How do financial frictions affect the spending multiplier during a liquidity trap?

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Abstract

We show that credit market imperfections substantially increase the government-spending multiplier when the economy enters a liquidity trap. This finding is explained by the tight association between capital goods and firms' collateral, a relationship that we highlight as the capital-accumulation channel. During a liquidity trap, a government spending expansion reduces the real interest rate, leading to a period of cheap credit. Entrepreneurs use this time to accumulate capital, which persistently improves their balance sheets and reduces their future costs of credit. A public spending expansion can thus encourage private investment, yielding consequently a large spending multiplier. This effect is further reinforced by Fisher's debt-deflation channel.

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1 Introduction

Expansionary fiscal policy has been widely used during the financial crisis against the contraction of economic activity. These public interventions have led to a heated debate, in which one of the major topics concerns the size of the government-spending multiplier.[1] Some contributions have become particularly relevant as very low interest rates and large credit spreads emerged in the wake of the financial turmoil. For instance, Eggertsson (2010) and Christiano, Eichenbaum and Rebelo (2011) argue that the spending multiplier can be large when the zero lower bound on the nominal interest rate binds. Credit market imperfections are also potential factors explaining large spending multipliers, as argued by Eggertsson and Krugman (2012) and Fernández-Villaverde (2010).[2] Despite the importance of these contributions, the literature remains fairly silent about the effects of the interaction between financial frictions and a liquidity trap on the size of the spending multiplier.[3]

In this paper, we argue that financial frictions have a much larger impact on fiscal multipliers during a liquidity trap, compared to a non-liquidity-trap environment. We show that the zero lower bound on nominal interest rate strengthens the financial-frictions channel of government spending. The interaction between these two rigidities yields a multiplier effect on capital purchases, in which public spending crowds-in investment. As a result, both rigidities combined contribute a larger amount to the multiplier than the sum of their contributions in isolation. To the best of our knowledge, this topic has not been fully addressed in the literature, especially regarding the effect of these two factors on firms' investment decisions.

We set our investigation in a New Keynesian model with capital and credit frictions *à la* Bernanke, Gertler, and Gilchrist (1999). In this model, inefficiencies in financial markets stem from asymmetric information between lenders and borrowers. Our aim is to evaluate the amount by which financial frictions raise the government-spending multiplier in different interest-rate regimes. To do so, we

[1] Some recent contributions include Cogan, Cwik, Taylor, and Wieland (2010) who show that the size of the multiplier is sensitive to modeling choices. Davig and Leeper (2011) study the outcomes of different monetary-fiscal policy regimes, while Erceg and Lindé (2010) focus on the timing and size of a fiscal stimulus during a liquidity trap. Other studies analyze the role of wealth effects (Monacelli and Perotti, 2008), spending reversals (Corsetti, Kuester, Meier, and Müller, 2010), public investment (Leeper, Walker and Yang, 2010), distortionary taxation (Drautzburg and Uhlig, 2011) and beliefs-driven traps (Mertens and Ravn, 2010).

[2] See also Canzoneri, Collard, Dellas and Diba (2011), who show that countercyclical financial frictions make government-spending multipliers larger during recessions.

[3] Eggertsson and Krugman (2012) and Erceg and Lindé (2010) consider cases in which a government-spending expansion occurs in the presence of financial frictions and a liquidity trap. However, they do not stress the feedback effects that originate from the interaction between these two distortions.

compute the present-value multiplier of a government-spending expansion for two models: one with financial frictions and one without. The difference between the two multipliers indicates the gains that are attributable to financial frictions. These gains are then compared in two regimes: a non-liquidity-trap regime, and a liquidity-trap regime. The liquidity trap is generated by a sudden shift from spending to saving by households (i.e., by a negative preference shock).[4]

We show that financial frictions have a much larger contribution to the size of the government-spending multiplier when the nominal interest rate is constrained by the zero lower bound (ZLB, hereafter). In a liquidity trap, an extra dollar spent by the government rises output by 1.82 dollars more in a model with financial frictions than in a model without. In a non-liquidity-trap regime, output rises by only 34 cents more. This result is explained by the spillover effect of the ZLB regime on the "capital-accumulation" channel of the financial accelerator. This channel stems from the tight relationship between capital accumulation and entrepreneurial collateral. A rise in investment today persistently enlarges the set of assets (capital) that can be used as collateral for several future periods. Therefore, entrepreneurs' creditworthiness persistently increases and credit spreads are reduced.

A government-spending expansion is propagated by the capital-accumulation channel as follows: a larger aggregate demand increases capital demand and the price of capital, which improves the value of borrowers' collateral. Financial frictions imply that credit spreads narrow, encouraging in turn investment demand. Therefore, the rise in capital stock boosts the value of borrowers' collateral for several periods and a *multiplier effect on investment* effectively emerges. The final effect of the fiscal stimulus depends on the reaction of the central bank. If the central bank reacts to the stimulus by increasing the nominal interest rate, the multiplier effect on investment vanishes because credit in general becomes expensive. Investment demand is discouraged and the price of capital is depressed; the capital-accumulation channel is thus weakened. If the nominal interest rate stays put for a few periods (at the ZLB, for instance), investors have incentives to accumulate capital. They know that the resulting extra investment has a long-lasting impact on their collateral and, in turn, on their future costs of credit. This expectation generates a large impact on investment demand. Consequently, the government-spending multiplier is larger when the ZLB binds for few periods, while the gains attributable to financial frictions are strictly larger than in a no-ZLB regime.

[4] This assumption has become standard in the literature. An outward shift of the savings supply leads to a fall in the natural rate of interest, which the nominal interest rate cannot follow due to the zero lower bound constraint (see Eggertsson and Woodford, 2003; Bodenstein, Erceg and Guerrieri, 2009; Eggertsson, 2010; Erceg and Lindé, 2010).

This explanation is novel in the literature, as we emphasize the multiplier effect on investment, rather than on consumption as a source of larger multipliers during a liquidity trap (see Christiano *et al.*, 2011). Not surprisingly, the crowding-in of investment by government spending in models in which financial frictions are not important is very difficult to obtain. In those models, the absence of credit spreads generates no reason to accumulate collateral. Interestingly, our prediction is in line with the empirical findings of Corsetti, Meier, and Müller (2010). Using SVAR evidence, these authors show that a government-spending expansion crowds-in investment during a financial turmoil, while in normal times investment is crowded-out.[5]

We argue that the capital-accumulation channel is the key determinant in explaining our results. We can easily verify this intuition by switching off this channel. The relationship between capital and collateral breaks down when the former is fully depreciated at the end of each period. Full capital depreciation implies that collateral cannot be accumulated. Credit spreads are thus practically invariant to past investment. As one could suspect in this case, financial frictions have a similar impact on fiscal multipliers, irrespective of whether the nominal interest rate hits the ZLB or not. Eggertsson and Krugman (2012) and Fernández-Villaverde (2010) stress the role of Fisherian debt-deflation (Fisher, 1933), or the presence of nominal debt contracts, as an alternative explanation of why financial frictions magnify the government-spending multiplier. We show that Fisherian debt-deflation indeed plays an important role, but it is not crucial for our result. When we turn this channel off, i.e. when we allow debt contracts to adjust to inflation, financial frictions still generate larger long-term output gains in the multiplier during the ZLB regime.

We complete our analysis by carrying out a sensitivity analysis. First, we perform a set of robustness exercises by modifying several model specifications. Second, we consider a fiscal policy that directly stimulates investment, namely a tax-rate cut on capital earnings. We find that our main results are robust to different modeling choices, while they also extend to capital taxes.

The remainder of the paper is organized as follows. Section 2 describes the baseline model. Section 3 discuss the calibration and solution strategy. Section 4 presents our main results based on a government spending expansion. Section 5 presents the sensitivity analysis by modifying different aspects of the model. Section 6 briefly discusses the alternative fiscal policy. The final section concludes.

[5] Corsetti *et al.* (2010) also report an estimated spending multiplier of 2 for the U.S., which roughly corresponds to our result.

2 The Model

Our framework builds on a standard New Keynesian model with capital, enriched with frictions in the credit market.[6] We adopt the financial frictions framework of Bernanke *et al.* (1999). We also consider three additional features. First, we impose a ZLB constraint on the nominal interest rate so as to study the implications of a liquidity trap. The latter is generated by a negative shock on preferences. Secondly, we allow the government to choose the level of public spending. And finally, we assume that government bonds and entrepreneurs loan contracts can be indexed to inflation. This assumption allows us to study the importance of the Fisherian debt-deflation channel on the transmission of fiscal shocks. As we show below, adjusting debt to realized inflation brings about certain modifications to the common financial accelerator model. We start the description of the model by highlighting these differences.

2.1 Entrepreneurs

Optimal Financial Contract Consider a continuum of risk neutral entrepreneurs indexed by $e \in [0, 1]$. At the end of period t, type$-e$ entrepreneur buys the stock of capital $k_{e,t}$. The nominal price of a unit of capital is Q_t. Capital expenditures are financed with internal resources and debt. Let $N_{e,t}$ be type-e entrepreneur's nominal net worth gathered at the end of period t and $B_{e,t}$ the amount of nominal debt borrowed from a financial intermediary (or lender, for short), hence $B_{e,t} = Q_t k_{e,t} - N_{e,t}$. Nominal debt is adjusted to inflation with an indexation coefficient specified below.

A unit of capital bought in period t is used in production in period $t + 1$. Accordingly, in time $t + 1$, type$-e$ entrepreneur rents $k_{e,t}$ out to intermediate firms, who pay the real rental rate z_{t+1}. After production occurs, entrepreneurs sell the un-depreciated capital at the current capital price Q_{t+1}. Thus, the gross nominal rate of returns of holding a unit of capital from t to $t + 1$ is

$$\begin{aligned} R^k_{t+1} &= \frac{P_{t+1} z_{t+1} + (1-\delta) Q_{t+1}}{Q_t} \quad \text{or,} \\ &\equiv (1 + r^k_{t+1})(1 + \pi_{t+1}), \end{aligned} \tag{1}$$

where δ is the rate of depreciation of capital, r^k_{t+1} is the real rate of capital returns, P_t is the price of the final good and $\pi_{t+1} \equiv \frac{P_{t+1}}{P_t} - 1$ is the inflation rate. Following Bernanke *et al.* (1999), we assume that type-e entrepreneur's returns are affected by an idiosyncratic disturbance, so that her

[6] The log-linear model is laid out in Appendix A. A hated variable denotes its deviation from the deterministic steady state. The full derivations are described in the online appendix available on http://users.ugent.be/~jcarrill.

nominal earnings in time $t+1$ equal $\omega_{e,t+1}R^k_{t+1}Q_tk_{e,t}$. The random variable $\omega_{e,t+1}$ is independent and identically distributed across time and types. The mean of ω_t equals one, while its variance is given by σ^2_ω. Variable ω's cumulative density function, $F(\omega)$, is a continuous and once-differentiable function defined over a non-negative support. The loan agreement is signed at period t, although $\omega_{e,t+1}$ is unknown to both the entrepreneur and the lender prior to the investment decision.

These conditions lead to the following optimal lending contract: for every possible realization of R^k_{t+1}, there exists a threshold $\bar{\omega}_{e,t+1}$ such that if $\omega_{e,t+1} \geq \bar{\omega}_{e,t+1}$, the entrepreneur repays her debt (which can be partially indexed to inflation) at rate $R^L_{e,t+1}$. Otherwise, the entrepreneur declares bankruptcy. The threshold $\bar{\omega}_{e,t+1}$ and $R^L_{e,t}$ are jointly defined by

$$\mathrm{E}_t\left\{\bar{\omega}_{e,t+1}R^k_{t+1}Q_tk_{e,t}\right\} = \mathrm{E}_t\left\{R^L_{e,t+1}B_{e,t}(1+\pi_{t+1})^{\gamma_b}\right\}, \tag{2}$$

where $(1+\pi_{t+1})^{\gamma_b}$ is a term that adjusts nominal debt to the realized inflation from t to $t+1$, and $\gamma_b \in [0,1]$ is the coefficient of debt indexation. In addition, E_t is the expectation operator conditional to the information available in period t.

