification scheme, some parameters were characterized by binding constraints. In a second step, we enforced these equalities and estimated the remaining parameters.

5.2.1 Monetary Policy Shock and Deep Parameters

The magnitude of the monetary policy shock, σ_e, is significantly estimated at between 0.15 and 0.16. This suggests the impulses in the economy is not strongly altered by the identification scheme.

We turn to investigate whether the estimates of taste and rigidity parameters vary with respect to the identification scheme. The probability of no price adjustment, α_p, is included in interval $[0.57, 0.70]$. This means that the average duration of price contract is around two and three quarters. This value is consistent with the results reported by Bils and Klenow (2004). The value of α_p is not significant in scheme $\mathcal{B}$ but this might be due to the difficulty confronting the model when it comes to replicating the response of inflation. In addition, the improvement of the model's fit in scheme $\mathcal{C}$ implies a higher degree of price rigidities. The probability of no wage adjustment, α_w, is estimated between 0.77 and 0.86 which is higher than Christiano et al. (2005) but consistent with Del Negro et al. (2007).

The preference parameters are given by the degree of habit consumption (b) and the curvature of the utility function (σ) which are closely linked together in the estimation. These parameters are precisely estimated and they do not vary with respect to the specification since b is estimated between 0.76 and 0.79 and σ is estimated between 0.13 and 0.15.

We now focus on the parameters related to the investment behavior, given by the investment adjustment costs parameter (κ) and the elasticity of capital utilization with respect to the rental rate of capital (σ_a^{-1}). The investment adjustment costs parameter, κ, is between 6.82 and 8.16 which is slightly higher than in Smets and Wouters (2005, 2007). In addition, in scheme $\mathcal{B}$, the algorithm estimation drives σ_a to a very small value. Following Christiano et al. (2005), we set $\sigma_a = 0.01$. In scheme $\mathcal{C}$, the estimated value of this parameter is 0.24, but it is not significant.

These small values of σ_a mean that capital utilization is highly sensitive to a variation of the rental rate of capital, as in Christiano et al. (2005).

Consequently, it appears that the estimation of the deep parameters is quite robust to the identification schemes. This fact suggests that the overall structure of the economy is not dependent on the assumed identification of monetary policy shocks. Given the usual sensitivity of New Keynesian models to the specification of monetary policy, this was not warranted. To some extent, this result is reassuring. It remains to be seen whether the remaining parameters are robust.

5.2.2 Monetary Frictions

We now turn to discuss the estimated degree of transaction frictions and money demand function. Firstly, we focus on the estimated real balance effects, τ, in order to investigate whether balances could have an impact on consumption behavior.[24] The estimates of τ imply that the real balance effects are small ($\tau = 0.07$ and 0.03) but significant, whatever the identification restrictions. This suggests that the money's role in the transmission of monetary policy shocks is not neutral.[25]

Now, we seek to emphasize how identification schemes alter the estimation of the money demand equation. It appears that the consumption elasticity of money demand (η_c) and the interest semi-elasticity of money demand (η_R) are significantly estimated and they are sensitive to a change of the identification scheme. Indeed, η_R is close to one in scheme $\mathcal{B}$, but it strongly increases when we assume simultaneity between the interest rate and money growth ($\eta_R = 3.10$). This result suggests that money demand is more sensitive to variation of the interest rate in scheme $\mathcal{C}$. This is not surprising if we look at the impact response of money growth to monetary policy shocks (figure 1). Indeed, this contemporaneous response is higher in scheme $\mathcal{C}$, implying a higher interest semi-elasticity of money demand. This higher elasticity results in a larger value of η_c in scheme $\mathcal{C}$ ($\eta_c = 3.49$) than scheme $\mathcal{B}$ ($\eta_c = 2.22$). This effect is due to the link between these

[24] Ireland (2004) points out that shifts in money demand have to be considered to measure the effect of variations of money on output and inflation. However, in this paper, we focus on the impact of money on consumption after monetary policy shocks. In this case, we do not need to take explicitly exogenous shifts in money demand into account since variations of money balances result from the monetary policy shock. Therefore, contrary to Ireland (2004), τ is only a monetary policy transmission channel.

[25] Let us recall that $\tau = \chi\sigma$. We estimate χ and σ and the standard error of τ is calculated using the numerical Delta method.

elasticities.[26] The estimated values of η_c are higher than usually assumed in the literature – i.e. a unity income elasticity of money demand – and emphasizes the wealth effect on money demand. Thus, assuming that the interest rate is simultaneously determined with money growth implies a significant increase of the sensitivity of broad monetary aggregate to consumption and the interest rate.

5.2.3 Monetary Policy Rule

The previous results can be confronted to the estimated monetary policy rule which is essential for our discussion. The degree of smoothing of the interest rate is high in schemes $\mathcal{B}$ and $\mathcal{C}$ (ρ_i = 0.86 and 0.95, respectively). This suggests that the behavior of the Federal Reserve is rather gradualist over our sample. However, results concerning a_p, a_y and a_m confirm the intuition that the propagation channels of the monetary policy shock vary with respect to the identification assumption. Under the non-simultaneity assumption (scheme $\mathcal{B}$), we find that the estimated monetary policy rule is significantly active ($a_p = 1.49$) which is consistent with the traditional view. In addition, the role of current output in the monetary policy rule seems to be negligible since the estimation algorithm drives a_y to zero in this scheme.

