

# Duration Dependent Rules and Nominal Inertia

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## Abstract

This paper develops a dynamic general equilibrium model where price and wage reset probabilities are duration dependent and analyses the effects of monetary shocks on inflation. The model is simulated for alternative reset probability distributions. It is found that such a model can explain the behaviour of inflation better than the standard Calvo model, in particular with respect to the delayed effect on inflation of a monetary policy shock. In this model, in fact, under certain conditions, the maximum impact occurs some time after the shock. Moreover, it is found that the presence of wage rigidities in addition to price rigidities is fundamental for the validity of the results.

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# 1 Introduction

Macroeconomic theory ideally requires models which are both based on solid microfoundations and consistent with the empirical evidence. In the recent past, New Neoclassical Synthesis (or New Keynesian) models have been successfully used to address a number of monetary policy issues. Their main innovation is the introduction of nominal rigidities in a dynamic general equilibrium framework based on the agents' optimising behaviour. This framework therefore provides a clear behavioural interpretation of the model's equations. However, New Neoclassical Synthesis models are not fully able to capture key features of the data, in particular with respect to inflation inertia.

Inflation inertia is defined as the delayed and gradual response of inflation to a shock. The empirical evidence in favour of inflation inertia is strong, but estimates of the magnitude of the phenomenon vary across studies. For example, Rotemberg and Woodford (1998) report a large impact of monetary policy on inflation after 2 quarters. However, most studies find that inflation peaks with a longer delay. Smets and Wouters (2003) estimate a dynamic general equilibrium model using Euro area data and find that inflation responds to a monetary shock in a hump-shaped fashion, with the maximum impact occurring after about 4 quarters. Christiano et al. (2005), using U.S. data, find that inflation peaks after 8 quarters following an expansionary monetary policy shock. A similar response of inflation is reported by Di Cecio and Nelson (2007) for the UK economy.

A number of studies have focused on the development of models able to ac-

count for this evidence. A first approach consists of assuming that price and wage contracts take effect at a later date than the one at which they are set (see, for example Bernanke and Woodford, 1997). A model where the new price or wage takes effect immediately but is set on the basis of old information, as in Mankiw and Reis (2002), has the same implications for the dynamics of inflation<sup>1</sup>. Another approach involves indexation: when agents are not allowed to review their contracts, prices or wages are automatically increased at the rate of past inflation. Christiano et al. (2005) introduce indexation in a dynamic general equilibrium model which also includes a set of real frictions. Among these, a key role is played by variable capital utilisation, which helps dampen the increase in the rental rate of capital (and therefore the rise in marginal cost) triggered by a monetary policy shock. Other models<sup>2</sup> generate inflation inertia assuming that, when allowed to review their contracts, a fraction of price (or wage) setters set a price according to a backward-looking rule of thumb. As in the cases discussed above, this generates a version of the Phillips curve which includes lags of the inflation rate.

However, in all the models above, inflation inertia follows from the assumption of elements of backward looking behaviour in the agents' decisions. This paper develops a model where the delayed response of inflation to a monetary policy shock is triggered by a price and wage setting framework which retains the features of forward lookingness and optimising behaviour.

In fact, the way nominal rigidities are modelled is crucial for the dynamics of

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<sup>1</sup>See Woodford (2003) for a discussion.

<sup>2</sup>See Galí and Gertler (1999).

inflation. One of the most widely used price setting frameworks is the Calvo (1983) model, on the basis of which firms and households reset prices and wages according to a constant, exogenous probability. However, the Calvo model cannot generate a hump shaped inflation response<sup>3</sup>. In addition, it has the unrealistic implication that a firm faces the same probability of price change regardless of how long the contract has been into place. This paper analyses the implications for the effects of monetary policy shocks of replacing the constant probability Calvo model with a duration dependent model, characterised by reset probabilities which vary with the age of the contract, within a dynamic general equilibrium model.

The effects of shocks to the economy in the presence of duration dependent contracts have been studied in a number of papers. Wolman (1999) focuses on the response of inflation to a marginal cost shock in a partial equilibrium model with duration dependent pricing. Levin and Coenen (2005) find that a model allowing for time varying hazard rates can, in combination with real rigidities, explain German macrodata. Mash (2006) derives a generalised version of the Phillips curve which allows the reset probability to vary with the time elapsed since the last price change and shows that such a model performs better than the simple Calvo model in terms of consistency with the evidence on inflation and output persistence. Dixon (2006) introduces a set of steady-state identities which make a comparison between alternative price and wage setting models possible and shows, among other results, that the Generalised Calvo model, allowing for time varying hazard rates, does

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<sup>3</sup>See Dixon and Kara (2006), who compare the performance of the Calvo model to that of other pricing models in terms of their implications for inflation persistence.

not generate a hump shaped response of inflation.

The main innovation of this paper is the introduction of a duration dependent structure for both prices and wages in a dynamic general equilibrium model. Other studies, in fact, tend to focus on the role of price or wage rigidities in isolation. However, as shown in this paper, not only price and wage rigidities have different implications for the dynamics of inflation, but their interaction determines the ultimate response of inflation to a shock. The model is calibrated for alternative empirical reset probability distributions and the impulse responses of inflation to an interest rate shock are analysed. It is found that the combination of price and wage rigidities is able to reproduce the delayed response of inflation, but that this result does not hold when wages are fully flexible.