In case of bankruptcy, the lender audits the entrepreneur and must pay a monitoring cost to observe the entrepreneur's earnings; entrepreneurs observe them without cost. The monitoring cost is given by $\mu\omega_{e,t+1}R^k_{t+1}Q_tk_{e,t}$, where $\mu \in [0,1]$. In order to enforce truth-telling, it is assumed that whenever the lender audits an entrepreneur, she gets to keep all of the entrepreneur's earnings, net of monitoring costs. The lender chooses to engage in a loan contract with the entrepreneur if her participation constraint is satisfied. The lender evaluates two options to place her funds: one is to lend to the entrepreneurs and the other is to buy government bonds, which are also indexed to realized inflation and pay the riskfree gross nominal interest rate R_t. In equilibrium, arbitrage ensures that that the expected returns of the two assets equalize.[7]

Similar to Bernanke *et al.* (1999), one can show that the equilibrium in the credit market implies that the discounted expected rate of capital returns, $\check{r}_t \equiv \mathrm{E}_t\left\{R^k_{t+1}/R_t\right\}$, equals the marginal cost of external finance.[8] The variable $\check{r}_t$ can be interpreted as the external finance premium. In the aggregate, the credit market equilibrium condition, in log-linearized terms, collapses to

$$\mathrm{E}_t\left\{\hat{R}^k_{t+1} - \hat{R}_t\right\} = \chi\hat{x}_t + \gamma_b\mathrm{E}_t\left\{\hat{\pi}_{t+1}\right\}, \tag{3}$$

where $x_t = Q_tk_t/N_t$, k_t is the economy's total stock of capital, N_t is the aggregate sum of entrepreneurs' net worth, and $\chi > 0$ is the elasticity of $\widehat{\check{r}}_t$ with respect to $\hat{x}_t$. The first term on the right-hand

[7] Assuming that both government bonds and entrepreneurs' debt are inflation-indexed ensures that there is no arbitrage opportunities between the two types of assets.

[8] The online appendix presents in detail the optimal financial contract.

side of the equation describes the usual financial accelerator mechanism. Consider for instance a recessionary shock that reduces both asset prices and investment. The fall in the value of collateral drives the credit spread up and magnifies in turn the drop in investment. The second term on the right-hand side of the equation adds new adjustments in the risk premium. Consider for instance a positive change in expected inflation. When entrepreneurs have to pay their debt in real terms, it is likely that more of them default. In addition, when government bonds are inflation-indexed, the participation constraint of the lender is harder to meet. These two facts drive the external finance premium up.

Entrepreneurs in General Equilibrium Every period, entrepreneurs supply one unit of labor in the labor market and earn the nominal wage W_t^e. Following Bernanke *et al.* (1999), we assume that entrepreneurs live for finite horizons. As such, each entrepreneur has a probability $1-\gamma$ of exiting the economy.[9] We let the birth rate of entrepreneurs be equal to their mortality rate so that the total number of entrepreneurs remains constant. Entrepreneurs leaving the economy consume a proportion ϱ of their own equity and transfer the remainder to households. They also transfer their wage to new entrepreneurs so that the nominal aggregate net worth equals

$$N_t = \gamma V_t + W_t^e, \tag{4}$$

where V_t denotes entrepreneurs aggregate equity, which is composed of the revenues from capital holdings net of expected monitoring costs, minus borrowing repayments

$$V_t = R_t^k Q_{t-1} k_{t-1} \left(1 - \mu G(\bar{\omega}_t)\right) - R_{t-1} B_{t-1} (1+\pi_t)^{\gamma_b}, \tag{5}$$

where $\mu G(\bar{\omega}_t) R_t^k Q_{t-1} k_{t-1}$ is the expected bankruptcy cost. For the sake of completeness, notice that the real aggregate entrepreneurial consumption, c_t^e, equals $\varrho(1-\gamma)\frac{V_t}{P_t}$, while entrepreneurs' transfers to households equal $\mathcal{A}_t \equiv (1-\varrho)(1-\gamma)V_t$.

Two propagation channels stand out from Equations (3) and (5). The first one is the Fisher's debt-deflation channel, naturally appearing in this environment when debt contracts are denominated in nominal terms ($\gamma_b = 0$). In such case, an unexpected positive change in inflation improves the value of equity since capital income adjusts to reflect the increase in general prices, while nominal debt remains constant (recall from Equation (1) that R_t^k reacts to realized inflation). For short, we refer to this channel as the "Fisher effect".

[9] This assumption deters entrepreneurs to accumulate enough wealth to be fully self-financed.

The second channel relates to capital accumulation and it results from the long-lasting influence that capital has on entrepreneur's collateral. Indeed, capital goods are assets that can be used as collateral by an entrepreneur during a loan agreement, as shown in Equation (5). Therefore, larger purchases of investment goods today will influence the external finance premium of an entrepreneur for several quarters in the future, provided that the depreciation rate of capital is small. We refer to this propagation device as the *capital-accumulation channel*.

2.2 Capital Producer

At the end of period $t-1$, capital producers sell to entrepreneurs the capital stock k_{t-1}. At time t, capital producers repurchase from entrepreneurs the un-depreciated capital $(1-\delta)\,k_{t-1}$. Capital producers build then the new stock k_t by combining investment goods, i_t, and the un-depreciated capital, so that

$$k_t = (1-\delta)\,k_{t-1} + S\left(i_t, i_{t-1}\right), \tag{6}$$

where adjustment costs in investment are given by $S\left(i_t, i_{t-1}\right) \equiv \left[1-\Phi\left(\frac{i_t}{i_{t-1}}\right)\right] i_t$, with $\Phi(1) = \Phi'(1) = 0$ and $\Phi''(1) = \varkappa > 0$. The representative capital producer chooses the level of investment that maximizes her profit. The presence of adjustment costs allows for a variable price of capital, which in turn contributes to the volatility of the net worth. In equilibrium, the relative price of capital, $q_t \equiv Q_t/P_t$, is given by

$$q_t = \left[1-\Phi\left(\frac{i_t}{i_{t-1}}\right) - \Phi'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}} + \beta \mathrm{E}_t\left\{\frac{\lambda_{t+1}q_{t+1}}{\lambda_t q_t}\left[\left(\frac{i_{t+1}}{i_t}\right)^2 \Phi'\left(\frac{i_{t+1}}{i_t}\right)\right]\right\}\right]^{-1}, \tag{7}$$

where $\beta\frac{\lambda_{t+1}}{\lambda_t}$ is the appropriate stochastic discount factor from the point of view of the representative household, and λ_t is its marginal valuation of wealth.

2.3 Households

Households maximize expected lifetime utility by selecting a sequence of consumption, c_t, labor supply, ℓ_t^h, and real savings, d_t. The representative household's optimization program reads

$$\max_{c_t, \ell_t^h, d_t} \quad \mathrm{E}_t \sum_{t=0}^{\infty} \beta^t \varepsilon_t \left\{\frac{\left(c_t^{\upsilon}\left(1-\ell_t^h\right)^{1-\upsilon}\right)^{1-\sigma} - 1}{1-\sigma}\right\}, \tag{8}$$

subject to the sequence of budget constraints

$$c_t + d_t \leq w_t \ell_t^h + R_{t-1}\frac{d_{t-1}(1+\pi_t)^{\gamma_b}}{1+\pi_t} - \frac{\Upsilon_t}{P_t} + \frac{\mathcal{A}_t}{P_t} + \mathrm{div}_t, \quad \forall t. \tag{9}$$

where w_t is the real wage, R_t is the riskfree gross nominal interest rate associated with one-period-maturity nominal deposits, D_t, so that real savings are given by $d_t \equiv D_t/P_t$. Notice that deposits are indexed to inflation since deposits and government bonds are perfect substitutes. Therefore, the arbitrage condition between the two assets makes their distinction irrelevant in the household budget constraint. Further, div_t, Υ_t, $\mathcal{A}_t$ denote real profits from monopolistic firms, nominal taxes to the government, and nominal transfers from entrepreneurs, respectively, each one redistributed as a lump sum. Finally, β is the subjective discount factor, $\sigma^{-1} > 0$ is the intertemporal elasticity of substitution and $\upsilon \in (0,1)$. The first order conditions from the household problem with respect to d_t, c_t and ℓ_t^h yields

$$\lambda_t = \beta \mathrm{E}_t \left\{ \frac{\lambda_{t+1} R_t (1+\pi_{t+1})^{\gamma_b}}{1+\pi_{t+1}} \right\}, \tag{10}$$

$$\varepsilon_t c_t^{\upsilon(1-\sigma)} (1-\upsilon) \left(1-\ell_t^h\right)^{(1-\upsilon)(1-\sigma)-1} = \lambda_t w_t, \tag{11}$$

$$\varepsilon_t \left(1-\ell_t^h\right)^{(1-\upsilon)(1-\sigma)} \upsilon c_t^{\upsilon(1-\sigma)-1} = \lambda_t. \tag{12}$$

The preference shock ε_t has a law of motion given by $\hat{\varepsilon}_t = \rho_\varepsilon \hat{\varepsilon}_{t-1} + \epsilon_{\varepsilon,t}$, where $\rho_\varepsilon \in (0,1)$, and $\epsilon_{\varepsilon,t} \sim$ i.i.d.$(0, \sigma_\varepsilon)$.

2.4 Intermediate and Final Good Sectors

The final good, y_t, is produced in a competitive market by combining a continuum of intermediate goods $y_{j,t}$, with $j \in [0,1]$, via a typical Dixit-Stiglitz aggregator. Profit maximization yields a sequence of input demand functions of the form

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\theta_p} y_t, \tag{13}$$

where θ_p is the input demand elasticity, and $P_{j,t}$ is the price of the intermediate good produced by firm j.

Type-j intermediate firm produces a differentiated good by assembling services of labor and capital using the technology $y_{j,t} = \ell_{j,t}^{1-\alpha} k_{j,t-1}^{\alpha}$. In turn, type-$j$ firm's total labor input, $\ell_{j,t}$, is composed by household labor, $\ell_{j,t}^h$, and entrepreneurial labor, $\ell_{j,t}^e \equiv 1$, such that $\ell_{j,t} = [\ell_{j,t}^h]^{\Omega} [\ell_{j,t}^e]^{(1-\Omega)}$, where $\Omega \in [0,1]$.

Nominal rigidities in price-setting follow Calvo (1983). Each period, a firm faces a constant probability $1-\alpha_p$ of being able to re-optimize its price. Let $P_{j,t}^{\star}$ denote the nominal price chosen in time t and $y_{j,t+k}^{\star}$ good j's specific demand k quarters after the last price re-optimization. Firm j

selects $P^{\star}_{j,t}$ so as to maximize the present discounted sum of profit streams, subject to its production technology and its specific demand function (13). The first order condition is given by

$$\mathrm{E}_t \sum_{k=0}^{\infty} (\beta\alpha_p)^k \lambda_{t+k} \frac{y^{\star}_{j,t+k}}{P^{\star}_{j,t}} \left\{ \frac{p^{\star}_{j,t}}{1+\pi_{t+k}} - \mu_p s_t \right\} = 0, \tag{14}$$

where $p^{\star}_{j,t} \equiv P^{\star}_{j,t}/P_t$ is type$-j$ intermediate producer relative price, $1+\pi_{t+k} \equiv P_{t+k}/P_t$ is total inflation from t to $t+k$, $\mu_p \equiv \theta_p/(\theta_p - 1)$ is the markup, and s_t is the real marginal cost.

2.5 Monetary and Fiscal Policy

We assume that the gross nominal interest rate R_t is set according to

$$R_t = \max\left(1, R^{not}_t\right), \tag{15}$$

where R^{not}_t is the desired (or notional) interest rate chosen by the central bank in response to inflation. As such, R^{not}_t follows a simple rule of the form

$$\frac{R^{not}_t}{R} = \left(\frac{R^{not}_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{1+\pi_t}{1+\pi}\right)^{a_\pi}\right]^{1-\rho_R}, \tag{16}$$

where $\rho_R \in (0,1)$ is a smoothing parameter, a_π is the elasticity of R^{not}_t with respect to inflation deviations, R is the steady-state gross nominal interest rate, and π is the central bank's steady-state inflation target. The central bank sets R_t equal to R^{not}_t if and only if its policy rule recommendation implies a non-negative level for the nominal interest rate. If this is not the case, the central bank simply fixes its target rate equal to zero.