In scheme $\mathcal{C}$, we allow for simultaneity between the interest rate and money growth, setting the interaction between these two monetary variables without priors, unlike the last scheme. In this case, we obtain that monetary authorities do not respond to past inflation ($a_p = 0$), whereas they significantly respond to current money growth disturbances ($a_m = 1.19$). In addition, the sensitivity of the interest rate to output is equal to 0.48 and it is significant at 10%. Therefore, assuming that the interest rate is simultaneously determined with money growth results in a high and significant value of a_m. This confirms that the theoretical model is able to capture the identification restrictions imposed in the SVAR model. Particularly, the assumption of simultaneity between money growth and the interest rate favors a money growth rule rather than a Taylor rule.[27] This result runs counter the usual thought that interest rate decisions are made only with respect to inflation and output paths. However, Christiano et al. (2005)

[26] Let us recall that $(1 - \beta b)\eta_c = \eta_R \chi \nu$. Since b, χ and ν are not strongly modified between schemes $\mathcal{B}$ and $\mathcal{C}$, most of the increase of η_R is offset by an increase of η_c.

[27] Indeed, since the monetary rule includes the interest rate and money growth and excludes inflation, we may interpret this relationship as a money growth rule. Output is also included in this relationship implying that we cannot interpret it as a money growth rule in the strict sense.

estimate a DSGE model by MDE on a monetary policy shock which is identified with the Cholesky decomposition and they show that the use of a money growth rule or a Taylor rule provides the same estimation results. Therefore, in the same spirit, we show that the money growth rule may be a valuable specification and we show that the identification restrictions are not neutral in the interpretation of the monetary policy decisions.

5.3 Discussion

By considering a DSGE model which closely corresponds to a particular SVAR model, we showed that the monetary policy representation in the estimated DSGE model is deeply changed by the identification restrictions made in the SVAR model. Precisely, the simultaneity assumption between the interest rate and money growth implies a high and significant interest elasticity to money supply. This result differs from the traditional view concerning the Taylor rule. However, we also showed that a variation of the real balances has a small but significant effect on consumption path, whatever the identification scheme. In view of these findings, we can infer that the question of money's role in the transmission of monetary policy shocks in a DSGE model is not straightforward. Therefore, we seek to investigate the real balance effect's contribution to these results. Indeed, if we show that the absence of a real balance effect does not change our estimation results, we could conclude that money only matters in the transmission of monetary policy shocks through the monetary policy rule and its does not have a causal role on consumption and inflation paths. Thus, in this section, we carry out a set of exercises in order to investigate to what extent our estimation results are affected by a change in the specification of money in the theoretical model.

We start by investigating how the interest elasticity to money supply matters for the dynamics of a standard DSGE model. Indeed, the significant value of a_m in scheme $\mathcal{C}$ implies that monetary authorities may take a great interest in broad monetary aggregate, especially since a_p is null. This intuition is confirmed when we reestimate the theoretical model, in scheme $\mathcal{C}$, subject to the constraint $a_m = 0$. In this case, the algorithm estimation drives a_p close to one and we encounter the usual indeterminacy issue.[28] This result confirms that money is essential in the monetary

[28] We do not report the details of the estimation because of the indeterminacy issue.

policy rule as soon as we leave the relationship between the interest rate and money growth free during the identification of monetary policy shocks. Henceforth, the estimated monetary policy rule runs counter to the Taylor rule. The primordial role of money in central bank's decisions is interpreted by Smets (2003) as the reflection that money contains information about future output and price and it can be viewed as a forward looking indicator. Although we are not able to confirm this interpretation with our DSGE model, we may think that money can be viewed as a useful indicator for monetary policy decisions in our DSGE model.

We now turn to focus on the role of money in the monetary policy transmission channels, through the real balance effects. The estimated real balance effects in each scheme are significant but they are also very small. This might suggest that the main role of money in the transmission of monetary policy shocks is not given by its impact on consumption and inflation paths. It is usual in the literature to find that money has no role in consumption behavior. More precisely, Woodford (2003) shows that the variables' dynamics are not modified by the omission of real balance effects in theoretical models. Some authors also emphasize the unimportant role of money in the overall behavior of the economy (McCallum, 2001; Ireland, 2004; Dotsey and Hornstein, 2003; Woodford, 2003; Andrés et al., 2006).[29] However, unlike these authors, we focus on money's role in the transmission of monetary policy shocks.

Therefore, we reestimate the model subject to $\tau = 0$.[30] Not surprisingly, in light of the previous result, we were confronted with identification issues in scheme $\mathcal{C}$, which resulted in the impossibility to identify a_m when $\tau = 0$. In other words, the relationship between money growth, inflation and consumption helps to identify a generalized Taylor's rule in which the interest rate can respond to broad monetary aggregate. Consequently, we reestimate our model subject to the constraints $a_m = \tau = 0$, in each scheme. First, figures (4) and (5) compare the SVAR-based and model-based IRFs in schemes $\mathcal{B}$ and $\mathcal{C}$, respectively, by assuming that money is supplied in the model only in order to clear the monetary market. In each scheme, the model is clearly unable to replicate the impulse response of money growth. This directly results from the constraint $\tau = 0$. Indeed, due to equation (10), the constraint $\tau = 0$ means that the interest semi-elasticity

[29] However, some papers have also highlighted the important role of money in the variables' dynamic (Favara and Giordani, 2002; Nelson, 2002,2003).