Another issue addressed by this paper is whether the constant probability Calvo model and the duration dependent model are a good approximation for a multi-sector economy. It is found that not only the dynamics of inflation following a monetary shock differs depending on whether the economy is modelled as a one sector or a multi-sector simple Calvo model, but it also varies widely with the values taken by the sectoral reset probabilities in the multi-sector model. On the contrary, it is found that the response of inflation in a one sector and a multi-sector duration dependent model is almost identical. This result suggests that the complication of dealing with a multi-sector duration dependent model can be avoided replacing it with a one sector model, giving up a relatively small amount of information.

The remainder of the paper is structured as follows. Section 2 describes the model and the price and wage setting framework incorporated. Section 3

reports the log-linearised model. Section 4 reports the impulse responses of inflation to a monetary shock and discusses the main results of the paper. Section 5 analyses the different implications of one and multi-sector models. Section 6 concludes.

## 2 The model

This section describes a dynamic general equilibrium model with monopolistic competition<sup>4</sup> and nominal rigidities both in the firms and the households sectors. Firms set prices and wages according to a generalised version of the Calvo (1983) model, which allows for duration dependent price and wage reset probabilities. Monetary policy is conducted by a central bank, which uses the short-term interest rate as an instrument, following a version of the Taylor rule.

### 2.1 Firms

There is a continuum of monopolistically competitive firms, indexed by  $i \in [0, 1]$ , producing differentiated goods,  $Y_t(i)$ . The goods are combined into a constant-elasticity-of-substitution output index, which is equal to:

$$Y_t = \left[ \int_0^1 Y_t(i)^{\frac{1}{1+\eta_p}} di \right]^{1+\eta_p} \quad (1)$$

where  $Y$  denotes aggregate output. The corresponding price index, which equals the minimum cost of a unit of the output index given the individual

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<sup>4</sup>Monopolistic competition gives firms a degree of market power, allowing them to take a decision about the price of the good produced.

goods' prices  $P_t(i)$ , is:

$$P_t = \left[ \int_0^1 P_t(i)^{-\frac{1}{\eta_p}} di \right]^{-\eta_p} \quad (2)$$

Total cost is minimised subject to (1), yielding the demand function for good  $y_t(i)$ :

$$Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\frac{1+\eta_p}{\eta_p}} Y_t \quad (3)$$

Every firm faces the same linear production function:

$$Y_t(i) = A_t N_t(i) \quad (4)$$

where  $A$  is the level of technology, common to all firms, and  $N$  denotes the labour input, which is firm specific. However unrealistic, a simple linear production function allows to isolate the role of the price setting framework in determining the effects of a shock to the economy. Real marginal cost is therefore equal to:

$$MC_t = \frac{W_t}{A_t P_t} \quad (5)$$

In any given period, firms reset prices with probability  $\omega_j$ , where  $j = 1, 2, \dots, J^5$  denotes the age of the contract. The probability depends on when the price was last reset, but is independent from the state of the economy, i.e. it is

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<sup>5</sup>The model can be easily extended to the case of an infinite horizon probability distribution. However, we focus on the truncated version of the model in order to calibrate it using empirical hazard rate distributions.

exogenously determined. This corresponds to the Generalised Calvo model as defined in Dixon (2006) and contains, as a special case, the simple Calvo model, where the hazard rate  $\omega_j$  is constant over the entire life of the contract. The profit functional for firm  $i$  is given by:

$$E_t \sum_{j=0}^{J-1} \prod_{k=0}^j (1 - \omega_k) \Psi_{t,t+j} [P_{t+j}(i) Y_{t+j}(i) - MC_{t+j}^n Y_{t+j}(i)] \quad (6)$$

where  $E_t$  denotes expectation conditional on the information available at time  $t$  and  $MC_t^n$  nominal marginal cost. In each period  $t+j$ , firms discount profits by the probability that they will not be allowed to reset their price  $(1 - \omega_j)$  and by the discount rate  $\Psi_{t,t+j}$ . The first order condition, derived from the maximization of profit subject to the demand function for  $Y_t(i)$ , takes the following form:

$$E_t \sum_{j=0}^{J-1} \prod_{k=0}^j (1 - \omega_k) \Psi_{t,t+j} \left[ \frac{1}{1 + \eta_p} P_{t+j}(i) Y_{t+j}(i) - MC_{t+j}^n Y_{t+j}(i) \right] = 0 \quad (7)$$

All firms resetting their price in the same period choose the same price. Consequently, the aggregate price index can be written as

$$P_t = \left[ \sum_{j=0}^{J-1} \alpha_j^s P_{t-j}(i)^{-\frac{1}{\eta_p}} \right]^{-\eta_p} \quad (8)$$

where  $\alpha_j^s$  is the fraction of firms whose price contract has age  $j$ .

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$${}^6 \alpha_j^s = \frac{\prod_{k=1}^{j-1} (1 - \omega_k)}{\sum_{j=1}^{J-1} \prod_{k=1}^{j-1} (1 - \omega_k)}$$

## 2.2 Households

Households are indexed by  $h \in [0, 1]$  and offer labour services, denoted by  $N_t(h)$ , to firms. These are aggregated into a labour index, equal to:

$$L_t = \left[ \int_0^1 N_t(h)^{\frac{1}{1+\eta_w}} dh \right]^{1+\eta_w} \quad (9)$$

The labour index is then bought by producers at the price

$$W_t = \left[ \int_0^1 W_t(h)^{-\frac{1}{\eta_w}} dh \right]^{-\eta_w} \quad (10)$$

The total demand for the individual household labour is then

$$N_t(h) = \left[ \frac{W_t(h)}{W_t} \right]^{-\frac{1+\eta_w}{\eta_w}} L_t \quad (11)$$

The utility function is additively separable between consumption and leisure.