Government spending evolves as $\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_{g,t}$, where $\rho_g \in [0,1]$, and $\epsilon_{g,t}$ is i.i.d fiscal shock. We assume that government spending is financed by lump-sum taxes.[10]

2.6 Resource Constraint and Equilibrium

The final good is allocated to investment, private and public consumption, and aggregate monitoring costs, such that $y_t = i_t + c_t + c^e_t + g_t + \mu G(\bar{\omega}_t) r^k_t q_{t-1} k_t$.

In the symmetric equilibrium, all entrepreneurs, households, and firms are identical and make the same decisions. The symmetric equilibrium is characterized by an allocation $\{y_t,\ c_t,\ c^e_t,\ i_t,\ \ell^h_t,\ k_t,\ N_t, V_t\}$ and a sequence of prices, wages, and co-state variables $\{P_t,\ W_t,\ W^e_t,\ R_t,\ R^k_t,\ Q_t,\ z_t,\ \lambda_t\}$ such that, for every realization of stochastic shocks, the optimization conditions in each sector, the monetary and fiscal rules, and the aggregate shocks' law of motions are satisfied.

[10] We could have chosen a scheme for public expenses financing based on public debt. However, adding distortionary tax rules for instance would imply effects in the dynamics that render the interpretation of our findings harder to achieve. We prefer a fiscal rule based on lump-sum taxes which has no implications to the model dynamics.

3 Calibration and Solution

In this section, we discuss the model calibration, how we induce a liquidity trap, and the solution strategy.

3.1 Calibration

Table 1 presents the model's parameters which are calibrated to fit the quarterly frequency.

[insert Table 1 here]

The preference and technology parameters closely follow the values adopted by Christiano *et al.* (2011). The subjective discount factor, β, is set to 0.99, implying an annual real interest rate of 4 percent. The degree of risk aversion, σ, is set to 2, while υ is chosen to ensure that household's labor in the steady state, ℓ^h, equals 1/3 (this implies that $\upsilon = 0.34$). The capital share in the intermediate sector technology, α, is set to 0.36; the depreciation rate, δ, equals 0.02; and the investment adjustment cost parameter, $\varkappa$, is set to 5.00. Further, we assume a mark-up of 10 percent in the intermediate sector (i.e., $\theta_p = 11$), and that firms re-optimize their prices on average once every 6 quarters (so $\alpha_p = 0.85$).

Regarding the financial sector, we adopt the original calibration of Bernanke *et al.* (1999). The entrepreneurial labor-income share is set to 0.01, implying a value of $\Omega = 0.9846$. The steady-state capital-to-net-worth ratio $x = k/n$, is calibrated to 2, so that half of capital purchases are done with debt. The steady-state gross external finance premium, $\check{r} = r^k/r$, is set to $1.02^{0.25}$, corresponding to an annual risk spread of 200 basis points. The annual business failure rate, $F(\bar{\omega})$, is set to 3 per cent. The idiosyncratic productivity shock, ω_t, obeys a log-normal distribution with an unconditional expectation equal to 1. These moments determine the survival probability of entrepreneurs, γ, the proportion of monitoring costs, μ, the standard deviation of ω_t, σ_ω, and the optimal financial contract threshold, $\bar{\omega}$. These quantities are, respectively, 0.98, 0.12, 0.28, and 0.50. The elasticity of the external finance premium with respect to leverage, χ, is deduced from the model's steady-state levels and equals 0.04. Interestingly, Gilchrist, Ortiz and Zakrajsěk (2009) obtain a similar value by estimating a financial accelerator model using Bayesian techniques.[11] As a benchmark, we assume that financial contracts are purely denominated in nominal terms ($\gamma_b = 0$).

[11] Appendix A describes the procedure to pick up the values of γ, μ, σ_ω, $\bar{\omega}$ and χ from the model's steady state.

We now turn to the calibrated values of monetary and fiscal policy parameters. The interest-rate smoothing parameter, ρ_R, is set to 0.50; the elasticity of the notional interest rate with respect to inflation, a_π, is set to 2. Steady-state inflation, π, equals zero. The steady-state share of government purchases in total output is calibrated to 0.20, which corresponds to the last decade's historical average for the U.S.. The persistence of the preference and the government spending shocks, ρ_ε and ρ_g respectively, are set to 0.80. The size of the preference shock is discussed below, while the size of the fiscal innovation is small enough to make sure that the duration of the ZLB regime does not change.

3.2 Liquidity Trap and Solution Strategy

A negative realization of ε_t implies that the marginal utility of consumption decreases. In this case, households find optimal to shift part of her spending towards savings, which results in a downward pressure on consumption, output, and the nominal interest rate. If the realization of ε_t is large enough, the nominal interest rate falls to zero. We choose the preference shock as the driver of the liquidity trap in order to compare two models: one with financial frictions and one without.[12] The deep recession scenario is shown in Appendix B for illustrative purposes.

We assume that the liquidity trap spans an equal number of periods in each model. Precisely, the ZLB binds from quarters 1 to 6. Normally, for a given size of the recessionary shock, a model with financial frictions is likely to stay longer in a liquidity trap than a standard New Keynesian model. However, the duration of the liquidity trap affects the size of the spending multiplier, as shown by Christiano *et al.* (2011). Since we do not want to pollute our comparison between multipliers with different liquidity-trap durations across models, we adjust the size of the recessionary shock in each model to have exactly the same duration. It is worth noticing that our results are not affected when we abandon this assumption.[13]

We now turn to the solution strategy. The ZLB constraint, described by Equation (15), introduces an important non-linearity into the system. We adopt a piecewise-linear approach to solve for the model dynamics, similar to Bodenstein, Erceg and Guerrieri (2009). In particular, we linearize all

[12] Other shocks are not as convenient. A financial shock does not allow us to compare both models. A positive technology shock decreases inflation but it implies an output boom. A negative investment-specific shock must be unrealistically large to generate a liquidity trap.

[13] In fact, our results are strengthened when we set the size of the shock equal accross models. These results can be found in the online appendix.

model equations around the steady-state, except for Equation (15). The piecewise-linear approach entails two different rules for the nominal interest rate. When $R_t^{not} > 1$, the percent deviation of the nominal interest rate, $\hat{R}_t$, follows a linearized version of Equation (16). When $R_t^{not} \leq 1$, then $\hat{R}_t$ is set to $-R$, where $R = 1/\beta$ is the steady-state level of the gross nominal interest rate.

The piecewise-linear approach implies an endogenous duration of the ZLB regime. In the analysis presented below, we were careful in ensuring that the liquidity trap has always a finite duration. We also disregard any solution path driven by an expectation trap (like in Mertens and Ravn, 2011) by assuming a deterministic environment after the occurrence of the preference shock. The last assumption implies that agents make their decisions knowing that in period $T+1$ the nominal interest rate will exit the ZLB regime.[14]

4 How Do Financial Frictions Affect the Spending Multiplier?

In this section, we show that the contribution of financial frictions to the size of the government-spending multiplier is larger when the ZLB binds. Capital accumulation plays a crucial role in this result. We start the discussion with our benchmark case, in which all financial contracts in the economy are set in nominal terms ($\gamma_b = 0$) and capital goods are stored and serve as future collateral ($\delta = 0.02$).

The negative preference shock that drives the economy into a recession happens in time 1. In the same quarter, the government temporally rises public spending. We measure the impact of this policy by using the present-value multiplier (see Leeper, Walker and Yang, 2010, and Uhlig, 2010). The latter is defined as

$$PVM_k = \frac{\mathrm{E}_t \sum_{j=0}^{k} \beta^{-j} y_{t+k}^{net}}{\mathrm{E}_t \sum_{j=0}^{k} \beta^{-j} g_{t+k}^{net}} = \frac{y}{g} \frac{\mathrm{E}_t \sum_{j=0}^{k} \beta^{-j} \hat{y}_{t+k}^{net}}{\mathrm{E}_t \sum_{j=0}^{k} \beta^{-j} \hat{g}_{t+k}^{net}}, \tag{17}$$

where y/g is the inverse of the steady-state public-spending-to-GDP ratio and $\hat{x}_t^{net}$ is the partial effect of a government-spending expansion on variable x, expressed in percentage points (i.e., the marginal response of variable x to the spending shock). $\hat{x}_t^{net}$ is defined as $\hat{x}_t^{net} \equiv \hat{x}_t^{fis} - \hat{x}_t^0$, where $\hat{x}_t^0$ is variable x's response to a negative preference shock, as a percent deviation from the steady state; $\hat{x}_t^{fis}$ is variable x's response to both the preference and the government-spending shocks together.[15]

[14] Alternative methods, such as a collocation method or spline functions (see Nakov, 2008; De Fiore and Tristani, 2011), can be very costly in terms of computation time for complex models like ours.

[15] Notice that, since $\hat{x}_t^{net}$ is the result of a difference, this variable is expressed in percentage points and not in percent deviations from its steady-state level.

Thus, PVM_k measures the expected and discounted marginal change in output due to an increase in government spending by 1 dollar, k-periods ahead in time.

In order to measure the multipliers' output gains that emerge from the presence of credit frictions, we compare the present-value multiplier for two models: one in which financial frictions matter, and one in which they do not. The first model corresponds to the one described in Section 2, and we referred to it as the *NK with f.f.* model. The second one is simply called the *standard NK* model.[16]

The fiscal multiplier's output gains attributed to the presence of financial frictions are thus measured as follows

$$\text{Output gains } k \text{ periods ahead} = \left(PVM_k|_{NK\ with\ f.f.} - PVM_k|_{Standard\ NK}\right).$$

In order to capture these gains at a value close to convergence, we compute them for $k = 100$. These gains represent the extra goods (or dollars) in the economy that result from the presence of financial frictions after a government spending expansion. The gains are computed for two scenarios: when the ZLB binds and when it does not.

4.1 Benchmark Case

The upper and medium panels of Figure 2 show the fiscal multipliers for the non-liquidity-trap regime and the liquidity-trap regime, respectively. The solid line stands for the *standard NK* model while the dashed line refers to *NK with f.f.* model. The bottom panel of the figure displays the multipliers' output gains for each of the two regimes mentioned.

[insert Figure 2 here]

Figure 2 confirms one fact emphasized by Christiano *et al.* (2011): the multiplier is always larger during a liquidity trap, irrespective of the model. For instance, in a non-liquidity trap regime, the impact multipliers of both models are below 1 while they are around 1.60 when the ZLB constraint is binding.[17] A government spending expansion rises output and expected inflation. If the nominal interest rate stays at zero for some periods, the increase in expected inflation drives down the real interest rate which stimulates in turn private spending and generates a further boost in output and expected inflation.

[16] In such an environment, the parameter χ in Equation (3) equals zero and the expected rate of capital returns equals the nominal interest rate.

[17] The spending multipliers reported in Figure 2 are smaller than in Christiano *et al.* (2011). In Section 5, we investigate some factors that affect the size of the multiplier.

Two other important facts, which relate to the presence of credit market frictions, emerge from Figure 2:

1. *The medium- and long-run multipliers are larger when credit frictions matter, irrespective of the interest-rate regime.* For instance, in a liquidity trap, the long-run spending multiplier reaches almost 3 in the *NK with f.f.* model while it is close to 1 in the *standard NK* model.

2. *The contribution of credit frictions to the size of the multiplier is strictly larger during a liquidity trap.* In the ZLB regime, an extra dollar spent by the government rises output by 1.82 dollars more in the *NK with f.f.* model than in the *standard NK* model. In the no-ZLB regime, output rises by only 34 cents more.