[30] More precisely, we set $\chi = 0$.

of money demand (η_R) is also constrained, which implies that the model is not able to fit the response of money growth. In addition, in scheme $\mathcal{C}$, the goodness-of-fit of the model is worse than in the unconstrained case, particularly concerning the impulse responses of inflation, wage inflation and the interest rate which are not persistent enough. In scheme $\mathcal{B}$, the deterioration of the goodness-of-fit is lower because this lack of persistence is less burdensome. These results mean that the presence of real balance effect in scheme $\mathcal{C}$ helps to generate a strong persistence of the interest rate. Indeed, due to the simultaneity between the interest rate and money growth, money's dynamics play an important role for those of the interest rate. Consequently, the model does not generate enough persistence of the interest rate because of its bad performance when it comes to replicating the response of money.

The reestimated parameters are reported in the last two columns of table 3. The estimates of the deep parameters are quite robust when we remove the real balance effects from the theoretical model. However, the estimates of the monetary policy rule and their accuracy are strongly changed by this specification. Indeed, in scheme $\mathcal{B}$, a_p tends to become very high ($a_p = 5.34$) and not significant. In scheme $\mathcal{C}$, the value of a_p is more consistent with the literature ($a_p = 1.17$) but it is also non significant. This result is not surprising since we showed in figure (5) that the model is unable to replicate the strong persistence of the interest rate. In addition, the sensitivity of the interest rate to output tends to be null in the two schemes. Therefore, we confirm our previous intuition that real balances can be viewed as a key mechanism in order to precisely estimate the monetary policy rule.

Conclusion

In this paper, we proceed in two steps. First, we estimate two monetary SVAR models which differ in terms of restrictions which are set to identify monetary policy shocks. A misspecification of an empirical SVAR model might result from the extreme assumptions on the interest elasticities of money supply and money demand, which are imposed when we use the Cholesky decomposition (Leeper and Roush, 2003). We thus compare the impulse responses of key variables according to the degree of simultaneity between the interest rate and money growth. Second, we consider a theoretical monetary model in which the monetary policy representation is consistent

with the corresponding SVAR model. Then, we investigate whether this standard DSGE model is able to replicate the higher persistence of inflation and output given by the assumption that the interest rate and money growth are simultaneously determined. In addition, we highlight how the estimates of the DSGE model are changed with respect to the identification scheme in several SVAR model.

We obtain many results. First, as in Leeper and Roush (2003), we show that the persistence and the magnitude of the SVAR-based impulse responses are changed as soon as we assume that the interest rate and money growth are simultaneously determined. Second, we show that the standard CEE-based model is perfectly able to replicate the stronger persistence of inflation and output which results from the simultaneity assumption. In addition, this good performance of the DSGE model does not result in a unreasonable degree of nominal rigidities and the estimation of the taste parameters is robust to the assumed identification scheme. Finally, the monetary policy representation in the estimated DSGE model is deeply changed by the identification restrictions made in the SVAR model. Precisely, the simultaneity assumption between the interest rate and money growth implies a high and significant interest elasticity to money in the interest rate rule whereas the interest rate is inelastic to past inflation. This result differs from the traditional view concerning the Taylor rule which usually excludes money from the monetary policy decisions. We also show that the real balance effects are essential to precisely estimate the monetary policy rule, whatever the identification scheme. These results suggest that we must be cautious about the identification of monetary policy shocks when we are interested in the estimation by MDE of DSGE models which contains both money and the interest rate. Indeed, in a standard CEE-based model, the estimates of the taste parameters and the degrees of nominal rigidities are quite robust to the identification restrictions made in the SVAR model. However, the estimated monetary policy representation is modified so as to capture the extra persistence implied by the simultaneity assumption between the interest rate and money growth.

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Model Appendix

Production Side and Price Setting

In the first sector, the overall aggregate good, $d_t = y_t + x_t$, is produced by competitive firms according to the production function proposed by Kimball (1995)

$$\int_0^1 G\left(\frac{d_t(\varsigma)}{d_t}\right) \mathrm{d}\varsigma = 1, \tag{21}$$

where $d_t(\varsigma)$ denote the overall demand addressed to the producer of intermediate good ς, and the function $G(\cdot)$ is increasing, strictly concave, and satisfies the normalization $G(1) = 1$. From the optimization program of the representative competitive firm, we can deduce the overall demand addressed to the producer of intermediate good $\breve{\varsigma}$

$$G'\left(\frac{d_t(\breve{\varsigma})}{d_t}\right) = \frac{P_t(\breve{\varsigma})}{P_t} \int_0^1 \frac{d_t(\varsigma)}{d_t} G'\left(\frac{d_t(\varsigma)}{d_t}\right) \mathrm{d}\varsigma. \tag{22}$$

Let $\theta_p(\xi_t)$ denote the elasticity of demand for a producer of intermediate good facing the relative demand $\xi_t = d_t(\varsigma)/d_t$. According to the implicit demand function (21), $\theta_p(\xi_t)$ obeys $\theta_p(\xi_t) = -G'(\xi_t)/[\xi_t G''(\xi_t)]$. This equality makes clear that intermediate good firms face a varying elasticity of demand for their output, implying a time varying markup, which is denoted by $\mu_p(\xi_t)$, and obeys $\mu_p(\xi_t) \equiv \theta_p(\xi_t)/[\theta_p(\xi_t) - 1]$. Afterwards, let θ_p denote the steady state elasticity of demand for a producer of intermediate good and μ_p is the steady state markup.