The discounted sum of utilities maximised by households takes the form

$$E_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{1}{1-\sigma} C_t(h)^{1-\sigma} - \frac{1}{1+\psi} N_t(h)^{1+\psi} \right] \quad (12)$$

where  $\beta$  is the discount factor and C denotes consumption. The budget constraint is given by:

$$P_t C_t(h) + R_{t,t+1} B_t(h) = W_t(h) N_t(h) + B_{t-1}(h) + \Lambda_t(h) - P_t T_t \quad (13)$$

At the beginning of each period households receive labour income  $W_t(h) N_t(h)$ .

$B_{t-1}(h)$  represents the quantity of bonds carried over from period t-1, while

$\Lambda_t(h)$  is the share of aggregate profits received by each household, on the assumption that it owns an equal share of all firms. Finally,  $T_t$  denotes lump-sum taxes or transfers.

Households maximise the utility functional subject to the labour demand function and the budget constraint. The combination of the first order conditions for consumption and bonds yields the Euler equation:

$$C_{t+k}(h)^{-\sigma} = E_t \left[ \beta(1 + i_{t+k}) C_{t+k+1}(h)^{-\sigma} \frac{P_{t+k}}{P_{t+k+1}} \right] \quad (14)$$

In addition, households set wages according to a duration dependent model analogous to the one followed by firms. Wages are renegotiated with probability  $\phi_j$ , where  $j = 1, 2, \dots, J$  denotes how long the contract has been in force. At any period  $t$  when the household is allowed to reset its wage, the utility functional is maximised with respect to  $W_t(h)$  and the following first order condition is obtained:

$$E_t \sum_{j=0}^J \beta^j \prod_{k=0}^j (1 - \phi_k) \left[ N_{t+j}(h)^\psi - \frac{1}{1 + \eta_w} \frac{W_t(h)}{P_{t+j}} C_{t+j}^{-\sigma} \right] N_{t+j}(h) = 0 \quad (15)$$

As for firms, all households resetting their contracts choose the same wage. Taking this into account, the aggregate wage index is written as:

$$W_t = \left[ \sum_{j=0}^{J-1} \mu_j^s W_{t-j}(h)^{-\frac{1}{\eta_w}} \right]^{-\eta_w} \quad (16)$$

where  $\mu_j^s$  is the fraction of households whose wage contract has age  $j$ . Finally, the marginal rate of substitution,  $MRS_t$ , is given by:

$$MRS_t = \frac{L_t^\psi}{C_t^{1-\sigma}} \quad (17)$$

### 2.3 Government and market clearing

The central bank sets the interest rate according to the following version of the Taylor rule:

$$i_t = (1 - \rho_i)\gamma\pi_t + \rho_i i_{t-1} + \epsilon_{t,m} \quad (18)$$

According to this rule, the nominal interest rate is set as a function of the gap between actual inflation and the inflation target.  $\epsilon_{t,m}$  is an i.i.d. monetary policy shock. The equation also allows for interest rate smoothing (carried out by central banks for the purpose of minimising financial markets fluctuations<sup>7</sup>), captured by the parameter  $\rho_i$ .

Taxes are lump-sum and the government pursues a Ricardian fiscal policy, under which tax policy has no impact on aggregate variables.

In the absence of capital, the aggregate resource constraint is:

$$Y_t = C_t \quad (19)$$

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<sup>7</sup>See, for example, Clarida et al. (2000).

### 3 Log-linearisation of the model

The equations of the model are log-linearised around a deterministic steady state with zero inflation. Lower case variables represent log-deviations from steady state values. The equation expressing real marginal cost is given by:

$$mc_t = (w_t - p_t) - a_t \quad (20)$$

The optimal reset price equation can be rewritten in real terms as:

$$E_t \sum_{j=0}^{J-1} \prod_{k=0}^j (1 - \omega_k) \Psi_{t,t+j} \left[ \frac{1}{1 + \eta_p} \frac{P_{t+j}(i)}{P_{t+j}} Y_{t+j}(i) - MC_{t+j} Y_{t+j}(i) \right] = 0 \quad (21)$$

which, in log-linearised terms, is equal to:

$$p_t(i) = \frac{E_t \sum_{j=0}^{J-1} \beta^j \prod_{k=0}^j (1 - \omega_k) [mc_{t+j} + p_{t+j}]}{E_t \sum_{j=0}^{J-1} \beta^j \prod_{k=0}^j (1 - \omega_k)} \quad (22)$$

The aggregate price index is log-linearised as:

$$p_t = \sum_{j=0}^{J-1} \alpha_j^s p_{t-j}(i) \quad (23)$$

The log-linearised Euler equation is given by:

$$c_t = E_t[c_{t+1}] - \frac{1}{\sigma} (i_t - \pi_{t+1}) \quad (24)$$

Equation (15) log-linearises as:

$$w_t(h) = \frac{E_t \sum_{j=0}^{J-1} \beta^j \prod_{k=0}^j (1 - \phi_k) \left[ \psi n_{t+j}(h) + p_{t+j} + \sigma c_{t+j} \right]}{E_t \sum_{j=0}^{J-1} \beta^j \prod_{k=0}^j (1 - \phi_k)} \quad (25)$$