These two facts highlight the importance of financial frictions on the size of the government-spending multiplier. We now explain the intuitions behind these two facts. Figure 3 displays selected partial impulse responses, following the definition of $\hat{x}_t^{net}$ above, to a 1 percent increase in government spending. The two models and the two interest-rates regimes are portrayed in the figure.

[insert Figure 3 here]

The first fact refers to the role of financial frictions on the government-spending multiplier through their medium-run effect on investment decisions. Figure 3 shows that, after 10 periods, the partial responses of investment are negative in the *standard NK* model, whereas they are positive in the *NK with f.f.* model. A first explanation of the crowding-in of investment by government spending has been provided by Eggertsson and Krugman (2012) and Fernández-Villaverde (2010). Accordingly, an increase in inflation ameliorates the balance sheet position of borrowers when debt contracts are denominated in nominal terms. A healthier financial position of firms improves their credit-worthiness, leading in turn to investment-enhancing conditions. This effect indeed corresponds to the debt-deflation channel.

In this paper we offer an alternative explanation using the capital-accumulation channel. The initial increase in the price of capital due to a higher aggregate demand induces a decrease in leverage and a rise in the value of collateral. Credit frictions then imply that the external finance premium falls, as shown in Figure 3. The narrowing of credit spreads encourages in turn investment demand and capital accumulation. A larger capital stock, as shown in Figure 3, further rises borrowers' collateral and reduces the external finance premium. This feedback loop results in a positive and

persistent *multiplier effect on investment*. From this perspective, the Fisher effect strengthens the capital-accumulation channel since an unexpected increase in inflation puts additional downward pressure on the external finance premium. Interestingly, when financial frictions are not important, investment is not encouraged and its fluctuations are moderated, as shown by the *standard NK* model. The absence of credit spreads in such model generates no reason to accumulate collateral, given that the price of credit depends only on the real interest rate.

The second fact refers to the role of financial frictions on the government-spending multiplier through their interaction with a liquidity trap. Credit frictions contribute more to the size of the multiplier during a liquidity-trap regime because of a substantial multiplier effect on investment in such regime. Figure 3 shows that the size of the investment responses in the *NK with f.f.* model is quite different across interest-rate regimes. The final multiplier effect of the capital-accumulation channel depends on the reaction of the central bank to the fiscal shock. If the central bank reacts to the stimulus by increasing the nominal interest rate, the multiplier effect on investment vanishes because credit in general becomes expensive. Figure 3 shows that, in such a case, investment demand is discouraged and the final effect on the price of capital is slightly negative. The capital-accumulation channel is thus weaker during a non-liquidity trap regime. In contrast, if the nominal interest rate stays put for a few periods (at the ZLB, for instance), the multiplier effect on investment is in full bloom. The expectation of a period of cheap credit provides incentives to accumulate capital. Entrepreneurs know that extra-capital goods have a long-lasting impact on their collateral and, in turn, on their future costs of credit. This expectation generates a large positive impact on investment demand and on the price of capital. Consequently, the government-spending multiplier is larger when the ZLB binds for a few periods, while the gains attributable to financial frictions are strictly larger than in the non-liquidity-trap regime.

In the next two subsections, we disentangle the importance of the capital-accumulation channel vis-à-vis the Fisher effect in explaining *Fact 2*.

4.2 Full Depreciation Case

We argue that the capital-accumulation channel provides entrepreneurs with intertemporal incentives to build collateral and influence their cost of credit. Therefore, if capital goods cannot be stored, entrepreneurs find not necessary to allocate too much resources into investment during a period of cheap credit. Consequently, it will not make a difference for them to acquire too many capital goods when the ZLB binds because this capital cannot be used as future collateral. In sum,

when investment goods cannot be stored, or they are fully depreciated after usage, any intertemporal spillover effects between the liquidity-trap regime and credit frictions vanishes.

We can confirm this intuition by assuming full capital depreciation ($\delta = 1$), as in Figure 4. The first two rows display selected partial responses to a government spending expansion.

[insert Figure 4 here]

Full capital depreciation effectively implies that current capital purchases will no longer influence the future net worth and the leverage of borrowers.[18] Entrepreneurs cannot accumulate capital to increase their creditworthiness, and have therefore no extra-incentives to invest heavily during a period of cheap credit. Therefore, an equal expansion of government spending generates an almost identical amount of extra goods in the *NK with f.f.* model when compared to the *standard NK* model, irrespective of the interest-rate regime, as shown by bottom-right panel of Figure 4 (the multiplier gains are 22 cents in no-ZLB regime, and 28 cents in ZLB regime). Notice that investment is crowded-in by government spending when $\delta = 1$ since investment demand and the input demand of capital are essentially the same (up to a delay of one period).

4.3 No-Fisher Effect Case

Eggertsson and Krugman (2012) and Fernández-Villaverde (2010) emphasize the key role of the Fisher effect in amplifying the spending multiplier. But how important is this effect at explaining the multiplier's output gains caused by credit imperfections in the medium-run? To provide an answer to this question, we turn off the debt-deflation channel by assuming that nominal debt adjusts to realized inflation ($\gamma_b = 1$).

Figure 5 presents this case. The first two rows display selected partial responses to the government spending shock, while the bottom row shows the present-value multipliers for the liquidity-trap and no-liquidity-trap cases, as well as the multiplier's output gains due to credit frictions. Notice that these gains are still larger during a liquidity trap.

[insert Figure 5 here]

Due to the absence of the Fisher effect, this gain is smaller than in the benchmark exercise. For instance, in a liquidity trap, an extra dollar spent by the government rises output by 50 cents more

[18] Investment adjustment costs *à la* Christiano, Eichenbaum, and Evans (2005) imply that the capital-accumulation channel is not fully removed even after assuming full capital depreciation. Indeed, investment today depends on past investment, among other factors. In our context, however, the adjustment costs of investment have negligible effects on the capital-accumulation channel. In the online appendix, we also consider a different type of capital adjustment cost. Our results are not affected by the alternative assumption.

in the *NK with f.f.* model than in the *standard NK* model. When entrepreneurs' debt is indexed to inflation, the external finance premium increases on impact, as shown by Equation (3) and Figure 5.[19] The rise in the external finance premium discourages in turn entrepreneurs from investing. However, entrepreneurs understand that maintaining a minimum level of collateral moderates their probability of default and subsequently their external finance premium. Therefore, entrepreneurs avoid de-accumulating a large amount of capital over the medium-run. This behavior is observed in the partial responses of investment as given by the *NK with f.f.* model in Figure 5. When the ZLB binds, the capital-accumulation channel is stronger for the same reasons given earlier, i.e., a period of cheap credit is the best occasion to buy capital and strengthen the balance sheet. The multiplier's output gains reach 34 cents in such case. When the ZLB does not binds, the capital-accumulation channel is not strong enough and the multiplier's output gains are slightly negative, reaching -2 cents.

5 Robustness Analysis

In this section, we test the robustness of our main finding, namely that financial frictions yield larger output gains in the spending multiplier during a ZLB regime in comparison with a no-ZLB regime. We consider a number of modifications with respect to our benchmark case. The alternative assumptions we discuss are the following: 1) separable preferences between consumption and labor, 2) a lesser degree of price stickiness, 3) no monetary policy inertia, and 4) a constant increase in government spending spanning *only* the duration of the liquidity trap. We then discuss the effect of these alternative specifications on the size of the spending multiplier.

5.1 Multiplier's Output Gains

Table 2 shows the results of our sensitivity analysis in terms of the multiplier's gains attributed to credit frictions.

[insert Table 2 here]

The first three rows display the benchmark case along with the two counterfactual exercises reviewed in the previous section. The rest of the table shows the results using the alternative specifications. In each case, we change one assumption at a time with respect to the benchmark model.[20] The table

[19] This is due to the fact the lender participation constraint is harder to meet and entrepreneurs pay their debt in real terms.

[20] The partial impulse response functions of selected variables for these robustness exercises along with the long-run present-value spending multipliers are provided in the online appendix.

confirms that financial frictions contribute more to the spending multiplier during a liquidity trap for every single alternative assumption we consider. Our main finding is thus robust to alternative modeling choices. The multiplier effect on investment due to the interaction between the capital-accumulation channel and the liquidty-trap period is indeed present in all of these alternative modeling specifications, and so the behavior of investment is not qualitatively affected. We next present in depth these alternative assumptions.

5.2 Alternative Assumptions and the Size of the Multiplier

Let us now discuss the details regarding the alternative assumptions as well as how do they affect the size of spending multiplier at the impact period.[21] We choose to talk about the impact multipliers for the following reason. Most of the discussion in the literature concerning the size of the fiscal multiplier is centered around its value at impact. For instance, Christiano *et al.* (2011) report impact spending multipliers close to 4 from a calibrated small-scale model, while Cogan *et al.* (2010) obtain values below 1 using a medium-scale model. This variability is an illustrative example of the importance of model uncertainty discussed in Cogan *et al.* (2010). In this subsection, we contribute to this discussion by stressing the role of different assumptions on the size of the spending multiplier in different interest-rate regimes. Table 3 displays the impact spending multiplier for the two models.

[insert Table 3 here]

Separable Preferences Our benchmark assumption regarding non-separable preferences implies that consumption and labor are complements (i.e., that the marginal utility of consumption increases with hours worked). As argued by Monacelli and Perotti (2008) and Christiano *et al.* (2011), in such a case the rise in labor demand and real wages resulting from the positive fiscal shock will lead to a crowding-in of private consumption as long as wealth effects in the labor supply are small. Consequently, fiscal multipliers will tend to be larger under non-separable preferences. We now depart from such specification by assuming a separable utility function between consumption and labor, which reads

$$\mathbb{U}(c_t, \ell_t^h) \equiv \frac{c_t^{1-\sigma} - 1}{1-\sigma} - \psi \frac{\left(\ell_t^h\right)^{1+\omega_w}}{1+\omega_w}, \tag{18}$$

[21] The intuitions provided below can be applied to both the impact and the long-run present-value multipliers. However, the former offers a clear-cut overview of the effects induced by the different modelling assumptions.

where $\sigma > 0$ is the coefficient of relative risk aversion, ω_w^{-1} is the Frisch elasticity of labor supply, and ψ is a normalizing constant. We set $\sigma = \omega_w = 1$ and we choose ψ to ensure that $\ell^h = 1/3$.

As expected, Table 3 shows that separable preferences generate lower spending multipliers for both models, irrespective of the interest-rate regime. For instance, considering a regime with a liquidity trap, the spending multiplier of a *NK model with f.f.* is 1.25 when preferences are separable and 1.63 when they are not. This result confirms that a larger wealth effect on labor supply mitigates the positive response of consumption to a government-spending expansion when the ZLB binds.

Price Rigidities The degree of nominal rigidities is another factor affecting the size of the multiplier. The second row of Table 3 shows the case of weaker nominal rigidities than the benchmark ($\alpha_p = 0.75$, instead of 0.85). Notice that in the no-ZLB regime, the multipliers are lower than the benchmark values, irrespective of the model. However, in the ZLB regime the opposite holds. Christiano *et al.* (2011) provide the intuition when the ZLB is not binding. Accordingly, after a rise in aggregate demand, nominal rigidities imply that prices increase by less than marginal costs. Consequently, markups effectively fall leading to an outward shift in the labor demand of *monopolistic* firms. The rise in labor demand following an increase in aggregate demand is indeed amplified when markups are countercyclical. Markups fall by less when prices are more flexible and therefore the effect on employment and consumption is moderate. As a result, the fiscal multiplier is smaller.

The transmission under lesser price stickiness is diametrically different during a liquidity trap. In such a regime, more flexible prices lead to larger spending multipliers. This is the case because, in the absence of an expansionary fiscal policy, agents expect a stronger deflation in response to a recessionary shock when prices are more flexible. These deflationary expectations drive up the real interest rate and generate an even larger recession. As pointed out by Christiano *et al.* (2011), a government spending expansion is more effective when the expected drop in output is larger.