In the second sector, monopolistic firms'ς produce the intermediate goods $\varsigma \in [0, 1]$. Given the demand function (22) of $d_t(\varsigma)$, we assume that monopolist ς faces the following production possibilities

$$\min \left\{ \frac{k_t(\varsigma)^{\phi} n_t(\varsigma)^{1-\phi} - \kappa}{1 - s_x}, \frac{x_t(\varsigma)}{s_x} \right\} \geq d_t(\varsigma), \tag{23}$$

where κ is a fixed production cost ensuring that aggregate profits are zero in its steady state. This specification, which includes material inputs in the production of intermediate goods, has been proposed by Rotemberg and Woodford (1995). Let $\mathbb{S}(d_t(\varsigma))$ denote the real cost of producing $d_t(\varsigma)$ units of good ς

$$\mathbb{S}(d_t(\varsigma)) = w_t n_t(\varsigma) + r_t^k k_t(\varsigma) + x_t(\varsigma). \tag{24}$$

Notice that since capital and aggregate labor are supplied in economy-wide markets w_t and r_t^k do not depend on ς. (Yun, 1996; Erceg et al., 2000, for instance). Minimization of the real production cost yields

$$s_t = (1 - s_x) \left(\frac{w_t}{1 - \phi} \right)^{1-\phi} \left(\frac{r_t^k}{\phi} \right)^{\phi} + s_x, \tag{25}$$

where s_t is the real marginal cost.

Following Calvo (1983), we assume that in each period of time, a monopolistic firm $\breve{\varsigma}$ can reoptimize its price with probability $1 - \alpha_p$, irrespective of the elapsed time since it last revised its price. If the firm cannot reoptimize its price, the latter is rescaled according to the simple revision rule

$$P_T(\breve{\varsigma}) = \delta_{t,T}^p P_t(\breve{\varsigma}), \quad \text{where} \quad \delta_{t,T}^p = \prod_{j=t}^{T-1} \pi^{1-\gamma_p} \pi_j^{\gamma_p}, \text{ if } T > t; \ \delta_{t,T}^p = 1, \text{ if } T \leq t. \tag{26}$$

Here, $\pi_t = P_t / P_{t-1}$ represents the inflation rate, π is the steady state inflation rate. Following Christiano et al. (2005) and Giannoni and Woodford (2004), we assume that $\gamma_p = 1$. Let $P_t^\star(\breve{\varsigma})$ denote the price chosen in period t, and let $d_{t,T}^\star(\breve{\varsigma})$ denote the production of good ς in period T if firm $\breve{\varsigma}$ last reoptimized its price in period t. In addition, we define $p_t^\star(\breve{\varsigma}) = P_t^\star(\breve{\varsigma}) / P_t$. Firm $\breve{\varsigma}$ selects $P_t^\star(\breve{\varsigma})$ so as to maximize the present discounted sum of profit streams

$$\mathrm{E}_{t-1} \sum_{T=t}^{\infty} (\beta \alpha_p)^{T-t} \frac{\lambda_T}{\lambda_t} \left\{ \frac{\delta_{t,T}^p P_t^\star(\breve{\varsigma})}{P_T} d_{t,T}^\star(\breve{\varsigma}) - \mathbb{S}\left(d_{t,T}^\star(\breve{\varsigma}) \right) \right\},$$

subject to the demand function

$$\frac{d_{t,T}^\star(\breve{\varsigma})}{d_T} = G'^{-1} \left(\frac{\delta_{t,T}^p P_t^\star(\breve{\varsigma})}{P_T} \int_0^1 \frac{d_t(\varsigma)}{d_t} G'\left(\frac{d_t(\varsigma)}{d_t} \right) \mathrm{d}\varsigma \right), \tag{27}$$

where $\beta \lambda_T / \lambda_t$ is the stochastic discount factor, and $\mathrm{E}_{t-1}\{\cdot\}$ is the expectation operator conditional on information available at $t - 1$. Standard manipulations yields equation (4). In addition, using the fact that $s = 1/\mu_p$, the loglinearized version of the real marginal cost defined in equation (25) is given by equations (5) and (6).

Households' Decisions

The economy is inhabited by differentiated households indexed on $\upsilon \in [0,1]$, each of which is endowed with a specific labor type. A typical household υ acts as a monopoly supplier of type-υ labor. Following Erceg et al. (2000), we assume for convenience that a set of differentiated labor inputs are aggregated into a single labor index ℓ_t by competitive firms. They produce the aggregate labor input according to the following CES technology

$$\ell_t = \left(\int_0^1 \ell_t(\upsilon)^{(\theta_w - 1)/\theta_w} \, \mathrm{d}\upsilon \right)^{\theta_w/(\theta_w - 1)}.$$

The typical household υ seeks to maximize

$$\mathrm{E}_{\Theta_t} \sum_{T=t}^{\infty} \beta^{T-t} \left[\mathbb{U}(c_T - bc_{T-1}, m_T) - \mathbb{V}(\ell_T(\upsilon)) \right], \tag{28}$$

where $m_T = M_T/P_T$ and M_T denotes nominal cash balances. In addition, E_{Θ_t} is an expectation operator conditioned on the particular information set available to the agent at the time she makes her decisions. Household υ maximizes (28) subject to the sequence of constraints

$$P_T[c_T + i_T + a(u_T)k_T] + M_T + \frac{B_T}{R_T} + P_T \mathrm{tax}_T \leq W_T(\upsilon)\,\ell_T(\upsilon) + P_T r_T^k u_T k_T + B_{T-1} + M_{T-1} + P_T \mathrm{div}_T, \tag{29}$$

$$k_{T+1} = (1-\delta)k_T + i_T \left(1 - F\left(\frac{i_T}{i_{T-1}} \right) \right), \tag{30}$$