The log-linearised labour demand equation is:

$$n_t(h) = \left( -\frac{1 + \eta_w}{\eta_w} \right) w_t(h) + \left( \frac{1 + \eta_w}{\eta_w} \right) w_t + l_t \quad (26)$$

while the log-linearised equation for the marginal rate of substitution is given by:

$$mrs_t = \psi l_t + \sigma c_t \quad (27)$$

The optimal wage setting equation is obtained combining the log-linearised first order condition with respect to wage and the equations for labour demand and marginal rate of substitution:

$$\left( 1 + \psi \frac{1 + \eta_w}{\eta_w} \right) w_t(h) = \frac{E_t \sum_{j=0}^{J-1} \beta^j \prod_{k=0}^j (1 - \phi_k) \left[ \psi \left( \frac{1 + \eta_w}{\eta_w} \right) w_{t+j} + p_{t+j} + mrs_{t+j} \right]}{E_t \sum_{j=0}^{J-1} \beta^j \prod_{k=0}^j (1 - \phi_k)} \quad (28)$$

Finally, aggregate nominal wage is given by:

$$w_t = \sum_{j=0}^{J-1} \mu_j^s w_{t-j}(h) \quad (29)$$

## 4 The effects of monetary shocks

This section analyses the response of inflation to a monetary policy shock in the model described in section 3. The model is calibrated for alternative hazard rate distributions and compared to the equivalent constant probability Calvo model. The focus is on the implications for inflation dynamics of replacing the simple Calvo model with a duration dependent price and wage setting structure.

### 4.1 Response of inflation to a monetary shock

In order to assess the impact of monetary policy in the duration dependent model, we focus on the impulse response of inflation to an interest rate shock. This is compared to the dynamics of inflation in a simple Calvo model with the same average duration of contracts across firms, as introduced by Dixon (2006). This is calculated starting from the completed contract lengths across the entire population of firms (or households) rather than across contracts. In fact, the distribution of durations across contracts only includes the cross-section of contracts starting at a certain point in time, excluding those contracts which have been set in previous periods but are still in place and thus overestimating the weight of shorter contracts.

The values assigned to the parameters are standard in the literature and the main results of this section are not qualitatively affected by their variations. The household utility parameters are assumed to be equal to  $\sigma = 1.5$  and  $\psi = 0.5$ . For simplicity, the discount factor  $\beta$  is set equal to 1. The price and wage markup rates are given by  $\eta_p = \eta_w = 0.2$ . The coefficients on the

interest rate rule are equal to  $\gamma = 1.5$  and  $\rho_i = 0.9$ . However, the simulations are carried out for alternative reset probability distributions<sup>8</sup>. This allows to isolate the implications of different price and wage setting models and different hazard rates distributions for inflation inertia.

Figure 1 compares the impulse response of inflation to a monetary shock in a model with a duration dependent framework to a simple Calvo model with the same completed duration of contracts. For the purpose of this simulation, the hazard rate distributions are derived from the microeconomic evidence reported in Mash (2006) and Taylor (1993). Based on survey data reported by Blinder et al. (1998) for the US economy, Mash (2006) derives a price change probability distribution with increasing hazards, which allows for price contracts to last for up to seven quarters<sup>9</sup>. This corresponds to an average completed duration across firms, calculated according to the formula  $\bar{T} = \bar{\omega} \sum_{i=1}^J i^2 \omega_i \prod_{k=0}^{i-1} (1 - \omega_k)$ , of 4.41 quarters. The equivalent simple Calvo probability, implying the same completed duration of contracts across firms, is  $\omega = 0.3694$ . The reset probabilities distribution for wages is taken from Dixon (2006), where it is derived starting from the empirical distribution of contract lengths estimated by Taylor (1993). Wage contracts last for up to eight quarters and the path of hazard rates is  $\{\phi_i\}_{i=0}^8 = \{0, 0.2017, 0.3430, 0.4213, 0.4986, 0.5682, 0.5849, 0.6038, 1\}$ . The average contract length across households is  $\bar{T} = 3.73$  and the corresponding simple Calvo probability is  $\omega = 0.4228$ .

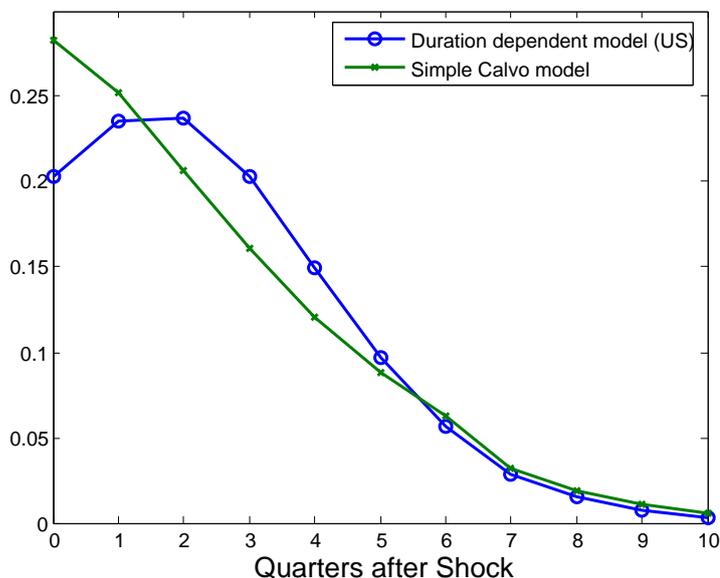
The monetary shock consists of a reduction of 25 basis points in the nom-

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<sup>8</sup>All simulations are performed using Dynare (see Juillard, 2001).