Interest-Rate Rule Policy inertia effectively changes the way agents form their beliefs regarding the future paths of the real interest rate and their personal income. It is thus a factor that alters spending decisions. The third row of Table 3 shows the impact multipliers in the absence of interest-rate inertia ($\rho_R = 0$). Similarly to the previous case, the effects of this alternative assumption on the multiplier varies with the presence of the liquidity trap. When the ZLB is not binding, the spending multiplier under 'no-policy inertia' is smaller than the benchmark. In contrast, when the

ZLB is binding, the multiplier under 'no-policy inertia' is larger than the benchmark.

Christiano *et al.* (2011) again offer an explanation for the no-ZLB regime. Accordingly, lower inertia relates to a less accommodative monetary policy. Therefore, the rise in inflation generated by the fiscal shock is limited when monetary policy acts promptly and does not smooth its target rate. As a result, the real interest rate raises quicker at impact, depressing investment and consumption. Consequently, the fiscal multiplier is smaller.

In contrast, the effects of a less accommodative monetary policy substantially differ during a liquidity trap. In such regime, the drop in output following the recessionary shock is larger when the interest-rate rule features no inertia. The reason behind this effect relies on the relationship between the expected future real interest rate and permanent income. No policy inertia implies that when the central bank is no longer constrained by the ZLB, it starts raising its target rate quicker and promptly. The expected real interest rate in the states in which the zero lower bound is not binding is therefore higher. Future income is thus discounted at a higher rate, which overall decreases permanent income. In anticipation, households reduce consumption by more in the wake of the recession, which further contracts output. As the government spending expansion is more effective when the expected drop in output is larger, so the spending multiplier is also bigger.

Brick-Shaped Fiscal Shock In our benchmark exercise, we assume that the government-spending shock follows an AR(1) process with a degree of smoothing of $\rho_g = 0.8$. Public purchases thus drop geometrically after the impact period. In contrast, Christiano *et al.* (2011) adopt an alternative specification. They assume that the government spending expansion is constant as long as the economy is in the liquidity trap, whereas it equals zero when the ZLB is not binding. Such assumption implies that the shape of the fiscal shock is basically a 'brick'. The fourth row of Table 3 shows the impact multipliers for this case. Notice that the multipliers corresponding to the brick-shaped shock are larger than those corresponding to the AR(1)-type shock. For instance, it equals 1.94 for the *NK model with f.f.* in a liquidity trap while it is 1.63 in the benchmark case.

The crucial difference between these two shocks is that the percentage of stimulus that occurs during the ZLB period is different. For the brick-shaped shock, 100 percent of the stimulus occurs during the liquidity trap, whereas with the AR(1)-type shock only a portion of the stimulus is spent while the rest is expected. Thus, the expected real interest rate is higher under the AR(1)-type shock, which reduces permanent income and induces a crowding-out of private spending. The spending multiplier is thus larger under the brick-shaped shock because the induced crowding-out is damped.

How Big can the Impact Multiplier be? Cogan *et al.* (2010) argue that the size of the government-spending multiplier depends on the adopted framework, the estimation strategy (in empirical works), or the assumed path for government spending. Using estimated structural models, they report multipliers that vary between 1.00 and 1.60. This interval roughly corresponds to the range obtained in this paper. To conclude this subsection, we investigate the size of the multiplier when we mix several factors that contribute to make fiscal policy more effective. The fifth row of Table 3 shows the impact multipliers when we consider a model with non-separable preferences, a lower degree of price rigidities ($\alpha_p = 0.75$), an interest-rate rule without inertia ($\rho_R = 0$), and a brick-shaped fiscal shock. This configuration is similar to the environment explored by Christiano *et al.* (2011). Notice that the multiplier is slightly above one for both models when the ZLB does not bind and it is much larger in the liquidity-trap regime. It equals 3.76 in the *standard NK model* and 5.25 in the *NK model with f.f.*[22]

6 Capital-Income Taxes

In this final section, we briefly discuss how the lessons learned for the government-spending case are also applicable to a cut in capital-income taxes. We have shown that a government-spending expansion is attractive during a liquidity trap because it can ameliorate the financial position of debtors. However, one might prefer measures that directly stimulate investment, such as a tax-rate cut on capital earnings. We now assume that entrepreneurs pay taxes on their capital earnings. The nominal rate of capital returns, given by Equation (1), becomes

$$R_t^k \equiv \frac{P_t}{P_{t-1}} \frac{(1-\tau_{k,t})(z_t - \delta q_t) + q_t}{q_{t-1}}, \tag{19}$$

where $\tau_{k,t}$ is the capital-income tax rate which follows $\log(\tau_{z,t}) = (1-\rho_z)\log(\tau_z) + \rho_z \log(\tau_{z,t-1}) + \epsilon_{z,t}$. The steady-state level of taxes, τ_z, equals 36 percent as in Drautzburg and Uhlig (2010), ρ_z is set at 0.8, and $\epsilon_{z,t}$ is the tax innovation. The term $\tau_{z,t}\delta q_t$ represents a depreciation allowance granted by the government.

In time 1, when the recessionary shock hits the economy, the government temporally reduces the capital-income tax rate by 1 percentage point.[23] We use the present-value tax-revenue multiplier

[22] Christiano *et al.* (2011) report multipliers of about 5 with a standard NK model with capital. The smaller value reported in our paper is explained by the calibrated value of the preference parameter, υ. These authors set a value of 0.29 while we set 0.34 to ensure that the steady-state number of hours is 1/3.

[23] We consider a small shock in order to keep the duration of the ZLB unchanged. The value of the multiplier is not affected by the size of the shock as long as it does not change the duration of the liquidity trap (see also Erceg and Lindé, 2010, and Christiano *et al.*, 2011).

to evaluate this policy. On the horizon of $k-$periods, the latter is given by

$$PVM_k^z = -\frac{\mathrm{E}_t \sum_{j=0}^{k} \beta^{-j} y_{t+k}^{net}}{\mathrm{E}_t \sum_{j=0}^{k} \beta^{-j} tax_{t+k}^{net}}. \tag{20}$$

The first two rows of Figure 6 display selected partial responses to a capital-income tax cut, while the bottom row shows the present-value multipliers, as well as the multiplier's output gains due to credit frictions.

[insert Figure 6 here]

Several points are in order. First, a capital income tax cut increases the nominal rate of capital returns, which stimulates capital demand and rises the price of capital, irrespective of the model or the interest-rate regime. The crowding-in effect on investment is then magnified in the *NK model with f.f.* due to the boost in firms' net worth and the reduction in credit spreads. Second, the two facts emphasized in Section 4 still hold. Namely, financial frictions increase the value of the multiplier in both interest-rate regimes. In addition, they indeed contribute more to size of the multiplier when the ZLB binds. At a horizon close to convergence ($k = 100$), the multipliers' gap is 1 dollar for every dollar of reduction in capital taxes in the no-ZLB regime, while it reaches 1.70 dollars in the ZLB regime. To conclude, in the online appendix we show that when capital is fully depreciated at the end of each period, the multipliers' gains are again quite similar irrespective of the interest-rate regime.[24] These results show how our findings naturally extend to the case of a cut in capital-income taxes.

7 Conclusion

This paper studies the spillover effects of a liquidity trap on the financial accelerator mechanism and their implications on the transmission of fiscal shocks. The interaction between these two types of rigidities is of particular interest because it provides entrepreneurs with incentives to accumulate capital, which influences their investment plans.

Our experiments throw the following findings. Fiscal policy can be very effective during a liquidity trap if it focuses on strengthening the balance sheets of debtors because, by doing so, it stimulates investment. This crowding-in effect is more likely to happen when there is asymmetric information in the credit market since debtors' collateral influences the supply of credit. It is also more likely to

[24] The gains reach in this case by around 20 cents for every dollar of reduction in capital taxes.

happen during a liquidity trap because monetary policy accommodates fiscal expansions, allowing for a reduction in the real interest rate which stimulates in turn private spending. We have also investigated the role of the Fisherian debt-deflation channel in amplifying fiscal shocks. While it remains an important channel in the short run, we show that the capital-accumulation channel of the financial accelerator (i.e., the relationship between capital accumulation and collateral) is the main factor for explaining the large multipliers caused by credit market imperfections.

Ramey (2011) shows that government spending expansions are usually anticipated, at least several months in advance. In an environment in which credit market imperfections matter, the expectation of future inflation might reinforce the capital-accumulation channel of the financial accelerator. We leave this task pending for future research.

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A Appendix: Log-linearized model

The following list provides a summary of the model that is laid out in Section 2. A hat above a variable denotes its deviation from the deterministic steady-state. A variable without a hat or a time subscript denotes its steady-state level.

Household

$$\hat{\lambda}_t - \hat{R}_t = \mathrm{E}_t \left\{ \hat{\lambda}_{t+1} - (1-\gamma_b)\, \hat{\pi}_{t+1} \right\} \qquad \text{and} \qquad \hat{r}_t = \hat{R}_t - \mathrm{E}_t\{\hat{\pi}_{t+1}\}. \tag{A.21}$$

$$\hat{\varepsilon}_t - \frac{\ell^h}{1-\ell^h} \hat{\ell}_t^h \,(1-\upsilon)\,(1-\sigma) + \hat{c}_t \,(\upsilon\,(1-\sigma) - 1) = \hat{\lambda}_t. \tag{A.22}$$

$$\left(\frac{\ell^h}{1-\ell^h} \right) \hat{\ell}_t^h = \hat{w}_t - \hat{c}_t. \tag{A.23}$$

Intermediate Good Sector

$$\hat{y}_t = (1-\alpha)\hat{\ell}_t + \alpha \hat{k}_t \qquad \text{and} \qquad \hat{\ell}_t = \Omega \hat{\ell}_t^h. \tag{A.24}$$

$$\hat{w}_t = \hat{s}_t + \hat{y}_t - \hat{\ell}_t^h, \qquad \hat{w}_t^e = \hat{s}_t + \hat{y}_t, \qquad \text{and} \qquad \hat{z}_t = \hat{s}_t + \hat{y}_t - \hat{k}_t \tag{A.25}$$

$$\hat{\pi}_t = \frac{(1-\alpha_p)\,(1-\beta\alpha_p)}{\alpha_p} \hat{s}_t + \beta \mathrm{E}_t \left\{ \hat{\pi}_{t+1} \right\}, \tag{A.26}$$

Entrepreneurs

Related parameters

$$\Gamma(\bar{\omega}) = \bar{\omega}\,[1 - F(\bar{\omega})] + \mathrm{E}(\omega) - F_N((\mu_\omega - \ln(\bar{\omega}))/\sigma_\omega + \sigma_\omega), \quad \text{where } \mathrm{E}(\omega) = 1.$$

$$\Gamma_\omega(\bar{\omega}) = [1 - F(\bar{\omega})]\,, \quad \text{and} \quad \Gamma_{\omega\omega}(\bar{\omega}) = -f(\bar{\omega}),$$

$$\mu G(\bar{\omega}) = \mu\,[\mathrm{E}(\omega) - F_N((\mu_\omega - \ln(\bar{\omega}))/\sigma_\omega + \sigma_\omega)]\,,$$

$$\mu G_\omega(\bar{\omega}) = \mu \bar{\omega} f(\bar{\omega}).$$

$$\begin{aligned} \chi &\equiv \left(\frac{1}{x-1} \right) \left[1 + \frac{f_2}{f_0 f_1} \right]^{-1}, \quad \text{and} \quad f_0 \equiv 1 - \check{r}\,[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]\,, \\ f_1 &\equiv \bar{\omega} \left[\frac{\Gamma_{\omega\omega}(\bar{\omega})}{\Gamma_\omega(\bar{\omega})} - \frac{\Gamma_{\omega\omega}(\bar{\omega}) - \mu G_{\omega\omega}(\bar{\omega})}{\Gamma_\omega(\bar{\omega}) - \mu G_\omega(\bar{\omega})} \right], \quad \text{and} \quad f_2 \equiv \bar{\omega} \left[\frac{\Gamma_\omega(\bar{\omega}) - \mu G_\omega(\bar{\omega})}{\Gamma(\bar{\omega}) - \mu G(\bar{\omega})} \right], \quad . \end{aligned}$$