where $w_T(\upsilon) \equiv W_T(\upsilon)/P_T$ is the real wage rate earned by type-υ labor; $b_T \equiv B_T/P_T$, where B_T denotes the nominal bonds acquired in period T and maturing in period $T+1$. In addition, div_T denotes profits redistributed by monopolistic firms and tax_T is a real lump-sum tax designed to finance the subsidies granted to monopolistic firms[31]. Finally, $a(u_T)$ denotes the real cost (in unit of consumption good) of setting the utilization rate of u_T and $F(\cdot)$ measures the adjustment costs related to investment. We assume that $a(u) = 0$, where u is the deterministic steady state value of u_t. Similarly, we assume that $F(1) = F'(1) = 0$, so that adjustment costs vanish along a deterministic balanced growth path. Let us denote $\lambda_t \equiv \Lambda_t P_t$ and $p_{k,t} \equiv \Upsilon_t/\lambda_t$, where Λ_t and

[31] As in Christiano et al. (2005), we assume that the government manages lump-sum taxes and it pursues a Ricardian fiscal policy. This implies that fiscal policy has no impact on the variables. Therefore, we do not specify this latter.

Υ_t are Lagrange multipliers associated to the budget constraint (29) and the law of motion of capital (30), respectively.

The loglinearized version of the FOCs with respect to B_t, c_t, m_t, k_{t+1}, u_t and i_t are given by equations (7), (8), (9), (11), (12), (13). In equation (8), we have $\sigma^{-1} = -\mathbb{U}_{cc}c/\mathbb{U}_c$. In equation (9), the parameters are given by $\eta_c = -\mathbb{U}_{mc}c/(\mathbb{U}_{mm}m)$ and $\eta_R = -(1-\beta b)\,\mathbb{U}_c/(\mathbb{U}_{mm}m)$. In equation (12) and (13), we have $\sigma_a \equiv ua''(u)/a'(u)$. and $\varkappa \equiv F''(1)$.

Households' Wage Setting

It is assumed that at each point in time only a fraction $1-\alpha_w$ of the households can set a new wage, which will remain fixed until the next time period the household is drawn to reset its wage. The remaining households simply revise their wages according to the simple rule

$$W_T(\upsilon) = \delta^w_{t,T} W_t(\upsilon), \quad \text{where } \delta^w_{t,T} = \prod_{j=t}^{T-1} \pi^{1-\gamma_w}\pi_j^{\gamma_w}, \text{ if } T > t; \ \delta^w_{t,T} = 1, \text{ if } T \leq t,$$

where we assume $\gamma_w = 1$. In the sequel, it will be convenient to define wage inflation $\pi^w_t \equiv W_t/W_{t-1}$. Let $W^\star_t(\upsilon)$ denote the wage rate chosen in date t, and $\ell^\star_{t,T}(\upsilon)$ denotes hours worked in period T if household υ last reoptimized its wage in period t. In period t, household υ chooses his wage rate $W^\star_t(\upsilon)$ in order to solve

$$\max_{W^\star_t(\upsilon)} \quad \mathrm{E}_{t-1}\sum_{T=t}^{\infty}(\beta\alpha_w)^{T-t}\left\{\lambda_T \frac{\delta^w_{t,T}W^\star_t(\upsilon)}{P_T}\ell^\star_{t,T}(\upsilon) - \mathbb{V}(\ell^\star_{t,T}(\upsilon))\right\},$$

subject to

$$\ell^\star_{t,T}(\upsilon) = \left(\frac{\delta^w_{t,T}W^\star_t(\upsilon)}{W_T}\right)^{-\theta_w}\ell_T.$$

Standard manipulations yield equation (15), where $\omega_w = \ell\mathbb{V}_{\ell\ell}/\mathbb{V}_\ell$.

Data Appendix

The SVAR model is estimated with quarterly data over the sample 1959Q2-2004Q4. All the series are seasonally adjusted except for the interest rate and commodity prices. The series are constructed as following: y_t is real output from non farm business sector (Source: Bureau of

Labor Statistics); i_t is fixed private investment (Source: Bureau of Economic Analysis). It is divided by the implicit price deflator of output; c_t is personal consumption of non durable goods and services (Source: Bureau of Economic Analysis). It is divided by the implicit price output deflator; π_t^w is growth rate of hourly compensation in the non farm business sector (Source: Bureau of Labor Statistics); π_t is the growth rate of the implicit price output deflator in the non farm business sector (Source: Bureau of Labor Statistics); R_t is Federal funds rate (Source: Federal Reserve Board); Δm_t is the growth money of M2 (Source: Federal Reserve Board). crb_t is the Commodity Research Bureau (CRB) spot commodity price index for raw industrials (Source: CRB). The variables y_t, i_t, c_t and Δm_t are expressed in per-capita terms by dividing by the civilian non institutional population, age 16 and over. In addition, $\log(y_t)$, $\log(i_t)$, $\log(c_t)$, $\log(crb_t)$ are linearly detrended.