<sup>9</sup> $\{\omega_i\}_{i=0}^7 = \{0, 0.09, 0.15, 0.29, 0.42, 0.54, 0.68, 1\}$ .

Figure 1: Response of inflation to a monetary shock



inal interest rate. As shown in Figure 1, the impulse response of inflation to the shock differs substantially between the duration dependent and the simple Calvo model. While in the latter the maximum inflation response is on impact, in the duration dependent model inflation responds to the shock in a hump-shaped fashion. The biggest effect occurs after two quarters and dies out more slowly than in the simple Calvo model until quarter 6. However, after then, the simple Calvo model is slightly more persistent. The inertia generated by the model is largely driven by the smoother response of real wages to a monetary policy shock occurring in the model with duration dependent price and wage setting as opposed to the real wage dynamics observed in the model with constant reset probabilities. The channel through which real wages impact on inflation is the real marginal cost.

## 4.2 Sensitivity analysis

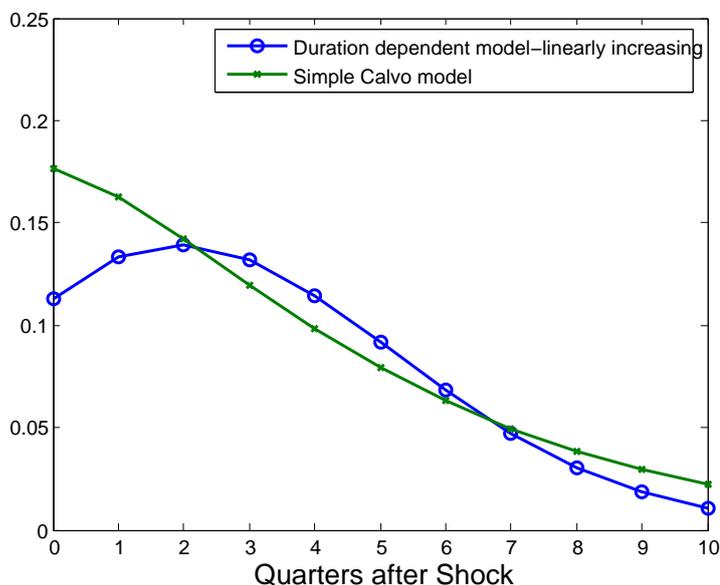
In the example illustrated by Figure 1, the implications of replacing the simple Calvo probability with a sequence of duration dependent hazard rates have been analysed with respect to two specific price and wage setting probability distributions. However, to what extent are these results valid and how do they vary with the values taken by the hazard rates? This section discusses the response of inflation to a monetary shock for alternative hazard rate distributions.

The evidence on the shape of the hazard function is mixed. Goette et al. (2005) and Sheedy (2007) estimate upward sloping hazard rate distributions. Similar results are obtained by Fougère et al. (2005). Other studies find evidence of a downward sloping hazard function pattern, e.g. Bils and Klenow (2004) for the US economy and Dhyne et al. (2005) for the Euro area. However, as underlined by Aucremanne and Dhyne (2005) and Baumgartner et al. (2005), there is evidence that declining hazard rates are mainly a result of aggregation of heterogenous price setters.

The first case considered is that of a model where price and wage reset probabilities are both linearly increasing in the time elapsed since the last price adjustment. In particular, it is assumed that price contracts last up to 14 periods and the reset probabilities take values determined by the formula  $\omega_j = 0.071 \cdot j$ , while wage contracts last up to 16 periods and wage reset probabilities vary according to the formula  $\phi_j = 0.0625 \cdot j$ .

The impulse response of inflation to a monetary shock is shown in Figure 2. As in the case illustrated in Figure 1, the maximum effect of the shock

Figure 2: Response of inflation to a monetary shock



occurs after 2 quarters. However, the response of inflation is more gradual and the shock is more persistent. This is due to the smoother path followed by the hazard rates, which increase more slowly, and to the longer maximum duration of the contracts as compared to the case described in paragraph 4.1.

An important issue to be addressed is whether the model requires monotonically increasing reset probabilities in order to generate a hump shaped response of inflation to a monetary shock. Figure 3 provides an illustration of the behaviour of inflation for alternative hazard rate distributions.

Figure 3a compares two models where contracts last up to 14 periods in the case of prices and up to 16 periods in the case of wages, as in the previous example. However, in the first model both price and wage reset probabilities

increase over time, while in the second model the hazard rates increase until quarter 7 (prices) and quarter 8 (wages) and then decrease according to the same function.