Optimal financial contract

$$\hat{x}_t = \hat{q}_t + \hat{k}_t - \hat{n}_t, \qquad \text{and} \qquad \widehat{\check{r}}_t = \mathrm{E}_t\{\hat{R}_{t+1}^k\} - \hat{R}_t, \tag{A.27}$$

$$\hat{n}_t = x\hat{q}_t + x\hat{k}_t - (x-1)\,\hat{b}_t \tag{A.28}$$

$$\widehat{\check{r}}_{t-1} = f_0 f_1 \widehat{\bar{\omega}}_t + \gamma_b \hat{\pi}_t, \tag{A.29}$$

$$\mathrm{E}_t \widehat{\check{r}}_t = \chi \hat{x}_t + \gamma_b \mathrm{E}_t \left\{ \hat{\pi}_{t+1} \right\}, \tag{A.30}$$

General equilibrium

$$\hat{R}_t^k = \hat{\pi}_t + \frac{z}{r^k} \hat{z}_t + \frac{(1-\delta)}{r^k} \hat{q}_t - \hat{q}_{t-1}. \tag{A.31}$$

$$\hat{n}_t = n_0 \hat{v}_t + [1 - n_0] \, \hat{w}_t^e \tag{A.32}$$

$$\hat{v}_t + \hat{\pi}_t = v_0 \left[\hat{R}_t^k + \hat{q}_{t-1} + \hat{k}_{t-1} \right] - [v_0 - 1] \left[\hat{R}_{t-1} + \hat{b}_{t-1} + \gamma_b \hat{\pi}_t \right] - v_1 \widehat{\bar{\omega}}_t, \tag{A.33}$$

$$\hat{v}_t + \hat{\pi}_t = v_0 \left[\hat{R}_t^k + \hat{q}_{t-1} + \hat{k}_{t-1} \right] - [v_0 - 1] \left[\hat{R}_{t-1} + \hat{b}_{t-1} + \gamma_b \hat{\pi}_t \right], \tag{A.34}$$

$$\frac{c^e}{k} \hat{c}_t^e = \frac{c^e}{k} \hat{v}_t \tag{A.35}$$

$$\begin{aligned} \text{where} \quad &: \quad v_0 \equiv \frac{1 - \mu G(\bar{\omega})}{1 - \Gamma(\bar{\omega})}, \; v_1 \equiv \frac{\bar{\omega} \mu G(\bar{\omega})}{1 - \Gamma(\bar{\omega})}, \\ &: \quad n_0 \equiv \gamma \left[1 - \Gamma(\bar{\omega}) \right] r^k x. \end{aligned}$$

and $\bar{\omega}$, σ_ω, γ and μ are chosen so as to satisfy the following system of steady-state equations:

$$F(\bar{\omega}) = 0.03/4; \quad x = 1 + \Gamma_\omega(\bar{\omega})[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]/[[1 - \Gamma(\bar{\omega})][\Gamma_\omega(\bar{\omega}) - \mu G_\omega(\bar{\omega})]];$$

$$(x-1)/x\check{r} = \Gamma(\bar{\omega}) - \mu G(\bar{\omega}); \quad n = \gamma[n r^k x[1 - \mu G(\bar{\omega})] - rn(x-1)] + (1-\tau) w^e.$$

Capital Producer

$$\hat{k}_t = \hat{k}_{t-1} (1 - \delta) + \delta \hat{\imath}_t, \tag{A.36}$$

$$\frac{1}{\varkappa} \hat{q}_t = (\hat{\imath}_t - \hat{\imath}_{t-1}) - \beta \left(\mathrm{E}_t \hat{\imath}_{t+1} - \hat{\imath}_t \right) \tag{A.37}$$

Resource Constraint

$$\hat{y}_t = \hat{c}_t \frac{c}{y} + \hat{\imath}_t \frac{i}{y} + \hat{g}_t \frac{g}{y} + \hat{c}_t^e \frac{c^e}{y} + \left[\hat{r}_t^k + \hat{q}_{t-1} + \hat{k}_{t-1} \right] \left[\mu G(\bar{\omega}) r^k \frac{k}{y} \right] + \widehat{\bar{\omega}}_t \bar{\omega} \mu G_\omega(\bar{\omega}) r^k \frac{k}{y}.$$

Monetary policy

No-ZLB regime

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \rho_\pi \hat{\pi}_t. \tag{A.38}$$

ZLB regime

$$\hat{R}_t = -r \equiv -\frac{1}{\beta} \tag{A.39}$$

B Appendix: Solving the model with a ZLB constraint

The methodology used in this paper to solve the model in the presence of a ZLB constraint follows Bodenstein *et al.* (2009). All the equations of the model are log-linearized, except for the ZLB constraint given by Equation (15).

The log-linearized model was solved by using the AIM algorithm (see Anderson and Moore, 1985). The equilibrium conditions are written in the matrix form

$$\mathbf{H}_{-1}\mathbf{y}_{t-1} + \mathbf{H}_0\mathbf{y}_t + \mathrm{E}_t\left\{\mathbf{H}_1\mathbf{y}_{t+1}\right\} + \mathbf{G}_0\boldsymbol{\epsilon}_t = 0, \tag{B.1}$$

where $\mathbf{y}_t$ is a vector $(n \times 1)$ of variables with n being the number of variables (including shocks), $\mathbf{H}_i$ refers to structural coefficient matrices $(n \times n)$, $\boldsymbol{\epsilon}_t$ is a vector $(k \times 1)$ of innovations with k as the number of innovation and $\mathbf{G}_0$ is a matrix $(n \times k)$.

To impose the ZLB constraint on this model, we proceeded in three steps. In System (B.1), let us assume that the line associated with R_t is the first line while R_t^{not} is associated to the second line. We assume that the ZLB constraint is hit from period $T^{low} = 1$ to $T^{up} = 6$.

Let consider the solution of Equation (B.1) for $t \geq T^{up} + 1$,

$$\mathbf{y}_t = \mathbf{F}\mathbf{y}_{t-1} + \mathbf{C}\boldsymbol{\epsilon}_t, \tag{B.2}$$

Then, let recursively solve the system for $T^{low} < t \leq T^{up}$. The system (B.1) is re-written as

$$\tilde{\mathbf{H}}_0\mathbf{y}_t = -\mathbf{H}_{-1}\mathbf{y}_{t-1} - \mathrm{E}_t\left\{\mathbf{H}_1\mathbf{y}_{t+1}\right\} - \mathbf{G}_0\boldsymbol{\epsilon}_t - \mathbf{g}, \tag{B.3}$$

where $\tilde{\mathbf{H}}_0$ is an $(n \times n)$ matrix, and $\mathbf{g}$ is an $(n \times 1)$ vector. The difference between $\tilde{\mathbf{H}}_0$ and $\mathbf{H}_0$ is that we set $\tilde{\mathbf{H}}_0(2,2) = 0$, implying that $R_t = R_{not}$. In addition, we impose $\mathbf{g}(2,1) = R$, implying that the notional interest rate is set to its steady-state value.

If $t = T^{up}$, then, plugging Equation (B.2) into Equation (B.3) yields

$$\tilde{\mathbf{H}}_0\mathbf{y}_{T^{up}} = -\mathbf{H}_{-1}\mathbf{y}_{T^{up}-1} - \mathbf{H}_1\mathbf{F}\mathbf{y}_{T^{up}} - \mathbf{g},$$

$$\Leftrightarrow \mathbf{y}_{T^{up}} = \boldsymbol{\Theta}_1\mathbf{y}_{T^{up}-1} + \tilde{\mathbf{g}}_1, \tag{B.4}$$

where $\Theta_1 \equiv -(\tilde{\mathbf{H}}_0 + \mathbf{H}_1\mathbf{F})\mathbf{H}_{-1}$ and $\tilde{\mathbf{g}}_1 = -(\tilde{\mathbf{H}}_0 + \mathbf{H}_1\mathbf{F})\mathbf{g}$.

Then we can solve the model from $T^{up} - 1$ to T^{low}, by backward induction. For $t = T^{up} - 1$, we can write Equation (B.3) as

$$\mathbf{y}_{T^{up}-1} = -\tilde{\mathbf{H}}_0^{-1}\mathbf{H}_{-1}\mathbf{y}_{T^{up}-2} - \tilde{\mathbf{H}}_0^{-1}\mathbf{H}_1\mathbf{y}_{T^{up}} - \tilde{\mathbf{H}}_0^{-1}\mathbf{g}, \tag{B.5}$$

and using Equation (B.4) yields

$$\mathbf{y}_{T^{up}-1} = \Theta_2 \mathbf{y}_{T^{up}-2} + \tilde{\mathbf{g}}_2. \tag{B.6}$$

where $\Theta_2 \equiv -\left[\mathrm{I}_n + \tilde{\mathbf{H}}_0^{-1}\mathbf{H}_1\mathbf{\Theta}_1\right]^{-1} \tilde{\mathbf{H}}_0^{-1}\mathbf{H}_{-1}$ and $\tilde{\mathbf{g}}_2 \equiv -\left[\mathrm{I}_n + \tilde{\mathbf{H}}_0^{-1}\mathbf{H}_1\mathbf{\Theta}_1\right]\left[\tilde{\mathbf{g}}_1 - \tilde{\mathbf{H}}_0^{-1}\mathbf{g}\right]$.
We generalize the expressions so that $\Theta_i \equiv -\left[\mathrm{I}_n + \tilde{\mathbf{H}}_0^{-1}\mathbf{H}_1\mathbf{\Theta}_{i-1}\right]^{-1} \tilde{\mathbf{H}}_0^{-1}\mathbf{H}_{-1}$
and $\tilde{\mathbf{g}}_i \equiv -\left[\mathrm{I}_n + \tilde{\mathbf{H}}_0^{-1}\mathbf{H}_1\mathbf{\Theta}_{i-1}\right]\left[\tilde{\mathbf{g}}_{i-1} - \tilde{\mathbf{H}}_0^{-1}\mathbf{g}\right]$.
Consequently, for $2 \leq i \leq T^{up} - T^{low}$, computing the dynamics of $\mathbf{y}_{T-i}$ amounts to compute Θ_t and $\tilde{\mathbf{g}}_t$ so that

$$\mathbf{y}_{T^{up}-i} = \Theta_{i+1}\mathbf{y}_{T^{up}-i-1} + \tilde{\mathbf{g}}_{i+1}. \tag{B.7}$$

If $T^{low} > 1$, the next step is to compute the dynamics for $0 \leq t < T^{low}$. For $T^{up} - T^{low} + 1 \leq i < T^{up}$, the dynamics are expressed as previously, except that $\tilde{\mathbf{H}}_0^{-1}$ is replaced by $\mathbf{H}_0^{-1}$.
Finally, for $i = T^{up}$, the impact response of the variables is given by

$$\mathbf{y}_0 = \tilde{\Theta}_{T^{up}+1}\boldsymbol{\epsilon}_0 + \tilde{\mathbf{g}}_{T^{up}+1}. \tag{B.8}$$

where $\tilde{\Theta}_{T^{up}+1} = -\left[\mathrm{I}_n + \mathbf{H}_0^{-1}\mathbf{H}_1\mathbf{\Theta}_{T^{up}}\right]^{-1} \mathbf{H}_0^{-1}\mathbf{G}_0$.
The choice of T, for $T = \{T^{low}, T^{up}\}$ is determined by computing R_T^{not} and R_{T+1}^{not} and ensuring that $\hat{R}_T^{not} < -R \leq \hat{R}_{T+1}^{not}$.

Deep recession scenario

For illustrative purposes, the following figure shows the effect of a negative preference shock that results in a liquidity trap. This is the deep recession scenario to which we apply the fiscal stimulus.