Table 1. Identification Schemes of Matrix A

Scheme $\mathcal{B}$	ε^y	ε^i	ε^c	ε^{π^w}	ε^π	ε^r	$\varepsilon^{\Delta m}$	ε^{crb}
η^y	1	0	0	0	0	0	0	0
η^i	$\times$	1	0	0	0	0	0	0
η^c	$\times$	$\times$	1	0	0	0	0	0
η^{π^w}	$\times$	$\times$	$\times$	1	0	0	0	0
η^π	$\times$	$\times$	$\times$	$\times$	1	0	0	0
η^R	$\times$	0	0	0	$\times$	1	0	0
$\eta^{\Delta m}$	0	0	$\times$	0	$\times$	$\times$	1	0
η^{crb}	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	1

Scheme $\mathcal{C}$	ε^y	ε^i	ε^c	ε^{π^w}	ε^π	ε^r	$\varepsilon^{\Delta m}$	ε^{crb}
η^y	1	0	0	0	0	0	0	0
η^i	$\times$	1	0	0	0	0	0	0
η^c	$\times$	$\times$	1	0	0	0	0	0
η^{π^w}	$\times$	$\times$	$\times$	1	0	0	0	0
η^π	$\times$	$\times$	$\times$	$\times$	1	0	0	0
η^R	0	0	0	0	0	1	$\times$	0
$\eta^{\Delta m}$	0	0	$\times$	0	$\times$	$\times$	1	0
η^{crb}	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	1

Notes: For each scheme, we represent the identification restrictions of matrix A. $\times$ denotes a freely estimated parameter. In scheme $\mathcal{B}$, the interest rate is predetermined for money growth. In scheme $\mathcal{C}$, the interest rate and money growth are simultaneously determined.

Table 2. Calibrated Parameters

Parameters	Value
β	0.99
ϕ	0.36
θ_p	11.00
s_x	0.55
ϵ_μ	0.20
θ_w	11.00
ω_w	1.00
δ	0.025
ν	1.15

Table 3. Results of MDE estimation (1959Q2-2004Q4)

	Unconstrained Model		Constrained Model	
Scheme	$\mathcal{B}$	$\mathcal{C}$	$\mathcal{B}$	$\mathcal{C}$
Monetary Policy Shocks				
σ_e	0.165 (0.008)	0.148 (0.018)	0.162 (0.008)	0.106 (0.010)
Taste and Rigidity Parameters				
α_p	0.572 (0.434)	0.705 (0.086)	0.467 (0.726)	0.523 (0.222)
α_w	0.859 (0.094)	0.775 (0.057)	0.913 (0.058)	0.824 (0.052)
b	0.762 (0.059)	0.790 (0.058)	0.819 (0.093)	0.881 (0.080)
σ	0.148 (0.038)	0.130 (0.032)	0.230 (0.080)	0.162 (0.068)
κ	8.157 (2.049)	6.819 (1.534)	7.463 (2.038)	11.208 (2.048)
σ_a	0.010 (*)	0.239 (0.238)	0.010 (*)	0.010 (*)
Monetary Frictions				
τ	0.074 (0.001)	0.028 (0.000)	–	–
η_R	0.955 (0.315)	3.097 (0.416)	1.027 (0.326)	3.771 (0.528)
η_c	2.225 (0.016)	3.490 (0.021)	–	–
Monetary policy rule				
ρ_R	0.859 (0.032)	0.953 (0.022)	0.922 (0.092)	0.924 (0.035)
a_p	1.491 (0.567)	0.000 (*)	5.341 (6.824)	1.171 (0.728)
a_y	0.000 (*)	0.485 (0.250)	0.091 (0.371)	0.000 (*)
a_m	–	1.195 (0.465)	–	–

Notes: *Unconstrained model:* Scheme $\mathcal{B}$: the interest rate predetermined for money growth. Scheme $\mathcal{C}$, the interest rate and money growth simultaneously determined. *Constrained model:* Scheme $\mathcal{B}$, $\tau = 0$. Scheme $\mathcal{C}$, $\tau = a_m = 0$. The numbers in parentheses are the standard errors of the parameters. A star refers to a constraint imposed during the estimation stage to avoid convergence issues.