As shown in Figure 3a, in both models the maximum impact of the shock is after 2 quarters. However, in the "symmetric" model there is more inflation persistence, which can be explained by the relatively low reset probabilities associated with older contracts as compared to the linearly increasing model. Finally, Figure 3b compares a model where the hazard rate increases in the first period of life of a contract to a model where the reset probabilities are monotonically decreasing. Figure 3b shows that the model can generate a hump shaped response of inflation even when the hazard rate distribution is upward sloping in the first period and then becomes downward sloping, but that if the hazard rates are declining the maximum effect of a monetary shock is on impact.

### **4.3 The role of wage rigidities**

In this model, wage rigidities have a crucial role in the explanation of inflation inertia. This is illustrated by Figure 4, which compares the impulse response of inflation to an interest rate shock in two models. The first is the duration dependent model described above, characterised by nominal rigidities in both the firms' and the households' sectors. The second model incorporates price rigidities, while wages are fully flexible. The main purpose of the comparison is to check whether the presence of price rigidities is sufficient to generate a hump shaped response of inflation to a monetary shock. The

Figure 3: Response of inflation to a monetary shock - Increasing and "symmetric" hazard distribution

Figure 3a

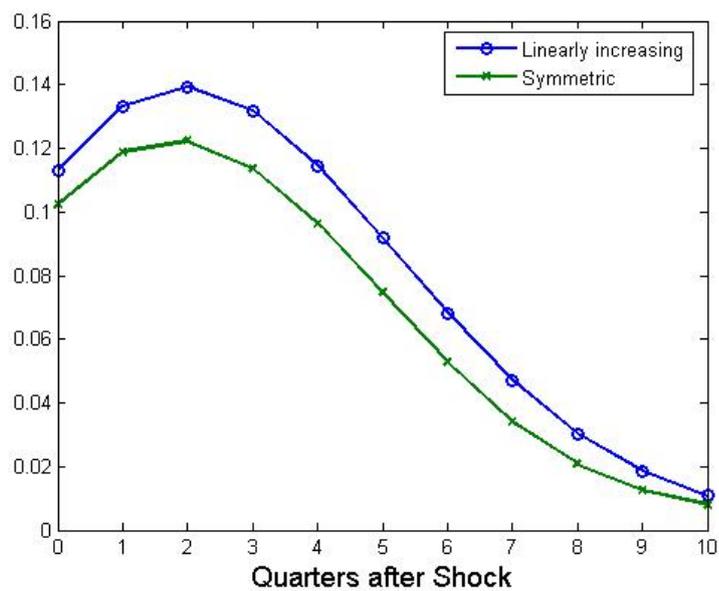


Figure 3b

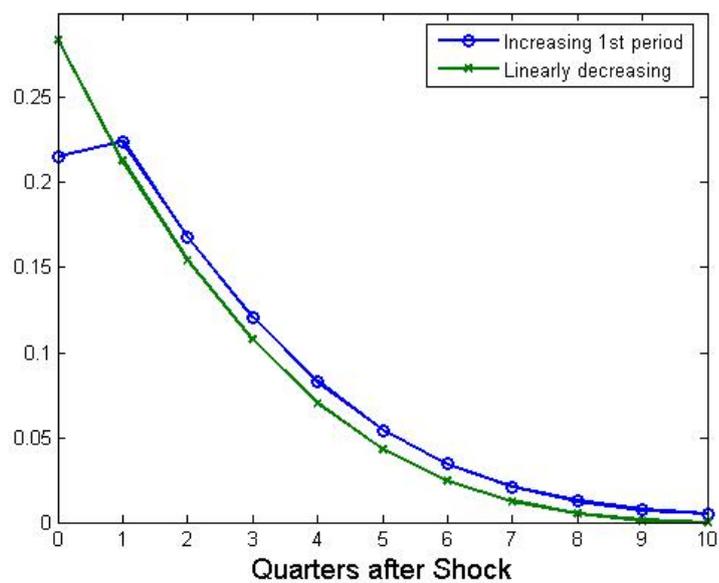


Figure 4: Response of inflation to a monetary shock - Wage rigidities vs flexible wages

Figure 4a

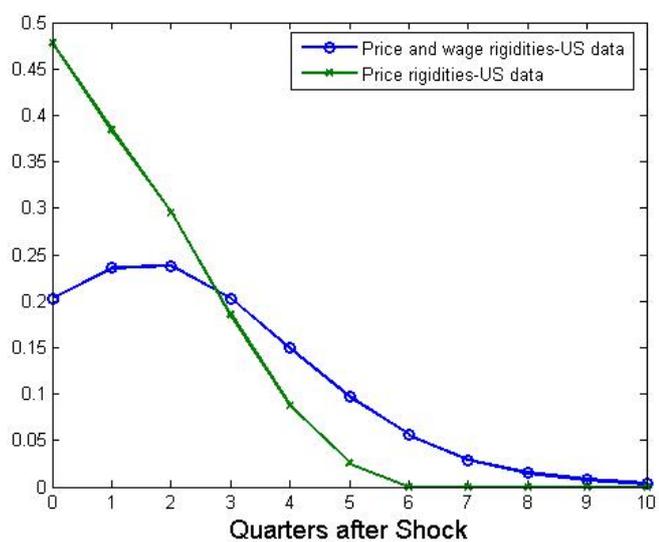
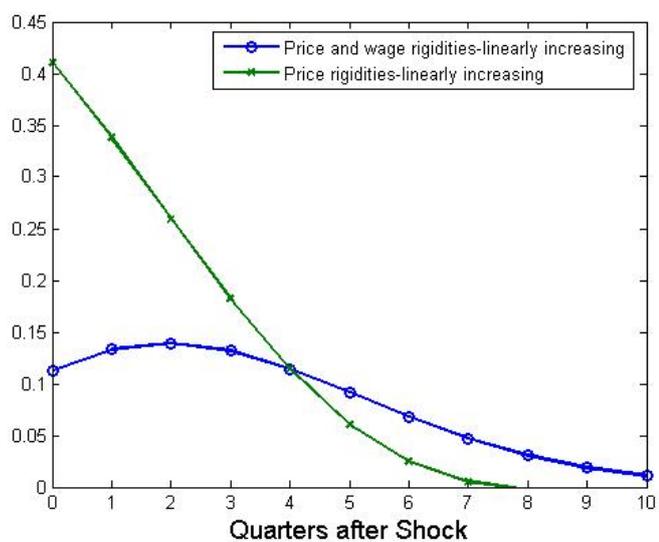