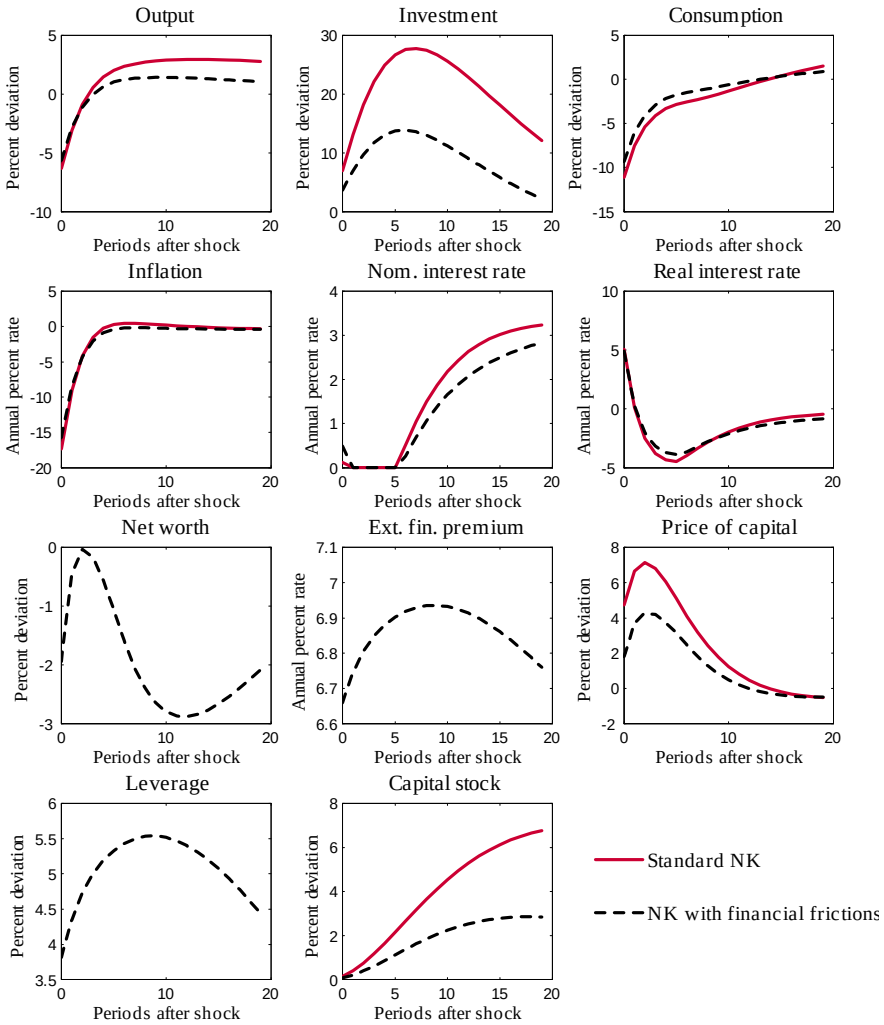

Figure 1: *Deep recession scenario.* Impulse response functions to a negative preference shock in a standard NK model with no financial frictions (solid line), and in a NK model with financial frictions (dashed line).

Table 1. Calibrated Parameters

Preferences and Technology		Value
β	Discount factor	0.99
σ	Degree of risk aversion	2.00
υ	Complementarity between consumption and leisure	0.34
α	Elasticity of value added wrt capital	0.36
δ	Capital depreciation rate	0.02
$\varkappa$	Investment adjustment cost	5.00
θ_p	Elasticity of substitution of goods	11.00
α_p	Degree of price stickiness	0.85
Financial Accelerator Mechanism		
χ	Elast. of risk premium wrt leverage ratio	0.0402
Ω	Proportion of household labor in aggr. labor	0.9846
x	Steady-state ratio of capital to net worth	2.00
$\check{r}$	Steady-state risk spread	$1.02^{0.25}$
γ	Survival rate of entrepreneurs	0.9834
$\bar{\omega}$	Threshold value of idiosyncratic shock	0.4982
σ_ω	Standard error of idiosyncratic shock	0.2764
$(1-\varrho)$	Transfers from failed entrepreneur to households	0.9999
μ	Monitoring cost	0.1175
Monetary and Fiscal Policy		
ρ_R	Interest rate smoothing	0.50
a_π	Elasticity of the interest rate wrt inflation	2.00
g/y	Share of government expenditure in output	0.20
Shocks		
ρ_g	Persistence of the gov. spending shock	0.80
ρ_ε	Persistence of preference shock	0.80

Table 2. Multiplier's output gains for different specifications

	No-liquidity trap regime	Liquidity trap regime
Benchmark	0.34	1.82
Full depreciation	0.22	0.28
No-Fisher effect	-0.02	0.38
(1): Separable preferences	0.27	1.48
(2): Weaker price rigidities	0.16	2.89
(3): No-policy inertia	0.25	2.38
(4): Brick shaped fiscal shock	0.77	2.61

Note: The multiplier's output gains are computed at horizon 100. In case (1), preferences are separable between consumption and labor. In cases (2) and (3), $\alpha_p = 0.75$ and $\rho_R = 0$, respectively. In case (4), government spending rises by a same amount during the ZLB and it is at its steady state otherwise.

Table 3. Impact multipliers for different specifications

	No-liquidity trap		Liquidity trap	
	Standard NK	NK with f.f.	Standard NK	NK with f.f.
(0): Benchmark	0.95	0.93	1.49	1.63
(1): Separable preferences	0.70	0.69	1.16	1.25
(2): Weaker price rigidities	0.79	0.79	1.78	2.07
(3): No-policy inertia	0.87	0.86	1.69	1.88
(4): Brick-shaped fiscal shock	1.24	1.23	1.73	1.94
(5): (2) + (3) + (4)	1.12	1.12	3.76	5.25

Note: In case (1), preferences are separable between consumption and labor. In cases (2) and (3), $\alpha_p = 0.75$ and $\rho_R = 0$, respectively. In case (4), government spending rises by a same amount during the ZLB and it is at its steady state otherwise. Case (5) is a mix of cases (2), (3) and (4).

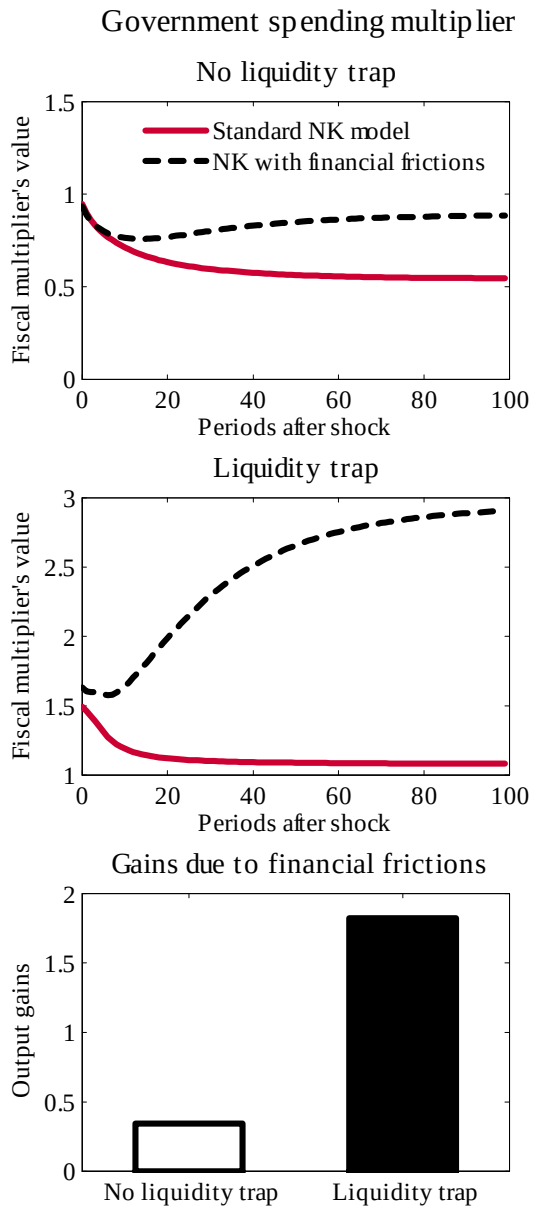

Figure 2: *Benchmark exercise*. Present-value government-spending multipliers and output gains attributable to the presence of financial frictions.

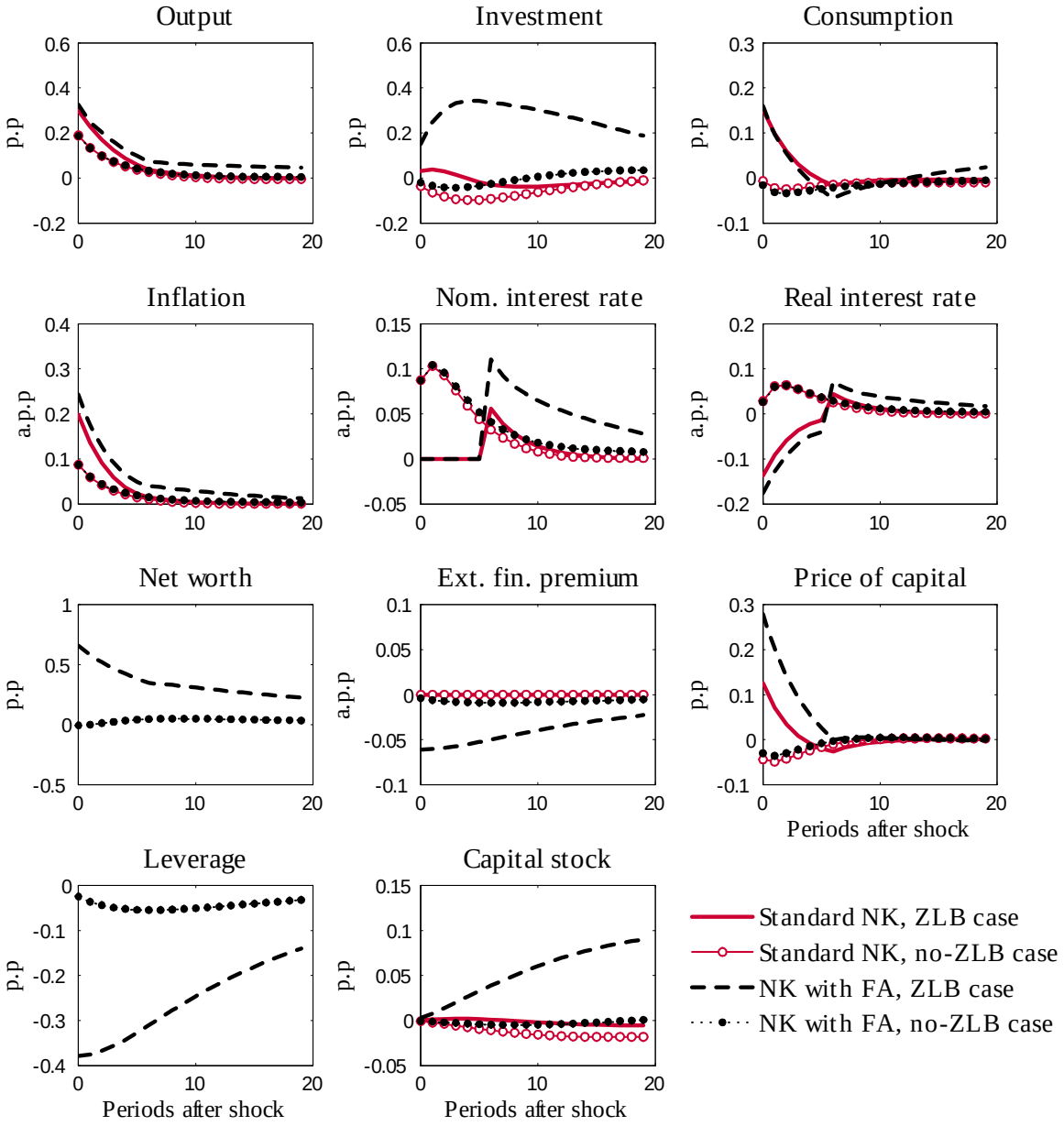

Figure 3: *Benchmark exercise.* Partial impulse response functions to a government spending expansion in the standard NK model (solid line and line with white circles) and the NK model with financial frictions (dashed line and line with black cirlces). The case for which the ZLB constraint binds is represented by the two lines with circles. The partial IRFs are expressed in percentage points (p.p) or annualized percentage points (a.p.p).