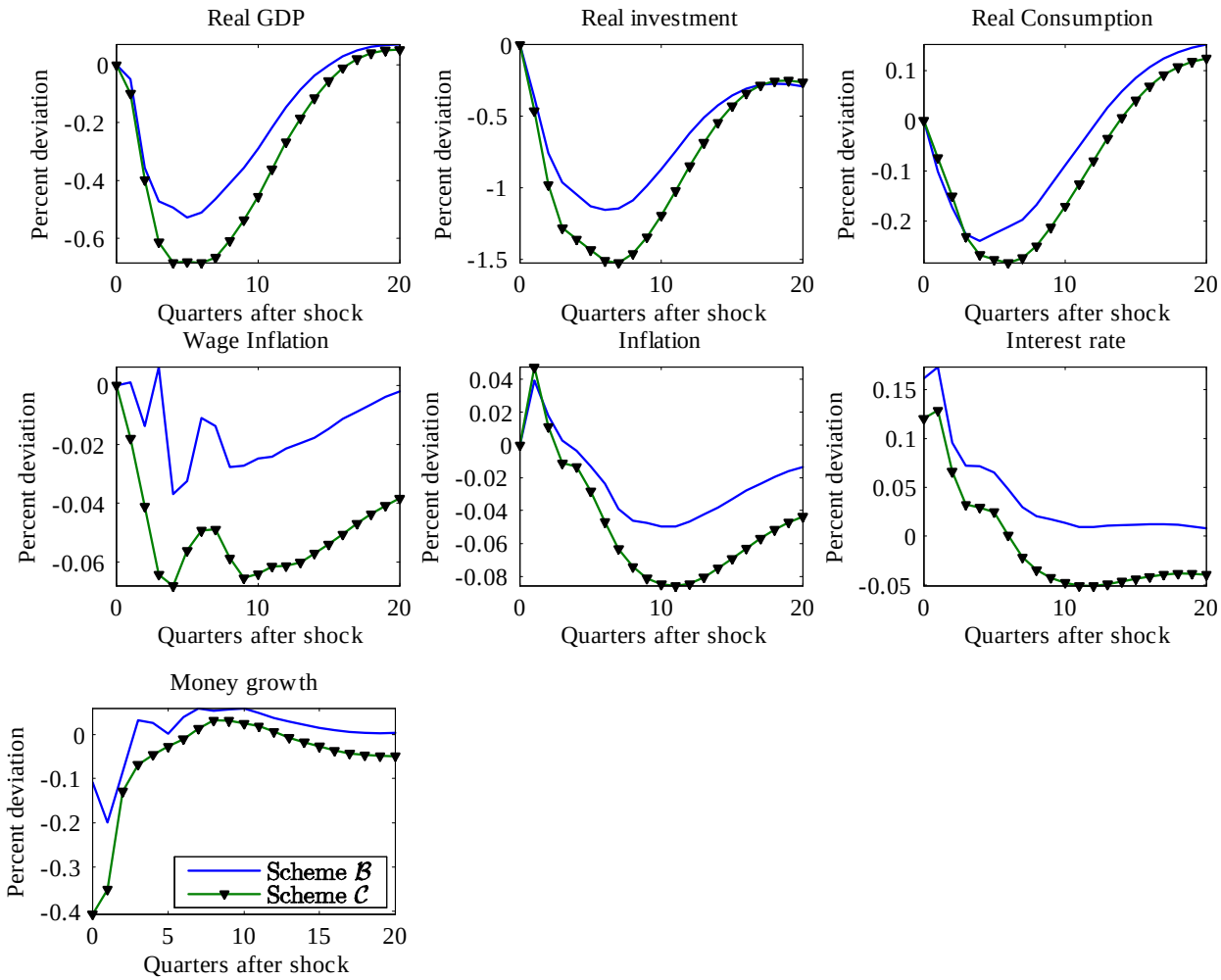

Figure 1: Comparison of the empirical impulse response functions (multiplied by 100) over the sample 1959Q2-2004Q4.

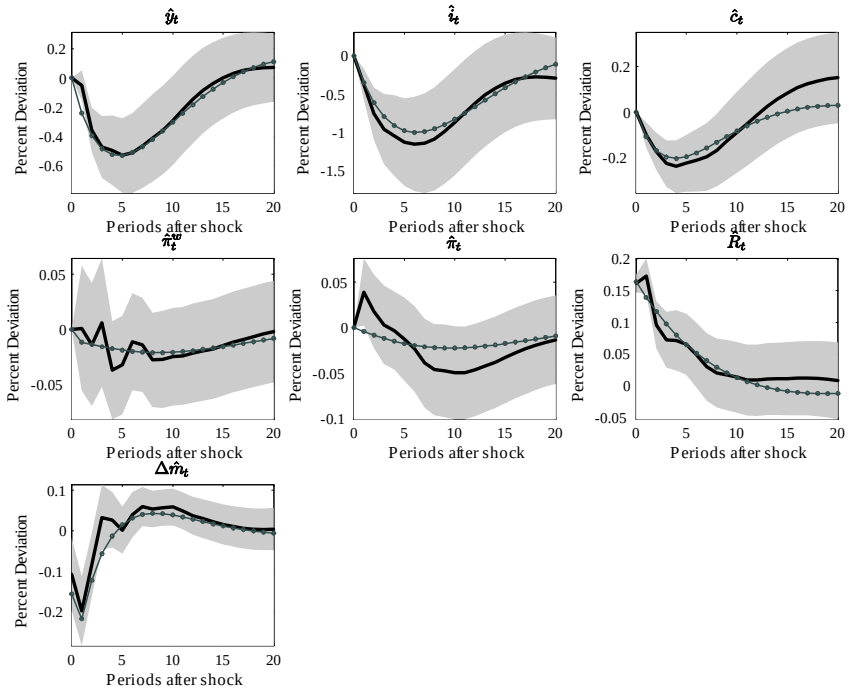

Figure 2: SVAR-based impulse responses (solid lines) and model-based impulse responses (lines with circle) – multiplied by 100 – to a monetary policy shock, in scheme $\mathcal{B}$. Grey area corresponds to the 90% confidence interval.