Figure 4b



difference between the two models is dramatic. When only the reset probability distribution for prices is duration dependent, the maximum effect on inflation occurs on impact and is much stronger than in the equivalent model to which wage rigidities are added. In the latter, the response of inflation shows a hump and is considerably more persistent.

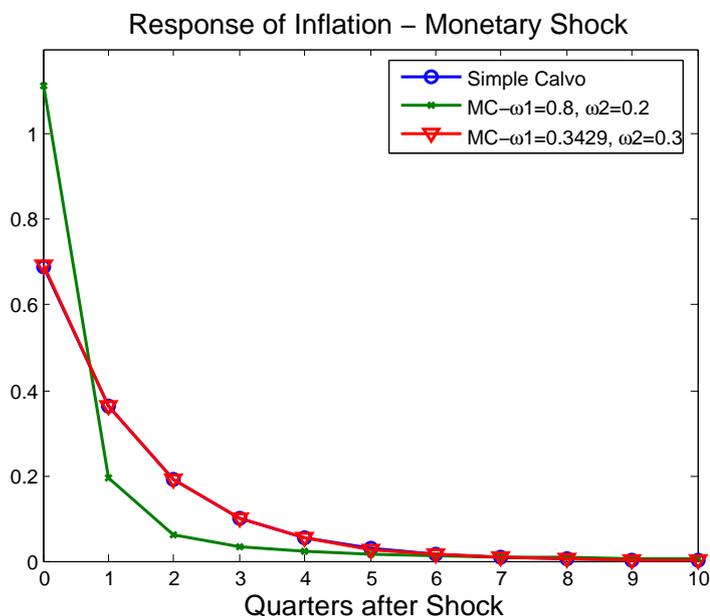
The combination of duration dependent price and wage setting seems therefore to be able to generate the hump shaped response of inflation which, as shown in Dixon (2006), is not triggered when price or wage rigidities are considered in isolation. This result is in line with the analysis in Christiano et al. (2005), who find that, in order to reproduce the dynamic response of inflation to a monetary shock, the critical nominal friction is wage rigidities.

## 5 One sector or many sectors?

The issue analysed in this section is whether the one-sector model with simple Calvo or duration dependent price setting is a good approximation for a multi-sector economy. For simplicity, only prices are assumed to be sticky while wages are taken to be fully flexible. This does not change the results for the purpose of the comparison between the one sector and the multi-sector Calvo economy.

Figure 5 compares the impulse responses of inflation to a monetary shock in three models with the same average completed duration of contracts. In the first model, prices are set according to a simple Calvo framework with a constant price reset probability equal to  $\omega = 0.32$  and a mean contract length of  $\bar{T} = 5.25$ . The other two are two-sector Multiple Calvo models

Figure 5: Simple vs Multiple Calvo



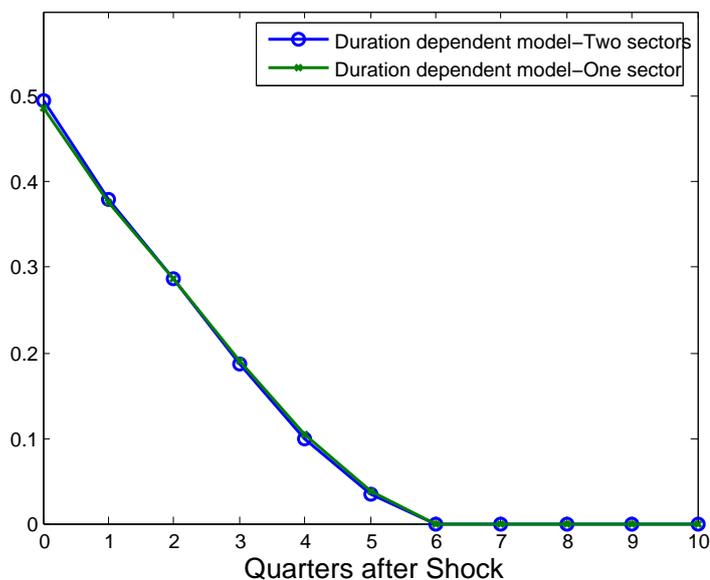
which differ from each other on the basis of the values taken by the reset probabilities. While both have an average contract length of 5.25 quarters and sector shares equal to  $\alpha_1 = 0.5$  and  $\alpha_2 = 0.5$ , in the first model there is a sector in which prices are much more flexible than in the other (the reset probabilities are equal to  $\omega_1 = 0.8$  and  $\omega_2 = 0.2$ ), while in the second the price setting probabilities are very close to each other ( $\omega_1 = 0.3429$  and  $\omega_2 = 0.3$ ).