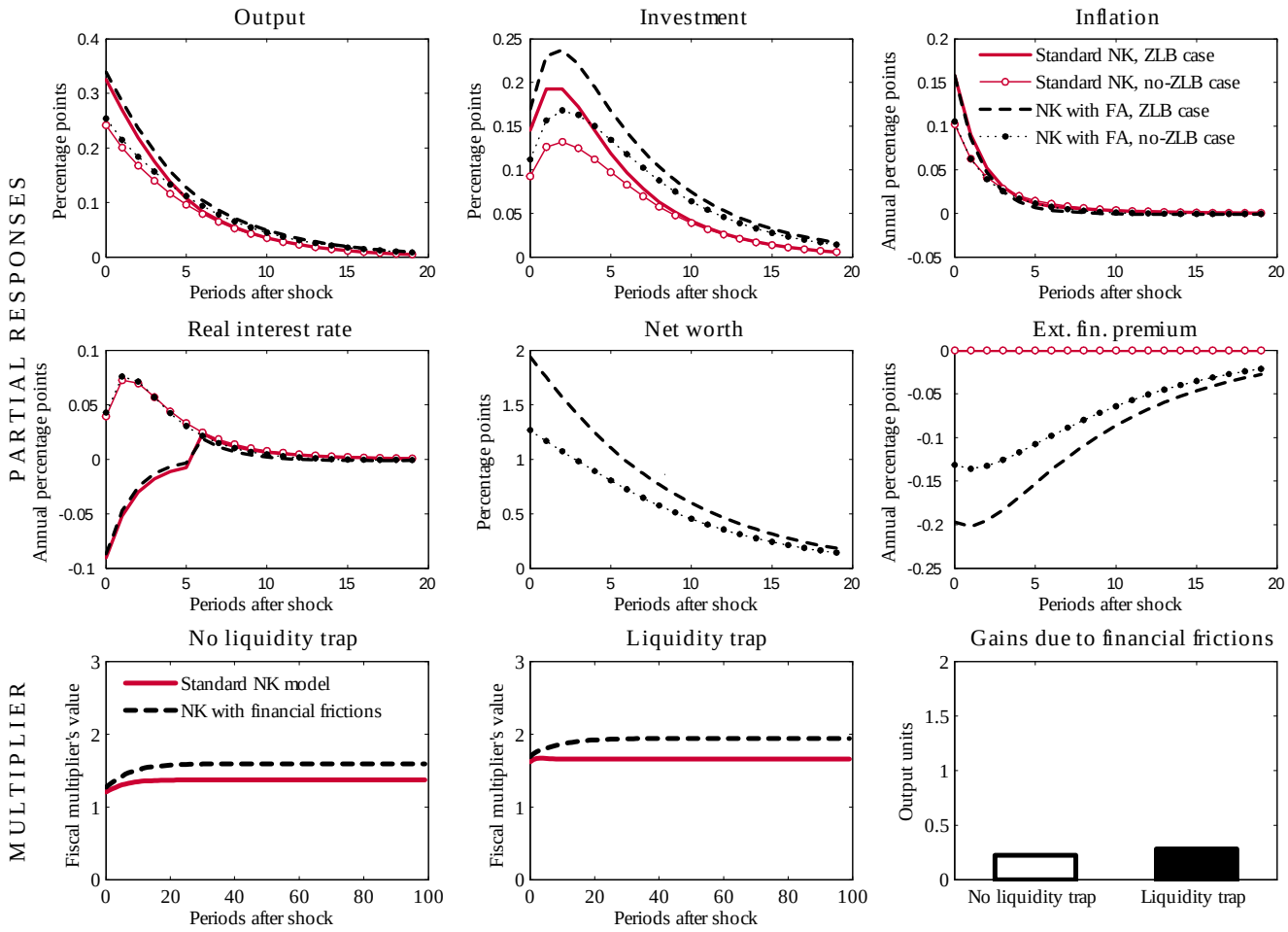

Figure 4: *Full-capital depreciation*. The first two rows display selected partial impulse responses to a government spending expansion. The bottom row displays the present-value multipliers and the output gains attributable to the presence of financial frictions.

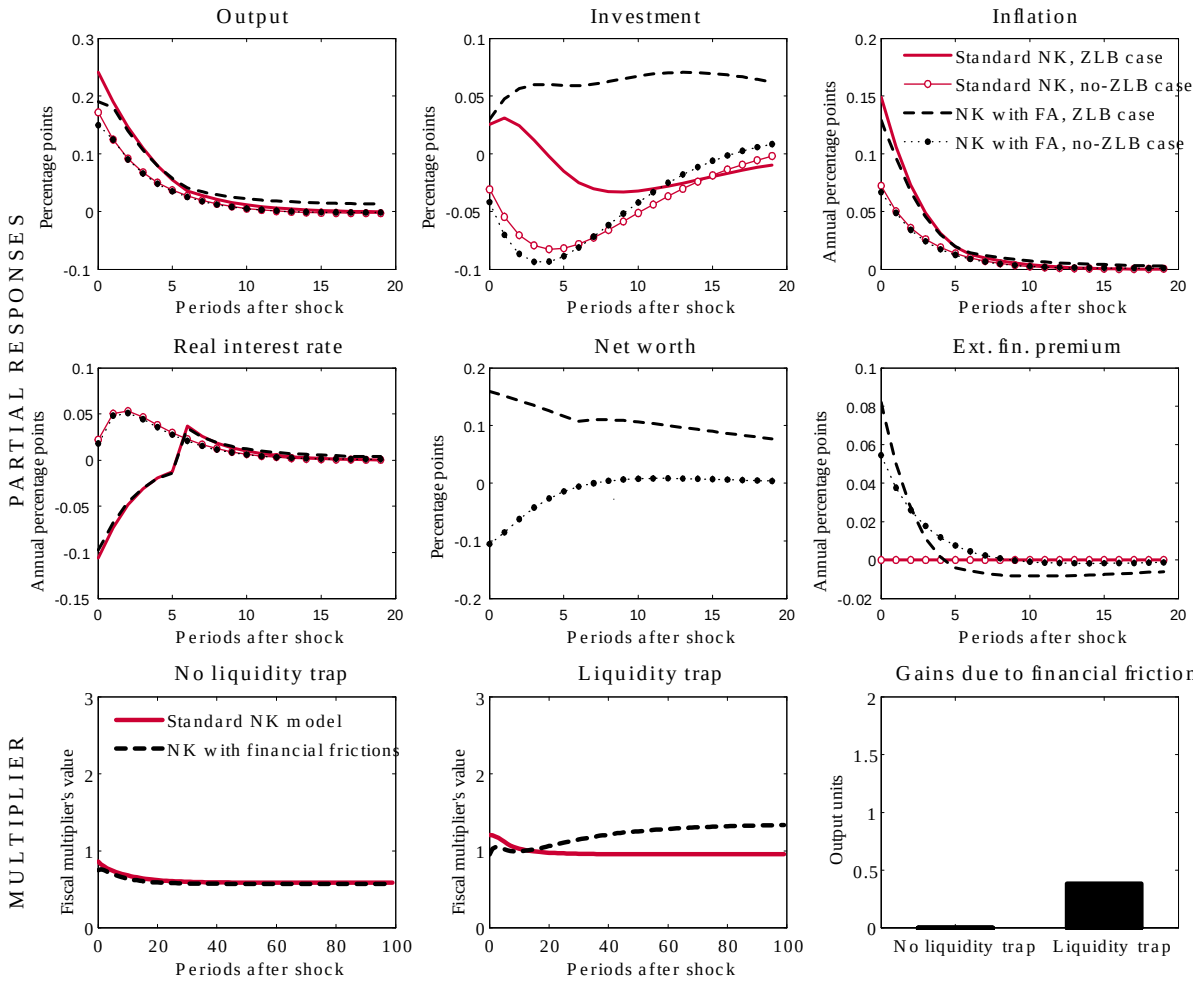

Figure 5: *No-Fisher effect*. The first two rows display selected partial impulse responses to a government spending expansion. The bottom row displays the present-value multipliers and the output gains attributable to the presence of financial frictions.

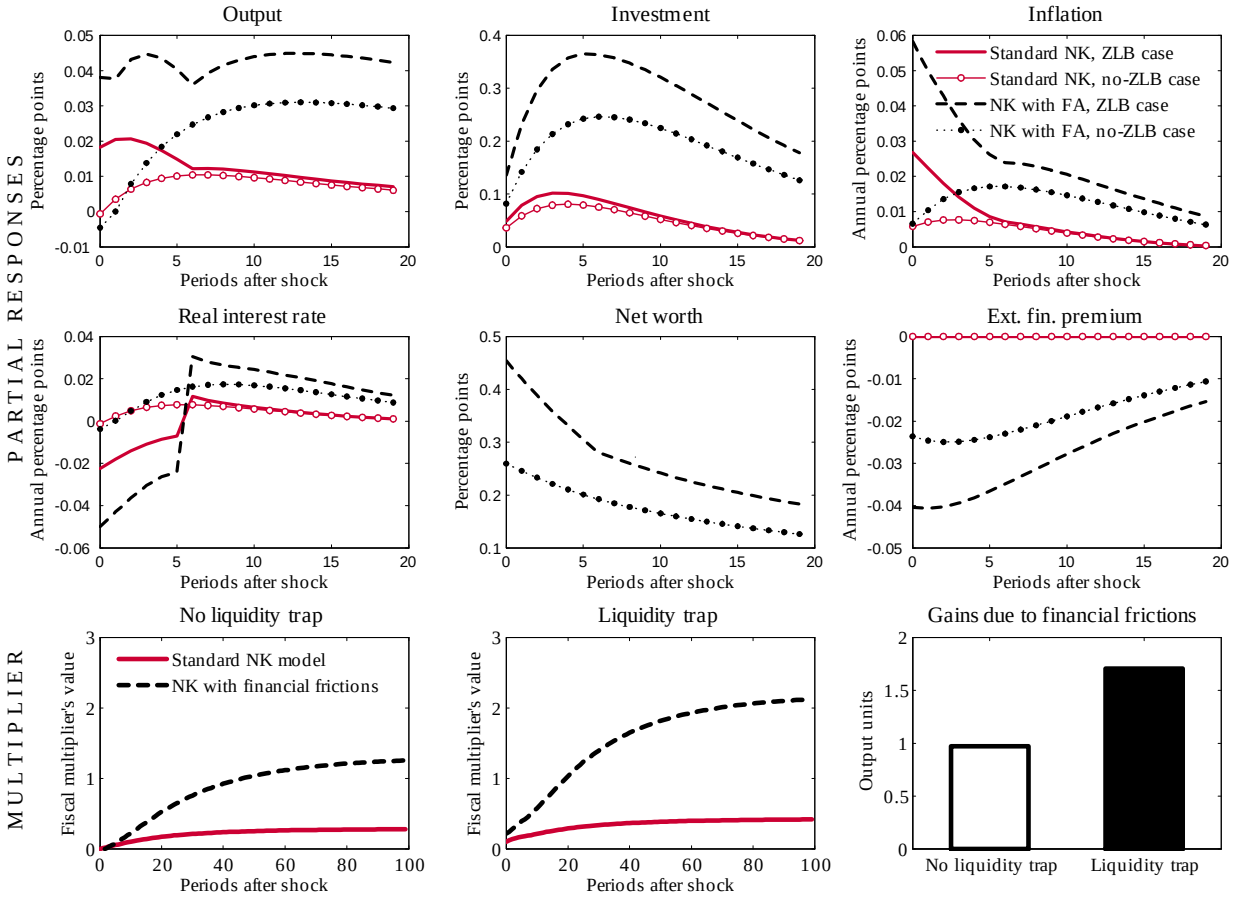

Figure 6: *Capital-income taxes.* The first two rows display selected partial impulse responses to a capital-income tax cut. The bottom row displays the present-value multipliers and the output gains attributable to the presence of financial frictions.