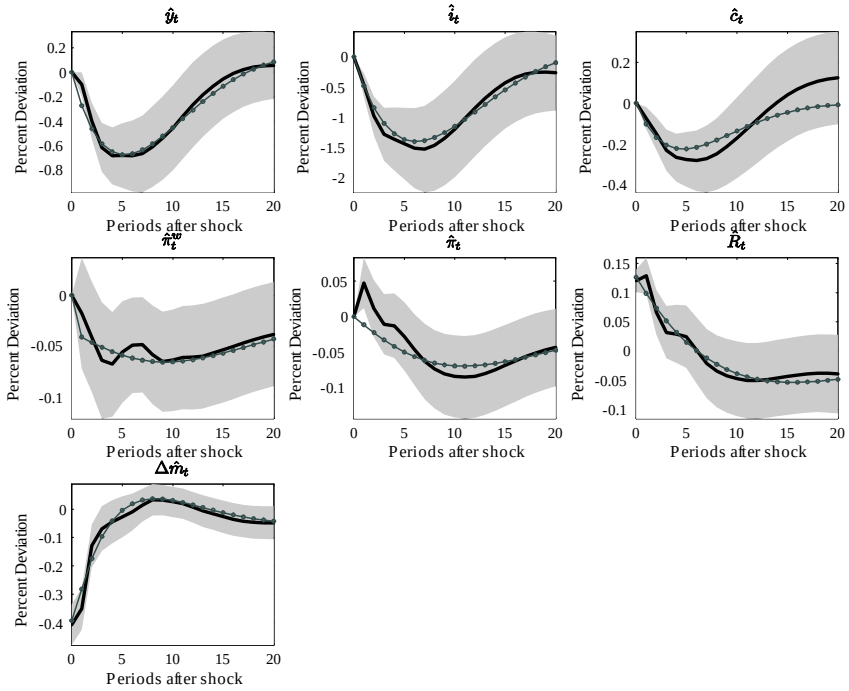

Figure 3: SVAR-based impulse responses (solid lines) and model-based impulse responses (lines with circle) – multiplied by 100 – to a monetary policy shock, in scheme $\mathcal{C}$. Grey area corresponds to the 90% confidence interval.

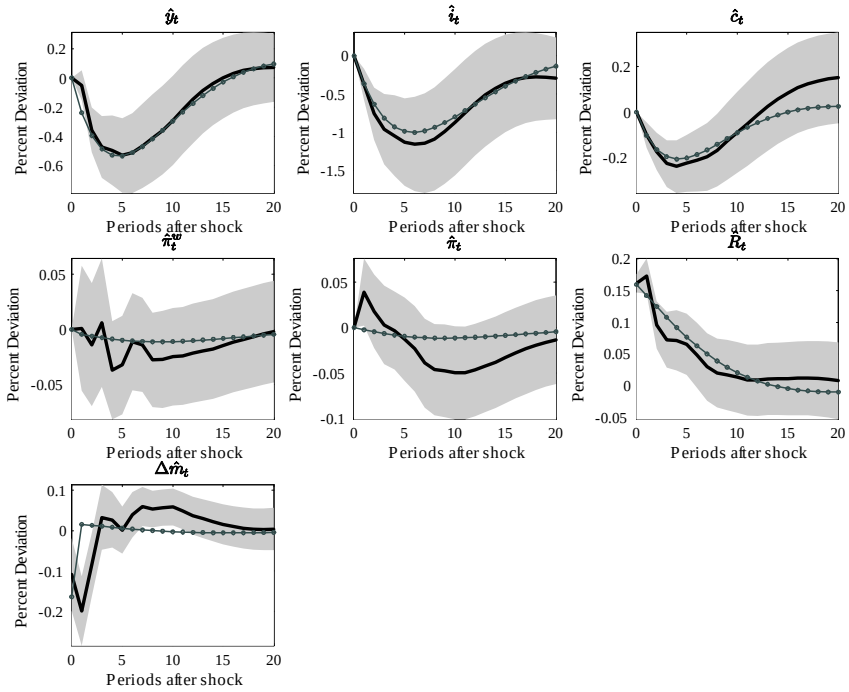

Figure 4: SVAR-based impulse responses (solid lines) and model-based impulse responses (lines with circle) – multiplied by 100 – to a monetary policy shock, in scheme $\mathcal{B}$, when we enforce $\tau = 0$. Grey area corresponds to the 90% confidence interval.

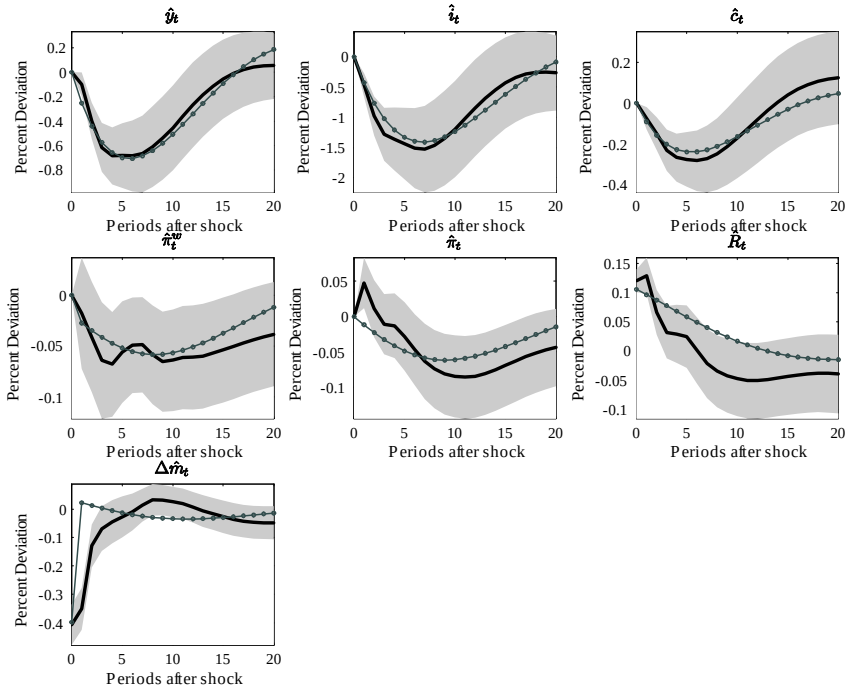

Figure 5: SVAR-based impulse responses (solid lines) and model-based impulse responses (lines with circle) – multiplied by 100 – to a monetary policy shock, in scheme $\mathcal{C}$, when we enforce $\tau = a_m = 0$. Grey area corresponds to the 90% confidence interval.