As shown in Figure 5, not only the dynamics of inflation following a monetary shock is very different depending on whether the same economy is modelled as a simple Calvo or a Multiple Calvo model, but it also varies widely with the values taken by the sectoral reset probabilities in Multiple Calvo models with the same mean duration of contracts. When the reset probabilities are

very different from each other, the presence of rather flexible contracts causes a monetary shock to have an immediate strong impact on inflation, which declines quickly but is then rather persistent due to the presence of firms with longer contracts. The response of inflation in the equivalent simple Calvo model is much smoother but slightly less persistent. Finally, the dynamics of inflation in a Multiple Calvo model where the sectoral reset probabilities tend to coincide is almost indistinguishable from that of a simple Calvo model. These results warn against the advisability of modelling the price (and wage) setting behaviour of a complex economy using a single constant probability. If this can be relatively harmless when there is not much sectoral variation with respect to the frequency of price change, a high degree of heterogeneity in terms of price setting behaviour (a finding supported by recent empirical studies, see among others Dhyne et al. (2005)) implies that a one-sector Calvo model is not able to replicate the actual dynamics of inflation. An interesting related issue is whether an economy consisting of a number of sectors in which price reset probabilities are duration dependent can be represented as a one-sector duration dependent model. As generating alternative duration dependent models with the same mean contract length would be a complex task, the comparison is carried out between a two-sector duration dependent model and a one sector duration dependent model where the reset probabilities are calculated according to the following formula:

$$\omega_i = \frac{\alpha_1 \omega_{1,i} \prod_{k=0}^{i-1} (1 - \omega_{1,k}) + \alpha_2 \omega_{2,i} \prod_{k=0}^{i-1} (1 - \omega_{2,k})}{\alpha_1 \prod_{k=0}^{i-1} (1 - \omega_{1,k}) + \alpha_2 \prod_{k=0}^{i-1} (1 - \omega_{2,k})} \quad (30)$$

Figure 6: Response of inflation to a monetary shock - One-sector duration dependent model vs multi-sector duration dependent model



As an example, we calibrate the sectoral probabilities using two of the hazard rate distributions reported by Mash (2006). The first one is the probability distribution for the US economy reported in paragraph 4.1. The second is a truncated version of the hazard rate distribution reported in Wolman (1999)<sup>10</sup>. The aggregate probabilities, used for the calibration of the one sector duration dependent model, are given by:  $\{\omega_i\}_{i=0}^7 = \{0, 0.055, 0.098, 0.2, 0.3, 0.41, 0.55, 1\}$ .

As illustrated in Figure 6, the dynamics of inflation following an interest rate shock is almost identical in the two models. The advantage of the duration dependent model over the constant probability model is that the aggregate probabilities are calculated period by period rather than over a relatively long

<sup>10</sup> $\{\omega_i\}_{i=0}^7 = \{0, 0.02, 0.05, 0.13, 0.22, 0.35, 0.5, 1\}$

period of time. This result suggests that the complication of dealing with a multi-sector duration dependent model can be avoided by replacing it with a one sector duration dependent model, giving up a relatively small amount of information. This finding is in line with the analysis of Dixon (2006), who argues that, in the presence of heterogeneous contract structures, price and wage setting models closed to aggregation, such as the Multiple Calvo and the Generalised Calvo (duration dependent), should be used.

## 6 Conclusions

This paper developed a dynamic general equilibrium model with nominal rigidities where price and wage reset probabilities are duration dependent, i.e. vary with the time elapsed since the last price and wage change. The main focus of the paper is on whether such a model can fit the evidence on inflation inertia better than the constant probability Calvo framework.

The main results are the following. First of all, it is shown that, for a set of empirical probability distributions, the response of inflation to an interest rate shock is considerably different between the duration dependent setting and the equivalent simple Calvo model. Time-varying reset probabilities cannot be approximated by a constant hazard rate.

Secondly, unlike a model with constant hazard rates, the model with duration dependent price and wage setting generates inflation inertia. While in a model with constant reset probabilities the maximum effect of a monetary shock is always on impact, in a duration dependent model the response of inflation is hump shaped. This finding is particularly meaningful as it does

not require any form of ad hoc indexation to past inflation. However, this result is conditional on the hazard rate function being upward sloping at least in the initial period. More evidence on the shape of the hazard function is needed in order to discriminate between alternative versions of the model.

Third, the presence of wage rigidities is crucial for the validity of the results. Price stickiness alone is not sufficient to generate a hump shaped response of inflation to a monetary shock. Wage stickiness dampens the response of real wages, and therefore of real marginal cost, to the shock, in turn affecting inflation.

Finally, it is found that a Calvo model with a single constant reset probability cannot, in most cases, satisfactorily approximate an economy consisting of heterogeneous sectors with different constant reset probabilities. On the contrary, the one-sector duration dependent model generates a dynamics of inflation very close to the one observed in a multi-sector economy with heterogeneous price and wage setters.

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